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The sea surface pressure formulation of rigid lid models. Implications for altimetric data assimilation studies

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Abstract

The sea surface pressure formulation of the rigid lid primitive equation oceanic problem is reviewed and clarified. The geostrophic limit for the sea surface pressure equation is then considered and a new diagnostic relationship is found that relates the surface pressure to the barotropic and baroclinic components of the subsurface flow field. We demonstrate that a direct insertion in the model equations of sea surface information, such as that provided by satellite altimetry, does not produce changes in the subsurface dynamics due to the divergenceless nature of the barotropic flow field.

The geostrophic limit of the sea surface pressure field computed from a standard general circulation model of the world ocean is presented and the barotropic/baroclinic components of the absolute dynamic topography of the global general circulation are discussed.

1. Introduction

Numerical ocean modelling has become one of the principal tools to investigate the dynamics of the ocean and predict its natural variabilities. In recent years, numerical models have also been used as part of an oceanic data assimilation system which may include subsurface hydrography, satellite altimeter, current meter data, floats, etc. After the advancements produced by the assimilation of conventional hydrography in primitive equations general circulation models (Derber and Rosati, 1989), satellite altimetry seems to be the next promising data set to be considered. It is then important to know if the conventional General Circulation Models (GCM) could be used to assimilate such data which consist of sea surface height measurements at very high spatial and temporal resolution.

In this note we try to point out the fundamental characteristics of the sea surface height field as computed from a classical GCM. We will show the connection between the sea surface height field and the internal flow field variables used in the GCM. The latter uses the rigid lid approximation at the sea surface and so there is some question as to how these models can properly assimilate sea surface topography information.

From the pioneering work of Marchuk and Sarkysian (1986), Demin and Ibraev (1989) and the more recent implementation of Dukowicz et al. (1993), we have experience with two rigid lid

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model formulations: the first, called the streamfunction formulation, has been widely used (Bryan, 1969) and the second, called the sea surface pressure formulation, is of more recent development. The latter uses sea surface height directly in the form of sea surface pressure to force the vertically integrated momentum equations. We show that even in this case the sea surface pressure is a purely diagnostic quantity and sea surface pressure information from a satellite would be of no use if purely inserted at the surface. The sea surface pressure relates to vertically integrated interior flow quantities (density and velocity) and cannot impose any constraint on their distribution in vertical. In this note we show formally that the sea surface pressure information cannot affect interior dynamical flow field variables in a standard rigid lid GCM.

For a review of altimeter data assimilation methods applied to quasigeostrophic and primitive equations models see Arnault and Perigaud (1992). Ezer and Mellor (1994) present the most advanced implementation of satellite altimeter data assimilation techniques to a free surface model of the Gulf Stream. All these works showed some success in altimeter data assimilation if the satellite altimeter data is converted into subsurface temperature or velocity profiles which can be directly assimilated in the GCM. Here we illustrate the dynamical reasons underlying the need to transform the sea surface height into interior dynamical variables. In other words, it appears to be necessary to use satellite altimeter data in an indirect way, e.g., convert these data into "representative" data for the GCM in order to be effective in the data assimilation procedure.

2. Splitting of barotropic / baroclinic components in primitive equations

The momentum and divergence equations for the oceanic primitive equations are written:

$$\boldsymbol{u}_t + L(\boldsymbol{u}) + f\hat{k} \times \boldsymbol{u} = -\frac{1}{\rho_0}\boldsymbol{\nabla} p + \boldsymbol{F} \quad (1)$$

$$\boldsymbol{F} = A_v \boldsymbol{u}_{zz} + A_h \nabla^2 \boldsymbol{u} \quad (2)$$

$$p_z = -\rho g \quad (3)$$

$$L(1) = 0 \quad (4)$$

where the curvature terms have been neglected for simplicity. The notation is the following: $\hat{k}$ is the unit vector in the z direction, $\boldsymbol{u} = (u,v)$ and w are the horizontal and vertical velocity field components, f is the Coriolis parameter, $f = 2\Omega n$, ρ and p the density and pressure, $m = sec\varphi$, $n = \sin\varphi$, $u = a\dot{\lambda}/m$, $v = a\dot{\varphi}$, $\Omega = 1.710^{-5}$ s^{-1}, φ indicates the latitude and λ the longitude. Furthermore, $L(\mu) = m/a[(u\mu)_\lambda + (v\mu/m)_\varphi] + (w\mu)_z$ and letter subscripts indicate partial differentiation. The thermodynamic equations are:

$$C_t + L(C) = F^c \quad (5)$$

where $C = (T, S)$ are the temperature, T, and salinity, S, tracers and

$$F^c = K_v C_{zz} + K_h \nabla^2 C \quad (6)$$

The system is closed by an equation of state of the form $\rho = f(T, S, p)$. The vertical boundary conditions for the rigid lid are, at $z = 0$:

$$w = 0$$
$$A_v \boldsymbol{u}_z = \boldsymbol{\tau} \quad (7)$$
$$k_v C_z = Q$$

where $\boldsymbol{\tau}$ is the wind stress and Q = (heat flux, salt flux) for $C = (T, S)$ respectively. At $z = -H$, where $H(x, y)$ is the topography, $w = -uH_x - vH_y$ and $A_v \boldsymbol{u}_z = \boldsymbol{\tau}_b$, where $\boldsymbol{\tau}_b$ is the bottom stress. At the lateral walls we assume the traditional no-slip boundary conditions, e.g., $(u, v) = 0$ and the condition of zero tracer flux (e.g., $\boldsymbol{\nabla} C \cdot \boldsymbol{n} = 0$, where $\boldsymbol{n}$ is the unit vector normal to the lateral boundaries).

Our intent is to show explicitly the connection between the sea surface pressure, p_s, and the interior flow dynamic variables predicted by the primitive Eqs. (1)–(5). Thus we split the pressure into:

$$p(x, y, z, t) = p_s(x, y, t) + g\int_z^0 \rho \, dz \quad (8)$$

The sea surface height, h, is related to p_s by the equality $h = p_s/\rho_0 g$.

