TRIAXIAL STRESS ANALYSIS MODEL (TRIAX1)

Theory and User's Manual

By

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February 1993
TR93-3

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1. Introduction

The triaxial stress analysis windows application program (TRIAX1) has been jointly developed by Maurer Engineering Inc. as part of the DEA-44 project to "Develop and Evaluate Horizontal Well Technology" and the DEA-67 project to "Develop and Evaluate Slim-Hole and Coiled-Tubing Technology." This program, coded in Visual Basic 1.0, is written for use with IBM or IBM compatible computers and must run with Microsoft Windows 3.0 or later version.

1.1 MODEL DESCRIPTION

The program calculates the limits of burst and collapse pressure and equivalent stress for the pipe body. Three pressure limit models are evaluated by the program: 1) triaxial, 2) biaxial, and 3) API. The program also performs triaxial stress sensitivity analysis for the factors of internal and external pressure, dogleg, and D/t ratio.

Axial load values from DDRAG (Torque and Drag Model) and internal and external pressure values from HYDMOD (Hydraulics Model) can be the input of this program to examine equivalent stress and pressure limits for a specific point in the drill string or coiled-tubing.

1.2 COPYRIGHT

Participants in DEA-44/67 can provide data output from this copyrighted program to third parties and can duplicate the program and manual for their in-house use, but will not give copies of the program or manual to third parties.

1.3 DISCLAIMER

No warranty or representation is expressed or implied, with respect to these programs or documentation, including their quality, performance, merchantability, or fitness for a particular purpose.

2. Theory and Equation

An element of material subjected to stress σ_x, σ_y, and σ_g in three perpendicular directions is said to be in a state of triaxial stress.

Tubes subjected to axial load and pressure (external and/or internal pressure) are in a triaxial stress state (Figure 2-1).

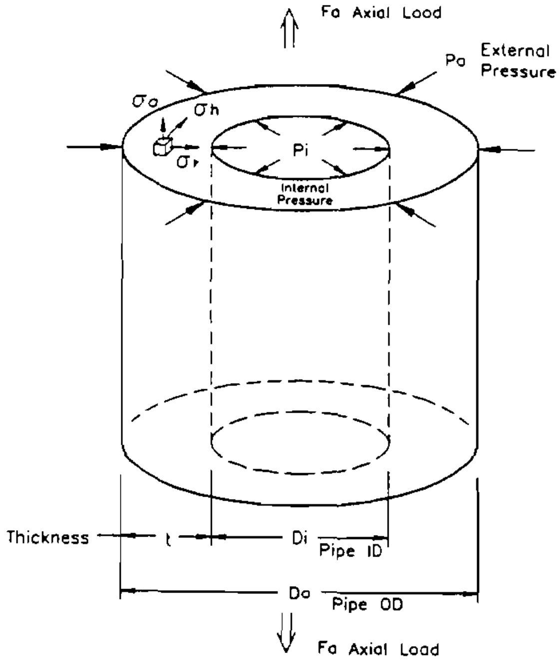

Figure 2-1. Triaxial Stress State

The generally accepted relationship for the effect of axial stress on collapse or burst is based on the distortion energy theory. A closed triaxial design procedure has been developed by using Von Mises's and Lame's equations. This present model does not consider torsional effects.

2.1 TRIAXIAL EQUATIONS

Let

E	=	Elastic modulus (psi)
D_o	=	Pipe OD (in.)
D_i	=	Pipe ID (in.)
r_o	=	Pipe outside radius (in.)
r_i	=	Pipe inside radius (in.)

σ_s	=	Yield stress (psi)
F_a	=	Axial force (lb)
DL	=	Dogleg (°/100 ft)
S_F	=	Safety factor
P_i	=	Internal pressure (psi)
P_o	=	External pressure (psi)

The pipe thickness is

$$t = 0.5 \cdot (D_o - D_i) \tag{2-1}$$

The cross area of pipe wall is

$$A = \pi \cdot (D_o^2 - D_i^2)/4 \tag{2-2}$$

The average axial stress is

$$\sigma_a = F_a / A \tag{2-3}$$

The maximum bending stress caused by the dogleg is

$$\sigma_{DL} = (\pi \cdot E \cdot DL \cdot D_o)/432000 \tag{2-4}$$

so that the minimum axial stress becomes

$$\sigma_{amin} = \sigma_a - \sigma_{DL} \tag{2-5}$$

and the maximum axial stress is

$$\sigma_{amax} = \sigma_a + \sigma_{DL} \tag{2-6}$$

The allowable maximum axial stress is

$$\sigma_{S-SF} = \sigma_s / SF \tag{2-7}$$

and this allowable maximum axial stress is based on zero pressure and zero bending stress. After converting above stress to force, the allowable maximum axial force is

$$F_{a-SF} = \sigma_{S-SF} \cdot A \tag{2-8}$$

According to Lame's equation for a thick tube, the hoop stress σ_h and the radial stress σ_r exerted by the internal and external pressure at the cylinder at radius equals r

$$\sigma_r = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (P_i - P_o)}{r^2 (r_o^2 - r_i^2)} \qquad (2\text{-}9)$$

and

$$\sigma_h = \frac{r_i^2 P_i - r_o^2 P_o}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (P_i - P_o)}{r^2 (r_o^2 - r_i^2)} \qquad (2\text{-}10)$$

