

# **User Manual for Stochastic Simulation Capability in GREET**

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# User Manual for Stochastic Simulation Capability in GREET

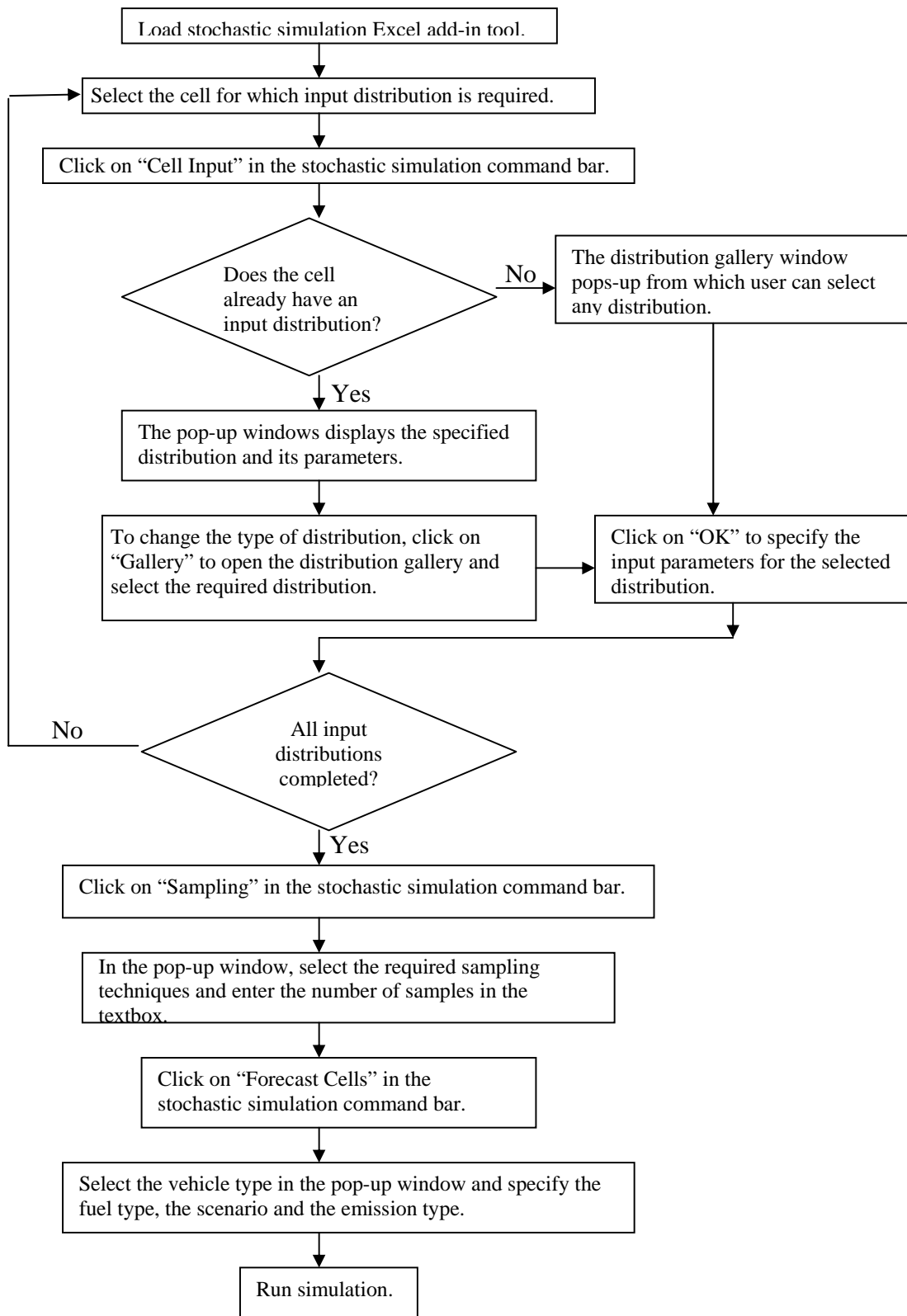
## 1. Introduction

This tool incorporates stochastic simulation capability into the GREET model. GREET is a complex model for estimating the full fuel-cycle energy and emission impacts of various transportation fuels and vehicle technologies. The GREET model incorporates large number of input parameters and a wide variety of output results. Many of the input parameter assumptions involve uncertainties, which require probability distributions to represent the trend of occurrence of the parameter over a specific range that define the uncertainty. Since the parameters in GREET are uncertain, the resulting output variables consequently have to be represented by distributions.

To address these uncertainties, a stochastic simulation tool has been developed to incorporate various sampling techniques. The tool has been built as a Microsoft® Excel add-in file, to assign probability distributions and perform sampling on the input parameters. The add-in file can be loaded whenever you need to perform a stochastic simulation within the GREET model. Broadly speaking, the software add-in tool allows you to:

- 1) Assign probability distribution functions to the input variables;
- 2) Specify the number of samples required and the sampling technique to be used;
- 3) Define the forecast variables (the tool provides you with various options to narrow down your preferences for forecast variables from approximately 3,000 choices);
- 4) Propagate the uncertainties; and
- 5) Statistically analyze the outputs.

Figure 1 shows a more detailed overview of the stochastic simulation process using the Excel add-in tool.

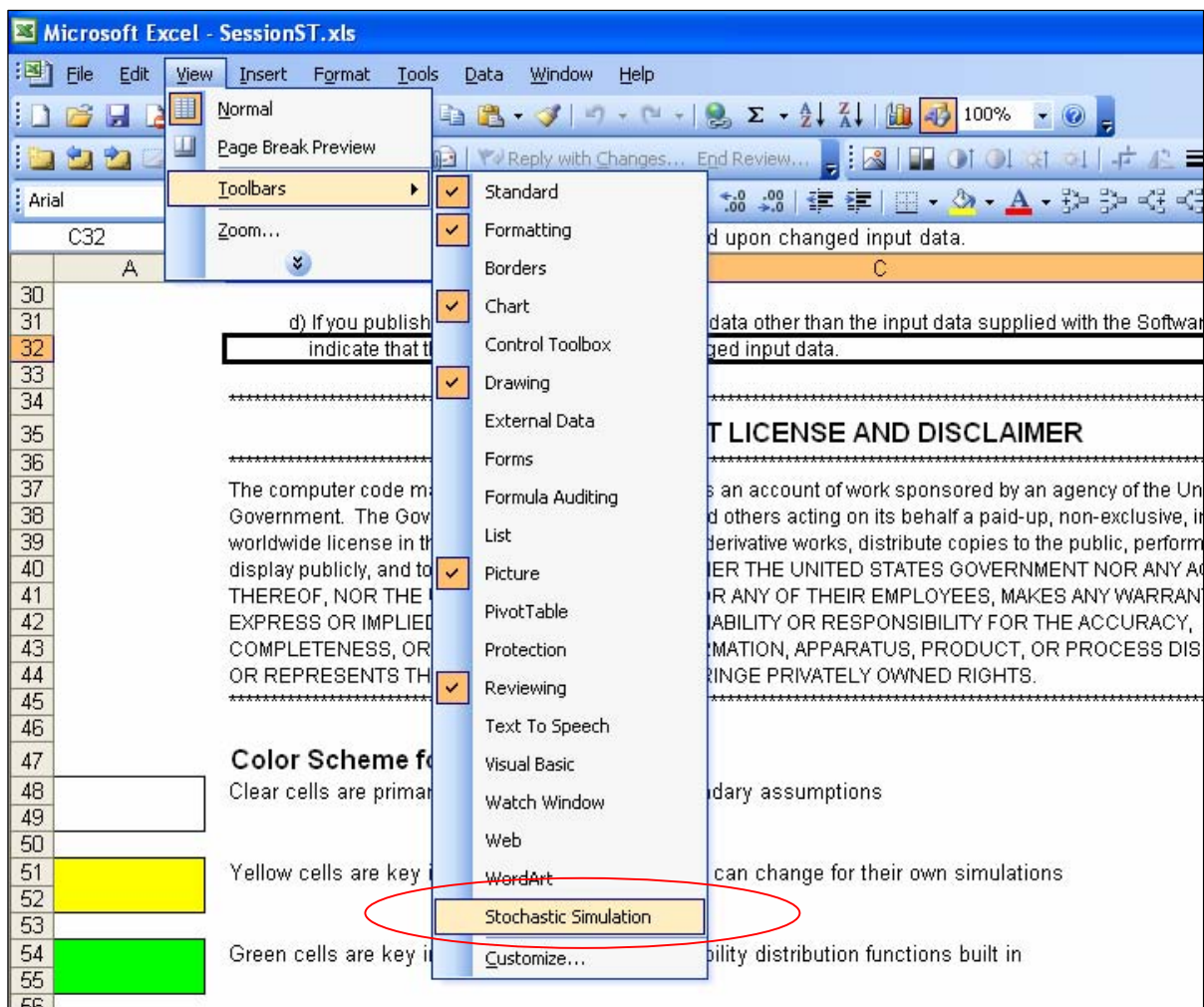


**Figure 1.** Overview of the stochastic simulation process

## 2. Loading the Stochastic Simulation Tool into GREET

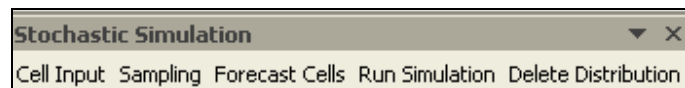
To load the stochastic simulation tool into the GREET model, perform the following steps:

- 1) Open the GREET Excel file that you are using for stochastic simulations.
- 2) Go to **View>Toolbars**.
- 3) Select the “**Stochastic Simulation**” toolbar, as shown in Figure 2.



**Figure 2.** Loading the “Stochastic Simulation” toolbar

- 4) A command bar with all the command buttons required for the stochastic simulation process appears as shown in Figure 3. The stochastic capability of the GREET model has been interfaced as a command bar containing five buttons for the five main steps of the uncertainty analysis process. Section 5 provides a detailed explanation of the functionality of each button in the stochastic simulation command bar.



**Figure 3.** Stochastic simulation command bar

### 3. Overview of Probability Distribution Functions

The tool contains eleven built-in probability distributions. The following paragraphs provide a brief description of each probability distribution.

#### 3.1 Beta Distribution

An important application of the Beta distribution is its use as a conjugate distribution for the parameter of a Bernoulli distribution. It is also used to describe empirical data. The general formula for the probability density function of the Beta distribution is

$$f(x) = \frac{\left(\frac{x}{s}\right)^{(\alpha-1)} \left(1 - \frac{x}{s}\right)^{(\beta-1)}}{\text{Beta}(\alpha, \beta)} \quad 0 < x < s; \alpha > 0; \beta > 0$$

Where,

$\alpha$  and  $\beta$  are the shape parameters,

's' is the scale, and

$\text{Beta}(\alpha, \beta)$  is the Beta function. The Beta function has the formula:

$$\text{Beta}(\alpha, \beta) = \int_0^1 t^{\alpha-1} (1-t)^{\beta-1} dt$$

#### 3.2 Normal Distribution

The normal distribution is the most commonly used distribution in the field of probability and statistics. The general formula for the probability density function of the normal distribution is

$$f(x) = \frac{1}{(\sqrt{2\pi})\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

Where,  $\mu$  is the mean and  $\sigma$  is the standard deviation. The case where  $\mu = 0$  and  $\sigma = 1$  is called the *standard normal distribution*.

### **3.3 Lognormal Distribution**

A variable  $x$  is log-normally distributed if the natural logarithm of  $x$ ,  $\ln(x)$ , is normally distributed. The general formula for the probability density function of the Lognormal distribution is

$$f(x) = \frac{1}{x(\sqrt{2\pi})\sigma} e^{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}} \quad 0 < x < \infty$$

Where,

$\mu$  is the logarithmic mean, and

$\sigma$  is the logarithmic standard deviation.

### **3.4 Uniform Distribution**

In this distribution, all the values between the minimum and maximum have equal chance of occurrence. The general formula for the probability density function of the uniform distribution is

$$f(x) = \frac{1}{B-A} \quad A \leq x \leq B$$

Where,

$A$  is the location parameter, and

$(B - A)$  is the scale parameter.

The case where  $A = 0$  and  $B = 1$  is called the *standard uniform distribution*.

### **3.5 Triangular Distribution**

Triangular distribution is usually used when there is insufficient data to fit any other distribution but the minimum, maximum and most likely values are known. The probability density function for a triangular distribution is given as:



$$f(x) = \frac{2(x-a)}{(b-a)(c-a)} \quad \text{for } a \leq x \leq b$$

$$= \frac{2(b-x)}{(b-a)(b-c)} \quad \text{for } c < x \leq b$$

Where,

$a$  is the minimum value,

$b$  is the likeliest value, and

$c$  is the maximum value.

### 3.6 Weibull Distribution

Weibull distribution is commonly used in reliability studies and it is a flexible distribution which can assume the properties of other distributions based on its input parameters. The formula for the probability density function of the general Weibull distribution is

$$f(x) = \left(\frac{\beta}{\alpha}\right) \left(\frac{x-L}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x-L}{\alpha}\right)^\beta} \quad \text{for } x \geq L$$

Where,

$L$  is the location parameter,

$\alpha$  is the scale parameter, and

$\beta$  is the shape parameter.

When  $\beta = 1$ , Weibull reduces to the Exponential distribution (to be discussed later).