We have to show what the p_s term controls in the momentum Eq. (1) in terms of barotropic and baroclinic components of the velocity field. It is in fact a common practice in ocean modelling to solve separately for internal (baroclinic) and external (barotropic or depth independent) velocities. In doing so it will be possible to write the analytical expression which relates gradients of p_s to the barotropic components of the velocity field. The velocity field is then divided into:

$$u = \bar{u} + u'$$
$$v = \bar{v} + v' \tag{9}$$

where $\bar{u}$, $\bar{v}$ are the components of the vertically integrated velocity field (barotropic) defined as:

$$(\bar{u}, \bar{v}) = \frac{1}{H}\int_{-H}^{0}(u, v)\,\mathrm{d}z \tag{10}$$

and (u', v') are the baroclinic velocity field components.

The equation for $\bar{\boldsymbol{u}}$ is obtained taking the vertical integral of Eq. (1), using Eq. (8), that is:

$$\bar{\boldsymbol{u}}_t + \frac{1}{H}\int_{-H}^{0} L(\boldsymbol{u}) + f\hat{k}\times\bar{\boldsymbol{u}} = -\frac{1}{\rho_0}\boldsymbol{\nabla}p_s - \frac{g}{\rho_0 H}\int_{-H}^{0}\int_{z'}^{0}\boldsymbol{\nabla}\rho\,\mathrm{d}z'' + \frac{1}{H}\int_{-H}^{0}\mathrm{d}z'\boldsymbol{F}\,\mathrm{d}z \tag{11}$$

The baroclinic velocity equations are obtained subtracting Eq. (11) from Eq. (1):

$$\frac{\partial}{\partial t}\boldsymbol{u}' + L(\boldsymbol{u}) - \frac{1}{H}\int_{-H}^{0} L(\boldsymbol{u}) + f\hat{k}\times\boldsymbol{u}' = -\frac{g}{\rho_0}\int_{z}^{0}\boldsymbol{\nabla}\rho\,\mathrm{d}z' + \frac{g}{\rho_0 H}\int_{-H}^{0}\mathrm{d}z'\int_{z'}^{0}\boldsymbol{\nabla}\rho\,\mathrm{d}z'' + \boldsymbol{F} - \frac{1}{H}\int_{-H}^{0}\boldsymbol{F}\,\mathrm{d}z \tag{12}$$

It is already clear that the $\boldsymbol{\nabla}p_s$ does not control the interior baroclinic velocity field because it has disappeared from Eq. (12). However it is present in the vertically integrated transport Eq. (11). We will show that even in the latter case, the $\boldsymbol{\nabla}p_s$ term does not produce any effect on the interior barotropic flow field. To do so we have to review the classical methods of solving for the $\bar{\boldsymbol{u}}$. For incompressible Navier–Stokes equations there are at least two approaches used (Gresho, 1991). Both of them derive an elliptic equation for an integral function, e.g., either the transport streamfunction ψ or the sea surface pressure, p_s. For a comparative review of the two methods in ocean modelling see Marchuk and Sarkisyan (1986).

2.1. Classical formulation: Streamfunction

This method has been widely used in oceanography from the work of Bryan (1969) to the most recent developments of Haidvogel et al. (1991). The numerical implementation of the elliptic problem and its associated boundary value problem has been shown to be robust for ocean current modelling.

The streamfunction formulation of the problem is obtained by taking the vertical component of the curl of Eq. (11). The streamfunction results from the fact that from Eq. (4):

$$\boldsymbol{\nabla}\cdot(H\bar{\boldsymbol{u}}) = 0$$

so that $\bar{\boldsymbol{u}} = 1/H\hat{k}\times\nabla\psi = 1/\mathrm{H}(-1/a\psi_\varphi, m/a\psi_\lambda)$.

The resulting elliptic equation for ψ_t is:

$$\frac{\partial}{\partial\lambda}\left(\frac{m}{aH}\frac{\partial}{\partial\lambda}\psi_t\right) + \frac{\partial}{\partial\varphi}\left(\frac{1}{aHm}\frac{\partial}{\partial\varphi}\psi_t\right) = \frac{\partial}{\partial\varphi}\left(\frac{G_u}{mH}\right) + \frac{\partial}{\partial\lambda}\left(\frac{G_v}{H}\right) \tag{13}$$

where:

$$G_u = \int_{-H}^{0} L(u) - fH\bar{v} - \int_{-H}^{0} F^{\lambda} + \frac{mg}{a\rho_0}\int_{-H}^{0}\frac{\partial}{\partial\lambda}\int_{z}^{0}\rho\,\mathrm{d}z'$$

$$G_v = -\int_{-H}^{0} L(v) - fH\bar{u} + \int_{-H}^{0} F^{\varphi} - \frac{g}{a\rho_0}\int_{-H}^{0}\frac{\partial}{\partial\varphi}\int_{z}^{0}\rho\,\mathrm{d}z$$

The boundary conditions for the elliptic Eq. (13) are deduced from the requirement that the normal velocity component is equal to zero, that is, $(\hat{k}\times\nabla\psi)\cdot\boldsymbol{n}\,|_{\partial\Omega} = 0$, or $\nabla\psi\cdot\boldsymbol{t}\,|_{\partial\Omega} = 0$, where $\boldsymbol{t}$

and $\boldsymbol{n}$ are the unit vectors in the direction parallel and normal to the boundaries. Thus ψ is a function of time along the boundaries. Its value is determined by an integral constraint condition resulting from the area average of Eq. (13). To impose the full no slip boundary condition we need to impose a consistency constraint so that also $\nabla\psi \cdot \boldsymbol{n}\,|_{\partial\Omega} = 0$.

