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BENCHMARKING: Evaluating Methods That Preserve Month-to-Month Changes

by

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ABSTRACT

This report has two main parts. The first part of the report presents the results of a study comparing three options for benchmarking monthly time series to annual surveys or censuses. The three penalty functions to be minimized are the sum of squares of the month-to-month changes of the unadjusted data (UA method), seasonally adjusted data (SA method), and relative month-to-month changes (REL method). We studied the differences between methods in terms of revisions (differences between the original and the benchmarked series) in levels, month-to-month changes in the unadjusted and seasonally adjusted data, and in the X-11 seasonal factors. We found no practical differences in the percent revisions in level between the SA and UA methods. The differences were at most 2.3 percent for one series and .59 percent or less for the rest of the series. We found that the UA method obtained lower median absolute month-to-month change revisions of the unadjusted data. The UA method also obtained smaller median absolute revisions of the seasonally adjusted data for a majority of the series studied. In comparing the REL method to the UA method the benchmarked series were identical to the second decimal place.

The second part is a user's guide to the new ASCII FORTRAN benchmarking computer program we have developed. This program includes the SA and REL methods as options, as well as the UA method used in the Bureau of the Census' current benchmarking computer program.

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I. INTRODUCTION

The dictionary defines a benchmark as a marked point of known or assumed elevation. This definition pertains to surveying, however -- in a time series context, a benchmark is a value or estimate produced from a survey that is assumed to be more reliable than the value or values from a more frequently taken periodic survey taken over the same time span. We will define the estimates from the more frequently taken survey as the original series **x**, the revised or benchmarked series **y**, and the estimates from the more reliable survey, the benchmarks, T. (Series will be in boldface or brackets $\{T_k\}$.) The benchmark may cover a single time point or several contiguous time points. In the some cases the benchmark is related to the sum of the original series taken over the benchmark period. Common applications of benchmarking are adjusting monthly survey estimates to annual survey estimates, or adjusting between-census figures to census figures. In this paper we will assume the series are monthly and the benchmarks are annual.

In a statistical framework, benchmarking could be viewed as the combination of information from two surveys measuring the same quantity; an original monthly survey and an annual survey which provides the benchmarks. To get **y**, the original and benchmarked series values could be weighted by the inverse of the variance of each survey's estimates. If the annual survey is more exhaustive then it will have a lower estimation variance and thus will influence **y** more. But benchmarking may also be motivated by a belief that **x** contains bias that is either not present in the benchmarks T_k, or is present to a lesser degree. In such a case we may not want **x** to influence the annual series. This could be achieved in a formal statistical framework by setting the variance of the T_k's to zero, which forces the y_t's to sum to their annual benchmark survey estimates. The main point is the more frequent estimates are forced to satisfy the benchmark constraints, i.e. behave as if the benchmark values are the true values.

Benchmarks are different for flow and stock series. Flow series measure accumulations over time such as a month or year. Examples are retail sales and new order series. Stock series are measures taken at a point in time; i.e., inventory series. On one hand, flow series benchmarks are values accumulated over a period of time. For example, annual sales are the sum of the sales over the year, and in particular the sum of the monthly sales over the year. On the other hand, stock series benchmarks are point estimates in time. The annual inventory is not the sum of the monthly inventories but the end-of-the-year value. When a flow series is benchmarked, the monthly figures are forced to sum to an annual total. When a stock series is benchmarked, one monthly value is forced to the year end value. Note that not all periods may be covered by benchmarks, as is true of stock series. Also, note that those values not covered by benchmarks may also be revised.

Just forcing a series to sum to its benchmark totals does not make a unique benchmark series; many different monthly values may sum to the same benchmark total. One way to construct a unique benchmarked series is to preserve some characteristic of the original series as closely as possible. This might be the original series levels, the unadjusted month-to-month changes, or the seasonally adjusted month-to-month changes. If the original levels are preserved, and there are large differences between the original series totals and their corresponding benchmarks, large discontinuities will exist between data points covered by different benchmarks. To smooth these discontinuities, Causey (1981) developed a method to preserve (unadjusted) month-to-month changes. However, in cases where the seasonally adjusted month-to-month changes are the most often used published figures, preserving the month-to-month changes in the seasonally adjusted data may be more reasonable.

In Section 2, we address different methods of benchmarking series - preserving levels, month-to-month changes in unadjusted data, relative month-to-month changes in the unadjusted data, and month-to-month changes in seasonally adjusted data. In Section 3 we analyze the benchmarked series produced using the methods that preserve month-to-month changes.

The Bureau of the Census has a computer program that produces benchmarked series by the unadjusted month-to-month changes method noted above. This program, written originally in ALGOL, has been rewritten in ASCII FORTRAN, and this new version of the Bureau's benchmarking program is described in Appendix A.

2. METHODS OF BENCHMARKING

2.1 Minimizing Revisions in Levels

The simplest form of benchmarking satisfies the benchmark constraints while preserving the levels in the original series as closely as possible according to some criterion. This can be done by using Lagrange multipliers where the function to be minimized is

$$\sum_{t=1}^{n} (y_t - x_t)^2 . \qquad (2.1.1)$$

x is the original series, y is the benchmarked or revised series, and t is the index for time in a series that exists from time point 1 to n. The benchmarked series is subject to the benchmark constraints

$$\sum_{i=b_k}^{e_k} y_t = T_k \qquad (2.1.2)$$

where T_k is the k^{th} benchmark covering the periods b_k to e_k for k = 1, ..., K non-overlapping benchmark periods. For stock series the benchmarks cover only single time points so in (2.1.2) $b_k = e_k$ for $k = 1, \cdots, K$. If we let $X_k = \sum_{i=b_k}^{e_k} x_i$ be the sum of the original series over the k^{th} benchmark period then the solution is

$$y_t = \begin{cases} x_t, & t \notin [b_k, e_k] \\ & \quad \text{for } k = 1 \text{ to } K \\ x_t + \dfrac{T_k - X_k}{e_k - b_k + 1}, & t \in [b_k, e_k] \end{cases} \qquad (2.1.3)$$

Thus, within a benchmark period k, the difference between the benchmark and the original series total for period k is spread evenly over the x_t's in period k. For stock series this simply sets $y_t = T_k$ for $t = b_k = e_k$. The benchmarked series values are the same as the original series if the values do not occur within any of the K benchmark periods.

2.2 Minimizing Revisions in Month-to-Month Changes in Unadjusted Data

We mentioned in the Introduction that preserving month-to-month changes, rather than levels, will smooth discontinuities between benchmark periods. In preserving month-to-month changes, the series is revised beyond the respective benchmark periods. This 'carry-over' effect smooths discontinuities between benchmark periods; in one case this is how benchmarking stock series affects data points other than the benchmarked end-of-the-year months. We find $\mathbf{y}^u$ such that:

$$F_{ua}(\mathbf{y}^u) = \min_{\mathbf{y}} F_{ua}(\mathbf{y}) \tag{2.2.1a}$$

subject to the benchmark constraints in the previous section, where

$$F_{ua}(\mathbf{y}) = \sum_{t=2}^{n} \left[\frac{y_t}{y_{t-1}} - \frac{x_t}{x_{t-1}} \right]^2 \tag{2.2.1b}.$$

Trager (1982) has demonstrated empirically that the solution, $\mathbf{y}_0^u$, the solution to

$$\min_{\mathbf{y}} F_{ua}^{Trager}(\mathbf{y}) \tag{2.2.2a}$$

subject to the same constraints, where

$$F_{ua}^{Trager}(\mathbf{y}) = \sum_{t=2}^{n} \left[\frac{y_t}{x_t} - \frac{y_{t-1}}{x_{t-1}} \right]^2 \tag{2.2.2b}$$

closely approximates $\mathbf{y}^u$ in (2.2.1). $\mathbf{y}_0^u$ in (2.2.2) has a closed form solution and the solution is discussed in Appendix B. Also, note that $\mathbf{y}_0^u$ is the first difference of the proportionate difference used by Denton (1981).

$F_{ua}(\mathbf{y})$ in (2.2.1) is minimized iteratively using $\mathbf{y}_0^u$ as starting values. Causey (1981) (and revised by Trager (1982)) provided a numerical algorithm using steepest descent to obtain $\mathbf{y}^u$ in (2.2.1), and their solution is discussed in Appendix C.

2.3 Month-to-Month Changes in Seasonally Adjusted Data

If the published figures are seasonally adjusted values it might be more reasonable to preserve month-to-month changes in seasonally adjusted, rather than in unadjusted data. Thus, we can find $\mathbf{y}^s$ such that

$$F_{sa}(\mathbf{y}^s) = \min_{\mathbf{y}} F_{sa}(\mathbf{y}) \tag{2.3.1a}$$

subject to the benchmark constraints where

$$F_{sa}(\mathbf{y}) = \sum_{t=2}^{n} \left[\frac{y_t / s_t}{y_{t-1} / s_{t-1}} - \frac{x_t / s_t}{x_{t-1} / s_{t-1}} \right]^2 \tag{2.3.1b}$$

where s_t is the seasonal factor for time t for the original series. Note that in (2.3.1) the seasonal factors are the same for the original and revised series. In theory, however, we would want to minimize

$$F_{sa^*}(\mathbf{y}) = \sum_{t=2}^{n} \left[\frac{y_t / s_t^*}{y_{t-1} / s_{t-1}^*} - \frac{x_t / s_t}{x_{t-1} / s_{t-1}} \right]^2 \qquad (2.3.2)$$

where s_t^* is the seasonal factor produced by a routine that performs the benchmarking and seasonal adjustment simultaneously. This would require a very complicated estimation routine that would have to do n seasonal adjustments just to calculate the numerical derivatives to take the next step in the nonlinear estimation of the benchmarked series. Since both s_t^* and y_t would change at each iteration, it is not clear that (2.3.2) would yield a unique benchmarked series $\mathbf{y}$, much less converge. Another alternative but still an approximation to (2.3.2) is to benchmark, then seasonally adjust at each iteration, thus using seasonal factors from the revised rather than from the original series. $F_{sa}(\mathbf{y})$ in this case would be

$$F_{sa(i)}(\mathbf{y}) = \sum_{t=2}^{n} \left[\frac{y_t^{(i)} / s_t^{(i)}}{y_{t-1}^{(i)} / s_{t-1}^{(i)}} - \frac{x_t / s_t^{(i)}}{x_{t-1} / s_{t-1}^{(i)}} \right]^2 \qquad (2.3.3)$$

In this case, the benchmarking and seasonal adjustment programs would have to be run iteratively, with i indicating the iteration in (2.3.3). Finally, note that (2.3.1b) or (2.3.3) may be written as a weighted version of (2.2.1b):

$$\sum_{t=2}^{n} \frac{s_{t-1}^2}{s_t^2} \left[\frac{y_t}{y_{t-1}} - \frac{x_t}{x_{t-1}} \right]^2 \qquad (2.3.4)$$

where s_t^2 are the squares of the original seasonal factors. Again, the benchmark constraints remain the same.

