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Climate Models and the Astronomical Theory of the Ice Ages*

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Attempts at determining the climatic response to perturbations in the Earth's orbital parameters are reviewed. The relationship between equilibrium and nonequilibrium responses and its implications for climatic sensitivity are discussed in the context of an empirical model due to Imbrie and Imbrie. Some counterintuitive features of the linear equilibrium response to the perihelion cycle in a simple energy balance model are then described in detail. The results of North and Coakley and of Pollard are examined as examples of results that are, respectively, discouraging and mildly encouraging for proponents of the astronomical theory of the ice ages. The attempt by Suarez and Held to address some of the deficiencies in the simplest energy balance models is reviewed, followed by a brief preview of some calculations of relevance to the ice age problem performed by Manabe and co-workers with an atmospheric general circulation model.

1. INTRODUCTION

The astronomical or Milankovitch theory of the ice ages interprets the Quaternary climatic variations on time scales from ~10,000 to 100,000 years as the response to perturbations in the Earth's orbital parameters. The work of Hays *et al.* (1976) suggests that the 40,000-year obliquity cycle and the 18,000- and 23,000-year cycles in the longitude of the perihelion are indeed observed in paleoclimatic indices extracted from deep-sea cores. Whether the larger amount of variance observed at longer (~100,000 year) periods has any connection with eccentricity variations on these longer time scales or, because of nonlinearities in the response, with the obliquity or perihelion cycles, however, is more controversial. Kominz and Pisias (1979), in particular, find little coherence between the paleoclimatic records utilized by Hays *et al.* and the known eccentricity variations.

Other theories have been offered for climatic variations on these time scales, most notably varying volcanic aerosol loading of the stratosphere (e.g., Pollack *et al.*, 1976) and stochastic theories that relate low-frequency variability to the forcing of a system with long relaxation times by white noise internal variations (Hasselmann, 1976; Lemke, 1977). The stochastic models predict variability with a red spectrum at frequencies higher than the reciprocal of the long relaxation time and a white spectrum at lower frequencies. Some of the spectra displayed by Hays *et al.* do at least superficially resemble a red noise continuum up to ~100,000-year periods, on which are superimposed modest peaks at the obliquity and perihelion frequencies. It may indeed be the case that perturbations in orbital parameters have a discernible influence on paleoclimates but that this influence accounts for only a rather modest portion of the Quaternary climatic variations.

Whether or not the astronomical forcing eventually explains enough of the variance to be of predictive value, it still has great potential significance if the orbital signal can be isolated. A much more familiar orbital variation, the seasonal cycle, provides invaluable tests of our understanding of climatic sensitivity to perturbations with time

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scales comparable to the annual period. The orbital variations on much longer time scales provide what are likely to be unique probes of the climatic responses of the more sluggish parts of the system that do not respond significantly to the seasonal cycle.

2. THE ORBITAL PARAMETERS

As the eccentricity, longitude of perihelion, and obliquity vary in time, the semimajor axis of the orbital ellipse, a, and the length of the year remain invariant. When the orbit is circular, its radius is therefore equal to a. For this circular orbit, we denote the solar irradiance on a horizontal surface at the top of the atmosphere, averaged over a diurnal cycle, by $Q\bar{s}\,(\Phi;\,\theta,\,t)$, where θ is latitude and t the time of year. The unit of time is chosen so that $2\pi = 1$ year, and the origin of time is chosen to be vernal equinox. The normalization condition

$$\frac{1}{2}\int_{-\pi/2}^{\pi/2} \bar{s}(\Phi;\,\theta,\,t)\cos(\theta)\,d\theta = 1 \qquad (1)$$

defines Q. The obliquity Φ is the angle between the axis of rotation and the normal to the orbital plane. The value of Φ does not affect the global mean irradiance at any time but only its distribution with latitude. Increasing Φ by a small amount above its present value of 23.5° results in larger seasonal variation and a smaller meridional gradient of the irradiance in the annual mean. The annual mean of $\bar{s}$ at the pole can be shown to be $4\sin(\Phi)/\pi$. The function $\partial\bar{s}(\Phi;\,\theta,\,t)/\partial\Phi$ evaluated at $\Phi = 23.5°$ is plotted in Fig. 13 of Suarez and Held (1979).

For an elliptical orbit of eccentricity ϵ and longitude of perihelion λ, the irradiance takes the form

$$Qs(\epsilon,\,\lambda,\,\Phi;\,\theta,\,t) = Q\,\bar{s}(\Phi;\,\theta,\,\omega(t))a^2/\tau^2(t), \qquad (2)$$

where $\tau(t) = a(1-\epsilon^2)/(1+\epsilon\cos(\omega(t)-\lambda))$ is the Earth–Sun distance and $\omega(t)$, the angle between the line connecting the Sun with the position of the Earth at time t and the line connecting the Sun with the position of the Earth at vernal equinox, is the solution to the equation

$$\frac{d\omega}{dt} = \frac{a^2}{\tau^2}(1-\epsilon^2)^{1/2} = \frac{(1+\epsilon\cos(\omega(t)-\lambda))^2}{(1-\epsilon^2)^{3/2}}, \quad (\omega(0)=0). \qquad (3)$$

Perihelion occurs when $\omega = \lambda$. These definitions are illustrated in Fig. 1. It follows from (2) and (3) that

$$s(\epsilon,\,\lambda,\,\Phi;\,\theta,\,t)\,\frac{dt}{d\omega} = \bar{s}(\Phi;\,\theta,\,\omega)/(1-\epsilon^2)^{1/2}, \qquad (4)$$

from which one sees that the annual mean irradiance at any latitude is independent of λ, and proportional to $(1-\epsilon^2)^{-1/2}$. The value of λ affects only the distribution of irradiance around the year at a particular latitude. The secular variation of Φ, ϵ, and $\epsilon\sin(\lambda)$ as computed from celestial mechanics for the past 100,000 and next 50,000 years are plotted in Fig. 2, using the convenient expressions provided by Berger (1978).