For most cases, the maximum equivalent stress is at the pipe inside surface. Therefore, the equation can simplified by letting $r = r_i$ and rewriting the equation in pressure and diameter terms so the above equations become

$$\sigma_r = -P_i \qquad (2\text{-}11)$$

and

$$\sigma_h = \left[\frac{d_o^2 + d_i^2}{d_o^2 - d_i^2}\right] P_i - \left[\frac{2 d_o^2}{d_o^2 - d_i^2}\right] P_o \qquad (2\text{-}12)$$

let $C = \frac{d_o^2}{2 \cdot t \cdot (d_o - t)}$ and Eq. 2-12 becomes

$$\sigma_h = (C - 1)\, P_i - C \cdot P_o \qquad (2\text{-}13)$$

Von Mises's equation is

$$2\,\sigma_V^2 = \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \qquad (2\text{-}14)$$

where σ_1, σ_2, and σ_3 are three principal stresses, and σ_V is the equivalent stress according to the three principal stresses. Because the stresses σ_r, σ_h, and σ_a are three principal stresses they can be inserted into Von Mises's equation. If the bending stress is disregarded, Eq. 2-14 becomes

$$2\,\sigma_V = \left[(\sigma_a - \sigma_h)^2 + (\sigma_h - \sigma_r)^2 + (\sigma_r - \sigma_a)^2\right] \qquad (2\text{-}15)$$

Combine Eqs. 2-11 and 2-13 into Eq. 2-15 and rearrange the terms in the equation.

$$P_o = \frac{-\sigma_a + 2CP_i - P_i \pm \sqrt{-3\sigma_a^2 - 6\sigma_a P_i - 3P_i^2 + 4\sigma_V^2}}{2C} \quad (2\text{-}16)$$

or

$$P_i = \frac{C\sigma_a - 2\sigma_a + 2C^2P_o - CP_o \pm \sqrt{-3C^2\sigma_a - 6C^2\sigma_a P_o - 3C^2P_o^2 + 4(C^2 - C + 1)\sigma_V^2}}{2(C^2 - C + 1)} \quad (2\text{-}17)$$

In Eq. 2-16, if σ_V is replaced with $\sigma_{S\text{-}SF}$ (Eq. 2-7), the solution P_o is the collapse pressure using the triaxial method.

Mathematically, there are two solutions for P_o from Eq. 2-16 (for positive and negative square roots), but in practicality only positive real number(s) represent the collapse pressure P_o. There can be one, two, or no solution(s) for the collapse pressure design. When bending stress is considered, which is caused by the wellbore dogleg, σ_a in Eq. 2-16 is replaced with σ_{amin} (Eq. 2-5) and σ_{amax} (Eq. 2-6). This results in the solution(s) for collapse pressure design with minimum and maximum bending stress effects.

Note, when σ_a is replaced by σ_{amin} and σ_{amax}, both σ_{amin} and σ_{amax} can have the positive square root solution. If this happens, the program uses the smaller value of the two positive square root solutions as the upper boundary of collapse design. In the same way, the program will take the larger value from the two negative square root solutions as the lower pressure boundary of the collapse design.

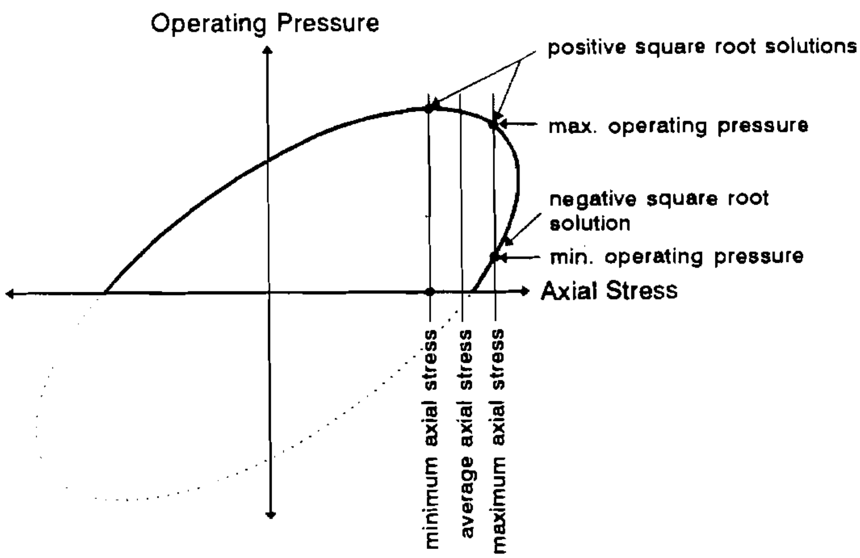

Figure 2-2. Bending Stress Effects on Burst Pressure Design

Buckling phenomenon, especially helical buckling, will cause bending stress. Buckling stress effects are not considered in this version.

In Eq. 2-17, if σ_V is replaced with $\sigma_{S\text{-}SF}$ (Eq. 2-7), and σ_a is replaced with σ_{amin} (Eq. 2-5), σ_{amax} (Eq. 2-6), the outcome is the burst design pressure for the triaxial stress method.

Figure 2-2 shows the relation between bending stress and operating pressure for the burst design.

2.2 BIAXIAL EQUATION

To disregard the internal pressure on collapse pressure design, let P_i = O and Eq. 2-16 is simplified.

$$P_o = \frac{\sigma_a \pm \sqrt{\sigma_a^2 - 4(\sigma_a^2 - \sigma_V^2)}}{-2C} \tag{2-18}$$

If σ_a and σ_V are replaced by σ_{amin} (Eq. 2-5), σ_{amax} (Eq. 2-6), and $\sigma_{S\text{-}SF}$ (Eq. 2-7), Eq. 2-18 produces the collapse design pressure for biaxial stress analysis.

Let P_o = O in Eq. 2-17 and it becomes

$$P_i = \frac{(C-2)\sigma_a \pm \sqrt{-3C^2\sigma_a^2 + 4(C^2 - C + 1)\sigma_V^2}}{2(C^2 - C + 1)} \tag{2-19}$$

The above equation is the burst design pressure for biaxial stress analysis.

where

$$C = \frac{d_o^2}{2 \cdot t \cdot (d_o - t)} \tag{2-20}$$

2.3 API EQUATION

API Bulletin, 5C3, 1989, "Formulas and Calculations For Casing, Tubing, Drill Pipe and Line Pipe Properties" (see for details) lists all API standard equations for axial stress limits, burst pressure limits, and four collapse pressure range limits.

