Chapter One

Progress and problems in large-scale atmospheric dynamics

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1.1 INTRODUCTION

A theory for the general circulation of the atmosphere has at its core a theory for the quasi-horizontal eddy fluxes of energy, angular momentum and water vapor by the *macro-turbulence* of the troposphere, as well as a theory for the much smaller scale convective motions that transport heat and water vertically, especially in the tropics. A few of the many issues related to convective vertical fluxes are discussed in Chapters 3, 4, 6, and 7 in this volume. The focus in this chapter, and of Chapters 2, 5, and 8-12, is primarily on the large-scale quasi-horizontal component of the problem. In the tropics, fluxes by large and small scale eddies are so tightly coupled that one cannot easily discuss one without simultaneously discussing the other. But outside of the tropics one can hope that a focus on large-scale dynamics in isolation is a meaningful starting point, and it is on the extratropical circulation that I concentrate here.

All of us would love to find a simple variational principle or "fundamental theorem of climate", that solves this problem in a single stroke, but I suspect that most of us are skeptical that such a principle exists. We assume, instead, that the best way of developing theories for a system of this complexity is to construct a hierarchy of models, of varying levels of comprehensiveness, chosen so as to capture the essential sources of complexity with minimal extraneous detail. When confronted with a theory claiming great generality, we expect to see a demonstration that it explains the behavior seen on a number of different levels of our model hierarchy.

An analogy with the use of "model organisms" in biology is informative. Nature has provided us with just the kind of hierarchy, from bacteria to fruit fly to mouse, needed to build up an understanding of our own complex biology. We have no such ready-made hierarchy in climate research, and must instead design and build our own. See Held (2005) for an extended discussion of this analogy and the consequences of the fact that our climate hierarchy is a theoretical construct while the biological hierarchy is provided by nature.

A key outstanding question in global circulation theory from this perspective is "what hierarchy of models should we study so as to best understand how global climate is controlled by external parameters and boundary conditions?" The choice of models is centrally important. Only if, as a community, we have selected appropriate models to study collectively will our understanding accumulate efficiently. I personally do not feel that appropriate models can be selected in a systematic way; our physical intuition must guide us towards the most informative models.

In this lecture I will refer to the classic two-layer quasi-geostrophic (QG) model, moist QG models, and particular idealized dry and moist primitive equation models on the sphere. The discussion revolves around the related problems of the poleward eddy heat flux, the effect of latent heat release on midlatitude eddies, and distinctions between the dynamics of the upper and lower troposphere. Considerable space is devoted to the simplest of these models, the two-layer QG model, in an especially simple horizontally homogeneous configuration.

I find this model of homogeneous QG turbulence useful from several perspectives, but there is no claim that the theory for the eddy fluxes in this model is of direct quantitative relevance to the atmosphere. When we talk about the need for a model hierarchy, we are implicitly assuming that the more idealized members of this hierarchy are missing some important ingredients, but that, in spite of these limitations, an understanding of these simpler models is a useful stepping stone to a useful understanding of their more complex relatives.

1.2 THE TWO-LAYER QG MODEL

The two-layer QG system provides us with what may be our simplest turbulent "climate" model. The state of this model is determined by the streamfunctions for the non-divergent component of the horizontal flow in two layers of fluid, meant to represent the flow in the upper (ψ_1) and lower (ψ_2) troposphere, the (eastward, northward) components of the velocity being $(u, v) = (-\partial\psi/\partial y, \partial\psi/\partial x)$. In the meteorological context we can think of two isentropic layers of ideal gas, with different entropies, or potential temperatures, θ, with $\theta_1 > \theta_2$ so as to represent a gravitationally stable system. Hydrostatic and geostrophic balance combine to create Margules' relation between the perturbations to the height of the interface between the two layers, η, and the difference between the two streamfunctions. In a Boussinesq fluid (with all potential temperatures assumed to be small perturbations away from a constant θ_0) this relation is $f(\psi_1 - \psi_2) = -g^*\eta$, where $g^* \equiv g(\theta_1 - \theta_2)/\theta_0$ is the reduced gravity, and f the Coriolis parameter. The dynamics reduces to the advection by these non-divergent flows of a scalar, the QG potential vorticity, q_k, within each layer, where

$$q_k = \nabla^2\psi_k + (-1)^k\lambda^{-2}(\psi_1 - \psi_2) + \beta y; \; k = 1, 2 \tag{1.1}$$

and λ is the radius of deformation, defined by $\lambda^2 = g^*H/f^2$, with H the resting depth of the two layers (assumed to be equal here). The final term in (1.1), with β a constant, is an approximation to the all-important vorticity gradient due to the increase in the radial component of the vorticity of solid body rotation with increasing latitude y. When relating this two-layer picture to a continuously stratified atmosphere, we think of $(g^*H)^{1/2} \to NH$, with $N^2 = (g/\theta)\partial\theta/\partial z$ and $-\eta$ as proportional to the vertically averaged potential temperature.

A simple way of creating a statistically steady state is to force the system with mass exchange between the two layers, this model's version of radiative heating, arranged so as to relax the interface to a "radiative equilibrium" shape with a zonally symmetric meridional slope. This mass exchange can be expressed in terms of potential vorticity sources in the two layers. One also invariably includes two types of dissipation: small scale diffusion is needed to mop up the vorticity variance that cascades to small scales; and surface friction, damping the low level vorticity, is needed to remove energy in a non-scale selective manner. Energy does not cascade to small scales in this model and cannot be removed realistically with horizontal diffusion.

Radiative equilibrium is a solution of these equations, with no flow in the lower layer and zonal flow in the upper layer, with the Coriolis force acting on the vertical shear $U = u_1 - u_2$ between the two layers balancing the pressure gradients created by the radiative equilibrium interface slope. This flow is unstable, in the absence of the dissipative terms, when the isentropic slope is large enough to overcome β and reverse the sign of the north-south potential vorticity gradient in one of the layers. In flows with temperature decreasing (interface slope rising) with increasing y, this reversal occurs in the lower layer. If the relative vorticity gradient of the zonal flow is negligible as compared to β, the criterion is the classic one discussed by Phillips more than half a century ago: $\xi \equiv U/(\beta\lambda^2) > 1$. (The supercriticality ξ is two-layer counterpart to the parameter S_c used in Chapter 10.) The existence of this critical slope presents us with a problem, since analogous models of inviscid baroclinic instability in continuously stratified atmospheres are unstable for any non-zero vertical shear (or isentropic slope). (In multi-layer models, the critical interfacial slope is simply proportional to the depth of the lowest layer.) We will need to return to this point.

