RESEARCH REPORT SERIES
(*Statistics #2005-08)*

Investigation of Alternative Estimators for the Quarterly Financial Report

Carol Caldwell[1], Irene Evans[1], Andromache Howe[3],
Michael Ikeda, Donald Luery[3], Terry Pennington[1],
Mark Sands[1], Richard Sigman[3], Katherine Thompson[3]

Statistical Research Division
U.S. Bureau of the Census
Washington D.C. 20233

[1] Company Statistics Division, [3] Economic Statistical Methods and Programming Division

Report Issued: October 3, 2005

Disclaimer: This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress. The views expressed are those of the authors and not necessarily those of the U.S. Census Bureau.

INVESTIGATION OF ALTERNATIVE ESTIMATORS FOR THE QUARTERLY FINANCIAL REPORT

Carol Caldwell, Irene Evans, Andromache Howe, Michael Ikeda,
Donald Luery, Terry Pennington, Mark Sands, Richard S. Sigman, Katherine Thompson

ABSTRACT

This report presents the research procedures and resultant recommendations for estimation in the Quarterly Financial Report (QFR). The research was motivated by the upcoming QFR migration to the Economic Directorate's Standard Economic Processing System (StEPS). This evaluation study was conducted by an interdivision team comprised of representatives from the Company Statistics Division (CSD), the Economic Statistical Methods and Programming Division (ESMPD), and the Statistical Research Division (SRD).

The QFR estimator of total differs from traditional design-based estimators in that the weight assigned to each sample corporation is not based on its initial probability of selection. Instead, corporations in the enumerated industry are assigned weights equal to the ratio of the *estimated* total number of corporations in the type of industry at the time of enumeration to the number of sample corporations in that type of industry at the time of enumeration. Because the final QFR weight changes each quarter, the QFR estimator is referred to as a "variable weight estimator." There have been several investigations into the statistical properties of this variable weight estimator but all prior studies have made simplifying assumptions. Instead, this workgroup attempted to account for all of the nuances of the QFR sample design and estimator via a Monte Carlo simulation study, comparing the existing estimator to several alternative variable weight estimators as well as to the Horvitz-Thompson estimator referred to by QFR as the "fixed weight estimator."

Ultimately, this study validated the current variable weight estimation method for QFR, showing it to have the lowest mean absolute error of the considered methods. Because of the its sample design, the QFR has a coverage bias that is usually negative. The study found that none of the alternative variable weight estimators reduced the coverage bias compared to the current estimator and that the coverage bias is approximately the same for the current estimator and the fixed weight estimator. Moreover, the quarter-to-quarter change estimates of sales constructed from the QFR variable weight estimates are more precise than the change estimates constructed from any of the considered alternative estimators, and the quarterly change in sales is a key economic statistic. Consequently, we recommend that QFR retain its current variable weight estimation methodology.

1. Introduction

The Quarterly Financial Report (QFR) is a sample survey of large corporations from the manufacturing, mining, wholesale trade, and retail trade sectors. The QFR sample is divided into

---

The following trademarked products mentioned in this report are hereby acknowledged: SAS, SAS/ETS, SAS/JMP and SAS/STAT are registered trademarks of SAS Institute, Inc. Base SAS refers to the basic module of the SAS system.

panels that are rotated into and out of the survey, and each non-certainty sampled corporation is interviewed for eight consecutive business quarters [certainty companies are included in the survey indefinitely]. For any given quarter, eight panels selected from up to three different frame years are in the survey. Each year, a new sample of corporate tax returns is selected from the most recent tax year data. This new sample is split into four panels. Each quarter, one of the four new panels is introduced, and the panel that has completed all eight interviews is dropped from the survey. This type of rotating panel design is employed to yield precise quarterly change estimates.

It is possible for a QFR company to conduct business in a different industry than indicated by the sampling frame. QFR estimates are tabulated by the company-reported industry (the enumerated industry), not the sample (frame) industry. The revised industry classification is referred to as the **enumerated** industry and is referred to in-house and throughout the remainder of this paper as the **post-stratum** (this is not the traditional post-stratified estimator, which adjusts survey estimates to control totals). To accommodate the industry-classification changes, the QFR estimator of total differs from traditional design-based estimators in that the weight assigned to each sample corporation is not based on its initial probability of selection. Instead, corporations in the enumerated industry are assigned weights equal to the ratio of the *estimated* total number of corporations in the type of industry at the time of enumeration to the number of sample corporations in that type of industry at the time of enumeration [Note: this is mathematically equivalent to the unweighted sample means multiplied by an estimate of population size for the enumerated industry/size-classification cell]. Because the final QFR weight changes each quarter, the QFR estimator is referred to as a "variable weight estimator."

The history of the development of the QFR variable weight estimator is not known. When the QFR program migrated to the U.S. Census Bureau from the Federal Trade Commission in the early 1980's, there were several efforts to study the properties of the estimator (Chapman, 1993) and compare it to a fixed weight, or Horvitz-Thompson, estimator (Chapman and Biemer, 1985, and Kott, 1992). These efforts indicate that there is a bias in the QFR variable weight estimator with respect to repeated sampling, but it may have a smaller variance than the Horvitz-Thompson estimator.

All of the earlier cited studies make simplifying assumptions. First, all assume complete response. Chapman and Biemer (1985) and Kott (1992) do not address the estimation problems caused by the panel design and the multiple frames. Chapman (1993) ignores the post-stratification effects.

This paper describes a more recent evaluation of the QFR estimator, which compared the existing estimator to several alternative variable weight estimators as well as to the Horvitz-Thompson estimator. This evaluation used a Monte Carlo simulation and accounted for the post-stratification effects, the rotation scheme, and population size changes. The evaluation showed that of the considered methods, the current QFR estimator has the lowest mean absolute error, but is usually negatively biased due to the coverage bias in non-decreasing populations.

A separate paper, Howe and Thompson (2005), describes research that was conducted on the simulated QFR data to investigate alternative variance estimators for the QFR. Based on this research, a delete-a-group jackknife variance estimator is recommended over the approximate sampling theory variance estimator in current use.

2. Background

2.a. The QFR Estimator

The Quarterly Financial Report (QFR) is a quarterly survey of mining, wholesale trade, and retail trade corporations with total assets of $50 million or more and manufacturing corporations with total assets of $250 thousand or more. The QFR collects income statement (e.g. sales, net income, depreciation, etc.) and balance sheet (cash, inventories, current assets, long term debt, retained earnings, total liabilities, etc.) data from each surveyed company. Among the most important characteristics from these collections are estimates of total quarterly sales and net income after taxes (NIAT); quarter-to-quarter percent change in sales and NIAT; and the quarterly ratio of NIAT/sales. Because NIAT can take on **all** real values, the QFR publishes an **absolute** difference on quarter-to-quarter change (and associated 90% confidence limits) instead of a percentage difference.

The sampling frame for the QFR survey comes from the file of United States Internal Revenue System (IRS) corporate tax returns. Every year, the Census Bureau receives a list of corporate tax returns for the previous year from the IRS and classifies all the companies by reported industry (sample industry) and total assets. Companies that have total assets of $250 million or more are included with certainty and are in the survey indefinitely. The remaining companies are stratified within sample industry. Units in the manufacturing sectors are further stratified within sample industry code by size; the within-industry size strata are referred to as the asset classes. The other sectors have one non-certainty stratum per sample industry. The QFR uses a stratified SRS-WOR sampling scheme with Neyman allocation in most strata: the allocation procedure is slightly modified in the largest non-certainty asset classes to reduce respondent burden (via time-in/time-out constraints). Section 5 provides more details on the QFR sampling procedure. We denote the sample sizes as n_{hi}, where the h index refers to the industry indicated on the sampling frame (the “sampling industry”).
This QFR sample is randomly split into four panels, each of which is introduced in a given quarter. The first panel from this new sample is introduced in the fourth quarter of the sampling year. At this point, companies in the four panels from the previous sample (selected from IRS returns two years prior) are mailed a questionnaire, as are three of the four panels from the previous sample (i.e., sample selected from the two-year prior frame constructed from IRS returns three years prior). In each quarter, as a new sample panel is introduced, the oldest sample panel (which has completed eight questionnaires) is dropped. So for any given quarter, there are up to three different sampling frames represented. At best, the QFR sample is drawn from sampling frames that are one and two years old. Thus, the QFR sample is subject to coverage bias because of eligible cases not included on the sampling frame.

It is possible for a QFR company to conduct business in a different industry than indicated on the sampling frame. Industry classification changes are determined via a nature of business questionnaire, administered **after** sample selection and generally completed by the first interview. The asset classification is rarely changed as a result of survey data. Subject-matter experts refer to enumerated industry "types" as "high mover," "medium mover," or "low mover," depending on the proportion of reclassified sample units. The industry reclassification adds variability to the QFR estimates, since sample sizes in the enumerated industries/asset classes (n_{ki}) are random variables, c.f. the fixed sample sizes (n_{hi}) in the sampling industries/asset classes.

The QFR does not use a Horvitz-Thompson estimator to produce estimates of quarterly totals (LEVELS). Instead, the level estimates are unweighted enumerated-industry level means multiplied by an estimate of population size for the enumeration industry/asset-classification post-stratum. This population estimate incorporates industry changes after sample selection, the rotation scheme, and the combined up-to-three possible frame population estimates. The formula for a QFR LEVEL estimate of item X in enumerated industry k and asset class i at time t is given by

$$\begin{aligned}
\widetilde{X}_{kit} &= \left[\frac{\frac{(4-b_{kit})}{Q_{kit}}\hat{N}_{kit}^{(-2)} + \frac{4}{Q_{kit}}\hat{N}_{kit}^{(-1)} + \frac{b_{kit}}{Q_{kit}}\hat{N}_{kit}^{(0)}}{n_{kit}^{(-2)} + n_{kit}^{(-1)} + n_{kit}^{(0)}} \right] \left[\sum_{hi \in ki} \sum_{j \in hki} I_{hkijt} x_{hkijt} \right] \\
&= \left[\frac{\hat{N}_{kit}}{n_{kit}} \right] \left[x_{kit} \right] \qquad (2.1) \\
&= \widetilde{W}_{kit} x_{kit}
\end{aligned}$$

where: $\hat{N}_{kit}^{(0)}$, $\hat{N}_{kit}^{(-1)}$, and $\hat{N}_{kit}^{(-2)}$, are the estimated population sizes at time t in enumerated industry k and asset class i for the sample from the current year frame (0), sample from prior year's frame (-1), and sample from prior prior year's frame (-2).

$n_{kit}^{(0)}$, $n_{kit}^{(-1)}$, and $n_{kit}^{(-2)}$ are the number of sampled cases in currently-interviewed panels at time t in enumerated industry k and asset class i from the (up to) three eligible sample frame years

Q_{kit} is the number of panels interviewed at time t in enumerated industry k and asset class i (usually 8)

b_{kit} is the number of active panels in the sample from the corresponding sample years

I_{hkijt} is an indicator variable indicating that company j was sampled in sampling industry h and enumerated in industry k/asset class i at time t

x_{hkijt} is the current data

The enumerated industry level "weight" $\left(\widetilde{W}_{kit}\right)$ approximates a sampling interval, using a weighted average of population estimates in the numerator and the actual observed sampled cases in the denominator. The population estimates for the *year-1* and *year-2* samples are Horvitz-Thompson (HT) estimates; the population estimate from the most recent sample frame year are frame totals from the sampling industry and asset class adjusted with survey estimates of in-movers (companies in an enumerated industry that were sampled from a different industry) and out-movers (companies sampled in a different industry than enumerated). The latter estimate also includes an adjustment for number of active panels. We refer to $\tilde{W}_{kit}$ as a "variable weight," and the QFR estimator of LEVELS as a variable-weight estimator, denoted by a tilde (~). The variable weight estimator can also be written as $\hat{N}_{kit}\bar{x}_{kit}$, where $\hat{N}_{kit}$ is the weighted-average population estimate defined above and $\bar{x}_{kit}$ is the unweighted cell mean at time *t*.

The QFR variable-weight estimates are further adjusted for non-response in the enumerated industry and asset class, using unweighted inverse response rates as advocated by Vartivarian and Little (2002).

2.b. Previous Research

There is a limited amount of previous research related to the QFR estimator. The earlier studies all make more extensive simplifying assumptions than the current study.

David W. Chapman and Paul Biemer (1985) performed a theoretical comparison of the current estimator and the fixed-weight estimator. The comparison ignores the rotation pattern, non-response, and coverage error. The main conclusion is that the current estimator is a biased estimator with the level of bias depending on the variation in the initial stratum sampling rates, while the fixed-weight estimator is an unbiased estimator. However, the current estimator may have a smaller variance because it equalizes the weights within each post-stratification cell. Chapman and Biemer suggest using the fixed-weight estimator because it is unbiased and is also a standard estimator.

Phil Kott (1992) also did a theoretical comparison between the current and fixed-weight estimators. Kott makes the same simplifying assumptions as Chapman does. He notes that the fixed-weight estimator is an unbiased estimator while the current estimator generally is not. However, if we can assume a model where units in the same post-stratification cell are independent with the same model mean and variance, then the current estimator is unbiased and has a smaller variance than the fixed-weight estimator.

David D. Chapman (1993) studied the properties of the current estimator using simulated data that assumed different patterns of change in universe size. The comparison ignored both stratum movers and non-response. The key finding is that the time lag between the sampling and the data collection results in bias in the estimated universe size when the universe size is changing. This bias is negative when the universe is expanding and positive when the universe is contracting. The time lag can also distort estimates of quarterly change. The bias due to the time lag is substantially reduced in magnitude for an estimator that uses sampling frames that are one year more recent than the frames used in the current estimator. Chapman notes that these effects might not be noticeable in published QFR data because of other factors, including the importance of certainty companies.

3. Annual Population Sizes

One of the primary goals of our research was to assess the relative performance of several different estimators under alternative economic scenarios. To do this, we generated seven different QFR annual populations (APOP files), each analogous to the annual frame data provided by the IRS used for the QFR sample. In each of these annual populations, expected population (count) proportions for sub-population cells (i.e., enumerated industry *x* sample industry *x* asset class) were modeled from two years of sample data estimates; the actual proportions in a year were randomly generated with expectation equal to the expected proportions. To avoid confounding different population effects with different estimator effects, we synchronized scenarios with the same population totals in initial years by using the same companies during these common initial years. Each population file contains 15 years of annual population counts for the **entire** QFR universe.

3.a. Scenarios

This section covers the scenarios, allocation of number of companies to strata, creation of company records, and the assignment of birth, living and death codes.

David D. Chapman (1993) analyzes the properties of the current QFR estimator under four different conditions. These conditions are

- The universe size (number of companies) increases constantly at a rate of 2 percent per quarter (8 percent per year),
- The universe size decreases constantly at a rate of 2 percent per quarter (8 percent per year).
- The universe size increases constantly at a rate of 2 percent per quarter (8 percent per year) for 8 years and then constantly decreases at a rate of 2 percent per quarter (8 percent per year) for 7 years.
- The universe size increases at a rate of 2 percent per quarter. In the 4^{th} quarter of year 1, the universe increases 30 percent in size. In the later quarters, the universe continues to increase 2 percent per quarter.

For the QFR estimator research discussed in this report, we adopted three of Chapman's conditions (the first, third and fourth) and added four additional ones. Hereafter, we refer to each population simulated under a given condition as a scenario. The size of these changes is larger than what would be expected in the QFR and so exaggerates the biases in the QFR estimator making it easier to identify differences between the estimators considered in this research. We started all scenarios with the actual QFR universe size in 2002. Singular changes in scenarios 2, 3 and 4 described below were initiated in year 6. The scenarios continue for 15 years allowing sufficient time for the effects of these singular changes to be absorbed. The scenarios are described below followed by Figure 1 showing a plot of the scenarios and Table 1 showing the universe sizes.

- Scenario 1, Trend – Annual 8 percent increases
- Scenario 2, Turning Point – Annual 8 percent increases for five years, followed by annual 8 percent decreases
- Scenario 3, Level Shift – Annual 8 percent increases for five years, a 30 percent increase in year six, followed by annual 8 percent increases

- Scenario 4, Outlier – Annual 8 percent increases for five years, a 30 percent increase in year six, a 30 percent decrease in year seven, followed by annual 8 percent increases
- Scenario 5, Annual Sawtooth – Alternating annual 8 percent increases and decreases
- Scenario 6, Biannual Sawtooth – Alternating increases and decreases of 16 percent over two years
- Scenario 7, Flat – No change in the universe sizes

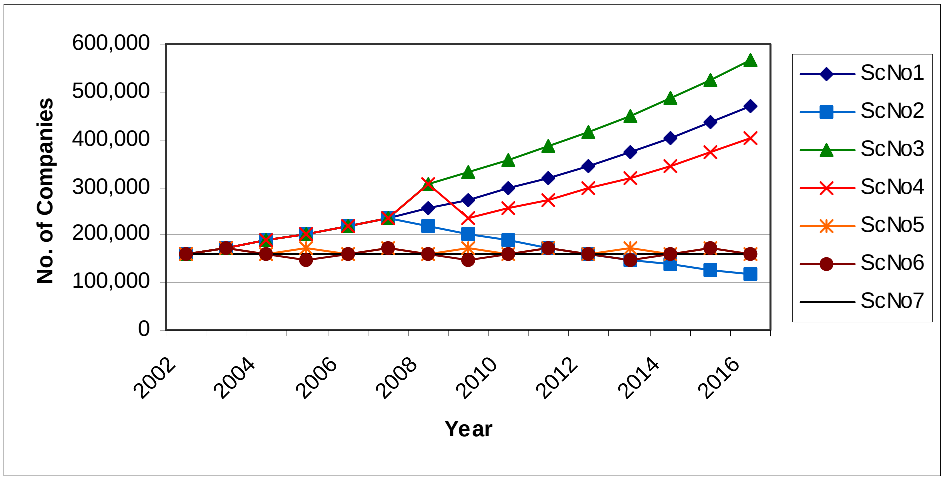


Figure 1: Scenarios for the QFR Research

3.b. Allocation of Number of Companies to Strata

The total number of companies in each year for each scenario is given in Table 1. The numbers of companies in the strata were drawn from a multinomial distribution where the total in the multinomial distribution is the total number of companies for a scenario and a year. The population proportions in each stratum were estimated from current QFR data. Simulated population sizes satisfied the following requirements.

- The strata for generating the number of companies are defined by asset class (AssetCl) × sample industry (SampInd) × enumerated industry (EnumInd).
- The expected proportions of companies by strata are the same for every scenario and year.
- The number of companies in a certainty stratum (asset class 18) in the first year equals the number of companies in the stratum in the 2002 QFR universe. The expected proportion of companies in a certainty stratum used in the subsequent years equals the number of companies in 2002 divided by 160,265, the common universe size in 2002 for all scenarios.
- The expected proportion of companies in a noncertainty stratum (asset classes other than 18) is proportional to the sum of the number of companies in the 1999 and 2000 QFR universes. If COUNT equals this sum for a noncertainty stratum, sumNE18 is the sum over all noncertainty strata, and sumEQ18 is the number of companies in the certainty strata then the expected proportion in a noncertainty stratum is

$$\pi = (\text{COUNT}/\text{sumNE18})\times(1-\text{sumEQ18}/160{,}265).$$

The factor on the right ensures that the sum of the proportions over both the certainty and non-certainty strata equals one.