3. Program Installation

3.1 BEFORE INSTALLING

3.1.1 Check the Hardware and System Requirements

TRIAX1 is written in Visual Basic®. It runs in either standard or enhanced mode of Microsoft Windows 3.0 or higher. The basic requirements are:

- Any IBM-compatible machine built on the 80286 processor or higher.
- Hard disk.
- Mouse.
- CGA, EGA, VGA, Hercules, or compatible display.
- MS-DOS version 3.1 or higher.
- Windows version 3.0 or higher in standard or enhanced mode.
- An 80486 processor and VGA display is recommended.

For assistance with the installation or use of **TRIAX1** contact:

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3.1.2 Check the Program Disk

The program disk is a 3½ inch, 1.44 MB disk containing thirteen files. These thirteen files are as follows:

SETUPKIT.DL_
VBRUN100.DL_
VER.DL_
GSWDLL.DLL
GSW.EXE
SETUP.EXE
SETUP1.EXE
TRIAX1.EXE
SETUP.LST
GRAPH.VBX

We recommend that all .VBX and .DLL files, that have the potential to be used by other DEA-44/67 Windows application, be installed in your Microsoft Windows\SYSTEM subdirectory. This applies to all the .VBXs and .DLLs included here. The TRIAX1 executable (TRIAX1.EXE) file should be placed in its own directory (default "C:\TRIAX1"). All these procedures will be done by a simple setup command explained in Section 3.2.

In order to run **TRIAX1**, the user must install all the files into the appropriate directory on the hard disk. Please see Section 3.2 to setup **TRIAX1**.

It is recommended that the original diskette be kept as a backup, and that working diskettes be made from it.

3.1.3 Backup Disk

It is advisable to make several backup copies of the program disk and place each in a different storage location. This will minimize the probability of all disks developing operational problems at the same time.

The user can use the COPY or DISKCOPY command in DOS, or the COPY DISKETTE on the disk menu in the **File Manager** in Windows.

3.2 INSTALLING TRIAX1

The following procedure will install **TRIAX1** from the floppy drive onto working subdirectories of the hard disk (i.e., copy from A: drive onto C: drive subdirectory **TRIAX1** and WINDOWS\SYSTEM).

1. Start Windows by typing "WIN" <ENTER> at the DOS prompt.
2. Insert the program disk in drive B:\.
3. In the File Manager of Windows, choose [Run] from the [File] menu. Type **B:\setup** and press Enter.
4. Follow the on-screen instructions.

This is all the user needs to setup **TRIAX1**. After setup, there will be a new Program Manager Group (DEA APPLICATION GROUP) which contains the icon (σ_{xyz}) for **TRIAX1** as shown in Figure 3-1.

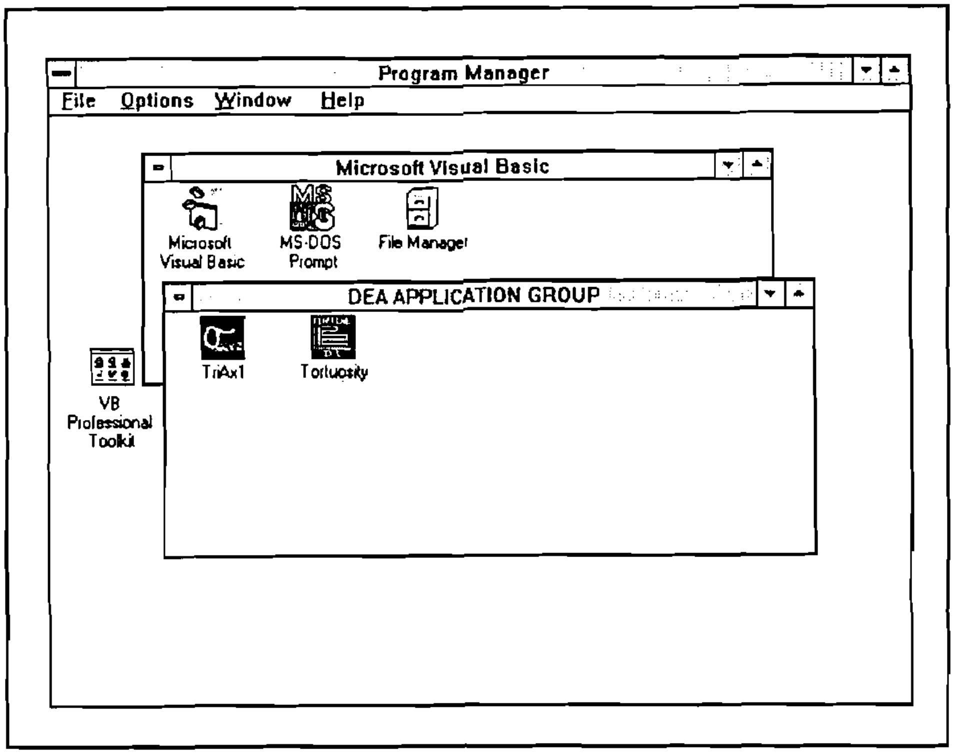


Figure 3-1. DEA APPLICATION GROUP and "TRIAX1" Icon

3.3 STARTING TRIAX1

3.3.1 Start TRIAX1 from Group Window

To run **TRIAX1** from the Group Window, the user simply double-clicks the "TRIAX1" icon, or when the icon is focused, press <ENTER>.

3.3.2 Use Command-Line Option from Windows

In the **Program Manager**, choose **[Run]** from the **[File]** menu. Then type C:\TRIAX1\TRIAX1.EXE <ENTER>.

3.4 ALTERNATIVE SETUP

When the SETUP procedure described before fails, follow these steps to install the program:

1. Create a subdirectory on drive C: C:\TRIAX1.
2. Insert the source disk in drive B: (or A:).
3. Type: CD\TRIAX1 <ENTER>.