The annual, global mean of the irradiance is proportional to $(1-\epsilon^2)^{-1/2} \approx 1 + \frac{1}{2}\epsilon^2$. Increasing ϵ from 0 to 0.04 results in less than a 0.1% increase in this quantity. The response of global mean surface tempera-

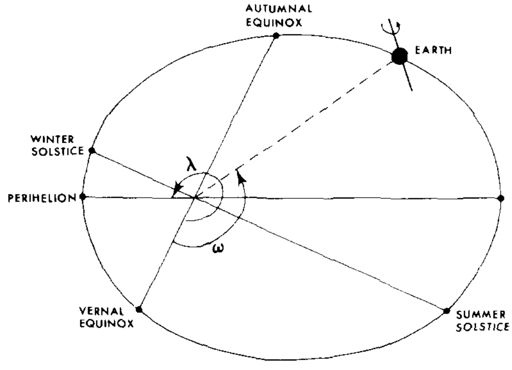

FIG. 1. A schematic of the Earth's orbit, illustrating the definitions of the angles λ and ω.

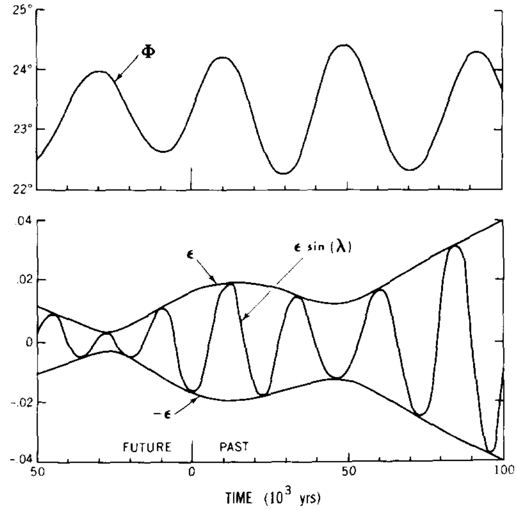

FIG. 2. The time evolution of Φ and ϵ sin(λ) computed from the expression in Berger (1978). Eccentricity variations are also shown as the envelope to the variations in ϵ sin(λ).

ture to a 1% change in solar constant obtained from the general circulation model calculations of Wetherald and Manabe (1975), for example, is less than 2°K. The temperature difference between the present climate and the peak of the last major glacial advance (~18,000 years ago) averaged over the ocean surface is ~2–3°K, according to the deep-sea core analysis of CLIMAP (1976), and factoring in the land temperatures would likely increase this difference. Thus, unless climatic sensitivity is being grossly underestimated, the effect of ϵ on the annual, global mean irradiance can be neglected.

The eccentricity has a much larger, $O(\epsilon)$ rather than $O(\epsilon^2)$, effect on the seasonal distribution of the solar flux, modulating the perihelion cycle through the combinations $\epsilon \cos(\lambda)$ and $\epsilon \sin(\lambda)$ in (2) and (3). The fractional difference in insolation at summer solstice between cases in which $\lambda = \pi/2$ and $\lambda = 3\pi/2$ (perihelion at summer and winter solstices, respectively) is 4ϵ, which is ≈6% at the present eccentricity and as large as 10–20% throughout much of the Quaternary.

3. EQUILIBRIUM AND NONEQUILIBRIUM RESPONSES

It is natural to begin an analysis of the climatic response to orbital parameter variations with the equilibrium or zero-frequency response, the difference between a climate consistent with one fixed set of orbital parameters and that consistent with another set of parameters. This equilibrium response will approximate the true response only if all of the relevant time scales in the system are much shorter than the time scales on which the orbital parameters vary. A quick comparison of the paleoclimatic record with Fig. 2 suffices to demonstrate that an equilibrium response theory cannot be entirely adequate. The values of the orbital parameters 20,000 years ago, near the peak of the last glaciation, are nearly identical to their present values. (This is so because of the ~20,000-year period of the perihelion cycle and the fact that the present value of the obliquity is near the average value in its cycle, which also recurs every ~20,000 years.) Glacial dynamics, the response of the solid Earth to varying glacial loading, the geochemical cycles that maintain the atmospheric CO_2 concentration, and the deep ocean circulation are all potential candidates for providing long time scales. The need for nonequilibrium behavior does not imply that the equilibrium responses are of no interest, however, as the following empirical nonequilibrium model due to Imbrie and Imbrie (1980) makes clear.