It is clear that in the streamfunction formulation $\bar{\boldsymbol{u}}$ is determined within the arbitrary field of ∇p_s since it disappears from Eq. (13). After the total barotropic flow field is computed as described above, the ∇p_s term can be computed diagnostically from Eq. (11). This is the demonstration that the $\bar{\boldsymbol{u}}$ field is dynamically independent from the surface pressure field. The barotropic flow field is produced solely by the external wind action, nonlinear/frictional and baroclinic–topographic effects in the fluid. This barotropic mode in turn produces a sea surface pressure anomaly on the rigid lid as a surface manifestation of the subsurface dynamics.

2.2. Sea surface pressure formulation

The sea surface pressure approach has been used only recently in oceanography (Demin and Ibraev, 1986, 1989; Dukowicz et al., 1993) while it has been widely applied elsewhere to solve incompressible Navier–Stokes equations. However, the explicit analytical formulation of the boundary conditions has not been written for the general oceanic case and we will present it here. Instead of taking the curl of Eq. (11) we take the divergence so that we obtain another elliptic problem:

$$\frac{1}{a\rho_0}\left[\frac{\partial}{\partial\lambda}\left(Hm\frac{\partial p_s}{\partial\lambda}\right)+\frac{\partial}{\partial\varphi}\left(\frac{H}{m}\frac{\partial p_s}{\partial\varphi}\right)\right] = \frac{\partial}{\partial\lambda}\left[Q^{\lambda}\right]+\frac{\partial}{\partial\varphi}\left[\frac{Q^{\varphi}}{m}\right] \qquad (14)$$

where:

$$Q^{\lambda} = -\int_{-H}^{0} L(u) + 2\Omega n\int_{-H}^{0} v + \int_{-H}^{0} F^{\lambda} - \frac{mg}{a\rho_0}\int_{-H}^{0}\frac{\partial}{\partial\lambda}\int_{z}^{0}\rho\,\mathrm{d}z'$$

$$Q^{\varphi} = -\int_{-H}^{0} L(v) - 2\Omega n\int_{-H}^{0} u + \int_{-H}^{0} F^{\varphi} - \frac{g}{a\rho_0}\int_{-H}^{0}\frac{\partial}{\partial\varphi}\int_{z}^{0}\rho\,\mathrm{d}z'$$

The time varying $\bar{\boldsymbol{u}}_t$ term has disappeared from the right hand of Eq. (14) because the $\bar{\boldsymbol{u}}$ field is divergenceless. Eq. (14) allows one to solve the p_s field as purely diagnostic variable and thereafter the $\bar{\boldsymbol{u}}$ field is solved by the finite time differencing of Eq. (11).

To solve Eq. (14) we need boundary conditions. Given Eq. (11) it is evident that the boundary conditions will consist of the partial derivatives specified at the domain boundaries. In particular, following the work of Gresho and Sani (1987) we claim that the appropriate boundary conditions for the elliptic Eq. (14) are Neumann boundary conditions. We write the normal component of ∇p_s as:

$$\frac{H}{\rho_0}\nabla p_s\cdot\boldsymbol{n} = -H\boldsymbol{u}_t\cdot\boldsymbol{n} - \int_{-H}^{0} L(\boldsymbol{u})\cdot\boldsymbol{n} - fH\boldsymbol{u}\cdot\boldsymbol{t} - \frac{g}{a\rho_0}\int_{-H}^{0}\left\{\int_{z}^{0}\nabla\rho\cdot\boldsymbol{n}\,\mathrm{d}z'\right\} + \int_{-H}^{0}\boldsymbol{F}\cdot\boldsymbol{n}\,\mathrm{d}z \qquad (15)$$

After a long controversy on which boundary conditions are most appropriate for the surface pressure approach, Gresho (1991) shows that for the incompressible Navier–Stokes equations the elliptic equation for surface pressure with Neumann boundary conditions has a well posed solution. Imposing the no slip boundary conditions and the condition of zero density diffusion through the boundaries, $\nabla\rho\cdot\boldsymbol{n} = 0$, we obtain:

$$\left.\frac{1}{\rho_0}\nabla p_s\cdot\boldsymbol{n}\right|_{\partial\Omega} = \left.\frac{(\boldsymbol{\tau}-\boldsymbol{\tau}_b)\cdot\boldsymbol{n}}{H}\right|_{\partial\Omega} + \left.\frac{A_h\int_{-H}^{0}(\nabla^2\boldsymbol{u})\cdot\boldsymbol{n}\,\mathrm{d}z}{H}\right|_{\partial\Omega} \qquad (16)$$

The no-slip surface pressure formulation of the oceanic problem consists of Eq. (14) with boundary conditions Eq. (16). It is interesting to note that the numerical implementation of Eq. (14) and Eq. (16) is straightforward because in a staggereed grid the imposition of Neumann boundary conditions can be made to disappear as discussed by Canuto et al. (1988) and Gresho (1991).

In conclusion we have shown that the p_s term enters only the barotropic velocity computations and that even for that part of the flow field the p_s is a purely diagnostic quantity. The time varying barogropic transport is determined either by directly timestepping the momentum Eq. (11) or by solving the elliptic problem Eq. (13) for the ψ_t. Both methods give the same solution for the barotropic component of the velocity field.

Given this equivalence, it is easy then to show that if:

$$p_s' = p_s + P_s^*$$

where P_s^* is a correction due to data insertion. To mantain the zero divergence constraint we have to constrain P_s^* such that:

$$\nabla \cdot (H \nabla P_s^*) = 0$$

which implies that there is no induced change in the barotropic field, given the same external forcing to the dynamical equations. It is then concluded that for the large scale oceanic circulation described by the primitive equations, the surface pressure is a purely diagnostic quantity which does not directly affect the dynamics of the interior flow field. The fundamental dynamical reason for this is that the surface pressure changes do not induce modifications in the other dynamical variables since the zero divergence constraint for the vertically integrated velocity field has to be enforced. The changes in surface pressure have to be inserted in the barotropic and/or baroclinic flow fields a priori, knowing what the changes in sea surface height are most likely to produce in the subsurface flow. In this respect pioneering work has been done for altimeter data assimilation by Hurlburt (1986) and De Mey and Robinson (1987).