4. At prompt C:\TRIAX1, type:
   Copy B:\TRIAX1.EXE <ENTER>

5. Type: CD\WINDOWS <ENTER>.

6. At prompt C:\WINDOWS>, type:
   Copy B:\VBRUN100.DL_ VBRUN100.DLL <ENTER>.

7. Type: CD\WINDOWS\SYSTEM <ENTER>.

8. At prompt C:\WINDOWS\SYSTEM>, type:
   Copy B:*.DLL <ENTER>
   Copy B:\GSW.EXE <ENTER>
   Copy B:*.VBX <ENTER>.

9. Type: CD.. <ENTER> then key in "WIN" <ENTER> to start Windows 3.0 or later version.

10. Click menu "File" under "PROGRAM MANAGER," select item [New...] click on [PROGRAM GROUP] option, then [OK] button.

11. Key in "DEA APPLICATION GROUP" after label "Description:," then key in "DEAMEI" after "Group File:," then click on [OK] button. A group window with the caption of "DEA APPLICATION GROUP" appears.

12. Click on menu [File] again, Select [NEW...] click on "PROGRAM ITEM" option, then, [OK] button.

13. Key in "TRIAX1" after label "Description," key in "C:\TRIAX1\TRIAX1.EXE" after label "COMMAND LINE," then click on [OK] button. The TRIAX1 icon appears.

14. Double-click the icon to start the program.

4. Running TRIAX1

4.1 INTRODUCTORY REMARKS

TRIAX1 runs in a windows environment. It is assumed that the user is familiar with windows, and the user's computer is equipped with Windows 3.0 or later version.

Some elements and terminology of windows are reviewed here:

4.1.1 Terminology and Elements of Windows

Figure 4-1 shows the Sensitivity Window (see Section 4.2.2 for details).

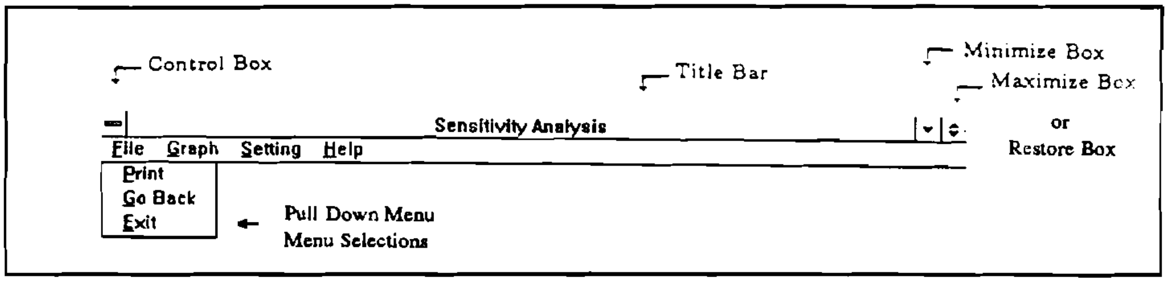


Figure 4-1. Title Bar in Window

1. **The Title Bar**

   The title bar serves two functions: one is to display the name of the current window and the other is to indicate which window is active. The active window is the one whose title bar is in color. (On monochrome monitors, the difference is shown by the intensity of the title bar). The user can make a window active by clicking anywhere within its border.

2. **The Control Boxes**

   At the left side of the title bar is the control box. It has two functions. First, it can display the CONTROL menu, which enables the user to control the window size using the keyboard. Second, double-clicking the control box will end the current program.

   During execution of TRIAX1, the control boxes are not needed. The program will run according to its own flow chart.

3. **Minimize and Maximize Boxes**

   At the right side of the title bar are the MINIMIZE and MAXIMIZE boxes. The box with the up arrow is the MAXIMIZE box. The box with the down arrow is the

MINIMIZE box. If a window has already been maximized, the MAXIMIZE box changes to a RESTORE box with both up and down arrows, as shown in Figure 4-1.

- Clicking on the MINIMIZE box will reduce the window to the size of an icon. The window's name in the title bar appears below the icon. To restore a window from an icon, double-click on the icon.
- Clicking on the MAXIMIZE box will make the window take up the total working area.
- Clicking on the RESTORE box will make the window take up a portion of the total working area, which is determined by how the user manually sizes the window.

4. **Text Boxes**

TEXT boxes can display the information that the user enters. Sometimes there will be text already typed in for the user. The user can use arrow keys to edit the existing text. Figure 4-2 shows a typical text box.

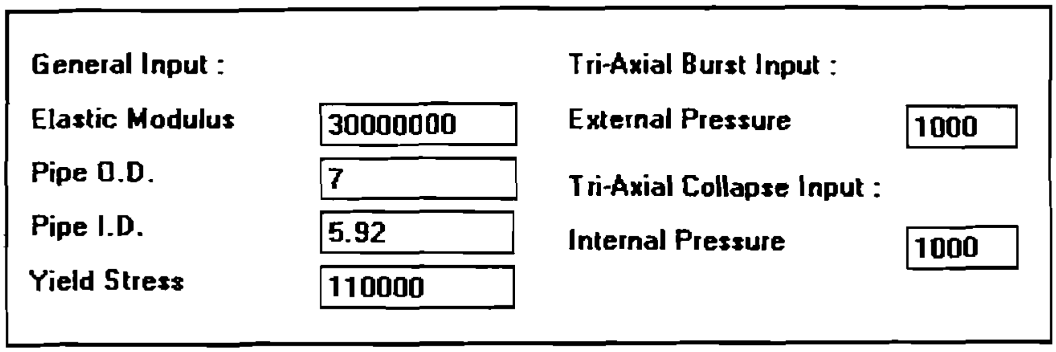


Figure 4-2. Text Boxes for Data Input and Display

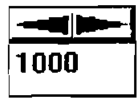
5. **Command Buttons**

A COMMAND button performs a task when the user chooses it, either by clicking the button or pressing a key. The caption of command buttons that are used in this program are: [OK], [CIRCULATING], [GRAPH], [GO BACK], [END], etc.

Using the [TAB] key or mouse point can switch the focus around the text boxes and command buttons within a window.