2.4 Relative Month-to-Month Changes

Laniel (1986) discusses minimizing month-to-month changes in unadjusted data using a similar function to (2.2.1). However, (2.2.1) is weighted by the month-to-month change in the original data. $\mathbf{y}^r$ is found by solving

$$\min_{\mathbf{y}} F_{rel}(\mathbf{y}) \tag{2.4.1a}$$

subject to the benchmark constraints where

$$F_{rel}(\mathbf{y}) = \sum_{t=2}^{n} \left[\frac{\dfrac{y_t}{y_{t-1}} - \dfrac{x_t}{x_{t-1}}}{x_t / x_{t-1}} \right]^2 \tag{2.4.1b}$$

He proposes its use in stock series. (2.4.1) can also be stated in a weighted form similar to (2.3.4):

$$\sum_{t=2}^{n} \frac{x_{t-1}^2}{x_t^2} \left[\frac{y_t}{y_{t-1}} - \frac{x_t}{x_{t-1}} \right]^2 \tag{2.4.2}$$

with $x_{t-1}^2 \,/\, x_t^2$ replacing $s_{t-1}^2 \,/\, s_t^2$ in (2.3.4). We analyze the impact of utilizing (2.4.1) on both stock and sales series, in Section 3.2.

3. EVALUATION OF METHODS THAT MINIMIZE MONTH-TO-MONTH CHANGES

For this study we obtained nine series with their benchmarks; three retail sales and three retail inventory series from Business Division (BUS), and three value shipped series from Industry Division (IND). Seasonal series that also might be hard to benchmark using the UA method were requested. All the series are seasonal except BGRCRS which we used as a control. The BUS retail inventory series were benchmarked as flow series; therefore, no series in this study are benchmarked as stock series. The series are listed in Table 3-1.

TABLE 3-1

Series Analyzed

BUSINESS DIVISION

RETAIL SALES (January 1967 - December 1985)

BDPTRS	S0000U531100	Department Stores
BDRGRS	S0000U591200	Drug and Proprietary Stores
BGRCRS	S0000U541100	Grocery Stores

RETAIL INVENTORIES (January 1967 - December 1985)

BGMRRI	I03000530000	General Merchandise
BTAPRI	I03000560000	Total Apparel
BTNDRI	I03000500007	Total Non-durable Goods

INDUSTRY DIVISION

VALUE SHIPPED (January 1976 - December 1983)

IOTEVS	S47VSN	Other Transportation Equipment
IPRFVS	S89VSN	Paving and Roofing Materials

VALUE SHIPPED (January 1958 - December 1984)

ITVRVS	S36VSN	Television and Radio Equipment

3.1 Analysis of UA and SA Methods

To analyze the differences between the methods that minimize unadjusted and seasonally adjusted month-to-month changes we looked at benchmark revisions, where revisions are the differences between the original and the benchmarked series. We analyzed the revisions in levels, unadjusted and seasonally adjusted month-to-month percent changes, and seasonal factors in percents. Minima, maxima, means, medians, and standard errors were calculated for the above benchmark revisions for each method and for the differences between methods. Only the medians and maxima are reported here. The means and medians followed similar patterns; medians are presented here because of their resistance to outliers. Maxima are presented to show the worst case scenario.

Hereafter, we will refer to minimizing $F_{ua}(\mathbf{y})$ as the UA method and minimizing $F_{sa}(\mathbf{y})$ as the SA method. Recall from (2.2.1) that $\mathbf{y}^u$ will be the benchmarked series produced using the UA method, and $\mathbf{y}^s$ from (2.3.1) the benchmarked series produced using the SA method.

3.1.1 Total Benchmark Revisions

The total benchmark revisions are due to the differences between the benchmarks, T_k, and the sum of the original series values within the same benchmark period, X_k.

Table 3-2 following displays the mean and maximum absolute percent total revisions.

TABLE 3-2

Percent Total Benchmark Revisions

Series	Mean	Maximum
BDPTRS	.86	2.48
BDRGRS	1.41	2.18
BGRCRS	.98	2.13
BGMRRI	14.05	19.61
BTAPRI	8.21	14.91
BTNDRI	9.94	15.72
IOTEVS	16.66	22.65
IPRFVS	12.11	19.59
ITVRVS	10.47	17.58

Table shows The absolute percent differences between original and benchmarked data by series. First column contains the means and the second column contains the maximums of the absolute percent total benchmark revision, $|100*(T_k-X_k)/X_k|$ over all K benchmarks. Note the maximum percent differences can be as large as 22 percent.

The difference between original and benchmark totals for each benchmark period (the total benchmark revision) is the bias removed by benchmarking. These changes are shown on the output from the Census Bureau benchmarking computer program output in the rightmost column by R-0, and their percentage differences are shown as the ratio R/0. With such large differences between the X_k's and the T_k's (up to a 22 percent difference) the user of the program should consider whether the original and the benchmark surveys are measuring the same quantity or targeting the same population. Also, figures 19 to 27 show the percent change in the total benchmark revisions. We will discuss the other aspects of these figures (benchmark revisions as a percent of the total benchmark revisions) at the end of this section. Finally, note that benchmarking has removed a strong upward trend in the ITVRVS series (see figures 9 and 27).

3.1.2 Levels

Next we look at the benchmark revisions in level, time point-by-time-point (month-by-month). Let L_t^u denote the benchmark percent revision in level when we benchmark using the UA method, and L_t^s be the analogous quantity for the SA method:

$$L_t^u = 100*(y_t^u - x_t)/x_t \qquad L_t^s = 100*(y_t^s - x_t)/x_t.$$

Then let the percentage difference between the SA and the UA methods be

$$D_t^{Level} = 100*(y_t^s - y_t^u)/x_t = L_t^s - L_t^u.$$

The medians of the absolute values of $\{L_t^u\}$, $\{L_t^s\}$, and $\{D_t^{Level}\}$ are shown in Table 3-3 as are the maximums of $\{D_t^{Level}\}$. $\{x_t\}$, $\{y_t^s\}$, and $\{y_t^u\}$ are graphed in figures 1 to 9, $\{L_t^u\}$ and $\{L_t^s\}$ are shown in figures 10 to 18.

TABLE 3-3

Benchmark Revisions and UA vs. SA Method Percent Differences

in Level

Series	Median UA Method Revisions	Median SA Method Revisions	Median Method Differences	Maximum Method Differences
BDPTRS	.81	.74	.06	.43
BDRGRS	1.40	1.39	.04	.25
BGRCRS	.87	.88	.005	.04
BGMRRI	15.25	15.24	.05	.38
BTAPRI	8.30	8.32	.06	.35
BTNDRI	9.62	9.62	.02	.13
IOTEVS	17.94	17.79	.06	.59
IPRFVS	12.53	12.52	.19	2.30
ITVRVS	10.91	10.94	.13	.50

Table shows the median absolute UA and SA method percent benchmark revisions, Med$|L_t^u|$ and Med$|L_t^s|$ respectively, in columns one and two, and the median and maximum absolute percent difference between methods, Med$|D_t^{Level}|$ and Max$|D_t^{Level}|$, in the third and fourth columns. Note the how close the revisions are for the UA and the SA methods and that the maximum method differences are less than one percent except for IPRFVS.

Since both methods are constrained within each benchmark period by the same benchmarks, the mean revision in the levels is going to be the same for both methods. The small differences in percent revision between the methods are seen in figures 9 to 18. The summary statistics in Table 3-2 also show the magnitude of these differences. Notice the small percent differences between the methods when compared to the median percent revisions themselves.

This comparison is valid since both the revisions and the method differences have the same denominator x_t. In general the differences in the revisions seem to occur at time points with strong seasonal fluctuations between one month and the next. This can be seen in figures 1, 2, 10, and 11 for BDPTRS and BDRGRS. But we could not detect any practical differences in the levels between the two methods (see figures 10 to 18). We did not find any relation between the method differences, D_t^{Level}, and the weights used in minimizing $F_{sa}(y)$ in (2.3.4), s_{t-1}^2/s_t^2.

Although any benchmarking method has the same total benchmark revisions, methods may differ in how they allocate the total revision to time points within each benchmark period. To see how each method allocated the total revision for each benchmark we looked at the benchmark revisions as a percent of the total benchmark revisions

$$P_t = \frac{100*(y_t - x_t)}{T_k - X_k}$$

where k is the benchmark that contains time t. From plotting $\{P_t^u\}$ for the UA method and $\{P_t^s\}$ for the SA method (seen in figures 19 to 27) we could not detect any consistent differences between methods. Also, in some of the series there are discontinuities from one benchmark period to another. Since the month-to-month changes are being minimized the procedure may require a much different revision on one side of the benchmark than on the other. Note that these same discontinuities do not occur when the revisions are plotted as a percentage of the original series. So we do not find the P_t's to be as helpful a diagnostic as the percent revisions (L_t's).