### 3.7 Gamma Distribution

The gamma distribution is commonly used in Bayesian reliability analysis. It is a flexible distribution and is related to other distributions like the lognormal and exponential distributions.

The general formula for the probability density function of the gamma distribution is

$$f(x) = \frac{\left(\frac{x-L}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x-L}{\alpha}\right)}}{\Gamma(\beta)\alpha} \quad \text{for } x \geq L$$

Where,

$L$  is the location parameter,

$\alpha$  is the scale parameter,

$\beta$  is the shape parameter, and

$\Gamma(\beta)$  is the gamma function given by:

$$\Gamma(\beta) = \int_0^{\infty} t^{\beta-1} e^{-t} dt$$

### **3.8 Extreme Value Distribution**

The extreme value distribution has two forms. One is based on the smallest extreme (skewed to the left) and the other is based on the largest extreme (skewed to the right).

For skew to the minimum:

$$f(x) = \left( \frac{1}{\beta} \right) \exp\left( \frac{\alpha - x}{\beta} \right) \exp\left( -\exp\left( \frac{\alpha - x}{\beta} \right) \right) \quad \text{for } \infty < x < \infty$$

For skew to the maximum:

$$f(x) = \left( \frac{1}{\beta} \right) \exp\left( \frac{x - \alpha}{\beta} \right) \exp\left( -\exp\left( \frac{x - \alpha}{\beta} \right) \right) \quad \text{for } \infty < x < \infty$$

Where,

$\alpha$  is the mode parameter, and

$\beta$  is the scale parameter.

### **3.9 Exponential Distribution**

The exponential distribution is usually used to depict events which occur at random like the time between the failures of equipment. The general formula for the probability density function of the exponential distribution is

$$f(x) = \lambda e^{-\lambda x} \quad \text{for } x \geq 0$$

Where,

$\lambda$  is the rate parameter.

### **3.10 Pareto Distribution**

The Pareto distribution is generally used to describe empirical phenomena like birth rate, income growth rate, etc. The general formula for the probability density function of the Pareto distribution is:

$$f(x) = \frac{\beta L^\beta}{x^{(\beta+1)}} \quad \text{for } x > L$$

Where,

$L$  is the location parameter, and

$\beta$  is the shape parameter.

### **3.11 Logistic Distribution**

The logistic distribution is used to model binary responses (e.g., Gender) and is commonly used in logistic regression. The logistic distribution is defined as:

$$f(x) = \frac{e^{-\left(\frac{x-\mu}{\alpha}\right)}}{\alpha \left(1 + e^{-\left(\frac{x-\mu}{\alpha}\right)}\right)^2} \quad \text{for } -\infty < x < \infty$$

Where,

$\mu$  is the mean parameter, and

$\alpha$  is the scale parameter.

## **4. Overview of Sampling Techniques**

The stochastic simulation tool has four sampling techniques incorporated into it [1]: 1) Monte Carlo Sampling; 2) Latin Hypercube Sampling; 3) Hammersley Sequence Sampling; and 4) Latin Hypercube Hammersley Sampling. The following paragraphs explain each sampling technique in more detail.

### **4.1 Monte Carlo Sampling (MCS)**

One of the most widely used techniques for sampling from a probability distribution is the Monte Carlo sampling technique, which is based on a pseudo-random generator used to approximate a uniform distribution (i.e., having equal probability in the range from 0 to 1). The specific values for each input variable are selected by inverse transformation over the cumulative probability distribution. A Monte Carlo sampling technique also has the important property that the successive points in the sample are independent.

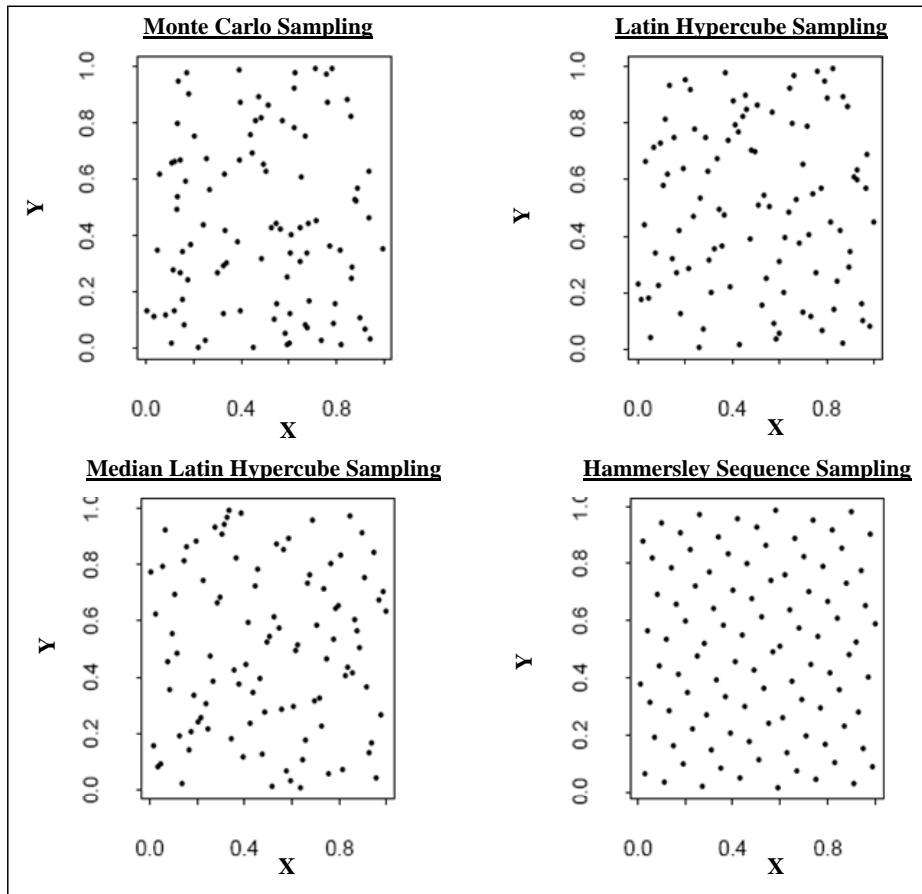
### **4.2 Median Latin Hypercube Sampling (MLHS)**

Latin Hypercube sampling is one form of stratified sampling that can yield more precise estimates of the distribution function. In Latin Hypercube sampling, the range of each uncertain parameter  $X_i$  is sub-divided into non-overlapping intervals of equal probability. In LHS, one value from each interval is selected at random with respect to the probability distribution in the interval. In MLHS, this value is the mid-point of the interval. The 'n' values thus obtained for  $X_1$  are paired in a random manner (i.e., equally likely combinations) with 'n' values of  $X_2$ . These n values are then combined with n values of  $X_3$  to form n-triplets, and so on, until 'n' k-tuplets are formed. The MLHS technique is used in the stochastic modeling tool that we developed.

### **4.3 Hammersley Sequence Sampling (HSS)**

In the late 1990s, an efficient sampling technique, Hammersley Sequence Sampling, based on Hammersley points, was developed [2], which uses an optimal design scheme for placing the ‘n’ points on a k-dimensional hypercube. Unlike Monte Carlo Sampling, the Latin Hypercube and its variant (the Median Latin Hypercube), the HSS sampling technique ensures that the sample set is more representative of the population, showing uniformity properties in multi-dimensions. Figure 4 graphs the samples generated by different techniques on a unit square. This provides a qualitative picture of the uniformity properties of the different techniques. It is clear from Figure 4 that the Hammersley points have better uniformity properties compared to other techniques. The main reason for this is that the Hammersley points are an optimal design for placing n points on a k-dimensional hypercube. In contrast, other stratified techniques such as the Latin Hypercube are designed for uniformity along a single dimension and then randomly paired for placement on a k-dimensional cube.

One of the main advantages of the Monte Carlo method is that the number of samples required to obtain a given accuracy of estimates does not scale exponentially with the number of uncertain variables. HSS preserves this property of Monte Carlo. Hammersley Sequence Sampling is estimated to be 3 to 100 times faster than the LHS and MCS and hence, is a preferred technique for uncertainty analysis as well as optimization under uncertainty [2, 3]. Recent findings show that the uniformity property of HSS for higher dimensions (more than 30 uncertain variables) gets distorted. HSS (and LHSS given below) is generated based on prime numbers as bases. In order to break this distortion, we introduced leaps in prime numbers for higher dimensions. This ‘leaped’ HSS and LHSS technique showed better uniformity than the basic HSS and LHSS techniques. For simplicity, we have leaped HSS and LHSS as a part of the HSS and LHSS techniques in the stochastic modeling capability. When the number uncertain variables exceeds 30, the switch occurs automatically. GREET applies this sampling method as the default sampling technique.



**Figure 4.** Sample points (100) on a unit square using four sampling techniques

#### **4.4 Latin Hypercube Hammersley Sampling (LHHS)**

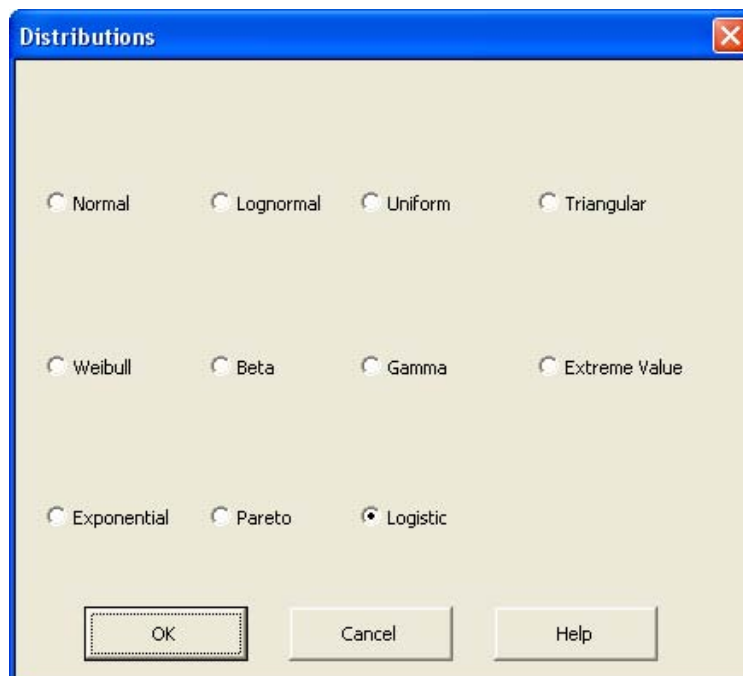
Latin Hypercube Hammersley Sampling [4] is a combination of HSS and LHS. It utilizes the one-dimensional uniformity of LHS and k-dimensional uniformity of HSS.

## 5. Stepwise Description of the Stochastic Simulation Process

The stochastic simulation command bar contains five buttons as shown in Figure 3, one for each step in the stochastic simulation process. The following paragraphs explain the functionality of each button in detail.

### 5.1 Cell Input

The first button, “Cell Input,” is for the specification of input probability distribution for each uncertain variable. Select one of the parametric assumption cells for which a probability distribution is to be specified, and click on “**Cell Input.**” The selected cell should have a nominal value and it shouldn’t be blank. If the cell is blank, an appropriate error message appears. Otherwise, a gallery window containing the built-in bank of probability distributions appears, as shown in Figure 5. You can select a type of distribution and click “**OK.**” The input parameter specification window for the particular distribution opens up. Once a cell has been assigned an input distribution, it turns green. The following paragraphs explain the input specification for each distribution.