6. **Spin Button**

   The spin button is used with text boxes to increment and decrement the number inside the text boxes. Click the right arrow to increment the value and click the left arrow to decrement the value.



7. **Getting Into Focus**

   Using the [TAB] key or by moving the mouse pointer, the user can move the cursor around the TEXT BOXES, COMMAND BUTTONS, and SPIN BUTTONS, etc.

   When the cursor stays on one element, for example the TEXT BOX, getting into focus. When the COMMAND BOX is getting into focus, the box will have a thick border, if you press <ENTER>, the command will be executed.

4.2 OVERVIEW

There are three major windows in TRIAX1:

1. Tubing Stress Analysis
2. Stress and Limited Pressure Graph
3. Sensitivity Analysis.

These three windows appear on screen in a series. The user can change the input data through each window, but the input data in each window will affect data in the other windows.

If using a color monitor, the input data text boxes always have a green background, and the output data text boxes always have a blue background.

4.3 GETTING STARTED

1. Call up Windows and select "DEA APPLICATION GROUP" as the active window, as shown in Figure 4-3.

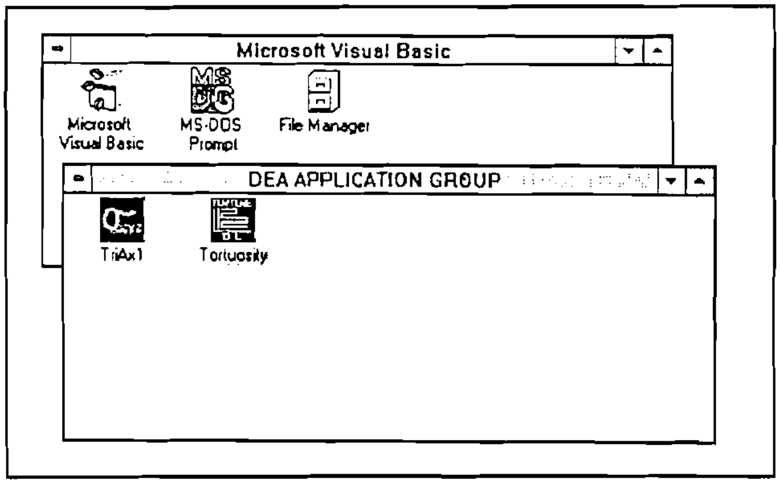


Figure 4-3. DEA Application Group (MEI) and TRIAX1 Icon

2. This window may contain more than one icon. Double-click on the TRIAX1 icon.

3. When the Disclaimer screen (Figure 4-4) appears, click on the [OK] button. This opens the "Tubing Stress Analysis " window (Figure 4-5).