In this study we used two diagnostics to look at the differences between the original and the benchmarked series, the percent benchmark revision, L_t (percent of the original series not the total benchmark revision, as are the P_t's) and the total percent benchmark revision. At each time point the differences between the original and the benchmarked series can most easily be seen by looking at the differences as a percentage the original series. This would be a helpful diagnostic tool to study the effect benchmarking on a series. Also, it is good practice to look at the percentage difference between the sums of the original series and their benchmarks to check on the accuracy of both the series and their benchmarks. Recall that IOTEVS had a 22 percent maximum total benchmark revision at one benchmark and for ITVRVS, benchmarking reduced a strong upward trend.

3.1.3 Month-to-Month Changes of the Unadjusted Data

Let U_t^u and U_t^s denote the difference at time t between the month-to-month change in the revised series $\{y_t\}$ produced using the UA and SA methods respectively and the month-to-month change in the original series $\{x_t\}$:

$$U_t^u = 100 \cdot \left[\frac{y_t^u}{y_{t-1}^u} - \frac{x_t}{x_{t-1}} \right] \qquad U_t^s = 100 \cdot \left[\frac{y_t^s}{y_{t-1}^s} - \frac{x_t}{x_{t-1}} \right]$$

for time points t = 2, ... , n. The sum of squares of the U_t's is $F_{ua}(y)$ (2.2.1b) which the UA method minimizes. To see if the SA method obtained the same or even smaller values of $F_{ua}(y)$ we would compare sums of squares of the U_t's. Another measure we could compare is median absolute revisions.

Let D_t^{unadj} denote the difference of the absolute revisions between the two methods:

$$D_t^{unadj} = \left| U_t^s \right| - \left| U_t^u \right| .$$

D_t^{unadj} is the difference of two absolute values instead of a simple difference because we are interested in which method has smaller revisions (i.e. smaller U_t's). If the SA method has smaller revisions then D_t^{unadj} will be negative and if the UA method revisions are smaller it will be positive. Median and maximum U_t's are displayed in the following table.

TABLE 3-4

Unadjusted Month-to-Month Percentage Changes

Median Changes

Series	UA Method Revisions	SA Method Revisions	Median Method Differences	Maximum Method Differences
BDPTRS	.04	.05	.02	-.25
BDRGRS	.10	.50	.03	.27
BGRCRS	.06	.07	.00	-.04
BGMRRI	.49	.50	.03	-.49
BTAPRI	.50	.54	.01	-.34
BTNDRI	.32	.33	.01	-.19
IOTEVS	.42*	.40*	.00	.88
IPRFVS	.37	.43	.04	-1.16
ITVRVS	.65	.84	.00	1.13

Table shows differences in the month-to-month percentage changes in the unadjusted data. UA Method Revisions are the median absolute $\{U_t^u\}$, SA Method Revisions are the median absolute $\{U_t^s\}$, Median Method Differences are the median absolute $\{D_t^{unadj}\}$, and the Maximum Method Differences are the signed maximum absolute $\{D_t^{unadj}\}$, where the maximum is negative if the SA had a smaller revision at that time point. Note that for all series the UA method obtains smaller median method differences.

*The mean revisions for IOTEVS are smaller for the UA method (.46 as compared with .47 for the SA method).

The median month-to-month percentage changes were lower for the UA method in all series. But the SA method did have smaller maximums in six of nine cases (the six negative Maximum Method Differences in Table 3-4). Note that by comparing the maximum of the method differences rather than the maximum revision for each method we are comparing methods time point by time point, thus we pair hard to adjust time points.

3.1.4 Month-to-Month Changes of the Seasonally Adjusted Data

Let A_t^u denote the difference in the month-to-month percent changes of the seasonally adjusted data, using the revised series produced by the UA method, and A_t^s similarly denote the difference using the revised series produced using the SA method:

$$A_t^u = 100 \cdot \left[\frac{y_t^u \,/\, s_t^u}{y_{t-1}^u / s_{t-1}^u} - \frac{x_t \,/\, s_t}{x_{t-1} \,/\, s_{t-1}} \right]$$

$$A_t^s = 100 \cdot \left[\frac{y_t^s \,/\, s_t^s}{y_{t-1}^s / s_{t-1}^s} - \frac{x_t \,/\, s_t}{x_{t-1} \,/\, s_{t-1}} \right].$$

Note that A_t^u and A_t^s are functions of not just s_t but also s_t^u and s_t^s, the seasonal factors from the seasonal adjustments of the benchmarked series so the sum of squares of the A_t^u's and A_t^s's is $F_{sa^*}(y)$ in (2.3.2) not $F_{sa}(y)$ in (2.3.1) which just uses s_t. Recall the SA method was developed to approximate $F_{sa^*}(y)$ in (2.3.2) by minimizing $F_{sa}(y)$ in (2.3.1b) so to see which method obtained smaller values of $F_{sa^*}(y)$ we would compare the sums of squares of the A_t's. Another measure we could compare is the median absolute revisions.

Let D_t^{adj} be the difference of the absolute revisions between A_t^u and A_t^s, a measure of the time point by time point difference between procedures:

$$D_t^{adj} = \left| A_t^s \right| - \left| A_t^u \right|$$

Again, we calculate D_t^{adj} to measure which method's revision is smaller. Table 3-5 displays the comparisons of seasonally adjusted month-to-month percentage changes:

TABLE 3-5

Seasonally Adjusted Month-to-Month Percentage Changes

Median Changes

Series	UA Method Revisions	SA Method Revisions	Median Method Differences	Maximum Method Differences
BDPTRS	.05	.05	.01	-.65
BDRGRS	.09	.10	01	-.34
BGRCRS	.07	.07	.00	-.04
BGMRRI	.48	.50	.02	-.53
BTAPRI	.49	.50	.02	-.42
BTNDRI	.30	.32	.01	.33
IOTEVS	.56	.53	-.03	-.52
IPRFVS	.67	.54	-.11	-13.07
ITVRVS	.87	.85	-.02	.51

Tables show differences in the month-to-month percent changes in the seasonally adjusted values. Descriptions are the same as Table 3-4.

Note that in 6 of the 9 series analyzed, the median differences are lower for the UA method, but all three of the IND series had lower median revisions when using the SA method. The SA method had lower maximum revisions in seven of the nine series. In all but IPRFVS the median differences of the percents are less than 0.1. The maximum method differences are less than 1 percent for all series except IPRFVS. The difference in IPRFVS seems to be due to different seasonal factors for January. Maybe this is not so much a difference of the benchmarking methods as it is how the X-11 program adjusted the revised series produced using the SA and UA methods. Partly because of this difference we examined the revisions in the seasonal factors.

3.1.5 Seasonal Factors

In benchmarking monthly time series to annual benchmark totals, the annual benchmark survey is used to get more accurate estimates of the level whereas the monthly series is used to get information on the pattern of month-to-month changes. In seasonal series this pattern is partly determined by the seasonal factors of a seasonal adjustment, say from X-11, so we looked at the revisions in the seasonal factors.

For each time point t, let $\{F_t^u\}$ and $\{F_t^s\}$ denote the changes in the seasonal factors, from original to revised series, for the UA and SA methods :

$$F_t^u = 100 \cdot (s_t^u - s_t) \qquad\qquad F_t^s = 100 \cdot (s_t^s - s_t)$$

The differences, D_t^{seas} between the two methods are

$$D_t^{seas} = \left| F_t^s \right| - \left| F_t^u \right|$$

Table 3-6 displays median and maximum calculations for the series $\{F_t^u\}$, $\{F_t^s\}$, and $\{D_t^{seas}\}$.

TABLE 3-6

Seasonal Factors as Percentages

Median Changes

Series	UA Method Revisions	SA Method Revisions	Median Method Differences	Maximum Method Differences
BDPTRS	.02	.01	-.01	-.08
BDRGRS	.04	.04	.00	-.50
BGRCRS	.02	.01	.00	.01
BGMRRI	.11	.06	-.02	-.26
BTAPRI	.09	.09	-.01	.09
BTNDRI	.05	.04	-.01	-.03
IOTEVS	.46	.52	.02	.27
IPRFVS	.41	.33	-.17	-5.40
ITVRVS	.24	.28	-.00	-.51

Table shows differences in seasonal factors. Descriptions are the same as in Table 3-4.

The SA method has smaller median D_t^{seas}'s in six series, but the differences are less than 0.1 percent except for IPRFVS. Also, the SA method had smaller maximums for six series.

In both the median and maximum method differences the IPRFVS series has values an order of magnitude larger than the rest of the series. The median method difference is -.17 and the maximum of -5.40 occurs January 1981. The preceding January values also have large method differences. The graph of the levels (figure 8) does not show visual differences but the plot of the percent level revisions (figure 17) shows 2 percent differences around the beginning of 1979. There is no visible seasonal pattern to the percent level. Maybe the quality of the seasonal adjustment of IPRFVS needs to be explored rather than the benchmarking method.

3.2 Analysis of Method Relative Month to Month Changes

In section 2.4, we mentioned the method of minimizing relative month-to-month changes in unadjusted data, as proposed by Laniel (1986). We also noted that (2.4.1) could be restated in a form similar to (2.3.4), where the weighting function contains month to month changes, $\left[\frac{x_{t-1}}{x_t}\right]^2$, rather than seasonal factors, $\left[\frac{s_{t-1}}{s_t}\right]^2$.

Eight series used in the study of the UA and SA methods were also used in the analysis of the method minimizing relative month-to-month changes. Hereafter, minimizing $F_{rel}(y)$ will be referred to as the REL method.

3.2.1 Comparison of Revised Series Produced Using the REL and UA Methods

Both the REL and UA methods minimize month-to-month changes in unadjusted data. However, the REL method (2.4.1) will place less weight on time points with large increases or decreases in level.