- The number of companies by stratum in year 2002 was deterministically generated. The number of companies by stratum in the subsequent years was sampled from a multi-nomial distribution with expected proportions specified above and universe sizes from Table 1.

Attachment A provides the algorithms used to generate population sizes.

Table 1: Universe Sizes for the Scenarios by Year

Year	ScNo1	ScNo2	ScNo3	ScNo4	ScNo5	ScNo6	ScNo7
2002	160,265	160,265	160,265	160,265	160,265	160,265	160,265
2003	173,086	173,086	173,086	173,086	173,086	173,086	160,265
2004	186,933	186,933	186,933	186,933	160,265	160,265	160,265
2005	201,888	201,888	201,888	201,888	173,086	148,394	160,265
2006	218,039	218,039	218,039	218,039	160,265	160,266	160,265
2007	235,482	235,482	235,482	235,482	173,086	173,087	160,265
2008	254,321	218,039	306,127	306,127	160,265	160,266	160,265
2009	274,667	201,888	330,617	235,482	173,086	148,394	160,265
2010	296,640	186,933	357,066	254,321	160,265	160,266	160,265
2011	320,371	173,086	385,631	274,667	173,086	173,087	160,265
2012	346,001	160,265	416,481	296,640	160,265	160,266	160,265
2013	373,681	148,394	449,799	320,371	173,086	148,394	160,265
2014	403,575	137,402	485,783	346,001	160,265	160,266	160,265
2015	435,861	127,224	524,646	373,681	173,086	173,087	160,265
2016	470,730	117,800	566,618	403,575	160,265	160,266	160,265

3.c. Creation of Company Records and Assignment of Birth, Living and Death (BLD) Codes

We used the following rules to create company records (i.e., frame/sampling units) for a stratum within a scenario.

- In year 2002, create company records equal to the stratum size. Assign all company records as births (BLD code = ‘B’).
- In each subsequent year, convert all births from the previous year to living (BLD code = ‘L’).
  - If the stratum size increases from the previous year, create additional company records equal to the stratum size increase. Assign these new company records as births.
  - If the stratum size decreases from the previous year, select a simple random sample of the living companies and assign them as deaths (BLD code = ‘D’).

- As for stratum size, as long as two scenarios have the same trajectory, the BLD codes will be the same in both scenarios.

4. Quarterly Population Data

Our simulation study used 60 consecutive quarters of population data for five key QFR variables: net income before taxes (NIBT), net income after taxes (NIAT), net property plant and equipment (NPPE), sales, and inventories (INV), with sales and NIAT being the primary items of interest. After preliminary data analysis on two years of QFR sample data, we decided to generate simulated data for a (hopefully representative) **subset** of the QFR enumerated industries, rather than try to generate the complete quarterly population data for all QFR enumeration industries.

The following sections describe how we selected test industries (Section 4.a.), how we synchronized the microdata (Section 4.b.), how we generated the initial two quarters of data items (Section 4.c.), and how we developed change models for generating the subsequent 58 quarters of data (Section 4.d.). Section 3 describes the procedure for generating the initial universes of population counts; this section describes how we simulated the associated microdata.

4.a. Selecting Industries

To select our test industries, we first characterized each enumerated industry in the QFR as high, medium, and low "movers" based on percentage of "in-mover" companies in every **enumerated industry** using two years of QFR sample data. Our classification boundaries were the mean percentage of "in-movers" for the QFR plus/minus one standard deviation. The upper limit of this interval was the cut-off value for high movers, the lower limit the cut-off value for low movers, and the interval itself contained medium mover industries. Figure 2 presents our distribution of QFR industries into mover categories.

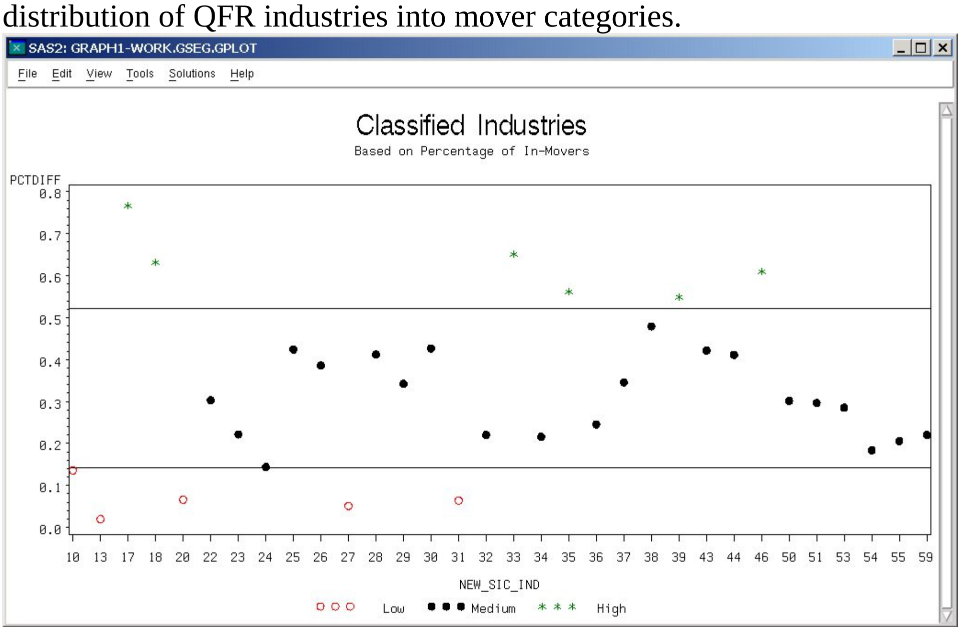


Figure 2: Distribution of QFR Enumerated Industries into High, Medium, and Low Categories (NEW_SIC_IND=Enumerated Industry)

To develop reasonable birth and change models for the quarterly data, we needed to select a set of test industries that contained sufficient quarterly QFR sample data for modeling (at least five observations per cell). Initially, we planned to develop quarterly data models within enumerated industry *x* sample industry *x* asset classification cells in our test industries. Because of the scarcity of data in cells where enumerated industry ≠ sample industry, however, we quickly abandoned this and instead used enumerated industry x asset class *x* "same" cell (same = 1 if enumerated industry=sample industry, 0 otherwise) as modeling cells, after verifying that the distributions of individual data items were in fact different within the same enumerated industry and asset class. After reviewing all industries that satisfied the population size criteria, the QFR Research and Methodology area selected the test industries listed in Table 2 for this study.

Table 2: Selected Test Industries

Classification	SIC Code	Description
Low	10	Metal Mining
	20	Food
Medium	28	Chemicals
	36	Electrical Machinery
	51	Wholesale Non-durables
	59	Residual of Retail Trade
High	18	Motor Vehicles
	35	Other Machinery

4.b. Synchronizing Quarterly Data

We synchronized unit response status and microdata values across all scenarios and all quarters. To do this, we first created a quarterly "superpopulation" file by combining all annual population (scenario) files and generating quarterly population data, assuming the following conventions:

- Births - units born in a given data year were randomly "born" in any one quarter of the birth year;
- Deaths - units that died in a given data year were randomly "killed" after quarter one of the death year;
- Living – four quarters per "continuing case" in each annual population year

The same company could have different birth/living/death status in the same quarter and year for different scenarios. This was not the case for the unit response value or simulated data values, where we generated **one** quarterly value per item and quarterly response code for each unique company.

We randomly assigned the unit response status (respondent/non-respondent) to each company in each quarter in the same proportion as QFR sample data. The proportions were calculated within each non-interview adjustment cell (enumerated industry x asset class). We used two separate approaches to model the key data items. First, we generated two initial quarters of data from the

QFR sample data using the non-parametric approach from Thompson (2002) described in Section 4.c. and Attachment B. Then, we developed different "change" models for each data item using mixed-model and time-series methods for all variables except NIBT; we used a regression model for NIBT (regressed on NIAT) to preserve correlation between those two items.

4.c. Data Simulation for Quarters 1 and 2

Modeling quarterly QFR data presented several challenges. First, only sample survey data were available; there was no comparable frame data available for variables of interest. Second, the sample data have an unknown multivariate distribution: three variables that are always positive and have low correlation within quarter (SALES, INV, NPPE) and two highly correlated variables that can take on all real values (NIAT, NIBT). Rather than attempt to develop individual multivariate parametric models for each modeling cell, we decided to resample the 4th quarter of the 1999 QFR sample data via a nonparametric algorithm described in Thompson (2000). In each modeling cell i, we generated a simulated population of size N_i by resampling the available n_i sample data cases, using the procedure outlined in Attachment B.

We verified the simulated population **distribution** of each item in modeling cell i by comparing the first quartile, the median, the third quartile, the 90th percentile, and the 95th percentile to the corresponding QFR statistics. We required that simulated population totals and distributions approximately equaled the corresponding QFR statistics, with an emphasis on NIAT and Sales.

4.d Change Models: Data Simulation For Quarters 3 through 60

4.d.1. Procedure for Modeling Sales, Inventory, and NPPE

In the QFR, data for each variable and each company represents a unique time series of quarterly data. We used change models to simulate the time series for each company from quarter three onward, using the eight quarters of QFR data to develop the models. For Sales, Inventory, and NPPE, we modeled the log of the current quarter to previous quarter ratio. This is very similar to modeling the percentage change. We modeled Sales, Inventory, and NPPE separately. There were three components to each change model: a zero observation model, a zero/non-zero change model, and a non-zero change model.

4.d.1.i. Zero Observation Model

The log of a zero does not exist so that a change model based on logs cannot predict a zero observation. Inspection of the QFR data indicated that if a value of an item for a company was zero in one quarter then it was usually zero in the other quarters. The model for zero observations followed the same logic – if the value for an item was zero for the first quarter, all subsequent quarters would be zero. If the first value was non-zero then the following two model components were used.

4.d.1.ii. Zero/non-zero change model

The non-zero change model described below is a model for continuous variables and, as such, simulated values of zero change have probability zero of occurring. However, small companies can often have zero changes and these zero changes do not occur at random. That is, whether a quarterly change is zero or not depends on whether the changes for previous quarters are zero or not. If the probability of a zero change only depends on whether the previous quarterly change is zero or non-zero then this is a Markov chain. We used a two-state random effects Markov chain model to determine whether a change will be zero or whether we use the non-zero change model described next to create the value of a change. This model was only used for inventory and NPPE for two asset classes (03 and 07). Sales and the other asset classes for inventory and NPPE had few zero changes so that a model of non-zero changes was used for them.

4.d.1.iii. Non-zero change model

We used a first-order autoregressive model for the non-zero change model, that is, a change is proportional to the previous change plus a random disturbance. Instead of modeling the random disturbances using a normal distribution, we used a *t*-distribution. Analysis of the distribution of non-zero changes showed that large (non-outlier) changes occur much more frequently than would be expected from a normal distribution. A *t*-distribution can be used to model this feature of the data. We established upper and lower bounds for the relative change that decreased with the size of the previous quarter's level. For example, a company with high sales in the previous quarter would be limited to a smaller relative increase than a company with low sales.

One objective in establishing the change model was to reflect both sample industry and enumeration industry along with asset class. Including one classification without the other might under-represent the variation caused by the reclassification in the simulated data. One approach would have been to stratify the data by asset class, sample industry, and enumeration industry and fit different models in each partition. However, the sample sizes in these would be relatively small for estimating the parameters in the change model. One way to improve the parameter estimation would be to borrow strength from the other industries.

To this end, SAS/Enterprise Miner was used to form tree partitions of the data. The partitioning variables were asset class, sample industry, and enumeration industry. The target variable for forming the partitions was the absolute value of the residual, where the residuals were obtained from an initial run of the mixed model described below. Partitions were formed so that they differed by the average absolute residuals. In this way, differences in variation among companies in different asset classes, sample industries, and enumeration industries could be explained by the model. An attempt was made to create partitions within a single asset class because asset class was not subject to change in the survey and substantial variation would be expected for different size companies. Initial partitions with few observations were combined to achieve a minimum of approximately 800 observations in each partition, in order to ensure adequate sample sizes to estimate the change model parameters well.

Initially, a simple autoregressive (AR) 1 model (Box and Jenkins, 1976) was fit to the log ratio change data for each of the partitions. The AR coefficients were usually negative indicating that the expected change in the current quarter would be in the opposite direction from the change in the previous quarter. Examination of the residuals and absolute residuals indicated, in general, negative correlations of each of these with the log of the previous quarter's data. The former would indicate that a company change in the current quarter would be less when the previous quarter's estimate was large and greater when it was small. The latter would indicate that the spread in the log ratio change (relative change in the original scale) would be less when the previous quarter's estimate was large and greater when it was small. Preliminary models were fit to examine this using loglinear variance models in JMP. (Harvey, 1976; Carroll and Ruppert, 1988). These models have the general form

Mean: $E(y) = \mathbf{X}b$,
Variance: $\log(Variance(y)) = \mathbf{Z}\lambda$.

Based on this preliminary modeling, the following mixed model was selected to model the QFR data for Sales, Inventory, and NPPE.

$$y_{it} = r_i\, y_{i,t-1} + \alpha + \beta \log\left(x_{i,t-1}\right) + e_{it} \tag{4.1}$$

where

i = company,
t = time,
x_{it} = reported data for company i and quarter t,
$y_{it} = \log(x_{it} / x_{i,t-1})$, log ratio change,
r_i = random autoregressive effect for company i with mean ρ and variance v,
e_{it} = t-distributed random effect for company i and time t with degrees of freedom df, mean 0, and variance parameter σ_i^2 where $\log\left(\sigma_i^2\right)$ is modeled as a linear function of $\log\left(x_{i,t-1}\right)$, i.e., $\log\left(\sigma_i^2\right) = a_\sigma + b_\sigma \log\left(x_{i,t-1}\right)$, $b_\sigma \le 0$,
and β and b_σ represent the inverse relationships of the log ratio, y_{it}, and $\log\left(\sigma_i^2\right)$ with $\log(x_{i,t\text{-}1})$.

The model fitting was conducted in two steps. First, the linear model was fit using SAS PROC MIXED. The residuals were output and the variance model for e_{it} was fit using SAS PROC NLP. Details and evaluation of the models are provided in Luery (2005).

The change model can generate extreme changes that fall outside the changes found in the data. This can have the consequence of generating level estimates well above the range of the observed data or generating microscopic level estimates. Inspection of the data indicated that the size of the largest positive change decreases as the level, $x_{i,t-1}$, increases and the size of the largest negative change decreases as the level, $x_{i,t-1}$, decreases.

Upper and lower bounds for the change in the log ratio were developed to address the extreme changes permitted by the change model. The upper and lower bounds are linear functions of the log of the level of the variable from the previous period. Let $z_{it} = \log(x_{it})$ and $z_{i,t-1} = \log(x_{i,t-1})$ be the logs of the variables at times t and t–1. The upper bound for the change, y_{it} is

$ub = a_u + b_u z_{i,t-1}$ and the lower bound is $lb = a_\ell + b_\ell z_{i,t-1}$. Given $z_{i,t-1}$, ub and lb are constants and will be treated as such in the following when taking expectations, that is, the expectations are conditional expectations given $z_{i,t-1}$.

Linear functions for the upper and lower bounds were developed by plotting the log ratio change (y_{it}) versus the log of the variable from the previous time ($z_{i,t-1}$). Using data from all industries within an asset class, upper and lower bounding points along the range of $z_{i,t-1}$ were selected by hand excluding observed outliers. For each asset class, separate regression lines were fit to these bounding points to generate the above regression coefficients. Because all of the points used to estimate the regressions were considered acceptable and using the estimated regressions lines would now identify 'half' of these bounding points as outliers, a constant of 0.1 was added to the upper bound and 0.1 was subtracted from the lower bound. The adjusted bounds, $ub = 0.1 + a_u + b_u z_{i,t-1}$ and $lb = -0.1 + a_\ell + b_\ell z_{i,t-1}$, included almost all of the bounding points. We used these adjusted bounds.

Despite the bounding of extreme changes, the generated level estimate at time t, $x_{it} = x_{i,t-1} y''_{it}$, still showed values noticeably outside the observed range of the QFR data. One final adjustment was used to correct for this. If x_{it} was greater than exp(IMAX) then it was set to exp(MAX). If it was less than exp(MIN) then it was set to exp(IMIN). MAX and MIN were by asset class and were identified using the QFR data omitting the data not in the study-enumerated industries since the data in the study-enumerated industries had a smaller range than the data as a whole. These bounds were rounded and adjusted so that they were monotonically increasing with asset class. No adjustment was made for the biasing due to this bounding.

4.d.2. Procedure for Modeling NIAT

We could not use the same change model approach for NIAT as for Sales, Inventories, and NPPE, because NIAT can assume both positive and negative values, and the log of the ratios are undefined for negative numbers. Our key requirements for the simulated quarterly **income** data were to:

- Preserve univariate distributions (1st quartile, median, 3rd quartile, 90th percentile) of key variables within quarter;
- Preserve correlation structures among key items within quarter; and
- Preserve quarter-to-quarter autocorrelations.

For NIAT, we fit the same time series model for all of our available sample QFR units:

$$NIAT_{ut} = \theta_{v(1)} NIAT_{u(t-1)} + \theta_{v(2)} NIAT_{u(t-2)} + \phi_{v(1)} \varepsilon_{u(t-1)} + \varepsilon_{ut} + \beta_v \gamma_{ut} \tag{4.2}$$

where u indexes the company, t indexes the time, $\varepsilon_i \sim (0,\sigma^2)$, and $\gamma_i \sim (0,1)$. This model was designed to satisfy the following assumptions:

- The expected value of individual unit's NIAT does not vary over time;

- The value of NIAT at time *t* is highly positively correlated with value of NIAT at time *t-1*;
- The correlation between consecutive measurements of NIAT on same unit decreases to zero over time.

Although we fit the same **model** for each company, the parameters differed. We obtained the Autoregressive Moving Average (ARMA) parameters ($\theta_{v(1)}$, $\theta_{v(2)}$,$\phi_{v(1)}$) and regression parameter (β_v) by fitting "RegARMA" models for each (true) sample QFR company *v* with at least six available quarters of data. We dropped from consideration all company models whose fitted parameters did not satisfy the following stationarity conditions:

$$\theta_{u(1)} + \theta_{u(2)} < 1$$
$$\theta_{u(2)} - \theta_{u(1)} < 1$$
$$-1 < \theta_{u(2)} < 1$$

Then, using PPS-WR sampling (with the unit measure-of-size = sampling weight of the model-originating company), we randomly assigned the remaining usable **models** to **simulated** units in the same asset class and enumerated industry **sector** (manufacturing – durable goods, manufacturing – nondurable goods, mining, wholesale trade, and retail trade) as the model-generating company. Thus, each sample QFR (true) company generates a unique set of parameters, but the same sets of parameters were used by multiple simulated data units.