**Figure 5.** Gallery of built-in distributions

### 5.1.1 Normal Distribution

Figure 6 shows a sample input parameter specification window for the normal distribution. The Probability Distribution Function (PDF) is plotted by taking the value of the active cell as the mean and standard deviation to be 10% of the mean. There are four portions in the input specification window:

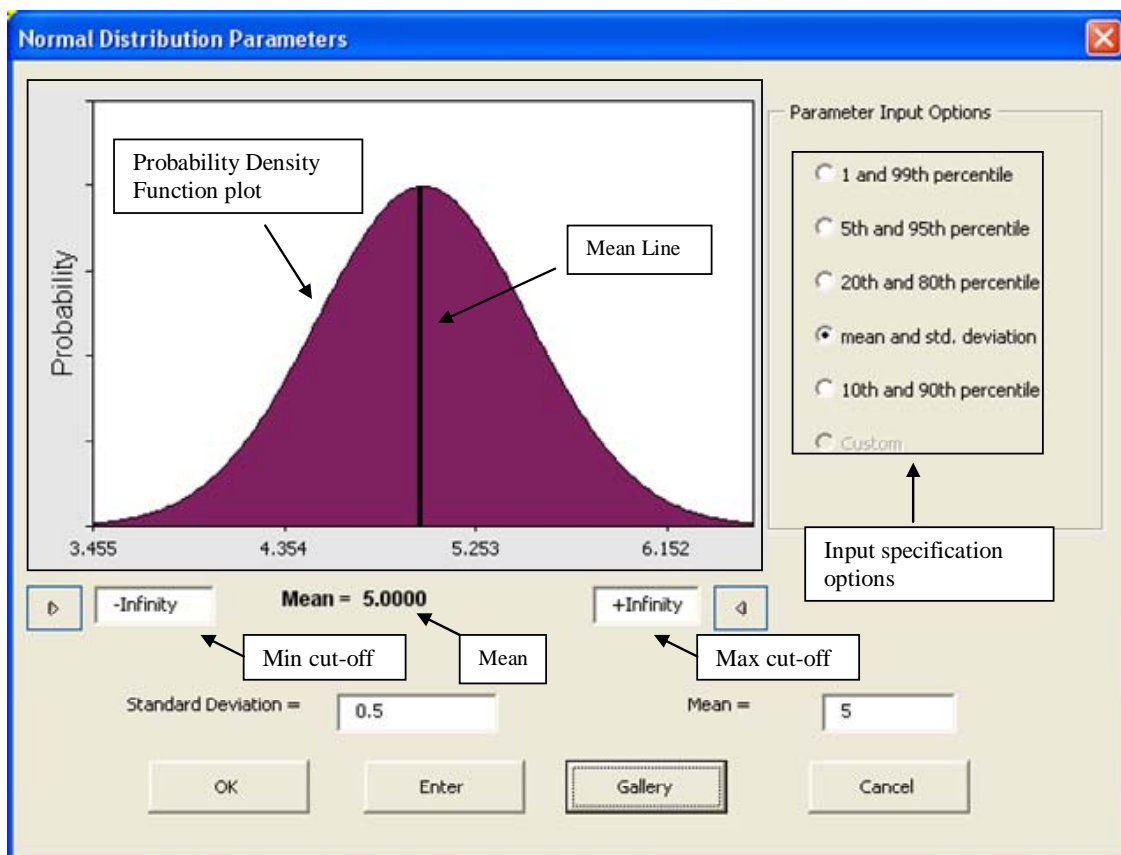
- 1) ***Input Specification frame at the right hand side:*** This portion consists of radio buttons which can be used to select the type of inputs specification. As seen from the figure, the normal distribution requires two input parameters, which can be selected from one of the following five input specification choices:
  - i. Mean and Standard deviation
  - ii. 1<sup>st</sup> and 99<sup>th</sup> percentile
  - iii. 20<sup>th</sup> and 80<sup>th</sup> percentile
  - iv. 5<sup>th</sup> and 95<sup>th</sup> percentile
  - v. 10<sup>th</sup> and 90<sup>th</sup> percentile

A “percentile” can be defined as a score location below which a specified percentage of the population falls. For example, if the 20<sup>th</sup> percentile of a test score in a class was 65, this means 20% of the class scored below 65. When the inputs are in terms of percentile, the code automatically estimates the values of the mean and standard deviation. When inputs are defined in terms of percentiles, care should be taken to provide feasible percentile values.

- 2) ***Input Parameters boxes above the control buttons:*** Once the type of input parameter is selected, the selected parameter automatically appears beside the input specification boxes. For example, in Figure 6, the mean and standard deviation input specification option has been selected and so they appear as labels of the input text boxes. There are certain requirements for proper input specification:
  - i. The inputs must be numeric,
  - ii. When inputs are specified in terms of percentiles, the input value for a lower percentile must be less than the input value for a greater percentile, and
  - iii. The standard deviation must be greater than 0.

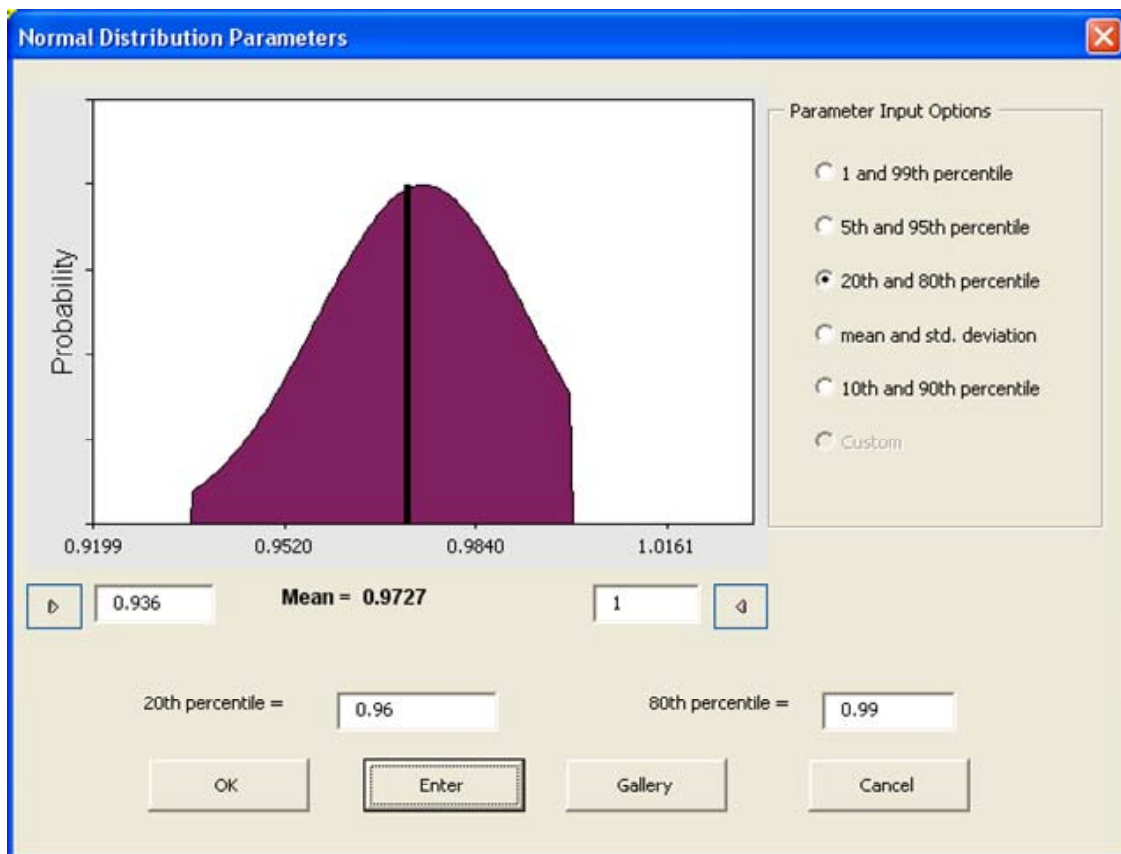


- 3) **Minimum and Maximum cut-off specification boxes below the PDF plot:** The default minimum and maximum cut-off values, in case of the normal distribution are “-Infinity” and “+Infinity,” respectively. These values are used in case you want to sample from the whole distribution. If you want to truncate the distribution so that samples cannot be less or greater than a particular value, you can truncate the distribution by specifying the particular values in these boxes. For example, when the uncertain variable is the efficiency of a process (which cannot be greater than 1), the maximum value of the distribution can be specified as 1 and the plot is truncated at this value. Figure 7 shows an example of such a scenario where a lower cut-off has also been specified at 0.936 for the purpose of demonstration. During execution, the samples for this particular uncertain variable would be between 0.936 and 1.00. If the minimum cut-off value is mistakenly specified to be greater than the maximum cut-off value, a message will pop-up indicating the error in the input specification.



**Figure 6.** Input specification window for normal distribution

- 4) **PDF plot portion in the middle:** Once the input parameters for the probability distribution has been specified, you can visualize the shape of the plot by clicking on the button captioned “**Enter.**” The plot is automatically redrawn according to the current input parameters. This is useful if you want to see the variation in the plot for various input parameters. The plot window also has a mean line that specifies the mean of the probability distribution function. For the full normal distribution plot, the mean line is right in the center of the graph. However, when the plot is truncated on the left side, the mean line shifts to the right; and vice-versa. In Figure 7, the plot is truncated on both sides, but the truncation in the right is greater than that in the left, and therefore, the net effect is the shifting of the mean line to the left.



**Figure 7.** Normal distribution truncated on both sides

Once all values pertinent to the specified distribution have been entered, click **“OK”** to confirm the input distribution for the uncertain parameter. **Note that it is not necessary to press “Enter” before clicking “OK.”** The “Enter” button is intended only to update and visualize the plot for the specified distribution inputs. If you decide to specify another type of distribution for the input parameter, you can click on the **“Gallery”** button, which displays a window containing all the available distributions as shown in Figure 5, and choose the desired probability distribution for that parameter.

### 5.1.2 Lognormal Distribution

When you select the Lognormal distribution in the gallery window and click “OK,” the input specification window for that distribution will be displayed as shown in Figure 8. The distribution is plotted by taking the active cell value as the mean, and 10 percent of mean as the standard deviation. All aspects of the Lognormal distribution are similar to those discussed above for the normal distribution, except for the fact that the values of samples of the lognormal distribution cannot be less than zero, and therefore, the minimum cut-off value is set to 0 instead of “–Infinity” as was the case for the normal distribution. This is because the equation for the lognormal distribution (see section 3.3) contains a natural logarithm term,  $\ln(x)$ , which goes to infinity for negative values of  $x$ . The following guidelines must be observed when specifying the inputs for the lognormal distribution:

- i) The inputs must be numeric,
- ii) The minimum cut-off value must be greater than 0, and
- iii) The standard deviation must be greater than 0.

As shown in Figure 9, the lognormal distribution can be truncated on either side.

