The Energy Budgets for the Eye and Eye Wall of a Numerically Simulated Tropical Cyclone

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Abstract

Energy budgets are analyzed for a tropical cyclone simulated previously in a quadruply nested mesh model (Kurihara and Bender, 1982). It will be shown that the eddy kinetic energy within the eye is comparable in magnitude to that of the mean kinetic energy. It is supplied by import from the eye wall regions as well as by the conversion from total potential energy. At the same time it is converted to the kinetic energy of the mean flow and also lost by the dissipation. The influx of mean kinetic energy from the outer radii to the eye wall region and the export of potential energy both to the outer radii and to the eye region play important roles in the energetics of the eye wall region. Many obtained features agree well with those of a coarser resolution model (Tuleya and Kurihara, 1975) in which the eye of the vortex could not be resolved. This suggests that the eye structure has little impact on the energetics in the eye wall and outer regions of a tropical cyclone.

1. Introduction

The purpose of this article is to supplement the previous contribution by Kurihara and Bender (1982, hereafter referred to as KB) in which various analysis results of the eye of a numerically simulated tropical cyclone were presented.

The numerical model used in the above simulation experiment was a quadruply nested mesh version of the Geophysical Fluid Dynamics Laboratory hurricane model. Finest grid resolution of the model was about 5 km. A compact and persistent region of subsidence within the storm center (defined as the eye) was maintained during the course of the integration, surrounded by a region of strong upward motion (defined as the eye wall). When the storm reached its greatest intensity at hour 43.6, the minimum surface pressure and the maximum surface wind were 924 mb and 76 m s^{-1}, respectively. A further description of the model as well as that of the time integration of the model can be found in Sections 2 and 3 of KB.

In KB, the azimuthal means as well as the asymmetry of the eye and eye wall structure were emphasized. The analysis of the budgets of heat, water vapor, radial wind and angular momentum revealed the role of the eddies and the mean radial-vertical circulation in the maintenance of the hurricane's mean structure. In the present paper, the discussion is continued by presentation of the budgets of the kinetic energy of the mean flow, the kinetic energy of the eddy motion, the total potential energy, and the latent energy.

The energy budget of a tropical cyclone in a three-dimensional model was previously investigated by Tuleya and Kurihara (1975, to be referred to as TK). Their model physics was similar to the present model but the grid resolution was coarser; their grid distance of 20 km in the center area prevented proper resolution of the eye. Also the storm size in their case was somewhat larger than in the present experiment. For example, the radius of maximum wind in their simulation was about 60 km compared to about 20 km at the mature stage of the present storm. Results from the above and the present model will be compared in order to see the impact of the presence of the eye on the energy budget of a model tropical storm. Although discussion in the present paper is restricted to model results, comparison with budgets obtained using real data can be found in TK.

2. Energy Budgets

The equations for the budgets of kinetic energy, total potential energy and latent energy were obtained using a cylindrical, sigma coordinate system (see TK, Section 3). The kinetic energy was divided into the kinetic energy of the azimuthally averaged flow or the mean kinetic energy K_M, and the eddy kinetic energy K_E defined as the difference between the total and the mean kinetic energy. The budget components for each of the four forms of energy were computed at each sigma surface and then vertically integrated. The azimuthal means of the various quantities used in the budget analysis were obtained at different radii from the storm center by first using a bilinear formula to interpolate the grid values onto 40 equally spaced points along a circle and then averaging these values together. With an azimuthal mean of a quantity denoted by an overbar and a deviation from it indicated by a prime, all product terms of the form $\overline{x'y'}$ were calculated from $\overline{xy}-\bar{x}\bar{y}$ in the manner described in KB, Section 6. The energy budgets were computed at five time levels within a period of 10.5 minutes after hour 45.4 of the time integration and were then averaged; this is the same procedure used in KB. It should be noted that the computed terms in the energy budgets did not balance exactly. This is probably due to inaccuracies introduced by the interpolations necessary to perform this analysis, especially at smaller radii. Also, although the storm was in approximately steady state at this time, short period fluctuations did exist which could not necessarily be averaged out during the small time interval over which the budgets were computed.

a. Mean kinetic energy

If we use the approximation that the surface pressure is a function of radius and time only, the equation for the mean kinetic energy K_M can be written as:

$$\frac{\partial}{\partial t}\{K_M\}=MA-ME+MC+MD \qquad (1)$$

where { } denotes the azimuthal average of a vertically integrated quantity and

MA : the flux convergence of K_M
ME : the conversion from K_M to K_E
MC : the generation of K_M from total potential energy
MD : the dissipation of K_M.

The actual forms of the various terms are defined explicitly in TK.