In all of the eight series studied, in terms of the ratio of the revised to original series for all months in the revision span, the revised series produced by the REL method were identical (to the second decimal place) to the series produced using the UA method. However, in 6 of the 8 series, the minimization procedure for the REL method failed to the extent that after several iterations the benchmark value solution calculated caused an increase, rather than a decrease in the estimate of $F_{rel}(y)$. The procedure stopped if this occurred. The amount of increase did not affect the <u>revised series</u>

produced. This breakdown in the minimization procedure did not occur using the UA method.

The algorithm derived by Trager and Causey listed in Appendix C computes a variable C at each iteration. C determines how much the revised series estimates will be changed in the direction of a computed direction vector D. They recommend setting the constant C to be 1. With the intent to more accurately locate the minimum, we investigated increasing the constant value to reduce the step size. Table 3-7 following displays the number of iterations needed to solve the problem for C=1 and C=2.

TABLE 3-7

ITERATION COMPARISONS FOR DIFFERENT BENCHMARKING OPTIONS

SERIES	UA METHOD C=1	UA METHOD C=2	SA METHOD C=1	SA METHOD C=2	REL METHOD C=1	REL METHOD C=2
IOTEVS	63	32	6 * (.99876)	10	3 * (.99997)	2 * (.99993)
ITVRVS	66	34	5	10	2 * (.98967)	2 * (.97062)
IPRFVS	201	71	3 * (.99965)	29	3	2 * (.99757)
BDPTRS	201	84	3	7	2 * (.99834)	2 * (.99791)
BTAPRI	61	28	13	21	4 * (.99957)	4
BGMRRI	56	28	3	4	7	7
BDRGRS	55	28	1	1	2 * (.99089)	2 * (.98847)
BTNDRI	39	21	5 * (.99999)	6	3 * (.99999)	3

* Indicates an increase in the minimization function and the ratio of the last two function values is displayed.

Table 3-7 shows that increasing the value of C, and thereby decreasing the step size, did not eliminate the occurrence of an increase in the penalty function. It is interesting to note that by changing C from 1 to 2 the number of iterations needed to solve the UA method is reduced by approximately 50 percent. It is also of some interest that for the original value of C=1, the breakdown in the minimization procedure caused an increase in the function value in 3 of the 8 series if the SA method is utilized. In the case of the SA method, increasing C eliminated the increases in the penalty function. Thus, the weighting of the penalty function, as utilized by the REL and SA methods, has something to do with the possibility of an increase in the penalty function. Relaxing the convergence criterion might be one possible way to address the problem of an increase in the penalty function. The effect of relaxing the criterion was not attempted in this study.

It is noted that while the discussion above assumes non-integer solutions, the revised series published frequently are rounded to integer solutions. Thus, any gain in more accurately minimizing the penalty function must be considered in the context of the revised series being rounded to integer values.

The ASCII FORTRAN version of the computer program that produces the revised series has been modified to inform the user if an increase in the penalty function has occurred. The revised series produced using the last two function values are displayed, as well as the ratio of the last two function values. A further discussion of the computer program, including an example of a series where an increase in the penalty function occurs, is included in Appendix A.

4. SUMMARY AND CONCLUSIONS

Since the general public most often pays attention to month-to-month changes in seasonally adjusted data, one might consider using a method that minimizes these changes. We have called this method the SA method, in contrast to the UA method, which minimizes month-to-month changes in the unadjusted data. However, we did not find any consistent differences between the UA and SA methods in the percent revisions of the levels. The UA method obtained equal or smaller median absolute revisions in the month-to-month changes than the SA method in the unadjusted data for all series, but the SA method had larger median absolute revisions for the seasonally adjusted month-to-month changes for a majority of the series. In IPRFVS we found relatively large differences (in January) between the two methods but this may be due to the seasonal adjustment and not the benchmarking. Because we could find no consistent practical differences between the methods, we do not recommend using the SA method, especially since it requires the extra step of seasonally adjusting the original series at the start of the benchmarking process.

We also examined a method proposed by Laniel (1986). This method we call the REL method for relative month-to-month changes in unadjusted data. The REL method places less weight than the UA method on time points with large increases or decreases in value. In the eight series used to compare the UA, SA, and REL methods, the revised series produced using the REL method were identical (to two decimal places) to the series produced using the UA method, as measured by the ratio of the revised to original series for all months in the revision span. If the REL method was used, it was observed that during the iterative minimization procedure rather than decreasing at every iteration, the penalty function increased at some step for a majority of the series studied. Sometimes this occurred after only a few steps. The procedure was programmed to stop if such an increase occurred. Decreasing the step size used in the iterative procedure of solving the penalty function did not eliminate the occurrence of increases in the function value. (It did reduce the number of iterations needed to solve the UA method.) The REL method's speed advantage over the UA method is therefore counter balanced by uncertainty about whether the penalty function has been adequately minimized.

It would be attractive to have a benchmarking method derived from statistical models which can respond to the individual features of each series. Hillmer and Trebelsi (1987) has proposed such a method, and we believe that their approach deserves further consideration.

5. REFERENCES

Hillmer, Steven C. and Abdelwahed Trabelsi (1987). "Benchmarking of Economic Time Series." Journal of the American Statistical Association, 84(400), 1064-1071.

Denton, Frank T. (1971). "Adjustment of Monthly or Quarterly Series to Annual Totals: An Approach Based on Quadratic Minimization." Journal of the American Statistical Association, 64, 99-102.

Laniel, Norman (1986). "Adjustment of Economic Production Sub-Annual Series to Corresponding Annual Series." Ottawa: Statistics Canada.

Lorenz, William and others (1986). "TIMEBASE: Time Series Data Base Management System." Unpublished Census Bureau reference manual.

Monsour, Nash J. and Mitch L. Trager (1979). "Revision and Benchmarking of Business Time Series", Proceedings of the Business and Economic Statistics Section, American Statistical Association.

APPENDIX A

THE ASCII FORTRAN Benchmarking Program

TABLE OF CONTENTS

1. INTRODUCTION

The ASCII Benchmarking computer program is an ASCII FORTRAN adaptation, with some modifications, of the ALGOL Benchmarking program used by the Bureau of the Census. In many respects, this ASCII version is the same.

There are four revision solution options. In one, the program will revise time series located in the ASCII TIMEBASE according to the mathematical method which minimizes the sum of squares of the differences in the ratios of the revised to the original series from one month to the next. That is, if **x** represents the original series, and **y** a revised one satisfying the benchmark constraints, the revision procedure minimizes

$$\sum_{t=2}^{n} \left[\frac{y_t}{x_t} - \frac{y_{t-1}}{x_{t-1}} \right]^2 \qquad \text{(Ratio Revision)}$$

It should be noted here that the ratio revision is referred to as the "relative revision" in Trager (1982). It is referred to as "ratio revision" in this document not to confuse it with the discussion of relative month-to-month changes in Bozik and Otto (1988).

In another revision solution option, the program will minimize the sum of squared differences in the month-to-month changes between the original and revised series:

$$\sum_{t=2}^{n} \left[\frac{y_t}{y_{t-1}} - \frac{x_t}{x_{t-1}} \right]^2 \qquad \text{(Trend Revision)}$$

Another revision solution option is minimizing the sum of squared month-to-month changes in seasonally adjusted data:

$$\sum_{t=2}^{n} \left[\frac{s_{t-1}}{s_t} \right]^2 \left[\frac{y_t}{y_{t-1}} - \frac{x_t}{x_{t-1}} \right]^2 \qquad \text{(Seasonally Adjusted Revision)}$$

Finally, a revision solution option is available that minimizes the sum of squared relative differences in the month-to-month changes, as proposed by Laniel (1986):

$$\sum_{t=2}^{n} \left[\frac{x_{t-1}}{x_t} \right]^2 \left[\frac{y_t}{y_{t-1}} - \frac{x_t}{x_{t-1}} \right]^2 \qquad \text{(Canadian Revision)}$$

It is referred to here as the Canadian revision method (proposed by Laniel of Statistics Canada), rather than the relative month-to-month changes method [the REL method in Bozik and Otto (1988)] because the option is referred to in the program by 'C'. 'R' refers to the ratio revision option.

1.1 New Features

As mentioned earlier, the ASCII Benchmarking program retains all of the features in the current ALGOL version. In addition, the program includes options for the seasonally adjusted and Canadian revision solutions. The ASCII program however, references the ASCII TIMEBASE, and not earlier versions. The ASCII program also accepts quarterly or annual data, and its output is tailored to the frequency of the data.

Bozik and Otto (1988) documented the breakdown of the minimization procedure to the extent that an increase, rather than a decrease may occur in the penalty function calculated if the seasonally adjusted or Canadian revision solution options are selected. The minimization procedure stops if an increase occurs. The program has been modified to print a message if an increase occurs, display results using both function values, and save into TIMEBASE (if requested) the revised series calculated according to the smaller function value.

1.2 Program Constraints

Several constraints have been placed on the program. They are as follows:

1. A maximum number of 20 non-overlapping benchmark or constraint periods may be specified as long as they are contained within the span of the series to be revised.

2. The length of the revision span in years must be no more than 20.

3. The length of the revision span must be specified. The user has the option selecting the 'W' option on the 'XQT' card, which indicates that the whole series is to be revised. If this option is used, the parameter card (discussed in the next section) indicating the span of the revision must not be used.

4. The data to be revised is assumed to exist in the ASCII TIMEBASE. Please note that this program does not reference earlier versions of TIMEBASE.

2. INPUT TO THE PROGRAM

The executable element for the program is available in

SRD*TSPROG.ASBMARK

Table 1 following lists the options available on the 'XQT' card, table 2 lists the options no longer available, and table 3 details the input ordering and formats.

TABLE 1 - ASCII PROGRAM OPTIONS

F - Attained value of the objective function will be displayed

W - Indicates the whole series is to be revised. See number 3 in section 1.2

B - The revised series is to be placed in TIMEBASE under the specified ID.