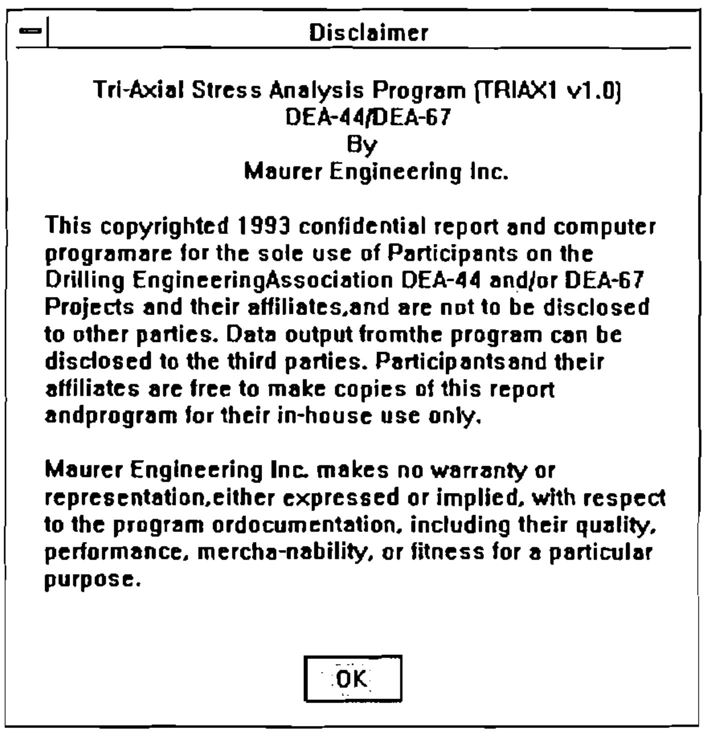


Figure 4-4. Disclaimer Window

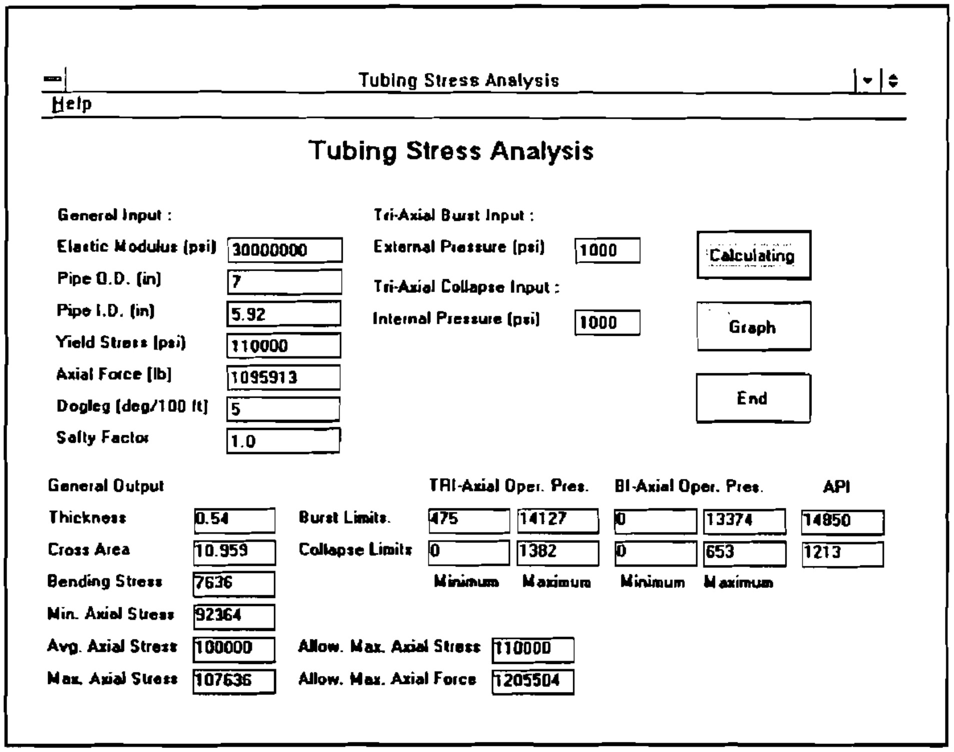


Figure 4-5. Tubing Stress Analysis Window

4.4 TUBING STRESS ANALYSIS

This window includes the default data. **Note that the "External Pressure" and "Internal Pressure" inputs are for the triaxial burst and the triaxial collapse calculations only.**

To change any of the input data, click on the desired text box. If you are not using a mouse, use the [TAB] key to move from one box to another and use the arrow keys to move within a field. When data has been input, click the [Calculating] button, the results will print in the lower part of the screen and the [Graph] command button will show on the screen also. All output data have a blue background. To end the program and return to the MS Window, click the [End] button.

Following are equation numbers shown in Chapter 2 relating to each output data:

Thickness	-	Equation 2-1
Cross Area	-	Equation 2-2
Average Axial Stress	-	Equation 2-3
Bending Stress	-	Equation 2-4
Minimum Axial Stress	-	Equation 2-5
Maximum Axial Stress	-	Equation 2-6
Allowable Maximum Axial Stress	-	Equation 2-7
Allowable Maximum Axial Force	-	Equation 2-8

Triaxial Minimum Burst Pressure - Equation 2-17
Triaxial Maximum Burst Pressure - Equation 2-17
Triaxial Minimum Collapse Pressure - Equation 2-16
Triaxial Maximum Collapse Pressure - Equation 2-16
Biaxial Minimum Burst Pressure - Equation 2-19
Biaxial Maximum Burst Pressure - Equation 2-19
Biaxial Minimum Collapse Pressure - Equation 2-18
Biaxial Maximum Collapse Pressure - Equation 2-18

4.5 STRESS AND LIMITED PRESSURE GRAPH

In the "Tubing Stress Analysis" window, the [Graph] command button will appear on the screen after calculating. Click the [Graph] button, then the "Stress and Limited Pressure Graph" window shows on the screen (Figure 4-6).

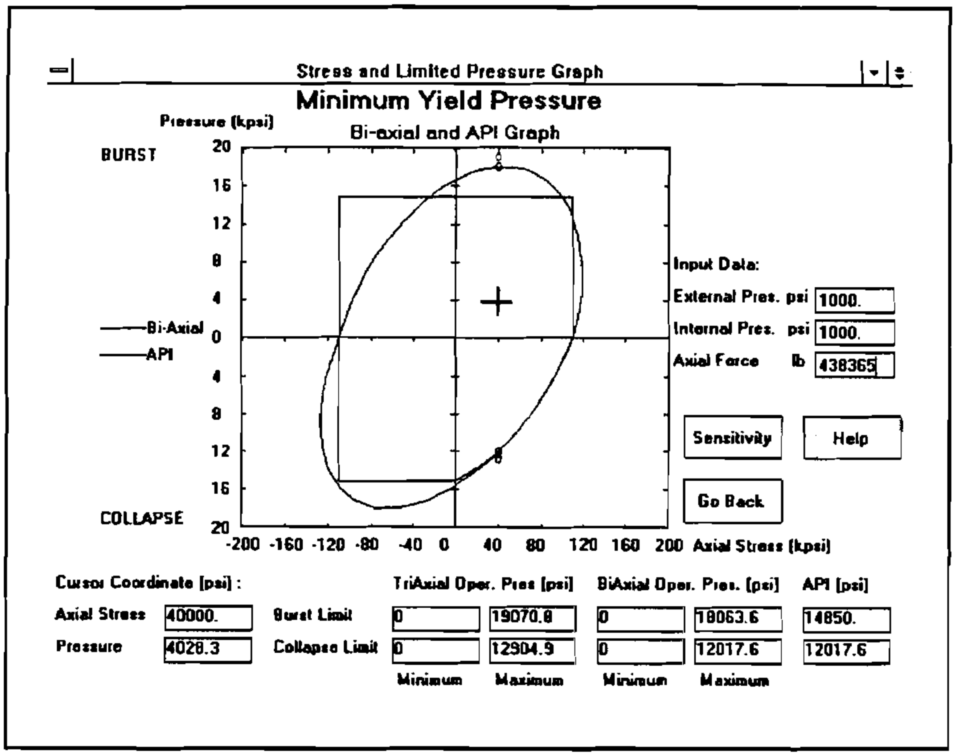


Figure 4-6. Stress and Limited Pressure Graph Window

On the right-hand side of the screen there are three input text boxes. The top two boxes, "External Pressure" (for burst) and "Internal Pressure" (for collapse) are for triaxial stress calculation only. The last input box, "Axial Force" relates to the cursor location.

This program converts "Axial Force" to "Axial Stress" and makes this axial stress the cursor's X-coordinate. Move the cursor into the "Biaxial and API Graph" area and a black-cross cursor will appear in the graph. While moving the cursor within the graph, the data in the cursor coordinate boxes "Axial Stress" and "Pressure," which are located on the left bottom of the screen, are updated. **Only the axial stress affects the burst pressure limits and collapse pressure limits. The pressure text box reflects only the cursor's Y-coordinate.** If there is any solution, the value will print on screen; otherwise, it shows "N/A" in the output box. When the tension or compression force is too large, there is not a solution. The pressure limit equations used here are the same in the "Tubing Stress Analysis" window (Figure 4-5). **If there are solutions, the red circles (for biaxial) and/or white circles (for triaxial) show on the screen to indicate the minimum yield pressure at current axial stress (i.e., cursor's X-coordinate). If both biaxial and triaxial have the same solution, the circle's color will be black.**

Note: If the mouse is moved too fast (this depends upon the capability of the user's PC), the circle drawing may not have enough time to respond.