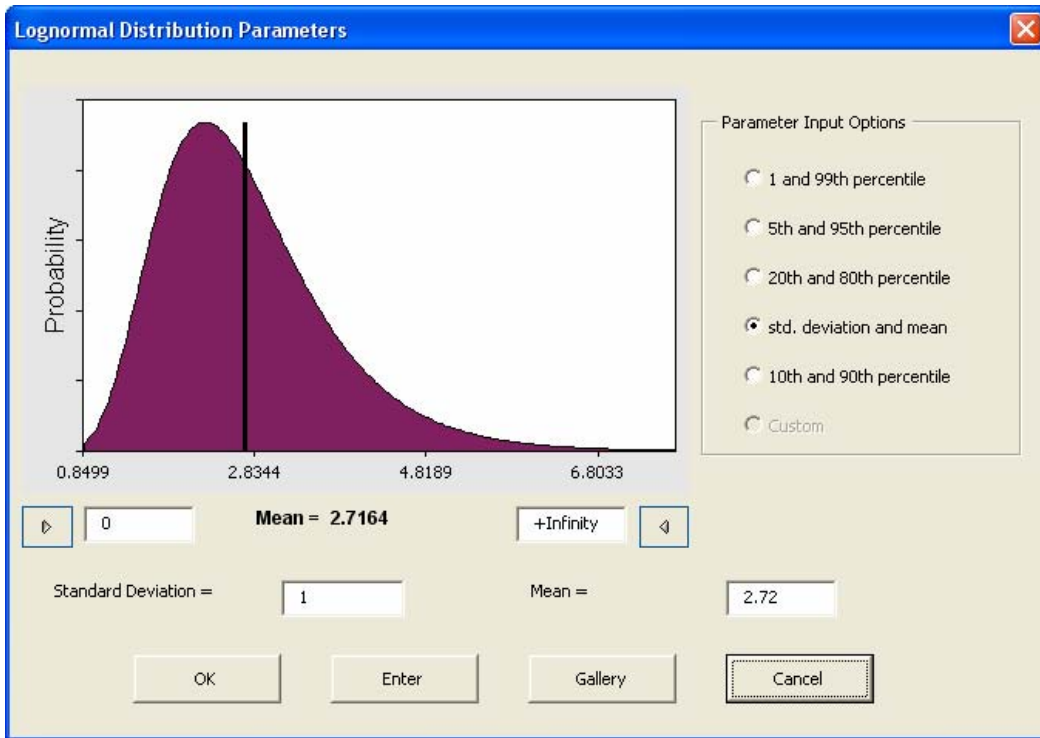


Figure 8. Lognormal input specification window

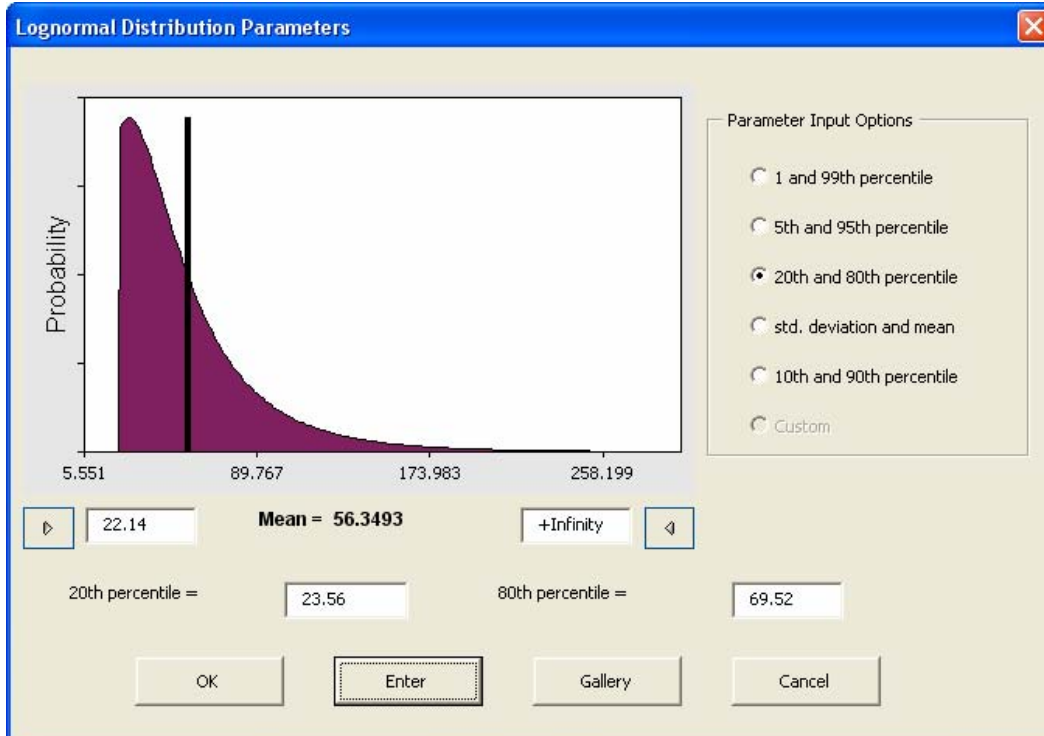
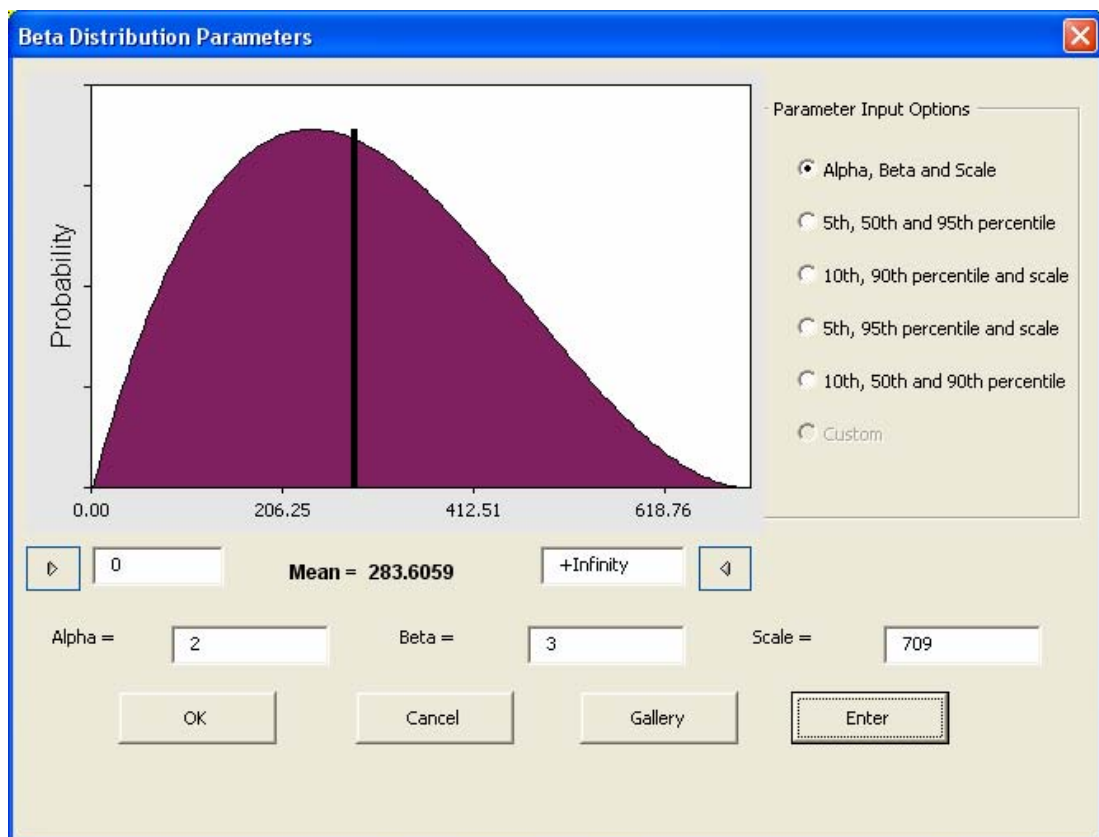


Figure 9. Lognormal distribution truncated on the left side

### 5.1.3 Beta Distribution

Figure 10 illustrates the input specification window for the beta distribution. Unlike the normal and lognormal, the beta distribution is a three-parameter distribution, i.e., it requires three parameters to define its shape. The default parameters are alpha, beta, and scale. The active cell value is taken as the default scale value, while 2 and 3 are taken as the default alpha and beta values, respectively. As with the normal distribution, the inputs for the beta distribution can be defined in terms of percentiles. Note that the minimum value of the beta distribution is zero and cannot be less than zero, as shown in Figure 10. Beta is a highly flexible distribution and can be used to simulate other distribution shapes based on the values of alpha and beta. When  $\alpha = \beta = 5$ , the shape is similar to the normal distribution. When  $\alpha = \beta = 1$ , the shape is similar to the uniform distribution. When  $\alpha = 1$  and  $\beta = 2$ , the shape is similar to the triangular distribution.



**Figure 10.** Input specification window for beta distribution

While alpha and beta define the shape of the beta distribution; the scale defines the range covered by the plot. Therefore, if alpha and beta were held constant and the scale was varied, the distribution shape would remain fixed and only the values in the x-axis would vary proportional to the scale. For example, if alpha = 2, beta = 5, and scale = 1, then the mean = 0.28, the 0<sup>th</sup> percentile = 0, and the 99<sup>th</sup> percentile = 0.87. If the scale was increased to 2, keeping alpha and beta the same, then the mean = 0.56, the 0<sup>th</sup> percentile = 0, and the 99<sup>th</sup> percentile = 1.74, while the distribution shape remains constant. The scale is essentially the maximum value of the distribution, assuming that the distribution is not truncated. The inputs for the beta distribution can be specified in one of three ways:

- 1) Alpha, beta, and scale
- 2) Three percentiles (10<sup>th</sup>, 50<sup>th</sup> and 90<sup>th</sup> percentiles or 5<sup>th</sup>, 50<sup>th</sup> and 90<sup>th</sup> percentile)
- 3) Two percentiles and scale (10<sup>th</sup>, 90<sup>th</sup> percentiles and scale or 5<sup>th</sup>, 95<sup>th</sup> percentiles and scale)

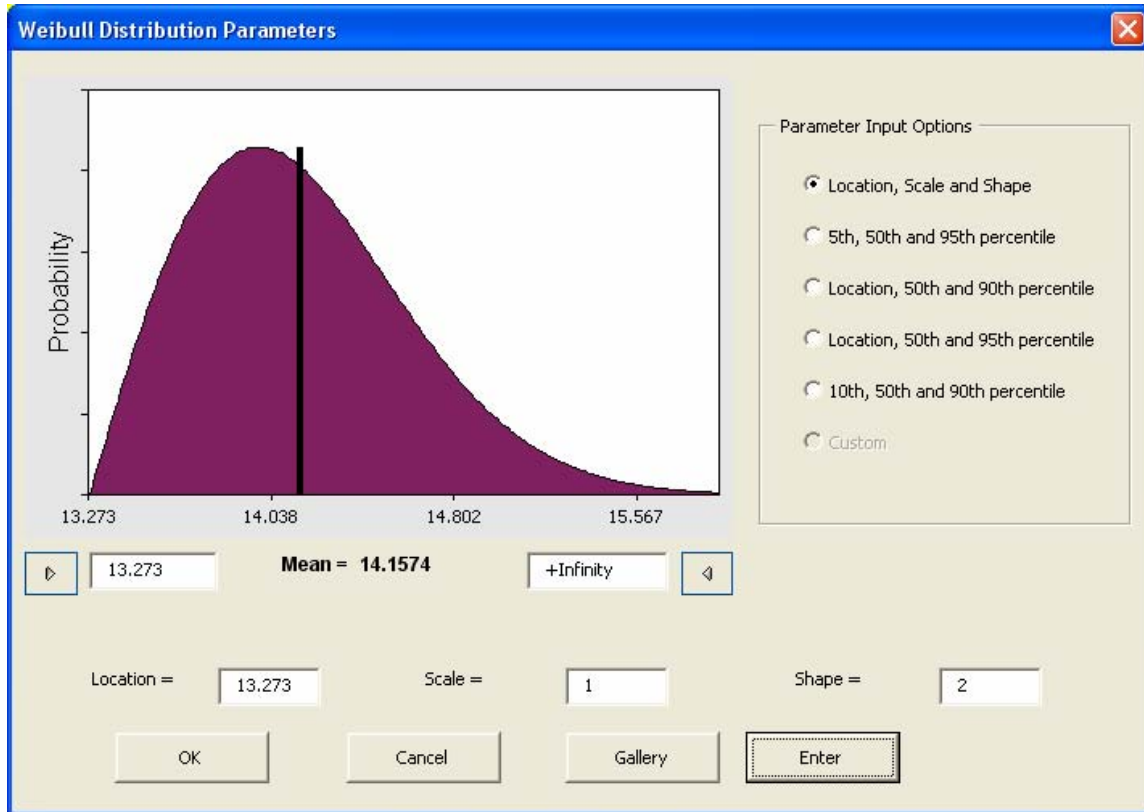
The distribution can be truncated on either side and the mean line would shift to either side depending on the level of truncation.

**Tips for proper input specification of the beta distribution:**

- a) Alpha > 0
- b) Beta > 0
- c) Scale > 0
- d) Minimum cut-off > 0
- e) When the input is specified in terms of 10<sup>th</sup> percentile, 90<sup>th</sup> percentile and Scale or 5<sup>th</sup> percentile, 95<sup>th</sup> percentile and scale, the value of scale must be greater than both percentile values.

### **5.1.4 Weibull Distribution**

The Weibull distribution is widely used in reliability and life data analysis. Figure 11 shows the input specification window for the Weibull distribution.

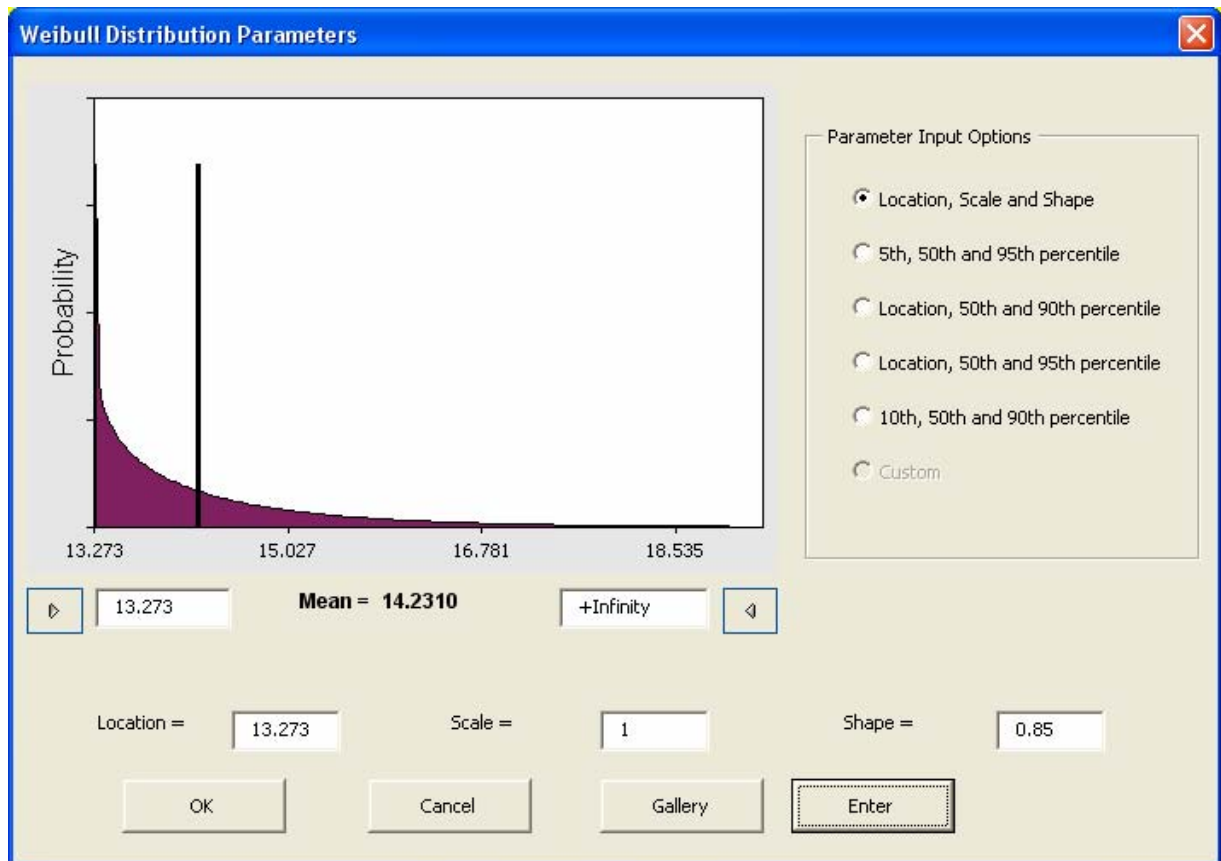


**Figure 11.** Input specification window for the Weibull distribution

As shown in Figure 11, the default input parameters for the Weibull distribution are the location, scale and shape. The location is the minimum value of the distribution (the 0<sup>th</sup> percentile). When you first select this distribution, the active cell value is taken as the location, which is also the minimum cut-off value of the distribution. The default values for the scale and shape are 1 and 2, respectively. The shape parameter alone defines the shape of the plot, while the scale parameter defines the range covered by the PDF and the location parameter defines the minimum value of the distribution. As was the case with the beta distribution, there are three types of input specification:

- 1) Location, Scale and Shape
- 2) Three percentiles (10<sup>th</sup>, 50<sup>th</sup> and 90<sup>th</sup> percentiles or 5<sup>th</sup>, 50<sup>th</sup> and 95<sup>th</sup> percentile)
- 3) Location and Two percentile (location, 10<sup>th</sup> and 90<sup>th</sup> percentiles or location, 5<sup>th</sup> and 95<sup>th</sup> percentiles)

When the value of the shape parameter is less than 1, the curve takes a concave shape, as shown in Figure 12, with  $f(x)$  tending to infinity as 'x' tends to the location value. Also note that there is a very long tail for the Weibull distribution. When the shape parameter = 1, the Weibull looks like an exponential distribution.