O - If using the B option and the output series already exists in TIMEBASE, then overwrite it with the newly revised series. NOTE -- The input and output series ID's must be different.

I - If using the B option, the insert the revised series into the existing output series. If the output series doesn't exist, then create a new one. AGAIN NOTE -- The input and output series ID's must be different.

T - The trend revision is desired. If the B option is specified, this solution will be placed in TIMEBASE.

R - The ratio revision is desired. If used without the T option, but with the B option, this solution will be placed in TIMEBASE. If both the T and B options are used, the trend and ratio solutions will be printed out but only the trend solution will be placed in TIMEBASE.

S - The seasonally adjusted version of the trend revision is desired. If used with the B option, the solution will be placed in TIMEBASE.

C - The revision option proposed by Statistics Canada is desired.

A - Annual data is to be benchmarked. Default is monthly data.

Q - Quarterly data is to be benchmarked. Default is monthly data.

TABLE 2 - OPTIONS ELIMINATED FROM THE ASCII PROGRAM

L - Long list of the table of revision results will be displayed. Default is a short print out of only the original and revised series. No comparisons.

NOTE - Only one print style is available.

Z - Suppresses all analytic tables.

TABLE 3 - INPUT FORMAT

Unless noted, FREE FORMAT is permissable.

@XQT,<options> SRD*TSPROG.ASBMARK

Number of benchmark periods (must be greater than 0 and no more than 20.)

TIMEBASE initials and password (columns 1-6 for initials, 7-18 for password)

TIMEBASE qualifyer & filename (columns 1-12 for qualifyer, and 13-24 for filename)

If not option 'W', specify the span of the revision, for example, 1 67 12 77

A note should be added here concerning option 'A', for annual data. If annual data is to be benchmarked, the periods entered for the span of revision must be 1.

Benchmark periods desired (one line for each benchmark period indicated above). For example,

```
1 67  1 67
1 68 12 68
1 76 12 76
1 77  6 77
```

if the number of benchmark periods entered was 4.

Again, if annual data is to be benchmarked, see the note above.

For each series to be revised (up to 100 per execution), enter the following:

1. Input series ID (in columns 1-12)
2. If option B is specified, the output series ID (in columns 1-12)
3. Benchmark values. (one for each benchmark period specified.) For example,

```
 6789
67898
99998
78787
```

if the number of benchmark periods entered was 4.

3. EXAMPLES

In this section examples of input and output are displayed for monthly, quarterly, and annual benchmarked series. The series names have been modified.

The first example, using a series from IND, updates only a single series. It also places the revised series into TIMEBASE by using the 'I' option. Example 2 updates two quarterly BUS series. Note this example does not place the benchmarked series into TIMEBASE. The third example, revising three BUS annual series, places the revised series into TIMEBASE using the 'O' option. Note the difference in the series placed in TIMEBASE depending if the 'I' or 'O' option is used. In this example, the existing series in TIMEBASE is deleted before the revised series is placed there. The final example illustrates the output in the case of a breakdown in the minimization procedure, where an increase in the penalty function occurs.

EXAMPLE 1

```
@ED,R J.TESTRUN/EX
READ-ONLY MODE
CASE UPPER ASSUMED
ED 16R1C-C FRI-08/22/86-10:06:05-(0,)
EDIT
0:
      1:@XQT,TFBI SRD*TSPROG.ASBMARK
      2:5
      3:INIT  PASSWORD
      4:QUAL        FILENAME
      5:1 77 12 81
      6:1 77 12 77
      7:1 78 12 78
      8:1 79 12 79
      9:1 80 12 80
     10:1 81 12 81
     11:INDEX1
     12:INDEX1BM
     13:6913
     14:7936
     15:8092
     16:8516
     17:8782
EOF:17
0:
END ED. NO CORRECTIONS APPLIED

@XQT,TFBI SRD*TSPROG.ASBMARK
```

SERIES INDEX1 REVISED SOLUTION OPTIONS SELECTED: TREND INTO SERIES: INDEX1BM
INPUT SERIES TITLE industry monthly series example 1
SERIES REVISED BY JB

DATE OF REVISION: 082286
TIME OF REVISION: 100623

SPAN OF REVISION: 1/1977 - 12/1981
BENCHMARK(S):
1/1977 - 12/1977 = 6913.
1/1978 - 12/1978 = 7936.
1/1979 - 12/1979 = 8092.
1/1980 - 12/1980 = 8516.
1/1981 - 12/1981 = 8782.
NUMBER OF ITERATIONS TO ATTAIN TREND SOLUTION 66

CARRY FORWARD FACTOR: .657364 ATTAINED VALUE OF TREND FUNCTION .00664735

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC	TOTAL	CODE
77	401.	485.	465.	394.	420.	541.	407.	524.	607.	670.	697.	640.	6251.	O
77	445.	539.	516.	437.	466.	600.	450.	579.	671.	739.	768.	703.	6913.	R
77	1.111	1.110	1.110	1.109	1.109	1.108	1.106	1.106	1.105	1.103	1.102	1.099	1.106	R/O
77	44.	54.	51.	43.	46.	59.	43.	55.	64.	69.	71.	63.	662.	R-O
77	.000	1.209	.959	.847	1.066	1.288	.752	1.287	1.158	1.104	1.040	.918		M/M-O
77	.000	1.209	.958	.847	1.065	1.287	.751	1.287	1.157	1.102	1.039	.916		M/M-R
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		Y/Y-O
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		Y/Y-R
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	CUM Y/Y-O
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	CUM Y/Y-R
78	455.	522.	547.	522.	516.	667.	519.	730.	779.	791.	803.	674.	7525.	O
78	498.	570.	595.	565.	555.	715.	549.	769.	813.	816.	819.	673.	7936.	R
78	1.094	1.092	1.088	1.082	1.076	1.071	1.057	1.053	1.044	1.032	1.019	.999	1.055	R/O
78	43.	48.	48.	43.	39.	48.	30.	39.	34.	25.	16.	-1.	411.	R-O
78	.711	1.147	1.048	.954	.989	1.293	.778	1.407	1.067	1.015	1.015	.839		M/M-O
78	.708	1.145	1.044	.949	.983	1.287	.768	1.400	1.058	1.004	1.003	.823		M/M-R
78	1.135	1.076	1.176	1.325	1.229	1.233	1.275	1.393	1.283	1.181	1.152	1.053		Y/Y-O
78	1.118	1.058	1.153	1.292	1.192	1.192	1.219	1.326	1.212	1.104	1.066	.957		Y/Y-R
78	1.135	1.103	1.128	1.172	1.183	1.193	1.204	1.231	1.239	1.231	1.221	1.204	1.204	CUM Y/Y-O
78	1.118	1.085	1.109	1.150	1.158	1.165	1.172	1.194	1.197	1.184	1.170	1.148	1.148	CUM Y/Y-R

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	,OCT	NOV	DEC	TOTAL	CODE
79	646.	690.	748.	548.	700.	867.	538.	787.	921.	910.	788.	643.	8786.	O
79	635.	670.	718.	514.	652.	803.	487.	709.	826.	812.	699.	568.	8092.	R
79	.982	.970	.960	.938	.932	.926	.905	.901	.897	.892	.887	.883	.921	R/O
79	-11.	-20.	-30.	-34.	-48.	-64.	-51.	-78.	-95.	-98.	-89.	-75.	-694.	R-O
79	.958	1.068	1.084	.733	1.277	1.239	.621	1.463	1.170	.988	.866	.816		M/M-O
79	.943	1.055	1.072	.716	1.269	1.231	.606	1.457	1.164	.983	.861	.812		M/M-R
79	1.420	1.322	1.367	1.050	1.357	1.300	1.037	1.078	1.182	1.150	.981	.954		Y/Y-O
79	1.275	1.175	1.206	.910	1.175	1.123	.887	.923	1.016	.994	.854	.843		Y/Y-R
79	1.420	1.367	1.367	1.286	1.301	1.300	1.264	1.234	1.226	1.216	1.189	1.168	1.168	CUM Y/Y-O
79	1.275	1.221	1.216	1.138	1.146	1.141	1.107	1.077	1.069	1.059	1.036	1.020	1.020	CUM Y/Y-R
80	801.	792.	759.	661.	635.	850.	674.	883.	1154.	1110.	1064.	807.	10190.	O
80	706.	696.	663.	572.	545.	725.	565.	734.	952.	899.	844.	616.	8516.	R
80	.882	.878	.873	.865	.858	.853	.838	.832	.825	.810	.793	.763	.836	R/O
80	-95.	-96.	-96.	-89.	-90.	-125.	-109.	-149.	-202.	-211.	-220.	-191.	-1674.	R-O
80	1.246	.989	.958	.871	.961	1.339	.793	1.310	1.307	.962	.959	.758		M/M-O
80	1.244	.985	.953	.863	.952	1.331	.779	1.301	1.296	.944	.939	.730		M/M-R
80	1.240	1.148	1.015	1.206	.907	.980	1.253	1.122	1.253	1.220	1.350	1.255		Y/Y-O
80	1.113	1.039	.923	1.113	.835	.903	1.160	1.035	1.152	1.107	1.207	1.085		Y/Y-R
80	1.240	1.192	1.129	1.145	1.095	1.071	1.092	1.096	1.119	1.131	1.152	1.160	1.160	CUM Y/Y-O
80	1.113	1.075	1.021	1.040	.998	.979	.999	1.004	1.024	1.034	1.050	1.052	1.052	CUM Y/Y-R
81	968.	983.	1115.	1008.	940.	1262.	859.	1042.	1282.	1152.	1105.	998.	12714.	O
81	727.	724.	809.	716.	655.	873.	579.	698.	854.	762.	728.	656.	8782.	R
81	.751	.736	.725	.710	.697	.692	.675	.670	.666	.662	.659	.657	.691	R/O
81	-241.	-259.	-306.	-292.	-285.	-389.	-280.	-344.	-428.	-390.	-377.	-342.	-3932.	R-O
81	1.200	1.015	1.134	.904	.933	1.343	.681	1.213	1.230	.899	.959	.903		M/M-O
81	1.181	.995	1.117	.885	.916	1.332	.663	1.205	1.224	.892	.955	.901		M/M-R
81	1.208	1.241	1.469	1.525	1.480	1.485	1.274	1.180	1.111	1.038	1.039	1.237		Y/Y-O
81	1.030	1.040	1.220	1.251	1.204	1.204	1.026	.951	.898	.848	.863	1.065		Y/Y-R
81	1.208	1.225	1.304	1.352	1.374	1.395	1.380	1.350	1.312	1.276	1.249	1.248	1.248	CUM Y/Y-O
81	1.030	1.035	1.094	1.128	1.141	1.153	1.137	1.111	1.078	1.048	1.029	1.031	1.031	CUM Y/Y-R