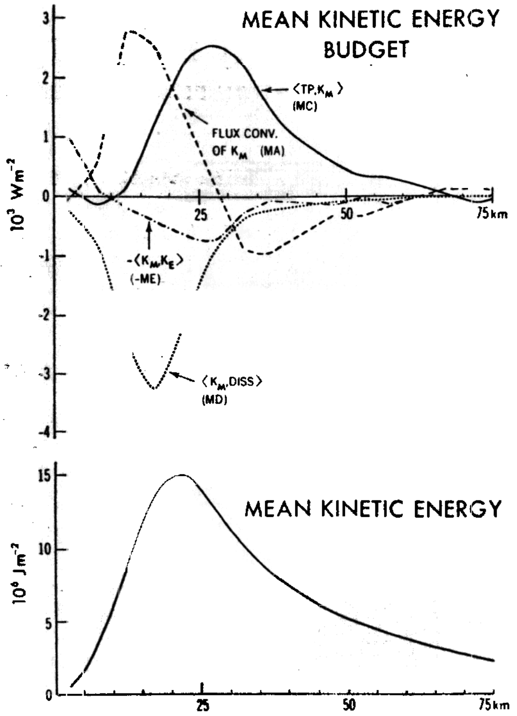

Fig. 1 Radial distribution of ~10 min average (at ~45.4 h) of the vertically integrated mean kinetic energy and mean kinetic energy budget components. Values are plotted for the changes due to the flux convergence (the term MA in (1), dashed line), conversion from K_E ($-ME$, dash-dotted line), conversion from TP (MC, solid line), and dissipation (MD, dotted line).

The radial distribution of the mean kinetic energy budget together with that of the mean kinetic energy is presented in Fig. 1. It was shown in KB that the eye, or the region of mean sinking motion, extended from the storm center outwards to about 7 km, and that the eye wall region surrounding it reached to about 35 km. The maximum of $\{K_M\}$ is found in the eye wall at about 22 km. Fig. 1 indicates that the mean kinetic energy is generated from the conversion of total potential energy, or the work done by mean pressure gradient force $-\bar{u}(\partial\bar{\phi}/\partial r)$, for a large extent of the tropical cyclone. However, at around 8 km radius from the center, this term MC becomes slightly negative due to radial countergradient outflow at the low levels from the eye to the inner eye wall region (Fig. 13, KB). In the region outside of the eye wall, the production term MC is largely offset by the export of K_M to the eye wall. In the eye wall, conversion from K_M to K_E takes place; this is an important source of K_E as mentioned later. At smaller radii in the eye wall, the flux convergence of K_M is the major source term balancing the loss of K_M due to dissipation. Qualitatively, the results described above agree well with those of TK. Note that the eye was not a persistent feature in TK. As shown in Fig. 1, in the eye of the present model tropical cyclone the conversion of K_E to K_M was strongly positive. This indicates that the eddies are supplying their kinetic energy to the mean flow in this region of the storm to maintain K_M against the effect of frictional dissipation. The role of the eddy motion in the maintenance of the mean structure within the eye was also extensively analyzed in KB.

b. Eddy kinetic energy

The equation used to compute the eddy kinetic energy budget can be written as

$$\frac{\partial}{\partial t}\{K_E\}=EA+ME+EC+ED \qquad (2)$$

where

EA : the flux convergence of K_E
ME : the conversion between K_M and K_E
EC : the generation of K_E from total potential energy
ED : the dissipation of K_E

The results are summarized in Fig. 2, with the eddy kinetic energy distribution also plotted, as a function of radius. The eddy kinetic energy, vertically integrated and then azimuthally averaged, was relatively large near the outer edge of the eye wall region, due to large asymmetric outflow at the upper levels. The eddy kinetic energy increases sharply inward from the middle of the eye wall to the eye area. This agrees well with the analysis by KB (Figs. 11 through 18 of KB) in which significant eddy activity within the eye was shown. This increase, however, is in strong contrast to the previous simulation by TK in which the eye could not be resolved and the eddy kinetic energy decreased monotonically to the center from the outer edge of the eye wall. The eddy kinetic energy in the eye in the present experiment was comparable in magnitude to the value of the mean kinetic energy. Elsewhere it was approximately one twentieth to one thirtieth of the mean kinetic energy.