4.6 SENSITIVITY ANALYSIS

Click the [Sensitivity] button. This will load the "Sensitivity Analysis" window. **This is the only window that the program outputs the results to the printer.**

There are five graphs shown:

1. "Internal Pressure Sensitivity Analysis"
2. "External Pressure Sensitivity Analysis"
3. Both "Internal Pressure Sensitivity Analysis and External Pressure Sensitivity Analysis"
4. "Dogleg Sensitivity Analysis"
5. "Diameter and thickness (D/t) Ratio Sensitivity Analysis."

Figure 4-7 shows the pull-down [Graph] menu.

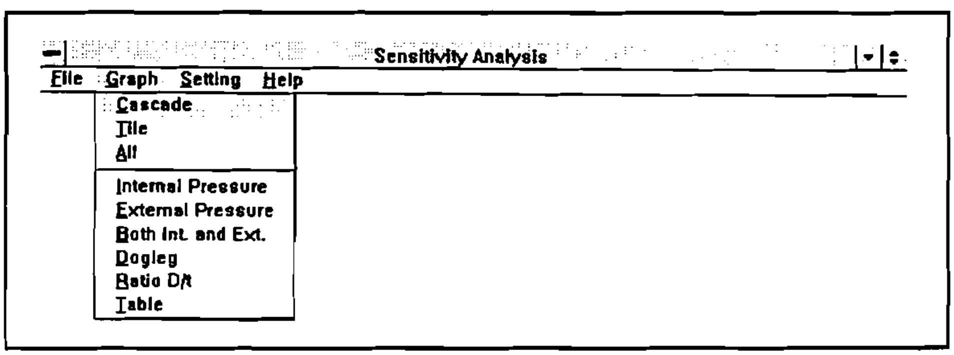


Figure 4-7. Pull Down (Graph) Menu in Sensitivity Analysis

The user can examine each graph by clicking the name of the graph. All five graphs and the data table are shown by clicking [All] (Figure 4-8). The graph and table are contained inside six sub-windows. To enlarge the graph size:

1. Activate the sub-window by clicking any where inside the window (the title bar of active sub-window will be in color).
2. When the mouse cursor is moved to the boundary of the sub-windows, the cursor becomes a double arrow (Figure 4-9).
3. Hold left button of mouse, then drag the boundary to the size you want.
4. For full screen graph, click the maximize box in the top-right corner of the sub-window.

NOTE: To change the graph size on the screen is important, because the printed graph size depends on the graph size shown on the screen.