Phillips (1956) constructed the first "general circulation model", or "climate model", based on this two-layer QG dynamics. Nowadays we might instead refer to this work as modeling the statistically steady state of a baroclinically unstable jet on a β-plane. Whatever we call it, this model still captures an impressive subset of the dynamics of the midlatitude storm tracks. Phenomena have been discovered in the solutions of these equations that have then been searched for and found in the atmosphere. The coherent baroclinic wave packets described in Lee and Held (1993) are an example from my own research. (Unfortunately, it is not obvious, in reading that paper, that we first encountered these wavepackets while experimenting with the two-layer QG system.)

As long as the dissipative terms are linear, a theory for the time mean geostrophic flow in this model reduces to a theory for the poleward eddy potential vorticity fluxes in the two layers, $\mathcal{P}_k \equiv \overline{v'_k q'_k}$, where an overbar refers to the zonal mean and a prime to deviations from this mean. We can relate these fluxes to the eddy momentum fluxes $\mathcal{M}_k \equiv \overline{v'_k u'_k}$ and the thickness (heat) fluxes $\mathcal{T}_k \equiv \overline{v'_k \eta'}$ (where $\mathcal{T}_1 = \mathcal{T}_2 \equiv \mathcal{T}$ from Margules' relation) ,

$$\mathcal{P}_1 = -\frac{\partial \mathcal{M}_1}{\partial y} + f\mathcal{T}/H; \quad \mathcal{P}_2 = -\frac{\partial \mathcal{M}_2}{\partial y} - f\mathcal{T}/H \tag{1.2}$$

The two potential vorticity fluxes cannot fully determine the three fluxes $(\mathcal{M}_1, \mathcal{M}_2, \mathcal{T})$; therefore, the eddy thickness and momentum fluxes are more than we need to know if we are only interested in the mean zonal flow and the interface displacement (temperature).

The most fundamental limitation of QG dynamics is that it assumes a reference static stability; in this two-layer model the potential temperature difference between the two layers is fixed. One is perilously close to throwing the baby out with the bath water in such a theory. What could be more fundamental to a theory of climate than an understanding of the mean stratification of the atmosphere? But perhaps we can develop theories for the QG fluxes, and then use these outside of the QG framework, to help as needed in determining the static stability. We illustrate this kind of argument below.

1.3 EDDY CLOSURE IN THE TWO-LAYER MODEL

What is the scale of the typical energy-containing eddy in this two-layer QG model? Linear theory points to the radius of deformation, as it is the zonal scale of the most rapidly growing linear waves. A classic assumption (Stone, 1972) is that the nonlinearity of the flow isotropizes the eddies in the horizontal and imprints this scale on the meridional as well as zonal eddy structure, and on eddy mixing lengths as well. An interesting implication is that there seems to be potential for scale separation in the horizontal, since this scale would then be independent of the mean flow inhomogeneity in the direction of the flux, in contrast to the situation in most laboratory turbulent flows.

If there is scale separation, one is justified in thinking in terms of local rather than global theories for the eddy fluxes. An example of a global theory is an approach referred to as *baroclinic adjustment*, in analogy with convective adjustment for gravitational instability (e.g., Stone, 1978a). Since the instability of the flow can be thought of as due to the reversal in sign of the lower layer potential vorticity gradient, suppose that the eddy fluxes are just sufficient to bring this gradient back to zero. Given a value of the radiative equilibrium shear and the width of the unstable region L_Q, the magnitude of the eddy potential vorticity flux required to destroy the gradient is proportional to L_Q^2 (since the rate of change of the mean gradient is proportional to the second derivative of the eddy flux). L_Q is a global piece of information. However, as L_Q is increased in numerical simulations, the eddy potential vorticity fluxes are found to grow more slowly than L_Q^2 and eventually to asymptote to values independent of L_Q (Pavan and Held, 1996). While baroclinic adjustment does not work in two-layer QG flows with large L_Q, it may very well be an adequate, indeed a very useful, approximation for the case of narrow regions of instability.

An example of a local theory is simple diffusion of potential vorticity, with a diffusivity determined by aspects of the local environment. We cannot expect a truly local diffusive theory to be exact. The relationship between eddy flux and environment must be non-local over the scale of the eddies at least. Additional non-locality is introduced if the production and dissipation of the eddies are not co-located. For example, the simplest diffusive picture does not work when applied locally in longitude in the zonally asymmetric midlatitude storm tracks (Marshall and Shutts, 1981; Illari and Marshall, 1983). Eddies are preferentially generated in the strongly baroclinic zones at the jet entrance regions and decay downstream in the jet exit regions. It is only when one averages zonally over these regions of predominant eddy growth and eddy decay that one has a reason to expect a local, diffusive picture to hold in some approximate sense.

Given a diffusivity D and radiative relaxation time, τ, we should not expect to reach the L_Q-independent asymptotic regime until $L_Q^2 > D\tau$, or $(L_Q/L)^2 > (\tau/T)$, where L and T are eddy length and time scales. The resulting scales are large compared to the radius of the Earth. But we do not need to be in this asymptotic regime to apply a diffusive theory; all that is required is scale separation $L_Q > L$. As in many applications of WKB-like theories, one can even hope that the local theory is adequate when $L_Q \approx L$.

The simplest scaling for the diffusivity is that suggested by Stone(1972): $D \sim VL \sim U\lambda$, where the eddy velocity scale V has been chosen proportional to the mean vertical shear U over the depth of the atmosphere. The assumption $V \sim U$ is equivalent to assuming that the eddy kinetic energy is proportional to the mean available potential energy (the increase in potential energy due to the interface slope) within a region of width λ. This diffusivity is itself proportional to the interface slope, or horizontal temperature gradient. If we can use this diffusivity for the sensible heat flux, following Stone, we obtain a heat flux proportional to the square of this gradient. Numerical experiments in the *homogeneous limit* described below clearly indicate that the eddy fluxes in this two-layer QG model are even more sensitive to the horizontal gradient; they also give us some guidance on how to incorporate β into the theory.

1.4 THE HOMOGENEOUS LIMIT

Given the potential for a local theory, one is led to artificially create a truly homogeneous environment in which to study eddy fluxes in the simplest possible context. QG theory allows one to do this in an elegant way by assuming that there is a uniform zonal flow in both layers, and, therefore, a uniform vertical shear and uniform potential vorticity gradients. One then assumes that the total flow consists of this environment, plus eddies constrained to be doubly periodic. One can think of this geometry as a generalization of the familiar QG β-plane to the case with potential vorticity gradients of opposite sign in the two layers.