**Figure 12.** Weibull distribution for shape parameter less than 1

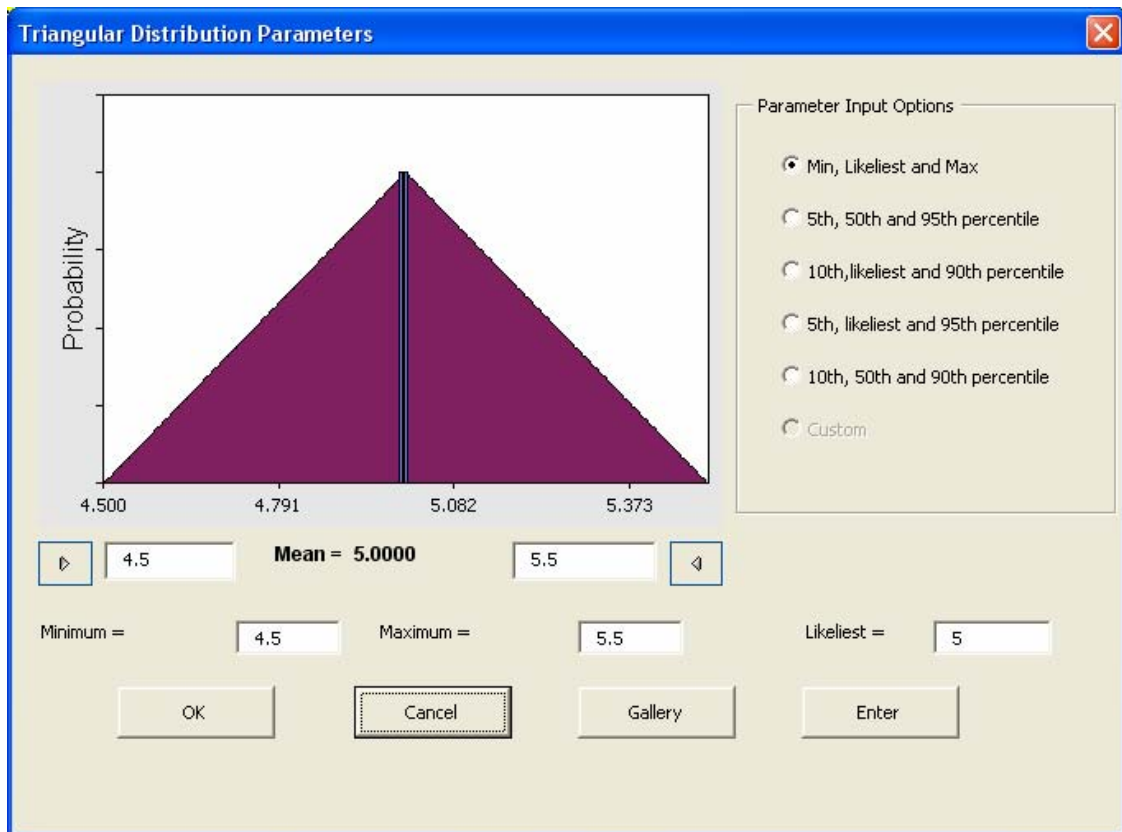
**Tips for proper input specification of the Weibull distribution:**

- a) Scale > 0
- b) Shape > 0
- c) When the input is specified in terms of 10<sup>th</sup> percentile, 90<sup>th</sup> percentile, and Location; or 5<sup>th</sup> percentile, 95<sup>th</sup> percentile, and Location, the value of Location must be less than both percentile values.



## 5.1.5 Triangular Distribution

Triangular distribution is usually used when there are insufficient data to use any other type of distribution but the minimum, maximum, and most likely values are known. Figure 13 shows the input parameter specification window for the Triangular distribution.



**Figure 13.** Input specification window for triangular distribution

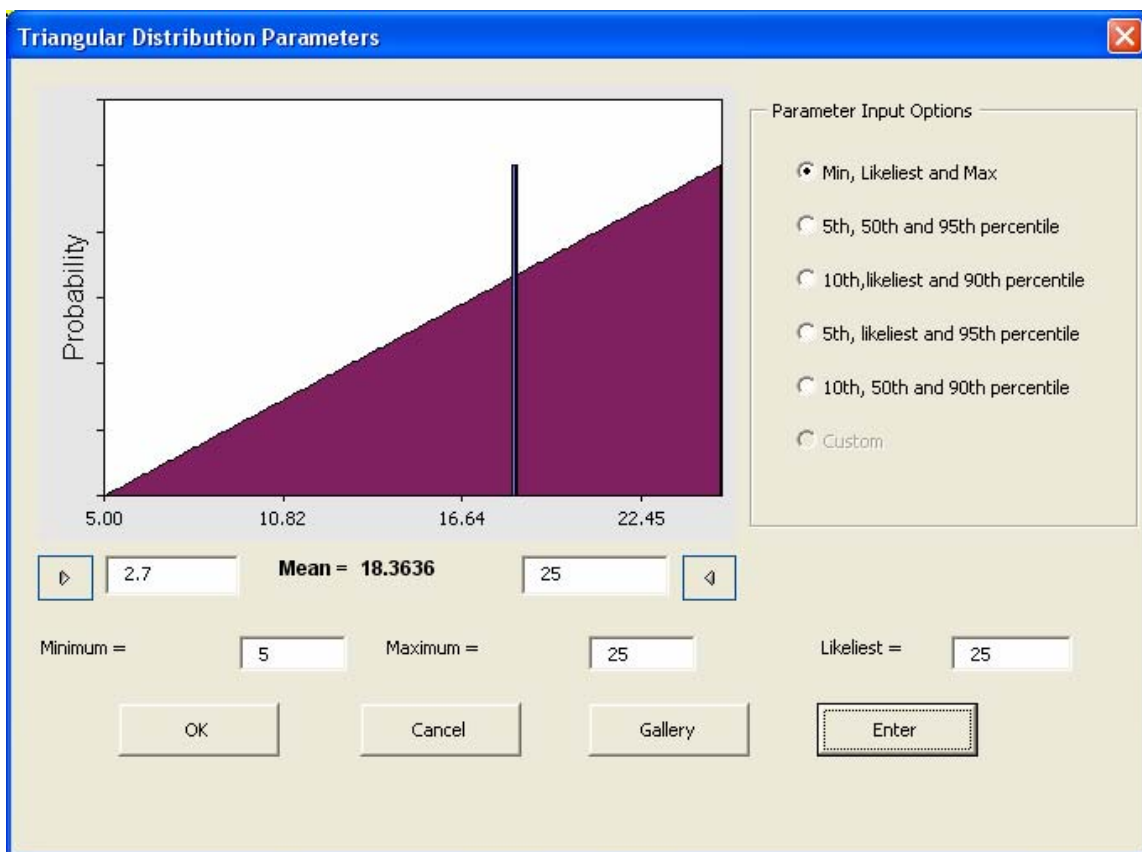
There are three ways to specify the input parameters for the triangular distribution:

- 1) Minimum, Likeliest, and Maximum
- 2) Three percentiles (10<sup>th</sup>, 50<sup>th</sup>, and 90<sup>th</sup> percentiles or 5<sup>th</sup>, 50<sup>th</sup>, and 90<sup>th</sup> percentile)
- 3) Likeliest and two percentiles (10<sup>th</sup> and 90<sup>th</sup> percentiles, or 5<sup>th</sup> and 95<sup>th</sup> percentiles)

Note that the minimum and maximum cut-off default values are equal to the minimum and maximum inputs specified for the distribution. If the minimum cut-off specified is lower than the

minimum input, then it will be ignored. If the minimum cut-off value specified is greater than the minimum input or the maximum cut-off value specified is lower than the maximum input, the distribution will be truncated at these values.

It is possible to have triangular distributions where the likeliest can be equal to the maximum or the minimum value, as shown in Figure 14.



**Figure 14.** Triangular distribution in which the likeliest value equal to the maximum value

**Tips for proper input specification of the triangular distribution:**

- a)  $\text{Minimum} \leq \text{Likeliest} \leq \text{Maximum}$
- b) When the inputs are specified in terms of percentiles, the specified values should fall between the minimum and maximum values.

### 5.1.6 Extreme Value Distribution

The extreme value distribution is usually used to describe the largest value of a response over a period of time. There are two forms of the extreme value distribution: one is based on the smallest extreme (skewed to the left) and the other is based on the largest extreme (skewed to the right). Figure 15 shows the input specification for the extreme value distribution of the first type. The distribution takes two standard inputs: mode and scale. The mode is the most likely value for the variable (the highest point on the probability distribution). The scale is proportional to the range of values covered by the distribution. Input parameters can also be specified in terms of percentiles. The distribution can be truncated on either side by specifying the minimum and maximum cut-off values.