The **simulated** NIAT data for quarters 3 through 60 were

$$NI\hat{A}T_{ut} = \hat{\theta}_{v(1)} NI\hat{A}T_{u(t-1)} + \hat{\theta}_{v(2)} NI\hat{A}T_{u(t-2)} + \hat{\phi}_{v(1)}^{t-2} r_v \zeta_u + \hat{\beta}_v \xi_{ut} \tag{4.3}$$

where r_v is the last fit residual from (true) QFR sample unit company *v*, ζ_u ~ N(0,1), and ξ_{ut} ~ N(0,1).

4.d.3. Procedure for Simulating NIBT

We did not use a time-series approach to simulate quarterly "change" data for NIBT. By definition, NIAT equals NIBT minus taxes, resulting in a very strong linear relationship within each quarter. Consequently, we decided to make each company's value of NIBT depend on same quarter's value of NIAT. Because our data analysis revealed a high proportion of cases whose NIBT = NIAT, our simulation procedure takes this into account as well.

To simulate NIBT data, we developed two separate models per modeling cell: a simple linear no-intercept regression model (Model 1) and an equivalence model (Model 2). We randomly applied these models to each sample unit within the modeling cell in the same proportion as in the sample data. The two models are

Model 1: $$NIBT_{iu} = \beta_i NIAT_{iu} + \varepsilon_{iu} \tag{4.4}$$

where *i* indexes the modeling cell (asset class *x* enumerated industry sector) and *u* indexes the company.

The **simulated** NIBT data for quarters 3 through 60 were

$$N\hat{I}BT_{iu} = \hat{\beta}_i NIAT_{iu} + \hat{\varepsilon}_{iu}, \tag{4.5}$$

where $\hat{\beta}_i$ is the weighted[1] least squares regression parameter obtained from QFR sample data in modeling cell *i* and $\hat{\zeta}_{iu} \sim N(0,\hat{\sigma}_i), \hat{\sigma}_i = \sqrt[3]{\text{sampling error (residuals)}_i}$. [The cubed-root of the residuals were normally distributed. However, the power-transformed error terms were too large, so we used the untransformed values for our simulation].

In some cells, the fitted value of $\hat{\beta}_i$ was less than 1, implying that that all companies in the cell were **receiving** (instead of **paying**) tax dollars. It was felt that this was an unrealistic model. In these cells, we set $\hat{\beta}_i = 1$ and randomly generated the error terms as $\hat{\xi}_{it} \sim N(\bar{x}_i, \hat{\sigma}_i)$, where $\bar{x}_i = \dfrac{\sum_{u \in i} w_u (NIBT_u - NIAT_u)}{n_i}$ and $\hat{\sigma}_i. = 1$.

Model 2: $$NIBT_{iu} = NIAT_{iu} + \hat{\varepsilon}_{iu} \tag{4.6}$$

To validate our models, we generated a simulated population of the same size as the QFR sample data population. We then calculated the total value of NIBT from the simulated population and compared it to the corresponding QFR sample estimate. We verified the simulated population **distribution** of NIBT in modeling cell *i* by comparing the first quartile, the median, the third quartile, the 90th percentile, and the 95th percentile to the corresponding QFR statistics. This verification procedure for NIBT indirectly verified the NIAT models as well due to the enforced modeled relationship between the two items.

5. Sampling

We selected samples from the simulated populations in accordance with the rotation scheme and sample sizes of the operational QFR survey. In particular, the operational survey annually selects a sample of size *n*=3304 and rotates the new sample in over four quarters, while rotating out the sample selected from the prior-prior (least recent) frame. The only difference between the operational survey sample and our simulation sampling procedure is the use of much simpler time-in/time-out constraints; we only required that **in-sample** non-certainty companies were ineligible for the new annual sample.

The set of in-sample companies each quarter consisted of all the certainty companies—that is, companies in asset class 18 -- plus a sample of non-certainty companies resulting from the following operations:

- Allocating the sample size (*n*=3304) to strata for each new annual sample,
- Selecting a new annual sample of non-certainty companies,

[1] by sample weight. $\hat{\beta}_i$ is a B.L.U.E. However, the SAS-calculated $SE(\hat{\beta}_i)$ are not correct because they do not take the sample survey variance-covariance structure into account in the calculations.

- Assigning the sampled non-certainty companies to quarterly panels and to random groups, and
- Rotating quarterly panels into and out of the set of in-sample companies

We used the SAS's PROC SURVEYSELECT to select each annual sample of non-certainty companies. As in the operational QFR survey, after a company enters the set of in-sample companies, it remains in sample for eight consecutive quarters. Sections 5.a. and 5.b. below provide more detail on our annual sample size allocation procedure and our panel assignment/random group assignment procedure.

5.a. Allocation of the Annual Sample Size to Strata

Define

N_{hit} = number of companies in sampling industry h and asset class i of the annual population file for year t, and

n_{hit} = number of companies sampled from sampling industry h and asset class i from the annual population file for year t.

We allocated stratum samples for asset classes 16 and 18 (large company size strata and certainty strata) as follows:

$n_{h,18,t} = N_{h,18,t}$ (i.e., allocation to certainty stratum)

$n_{h,16,t} = \text{Min}[\ (N_{h,16,2002})/4, (N^*_{h,16,t})/2\ ]$,

where

N^*_{hit} = number of companies in sampling industry h and asset class i of the annual population file for year t <u>that are not in sample at the time a new sample is selected from the population file for year</u> t.

In manufacturing industries, we allocated the balance of the annual sample size--that is, $[3304 - \sum_h (n_{h,18,t} + n_{h,,16,t})]$ -- using the following three-step procedure:

n_{hit} = Neyman allocation to sampling industry stratum h and asset class i, where the stratum sizes used for the Neyman allocation are the N^*_{hit} values and the stratum variances are the average variances of Net Income Before Taxes calculated from operational QFR data,

n'_{hit} = multiplicative adjustment of n_{hit} so that $n_{hit} \le N^*_{hit}$ and $\Sigma_{\text{hi}}\, n'_{hit} = \Sigma_{\text{hi}}\, n_{hit}$.

n''_{hit} = multiplicative adjustment of n'_{hit} so that $n''_{hit} \ge \min(8, N^*_{hit})$ and $\Sigma_{\text{hi}}\, n''_{hit} = \Sigma_{\text{hi}}\, n_{hit}$.

5.b. Assignments to Panels and to Random Groups

After each selection of a new annual sample of non-certainty companies, we assigned the sampled companies to quarterly panels and to random groups. To assign the sampled companies to quarterly panels, we arranged them in random order within their sampling industry and asset class and then systematically assigned them to one of four quarterly panels—the first company to the first panel, the second company to the second panel,…, the fourth company to the fourth panel, the fifth company to the first panel, etc.

Each sampled company was assigned to one of 15 random groups. In order to prevent sample rotation from causing large changes in sizes of the random groups, we assigned companies to panels and to random groups so that each quarter when we rotated a certain number of companies out of a random group we were at the same time rotating approximately the same number of companies into the random group. We were able to achieve this situation by assigning sampled companies to panels and random groups one way in odd years and a different way in even years. Because there is a two-year difference between the time when a company rotates into and out of sample, this made the random group assignments of companies being rotated out of sample similar to the random group assignments of companies being rotated in. The different orders of assignment to random groups for even and odd years distributed the random group assignments more evenly across the random groups than if the same order of assignment to random groups had been used each year.

6. Estimators Considered

Altogether, we considered eleven different estimators of quarterly level: the current method, nine modified variable weight estimators, and a fixed weight estimator. All level estimates are adjusted for non-response. The candidate estimators and non-response adjustment procedures are described below.

6.a. Current Variable Weight Estimator (VWE1)

As described in Section 2 (equation 2.1), the current variable weight estimator, or VWE1, constructs an enumerated industry level weight using a weighted average of (up to) three population total estimates divided by the total sample size, then multiplies sample data by this variable weight $\left(\tilde{W}_{ki}\right)$. Table 3 provides the weight applied to each estimated population size in the current variable weight estimator.

Table 3. Weighting Of Estimated Population Sizes In Current Variable-Weight Estimator

Quarter	Weight applied to estimated population sizes based on:		
	Prior prior year's sample (4-*b*)/Q	Prior year's sample (4/Q)	Most recent sample (b/Q)
4	0.375	0.5	0.125
1	0.250	0.5	0.250
2	0.125	0.5	0.375
3	0.000	0.5	0.500

6.b. Modified Variable Weight Estimators (VWE2 –VWE10)

The modified variable weight estimators modify the **numerator population total estimates** used in the current VWE. The modifications are as follows:

- **Adjustments to prior-period population totals (VWE2-VWE3)**. Applies a correction factor to the two prior period population estimates to account for the changed population between frame years. The correction factors are ratios of the population total in the sample industry and the asset class to the corresponding frame data population totals from the prior and prior-prior periods' sample frames. Note that the most recent population estimate ($\hat{N}^{(0)}$) is not affected.

  - The VWE2 estimator matches the sampling industry/asset class level correction factor to estimates of $\hat{N}^{(-1)}$ and $\hat{N}^{(-2)}$ that have the same enumerated industry/asset class codes. The variable weight is given by

$$\tilde{W}_{VWE2,ki} = \frac{\frac{(4-b)}{Q}\frac{M_{hi}^{0}}{M_{hi}^{(-2)}}\hat{N}_{ki}^{(-2)} + \frac{4}{Q}\frac{M_{hi}^{0}}{M_{hi}^{(-1)}}\hat{N}_{ki}^{(-1)} + \frac{b}{Q}\hat{N}_{ki}^{(0)}}{n_{ki}^{(-2)} + n_{ki}^{(-1)} + n_{ki}^{(-0)}} \tag{6.1}$$

  - The VWE3 estimator matches the correction factors to sample estimates of $\hat{N}^{(-1)}$ and $\hat{N}^{(-2)}$ with the same sample industry and asset class, then sums these estimates to the enumerated industry level. The variable weight is given by

$$\tilde{W}_{VWE2,ki} = \frac{\frac{(4-b)}{Q}\sum_{h}\frac{M_{hi}^{0}}{M_{hi}^{(-2)}}\hat{N}_{ki}^{(-2)} + \frac{4}{Q}\sum_{h}\frac{M_{hi}^{0}}{M_{hi}^{(-1)}}\hat{N}_{ki}^{(-1)} + \frac{b}{Q}\hat{N}_{ki}^{(0)}}{n_{ki}^{(-2)} + n_{ki}^{(-1)} + n_{ki}^{(-0)}} \tag{6.2}$$

  where the sums are over h such that $\hat{N}_{hki}^{(-g)} \neq 0$.

  Note that the values of b and Q for each quarter are the same as in the current variable weight estimator.

- **Revised weighted average of population totals (VWE4-VWE10).** These estimators combine the three separate estimates of population totals using different weighted average schemes, each combining the proportion of active panels in the current sample from each frame with other averaging factors, according to the following formula

$$\widetilde{W}_{VWE4-VWE10,ik} = \frac{(1-c)a\hat{N}_{ki}^{(-2)} + ((1-c)(1-a)+cb)\hat{N}_{ki}^{(-1)} + c(1-b)\hat{N}_{ki}^{(0)}}{n_{ki}^{(-2)} + n_{ki}^{(-1)} + n_{ki}^{(0)}} \tag{6.3}$$

where $c = \begin{cases} 0.25, & q=4 \\ 0.50, & q=1 \\ 0.75, & q=2 \\ 1.00, & q=3 \end{cases}$ and the values for *a* and *b* are given by

Estimator #	a	b	***Estimator #***	a	b
4	0.40	0.40	8	0.00	0.40
5	0.25	0.25	9	0.00	0.25
6	0.10	0.10	10	0.00	0.10
7	0.00	0.50			

Table 4 provides the weight applied to each estimated population size for these estimators.

Table 4. Weighting Of Estimated Population Sizes In Modified Variable-Weight Estimator With Revised Weighting Of Population Totals

Estimator	**Quarter**	**Weight applied to estimated population sizes based on:**		
		Prior prior year's sample	**Prior year's sample**	**Most recent sample**
VWE4	4	0.3	0.55	0.15
	1	0.2	0.50	0.30
	2	0.1	0.45	0.45
	3	0.0	0.40	0.60
VWE5	4	0.1875	0.625	0.1875
	1	0.1250	0.500	0.3750
	2	0.0625	0.375	0.5625
	3	0.0000	0.250	0.7500
VWE6	4	0.075	0.7	0.225
	1	0.050	0.5	0.4500
	2	0.025	0.3	0.6750
	3	0.000	0.1	0.9000
VWE7	4	0.0	0.875	0.125
	1	0.0	0.750	0.250
	2	0.0	0.625	0.375
	3	0.0	0.500	0.500
VWE8	4	0.0	0.85	0.15
	1	0.0	0.70	0.30
	2	0.0	0.55	0.45
	3	0.0	0.40	0.60

Estimator	Quarter	Weight applied to estimated population sizes based on:		
		Prior prior year's sample	Prior year's sample	Most recent sample
VWE9	4	0.0	0.8125	0.1875
	1	0.0	0.6250	0.3750
	2	0.0	0.4375	0.5625
	3	0.0	0.2500	0.7500
VWE10	4	0.0	0.775	0.225
	1	0.0	0.550	0.450
	2	0.0	0.325	0.675
	3	0.0	0.100	0.900

6.c. Fixed Weight Estimator (FWE)

The fixed weight estimator constructs enumerated industry level estimates by summing data multiplied by associated **sampling weights.** This estimator is also referred to as the unbiased estimator or Horvitz-Thompson estimator in the sampling literature.

6.d. Non-response Adjustment Procedures

We considered two different non-response adjustment procedures. The unweighted non-response adjustment procedure (the current method) is given by

$$A_{ki} = \frac{\sum_{hij \in ki} I_{a,hkij}}{\sum_{hij \in ki} I_{r,hkij}} = \frac{a_{ki}}{r_{ki}} \tag{6.4}$$

where a_{ki} = the number of active cases in **enumerated** industry, asset class cell (k,i), i.e., the number of cases that could have responded.
r_{ki} = the number of respondents in **enumerated** industry, asset class cell (k,i).

$$\mathrm{I}_{a,hkij} = \begin{cases} 1 & \text{if unit } j \text{ is an active case in sampling industry } h\text{, eumerated industry } k\text{, asset class } i \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{I}_{r,hkij} = \begin{cases} 1 & \text{if unit } j \text{ is an active case in sampling industry } h\text{, enumerated industry } k\text{, asset class } i \text{ that responded} \\ 0 & \text{otherwise} \end{cases}$$

This procedure is advocated by Vartivarian and Little (2002).

The weighted non-response adjustment procedure is given by

$$A_{ki} = \frac{\sum_{hij \in ki} SAMPWT_{hi} \mathrm{I}_{a,hkij}}{\sum_{hij \in ki} SAMPWT_{hi} \mathrm{I}_{r,hkij}} \tag{6.5}$$

This procedure is described in Kalton and Flores-Cervantes (2003).

7. Evaluation Statistics

The evaluation of the statistical properties of the candidate estimators focused primarily on the following two measurements:

- bias, measuring both distance and direction of estimates from "truth"
- mean absolute error (MAE), measuring magnitude of distance of estimates from truth.

We examined properties of the absolute bias and MAE and the relative bias (RelBias) and relative MAE (RMAE) of each estimator for quarterly estimates and for aggregated quarterly estimates.

$$MAE(\hat{X}_{im}) = \sum_{s} \left| \hat{X}_{ims} - X_i \right| / n_s \qquad (7.1)$$

where n_s is the number of samples, X_i is the true value of item X in enumerated industry/asset class i, and $\hat{Y}_{ims}$ is the estimate calculated using method m in sample s and enumerated industry/asset class i.

RMAE is the quotient of the MAE and "truth." We examined RMAE when analyzing **level** estimates of sales, inventories, and NPPE. For the non-linear statistics (change estimates and ratios), "truth" could be close to or equal to zero, so we examined MAE. The RMAE was an inconsistent measure of error for NIAT and NIBT, since those characteristics can have positive or negative values, hence we used MAE for their analyses.

Besides examining the statistical properties of each estimator, we directly compared each estimator's statistical properties over repeated samples with the comparable value obtained using the current variable-weight estimator (VWE1). This measures **improvements** in statistical properties from the current estimator, i.e., expected gains in precision. Additionally, we calculated the mean squared error (MSE) and the square-root-MSE (RMSE) of each estimator, but ended up using MAE as our measure of error since both MAE and RMSE maintain the same error properties.

8. Results

8.a. Sources of Error

Total error (mean squared error) has a bias component and a variance component. This section evaluates the bias component.

A useful way of breaking down the bias is to decompose the relative bias into four components: the non-response adjustment relative bias, the relative bias of the estimator for covered respondents, the coverage error relative bias, and the remainder. The relative bias due to the non-response adjustment is generally small because the assignment of response status assumed that non-response was missing at random within adjustment cell. In addition, this bias is the same for all estimators since it is a characteristic of the population. Coverage error is due to undercoverage of recent births in the sampling frame. For estimates that are always positive, bias due to coverage error is always negative. Bias due to coverage error is the same for the fixed-weight estimator and the current estimator. The modified variable weight estimator should reduce the coverage error to some extent, since it overweights more recent data in calculating estimates of population sizes that are used in assigning estimation weights.

We calculated the components of relative bias for estimates of levels for SALES and NIAT in Scenarios 1, 2, 6, and 7 (other scenarios were dropped for reasons discussed in Section 8). Attachment C displays our results for SALES averaged over quarters and industries. Attachment C omits the remainder component because when rounded to the nearest one tenth of one percent it was always either 0.0 percent or -0.1 percent. The components of relative bias for NIAT did not provide useful information. The relative bias results presented in Attachment C are restricted to the fixed weight estimator (FWE), the current variable weight estimator (VWE1) which uses three years' of population estimates, and one of the variable weight estimators (VWE10) that uses only the current and prior years' population estimates. The other candidate estimators were dropped after the analyses described in Section 8.c., so their results are omitted here for the sake of brevity.

For SALES, the low and medium movers show a different pattern than the high movers. For the low and medium movers, coverage error is generally the most important component of relative bias. The exception is for the modified variable weight estimator in quarters 28+ in Scenario 2 where the relative bias for covered respondents is roughly equal to coverage error. The modified variable weight estimator tends to have slightly less negative coverage error than the other two estimators. However, except for Scenario 1, the relative bias for covered respondents is slightly more negative for the modified variable weight estimator than for the other two estimators. The fixed-weight estimator and the current estimator have similar relative bias components for all four scenarios for both low and medium movers. Overall, all three estimators tend to have similar relative bias for low and medium movers. The modified variable weight estimator is slightly better in Scenario 1 and slightly worse in Scenario 2 (quarters 28+).