In this geometry, the eddy fluxes are horizontally homogeneous. Therefore, according to (1.2), the potential vorticity fluxes reduce to the eddy thickness (heat) flux and are equal and opposite in the two layers. The momentum fluxes must also vanish if the climate is unique, since the equations are symmetric with respect to reflection in y, and the momentum fluxes change sign upon reflection. The central simplification is that one can study how the eddy fluxes are controlled by environmental parameters without simultaneously being concerned with the effect of these eddy fluxes on their environment.

A problem immediately arises from the inverse energy cascade, a cascade to larger rather than smaller scales. Calculations show unambiguously that the dominant eddy scale in the fully turbulent statistically steady state is generally larger than the radius of deformation, due to this inverse cascade. It is useful to rearrange the two vertical degrees of freedom of this model into the barotropic $(\psi_1 + \psi_2)$ and the baroclinic $(\psi_1 - \psi_2)$ modes. The picture of the energy flows as a function of wavenumber in this modal basis has been described by Rhines(1977), Salmon(1978,1980), and Larichev and Held(1995). The barotropic mode is energized by transfer from the baroclinic mode near the radius of deformation. The inverse cascade takes place in the barotropic mode, and energy is dissipated by surface friction on the scales to which this cascade carries the energy. If the cascade is extensive, the barotropic mode dominates the kinetic energy, so that the baroclinic potential vorticity (dominated on large scales by the thickness variations) can then be thought of as advected passively by the barotropic mode (since it does not induce the flow by which it is advected). The available potential energy, or thickness variance, is generated on these large energy-containing scales by extraction of energy from the environmental potential energy through downgradient thickness (heat) fluxes, just as in two-dimensional downgradient turbulent diffusion of a passive scalar, and cascades to smaller scales back towards the radius of deformation, completing the cycle.

Albeit directly applicable only for a rather special situation, it is striking how little this homogeneous turbulence picture has left in it that bears any resemblance to the scales and concepts familiar from linear theory.

As in Kolmogorov‘s classic work on the direct cascade of energy in 3D turbulence, the key element of the two-dimensional inverse cascade, as described by Kraichnan(1971), is the rate of transfer of energy through the spectrum, ϵ. Together with the wavenumber k, ϵ determines the energy level of the flow and the characteristic time scale of the eddies. The key question is the scale at which the inverse energy cascade is halted.

At this point we take advantage of the insight of Rhines (1975) that the presence of an environmental barotropic vorticity gradient β can effectively stop the cascade, given the property of the Rossby wave dispersion relation that larger waves have larger intrinsic phase speeds (β/k^2). When these phase speeds become comparable to the characteristic velocity of the flow, the eddies morph into linear waves. Stopping the cascade in this way produces a flow that is simultaneously marginally turbulent and marginally wave-like, an elegant qualitative description of midlatitude eddies. From β and ϵ one forms length and time, or velocity, scales, or one can proceed directly to a diffusivity with units of $(length^2)/time$

$$D \sim \epsilon^{3/5}\beta^{-4/5} \tag{1.3}$$

Surface friction must eventually remove energy from the model. In the presence of β, the flow forms zonal jets which store the energy until it is dissipated.

The potential energy extracted from the environment can be written in terms of the eddy potential vorticity flux in either layer

$$\epsilon = \sum_i U_i \mathcal{P}_i = U\mathcal{P}_1 = -U\mathcal{P}_2 = U\beta D_1(1+\xi) = U\beta D_2(1-\xi) \tag{1.4}$$

where $U = U_1 - U_2$. We equate this production to the rate of energy transfer through the inverse energy cascade. We define a diffusivity in each layer as the eddy potential vorticity flux divided by the mean potential vorticity gradient. As $\beta \to 0$, $\xi \to \infty$, and $D_1 \to D_2$ (see Vallis, 1988). Equating D in (1.3) with either D_1 or D_2, in this limit we have

$$\epsilon = \frac{D}{T^2}; \quad T \equiv \frac{NH}{fU} \tag{1.5}$$

where T^{-1} is often referred to as the Eady growth rate, though it does not enter here through any connection to linear theory. Combining with (1.3) one arrives at

$$D \sim \frac{1}{\beta^2 T^3} \tag{1.6}$$

or

$$\frac{D}{\beta\lambda^3} \sim \xi^3 \tag{1.7}$$

This is the scaling presented by Held and Larichev (1996). A more accurate fit to numerical experiments is provided by the modified formulation in Lapeyre and Held (2003), for which a satisfactory justification has yet to be provided. The proposal is simply to equate D with the lower-layer diffusivity D_2, irrespective of the value of β. (I return to the motivation for this assumption in section 1.9.) The result is

$$\frac{D}{\beta\lambda^3} \sim \xi^{3/2}(\xi^{3/2} - 1) \tag{1.8}$$

This has the same $\xi \to \infty$ limit as (1.7). The fit to the numerical results is shown in Fig 1. This form also has the advantage that the diffusivity vanishes as $\xi \to 1$, consistent with the criterion for instability. This is my best shot at present for a qualitative explanation of the baroclinic eddy fluxes in this idealized homogeneous environment. As $\beta \to 0$ and $\xi \to \infty$, the eddy length scale increases without bound in this theory, implying that some other scales, determined by the surface friction or the domain geometry must come into play.