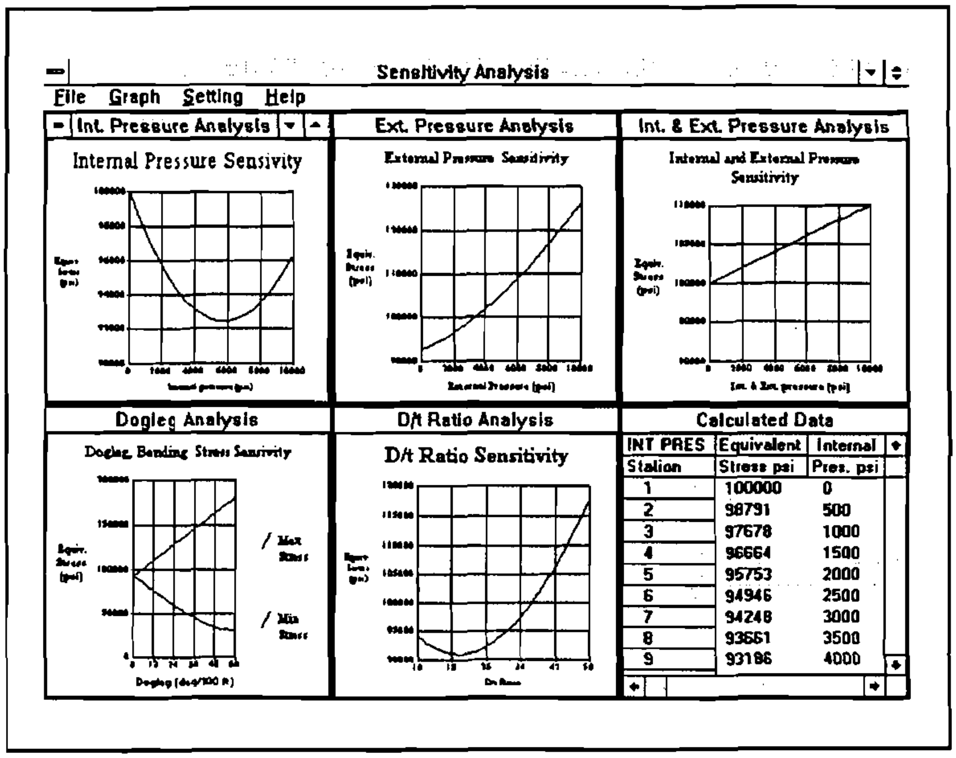


Figure 4-8. All Sensitivity Graphs and Table

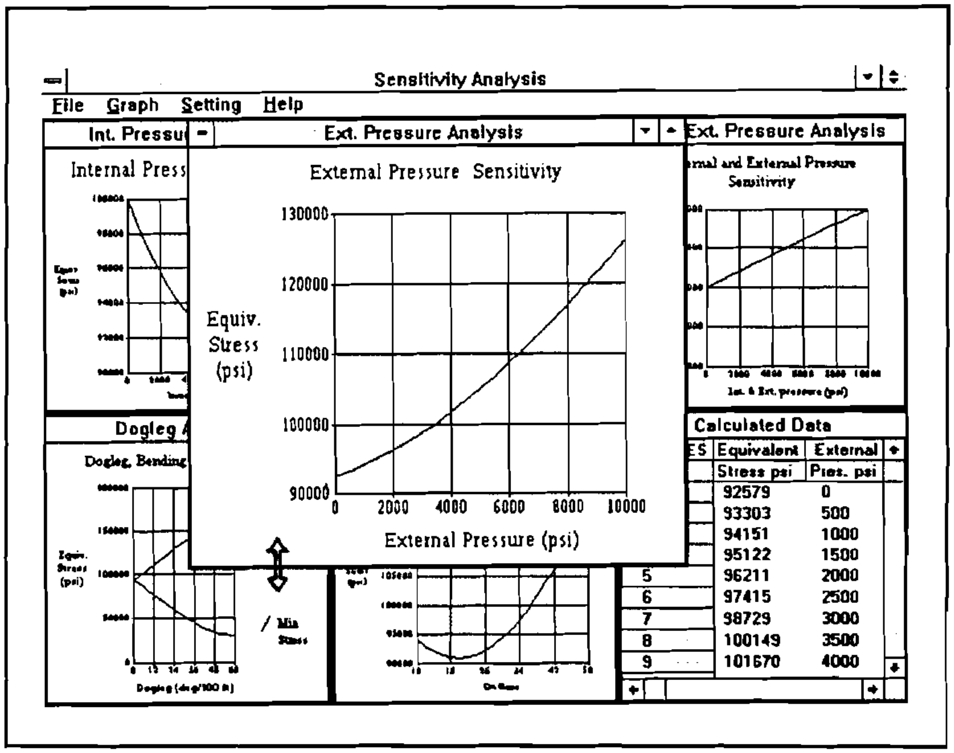


Figure 4-9. Using Mouse to Change Window Size

The "calculated data table" will show data for the active sub-window's graph. To send results to the printer:

1. Activate the desired sub-window
2. Pull down the [File] menu
3. Click [Print]

To leave Sensitivity Window:

1. Pull down the [File] menu.
2. Click [Go Back] to go back to "Stress and Limited Pressure Graph" window.
3. Click [Exit] to return to the MS windows.

Figure 4-1 shows the pull-down [File] menu.

4.7 SETTING SENSITIVITY DATA

Click [Setting] and [Setting Range and Data] on the menu bar, a sub-window of data settings will appear (Figure 4-10).

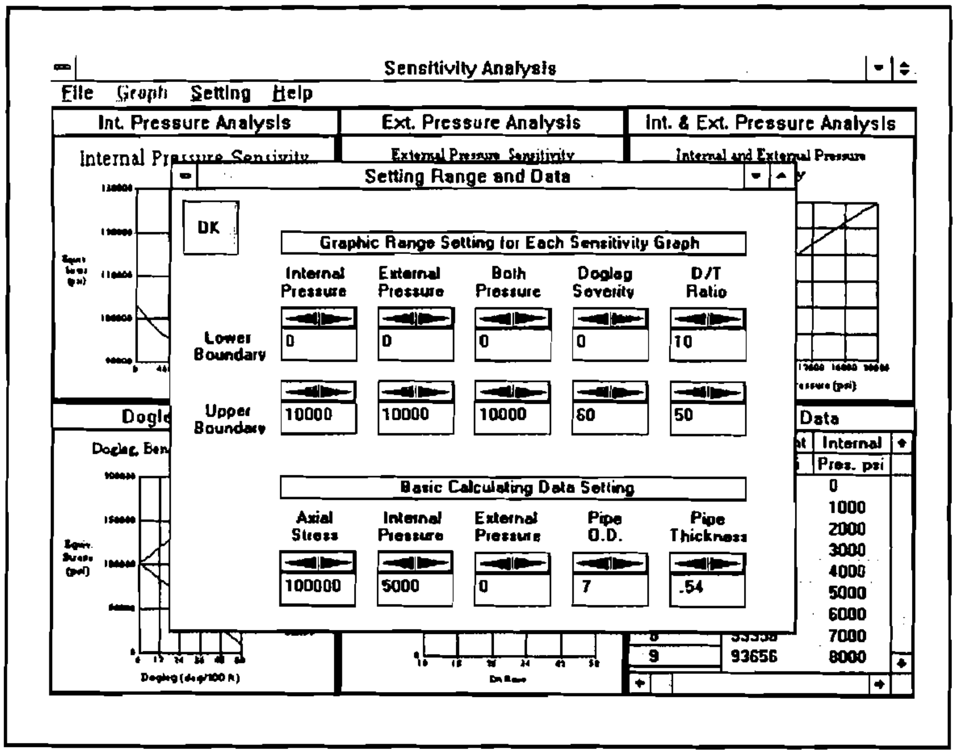


Figure 4-10. Data Setting Window for Sensitivity Analysis

The user can set up two groups of data:

1. Graphic range for each sensitivity graph's X-coordinate, on the top-side of the Setting window.
2. Basic calculating data, on the bottom-side of the Setting window.

The setting of both groups will affect the graphic output. Each sensitivity graph uses only part of the set data. For example, the [BASIC CALCULATING DATA] group:

1. [INTERNAL PRESSURE] sensitivity graph uses the data: axial stress, external pressure, pipe OD, and pipe thickness.

2. [EXTERNAL PRESSURE] sensitivity graph uses the data: axial stress, internal pressure, pipe OD and pipe thickness.

3. [INTERNAL AND EXTERNAL PRESSURE] sensitivity graph uses the data: axial stress,

4. [DOGLEG] sensitivity graph uses all data in the group.

5. [D/t RATIO] sensitivity graph uses the data: axial stress, internal pressure, and external pressure.

6. The X-axial data range for each graph depends on the "lower boundary" value and "upper boundary" value underneath each graph's title in the Setting window.

Each time the data is changed, the user must click the [OK] button to redraw the graph with current settings.