For SALES for the high movers, coverage error is still clearly the most important component of relative bias for the fixed-weight estimator. For both variable-weight estimators, the relative bias for covered respondents is substantially more negative than for the fixed-weight estimator and tends to be similar in magnitude to coverage error relative bias. Except for Scenario 1, the relative bias for covered respondents is slightly more negative for the modified variable weight estimator than for the current estimator. The modified variable weight estimator, however, tends to have slightly less negative coverage error than the other two estimators. Overall, the fixed-weight estimator tends to have less of a negative bias than the variable-weight estimators for high movers. The modified variable weight estimator does slightly better than the current estimator in Scenario 1 but slightly worse in Scenario 2.

8.b. Non-response (Weighted vs. Unweighted)

This section describes a comparison of estimates using unweighted non-response adjustment with estimates using weighted non-response adjustment (see Section 6.d.). This analysis compared weighted and unweighted non-response adjusted estimates for each combination of item by type of estimate using a paired t-test. This comparison proceeded as follows.

1. Take the difference of estimates using unweighted and weighted non-response adjustment factors. These estimates are by item, type of estimate, quarter, scenario, estimation method, enumeration industry, and sample.
2. Average these estimates by item, type of estimate, and sample. These averages are the observations used in the t-tests because the estimates by quarter, scenario, estimation method, and enumeration industry are not independent. For a t-test to be valid, the observations need to be independent. Because the samples are independent, these averages by sample within each combination of item and type of estimate are independent.
3. Calculate t-tests of the hypotheses that the differences are zero for each combination of item and type of estimate.

In general, we found that the differences between corresponding unweighted and weight non-response adjustment factors are very small compared to the sizes of the estimates. When the difference was <u>not</u> small, the variability among the averages is so large that the t-test did not detect a significant difference. The former condition can be seen in the level estimates where the differences have a high statistical significance but are trivial compared to the level of the estimates. The latter condition can be seen in the NIAT and NIBT estimates where the quarterly trend (TRENDQ) and annual trend (TRENDY) differences are large but not statistically significant. This can be attributed to the instability of these estimates since they are ratios in which the denominators can be very close to zero. The results are shown in Attachment D.

This analysis did not reveal substantive differences between the weighed and unweighted non-response adjustment methods, so we concluded that there is no reason to change from QFR's practice of using an unweighted non-response adjustment.

8.c. Estimators

Originally, the scope of this project was quite large. We were comparing the statistical properties of eleven different estimators in seven different population scenarios (each with three different type of industry mover categories) for five different data items on four key statistics (Quarter-to-Quarter Change; Quarterly Level; Quarterly Ratio (NIAT/SALES); and Year-to-Year Change). It was impossible to find a single estimator with optimal statistical properties in all situations. Ultimately, we decided to concentrate on the two key QFR variables of SALES and Net Income After Taxes (NIAT) in the medium-mover industries in Scenario 1 (monotone increase in population size), Scenario 2 (monotone increase until a turning point, then monotone decrease), 6 ("see-saw," i.e., repeated cycles of monotone increase for two years followed by monotone decrease for two years), and Scenario 7 (no change in population size). Since the quarterly change estimates are economic indicators, our final decision focused on their statistical properties first.

As described in Section 7, our primary evaluation examined mean absolute error (MAE) or relative mean absolute error (RMAE) properties for each item/statistic relative to the current variable weight estimator. MAE and RMAE measure the magnitude of the deviation of the considered estimate from "truth."

The first stage of our analysis reviewed time-series plots comparing the eleven different average MAE or RMAE values per characteristic/statistic using 50 samples per scenario. This analysis uncovered some undesirable properties of two of the candidate variable-weight estimators (VWE2 and VWE3) that were confirmed analytically, so these two estimators were dropped from consideration. Further review of these time-series plots combined with distributional comparisons showed almost uniformly comparable results for the remaining eight candidate modified-variable weight estimators. Consequently, we decided to restrict our final comparisons to the current method, the fixed-weight estimator (whose performance generally differed from the modified variable-weight estimators), and a modified variable-weight estimator that used the current and prior sample years' population size estimates (c.f., the current method, which uses up to three years' population estimates). This estimator (VWE10) had the best theoretical properties in terms of reduced expected coverage bias. At this point, we also decided that 50 samples was insufficient and selected an additional 350 samples from the Scenario 1, 2, 6, and 7 data, resulting in n_s = 400 samples for analysis.

We measured whether an alternative estimator was obtaining more reliable estimates than the corresponding current-estimation-method value by examining distributions of the difference in MAE or RMAE, specifically comparing the 10^{th} percentile, 90^{th} percentile, and median of quarterly difference between the two estimators' MAE or RMAE. We made our distributional comparisons for each statistic and characteristic within industry-mover category (Low, Medium, and High), focusing primarily on the medium mover industries; the statistical properties of all three competing estimators are approximately the same in the low-mover industries; and the problems encountered in high-mover category will generally be addressed by improving the frame prior to sample selection.

Figure 3 illustrates our comparison procedure. This graph plots the distribution of quarterly difference between the new and current method for a given item/statistic. A negative difference indicates that the "new" method is more precise than the current method. Thus, when the 90^{th} percentile is negative, the new estimator generally yields more precise estimates (smaller mean absolute errors) than the current method, showing a "strong preference" for the new method. If the 90^{th} percentile is positive, **but** the median is negative, then at least half of the time the new method yields more precise estimates than the current method, showing a "weak preference" for the new method. In contrast, if the 10^{th} percentile is positive, then the current method generally yields more precise estimates than the new method, showing a "strong preference" for the current method over the new method. Finally, if the median difference is positive, but the 10^{th} percentile is negative, then at least half the time the current method yields more precise estimates than the new method, showing a "weak preference" for the current method.

The distributional comparison results for all statistics and characteristics are attached. The distributional comparisons for Scenario 2 only cover quarters 28 through 60. Noncertainty companies from the peak population year in Scenario 2 first enter the sample in quarter 28. Attachment E provides a summary of our comparisons by item. Attachment F compares both the fixed-weight and modified variable weight estimators to the current method, then to each other. In both attachments, an uppercase letter indicates a strong preference and a lowercase letter indicates a weak preference. "F" or "f" indicates the fixed-weight estimator, "V1" or "v1" indicates the current variable weight estimator, and "V10" or "v10" indicates the proposed modified variable weight estimator. We used the following logic to obtain the summary results for each characteristic/statistic in Attachment E from Attachment F:

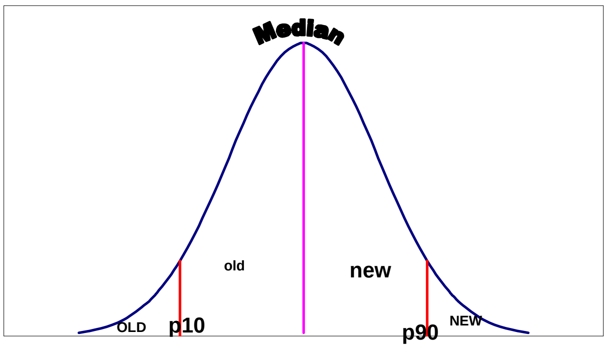


Figure 3: Distributional Comparison

- If one alternative method (fixed or modified variable weight) was better than the current method, then Attachment F contains the alternative method;
- If both alternative methods are better than the current method, then Attachment F contains the alternative method indicated by the "V10 versus Fixed" column;
- If the current method shows "strong preference" over both alternative methods, then Attachment F contains "V1"; and
- Otherwise, Attachment F contains "v1"

Table 5 summarizes the aggregate preference scores for the medium mover industry categories. Boldface **indicates that at least half the observations are classified into this category**.

Table 5: Aggregate Preference Scores for Medium Mover Industries

Description	v1 or V1	v10 or V10	f or F
All Items, All Estimators	**40/64**	18/64	6/64
All Items, Quarterly Change	**13/20**	5/20	2/20
All Items, Levels	9/20	8/20	3/20
Sales and NIAT, All Estimators*	**15/28**	9/28	4/28
Sales and NIAT, Quarterly Change	**5/8**	2/8	1/8
Sales and NIAT, Levels	2/8	**4/8**	2/8

** Includes SALES/NIAT*

In most cases, the current method yields the most precise estimates, the notable exception being level estimates of Sales and NIAT. In these cases, however, the difference in precision is often very small in terms of practical impact. Attachments E and F present time-series plots of MAE and RMAE for SALES and NIAT in medium-mover industries.

For the SALES statistics, the graphs of quarterly and yearly change are nearly overlays with little visible difference between the methods. The same is true for the graph of level in Scenario 7. In

Scenarios 1, 2, and 6 we do see some separation between the modified variable weight estimator and the other two estimators. The modified variable weight estimator has lower RMAE in Scenario 1. In Scenario 2 the modified variable weight estimator has lower RMAE in the increasing phase and higher RMAE in the decreasing phase. This difference in MAE is small, however--approximately 0.25 percent. Population size is maximized in Scenario 2 in sequential quarters 21 through 24; from sequential quarters 25 onward, the population size is monotone decreasing. It appears the point at which the modified variable weight estimate of sales changes from a lower RMAE (i.e., more precise estimate of sales than the current method) to a higher RMAE (i.e., less precise method than the current method) is sequential quarter 33. Finally, in Scenario 6 the modified variable weight estimator alternates between lower and higher relative MAE in what presumably reflects the alternation (or, more likely, a lagged alternation) between increasing and decreasing phases.

The NIAT time-series plots of quarterly and yearly change are less consistent, basically a series of spikes which seem more or less random. Removing clearly visible outliers does very little to change the overall random pattern. The level estimate plots are nearly always overlays; there simply isn't much visible difference between the methods.

9. **Conclusion**

The purpose of this study was to evaluate the statistical properties of the QFR variable weight estimator, along with several proposed alternative estimators, without making any simplifying assumptions about survey design (particularly the rotation scheme), "post-stratification" effects, or population size changes. Our analysis of estimator properties relied primarily on Mean Absolute Error and Relative Mean Absolute Error.

Ultimately, this study validated the current variable weight estimation method for QFR, showing it to have the lowest mean absolute error of the considered methods, although the totals are usually negatively biased due to the coverage bias in non-decreasing populations; however, the coverage bias is approximately the same for the current variable weight estimator and the fixed weight estimator. Moreover, the quarter-to-quarter change estimates of sales constructed from the QFR variable weight estimates are more precise than the change estimates constructed from any of the considered alternative estimators.

The very specific design of this study renders the results extremely useful for the Quarterly Financial Report program. At the same time, our simulation results may not be applicable to other surveys, even those that employ rotating panel designs. Implementation of the variable weight estimator for any of the U.S. Census Bureau economic surveys is not a problem since all of the associated calculations for this estimator are available in our Standardized Economic Processing System (StEPS). However, the desirability of using this type of estimator with other surveys cannot be determined without further research with less specifically designed data.

Acknowledgements

The authors of this report thank Patrick Cantwell for his contribution to this research, and John Chantis, David Dickerson, Rita Petroni, Lynn Weidman, and Tommy Wright for their useful

comments on earlier versions of this manuscript. This paper reports the results of research and analysis undertaken by the U.S. Census Bureau staff. It has undergone a Census Bureau review more limited in scope than that given to official Census Bureau publications. This report is released to inform interested parties of ongoing research and to encourage discussion of work in progress.

References

Box, George P. and Gwilym M. Jenkins (1976). *Time Series Analysis: Forecasting and Control*. San Francisco: Holden-Day.

Carroll, R.J. and Ruppert, D. (1988). *Transformation and Weighting in Regression*. New York: Chapman and Hall.

Chapman, David D. (1993). Quarterly Financial Report (QFR) Research: Investigation of the Properties of the Current QFR Estimator. Washington, D.C. : internal Census Bureau memorandum (Technical report QFR-21-A-16).

Chapman, David W. and Paul Biemer (1985). A Comparison of Two Estimators for the Quarterly Financial Report Survey. Washington, DC: internal Census Bureau memorandum (Technical report SRD/RR-85-20, available from the Statistical Research Division).

Harvey, A.C. (1976). Estimating Regression Models with Multiplicative Heteroscedasticity. *Econometrica*, **44,** pp. 461-465.

Howe, Andromache R. and Katherine J. Thompson (2005). Investigation of Alternative Variance Estimators for the Quarterly Financial Report. *Proceedings of the Section on Survey Research Methods*, American Statistical Association, forthcoming.

Kalton, Graham and Ismael Flores-Cervantes (2003). Weighting Methods. *Journal of Official Statistics*, 2003, **19**, pp. 81-97.

Kott, Phillip S. (1992). Post-Stratifying with Sample Data Only: Can Using the Obvious Model Help? *Proceedings of the Section on Survey Research Methods*, American Statistical Association, pp. 632 – 636.

Luery, Donald (2005). Data Modeling for a Simulation Study of the Quarterly Financial Report Estimator. *Proceedings of the Section on Survey Research Methods*, American Statistical Association, forthcoming.

SAS Institute (1998). *SAS/ETS User's Guide, Version 6, First Edition*. Cary, NC: SAS Institute Inc.

SAS Institute (1999). *SAS/Stat User's Guide, Version 8, First Edition.* Cary, NC: SAS Institute Inc.

Thompson, J. R. *Simulation: A Modeler's Approach.* New York: Wiley, 2000.

Vartivarian, S. and Little, R.J. On the Formation of Weighting Adjustment Cells for Unit Non-response. *Proceedings of the Section of Survey Research Methods*, American Statistical Association, 2002, pp. 3553 - 3558.

Attachments

Attachment A	Algorithms for Generating Population Scenarios
Attachment B	Algorithm for Generating Quarters 1 and 2 of Microdata
Attachment C	Relative Bias Of Level Estimates For Sales Averaged Over Quarters And Industries
Attachment D	Comparison of Unweighted and Weighted Non-response Adjustments Averaged Over Quarters, Scenarios, Estimation Methods, and Enumeration Industries Prior To Tests
Attachment E	Summary of Distributional Comparisons Of Mean Absolute Error (MAE)
Attachment F	Pairwise Distributional Comparisons of Mean Absolute Error
Attachment G	Time-series plots of MAE and RMAE for SALES in Medium-mover Industries
Attachment H	Time-series plots of MAE for NIAT in Medium-mover Industries

Algorithms for Generating Population Scenarios

Let n_t be the universe size for a scenario for year t, d the number of strata, and π_1, π_2, …, π_d the expected proportions. Let y_{1t}, y_{2t}, …, y_{dt} be the stratum sizes to be generated.

Algorithm to Generate Stratum Sizes for 2002

i. The stratum size for the first stratum is y_{1t} = round ($\pi_1 \times n_t$).

ii. Suppose we have generated k stratum sizes y_{1t}, y_{2t}, …, y_{kt}. The stratum size for stratum k+1 is $y_{k+1,t} = \text{round}\left(\left(1-\sum_{i=1}^{k}\pi_i\right)^{-1}\pi_{k+1}\times\left(n_t-\sum_{i=1}^{k}y_{it}\right)\right)$.

iii. Stop when $k = d$.

Algorithm to Generate Stratum Sizes for Each Year After 2002

i. Generate binomial random variate y_{1t} ~ Bin (π_1, n_t). This is the stratum size for stratum 1.

ii. Suppose we have generated k binomial random variates y_{1t}, y_{2t}, …, y_{kt}. Generate the stratum size for stratum k+1 as $y_{k+1,t} \sim \text{Bin}\left(\left(1-\sum_{i=1}^{k}\pi_i\right)^{-1}\pi_{k+1},\, n_t-\sum_{i=1}^{k}y_{it}\right)$.

iii. Stop when $k = d$.

This will generate a multinomial allocation.

Algorithm for Generating Quarters 1 and 2 of Microdata

We modified the non-parametric resampling algorithm described in Thompson (2000) as follows to generate a simulated population of size N_i by resampling the available n_i sample data cases using the following procedure:

1. For each modeling cell i, create a 5 x n_i matrix containing the response data from the sampled individual companies as

$$X = \begin{bmatrix} X_{11} & X_{21} & X_{31} & X_{41} & X_{51} \\ X_{12} & \ddots & & & \vdots \\ \vdots & & & & \\ \vdots & & & & \\ X_{1n} & \dots & & & X_{5n} \end{bmatrix}$$

   where {$\mathbf{X_1}$ $\mathbf{X_2}$ $\mathbf{X_3}$ $\mathbf{X_4}$ $\mathbf{X_5}$**} = {SALES INV NIBT NPPE NIAT}**

2. Within each modeling cell, pick a value of m_i, the number of nearest neighbors to be used in calculation.

   We tested several values of m_i in each modeling cell and selected the value that yielded population quartiles that most closely matched the corresponding sample data quartiles, giving priority to the values of SALES and NIAT.

3. Calculate (Euclidean) distance for each unit (company) j to the remaining $(n_i - 1)$ companies. For each unit j, select the $(m_i - 1)$ nearest neighbors. Calculate

   $\overline{X}_{rj} = \sum_{k=1}^{m_i} \frac{X_{rjk}}{m_i}$, where r = 1,2,…,5 (sales, inventories, etc.).

4. "Center" the $(m_i - 1)$ nearest neighbors $\{k = 2,3,\ldots,m_i\}$ and the company response $\{\mathrm{k} = 1\}$ for each variable X_r for each unit j, using $\{X'_{rjk}\} = \{X_{rjk} - \overline{X}_{rj}\}_{k=1}^{m_i}$.

5. Obtain N_i initial population values by selecting a PPS-WR sample of size N_i from the n_i sample units ($N_i \geq n_i$), using a selection probability of $\frac{W_j}{\sum_{j=1}^{n_i} W_j}$ for each unit *j*. This initial population will contain several observations with identical values of X'_{rs} and $\overline{X}_{rs}$ for each unit *s* (*s* = 1, 2, ..., N_i).

6. Generate m_i random numbers (u_{rsk}) from the uniform distribution given by $U\left(\frac{1}{m_i} - \sqrt{\frac{3(m_i - 1)}{m_i^2}}, \frac{1}{m_i} + \sqrt{\frac{3(m_i - 1)}{m_i^2}}\right)$ for each unit *s*.

7. Randomly generate a new centered value as $X'_{rs} = \sum_{k=1}^{m_i} u_{rsk} X'_{rsk}$. Obtain the final simulated data point $(\tilde{X}_{rs})$ by adding $\overline{X}_{rs}$ to the new centered value: $\tilde{X}_{rs} = X'_{rs} + \overline{X}_{rs}$.

8. Check simulated values to make sure that the values for inventory, sales, and NPPE are all positive. If any one of those items has a negative value, repeat steps 7 and 8 until all simulated values are positive.

Note: To validate our selected values of m_i, we simulated populations of the **same size** as the QFR sample in the modeling cell, then compared item **totals** of the simulated population data to the corresponding QFR estimates.

Our simulation procedure was slightly different for the asset class 18 cases. Since these companies are self-representing, we tried to use as much sample data as possible. We combined two consecutive files of quarterly QFR sample data by company ID and assigned all of these cases to a unique simulated population unit. We used the non-parametric algorithm (with m_i = 2) described above to populate the remaining cases in the simulated population. To balance the simulated data, we stratified the asset class sample data into three strata by reported sales value before selecting our with-replacement sample.