Imbrie and Imbrie fit a simple model to the variations in global ice volume, I, as inferred from oxygen isotope ratios in deep-sea cores. They assume that the equilibrium response to the orbital variations takes the form

$$I_{eq} = I_0 - \gamma \frac{\delta\Phi}{\Delta\Phi} - \beta \frac{\epsilon}{\Delta\epsilon} \cos(\lambda - \phi). \quad (5)$$

I_0 is the ice volume at zero eccentricity and some standard value of the obliquity Φ_0($\delta\Phi \equiv \Phi - \Phi_0$). The parameters γ and β set the magnitude of the response to obliquity perturbations and the perihelion cycle, respectively ($\Delta\Phi$ and $\Delta\epsilon$ being the rms variations of Φ and ϵ over the past 500,000 years), and ϕ determines the phase of the perihelion cycle at which the ice is minimum. As illustrated below for a simple energy balance model, (5) is precisely the form of any *linear* equilibrium response to the orbital parameters. Imbrie and Imbrie then assume that I relaxes to I_{eq} on a time scale which depends on whether the ice is growing or decaying,

$$dI/dt = -(I - I_{eq})/\tau, \qquad (6)$$

where

$$\tau = \tau_c \quad \text{if } I < I_{eq},$$
$$= \tau_w \quad \text{if } I > I_{eq}.$$

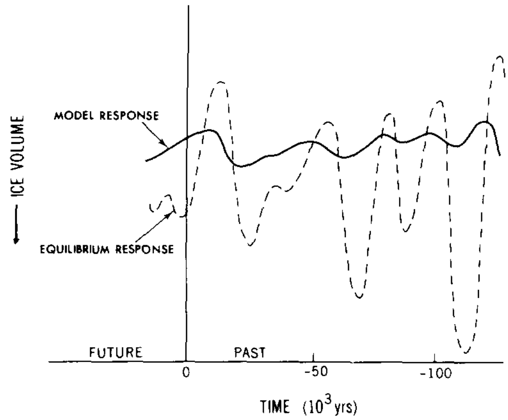

FIG. 3. The response of the Imbrie and Imbrie (1980) model to orbital variations (solid line), and the equilibrium response to which the solution attempts to relax (dashed line). Ice volume increases downwards.

An asymmetry between slow growth of ice and fast decay is evident upon inspection of the ice volume record. The best fit is obtained for the parameter values $\tau_c = 42{,}000$ years, $\tau_w = 10{,}600$ years, $\gamma > 0$, $\beta/\gamma = 2$, and $\phi = 125°$. The implication is that if the climate had time to adjust, smallest ice volumes would occur for large obliquity and for perihelion between summer solstice and autumnal equinox, with the obliquity perturbations and the perihelion cycle being of comparable importance. The resulting $I(t)$ and $I_{eq}(t)$ curves are shown in Fig. 3. The long time scales needed to produce significant phase lags have also succeeded in reducing the amplitude to 10–20% of the equilibrium response. As is apparent in the figure, this model predicts that the equilibrium climate consistent with the present parameters is an ice age more severe than any on record. If this model does indeed reflect the physics of the system, then we should all be thankful that the orbital parameters change as rapidly as they do!

The extreme sensitivity of equilibrium states implied by this empirical model is striking. It is difficult enough to obtain equilibrium responses of ice age magnitude from climate models perturbed by the orbital parameters, let alone responses nearly an order of magnitude larger. Models with two or more "slow" variables, such as the ice sheet plus crustal deformation models of Birchfield and Weertman (1978), Källén *et al.* (1979) and Oerlemans (1980), have the potential for generating realistic glacial cycles without requiring such large equilibrium sensitivities. They also have the potential for producing the predominant ~100,000-year peak in the variance. (Equilibrium response models produce negligible variance at 100,000 years; the Imbrie and Imbrie asymmetric decay model produces some variance at these low frequencies but not as much as is present in the ice volume record.) However, the analysis of such models and, more importantly, the critical evaluation of the assumptions made in the "slow" physics are still in their infancy. In particular, the recent CO_2 measurements in Antarctic ice cores (Berner *et al.*, 1980; Delmas *et al.*, 1980) indicate that the dependence of the geochemical cycles maintaining the CO_2 concentration in the atmosphere on ice volume and other climatic variables may be the crucial "slow" physics in the problem.

Despite its limitations, it is instructive to consider in more detail the form of the linear equilibrium response in the following simple energy balance model:

$$C\frac{\partial T}{\partial t} = Qs(\epsilon, \lambda, \Phi; \theta, t)\alpha(T) - (A + BT) + \frac{D}{\cos(\theta)}\frac{\partial}{\partial \theta}\left(\cos(\theta)\frac{\partial T}{\partial \theta}\right). \quad (7)$$

$T(\theta, t)$ is the surface temperature as a function of latitude and time, C a heat capacity needed to produce a reasonable amplitude of the seasonal temperature cycle, D the effective meridional heat diffusivity, A and B constants defining the temperature dependence of the long-wave flux escaping to space, and $\alpha(t)$ a temperature-dependent planetary co- albedo. We set $\alpha(T) = \alpha_1$ if $T < T_0$ and $\alpha(t) = \alpha_2$ if $T > T_0$, where T_0 can be thought of as the freezing temperature. This time-dependent version of the model described in North (1975) is not put forward as particularly appropriate for the ice age problem, but rather as a means of exhibiting certain features of the linear response in a simple context.