3. The geostrophic limit for the sea surface pressure equation

In this section we show now that the uncontrollability of the rigid lid primitive equations by the surface pressure field can be extended to the quasigeostrophic modelling framework. In order to show this we convert the surface pressure in the traditional surface streamfunction writing the geostrophic limit of Eq. (14). We rewrite the surface pressure equation as:

$$\frac{1}{\rho_0}\nabla \cdot (H \nabla p_s) = -\frac{g}{\rho_0}\nabla \cdot \int_{-H}^{0} (z+H)\nabla \rho \, dz - \nabla \cdot \left(fH\hat{k} \times \boldsymbol{u}\right) + \nabla \cdot \boldsymbol{R} \qquad (17)$$

where the term of $\int_{-H}^{0} dz \nabla \int_z^0 \rho dz'$ in Eqs. (11) and (12) is now transformed to $\int_{-H}^{0}(z+H)\nabla \rho dz$, and $\boldsymbol{R} = -(\int_{-H}^{0} L(\boldsymbol{u}) - A_h \nabla^2 \boldsymbol{u}) + \boldsymbol{\tau}_b$.

We nondimensionalize Eq. (17) by using: $t = d/u_0 t'$, $\bar{\boldsymbol{u}} = u_0 \bar{\boldsymbol{u}}'$, $p_s = (f_0 u_0 d \rho_0) p_s'$, $z = h_0 z'$, $f = f_0 f'$, $(x, y) = d(x', y')$, $w = h_0 u_0/dw'$, $\hat{p} = (f_0 u_0 d \rho_0) p'$, $\rho = (f_0 u_0 d \rho_0 / g h_0)\rho'$, $\boldsymbol{\tau} = \tau_0 \boldsymbol{\tau}'$, $\boldsymbol{\tau}_b = \tau_0 \boldsymbol{\tau}'$, where h_0, d, u_0, f_0 are the vertical and horizontal scales, the velocity and Coriolis parameter scales respectively. The resulting Rossby number for the system is $\epsilon = u_0/f_0 d$, The nondimensional form of Eq. (17) is:

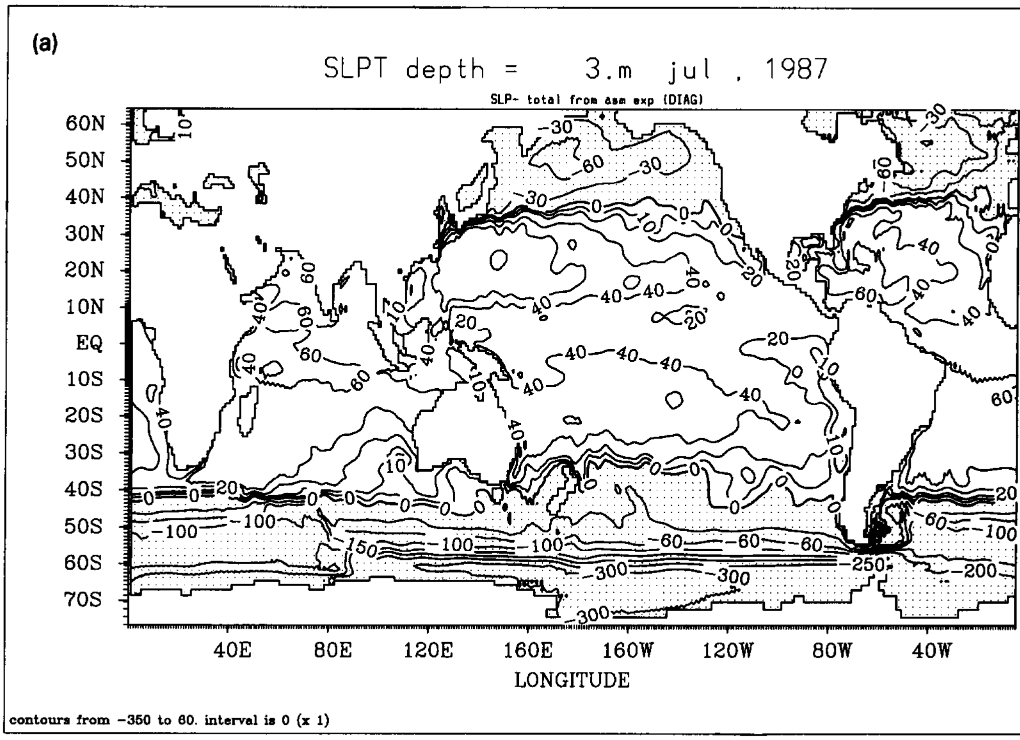
$$\nabla \cdot (H \nabla p_s) = -\nabla \cdot \int_{-H}^{0} (z+H)\nabla \rho \, dz - \nabla \cdot \left(fH\hat{k} \times \boldsymbol{u}\right) + \epsilon \nabla \cdot \boldsymbol{R} \qquad (18)$$

where the primes have been dropped and

$$\boldsymbol{R} = -\left(\int_{-H}^{0} L(\boldsymbol{u}) - \frac{A_h}{d^2 u_0}\nabla^2 \boldsymbol{u}\right) + \frac{\tau_0 d}{h u_0^2}(\boldsymbol{\tau} - \boldsymbol{\tau}_b)$$