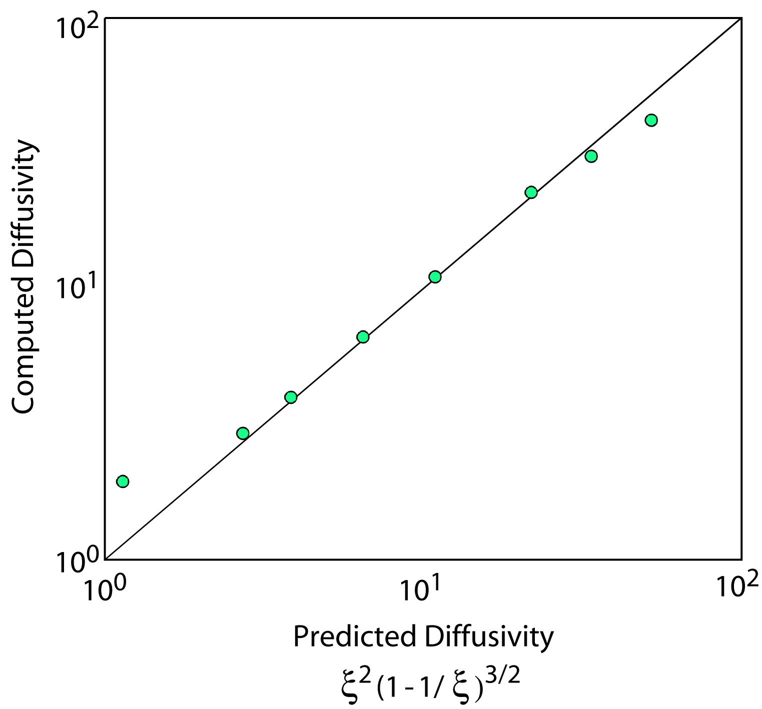

Figure 1.1 Dots are the diffusivity computed numerically in a 1024x1024 spectral simulation of the homogneous two-layer model, after Lapeyre and Held (2003), compared with the theoretical scaling provided by Eq.8. The departures at large supercriticality are probably related to the finite size of the domain.

Whether or not the details are right, this line of argument points to a flux that is very sensitive to environmental gradients; Eq. (1.7) yields a diffusivity proportional to the third power, a flux proportional to the fourth power, and an energy cycle, ϵ, proportional to the fifth power of the horizontal temperature gradient. Eq. (1.8) only increases this senstivity to ξ. In practice, this means that it is very hard, in this two-layer model, to change the gradient, to the extent that the system has difficulty supplying energy at the rate required.

As the width of the unstable region increases, the flow typically makes a transition from one to two and then to multiple jets, with a storm track associated with each jet. One is tempted to assume that the homogeneous limit cannot be relevant to the one-jet case, but only begins to become appropriate for the case of two or more jets, the former being too inhomogeneous. The implication would be that the theory is irrelevant for the Earth, which has only one eddy-driven jet per hemisphere. An argument along similar lines starts with the observations that momentum fluxes vanish in the homogeneous system, averaged over space, or, presumably, averaged over time at each point in space. But momentum fluxes appear to play a significant role in the stabilization of flows in the troposphere, as encapsulated in the *barotropic governor* mechanism of James (1987). The essence of this mechanism is that, as barotropic shears are generated by the momentum fluxes, they progressively interfere with the baroclinic production mechanism and thereby limit growth.

My impression, in contrast, is that the equilibration mechanisms in one-jet and multiple-jet flows, and in this homogeneous model, are essentially the same, the dominant process being a generalized version of the barotropic governor, in which it is not only zonally averaged barotropic shears but the energy containing barotropic mode, whether jet-like or not, that interferes with baroclinic production (see Salmon, 1980)). One does not need time-averaged momentum fluxes to create a barotropic governor; instantaneous shears are adequate.

On the other hand, the approach to the homogeneous limit is not likely to be simple. For example, Lee (1997) has shown that eddy statistics undergo non-monotonic evolution as one increases the width of the unstable region so as to make the transition from one jet to two. Relatively little has been achieved with regard to how one might use the homogeneous limit as a starting point for inhomogeneous theory. See in this regard Pavan and Held (1997).

1.5 STATIC STABILITY MAINTENANCE

A key question in general circulation theory is whether or not the slope of the mean isentropes in the troposphere is strongly constrained. The observed slope is close to the aspect ratio of the troposphere: an isentropic surface that is near the ground in the tropics rises to the tropopause in polar latitudes. Is this a coincidence, or is this particular slope favored?

Using the scaling from the previous section for the diffusivity due to baroclinic eddies, one can, in the spirit of Stone (1972), try to develop a theory for the static stability. In a stratified atmosphere, the expression (1.7) for the diffusivity, for example, implies that $D \sim \Delta_H^3 \Delta_V^{-3/2}$, where Δ_H and Δ_V are the horizontal and vertical potential temperature gradients respectively. To obtain the horizontal eddy heat flux $\mathcal{H}$, one multiplies by another factor of Δ_H. To estimate the vertical eddy heat flux $\mathcal{V}$ (ignored in QG theory) one can assume that the total flux is aligned along isentropic surfaces, averaged over the troposphere, so that $\mathcal{V}\Delta_V \sim \mathcal{H}\Delta_H$, or $\mathcal{V} \sim \Delta_H^5\Delta_V^{-5/2}$. We next need to assume that the static stability is maintained by a balance between this eddy vertical heat flux and the destabilization by radiation. If we just assume that radiation relaxes Δ_V to 0 on some specified time scale, and that the vertical scale of the eddies is fixed, then $\Delta_V \sim \mathcal{V}$, resulting in the estimate $\Delta_V \sim \Delta_H^{10/7}$ and $D \sim \Delta_H^{6/7}$. The point of this manipulation is not to make a case for this specific result, but to illustrate how allowing the stability to adjust to changing eddy fluxes can potentially alter the sensitivity of the fluxes to the horizontal temperature gradient.

This scaling suggests that the isentropic slope can be altered by modifying the horizontal temperature gradient, albeit with some difficulty: $\Delta_H/\Delta_V \sim \Delta_H^{-3/7}$. If we use (1.8) instead of (1.7) the result is a much stronger constraint on the isentropic slope, when the system is in the proximity of the critical slope. But is this legitimate, given the seemingly artificial character of the two-layer model's critical slope?

As an alternative to thinking in two-layer terms, I have suggested that one needs to couple the prediction of the static stability with a prediction of the tropopause height, and that by doing so one introduces a stronger constraint on the isentropic slope in the continuously stratified case (Held, 1982). The essence of this argument can be understood by thinking of a continuously stratified QG model with fixed static stability and vertical shear; in this system the claim is that the distance that the eddy fluxes extend above the surface scales as $h \sim f^2\partial U/\partial z/(\beta N^2$, which is equivalent to $\xi \sim 1$. See Thuburn and Craig (1997) for a critique of this claim, and Schneider (2004) and Schneider and Walker (2005) who provides strong support for a refined version of this argument, (while simultaneously calling into question the relevance of continuously stratified QG theory).

1.6 THE ENTROPY BUDGET

In a comparison of theories for the poleward heat flux with various scaling arguments, Barry et al (2002) combine the Rhines scale-inverse energy cascade relation (1.3) with an estimate of ϵ from a global entropy budget, rather than an energy budget. It is useful to understand how these approaches are related, as the entropy perspective may be especially useful in the presence of latent heat release.