In light of (4), it is convenient to change independent variables from t to ω. To first order in ϵ, (3) implies that $dt/d\omega = 1 - 2\epsilon \cos(\omega - \lambda)$, so that

$$C\frac{\partial T}{\partial \omega} = Q\,\bar{s}(\Phi; \theta, \omega)\alpha(T) + (1 - 2\epsilon\cos(\omega - \lambda))L(T), \quad (8)$$

where $L(T) \equiv \frac{D}{\cos(\theta)}\frac{\partial}{\partial \theta}(\cos(\theta)T) - (A + BT)$. Denoting the equilibrium solution for a circular orbit and standard obliquity by $\bar{T}$, the solution for small ϵ and $\delta\Phi$ is

$$\bar{T}(\theta, \omega) + \delta\Phi T_\Phi(\theta, \omega) + 2\epsilon Re[T_\lambda(\theta, \omega)e^{-i\lambda}],$$

where T_Φ and T_λ satisfy

$$C\frac{\partial T_\Phi}{\partial \omega} = Q\frac{\partial \bar{s}}{\partial \Phi}\alpha(\bar{T}) + Q\bar{s}\frac{d\alpha(\bar{T})}{dT}T_\Phi + L(T_\Phi)$$

and

$$C\frac{\partial T_\lambda}{\partial \omega} = L(\bar{T})e^{i\omega} + Q\bar{s}\frac{d\alpha(\bar{T})}{dT}T_\lambda + L(T_\lambda),$$

as can be confirmed by substitution. For the albedo temperature dependence chosen, the second term on the right-hand side of these equations reduces to a Dirac δ-function at the unperturbed ice boundary. If $T_\lambda \equiv \Gamma e^{i\phi}$, then $2\epsilon\Gamma$ and ϕ are the amplitude and phase of the response to the perihelion cycle. For example, the temperature at 45°N at summer solstice reaches its maximum value when the longitude of perihelion equals $\phi(45°\text{N}, \pi/2)$. This linearization is an excellent approximation as long as the movement of the ice margin is only a few degrees of latitude, but it retains its qualitative value for much larger perturbations.

Figure 4 shows the nondimensional perturbation amplitude, $B\Gamma(\theta, \omega)/(A + BT_0)$, and the phase, $\phi(\theta, \omega)$, for the following choice of nondimensional parameters: $D/B = 0.30$, $Q/(A + BT_0) = 1.72$, $C/B = 0.75$, $\alpha_1 = 0.4$, and $\alpha_2 = 0.7$. One finds that in all seasons temperatures in high latitudes reach their maximum values when perihelion is close to winter solstice ($\lambda \approx 3\pi/2$). This is the opposite of the relation assumed by Milankovitch (1941) and obtained in other energy balance models (see Section 4). In this particular calculation, albedo feedback in the wintertime evidently dominates over the feedback of opposite sign in the summer. One should not automatically assume that the warmest high-latitude temperatures occur with perihelion at summer solstice; the phase of the response can be counterintuitive and should be deduced from one's modeling assumptions.

From Fig. 4 one also finds that in low latitudes at, say, summer solstice, warmest temperatures occur when perihelion is at vernal equinox. More generally, warmest temperatures at a particular time of year occur when perihelion occurs 3 months earlier. The value of C needed to produce a seasonal cycle of reasonable magnitude is sufficiently large to produce nearly a 3-month phase lag between insolation and temperature, so the latter behavior is to be expected in the absence of albedo feedback. While the details of the phase of the

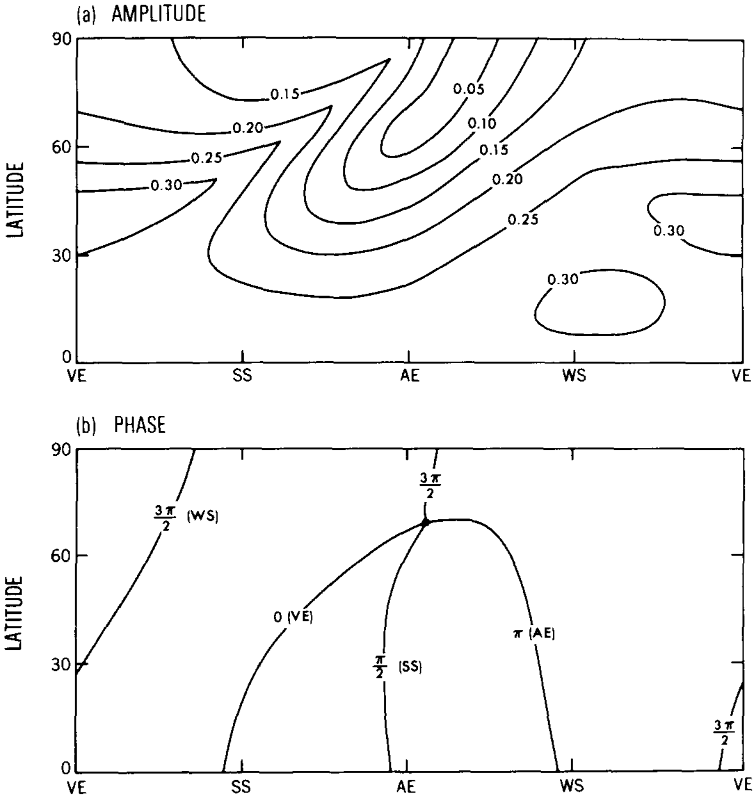

FIG. 4. The nondimensional amplitude, $B\Gamma(\theta, \omega)/(A + BT_0)$, and phase, $\phi(\theta, \omega)$, of the temperature response to the perihelion cycle in the simple diffusive energy balance model described in the text. (VE, SS, AE, WS) = (Vernal equinox, summer solstice, autumnal equinox, winter solstice).