The geostrophic balance form of Eq. (18) results by taking the zero-th order balance of the equation after the dynamical variables $\phi = (U, V, p_s, p, \rho)$ have been expanded in series of ϵ, e.g., $\phi = \phi^{(0)} + \epsilon \phi^{(1)} + \ldots$.. The geostrophic diagnostic relationship results:

$$\nabla \cdot \left(H \nabla p_s^{(0)}\right) = -\nabla \cdot \int_{-H}^{0} (z+H)\nabla^{(0)} \rho \, dz - \nabla \cdot \left(fH\hat{k} \times \bar{\boldsymbol{u}}^{(0)}\right) \qquad (19)$$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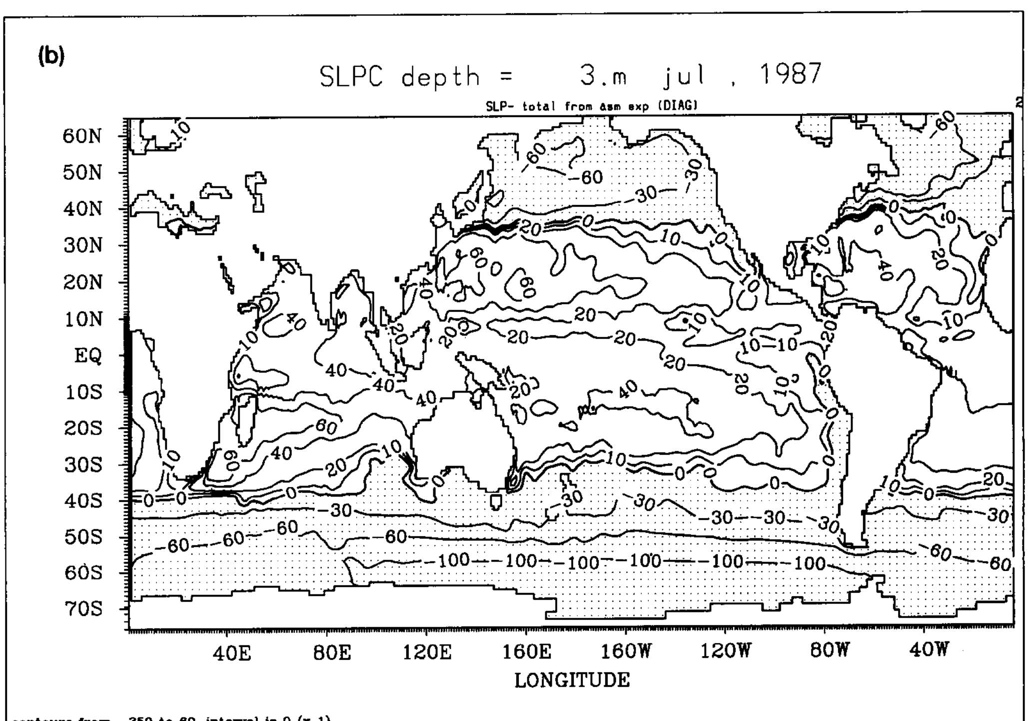
LONGITUDE
contours from −350 to 60. interval is 0 (x 1)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EQ
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20S
30S
40S
50S
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LONGITUDE
contours from −350 to 60. interval is 0 (x 1)

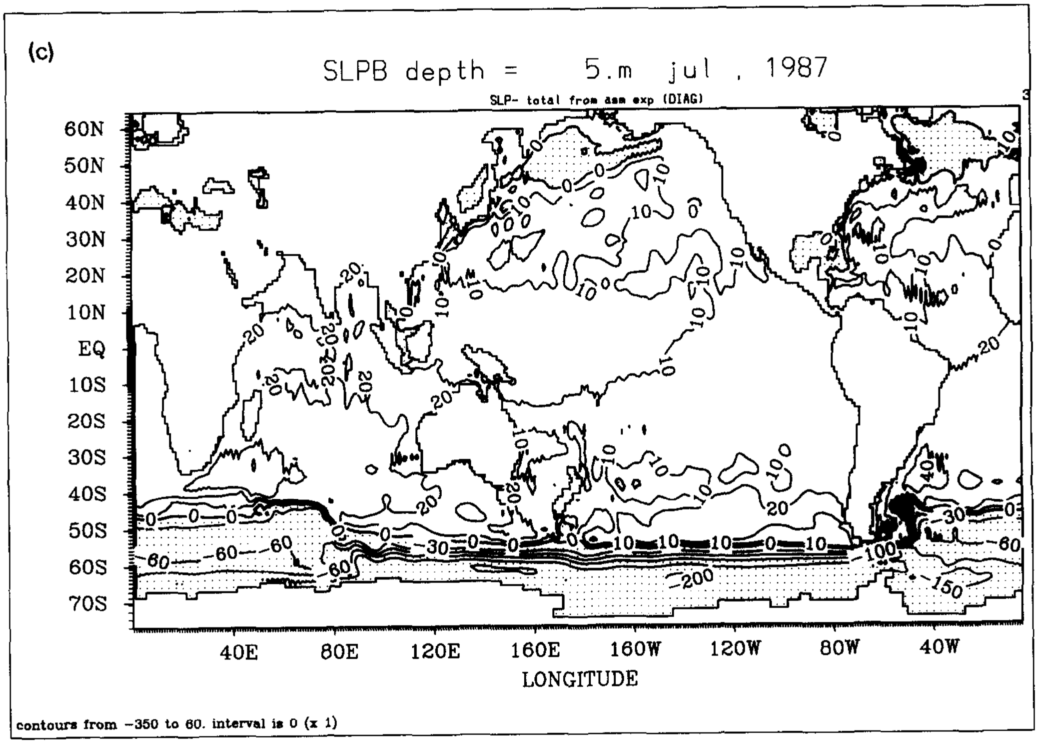


Fig. 1. Sea surface height for July 1987 computed from the global ocean model. (a) the solution of Eq. (14); (b) the baroclinic component from Eq. (26) and (c) the barotropic component from Eq. (25). Units are cm.