Consider a dry atmosphere forced by the time mean heating/cooling Q. The forcing decreases the entropy at a rate determined by averaging Q/T over the atmosphere. (From this point on, the symbol T refers to temperature, not to an eddy time scale.) This is a decrease in entropy because Q creates temperature gradients by warming (cooling) regions that are already relatively warm (cool). In a steady state this entropy destruction is balanced by production due to irreversible processes, the dominant one in a dry atmosphere being the dissipation of kinetic energy (that is, the diffusion of momentum), the rate of kinetic energy dissipation being ϵ once again. We ignore radiative damping of transients due to the correlation in time between Q and T, which will create entropy, and we also ignore diffusion of temperature. The latter tends to be small because temperature, in balanced flows, cascades to small scales only at the surface and not in the interior of the troposphere. Therefore,

$$-\int \frac{Q}{T} \approx \frac{\epsilon}{T_\epsilon} \tag{1.9}$$

where T_ϵ is the average temperature at which the energy dissipation occurs and all integrations are over mass of the atmosphere. We assume small departures of T and T_ϵ from a reference temperature T_0 as needed.

Barry et. al. estimate ϵ in their model by taking the distribution of Q as given. This may seem like one is giving oneself too much information, in that Q is dominated by the divergence of the eddy heat flux for which one is trying to develop a theory. But suppose one has a theory for the eddy diffusivity, and eddy heat or potential vorticity fluxes, that depends on ϵ. Given ϵ, one determines the fluxes and temperatures, and therefore, Q; one can then iterate to obtain self-consistency. A difficulty with this approach is that one loses the sense of a local theory, ϵ being determined by a global integral.

But one can regain a local perspective by setting $Q = \nabla \cdot F$, where F is the flux of dry static energy, and then integrating by parts:

$$\frac{\epsilon}{T_\epsilon} \approx \int \frac{1}{T^2} F \cdot \nabla T = \int \frac{1}{T} F \cdot \nabla \ln T = \int \frac{1}{T} F_H \frac{\partial \ln T}{\partial y}|_M \tag{1.10}$$

In the final expression, we have assumed that the climate is zonally symmetric, so that F is a vector in the $y - z$ (or $y - p$) plane, with horizontal component F_H, and have let M denote a coordinate that is constant on the surface along which F is aligned. One can now apply this locally, setting the local ϵ-density equal to the integrand. To see the connection with the QG arguments above, one needs to assume that $M \approx \theta$. Letting S be the isentropic slope,

$$F_H \frac{\partial \ln T}{\partial y}|_\theta = F_H \frac{R}{c_p} \frac{\partial \ln p}{\partial y}|_\theta = F_H \frac{R}{c_p} S \frac{\partial \ln p}{\partial z} = F_H \frac{g}{c_p T} S \tag{1.11}$$

Substituting for $F_H \approx c_p \overline{v'T'}$ and setting

$$S = -\frac{\partial_y \theta}{\partial_z \theta} \tag{1.12}$$

and $(g/T)\overline{v'T'} = D f \partial_z U$, we regain Eq (1.5). For later reference, notice that the static stability makes its only appearance in this argument at the point when the mixing slope is set equal to the isentropic slope.

Entropy and available potential energy budgets are not equivalent in general, but they are closely enough related that they lead to essentially the same scaling approximations.

1.7 MOIST EDDIES

We would like our theories for midlatitude eddy fluxes to help us understand the implications of the increase in moisture content in the atmosphere that will accompany global warming. We would also like to make use of the seasonal cycle to test our theories for these eddy fluxes (e.g., Stone and Miller(1980)), but these tests are not very convincing as long as one is ignoring the effects of latent heat release, which vary seasonally in tandem with the variations in the large-scale temperature gradients. The length scale of midlatitude eddies is observed to be larger in Northern winter than in summer. Is this due to the larger eddy energies in winter, which result in a larger Rhines scale, or is it that eddies are smaller in summer because of a reduction in an *effective static stability* due to latent heat release. A central theoretical issue is whether there are ways of using concepts like moist entropy (Emanuel and Bister(1996)) or moist available potential energy (Lorenz(1978)) so as to carry some of the lines of argument developed for dry eddies over to the moist case.

Lapeyre and Held (2004) construct a relatively simple moist model by adding a water vapor variable to the two-layer QG model. To obtain a consistent energetics, they treat moisture in an analogous way to temperature (or thickness), by requiring the moisture field to be a small perturbation away from a prescribed mean value that is uniform within each layer. Despite this limitation, the form of this model's energetics is of interest. Here I provide a brief sketch of QG moist energetics more generally, because it has a feature that is counterintuitive (for me) and may have interesting implications for how we think about moist eddies.

In a dry QG model the available potential energy APE is proportional to the variance of the interface displacement. This form follows from the QG thermodynamic equation of the form

$$\frac{\partial b}{\partial t} = -N^2 w - J(\psi, b) \tag{1.13}$$

We use the Boussinesq approximation for simplicity, with b the buoyancy; the final term represents horizontal advection by the geostrophic flow. The conversion of potential to kinetic energy is $[bw]$, where brackets denote a global mean. One manipulates the buoyancy equation to have the same expression on the RHS by multiplying by b/N^2 and averaging:

$$\frac{\partial APE}{\partial t} = -[wb]; \quad APE \equiv [\frac{b^2}{2N^2}] \tag{1.14}$$

In a moist QG model, one has instead, schematically,

$$\frac{\partial b}{\partial t} = -N^2 w - J(\psi, b) + LP \tag{1.15}$$

where P is the condensation rate and L the latent heat, and

$$\frac{\partial q}{\partial t} = -(\partial_z Q) w - J(\psi, q) - P \tag{1.16}$$

where q is now the moisture perturbation (not potential vorticity) and $Q(z)$ the reference moisture. Forming an equation for the buoyancy variance results in the term $[Pb]$ on the RHS, which we wish to avoid. One can eliminate P by forming a moist enthalpy equation for $h \equiv b + Lq$, but forming an equation for the variance of h generates a term proportional to $[hw]$ rather than $[bw]$. One can try to remedy this problem by forming an equation for the variance of the moisture, but this reintroduces the precipitation on the RHS through $[qP]$. The successful manipulation uses the variance of the saturation deficit, $d = q_s - q$, where one assumes that precipitation occurs when one is saturated, so that $[dP] = 0$. Here I describe the simplest case, in which q_s is a constant, independent of temperature (the case in which the saturation vapor pressure is a function of temperature (b) is a bit more involved). We can then set this $q_s = 0$ (recall that q is here the departure from the reference $Q(z)$). We finally obtain an equation of the form

$$\frac{\partial QAPE}{\partial t} = -[wb]; \quad QAPE \equiv \frac{1}{2}\left[\frac{h^2}{(N^2 - L|\partial_z Q|)} + L\frac{d^2}{|\partial_z Q|}\right] \tag{1.17}$$