response are parameter and model dependent, the differences between the high- and low-latitude responses are a reminder that different parts of the climatic system need not be so strongly coupled that they all have the same phase of response to the perihelion cycle.

The amplitudes in Fig. 4 are as large as 0.3. Given that $B/(A + BT_0) \approx 0.01$, for $\epsilon = 0.04$ this implies a peak-to-peak temperature change in the perihelion cycle of $(4\epsilon)((A + BT_0)/B)(0.3) \approx 5°\text{K}$ at a particular time of year. Changes in annual mean temperatures are much smaller, however. Figure 5 is a plot in the complex plane of the amplitude and phase of the annual mean temperature response to the perihelion cycle,

$$\frac{B}{A + BT_0} \int_0^{2\pi} d\omega \, (T_\lambda - e^{i\omega} \bar{T}),$$

as a function of latitude. The largest responses are now in high latitudes and correspond to a peak-to-peak annual mean temperature change of $\approx 2°\text{K}$ for $\epsilon = 0.04$. The global mean response is roughly a factor of 3 smaller. For these same parameters, the response of global mean temperatures to a 1% increase in solar constant is $\approx 2.9°\text{K}$. With fixed albedos the response is $\approx 1°\text{K}$, so we are considering a system with strong albedo feedback. Even with this strong feedback, the temperature responses are still of less than ice age proportions.

According to Fig. 5, the warmest annual mean temperatures at all latitudes occur

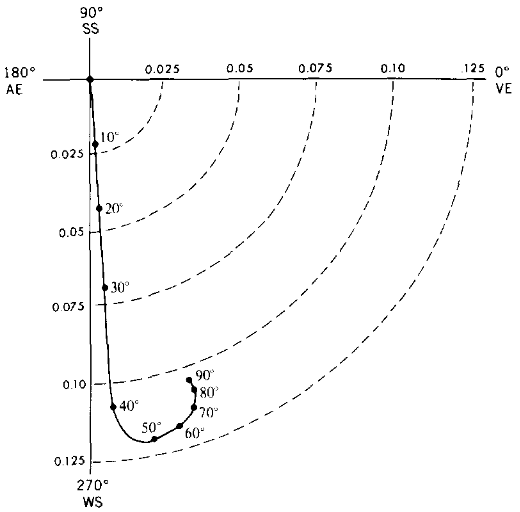

FIG. 5. The nondimensional amplitude and phase of the *annual mean* temperature response to the perihelion cycle for the same model as in Fig. 4, plotted in the complex plane. The responses at different latitudes are labeled.

with perihelion near winter solstice. However, there is a phase difference of ≈15° between the low- and high-latitude responses. As a result, warmest annual mean low-latitude temperatures will occur ≈800 years earlier in the 20,000-year cycle than high-latitude temperatures. The appearance of such a phase lag in the paleoclimatic record could easily be misinterpreted as evidence for "slow" physics and nonequilibrium behavior. Fortunately, no such ambiguity affects the obliquity signal. A phase lag between obliquity and ice volume, such as that obtained by Hayes *et al.* (1976), by isolating that part of the response with 40,000-year period, is unambiguous evidence for a long time scale. In general, the leads and lags between different parts of the climatic system in a particular climatic change may be partly due to slow physics and partly due to lags inherent in the equilibrium response to the perihelion cycle.

4. CLIMATE MODELING

Climate models that have been utilized for sensitivity studies can be divided into two distinct classes. The distinction arises because there exists no satisfactory theory for the general circulation of the atmosphere and, more particularly, for the transports of heat, momentum, and moisture by large-scale eddies. Therefore, one has the choice of working with semiempirical models in which one makes the simplest plausible assumptions concerning atmospheric dynamics, linear meridional diffusion of heat being a good example, or one can work with numerical general circulation models (GCMs) in which the life cycles of individual eddies are explicitly computed. (The impact of oceanic circulation models on estimates of climate sensitivity has not been significant to date, but this may soon change.) GCMs have for the most part addressed the ice age problem through diagnostic calculations of the atmospheric circulation consistent with ice age boundary conditions, rather than by attempting a direct evaluation of the astronomical theory. Several semiempirical energy balance model calculations are described below, followed by a description of recent work with a GCM by Manabe and co-workers.