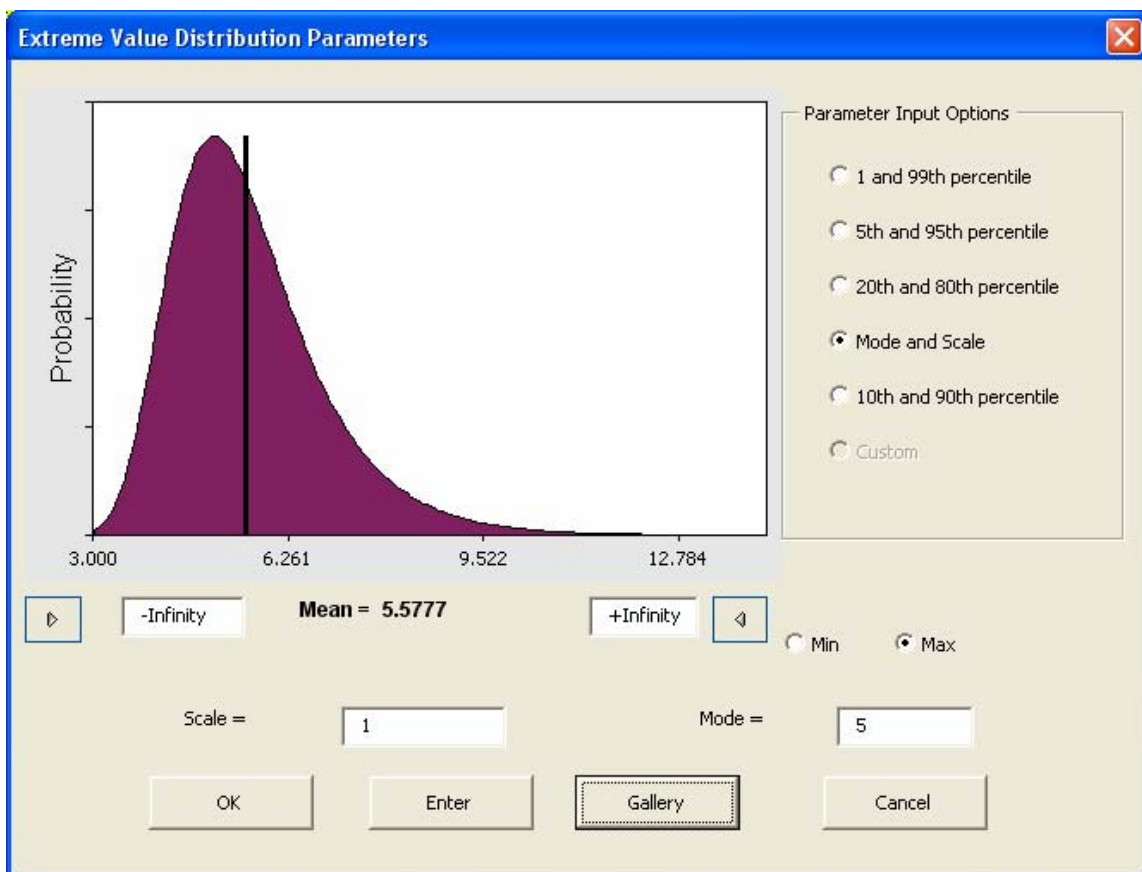


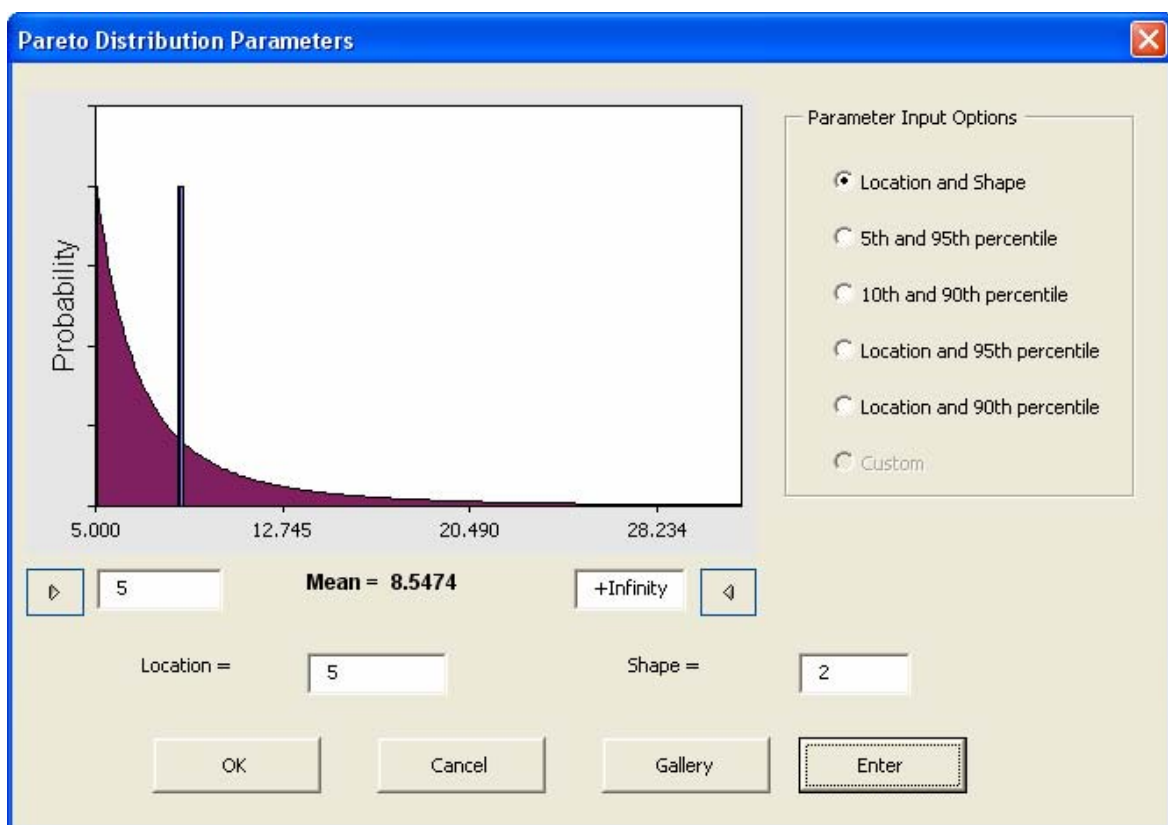
Figure 15. Input specification window for extreme value distribution (Type 1)

**Tips for proper input specification of the extreme value distribution:**

- a) Scale > 0
- b) When the input is specified in terms of mode and 90<sup>th</sup> or mode and 95<sup>th</sup> percentile, the value of mode must be lesser than the percentile value.

**5.1.7 Pareto Distribution**

The Pareto distribution is generally used to describe empirical phenomena like birth rate, income growth rate, etc. Figure 16 shows the input specification window for the Pareto distribution. Note that the Pareto distribution has a long tail to the right, which decreases as the shape parameter increases.



**Figure 16.** Input specification window for the Pareto distribution

The Pareto distribution features two standard parameters: location and shape. The location parameter is the lower bound for the distribution, while the shape parameter defines the distribution shape. As the shape parameter decreases, the concavity of the distribution increases, i.e., the curve becomes inwardly steeper. Inputs can also be specified in terms of:

- 1) Percentiles (5<sup>th</sup> and 95<sup>th</sup> percentiles or 10<sup>th</sup> and 95<sup>th</sup> percentiles)
- 2) Location and a percentile

**Tips for proper input specification of the Pareto distribution:**

- a) Location  $> 0$
- b) Shape  $> 0$
- c) Minimum cut-off value  $> 0$
- d) When the input is specified in terms of location and 90<sup>th</sup> or location and 95<sup>th</sup> percentile, the value of location must be less than the percentile value.

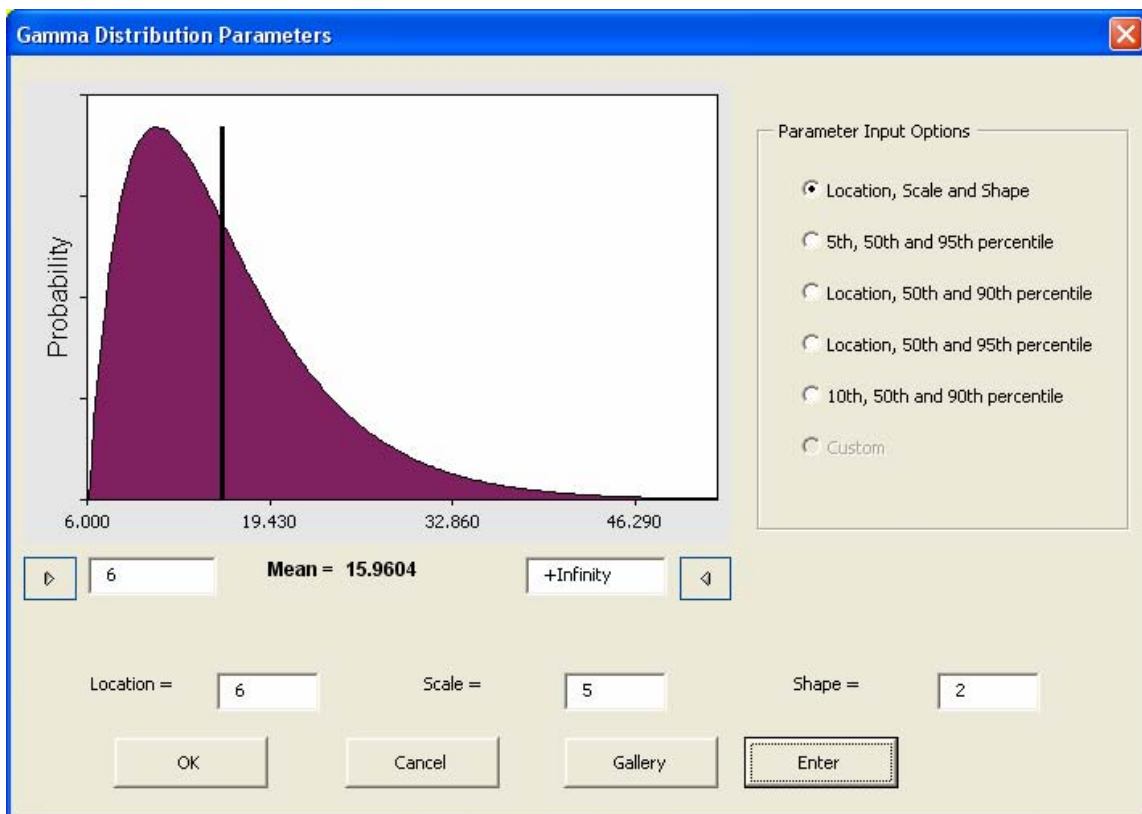
### **5.1.8 Gamma Distribution**

The Gamma distribution can be used to fit failure data. It occurs naturally as the time-to-first failure distribution for a system with standby exponentially distributed backups. Figure 17 shows the input specification window for this distribution. There are three standard parameters for the Gamma distribution specification: location, scale, and shape. The shape parameter alone defines the shape of the plot, while the scale parameter defines the range covered by the PDF and the location parameter defines the minimum value of the distribution (the lower bound of the distribution). When the shape parameter = 1, the gamma distribution reduces to the exponential distribution. The input parameters can also be specified as:

- 1) Three percentiles (10<sup>th</sup>, 50<sup>th</sup>, and 90<sup>th</sup> percentiles, or 5<sup>th</sup>, 50<sup>th</sup>, and 95<sup>th</sup> percentile)
- 2) Location and two percentile (10<sup>th</sup> and 90<sup>th</sup> percentiles, or 5<sup>th</sup> and 95<sup>th</sup> percentiles)

**Tips for proper input specification for the Gamma distribution:**

- a) Scale > 0
- b) Shape > 0
- c) When the input is specified in terms of 10<sup>th</sup> percentile, 90<sup>th</sup> percentile and Location or 5<sup>th</sup> percentile, 95<sup>th</sup> percentile and Location, the value of Location must be less than both percentile values.



**Figure 17.** Input specification window for the Gamma distribution

### 5.1.9 Logistic Distribution

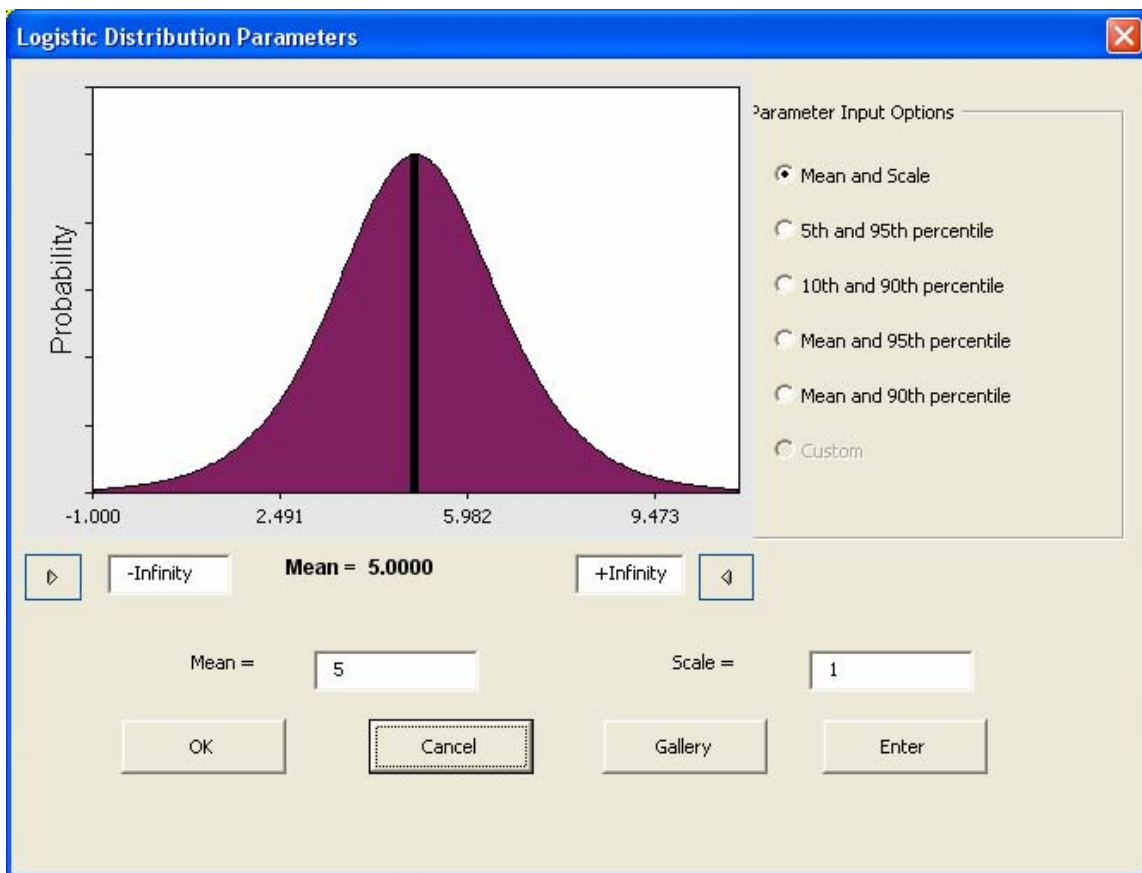
The logistic distribution is used to describe growth of population. Figure 18 shows the input specification window for the logistic distribution. The logistic distribution is specified by two standard parameters: mean and scale. The distribution is a symmetric distribution and hence mode = median = mean. Scale denotes the range of values covered by the distribution.

Inputs can also be specified in terms of:

- 1) Two percentiles (10<sup>th</sup> and 90<sup>th</sup> percentiles, or 5<sup>th</sup> and 95<sup>th</sup> percentiles)
- 2) Mean and 90<sup>th</sup> percentile or mean and 95<sup>th</sup> percentile

**Tips for proper input specification of the logistic distribution:**

- a) Scale > 0
- b) When the input is specified in terms of mean and 90<sup>th</sup> or mean and 95<sup>th</sup> percentile, value of mean must be less than the percentile value.