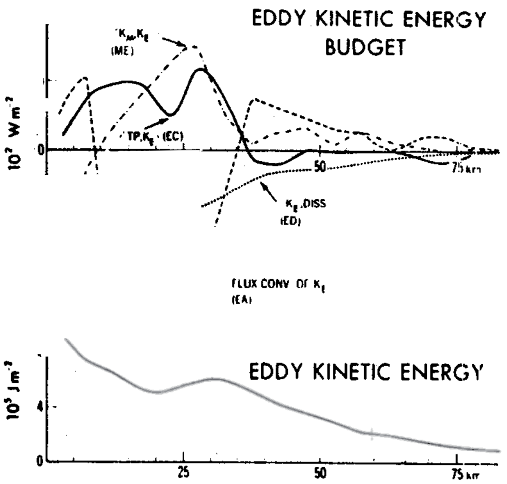

Fig. 2 Radial distribution of ~10 min average (at ~45.4 h) of the vertically integrated eddy kinetic energy and eddy kinetic energy budget components. Values are plotted for the changes due to the flux convergence (the term EA in (2), dashed line), conversion from K_M (ME, dash-dotted line), conversion from TP (EC, solid line), and dissipation (ED, dotted line).

In the distribution of the energy budget components, eddy kinetic energy is shown to be imported into the eye from the eye wall region. This transport is made by the mean as well as by the eddy motions, although the flux convergence by the eddy motion appears to dominate in both the eye and the eye wall. This positive contribution of the term EA in the eye along with EC, *i.e.*, the generation of K_E from the total potential energy, serves to balance the eddy energy lost by the conversion to the mean flow, ME, and by the dissipation, ED. Within the eye wall region the conversions from mean kinetic energy and total potential energy, namely the terms ME and EC, are almost equal in importance as a source of K_E to balance the eddy kinetic energy exported to the eye and the outer radii and lost to dissipation. The abovementioned term EC represents the combined effect of the release of total potential energy TP through the overturning process $(-\overline{\omega'\alpha'})$ and the radial redistribution due to the pressure work $-\partial(r\overline{u'\phi'})/(r\partial r))$. In the present case, it is found that $-\overline{\omega'\alpha'}$ is greater than $1.5\times10^2\,\mathrm{Wm}^{-2}$ between 10 km and 35 km radii, *i.e.*, within the eye wall, with a single maximum of $6.4\times10^2\,\mathrm{Wm}^2$ located at the 23 km radius. The release of TP was hinted in KB from the correlation between the temperature and vertical motion fields within the eye wall (Fig. 9 of KB).

The two maxima in the distribution of *EC* in Fig. 2 apparently result from the contribution of pressure work.

c. *Total potential energy*

The equation for the total potential energy *TP* derived from the 1st law of thermodynamics becomes:

$$\frac{\partial}{\partial t}\{TP\} = TA + TB + TC - TD + TE + TF + LT \qquad (3)$$

where

- *TA* : the flux convergence of internal energy
- *TB* : the flux convergence of potential energy
- *TC* : the flux convergence of pressure work
- *TD* : generation of kinetic energy from total potential energy
- *TE* : horizontal diffusion of *TP*
- *TF* : vertical diffusion of *TP* (surface heat flux)
- *LT* : gain due to release of latent energy.

The radial distribution of the budget of total potential energy is given in Fig. 3. The changes of *TP* through conversion to kinetic energy, $-TD$, as well as through both horizontal and vertical diffusion, *TE* and *TF*, were negligible compared to the other terms in the budget and were not plotted. Consistent with results obtained by TK, export of potential energy from the eye wall region is compensated by the pressure work, latent heat release and import of internal energy. Within the eye, the term *TB* was difficult to accurately obtain due to a computational problem related to the large magnitude of the geopotential at the upper levels. However, the sign of this term, which indicates an import of potential energy, seems reliable; this term counterbalances the negative contributions of the other terms in the potential energy budget. In Fig. 3, the flux term was not partitioned into the transports by the mean flow and eddies. It was shown in KB that the warming effect due to the mean flow in the eye was largely compensated by the cooling effect due to the horizontal eddy motion.

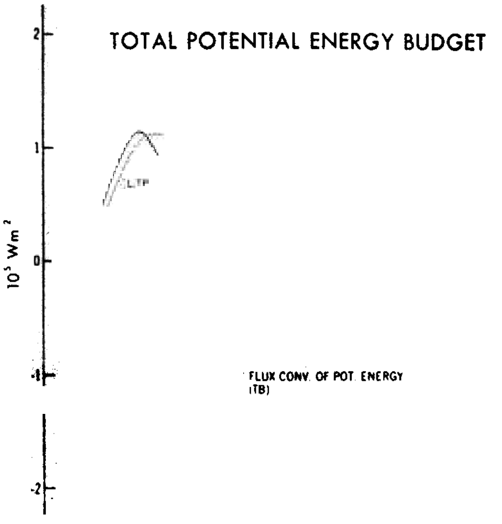

Fig. 3 Radial distribution of ~10 min average (at ~45.4 h) of the total potential energy budget components. Values are plotted for the changes due to the flux convergence of internal energy (the term *TA* in (3), dotted line), release of latent energy (*LT*, solid line), pressure work (*TC*, dash-dotted line), and flux convergence of potential energy (*TB*, dashed line). Other quantities are too small to be plotted.