Thus, our moist available potential energy (QAPE) has one term proportional to the variance of the moist enthalpy, divided by a moist stability, plus an additional term proportional to the variance of the saturation deficit, or dew point depression. This form is presumably related to Lorenz's general form for moist APE, specialized to the case of small interface displacements and small moisture deficits. The implications for moist energetics of the presence of the term proportional to the saturation deficit variance are obscure but intriguing. *There is an energetic cost to an increase in understaturation.* I find it difficult to understand this statement intuitively. See Frierson, et al, (2004) for an application of an analogous expression to a shallow water model.

The sources/sinks of QAPE also have additional terms not present in the dry case. Evaporation into unsaturated air and diffusion of water are both important sinks of QAPE and have no direct counterparts in the dry case. Unlike temperature, water mixing ratio does cascade to small scales in this QG flow, so the diffusive loss of mixing ratio variance is both significant and energetically important from the perspective of QAPE. There is an intriguing resemblance between these sinks of QAPE and the sources of irreversibility in a moist entropy budget. As discussed by Emanuel and Bister(1996) and Pauluis and Held (2002) for tropical convection, the efficiency of the kinetic energy cycle is reduced by diffusion of moisture, either due to a cascade of variance to small scales, or to evaporation into unsaturated air (microscopically, the latter is simply diffusion down the gradient between the saturated air in contact with the liquid and the air a bit further removed). Eq (9) is replaced by

$$\frac{\epsilon}{T_\epsilon} \approx -\int \frac{Q}{T} - R \tag{1.18}$$

where R is the positive definite generation of entropy due to diffusion of vapor, and Q now includes radiative cooling and surface evaporation plus surface sensible heating (but not latent heat release!). It is likely that the term R reduces the efficiency of midlatitude eddy dynamics substantially (especially in summer) just as it does the efficiency of tropical convection.

How would latent heat release modify the kinds of scaling arguments described earlier? We can try to work from either a moist available potential energy or a moist entropy perspective, but the latter may be simpler, especially since the QG version of QAPE is undoubtedly too restrictive. Setting Q equal to the divergence of the eddy moist static energy flux, F, with horizontal component F_H equal to the flux of moist enthalpy $\overline{v'h'} = c_p\overline{v'T'} + L\overline{v'q'}$, we can write

$$\frac{\epsilon}{T_\epsilon} = \int \overline{v'h'}\frac{1}{T}\frac{\partial \ln(T)}{\partial y}|_M - R \tag{1.19}$$

where the derivative is taken along the mixing surface, defined by the direction of the eddy moist static energy flux. One can then diffuse moist enthalpy down the mean moist enthalpy gradient, and combine this expression with (3) or its equivalent. Thus, moisture and latent heat release affect the theory of Held and Larichev, or Barry et al, in three ways: by reducing efficiency through the term R, by increasing the mixing slope (i.e. reducing the effective static stability) and by replacing the dry enthalpy by the moist enthalpy as the quantity being diffused. Without expressions for the mixing slope and the efficiency reduction this is not a closed theory, but it gives us some feeling for what such a theory might look like.

1.8 AN IDEALIZED MOIST MODEL ON THE SPHERE

Does latent heat release reduce the mean length scale of the energy containing eddies? The Eady model linear theory of Emanuel et al (1987) indicates potential for a reduction by a factor of 2 or so. But if one thinks in terms of the Rhines scale one might guess that a reduction in effective static stability likely increases the scale by increasing the eddy kinetic energy.

Frierson et al (2006, hereafter FHZ) have constructed an idealized moist GCM on the sphere in part to address questions of this kind. The moist general circulation is generally addressed with comprehensive atmospheric climate models in which clouds, convection, and radiative transfer interact in a host of subtle ways that are only dimly appreciated, and in which there are sensitivities to resolution, time-stepping, and (often undocumented) details in the closure schemes that makes it difficult to reproduce model results. In FHZ, the radiation is a function of temperature only, there is no condensate, and the boundary layer and convective closures are kept simple enough to encourage tests of reproducibility and sensitivity to resolution. In the simplest case, the model is run with large-scale condensation only, with no convective closure scheme.

The initial results with this model (FHZ) show surprising insensitivity of the eddy scale to the amount of moisture in the atmosphere, and, therefore, to the amount of latent heat release. There is essentially no difference in the midlatitude eddy spectrum between the dry limit of this model and and a control run with realistic moisture content. The dry static stability increases with increasing moistures, to prevent large changes in moist stability, so the constancy of the eddy length scale is in disagreement with any theory based on an effective stability that scales with the dry stability. The Rhines scale, on the other hand, does predict the constancy of this scale as the moisture increases if, it turns out, one allows oneself to compute it at the position of the maximum eddy kinetic energy (FHZ). This latitude moves polewards as moisture increases, and the Rhine's scale remains unchanged only because of the canceling effects of a reduction in eddy energy and a reduction in β. But this does not explain whether the cancellation is a coincidence or a result of some dynamical constraint.

The results in FHZ are also intriguing with respect to the question of the partitioning of the poleward heat flux between latent and sensible parts. As shown in Fig. 2, the total poleward flux in this model stays remarkably constant (to within 1%) as the amount of water vapor, and the poleward flux of latent energy, increases from the dry limit to a realistic value. A reduction in sensible flux cancels the increase in latent flux. It is sometimes argued (following Stone, 1978b) that the total atmospheric flux is more or less as large as it can possibly be, since the profile of outgoing infrared flux is much flatter than that of the absorbed solar flux. But analysis shows that there is substantial room for an increase in the FHZ model. In any case, why should the atmosphere be incapable of reversing the sign of the outgoing long wave gradient? It is not difficult to construct a model that does precisely this (D. Frierson, personal communication).