Attachment C

Relative Bias Of Level Estimates For Sales Averaged Over Quarters And Industries

Scenario	Estimation method	Total relative bias	Non-response adjustment relative bias	Covered estimation relative bias	Coverage error relative bias	Coverage error proportion of total bias
Low-mover industries						
1	FWE	-2.1%	0.1%	0.0%	-2.1%	1.00
	VWE1	-2.0%	0.1%	0.0%	-2.1%	1.00
	VWE10	-1.7%	0.1%	0.0%	-1.8%	1.00
2 (Qrtr>27)	FWE	-0.7%	0.0%	-0.1%	-0.5%	0.71
	VWE1	-0.7%	0.0%	-0.1%	-0.5%	0.71
	VWE10	-0.9%	0.0%	-0.4%	-0.4%	0.44
6	FWE	-2.2%	0.1%	-0.2%	-2.0%	0.91
	VWE1	-2.2%	0.1%	-0.2%	-2.0%	0.91
	VWE10	-2.1%	0.1%	-0.5%	-1.8%	0.86
7	FWE	-1.6%	0.1%	0.0%	-1.5%	0.94
	VWE1	-1.5%	0.1%	0.0%	-1.5%	1.00
	VWE10	-1.5%	0.1%	-0.3%	-1.3%	0.87
Medium-mover industries						
1	FWE	-2.6%	0,0%	-0.1%	-2.5%	0.96
	VWE1	-2.6%	0.0%	0.0%	-2.5%	0.96
	VWE10	-2.2%	0.0%	0.0%	-2.2%	1.00
2 (Qrtr>27)	FWE	-1.2%	-0.1%	-0.2%	-0.9%	0.75
	VWE1	-1.1%	-0.1%	-0.2%	-0.9%	0.82
	VWE10	-1.4%	-0.1%	-0.6%	-0.8%	0.57
6	FWE	-2.0%	0.0%	-0.3%	-1.7%	0.85
	VWE1	-1.9%	0.0%	-0.2%	-1.7%	0.89
	VWE10	-2.0%	0.0%	-0.5%	-1.5%	0.75
7	FWE	-2.1%	0.0%	-0.3%	-1.8%	0.86
	VWE1	-2.1%	0.0%	-0.3%	-1.8%	0.86
	VWE10	-2.2%	0.0%	-0.6%	-1.6%	0.73
High Mover Industries						
1	FWE	-3.6%	0.1%	0.0%	-3.5%	0.97
	VWE1	-4.5%	0.1%	-1.2%	-3.5%	0.78
	VWE10	-4.0%	0.1%	-1.1%	-3.0%	0.75
2 (Qrtr>27)	FWE	-1.9%	0.2%	-0.2%	-1.2%	0.63
	VWE1	-3.1%	0.2%	-1.6%	-1.2%	0.39
	VWE10	-3.4%	0.2%	-2.3%	-1.0%	0.29
6	FWE	-3.5%	0.1%	-0.5%	-3.0%	0.86
	VWE1	-4.3%	0.1%	-1.5%	-3.0%	0.70
	VWE10	-4.3%	0.1%	-1.9%	-2.6%	0.60
7	FWE	-2.6%	0.1%	-0.6%	-2.1%	0.81
	VWE1	-3.4%	0.1%	-1.5%	-2.1%	0.62
	VWE10	-3.5%	0.1%	-1.8%	-1.8%	0.51

Comparison of Unweighted and Weighted Non-response Adjustments Averaged Over Quarters, Scenarios, Estimation Methods, and Enumeration Industries Prior To Tests

Item	Average Unweighted Non-response Adjusted	Average Weighted Non-response Adjusted	Difference	T Value	P Value
Type of Estimate=LEVEL					
INV	63,343,290	63,349,225	-5,935	-9.61	0.0000
NIAT	146,221	146,285	-64	-3.60	0.0007
NIBT	224,465	224,536	-71	-3.97	0.0002
NPPE	117,161,631	117,166,712	-5,081	-9.38	0.0000
SALES	151,925,068	151,936,411	-11,343	-11.32	0.0000
Type of Estimate=RATIO					
RATIO	0.0040	0.0040	0.0000	0.39	0.7017
Type of Estimate=TRENDQ					
INV	0.0060	0.0060	0.0000	3.61	0.0007
NIAT	0.5446	-0.3088	0.8534	1.04	0.3027
NIBT	7.0522	1.4979	5.5542	0.81	0.4200
NPPE	0.0017	0.0017	0.0000	7.42	0.0000
SALES	0.0193	0.0193	0.0000	8.61	0.0000
Type of Estimate=TRENDY					
INV	0.0084	0.0084	-0.0000	-5.41	0.0000
NIAT	0.8260	-0.3766	1.2026	1.49	0.1437
NIBT	3.2554	-0.3791	3.6345	0.94	0.3518
NPPE	0.0020	0.0020	0.0000	0.55	0.5863
SALES	0.0643	0.0642	0.0000	3.90	0.0003

INVENTORIES
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	Quarter-to-Quarter Change		
	Low	Medium	High
1	v1	v1	V1
2	v10	f	F
6	v1	v1	v1
7	f	v1	v1

Scenario	Level		
	Low	Medium	High
1	V10	V10	F
2	V1	v10	v1
6	v1	v1	f
7	f	f	F

Scenario	Year-to-Year Change		
	Low	Medium	High
1	v1	v1	v1
2	f	v1	v1
6	v1	v1	v1
7	f	v1	v1

NIAT
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	Quarter-to-Quarter Change		
	Low	Medium	High
1	v1	v1	v1
2	v10	v10	v10
6	v1	v10	v10
7	v1	v1	v10

Scenario	Level		
	Low	Medium	High
1	v1	v1	V1
2	v10	v10	v10
6	v10	v10	v10
7	v1	v10	v10

Scenario	Year-to-Year Change		
	Low	Medium	High
1	v1	v1	v1
2	v10	v10	v10
6	f	v10	v10
7	v1	v1	v10

NIBT
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	Quarter-to-Quarter Change		
	Low	Medium	High
1	v1	v1	V1
2	f	v10	v10
6	v1	v10	v10
7	v1	v10	v10

Scenario	Level		
	Low	Medium	High
1	f	V1	V1
2	v10	v10	v10
6	f	v1	v10
7	f	v1	v1

Scenario	Year-to-Year Change		
	Low	Medium	High
1	v1	v1	v1
2	v10	v10	v10
6	f	v10	v10
7	v10	v1	v1

NPPE
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	Quarter-to-Quarter Change		
	Low	Medium	High
1	f	v1	V1
2	v1	v1	F
6	v1	v1	v1
7	v1	v1	V1

Scenario	Level		
	Low	Medium	High
1	V10	V10	F
2	v10	v1	v1
6	v1	v1	F
7	v10	v1	F

Scenario	Year-to-Year Change		
	Low	Medium	High
1	v1	v1	v1
2	v10	v1	v1
6	v1	v1	v1
7	v1	v1	v1

SALES
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	Quarter-to-Quarter Change		
	Low	Medium	High
1	f	v1	V1
2	v1	f	F
6	v1	v1	v1
7	v1	v1	v1

Scenario	Level		
	Low	Medium	High
1	V10	V10	F
2	v10	v1	v1
6	v1	f	F
7	v10	f	F

Scenario	Year-to-Year Change		
	Low	Medium	High
1	v1	v1	v1
2	v10	v1	V1
6	f	v1	v1
7	f	v1	v1

SALES/NIAT (RATIO)
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	Low	Medium	High
1	v1	v10	f
2	v10	v1	v10
6	f	f	f
7	v1	v1	f

INVENTORIES
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	QUARTER-TO-QUARTER CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	V1	F	V1	V1	f	V1	V1	v10
2	f	v10	v10	F	v1	f	F	v1	F
6	v1	v1	F	V1	v1	v10	v1	v1	v10
7	f	v1	F	V1	v1	v10	v1	v1	v10

Scenario	LEVEL								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	V10	V10	F	V10	V10	F	V10	f
2	v1	v10	f	V1	v10	v10	V1	v1	v10
6	v1	v1	f	V1	v1	f	f	v1	f
7	f	v1	f	F	v1	f	F	v1	F

Scenario	YEAR-TO-YEAR CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	f	V1	V1	f	v1	V1	f
2	f	v10	f	V1	v1	v10	v1	v1	v10
6	v1	v1	f	V1	v1	f	v1	v1	v10
7	f	v1	f	V1	v1	f	v1	v1	v10

NIAT
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	**QUARTER-TO-QUARTER CHANGE**								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	f	V1	v1	f	v1	V1	v10
2	f	v10	v10	V1	v10	V10	v1	v10	V10
6	v1	v1	f	V1	v10	v10	v1	v10	v10
7	v1	v1	v10	V1	v1	v10	v1	v10	v10

Scenario	**LEVEL**								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	V1	F	v1	V1	f	F	V1	v10
2	f	v10	v10	v1	v10	v10	v1	v10	v10
6	v1	v10	v10	v1	v10	v10	V1	v10	V10
7	v1	v1	v10	v1	v10	v10	V1	v10	V10

Scenario	**YEAR-TO-YEAR CHANGE**								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	f	v1	v1	f	v1	V1	v10
2	f	v10	v10	v1	v10	v10	v1	v10	v10
6	f	v1	f	v1	v10	v10	v1	v10	v10
7	v1	v1	v10	v1	v1	v10	v1	v10	v10

NIBT
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	QUARTER-TO-QUARTER CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	v10	v1	v1	v10	V1	V1	V10
2	f	v10	f	V1	v10	V10	V1	v10	V10
6	f	v1	v10	v1	v10	v10	v1	v10	v10
7	v1	v1	v10	v1	v10	v10	V1	v10	v10

Scenario	LEVEL								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	f	v1	f	V1	V1	V10	V1	V1	V10
2	v1	v10	v10	v1	v10	v10	v1	v10	v10
6	f	v1	f	V1	v1	V10	V1	v10	V10
7	f	v10	f	V1	v1	V10	V1	v1	V10

Scenario	YEAR-TO-YEAR CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	v10	v1	v1	v10	V1	v1	V10
2	v1	v10	v10	v1	v10	v10	v1	v10	v10
6	f	v10	f	v1	v10	v10	v1	v10	v10
7	v1	v10	v10	v1	v1	v10	V1	v1	v10

NPPE
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	QUARTER-TO-QUARTER CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	f	v1	F	v1	V1	F	V1	V1	v10
2	V1	v1	f	V1	v1	f	F	v1	F
6	v1	v1	v10	v1	v1	f	V1	v1	v10
7	v1	v1	f	v1	v1	f	V1	V1	v10

Scenario	LEVEL								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	V10	V10	V1	V10	V10	F	V10	F
2	f	v10	v10	v1	v1	v10	V1	v1	v10
6	v1	v1	f	V1	v1	v10	F	v1	F
7	v1	v10	v10	v1	v1	f	F	v1	F

Scenario	YEAR-TO-YEAR CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	V1	f	v1	V1	F	v1	V1	v10
2	f	v10	v10	v1	v1	f	v1	v1	f
6	v1	v10	f	v1	V1	F	v1	v1	f
7	v1	v1	f	v1	v1	f	V1	v1	v10

SALES
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

Scenario	QUARTER-TO-QUARTER CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	f	V1	F	v1	V1	f	V1	V1	v10
2	v1	v1	f	F	v1	f	F	v1	F
6	v1	v1	f	v1	v1	f	v1	v1	v10
7	v1	v1	f	v1	v1	v10	v1	v1	v10

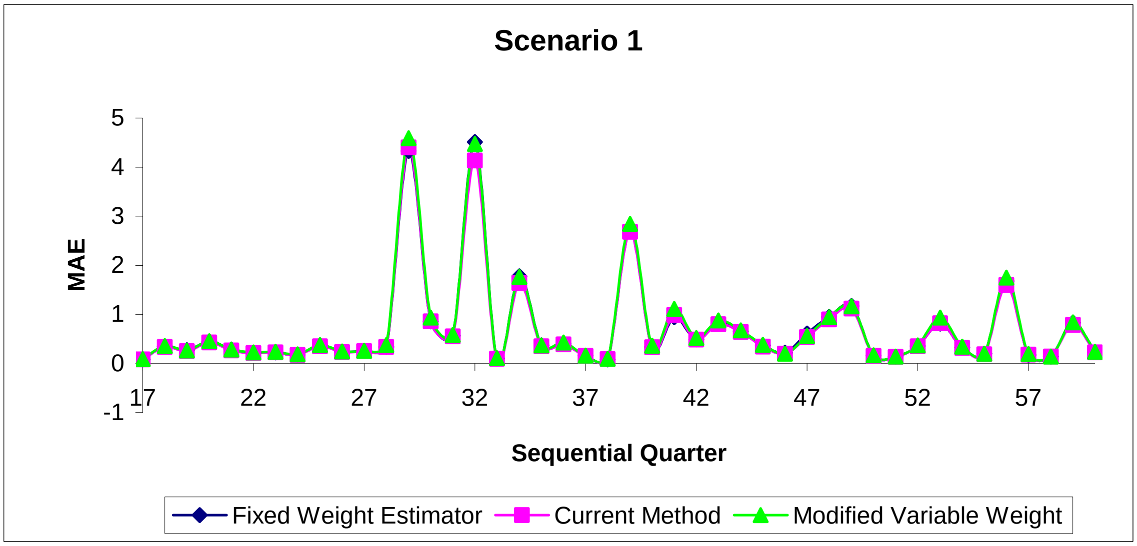
Scenario	LEVEL								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	V10	V10	F	V10	V10	F	V10	F
2	f	v10	v10	v1	v1	v10	V1	v1	V10
6	v1	v1	f	F	v1	f	F	v1	F
7	v1	v10	v10	F	v1	f	F	v1	F

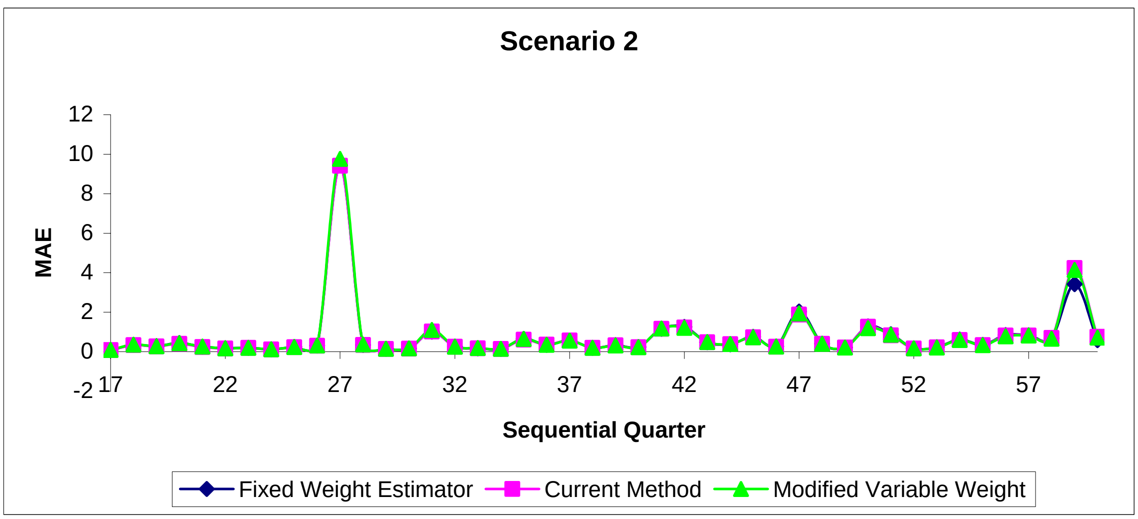
Scenario	YEAR-TO-YEAR CHANGE								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	f	v1	V1	f	v1	v1	f
2	f	v10	v10	v1	V1	f	V1	V1	v10
6	f	v1	f	v1	v1	f	v1	v1	v10
7	f	v1	f	v1	v1	f	v1	V1	v10

SALES/NIAT (RATIO)
DISTRIBUTIONAL COMPARISONS OF MEAN ABSOLUTE ERROR (MAE)

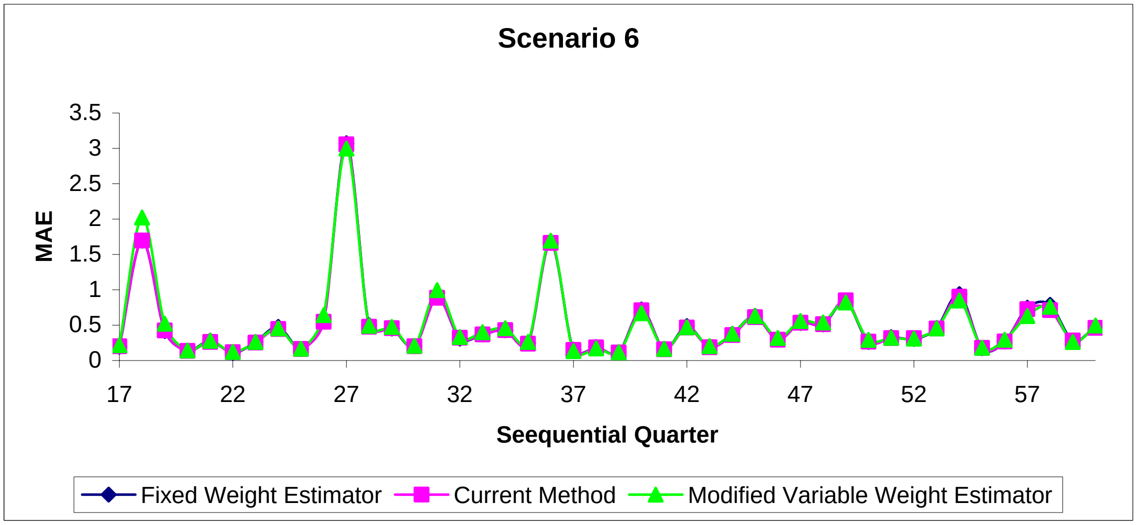
Scenario	**LEVEL**								
	LOW			MEDIUM			HIGH		
	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED	V1 VERSUS FIXED	V1 VERSUS V10	V10 VERSUS FIXED
1	v1	v1	f	v1	v10	v10	f	v10	f
2	f	v10	V10	V1	v1	V10	v1	v10	V10
6	f	v10	f	F	v1	f	f	v1	f
7	v1	v1	f	v1	v1	v10	f	v10	f