d. *Latent energy*

Fig. 4 summarizes the distribution of the latent energy budget. The equation for the tendency of latent energy *LR* is written as:

$$\frac{\partial}{\partial t}\{LR\} = LA_M + LA_E + LB + LC - LT. \qquad (4)$$

where

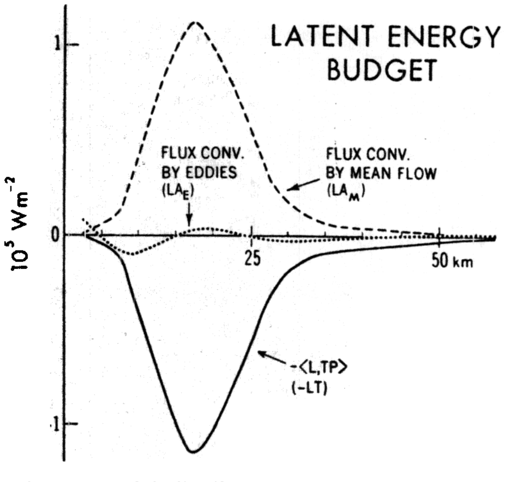

Fig. 4 Radial distribution of ~10 min average (at ~45.4 h) of the latent energy budget components. Values are plotted for the changes due to the flux convergence by the mean flow (the term LA_M in (4), dashed line), flux convergence by eddies (LA_E, dotted line), and the condensation ($-LT$, solid line). Evaporation and horizontal diffusion are too small to be plotted.

LA_M : the flux convergence of latent heat by the mean flow
LA_E : the flux convergence of latent heat by the eddy motion
LB : horizontal diffusion of latent energy
LC : vertical diffusion of LR (evaporation)
LT : release of latent energy through condensation.

Although the evaporation is essential for the moisture balance in the boundary layer and hence for the evolution of hurricanes, its contribution to the tendency of the vertically integrated latent energy, *i.e.*, the term LC, is small compared to the other budget components and was neglected in Fig. 4 along with the term LB. We find from the figure that the large loss of moisture due to condensation processes, represented by $-LT$, was balanced by the import of latent energy by the mean flow LA_M. The tendency due to the eddy term LA_E is consistent with the distributions of the water vapor budget presented in KB, which showed an import of moisture by the eddies from the eye wall region to the eye.

3. Summary

Energy budgets were presented for a tropical cyclone simulated numerically in a nested mesh model with finest grid resolution of about 5 km. The improved grid resolution in comparison to earlier experiments by TK enabled resolvability of the eye region. The eddy kinetic energy within the eye was found to be comparable in magnitude to the mean kinetic energy. It was shown that the eddy kinetic energy and total potential energy were imported into the eye from the eye wall. Within the eye, the eddies served as an important source to maintain the kinetic energy for the mean flow, while the mean flow supplied kinetic energy to the eddies throughout most of the storm domain. In the eye wall and at the outer radii of the storm, the analysis results from the present experiment agreed well with those obtained by TK, suggesting that the existence of eye has little impact on the energetics of a storm as a whole. Also, our analysis suggests that the existence of distinctly different energetic regions, one within the eye and the other within the eye wall. It should be noted however that the model storm in both experiments evolved without the influence of a basic flow. Although these circumstances simplified the analysis of the results, a more realistic environmental condition cou'd have a significant influence on the overall energy budgets of the tropical cyclone.

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数値的にシミュレートした熱帯低気圧の眼と眼の壁におけるエネルギー収支

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4重格子数値モデルでシミュレートした熱帯低気圧（栗原・Bender, 1982）についてエネルギー収支を解析した。眼の中では，擾乱の運動エネルギーが平均流の運動エネルギーと同程度であること，それが眼の壁からの輸入と全位置エネルギーからの変換で補給されていること，そして一般流の運動エネルギーへの変換と粘性散逸で消耗されていることが分かった。眼の壁のエネルギー収支で重要な役割をもつのは，眼の壁の外からの運動エネルギーの流入と，位置エネルギーの外域および眼の領域への流出である。得られた解析結果は，多くの点で，分解能の粗いモデル（Tuleya・栗原, 1975）で得られた特色とよく一致する。このことは，眼の構成は眼の壁とその外域のエネルギー収支にはあまり影響を及ぼさないことを示唆している。