I have recently examined this compensation in the comprehensive climate model at GFDL when run in aqua-planet mode (over a uniform boundary condition of slab ocean with fixed heat capacity) and find about 80%, rather than near perfect, compensation when the atmospheric CO_2 is doubled. My impression it that this level of compensation is typical in comprehensive climate models (e.g., Manabe and Bryan, 1985). We suspect that the key to the near-perfect cancellation in FHZ is the fact that the radiation is a function of temperature only.

A final question that is addressed by FHZ model is that of the role of latent heating, and moist convection more specifically, in maintaining the static stability in midlatitudes. The claim is that this idealized model supports the picture of Juckes(2000), who argues that the large scale eddy fluxes are not capable of stabilizing the atmosphere to the point of preventing moist convection in the warm sectors of extratropical cyclones. A ppossible implication is that the mean static stability of the extratropical troposphere is maintained by this moist convection so that the favorable sectors of extratropical cyclones are moist neutral. The average moist stability of the atmosphere is then determined by the difference between the average boundary layer moist enthalpy and the maximum value of this boundary layer moist enthalpy within the eddies, or equivalently, by the rms moist enthalpy in the

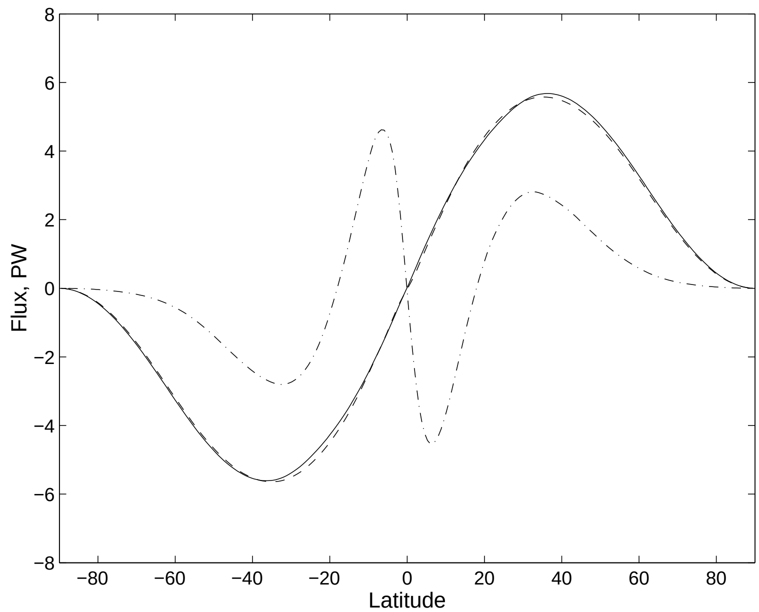

Figure 1.2 Poleward energy fluxes in moist and dry model. Solid: total energy flux in moist model; dashed; dry static energy flux(and, therefore, total flux) in dry model; dash-dot: latent heat flux in moist model.

boundary layer. The latter, in turn, is presumably determined by the large scale eddy mixing length and the mean moist enthalpy gradient.

Clearly, we have just scratched the surface of many central climatic questions involving the effects of moisture on the large-scale circulation, many of which are important in understanding global warming simulations. Idealized models of the moist general circulation are sorely needed, to make contact with our even more idealized dry models and with the high-end comprehensive models that play the predominant role when we apply climate models to real world problems.

1.9 UPPER VS. LOWER TROPOSPHERIC DYNAMICS

There is an important qualitative distinction between the upper and lower troposphere that impacts the general circulation in numerous ways: the upper troposphere is more wave-like than the lower troposphere. This distinction must fundamentally be due to β, the environmental vorticity gradient. In the two-layer model, for example, β adds to the contribution of vertical shear to the upper layer potential vorticity gradient, while it tends to cancel this contribution in the lower layer. The result is that potential vorticity gradients are larger in magnitude in the upper than in the lower layer.

These gradients create the restoring forces for Rossby waves. A disturbance of a given scale will propagate westward with respect to the upper level flow more strongly than it will propagate eastward with respect to the lower level flow (in the lower layer the potential vorticity gradient is negative, causing Rossby waves to propagate eastward rather than westward.) As one consequence, β pushes the steering level for baroclinic instabilities, where the phase speed matches the environmental flow, into the lower troposphere.

When waves on shear flows grow they typically break when the flow perturbations, u', become comparable to $\overline{u} - c$, the phase speed of the wave with respect to the environment. So eddies of the same amplitude will break first in the lower layer, and the upper layer will remain more linear. To the extent that they are determined by this kind of breaking criterion, eddy amplitudes should be larger in the upper than in the lower layer. The flow in each layer can be thought of as induced by the potential vorticity in both layers, but if the eddy amplitudes are larger aloft, the lower level flow will be primarily induced by the potential vorticity in the upper layer – while the upper layer flow will be primarily self-induced, allowing more wave-like evolution. I suspect that this has something to do with the fact that the two-layer closure theory works best when based on lower layer diffusion of potential vorticity, leading to Eq (1.8).

The distinction between upper and lower troposphere dynamics, with the latter more turbulent and the former more wave-like, is central to any discussion of eddy momentum fluxes. *That the eddy momentum fluxes are almost entirely confined to the upper troposphere is a consequence of this distinction.* Rossby waves propagating away from their midlatitude source on a positive potential vorticity gradient (as in the upper layer of a two-layer model) converge eastward (positive) angular momentum into midlatitudes; Rossby waves propagating on a negative vorticity gradient (as in the lower layer of the two-layer model) converge negative momentum into the source latitudes. Because almost all of the propagation in fact occurs in the upper troposphere, surface westerlies are generated in midlatitudes to remove the positive momentum flux convergence. If lower tropospheric propagation were dominant, surface easterlies would be generated in midlatitudes. All of the profound consequences for the atmosphere and the oceans that follow from the existence of midlatitude surface westerlies result from this asymmetry between upper and lower tropospheres.

The simplest picture of linear midlatitude eddies in the upper troposphere starts with a barotropic westerly point jet, $u(y) = -\Lambda|y|$, the corresponding vorticity distribution being a single contour separating two homogenized regions, with jump $\Delta = 2\Lambda$ across the contour. This flow supports the simplest Rossby edge waves with dispersion relation $c = U - \Delta/(2k)$. One can usefully speak of a *capacity* of this jet, the amplitude of the waves that can propagate along this contour without significant breaking. Using the criterion $u' \sim \overline{u} - c$ for overturning streamlines in the frame of reference of the wave, one gets $u' \sim \Delta/k$. The corresponding trajectory displacements are also of the order of the inverse wavenumber k^{-1}. If we now think of the homogenized regions on each side of this contour, and, therefore, the size of the jump Δ as having been created by the eddies themselves from the environmental gradient β, we are led to assume that $\Delta \sim \beta k^{-1} = \beta L$, or $u' \sim \beta L^2$. The resulting relation between the eddy scale and eddy energy is just that proposed by Rhines, even though there is no association here with an inverse cascade. This picture may help us understand why it is the Rhines scale *at the latitude of the jet* that seems to be the relevant scale for the eddies in FHZ.