carry forward factors
INDEX1BM .657364

EXAMPLE 2

```
@ED,R J.TESTRUN/EX
READ-ONLY MODE
ED 16R1C-C FRI-08/22/86-09:49:23-(0,)
CENSUS ENFORCEMENT: OUTPUT WILL BE ASCII
EDIT
0:
      1:@XQT,TFQ SRD*TSPROG.ASBMARK
      2:8
      3:init  password
      4:QUAL          FILENAME
      5:1 76 4 83
      6:1 76 4 76
      7:1 77 4 77
      8:1 78 4 78
      9:1 79 4 79
     10:1 80 4 80
     11:1 81 4 81
     12:1 82 4 82
     13:1 83 4 83
     14:BUSQTREX1
     15:446690
     16:505160
     17:547570
     18:588160
     19:640390
     20:645850
     21:691830
     22:767660
     23:BUSQTREX2
     24:73400
     25:85660
     26:91430
     27:98200
     28:110230
     29:111830
     30:123910
     31:136530
EOF:31
0:
END ED. NO CORRECTIONS APPLIED

@XQT,TFQ SRD*TSPROG.ASBMARK
```

YEAR	FIRST	SEC	THIRD	FOURTH	TOTAL	CODE
81	29519.	30991.	34168.	36328.	131006.	O
81	25850.	26465.	28842.	30673.	111830.	R
81	.876	.854	.844	.844	.854	R/O
81	-3669.	-4526.	-5326.	-5655.	-19176.	R-O
81	.909	1.050	1.103	1.063		Q/Q-O
81	.864	1.024	1.090	1.063		Q/Q-R
81	1.095	1.108	1.116	1.119		Y/Y-O
81	1.028	1.003	1.003	1.025		Y/Y-R
81	1.095	1.101	1.107	1.110	1.110	CUM Y/Y-O
81	1.028	1:015	1.011	1.015	1.015	CUM Y/Y-R
82	33275.	34251.	36399.	37160.	141085.	O
82	28605.	29865.	32157.	33284.	123910.	R
82	.860	.872	.883	.896	.878	R/O
82	-4670.	-4386.	-4242.	-3876.	-17175.	R-O
82	.916	1.029	1.063	1.021		Q/Q-O
82	.933	1.044	1.077	1.035		Q/Q-R
82	1.127	1.105	1.065	1.023		Y/Y-O
82	1.107	1.128	1.115	1.085		Y/Y-R
82	1.127	1.116	1.098	1.077	1.077	CUM Y/Y-O
82	1.107	1.118	1.117	1.108	1.108	CUM Y/Y-R
83	33517.	35116.	38325.	41160.	148118.	O
83	30531.	32300.	35473.	38226.	136530.	R
83	.911	.920	.926	.929	.922	R/O
83	-2986.	-2816.	-2852.	-2934.	-11588.	R-O
83	.902	1.048	1.091	1.074		Q/Q-O
83	.917	1.058	1.098	1.078		Q/Q-R
83	1.007	1.025	1.053	1.108		Y/Y-O
83	1.067	1.082	1.103	1.148		Y/Y-R
83	1.007	1.016	1.029	1.050	1.050	CUM Y/Y-O
83	1.067	1.075	1.085	1.102	1.102	CUM Y/Y-R

carry forward factors

BUSQTREX1	.928318
BUSQTREX2	.928717

YEAR	FIRST	SEC	THIRD	FOURTH	TOTAL	CODE
78	22192.	23944.	26500.	28246.	100882.	O
78	20901.	21874.	23711.	24944.	91430.	R
78	.942	.914	.895	.883	.906	R/O
78	-1291.	-2070.	-2789.	-3302.	-9452.	R-O
78	.939	1.079	1.107	1.066		Q/Q-O
78	.891	1.047	1.084	1.052		Q/Q-R
78	1.162	1.166	1.211	1.195		Y/Y-O
78	1.090	1.048	1.070	1.064		Y/Y-R
78	1.162	1.164	1.181	1.185	1.185	CUM Y/Y-O
78	1.090	1.068	1.069	1.067	1.067	CUM Y/Y-R
79	25300.	26855.	28461.	29861.	110477.	O
79	22231.	23662.	25311.	26995.	98200.	R
79	.879	.881	.889	.904	.889	R/O
79	-3069.	-3193.	-3150.	-2866.	-12277.	R-O
79	.896	1.061	1.060	1.049		Q/Q-O
79	.891	1.064	1.070	1.067		Q/Q-R
79	1.140	1.122	1.074	1.057		Y/Y-O
79	1.064	1.082	1.067	1.082		Y/Y-R
79	1.140	1.130	1.110	1.095	1.095	CUM Y/Y-O
79	1.064	1.073	1.071	1.074	1.074	CUM Y/Y-R
80	26963.	27978.	30609.	32462.	118012.	O
80	25155.	26385.	28768.	29922.	110230.	R
80	.933	.943	.940	.922	.934	R/O
80	-1808.	-1593.	-1841.	-2540.	-7782.	R-O
80	.903	1.038	1.094	1.061		Q/Q-O
80	.932	1.049	1.090	1.040		Q/Q-R
80	1.066	1.042	1.075	1.087		Y/Y-O
80	1.132	1.115	1.137	1.108		Y/Y-R
80	1.066	1.053	1.061	1.068	1.068	CUM Y/Y-O
80	1.132	1.123	1.128	1.123	1.123	CUM Y/Y-R

SERIES BUSQTREX2 REVISED SOLUTION OPTIONS SELECTED: TREND INTO SERIES:
INPUT SERIES TITLE business quarterly series example 2
SERIES REVISED BY jb

DATE OF REVISION: 082286
TIME OF REVISION: 094944

SPAN OF REVISION: 1/1976 - 4/1983
BENCHMARK(S):
1/1976 - 4/1976 = 73400.
1/1977 - 4/1977 = 85660.
1/1978 - 4/1978 = 91430.
1/1979 - 4/1979 = 98200.
1/1980 - 4/1980 = 110230.
1/1981 - 4/1981 = 111830.
1/1982 - 4/1982 = 123910.
1/1983 - 4/1983 = 136530.
NUMBER OF ITERATIONS TO ATTAIN TREND SOLUTION 15

CARRY FORWARD FACTOR: .928717 ATTAINED VALUE OF TREND FUNCTION .01230586

YEAR	FIRST	SEC	THIRD	FOURTH	TOTAL	CODE
76	17866.	18791.	19947.	20542.	77146.	O
76	16722.	17693.	19013.	19972.	73400.	R
76	.936	.942	.953	.972	.951	R/O
76	-1144.	-1098.	-934.	-570.	-3746.	R-O
76	.000	1.052	1.062	1.030		Q/Q-O
76	.000	1.058	1.075	1.050		Q/Q-R
76	.000	.000	.000	.000		Y/Y-O
76	.000	.000	.000	.000		Y/Y-R
76	.000	.000	.000	.000	.000	CUM Y/Y-O
76	.000	.000	.000	.000	.000	CUM Y/Y-R
77	19091.	20528.	21887.	23627.	85133.	O
77	19181.	20864.	22166.	23449.	85660.	R
77	1.005	1.016	1.013	.992	1.006	R/O
77	90.	336.	279.	-178.	527.	R-O
77	.929	1.075	1.066	1.079		Q/Q-O
77	.960	1.088	1.062	1.058		Q/Q-R
77	1.069	1.092	1.097	1.150		Y/Y-O
77	1.147	1.179	1.166	1.174		Y/Y-R
77	1.069	1.081	1.087	1.104	1.104	CUM Y/Y-O
77	1.147	1.164	1.164	1.167	1.167	CUM Y/Y-R

YEAR	FIRST	SEC	THIRD	FOURTH	TOTAL	CODE
81	177667.	184822.	195316.	207472.	765277.	O
81	152027.	155829.	163687.	174307.	645850.	R
81	.856	.843	.838	.840	.844	R/O
81	-25640.	-28993.	-31629.	-33165.	-119427.	R-O
81	.922	1.040	1.057	1.062		Q/Q-O
81	.894	1.025	1.050	1.065		Q/Q-R
81	1.071	1.062	1.072	1.076		Y/Y-O
81	1.013	.991	1.004	1.025		Y/Y-R
81	1.071	1.066	1.068	1.071	1.071	CUM Y/Y-O
81	1.013	1.002	1.002	1.009	1.009	CUM Y/Y-R
82	188527.	194291.	200948.	210364.	794130.	O
82	160845.	167921.	176081.	186983.	691830.	R
82	.853	.864	.876	.889	.871	R/O
82	-27682.	-26370.	-24867.	-23381.	-102300.	R-O
82	.909	1.031	1.034	1.047		Q/Q-O
82	.923	1.044	1.049	1.062		Q/Q-R
82	1.061	1.051	1.029	1.014		Y/Y-O
82	1.058	1.078	1.076	1.073		Y/Y-R
82	1.061	1.056	1.047	1.038	1.038	CUM Y/Y-O
82	1.058	1.068	1.071	1.071	1.071	CUM Y/Y-R
83	192972.	202348.	213425.	226064.	834809.	O
83	174918.	185575.	197308.	209859.	767660.	R
83	.906	.917	.924	.928	.920	R/O
83	-18054.	-16773.	-16117.	-16205.	-67149.	R-O
83	.917	1.049	1.055	1.059		Q/Q-O
83	.935	1.061	1.063	1.064		Q/Q-R
83	1.024	1.041	1.062	1.075		Y/Y-O
83	1.087	1.105	1.121	1.122		Y/Y-R
83	1.024	1.033	1.043	1.051	1.051	CUM Y/Y-O
83	1.087	1.097	1.105	1.110	1.110	CUM Y/Y-R