Estimates of climatic responses to orbital variations have often been discouragingly small, an example being the results of North and Coakley (1979). This model predicts one land and one ocean temperature at each latitude with two equations identical to (7) except for the addition of a term representing zonal heat transport between land and ocean. As in Sellers (1973), this transport is assumed to be proportional to the land–ocean temperature difference. Over the ocean, the heat capacity is assumed to be that of 75 m of water; the heat capacity over land, which is assumed to be that of the atmosphere only, is a factor of 30 smaller. Some such distinction is needed to obtain a reasonable phase of the seasonal cycle over land. Some interaction between land and ocean is then needed to reduce the amplitude of the cycle over land. The model's high-albedo "snowcover" over land extends to the seasonally varying 0°C isotherm, and a permanent ice cap over land and ocean is located poleward of the $-10°C$ *annual mean* isotherm. The height of the ice sheets is assumed to have no effect on the energy balance. The model produces a modest 3° latitude shift in the permanent ice cap and a global mean temperature change of a small fraction of a degree for a change in obliquity of $\approx 1°$, and even smaller changes for the perihelion cycle.

One expects this insensitivity of the ice cap to the perihelion cycle to be shared by all energy balance models in which the ice cap size is linked to annual mean temperatures. In such models, the redistribution of insolation by the perihelion cycle only affects the annual mean temperatures to the extent that the insolation anomalies are rectified by the seasonally varying albedos. To illustrate the difficulty in obtaining significant responses from such a model, we note that the annual mean temperature change due to a similar rectification of the seasonal cycle itself (the temperature difference between models forced by seasonally varying and annually averaged irradiance) in the idealized GCM of Wetherald and Manabe (1981) is only $\approx 1°K$ in the global mean and 4–6°K in high latitudes. These temperature changes result in only $\approx 5°$ latitude shift in the position of the 270°K annual mean isotherm. One must expect a proportionally smaller rectification from the much smaller redistribution due to a change in perihelion.

Fortunately for the astronomical theory, there is no reason to suspect that the location of the ice cap boundary is determined primarily by annual mean temperatures. The sensitivity of the ice cap increases dramatically if it is tied to summertime temperatures, which would, for example, be the case for the following highly idealized model. Suppose that snow falls whenever $T < 0°C$, but at a very slow rate. When temperatures rise above 0°C, complete melting occurs almost instantaneously because of the small accumulation. Ignoring glacial flow, snow can only accumulate poleward of the latitude of maximum poleward extension of the 0°C isotherm over land. This summertime position of the isotherm over land will, in turn, be most sensitive to summertime insolation because of the small heat capacity of the land surface. [If one actually tries to incorporate this modification into the North–Coakley model, one finds it difficult to prevent snow from melting in the summer for reasonable parameter choices. Suarez and Held (1979) have the same difficulty in their energy balance model, described below. This may simply imply that the initiation of glaciers can occur only in maritime climates, with abundant snowfall and moderate summertime temperature, or, perhaps, in highlands.]

Pollard *et al.* (1980) consider a model consisting of an energy balance essentially identical to (7) coupled to an ice sheet. The ice sheet model is that discussed by Weertman (1964), which assumes perfectly plastic flow and yields a parabolic height profile. Snowmelt is computed from an empirical relation dependent on insolation and temperature of the ice sheet surface, which in turn is computed from the sea-level temperature by assuming a constant lapse rate. Snowfall is set equal to the observed climatological annual mean precipitation as a function of latitude whenever the surface temperature drops below freezing. The ice budget is integrated with a time step of 2,000 years; at each time step the energy balance model is integrated to equilibrium to diagnose the atmospheric state and the resulting snowfall and melt.

A variety of problems arise when trying to incorporate ice sheets into energy balance models. Pollard *et al.* take the variable in (7) to be sea-level temperature, except for the albedo term, which is evaluated with surface temperature. Given the assumption of a constant lapse rate between sea level and mid-troposphere, one could try to justify the use of sea-level temperature in the outgoing long-wave and meridional heat transport parameterizations by arguing that both are dependent primarily on mean or mid-tropospheric temperatures. Using a detailed radiative model, however, K. Bowman (personal communication) has computed the effect of ice sheet height on the long-wave flux escaping to space, with temperatures a fixed function of height above sea level, and found a substantial decrease in flux with increasing ice sheet height. Incorporating these radiative calculations into an annual mean energy balance model, he finds that this effect has a strong *stabilizing* influence: colder temperatures result in a larger and higher ice cap, the increase in height resulting in even colder surface temperatures, a smaller outgoing flux, and, therefore, less of a cooling at sea level than if the height increase were absent. The effects of the ice sheet on the horizontal heat flux are also potentially important, but much more difficult to estimate.

The Pollard *et al.* model responds to the time-dependent orbital perturbations with movements of the ice boundary of nearly 10° latitude, which the authors argue is of the right size to explain the secondary variations producing the spectral peaks at 18,000, 23,000, and 40,000 years in the ice volume record, but not at the dominant ~100,000-year glacial–interglacial cycles. The model ice cap grows when obliquity is small or perihelion is near winter solstice. These results are more promising than those of North and Coakley, presumably because of the sensitivity of the ice sheet model to summertime temperature.