Knowing that $H\bar{u}^{(0)} = \hat{k} \times \nabla\psi$, where ψ is the transport streamfunction, we can write

$$\nabla \cdot (H \nabla p_s) = -\nabla \cdot \int_{-H}^{0} (z+H) \nabla \rho \, dz + \nabla \cdot (f \nabla \psi) \quad (20)$$

where the superscripts have been dropped. This relation shows that the sea surface pressure is composed of barotropic and baroclinic contributions from the interior flow fields. The baroclinic contribution results from a vertical integral of the density structure of the ocean. This could have been expected since we usually compute dynamic height at the surface to compare with surface pressure satellite data. Mellor et al. (1982) deduced a somewhat equivalent form of Eq. (20) for the velocity field and called part of the first term in the r.h.s. of Eq. (20) the potential energy.

If we examine now the case of open ocean flat regions of limited extension for which we can consider $f = 1$, p_s can be split explicitly in barotropic and baroclinic components, e.g.:

$$p_s = \frac{\psi}{H} - \frac{\int_{-H}^{0} (z+H) \rho \, dz}{H} \quad (21)$$

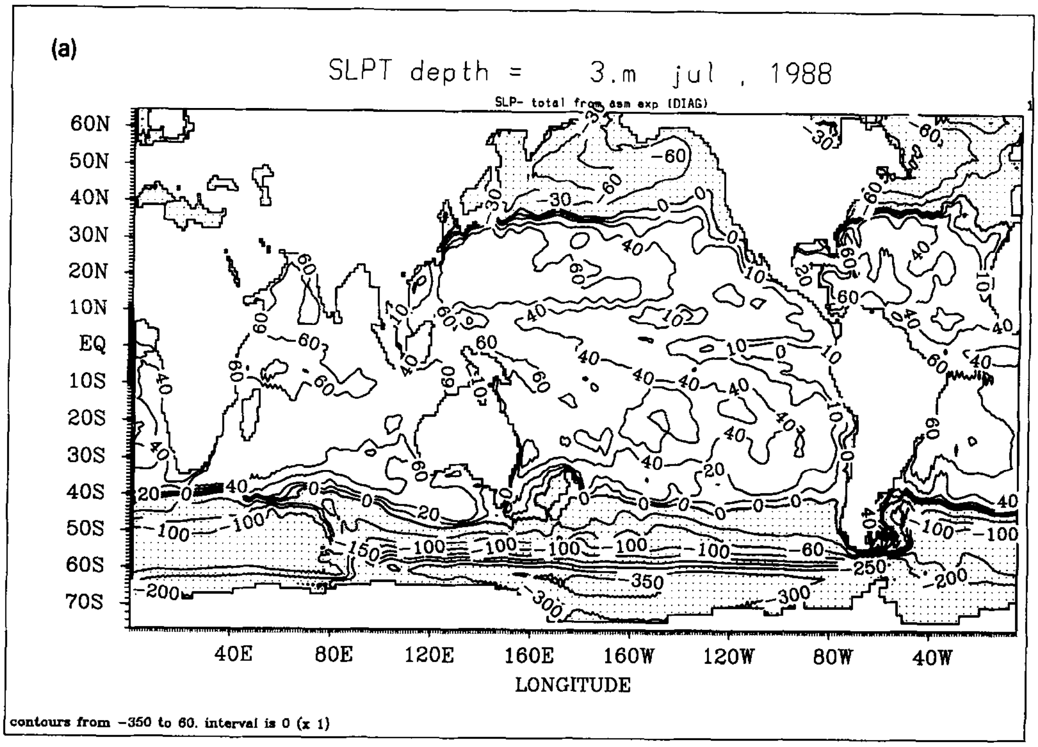
or dimensionally:

$$\frac{1}{\rho_0} p_s = \frac{f_0 \psi}{H} - \frac{g}{\rho_0 H} \int_{-H}^{0} (z+H) \rho \, dz \quad (22)$$

Since $h = p_s / \rho_0 g$ then:

$$h = \frac{f_0}{g} \frac{\psi}{H} - \frac{1}{\rho_0 H} \int_{-H}^{0} (z+H) \rho \, dz \quad (23)$$

We want to transform Eq. (23) in the familiar relationship between h and the surface geo-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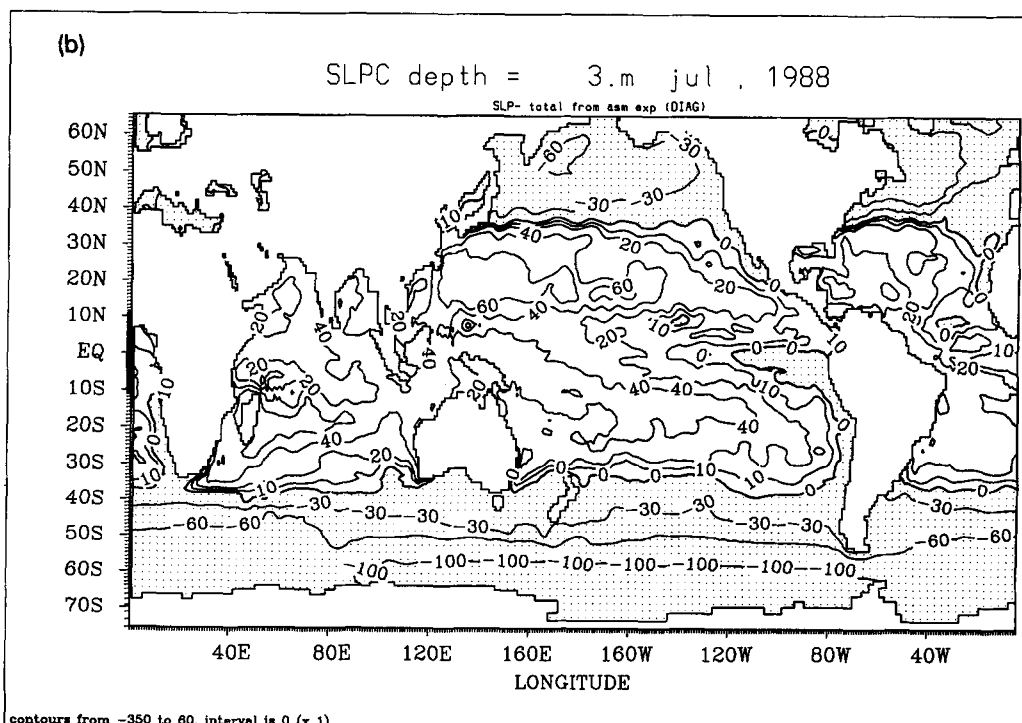
LONGITUDE
contours from -350 to 60. interval is 0 (x 1)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50N
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EQ
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20S
30S
40S
50S
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LONGITUDE
contours from -350 to 60. interval is 0 (x 1)