The homogeneous turbulence theory outlined above can be thought of as consisting of three relations between three unknowns: the strength of the energy generation/dissipation ϵ, an eddy length scale L, and an eddy velocity scale V (or a diffusivity VL). The three relations are 1) an entropy or available potential energy budget that relates ϵ and D, 2) the Rhines scale relation between V and L, and 3) the turbulent cascade scaling $\epsilon \approx V^3/L$. (One can combine 2) and 3) to give Eq (3).) In light of the results described by Schneider (2004) for a primitive equation model on the sphere in which the static stability adjusts to prevent a significant inverse cascade, it may be desirable to try to retain 1) and 2), but to replace 3) with a non-turbulent alternative, a choice made palatable by this alternative argument for the Rhines scale.

A picture that emerges is of an upper level waveguide fed by baroclinic eddy production, with an eddy sink given by the sloughing off of excess wave activity, and fed by baroclinic eddy production that is, in turn, determined by the diffusion of low level PV (or heat) controlled by the upper level eddy amplitudes. One can try to expand this picture into a theory for the zonally asymmetric storm tracks (see in this regard Swanson et al (1997) and Chapter 8 inthis volume) in which the key new ingredient is the zonally varying capacity of the jet.

While we have some useful pictures of upper level dynamics that help us understand the eddy momentum fluxes, and even some simple linear models that fit the eddy momentum fluxes quantitatively, given the low level eddy stirring (DelSole(2001), our understanding of the location of the surface westerlies is far from complete. This is evident when we perturb the system and try to understand how and why the surface westerlies (and the associated eddy momentum flux convergence) move. An excellent example is provided by experiments in which the strength of the surface friction is modified; as the friction is weakened the westerlies move poleward (Robinson, 1997). Fig. 3 is from unpublished work by G. Chen (personal communication, 2005), using the dry dynamical core benchmark of Held and Suarez (1994). The theory for this shift is still undeveloped. Robinson has suggested that a barotropic governor mechanism is the key: as the surface friction is reduced, surface winds and horizontal shears increase, and, it is argued, the resulting stabilization by these shears is larger on the equatorward side. How one would go about making this hypothesis quantitative and then testing it remains a challenge.

Poleward displacement of the surface westerlies and storm tracks is also

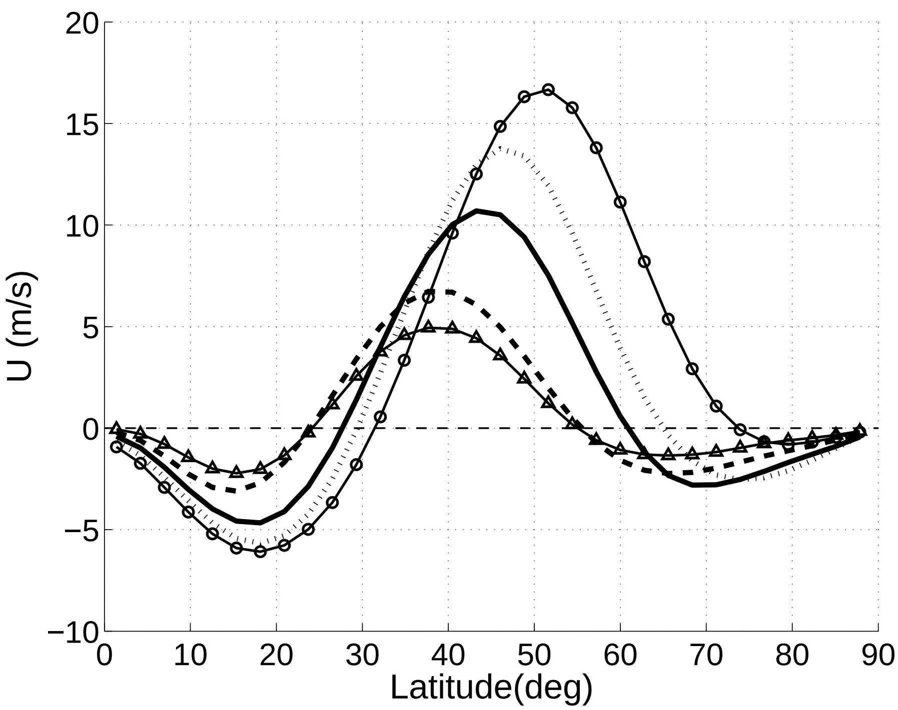

Figure 1.3 The near-surface zonal mean wind field in the climate of an idealized dry GCM, for several values of the strength of the surface friction. In this model, the surface friction is simply a linear drag, with relaxation time (0.5, 0.75, 1, 1.25, 1.5) days in the different cases. The longer relaxation times produce stronger winds and a poleward displacement of the westerlies.

seen in global warming simulations. Several alternative explanations have been offered for this shift, some involving the increase in latent heat release. It will be a challenge to our theories, and our ability to develop the appropriate hierarchy of idealized models, to cleanly isolate the dynamics underlying this shift.

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FIGURE CAPTIONS

Figures 1: Comparing a theory for eddy heat fluxes in a homogeneous two-layer model with numerical simulations. Dots are the diffusivity computed numerically in a 1024x1024 spectral simulation, after Lapeyre and Held (2003), compared with the theoretical scaling provided by Eq.8. The departures at large supercriticality are probably related to the finite size of the domain.

Figures 2: Poleward energy fluxes in moist and dry idealized models. Solid: total energy flux in moist model; dashed; dry static energy flux(and, therefore, total flux) in dry model; dash-dot: latent heat flux in moist model.(Provided by D. Frierson)

Figures 3: The near-surface zonal mean wind field in the climate of an idealized dry GCM, for several values of the strength of the surface friction. The surface friction is a linear drag in the lower troposphere, with relaxation time (0.5, 0.75, 1, 1.25, 1.5) days in the different cases. The longer relaxation times produce stronger winds and a poleward displacement of the westerlies. (Provided by G. Chen)