Equilibrium responses are not discussed by Pollard *et al.*, in any detail, but it is clear from their Fig. 4 that they are considerably larger than the responses to the time-varying perturbations, ranging from no ice cap at all to an ice cap boundary at 45°N. The "inertia" of the ice sheet has significantly reduced the ice fluctuations, in qualitative agreement with the Imbrie and Imbrie fit. However, analysis of the equilibrium responses is complicated by the instability of small ice sheets in this model (those with boundary poleward of ≈55°N) and the associated nonuniqueness of equilibrium climates. A completely deglaciated Northern Hemisphere is a stable equilibrium state in their model, just as is the partially glaciated state to which all of the results described above refer. The time-varying orbital perturbations are not sufficient to force a transition from one stable state to the other; however, if the model is given sufficient time to relax to its equilibrium response, then the transition from partially glaciated to unglaciated state would occur, after which the model would remain unglaciated. This instability and nonuniqueness is sensitive to the modeling assumptions. In particular, the small caps can be stabilized by forcing the precipitation to decrease sufficiently rapidly with increasing ice cap size. To the extent that this small ice cap instability is related to a similar instability in the simplest annual mean models (Held and Suarez, 1974; North, 1975; Held *et al.*, 1981), it should also be very sensitive to the magnitude of the effective heat diffusivity in high latitudes and to the albedo formulation. One suspects that the stability or instability of small ice sheets may be of central importance for the ice age problem.

Suarez and Held (1976, 1979) describe equilibrium responses in a somewhat more complex energy balance model designed to rectify what the authors view as some of the serious deficiencies of models based on (7). One deficiency becomes apparent when the observed seasonal variation of the long-wave flux escaping to space is compared with the seasonal variation of surface temperatures. If the data at each latitude are separately fit with a straight line, flux = $A + BT$, the parameter B is found to be twice as large in low as in high latitudes, disregarding the deep tropics where the outgoing flux is more a function of the position of the high clouds associated with the intertropical convergence zone than with surface or atmospheric temperatures (Held and Suarez, 1974). As a result, one expects models based on (7) to overestimate sensitivity in low latitudes and underestimate it in high latitudes. This is consistent with the calculations of sensitivity to the solar constant in a highly truncated GCM with fixed surface albedos described in Held (1978), which shows surface temperatures in high latitudes twice as sensitive as in low latitudes. In the same circumstances, (7) predicts slightly larger sensitivity in low latitudes. This deficiency is also apparent in attempts to fit GCM results with models based on (7), such as that in Coakley and Wielicki (1979) and in North and Coakley (1979).

These latitudinal differences in sensitivity are related to changes in the tropospheric static stability. In the GCM, low-latitude stability increases with increasing insolation because the moist adiabatic lapse rate is smaller at the higher temperatures, whereas in high latitudes heat is not transferred efficiently from the lower to upper troposphere and the stability decreases with increasing insolation (as one might intuitively expect for an atmosphere heated primarily from below). Modifying (7) by making B latitude dependent would be of limited utility, since one expects the stable atmosphere that prevents efficient vertical mixing to expand equatorward as the ice cap expands and the region dominated by moist convection to contract toward the equator. Suarez and Held attempt instead to predict the static stability by dividing the troposphere into two layers. The model's static stability, the difference in potential temperature between the two layers, is maintained by a balance between radiative fluxes, differential horizontal diffusion in the two layers, and vertical convective heat fluxes approximated with a simple moist convective adjustment.

Whatever the merits or deficiencies of the particular scheme in this model, it seems clear that prediction of the tropospheric static stability is one of the most important ways in which simple climate models must develop, not only because of the direct effect of stability changes on surface temperature sensitivity, but also because all dynamical arguments for the magnitude of the horizontal heat flux involve the static stability in some way. The static stability affects the scale of the most unstable baroclinic waves through its effect on the radius of deformation, and also affects the amplitude to which these waves grow if, as seems plausible, this amplitude is related to the available potential energy in the flow, which, in turn, is inversely proportional to the static stability.

Another important difference between

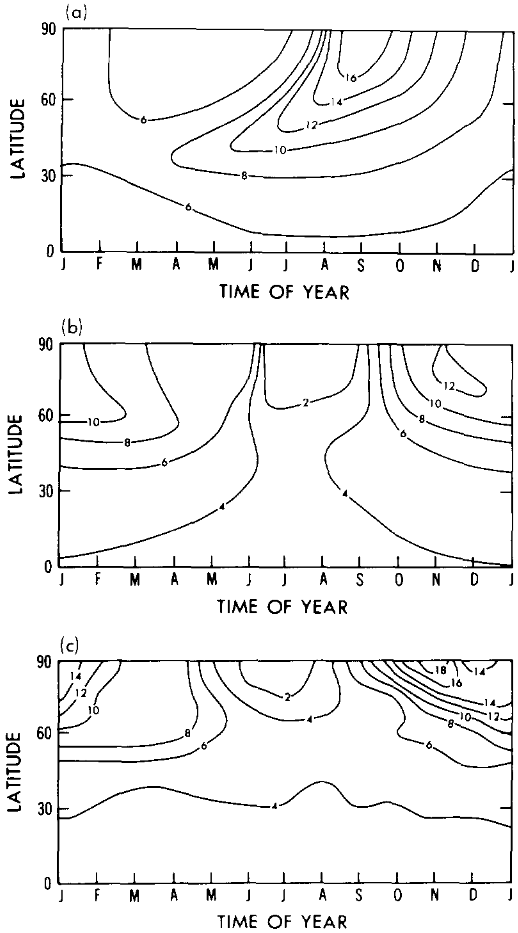

FIG. 6. (a) Surface temperature change due to a 4% increase in solar constant in a one-layer diffusive energy balance model. (b) Same as (a) for an energy balance model with variable static stability, a surface energy balance, and a sea ice model, as in Suarez and Held (1979). (c) Surface temperature change due to fourfold increase in CO_2 in the GCM of Manabe and Stouffer (1980). Temperature changes are in °C.