YEAR	FIRST	SEC	THIRD	FOURTH	TOTAL	CODE
78	136218.	145898.	154435.	163682.	600233.	O
78	126446.	133624.	140022.	147479.	547570.	R
78	.928	.916	.907	.901	.912	R/O
78	-9772.	-12274.	-14413.	-16203.	-52663.	R-O
78	.951	1.071	1.059	1.060		Q/Q-O
78	.930	1.057	1.048	1.053		Q/Q-R
78	1.111	1.126	1.145	1.142		Y/Y-O
78	1.087	1.078	1.085	1.085		Y/Y-R
78	1.111	1.119	1.128	1.132	1.132	CUM Y/Y-O
78	1.087	1.082	1.084	1.084	1.084	CUM Y/Y-R
79	151675.	158347.	166845.	177597.	654464.	O
79	136253.	142132.	149873.	159902.	588160.	R
79	.898	.898	.898	.900	.899	R/O
79	-15422.	-16215.	-16972.	-17695.	-66304.	R-O
79	.927	1.044	1.054	1.064		Q/Q-O
79	.924	1.043	1.054	1.067		Q/Q-R
79	1.113	1.085	1.080	1.085		Y/Y-O
79	1.078	1.064	1.070	1.084		Y/Y-R
79	1.113	1.099	1.092	1.090	1.090	CUM Y/Y-O
79	1.078	1.070	1.070	1.074	1.074	CUM Y/Y-R
80	165829.	174090.	182154.	192782.	714855.	O
80	150077.	157229.	163092.	169992.	640390.	R
80	.905	.903	.895	.882	.896	R/O
80	-15752.	-16861.	-19062.	-22790.	-74465.	R-O
80	.934	1.050	1.046	1.058		Q/Q-O
80	.939	1.048	1.037	1.042		Q/Q-R
80	1.093	1.099	1.092	1.086		Y/Y-O
80	1.101	1.106	1.088	1.063		Y/Y-R
80	1.093	1.096	1.095	1.092	1.092	CUM Y/Y-O
80	1.101	1.104	1.098	1.089	1.089	CUM Y/Y-R

SERIES BUSQTREX1 REVISED SOLUTION OPTIONS SELECTED: TREND INTO SERIES:
INPUT SERIES TITLE business quarterly series example 1
SERIES REVISED BY jb

DATE OF REVISION: 082286
TIME OF REVISION: 094940

SPAN OF REVISION: 1/1976 - 4/1983
BENCHMARK(S):
1/1976 - 4/1976 = 446690.
1/1977 - 4/1977 = 505160.
1/1978 - 4/1978 = 547570.
1/1979 - 4/1979 = 588160.
1/1980 - 4/1980 = 640390.
1/1981 - 4/1981 = 645850.
1/1982 - 4/1982 = 691830.
1/1983 - 4/1983 = 767660.
NUMBER OF ITERATIONS TO ATTAIN TREND SOLUTION 13

CARRY FORWARD FACTOR: .928318 ATTAINED VALUE OF TREND FUNCTION .00421205

YEAR	FIRST	SEC	THIRD	FOURTH	TOTAL	CODE
76	114533.	119850.	122590.	128178.	485151.	O
76	104445.	109680.	113050.	119515.	446690.	R
76	.912	.915	.922	.932	.921	R/O
76	-10088.	-10170.	-9540.	-8663.	-38461.	R-O
76	.000	1.046	1.023	1.046		Q/Q-O
76	.000	1.050	1.031	1.057		Q/Q-R
76	.000	.000	.000	.000		Y/Y-O
76	.000	.000	.000	.000		Y/Y-R
76	.000	.000	.000	.000	.000	CUM Y/Y-O
76	.000	.000	.000	.000	.000	CUM Y/Y-R
77	122564.	129543.	134877.	143296.	530280.	O
77	116345.	123908.	128994.	135913.	505160.	R
77	.949	.957	.956	.948	.953	R/O
77	-6219.	-5635.	-5883.	-7383.	-25120.	R-O
77	.956	1.057	1.041	1.062		Q/Q-O
77	.973	1.065	1.041	1.054		Q/Q-R
77	1.070	1.081	1.100	1.118		Y/Y-O
77	1.114	1.130	1.141	1.137		Y/Y-R
77	1.070	1.076	1.084	1.093	1.093	CUM Y/Y-O
77	1.114	1.122	1.129	1.131	1.131	CUM Y/Y-R

```
@ED,R J.TESTRUN/EX
READ-ONLY MODE
ED 16R1C-C FRI-08/22/86-09:38:05-(2,)
CENSUS ENFORCEMENT: OUTPUT WILL BE ASCII
EDIT
0:
      1:@XQT,TFBOA SRD*TSPROG.ASBMARK
      2:2
      3:init  password
      4:QUAL         FILENAME
      5:1 76 1 83
      6:1 76 1 76
      7:1 81 1 81
      8:BUSANNN01
      9:BUSANNN01BM
     10:56468
     11:97148
     12:BUSANNN02
     13:BUSANNN02BM
     14:147759
     15:230142
     16:BUSANNN03
     17:BUSANNN03BM
     18:23196
     19:36152
EOF:19
0:
END ED. NO CORRECTIONS APPLIED

@XQT,TFBOA SRD*TSPROG.ASBMARK
 SERIES  BUSANNN01BM   DELETED
```

EXAMPLE 3

SERIES BUSANNNO3 REVISED SOLUTION OPTIONS SELECTED: TREND INTO SERIES: BUSANNNO3BM
INPUT SERIES TITLE
SERIES REVISED BY jb

DATE OF REVISION: 082286
TIME OF REVISION: 093827

SPAN OF REVISION: 1/1976 - 1/1983
BENCHMARK(S):
1/1976 - 1/1976 = 23196.
1/1981 - 1/1981 = 36152.
NUMBER OF ITERATIONS TO ATTAIN TREND SOLUTION 8

CARRY FORWARD FACTOR: 1.005144 ATTAINED VALUE OF TREND FUNCTION .00000628

1976	1977	1978	1979	1980	1981	1982	1983	CODE
23196.	25378.	28173.	30613.	33593.	35967.	39845.	42954.	O
23196.	25404.	28230.	30707.	33730.	36152.	40050.	43175.	R
1.000	1.001	1.002	1.003	1.004	1.005	1.005	1.005	R/O
0.	26.	57.	94.	137.	185.	205.	221.	R-O
.000	1.094	1.110	1.087	1.097	1.071	1.108	1.078	Y/Y-O
.000	1.095	1.111	1.088	1.098	1.072	1.108	1.078	Y/Y-R

carry forward factors

BUSANNNO1BM	.938202
BUSANNNO2BM	.986350
BUSANNNO3BM	1.005144

SERIES BUSANNNO2 REVISED SOLUTION OPTIONS SELECTED: TREND INTO SERIES: BUSANNNO2BM
INPUT SERIES TITLE
SERIES REVISED BY jb

DATE OF REVISION: 082286
TIME OF REVISION: 093825

SPAN OF REVISION: 1/1976 - 1/1983
BENCHMARK(S):
1/1976 - 1/1976 = 147759.
1/1981 - 1/1981 = 230142.
NUMBER OF ITERATIONS TO ATTAIN TREND SOLUTION 13

CARRY FORWARD FACTOR: .986350 ATTAINED VALUE OF TREND FUNCTION .00004517

1976	1977	1978	1979	1980	1981	1982	1983	CODE
147759.	164279.	185847.	206768.	222432.	233327.	242362.	257761.	O
147759.	163839.	184872.	205135.	220050.	230142.	239054.	254242.	R
1.000	.997	.995	.992	.989	.986	.986	.986	R/O
0.	-440.	-975.	-1633.	-2382.	-3185.	-3308.	-3519.	R-O
.000	1.112	1.131	1.113	1.076	1.049	1.039	1.064	Y/Y-O
.000	1.109	1.128	1.110	1.073	1.046	1.039	1.064	Y/Y-R

SERIES BUSANNNO3BM DELETED

SERIES BUSANNNO1 REVISED SOLUTION OPTIONS SELECTED: TREND INTO SERIES: BUSANNNO1BM
INPUT SERIES TITLE
SERIES REVISED BY jb

DATE OF REVISION: 082286
TIME OF REVISION: 093823

SPAN OF REVISION: 1/1976 - 1/1983
BENCHMARK(S):
1/1976 - 1/1976 = 56468.
1/1981 - 1/1981 = 97148.
NUMBER OF ITERATIONS TO ATTAIN TREND SOLUTION 28

CARRY FORWARD FACTOR: .938202 ATTAINED VALUE OF TREND FUNCTION .00100080

1976	1977	1978	1979	1980	1981	1982	1983	CODE
56468.	60546.	75103.	97033.	107670.	103547.	105374.	106015.	O
56468.	59709.	73301.	93811.	102763.	97148.	98862.	99463.	R
1.000	.986	.976	.967	.954	.938	.938	.938	R/O
0.	-837.	-1802.	-3222.	-4907.	-6399.	-6512.	-6552.	R-O
.000	1.072	1.240	1.292	1.110	.962	1.018	1.006	Y/Y-O
.000	1.057	1.228	1.280	1.095	.945	1.018	1.006	Y/Y-R

SERIES BUSANNNO2BM DELETED

```
@ED,R J.TESTRUN/EX
READ-ONLY MODE
ED 16R1C-C TUE-12/30/86-12:22:13-(2,)
CENSUS ENFORCEMENT: OUTPUT WILL BE ASCII
EDIT
0:
      1:@XQT,CF SRD*TSPROG.ASBMARK
      2:5
      3:init  password
      4:QUAL          FILENAME
      5:1 77 12 81
      6:1 77 12 77
      7:1 78 12 78
      8:1 79 12 79
      9:1 80 12 80
     10:1 81 12 81
     11:INDEX1
     12:6913
     13:7936
     14:8092
     15:8516
     16:8782
EOF:16
0:
END ED. NO CORRECTIONS APPLIED

@XQT,CF SRD*JBBENCH.RUN
```

EXAMPLE 4

SERIES INDEX1 REVISED SOLUTION OPTIONS SELECTED: CANADIAN INTO SERIES:
INPUT SERIES TITLE RADIO AND TV - SHIPMENTS
SERIES REVISED BY jb

DATE OF REVISION: 123086
TIME OF REVISION: 122225

LAST ITERATION PRODUCED AN INCREASE IN THE FUNCTION VALUE. OLD VALUE IS .0074473 NEW ONE IS .0076727
OLD SOLUTION WILL BE USED IF SERIES IS SAVED IN TIMEBASE. REVISED SERIES USING NEW SOLUTION IS IDENTIFIED BY 'R(L)'.