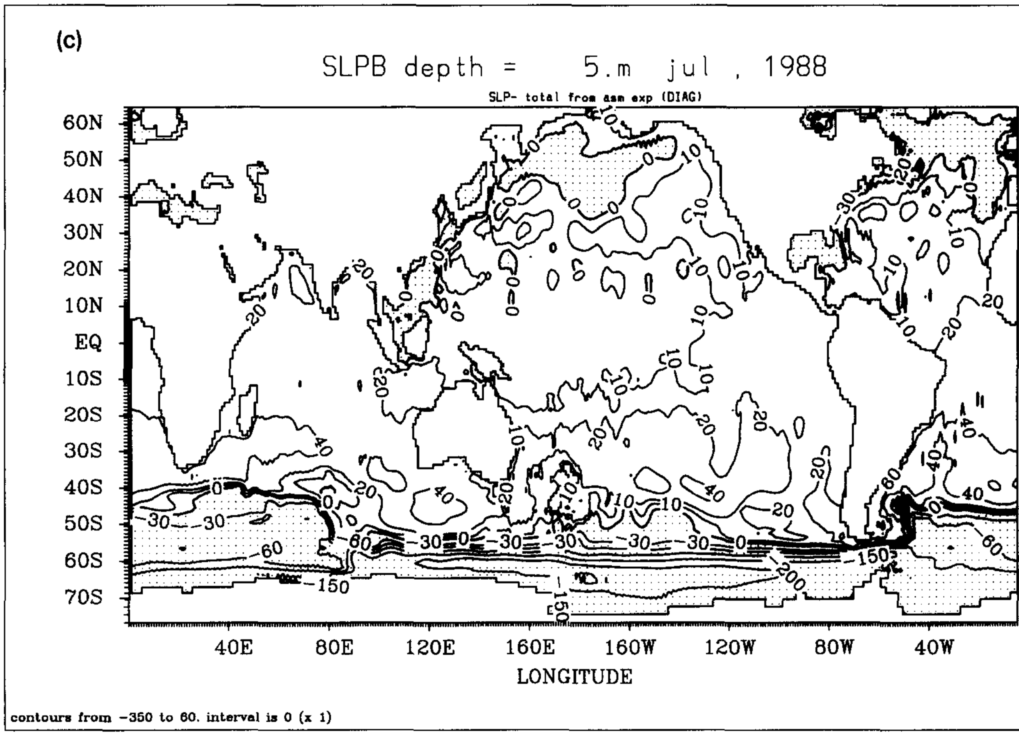

Fig. 2. Sea surface height for July 1988 computed from the global ocean model. (a) the solution of Eq. (14); (b) the baroclinic component from Eq. (26) and (c) the barotropic component from Eq. (25). Units are cm.

strophic streamfunction, called $\phi(x, y, z, t)|_{z=0}$. To achieve this, we use the geostrophic balance in the hydrostatic relationship which results in:

$$\rho^{(0)} = -\phi_z \frac{\rho_0 f_0}{g}$$

and the equality, $\int_{-H}^{0} \phi(x, y, z, t)\, dz = \psi/H$ for the geostrophic barotropic mode. We obtain:

$$\begin{aligned} h &= \frac{f_0}{g}\int_{-H}^{0} \phi(x, y, z, t)\, dz \\ &\quad + \frac{f_0}{g}\left\{\phi(x, y, z, t)|_{z=0} - \int_{-H}^{0} \phi\, dz\right\} \\ &= \frac{f_0}{g}\phi(x, y, z, t)|_{z=0} \end{aligned} \tag{24}$$

Thus this classical formula is valid in the limit of $f \sim f_0$ and small topographic changes. The usage of surface altimeter data as surface streamfunction to control the dynamical evolution of the flow fields has shown different degrees of uncontrollability (Haines, 1991). This depends crucially on the number of levels used, e.g., the number of baroclinic vertical modes used in each model, as shown recently by Rienecker (1994, pers. commun.).

In the following we will discuss the baroclinic and barotropic contribution to the surface pressure in the geostrophic limit but for finite amplitude topography and variable Coriolis parameter.

4. Barotropic and baroclinic components of the sea surface pressure

Here we will show the results of using the diagnostic sea surface pressure Eq. (14) to identify barotropic and baroclinic components of the ocean sea surface pressure field. The model here is the GFDL-MOM adapted to the world ocean geometry by Rosati and Miyakoda (1988) and used for data assimilation studies by Derber and Rosati (1989). The experiments use realistic sea surface boundary conditions as described in Rosati and Miyakoda (1988) and they were integrated for about a decade. We examined the results of integrations from 1987 to 1988 and computed the sea surface pressure diagnostically with Eq. (17). Furthermore we split the geostrophic relationship Eq. (20) into its barotropic component:

$$\nabla \cdot \left(H \nabla p_s^{(1)}\right) = \nabla \cdot (f \nabla \psi) \tag{25}$$

and baroclinic component:

$$\nabla \cdot \left(H \nabla p_s^{(2)}\right) = -\nabla \cdot \int_{-H}^{0} (z+H) \nabla \rho \, \mathrm{d}z \tag{26}$$

so that the total geostrophic pressure is $p_s = p_s^{(1)} + p_s^{(2)}$. Numerically we solve the two elliptic problems Eqs. (25) and (26).