the Suarez–Held model and the simpler models based on (7) is the separation of the atmospheric and surface energy balances in the former. Among other things, this separation allows one to predict sea ice extent and thickness in a natural way. That sea ice can play an important role in climatic responses is apparent in Fig. 6. The upper plot shows the response to a 4% increase in solar constant in the one-level diffusive model (7), as a function of latitude and time of year; the middle plot shows the same result for a model with the same basic structure as that of Suarez and Held, including the sea ice model; the lower plot shows the response to a fourfold increase in the CO_2 concentration in the GCM of Manabe and Stouffer (1980). In the first model, the largest sensitivities are found in high latitudes in late summer. This is partly due to the smaller meridional temperature gradient in summer, resulting in a larger displacement of a given isotherm for a given temperature change, and partly to the strong summer insolation that enhances albedo feedback. (The result shown is for a case in which the colder of the two experiments has "snowcover" year round at high latitudes. The pattern is rather different if one compares two states with no summer snowcover—the maximum sensitivity shifts to the time of spring snowmelt.)

In the solar constant response of the model with sea ice and in the CO_2 response of the GCM, maximum sensitivity occurs in high latitudes in winter, with a sharp minimum in summer, a difference that can be attributed to the sea ice. In the summer, the heat added to the high-latitude oceans is used to melt ice rather than to raise temperatures. The thinner ice then allows more heat flux from ocean to atmosphere in winter, enhancing the sensitivity of wintertime temperature. One can think of the thinner sea ice as providing less insulation of the atmosphere from the ocean, resulting in a larger effective heat capacity and smaller seasonal variation. While there is no simple correspondence between the response to orbital perturbations and this seasonal dependence of the response to an increase in solar constant, these results do suggest that there is ample potential for error in models not incorporating the effects of sea ice.

The equilibrium calculations described in Suarez and Held show large movements of the permanent snowline over land (the latitude poleward of which snow is present year round) in response to the orbital perturbations, from 85 to 60°N latitude. Warmest climates occur at high obliquity and perihelion between vernal equinox and summer solstice. The difference between the extreme Northern Hemisphere mean temperatures over the past 150,000 years is $\approx 2°C$. However, the model is exceptionally sensitive to solar constant ($\approx 4°C$ for 1% change in Q) due to very strong albedo feedback resulting from excessive ice and snow. Further calculations have shown that one still finds substantial ($>10°$ latitude) movements of the Northern Hemisphere permanent snowline over land when the model is made less sensitive to the solar constant (by decreasing the diffusivity, for example), as long as one modifies the model in other ways to maintain year-round snowcover in high latitudes, but this decrease in sensitivity to Q is accompanied by much smaller annual and global mean temperature responses to the orbital perturbations. It appears to be much more difficult to obtain temperature changes of the magnitude found by CLIMAP in response to orbital perturbations than to obtain ice age-sized shifts in the ice and snow margins.

Recent calculations with an atmospheric GCM by Manabe and co-workers promises to shed some light on the difficulties experienced by the energy balance models. Previous GCM studies of the ice age climate (Williams, 1974; Gates, 1976; Manabe and Hahn, 1977) have taken the ice age ocean temperatures, as well as the continental glaciers, as given, and attempted to compute the atmospheric state consistent with these boundary conditions. More recently, Manabe and Hahn (personal communication) have taken only the continental glaciers as given, using CLIMAP (1976) data for the last glacial maximum, and computed the atmospheric climate and sea surface temperature using a GCM coupled to a simple mixed-layer model of the ocean. The model is identical to that utilized by Manabe and Stouffer (1980) in a CO_2 sensitivity study. The hope is that the model's sensitivity can be verified by comparing the predicted temperature difference over the oceans between glacial maximum and the present against the difference inferred by CLIMAP (1976).

Manabe and Hahn find that their model predicts temperature changes in the North Atlantic comparable to but slightly smaller than CLIMAP's (8 as compared to 12°K) and changes in the North Pacific and southern oceans that are much too small. Temperature changes in the Southern Hemisphere are less than 0.5°K almost everywhere. The comparison improves noticeably if the CO_2 concentration in the glacial maximum calculation is reduced to two-thirds of its present value [taking at face value the measurements of Berner *et al.* (1980)]. Even in this case, temperature changes in the southern ocean are still deficient, but this may be due to the fact that the standard model produces insufficient sea ice around Antarctica. The warmer subtropical oceans at glacial maximum found by CLIMAP (1976) are also not present in the model, perhaps because these result from enhanced poleward oceanic transport in low latitudes. Despite the model's limitations, the lack of oceanic transport and the assumption of fixed cloudiness being the most obvious, these results do suggest that a CO_2 decrease or some other comparable change in the Earth's radiation balance, in addition to the albedo change due to the larger ice and snowcover, is needed to explain the colder ocean temperatures at the last glacial maximum.

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