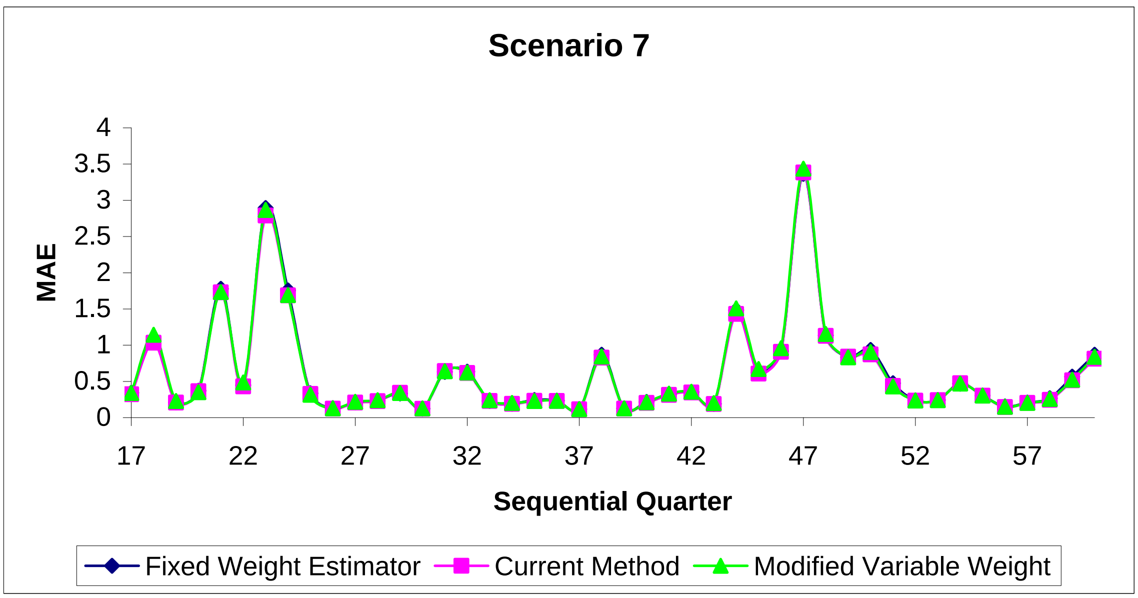
SALES
QUARTER-TO-QUARTER CHANGE Estimates
Medium Mover Industries



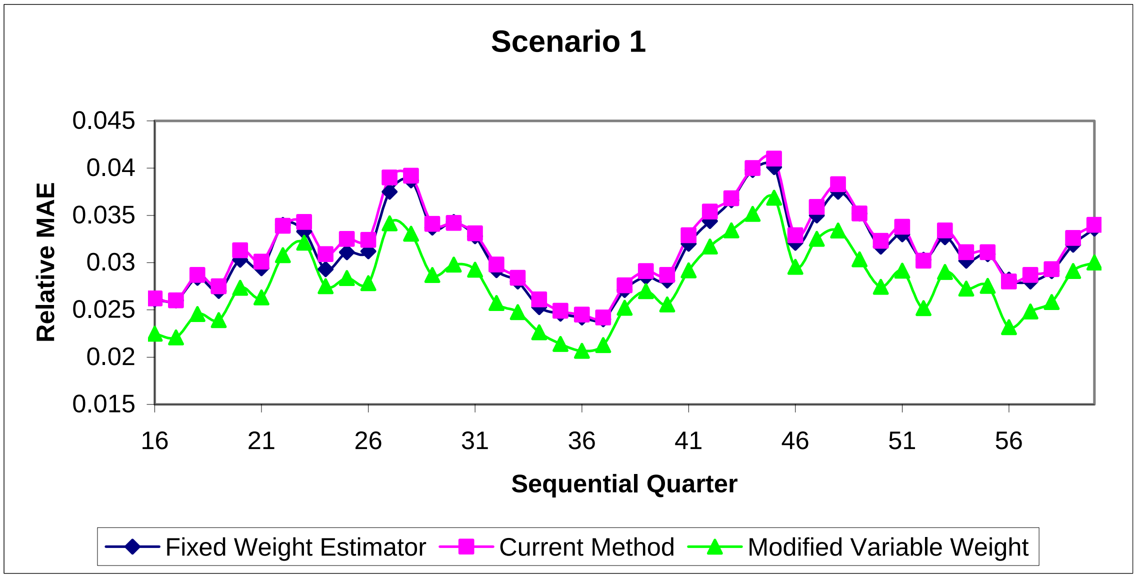


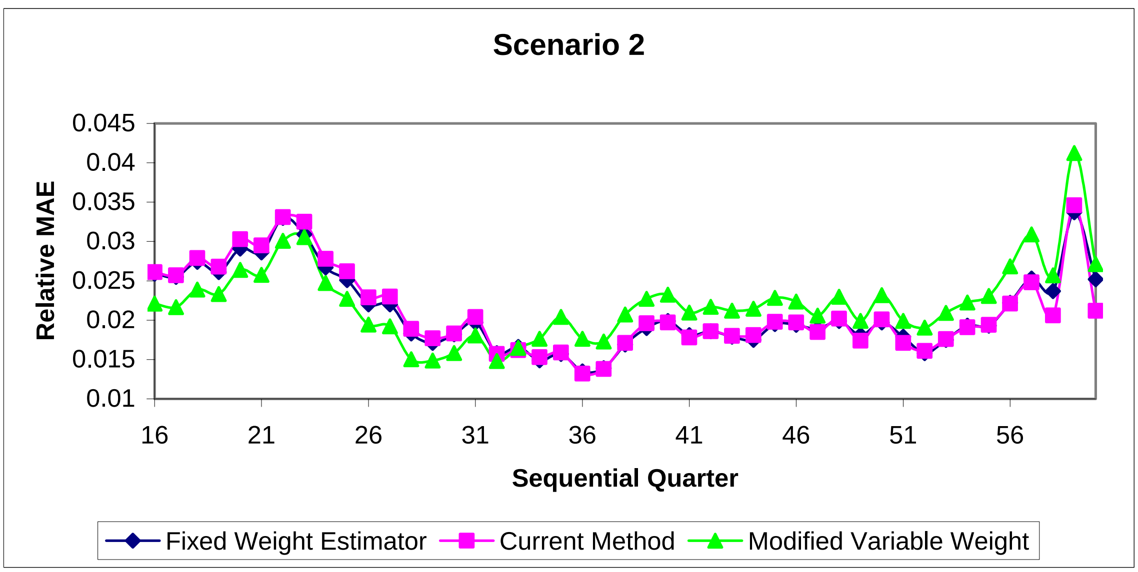
SALES
QUARTER-TO-QUARTER CHANGE Estimates
Medium Mover Industries



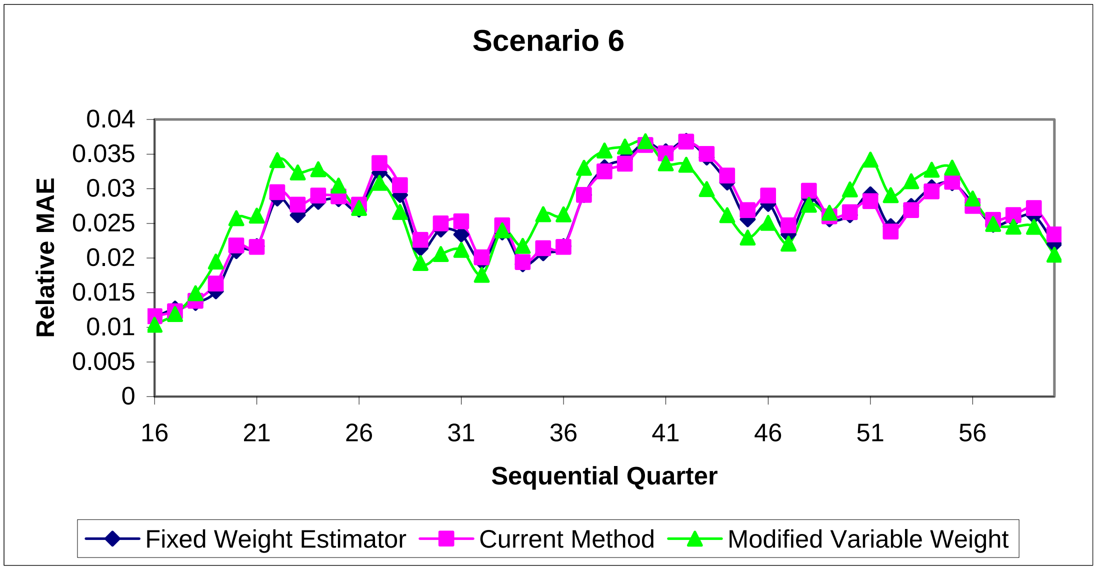


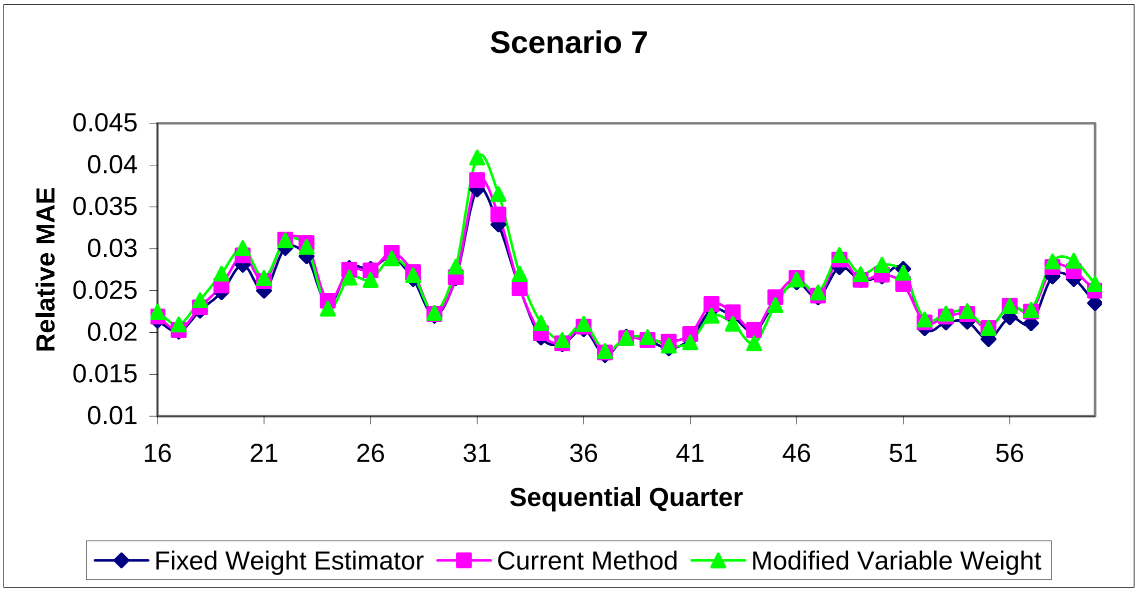
SALES
LEVEL Estimates
Medium Mover Industries



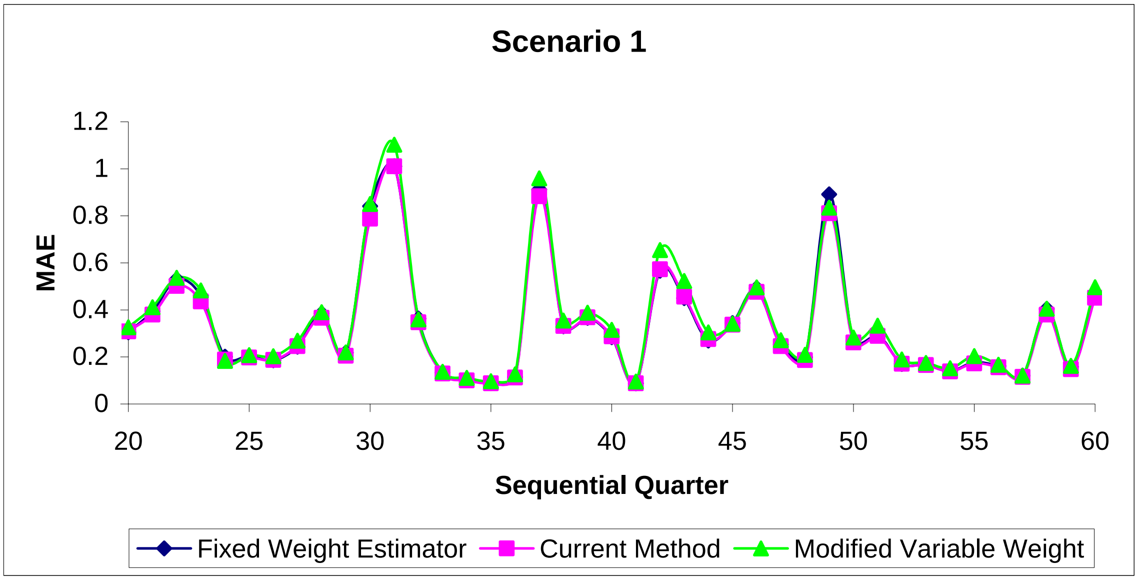


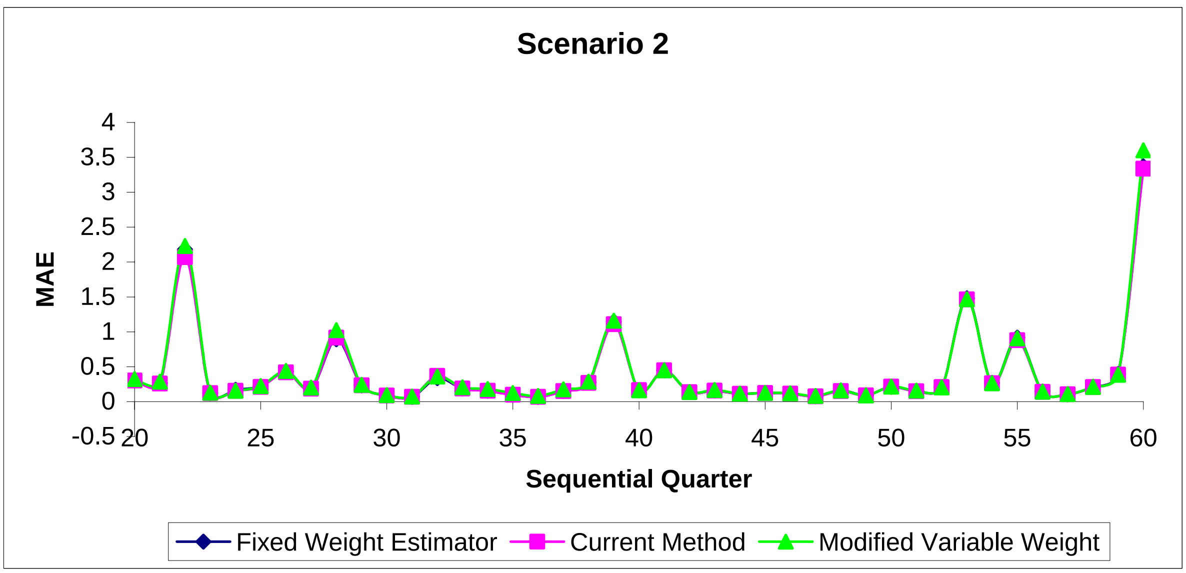
SALES
LEVEL Estimates
Medium Mover Industries



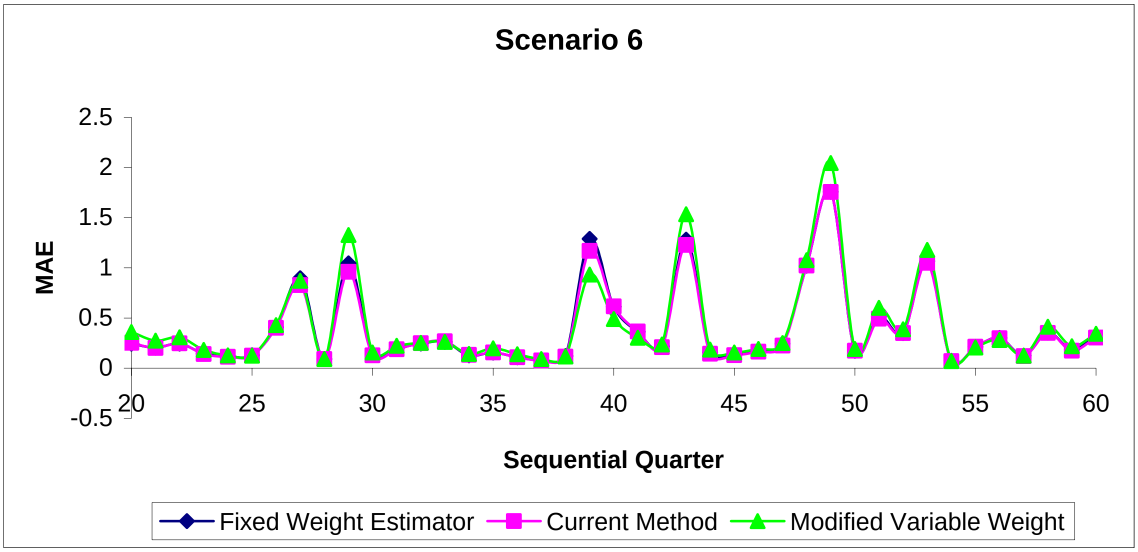


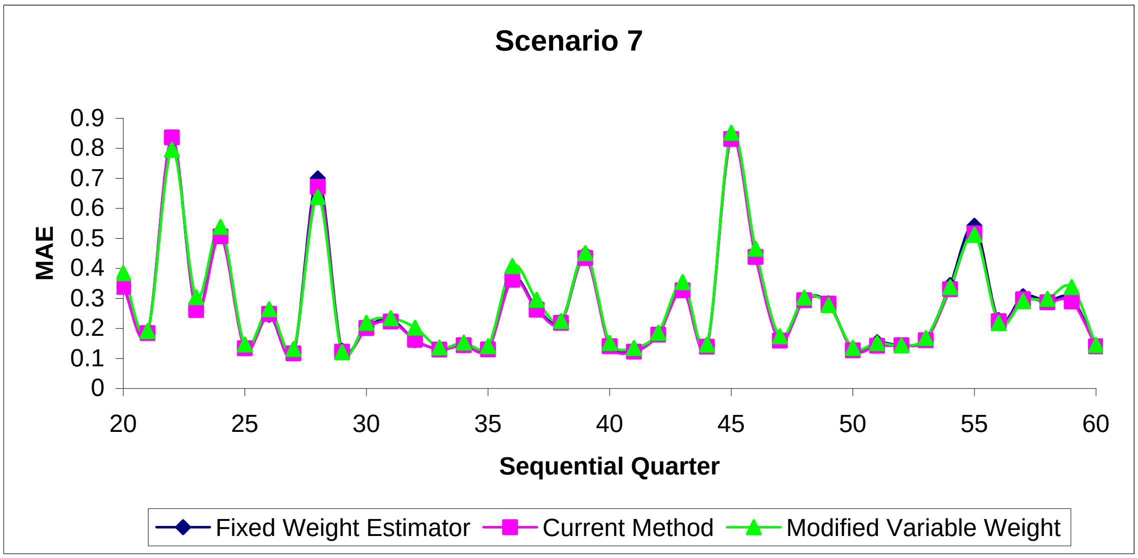
SALES
YEAR-TO-YEAR CHANGE Estimates
Medium Mover Industries