**Figure 18.** Input specification window for the logistic distribution

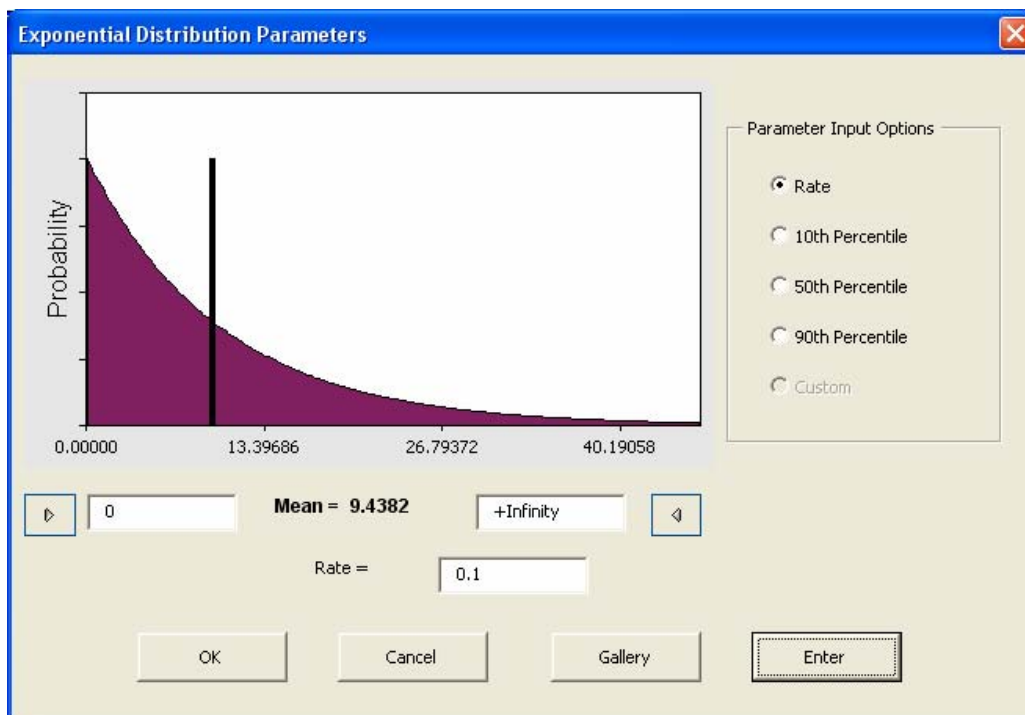
### 5.1.10 Exponential Distribution

The exponential distribution is used to depict events which occur at random, like the time between failures of equipment. Figure 19 illustrates the input specification window for the exponential distribution. The distribution can also be truncated by either specifying the truncation value at the minimum cut-off box, maximum cut-off box, or both. The samples are then chosen only from the shaded region.

The standard input for the exponential distribution is the rate. The active cell value is taken as the rate parameter to construct the distribution curve. Input can also be specified as 10<sup>th</sup>, 50<sup>th</sup>, or 90<sup>th</sup> percentile. The lower bound for the distribution = 0.

#### **Tips for proper input specification of the exponential distribution:**

- a) Rate > 0
- b) Minimum cut-off > 0



**Figure 19.** Input specification window for the exponential distribution

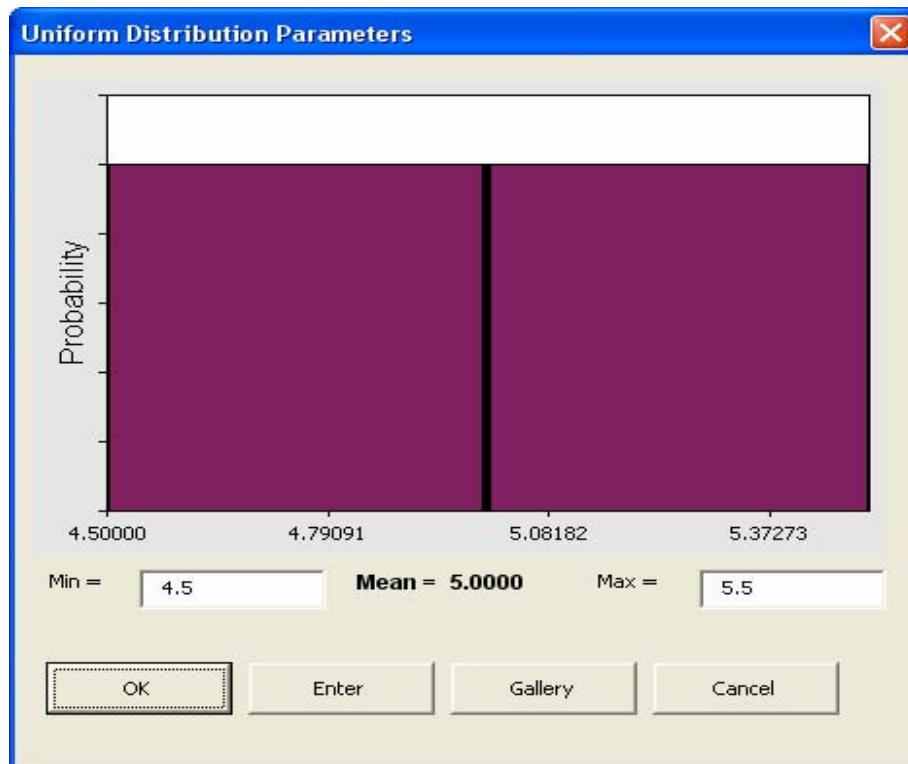


### 5.1.11 Uniform Distribution

The uniform distribution is used when there is equal probability of occurrence of an event between a minimum and maximum values. Figure 20 shows the input specification parameter window for this distribution.

**Tip for proper input specification of the uniform distribution:**

Minimum value < Maximum value



**Figure 20.** Input specification window for the uniform distribution

### **General guidelines to be followed during any distribution input specification**

Improper input values are met with appropriate error messages. Proper input requirements common to all distribution include the following:

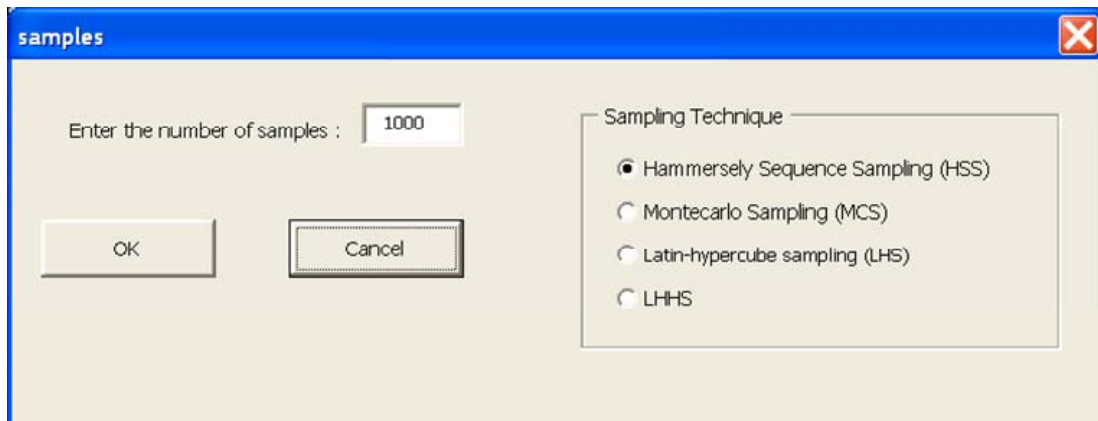
- a) Inputs must be numeric.
- b) When inputs are specified in terms of percentiles, input value for a less percentile must be less than input value for a greater percentile.
- c) Minimum cut-off value must be less than maximum cut-off value.

## **5.2 Sampling**

Once the distributions for all the uncertain parameters have been specified, the next step is to specify the sampling technique to be used and the number of samples required. When you click on “**Sampling**” in the stochastic simulation command bar, the window shown in Figure 21 appears. You can select from one of four sampling techniques:

- a) Hammersley Sequence Sampling [Default number of samples = 1000]
- b) Monte Carlo Sampling [Default number of samples = 4000]
- c) Latin Hypercube Sampling [Default number of samples = 2000]
- d) Latin Hypercube Hammersley Sampling [Default number of samples = 1000]

The sampling techniques have been explained in detail in section 4. MCS is the conventional sampling technique in many stochastic simulations. The new and efficient HSS sampling technique typically requires  $1/4^{\text{th}}$  the number of samples required by the MCS technique. Latin-hypercube sampling performs better than MCS but is not as efficient as HSS. Most of the time, LHHS performs better than HSS. However, unlike MCS or HSS, the performance measure for LHHS is not independent of number of variables or type of functionality used to compute the output distributions. When you select a sampling technique, the default number of samples (based on the assumption that number of uncertain variables are more than 100 and less than 500) required automatically appears in the corresponding textbox. You can change the number of samples according to your preference.



**Figure 21.** Window to specify the sampling technique and the number of samples

You can also retrieve the specified sampling technique and number of samples at any time during the stochastic simulation by clicking the “Sampling” button in the command bar.

### **5.3 Forecast Cells**

The next step is to select those variables whose values will be forecasted. GREET includes approximately 3,000 forecast variables. A special algorithm has been created to enable you to easily select the forecast variables for the pathways of interest through four simple steps:

1. Select the vehicle technologies.
2. Specify the transportation fuels.
3. Specify the well-to-wheels (WTW) simulations and/or well-to-pump (WTP) simulations.
4. Select the energy and emission forecast groups.

To begin, click on “**Forecast Cells**” in the command bar, to display the forecast window as shown in Figure 22. Step 1 of the forecast selection provides a list of vehicle types.

When you select a checkbox for a particular vehicle technology, e.g., “Conventional Spark Ignition,” the window shown in Figure 23 is displayed. This window contains three frames:

- a) Fuel type specification
- b) WTW simulation option and/or WTP simulations option
- c) Energy and emission groups

For any fuel type, the WTW and WTP boxes, and the energy and GHG forecasts are selected by default. If you do not select any fuel type before clicking “OK,” these default selections will be ignored when the stochastic simulation is executed.

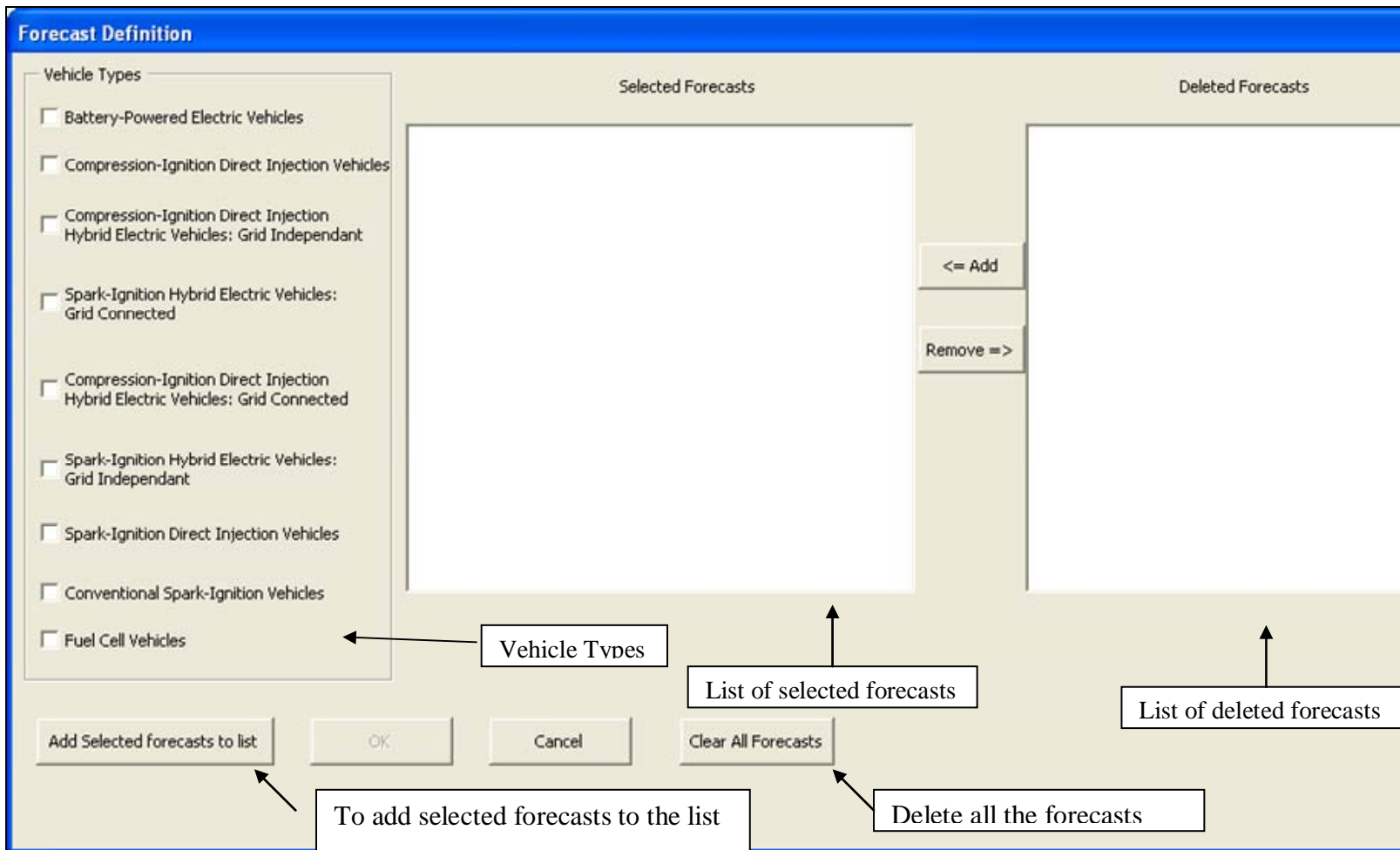
After making the necessary selections in this window, click “**OK**” and repeat the process for other vehicle technologies as needed.

Once you finish defining the forecasts, click on the button captioned “**add forecasts to list**” in the "Forecast Definition" window to add the defined forecasts to the “Selected” listbox (see Figure 22). This adds the selected forecasts to the “selected” listbox, enabling you to remove/add individual forecasts as needed using the “Remove =>” and “<= Add” buttons in that window. This process is shown in Figure 24.