In Figs. 1 and 2 we show the corresponding ($h = p_s / \rho_0 g$) sea surface height fields for the July months of 1987 and 1988. The mean surface level is unknown in our system and thus we have subtracted the horizontal average of each field. The two months of July correspond to the two opposite phases of the large El Niño event which started in December 1986–January 1987.

In the tropical region (~ 10 $S - 10°N$) we notice the distinct change in sea level height at the Eastern boundary going from an El Niño (July 1987) to La Niña phases of ENSO. The total change is of the order of 20 cm. During El Niño phase the northern and southern Subtropical Gyres are disconnected (see the 40 cm isoline in Fig. 1a and b) while they connect across the equator in July 1988.

The subtropical and subpolar gyres are well defined in the Pacific while the Atlantic subtropical gyre is less defined in terms of closed streamlines. The Atlantic subpolar gyre is deeper than the corresponding Pacific one.

The Antarctic Circumpolar Current (ACC) is strong and it meanders on several locations. The southern Indian ocean shows a well defined subtropical gyre which composes part of the Agulhas retroflection region.

It is interesting to compare the relative importance of barotropic versus baroclinic contributions to the total sea surface pressure signal at the geostrophic level. The baroclinic contribution dominates everywhere except in the ACC region where the barotropic contribution is over 50% of the total signal. The subpolar gyres have also a nonneglegible barotropic component even though the baroclinic dominates. The subtropical areas of the southern Atlantic and Inidian Oceans show also relevant contributions from the barotropic component.

The time variability of the sea surface pressure is contained both in the barotropic and baroclinic components, as shown by comparing Fig. 1 and Fig. 2. The largest baroclinic changes are in the equatorial regions, as expected, but relevant amplitude variability occur also in the North Atlantic subpolar gyre.

5. Summary

We have shown the analytical surface pressure formulation of the oceanic primitive equation problem in the rigid lid approximation. We have demonstrated the diagnostic relationship between the sea surface height and the interior dynamical variables which shows that the sea surface pressure cannot control by itself the internal dynamical structure of the ocean. It is then concluded that for these dynamical reasons, the sea surface pressure cannot be simply inserted at the surface of GCM, expecting the associated changes in the subsurface interior flow field.

The geostrophic limit of the surface pressure equation has been demonstrated for finite amplitude topography and variable Coriolis parameter (the standard limit of small topographic excursions and $f \sim f_0$ has also been recovered). It is also concluded that quasigeostrophic models are uncontrollable by insertion of surface information only. The geostrophic sea surface height for the global ocean has been studied with respect to its baroclinic and barotropic components and we show that subtropical and subpolar gyres have large interannual changes in amplitude. The Antarctic Circumpolar Current is shown to be the major current system to have relevant contribution at the sea surface from the vertically integrated velocity field.

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References

Arnault, S. and Perigaud, C., 1992. Altimetry and models in the tropical oceans: a review. Oceanol. Acta, 15: 411–430.

Bryan, K., 1969. A numerical method for the study of the circulation of the world ocean. J. Comput. Phys., 4: 347–376.

Canuto, C., Hussaini, M.Y., Quarteroni, A. and Zhang, T.A., 1988. Spectral Methods in Fluid Dynamics. Springer, Berlin, 2nd ed., 547 pp.

De Mey, P. and Robinson, A.R., 1987. Assimilation of altimeter eddy fields into a limited area quasi-geostrophic model. J. Phys. Oceanogr., 17(12): 2280–2293.

Derber, J. and Rosati, A., 1989. A global oceanic data assimilation system. J. Phys. Oceanogr., 19: 1333–1347.

Demin, Y.L. and Ibraev, R.A., 1986. Basin level boundary problem in Sea Current Models. Izv. Atm. Ocean. Phys., 22(7): 585–590.

Demin, Y.L. and Ibraev, R.A., 1989. A numerical method of calculation of currents and sea surface topography in multiply conneted domains of the ocean. Sov. J. Numer. Anal. Math. Modell., 4(3): 211–225.

Dukowicz, J.K., Smith, R.D. and Malone, R.C., 1993. A reformulation and implementation of the Bryan–Cox–Semtner ocean model on the connection machine. J. Atmos. Oceanol. Techn., 10: 195–208.

Ezer, T. and Mellor, G.L., 1994. Continuos assimilation of Geosat altimeter data into a three dimensional primitive equation Gulf Stream model. J. Phys. Oceanogr., 24: 832–847.

Gresho, P.M. and Sani, R.L., 1987. On pressure boundary conditions for the incompresible Navier–Stokes equations. Int. J. Numer. Meth. Fluids, 7: 1111.

Gresho, P.M., 1991. Some current CFD issues relevant to the incompressible Navier Stokes equations. Comput. Meth. Appl. Mech. Eng., 87: 201–252.

Haidvogel, D.B., Wilkin, J.L. and Young, R., 1991. A semi-spectral primitive equation ocean circulation model using vertical sigma and orthogonal curvilinear horizontal coordinates. J. Comput. Phys., 94: 151–185.

Haines, K., 1991. A direct method for assimilating sea surface height data into ocean models with adjustment to the deep circulation. J. Phys. Oceanol., 21, 6: 843–868.

Hurlburt, H.E., 1986. Dynamic tranfer of simulated altimeter data into subsurface information by a numerical ocean model. J. Geophys. Res., 91: 2372–2400.

Marchuk G.I. and Sarkisyan, A.S., 1986. Mathematical Modelling of Ocean Circulation. Springer, Berlin, 292 pp.

Mellor, G.L. and Ezer, T., 1991. A Gulf Stream model and an altimetry assimilation scheme. J. Geophys. Res., 96: 1171–1192.

Mellor, G.L., Mechoso, C.R. and Keto, E., 1982. A diagnostic calculation of the general circulation of the Atlantic ocean. Deep-Sea Res., 29(10A): 1171–1192.

Rosati, A. and Miyakoda, K., 1988. A general circulation model for upper ocean simulation. J. Phys. Oceanogr., 18: 1601–1626.