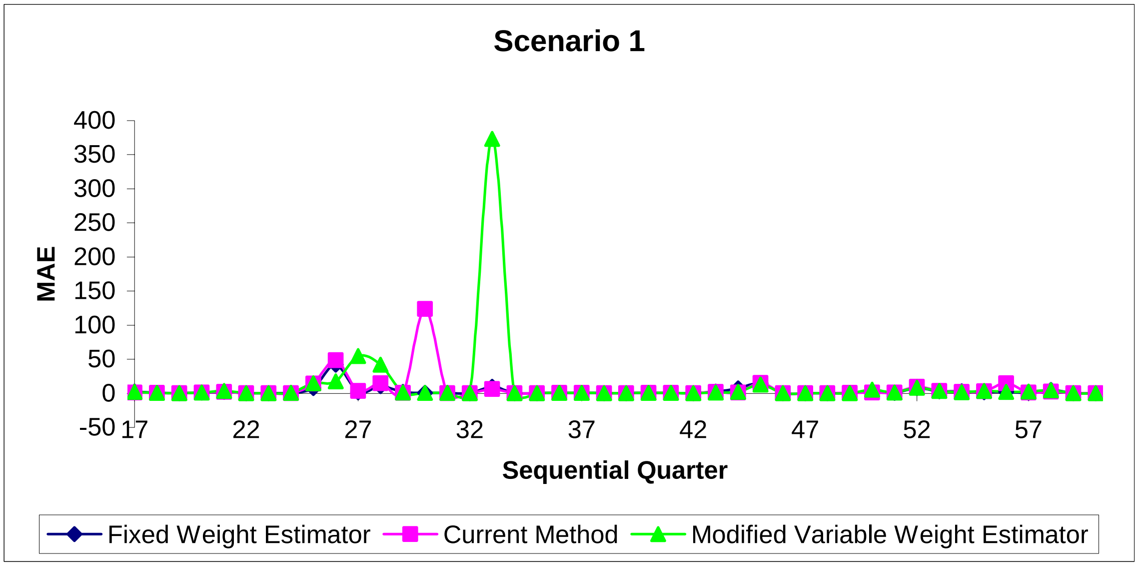


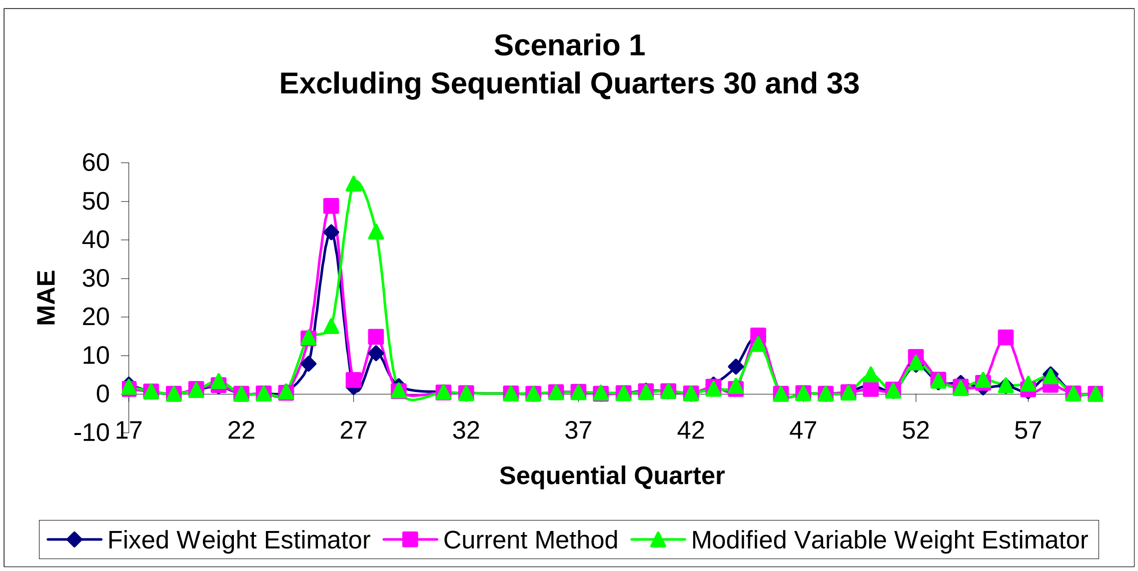
SALES
YEAR-TO-YEAR CHANGE Estimates
Medium Mover Industries



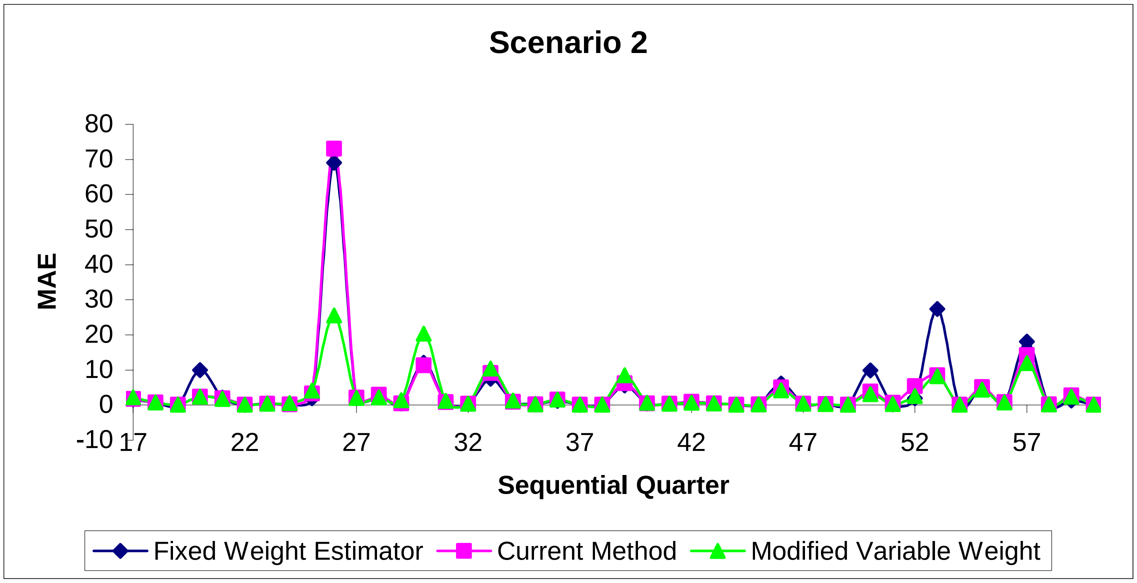


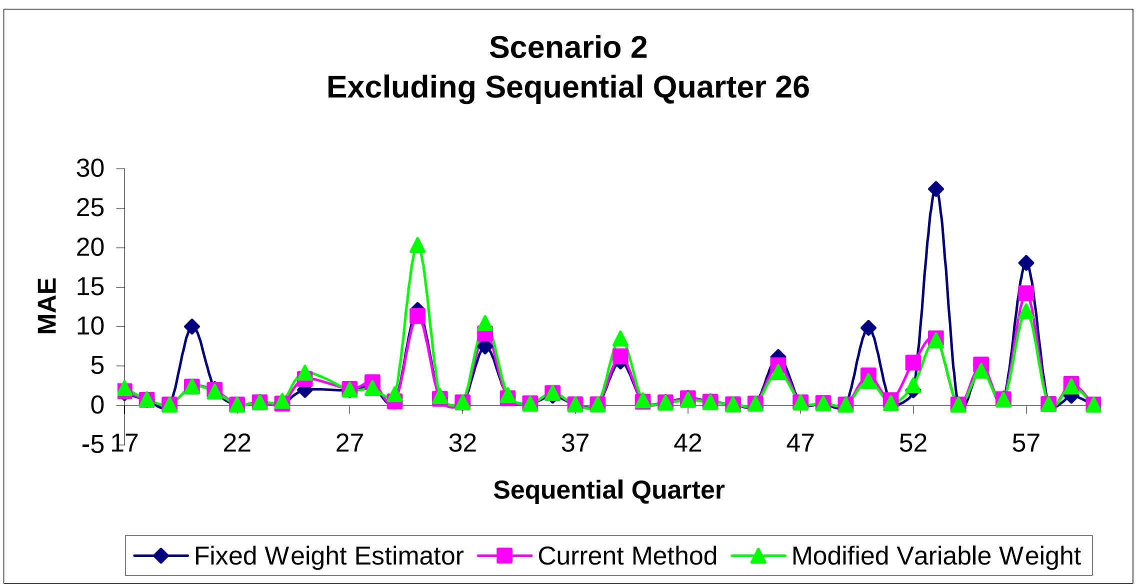
NET INCOME AFTER TAXES (NIAT)
QUARTER-TO-QUARTER CHANGE Estimates
Medium Mover Industries



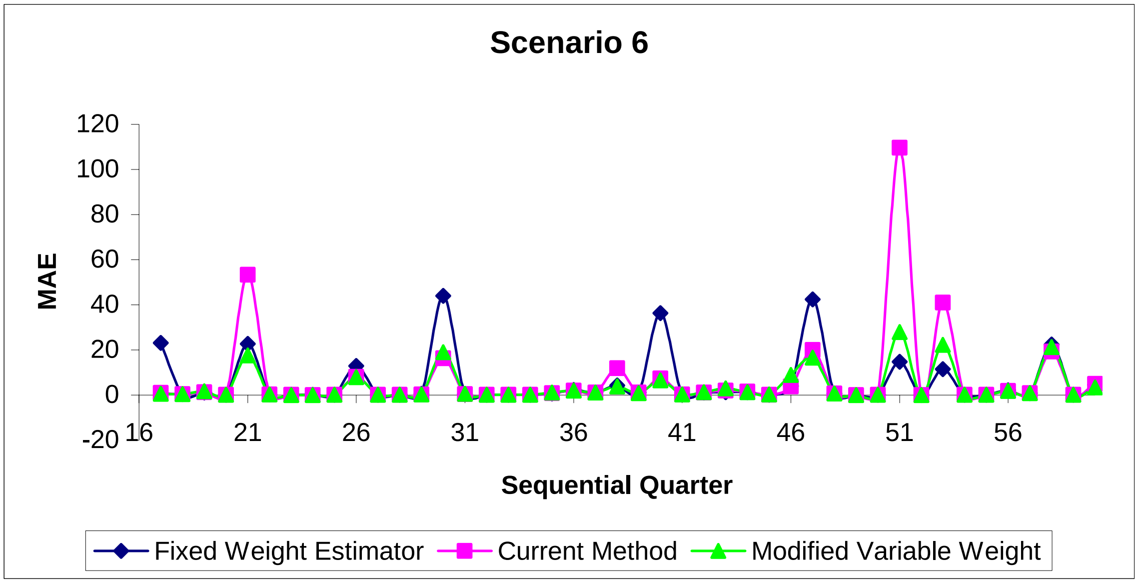


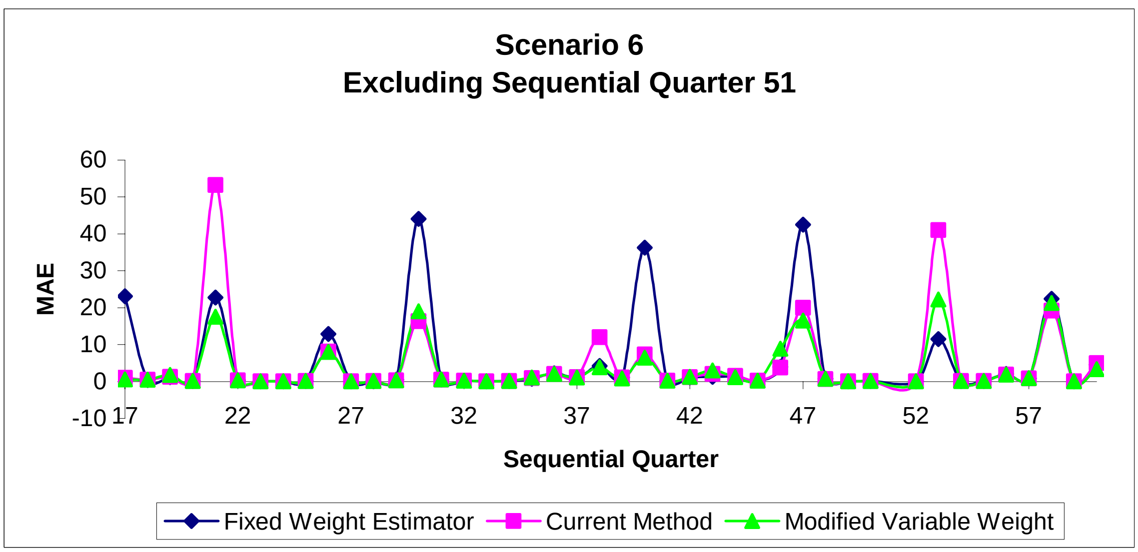
NET INCOME AFTER TAXES (NIAT)
QUARTER-TO-QUARTER CHANGE Estimates
Medium Mover Industries



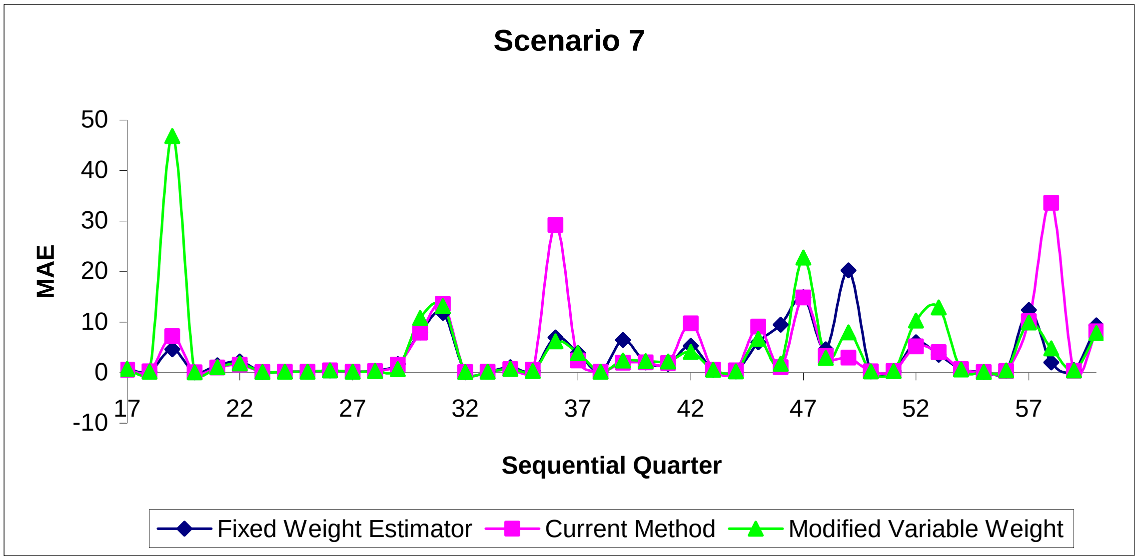


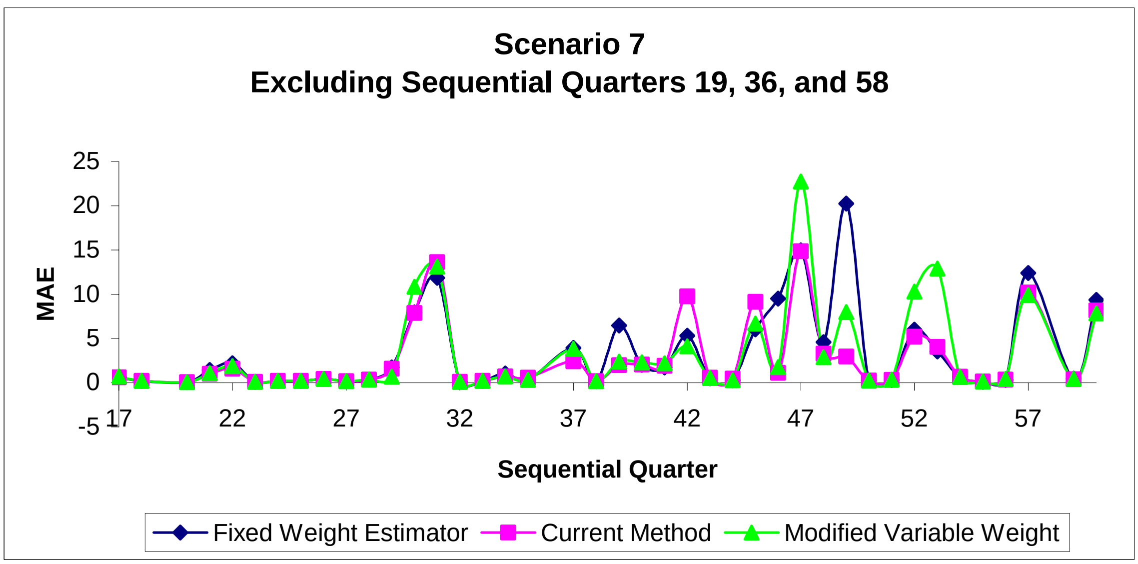
NET INCOME AFTER TAXES (NIAT)
QUARTER-TO-QUARTER CHANGE Estimates
Medium Mover Industries