Once the forecasts are added to the “Selected” listbox, individual forecasts can be moved back and forth to and from the “Deleted” listbox. When a vehicle technology is unchecked, the corresponding forecasts are automatically deleted from the list.

The naming convention for the forecasts is “Vehicle Technology – Transportation Fuel – WTW and/or WTP – Energy and Emission Forecast.” For example, “CIDI-DME-WTW-N<sub>2</sub>O” can be interpreted as the well-to-wheels N<sub>2</sub>O emission results for the CIDI vehicle fueled with DME. The forecasts listed in the list box titled “Selected” are the forecasts which would be predicted at the end of the stochastic simulation.

After completing the forecast selection process, click the “**OK**” button to confirm the list of selected forecasts.



**Figure 22.** Stage 1 of forecast definition: window listing all the vehicle types

**UserForm1** [Close]

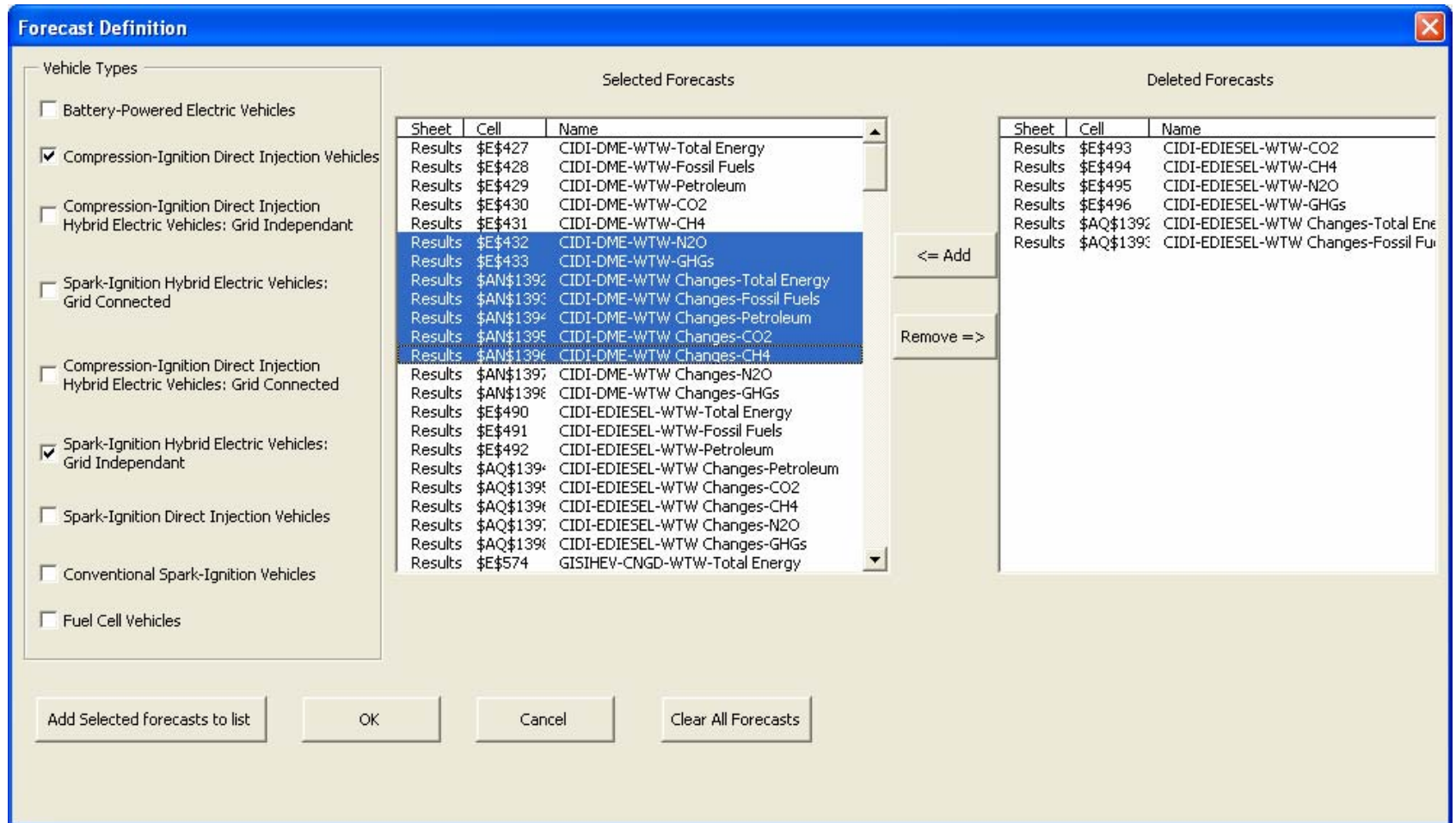
**Fuel Types**

- Re-Formulated / Conventional Gasoline
- California Re-Formulated Gasoline
- E10
- Compressed Natural Gas (Dedicated)
- Liquid Natural Gas (Dedicated)
- Liquefied Petroleum Gas (Dedicated)
- Gaseous Hydrogen
- Liquid Hydrogen
- M90 (Dedicated)
- E90 (Dedicated)
- E85 Flexible Fuel
- M85 Flexible Fuel
- Compressed Natural Gas (bi-fuel)

**Forecast Definition Options**

- WTW Forecast Cells
  - Energy and Emission for WTW forecast
    - Energy Forecast Cells
    - GHGs Forecast Cells
    - Criteria Pollutant (Total) Forecast Cells
    - Criteria Pollutant (Urban) Forecast Cells
- WTP Forecast Cells
  - Energy and Emission for WTP forecast
    - Energy Forecast Cells
    - GHGs Forecast Cells
    - Criteria Pollutant (Total) Forecast Cells
    - Criteria Pollutant (Urban) Forecast Cells

**Figure 23.** Forecast definition example for the “Conventional Spark Ignition” vehicle type



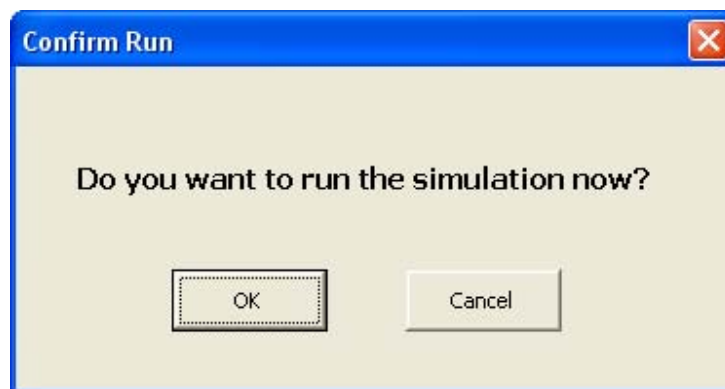
**Figure 24.** Removal and addition of individual forecasts

## 5.4 Delete Distributions

For any parametric assumption cell with a probability distribution, if you decide to just assign a point value to that cell, the probability distribution can be deleted by selecting the cell and clicking on the “Deleted Distribution” button. The input distribution is automatically deleted and the cell color turns from green to white.

## 5.5 Run Simulation

After all the required inputs and forecast selections have been finalized, the “Run Simulation” button is enabled to click to begin execution of the stochastic simulation. When the “Run Simulation” is clicked, you will be asked to confirm that the simulation should begin, as shown in Figure 25.



**Figure 25.** Confirmation window to begin simulation

After the simulation run is completed, the forecasts are exported to another Excel file and statistical values like the mean, standard deviation, and 0<sup>th</sup> to 100<sup>th</sup> percentile are calculated automatically for each forecast as shown in Figure 26. You can save the output file in the directory of your choice.



### Nomenclature for Figure 26

Column A Label	Description
<b>Mean</b>	Value of mean for the forecast.
<b>S.D.</b>	Value of standard deviation.
<b>P0</b>	Value of 0 <sup>th</sup> percentile. The value means that there is a probability of zero that actual values would be equal to or below the P0 value.
<b>P10</b>	Value of 10 <sup>th</sup> percentile. The value means that there is a probability of 10% that actual values would be equal to or below the P10 value.
<b>P20:</b>	Value of 20 <sup>th</sup> percentile. The value means that there is a probability of 20% that actual values would be equal to or below the P20 value.
<b>P30</b>	Value of 30 <sup>th</sup> percentile. The value means that there is a probability of 30% that actual values would be equal to or below the P30 value.
<b>P40</b>	Value of 40 <sup>th</sup> percentile. The value means that there is a probability of 40% that actual values would be equal to or below the P40 value.
<b>P50</b>	Value of 50 <sup>th</sup> percentile. The value means that there is a probability of 50% that actual values would be equal to or below the P50 value.
<b>P60</b>	Value of 60 <sup>th</sup> percentile. The value means that there is a probability of 60% that actual values would be equal to or below the P60 value.
<b>P70</b>	Value of 70 <sup>th</sup> percentile. The value means that there is a probability of 70% that actual values would be equal to or below the P70 value.
<b>P80</b>	Value of 80 <sup>th</sup> percentile. The value means that there is a probability of 80% that actual values would be equal to or below the P80 value.
<b>P90</b>	Value of 90 <sup>th</sup> percentile. The value means that there is a probability of 90% that actual values would be equal to or below the P90 value.
<b>P100</b>	Value of 100 <sup>th</sup> percentile. The value means that there is a probability of 100% that actual values would be equal to or below the P100 value.

*Note that it may take several minutes to more than an hour to finish a particular stochastic simulation run depending on many factors, such as the number of forecast cells selected, the number of samples selected, and the hardware configuration of your computer.*

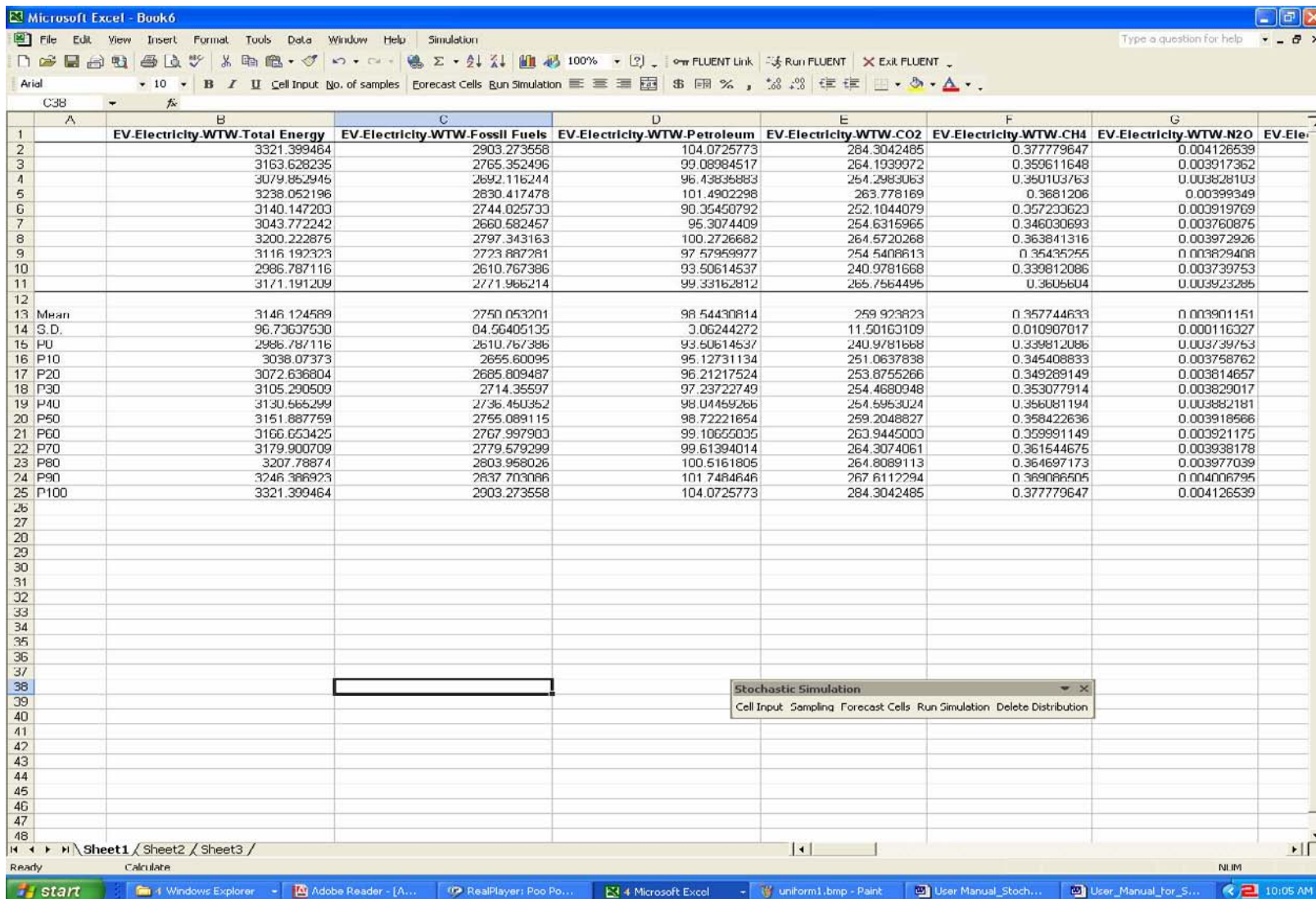


Figure 26. Format for forecast values listing in the output file

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