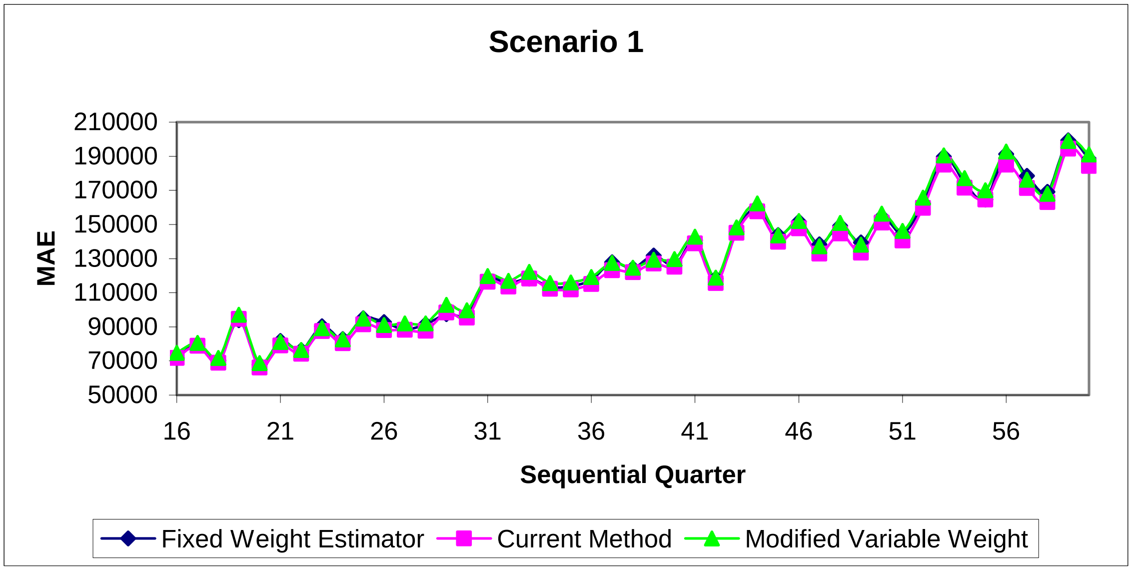


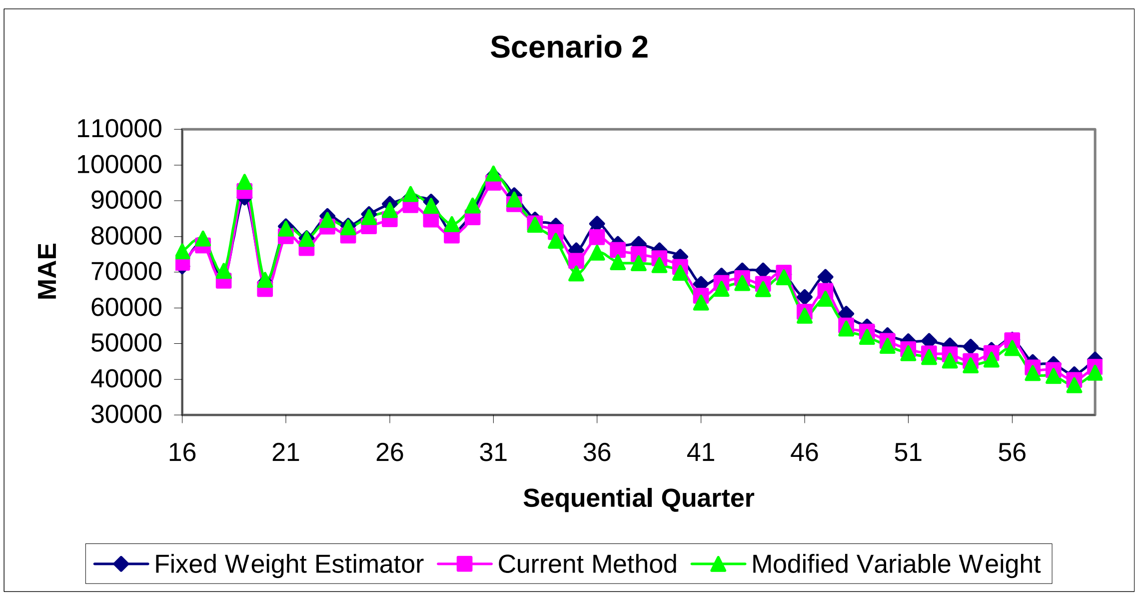
NET INCOME AFTER TAXES (NIAT)
QUARTER-TO-QUARTER CHANGE Estimates
Medium Mover Industries



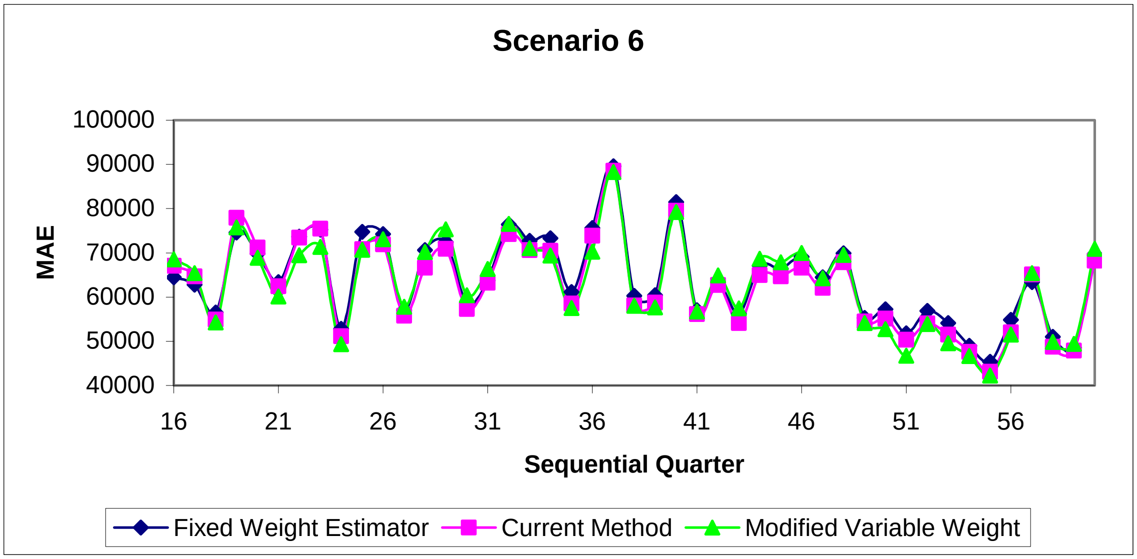


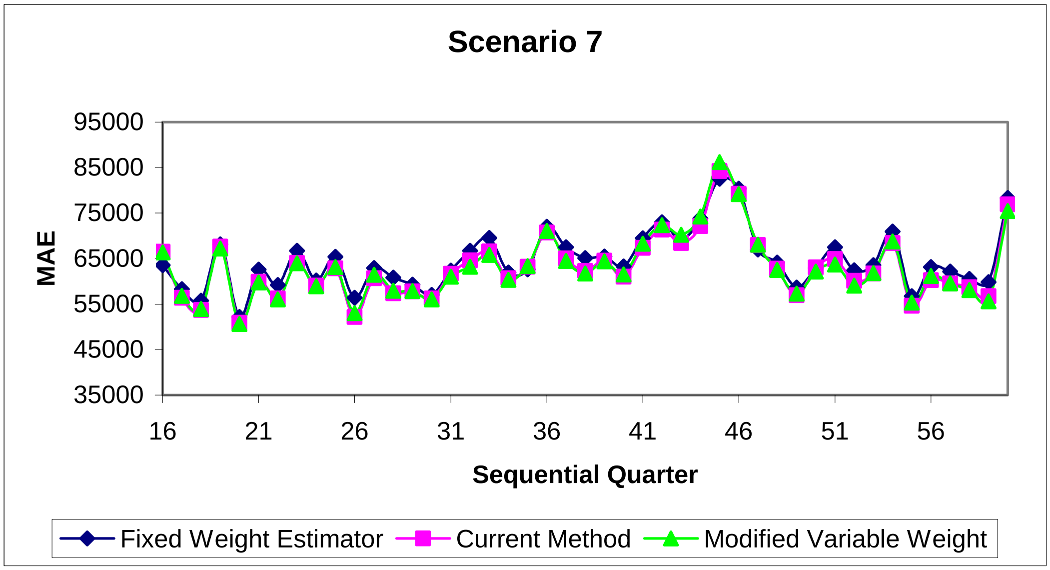
NET INCOME AFTER TAXES (NIAT)
LEVEL Estimates
Medium Mover Industries



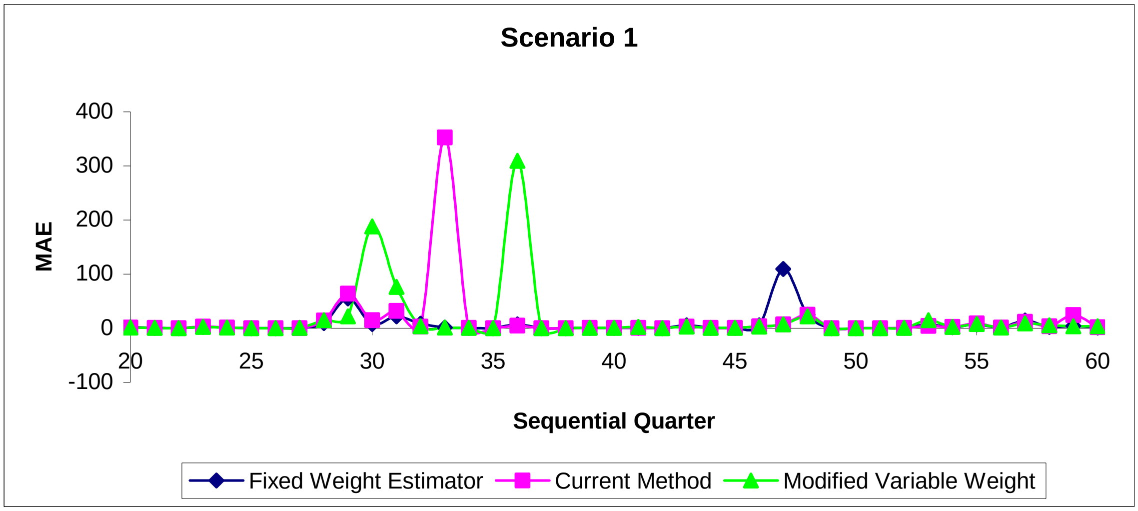


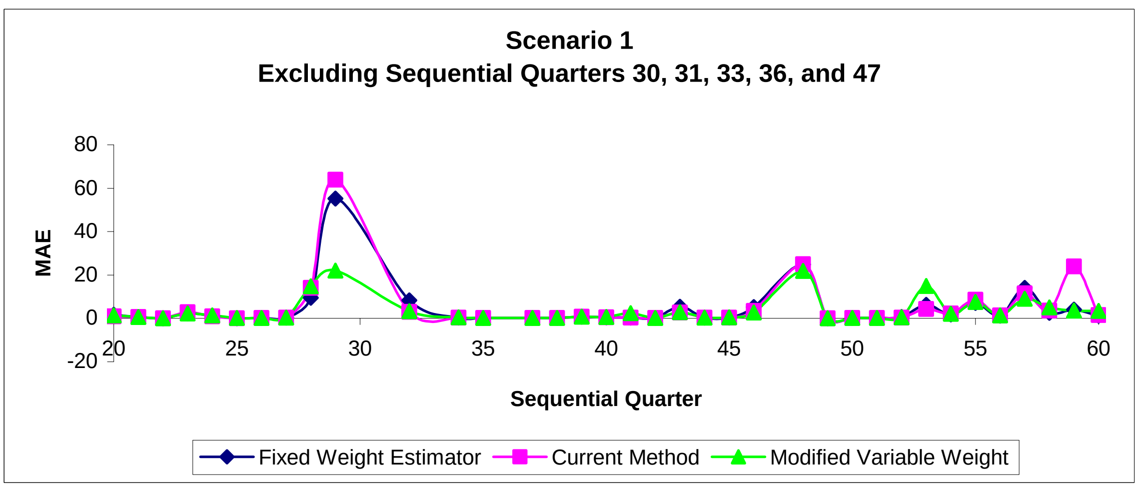
NET INCOME AFTER TAXES (NIAT)
LEVEL Estimates
Medium Mover Industries



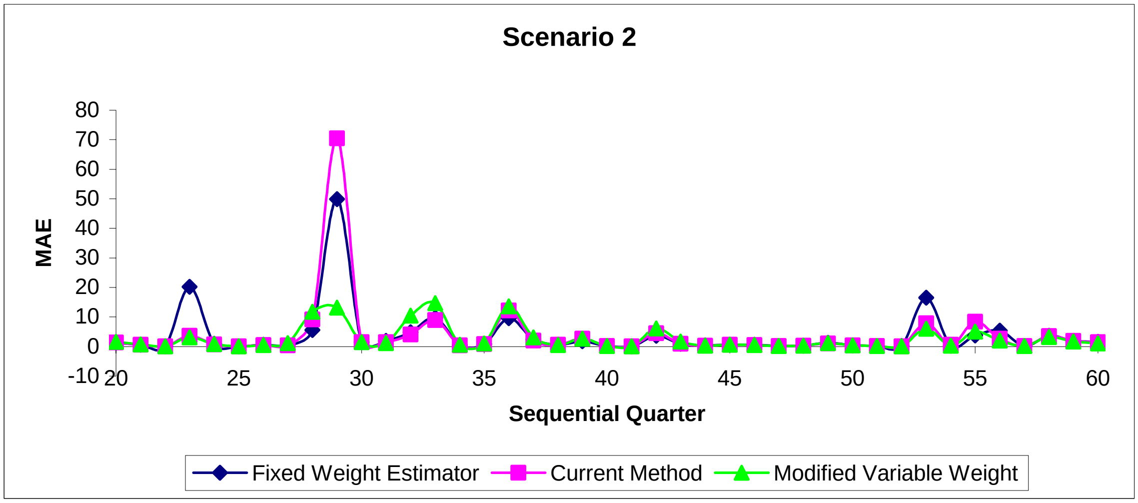


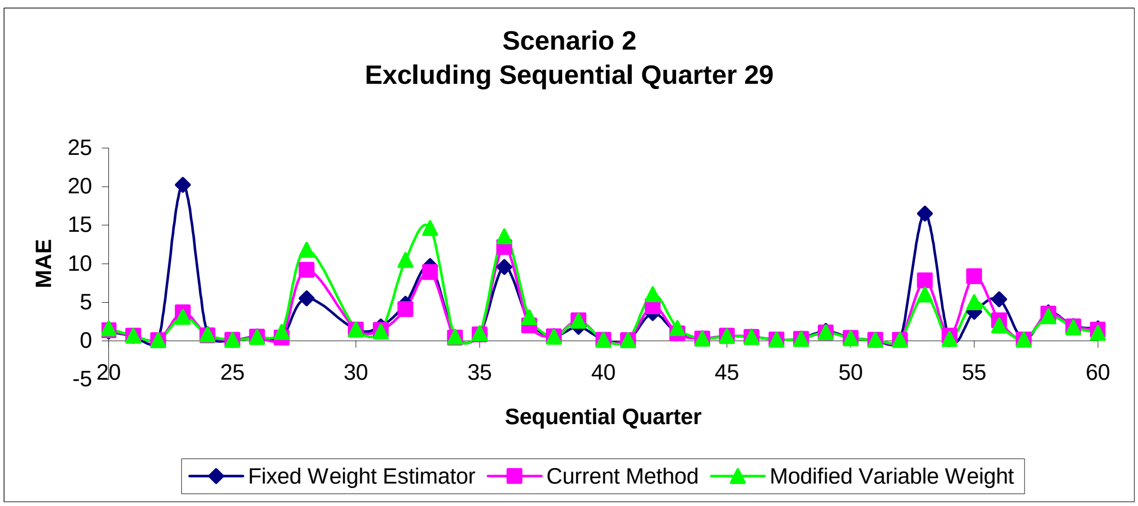
NET INCOME AFTER TAXES (NIAT)
YEAR-TO-YEAR CHANGE Estimates
Medium Mover Industries



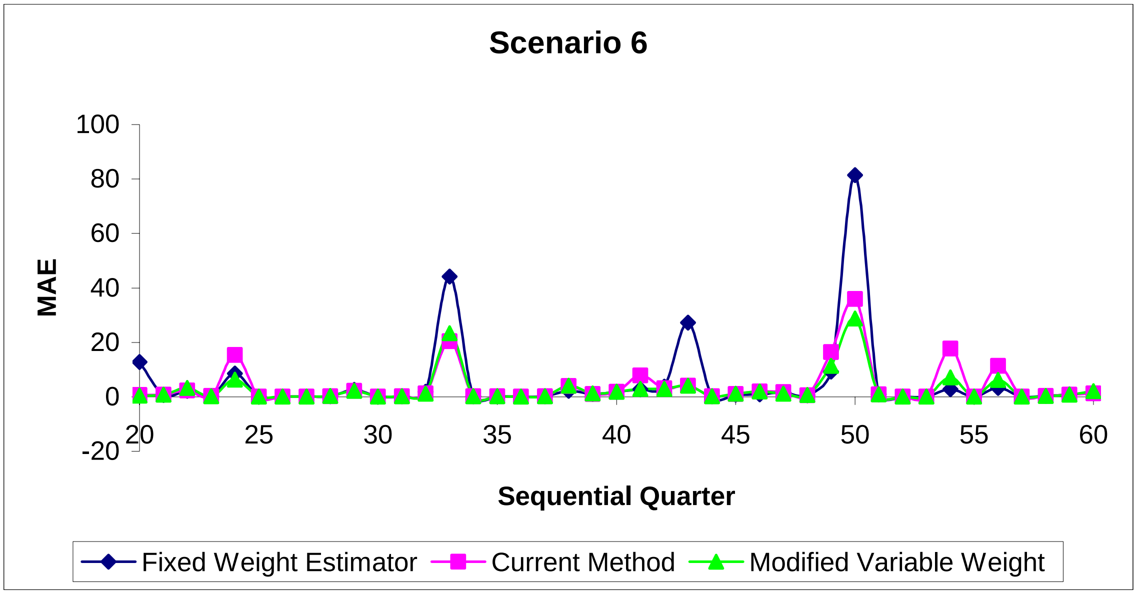


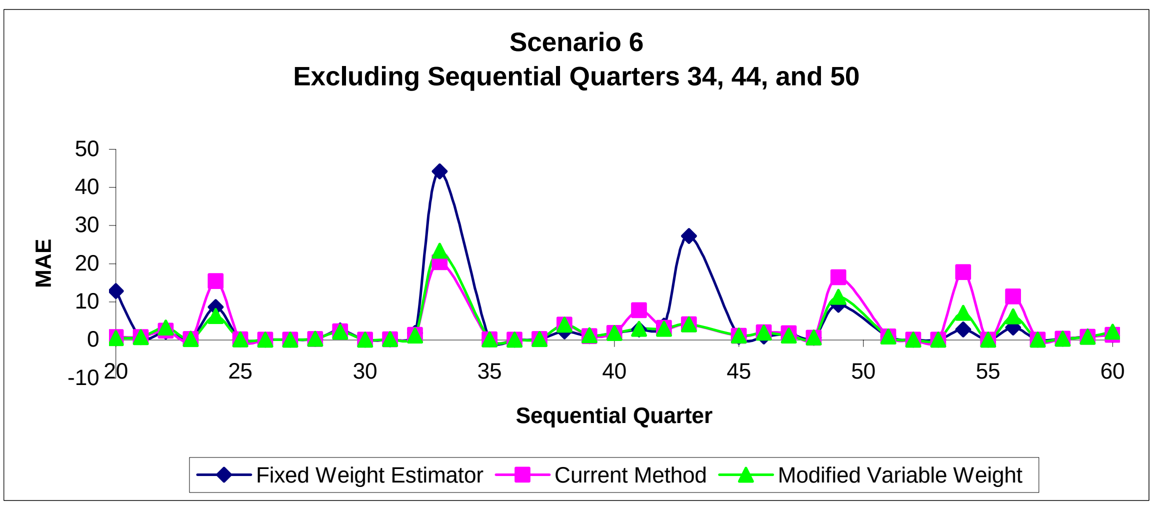
NET INCOME AFTER TAXES (NIAT)
YEAR-TO-YEAR CHANGE Estimates
Medium Mover Industries





NET INCOME AFTER TAXES (NIAT)
YEAR-TO-YEAR CHANGE Estimates
Medium Mover Industries





**NET INCOME AFTER TAXES (NIAT)
YEAR-TO-YEAR CHANGE Estimates
Medium Mover Industries**