SPAN OF REVISION: 1/1977 - 12/1981
BENCHMARK(S):
1/1977 - 12/1977 = 6913.
1/1978 - 12/1978 = 7936.
1/1979 - 12/1979 = 8092.
1/1980 - 12/1980 = 8516.
1/1981 - 12/1981 = 8782.
NUMBER OF ITERATIONS TO ATTAIN CANADIAN SOLUTION 2

CARRY FORWARD FACTOR: .660201 ATTAINED VALUE OF CANADIAN FUNCTION .00767265

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC	TOTAL	CODE
77	401.	485.	465.	394.	420.	541.	407.	524.	607.	670.	697.	640.	6251.	O
77	446.	539.	516.	437.	466.	600.	451.	580.	670.	739.	767.	703.	6913.	R
77	445.	539.	516.	437.	466.	600.	451.	579.	670.	739.	767.	704.	6913.	R(L)
77	1.111	1.111	1.110	1.110	1.109	1.108	1.107	1.106	1.104	1.103	1.101	1.098	1.106	R/O
77	45.	54.	51.	43.	46.	59.	44.	56.	63.	69.	70.	63.	662.	R-O
77	.000	1.209	.959	.847	1.066	1.288	.752	1.287	1.158	1.104	1.040	.918		M/M-O
77	.000	1.209	.958	.847	1.065	1.287	.752	1.286	1.157	1.102	1.038	.916		M/M-R
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		Y/Y-O
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000		Y/Y-R
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	CUM Y/Y-O
77	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	CUM Y/Y-R
78	455.	522.	547.	522.	516.	667.	519.	730.	779.	791.	803.	674.	7525.	O
78	500.	572.	596.	566.	556.	714.	551.	768.	811.	814.	815.	672.	7936.	R
78	499.	572.	597.	566.	556.	714.	551.	768.	811.	814.	814.	673.	7936.	R(L)
78	1.099	1.095	1.090	1.085	1.078	1.071	1.062	1.052	1.041	1.029	1.015	.998	1.055	R/O
78	45.	50.	49.	44.	40.	47.	32.	38.	32.	23.	12.	-2.	411.	R-O
78	.711	1.147	1.048	.954	.989	1.293	.778	1.407	1.067	1.015	1.015	.839		M/M-O
78	.711	1.143	1.043	.949	.982	1.284	.772	1.394	1.056	1.003	1.001	.825		M/M-R
78	1.135	1.076	1.176	1.325	1.229	1.233	1.275	1.393	1.283	1.181	1.152	1.053		Y/Y-O
78	1.122	1.061	1.155	1.295	1.194	1.191	1.223	1.326	1.210	1.101	1.062	.957		Y/Y-R
78	1.135	1.103	1.128	1.172	1.183	1.193	1.204	1.231	1.239	1.231	1.221	1.204	1.204	CUM Y/Y-O
78	1.122	1.089	1.112	1.153	1.161	1.167	1.174	1.196	1.198	1.185	1.170	1.148	1.148	CUM Y/Y-R

YEAR	JAN	FEB	MARCH	APRIL	MAY	JUNE	JULY	AUG	SEPT	OCT	NOV	DEC	TOTAL	CODE
79	646.	690.	748.	548.	700.	867.	538.	787.	921.	910.	788.	643.	8786.	O
79	633.	667.	714.	515.	652.	799.	491.	712.	827.	812.	700.	570.	8092.	R
79	634.	667.	713.	516.	651.	799.	491.	712.	827.	812.	700.	569.	8092.	R(L)
79	.981	.967	.954	.941	.931	.921	.912	.905	.898	.892	.889	.886	.921	R/O
79	-13.	-23.	-34.	-33.	-48.	-68.	-47.	-75.	-94.	-98.	-88.	-73.	-694.	R-O
79	.958	1.068	1.084	.733	1.277	1.239	.621	1.463	1.170	.988	.866	.816		M/M-O
79	.942	1.053	1.070	.722	1.264	1.226	.614	1.451	1.161	.982	.862	.814		M/M-R
79	1.420	1.322	1.367	1.050	1.357	1.300	1.037	1.078	1.182	1.150	.981	.954		Y/Y-O
79	1.267	1.167	1.197	.910	1.171	1.119	.891	.927	1.019	.998	.860	.848		Y/Y-R
79	1.420	1.367	1.367	1.286	1.301	1.300	1.264	1.234	1.226	1.216	1.189	1.168	1.168	CUM Y/Y-O
79	1.267	1.214	1.208	1.132	1.140	1.136	1.102	1.074	1.066	1.058	1.036	1.020	1.020	CUM Y/Y-R
80	801.	792.	759.	661.	635.	850.	674.	883.	1154.	1110.	1064.	807.	10190.	O
80	708.	697.	664.	574.	547.	725.	568.	734.	946.	894.	839.	619.	8516.	R
80	708.	697.	664.	574.	547.	725.	568.	734.	946.	894.	838.	621.	8516.	R(L)
80	.883	.880	.875	.869	.861	.853	.842	.832	.820	.805	.789	.768	.836	R/O
80	-93.	-95.	-95.	-87.	-88.	-125.	-106.	-149.	-208.	-216.	-225.	-188.	-1674.	R-O
80	1.246	.989	.958	.871	.961	1.339	.793	1.310	1.307	.962	.959	.758		M/M-O
80	1.242	.985	.953	.865	.952	1.325	.783	1.294	1.288	.945	.939	.738		M/M-R
80	1.240	1.148	1.015	1.206	.907	.980	1.253	1.122	1.253	1.220	1.350	1.255		Y/Y-O
80	1.117	1.044	.930	1.114	.839	.907	1.157	1.032	1.144	1.101	1.199	1.087		Y/Y-R
80	1.240	1.192	1.129	1.145	1.095	1.071	1.092	1.096	1.119	1.131	1.152	1.160	1.160	CUM Y/Y-O
80	1.117	1.080	1.027	1.045	1.003	.983	1.003	1.007	1.026	1.035	1.050	1.052	1.052	CUM Y/Y-R
81	968.	983.	1115.	1008.	940.	1262.	859.	1042.	1282.	1152.	1105.	998.	12714.	O
81	726.	722.	804.	714.	655.	868.	584.	701.	855.	764.	730.	659.	8782.	R
81	726.	722.	804.	714.	655.	868.	584.	701.	855.	764.	730.	659.	8782.	R(L)
81	.750	.734	.721	.708	.697	.688	.680	.673	.667	.663	.661	.660	.691	R/O
81	-242.	-261.	-311.	-294.	-285.	-394.	-275.	-341.	-427.	-388.	-375.	-339.	-3932.	R-O
81	1.200	1.015	1.134	.904	.933	1.343	.681	1.213	1.230	.899	.959	.000		M/M-O
81	1.172	.995	1.114	.888	.918	1.325	.672	1.201	1.219	.894	.956	.000		M/M-R
81	1.208	1.241	1.469	1.525	1.480	1.485	1.274	1.180	1.111	1.038	1.039	.000		Y/Y-O
81	1.026	1.036	1.211	1.243	1.198	1.198	1.028	.954	.904	.854	.870	.000		Y/Y-R
81	1.208	1.225	1.304	1.352	1.374	1.395	1.380	1.350	1.312	1.276	1.249	1.248	1.248	CUM Y/Y-O
81	1.026	1.031	1.089	1.122	1.135	1.147	1.132	1.107	1.076	1.048	1.029	1.031	1.031	CUM Y/Y-R

carry forward factors
INDEX1 .660201

4. PROGRAM ENHANCEMENTS

Several enhancements to the existing ASCII program are planned. A significant improvement planned is to remove the program's dependency on the ASCII TIMEBASE for input and output, thereby allowing series to be read in, and saved, outside of the ASCII TIMEBASE. This modification, however, must be accompanied by detailed verification of the data that is now handled by TIMEBASE. In addition, the inclusion of graphics output is being considered.

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APPENDIX B

Derivation of Solution to the Benchmarking Problem: Relative Revision

MTrager:ra
8/10/82

DERIVATION OF SOLUTION TO THE BENCHMARKING PROBLEM

RELATIVE REVISION

Let x_i = original series for period i, i = 1, 2, ..., z

y_i = revised series for period i

$$A_i = \frac{y_i}{x_i}$$

$$F = \sum_{i=1}^{z-1} (A_{i+1} - A_i)^2$$

Problem: Minimize F ∋

$$\sum_{i=b_k}^{e_k} y_i = T_k \quad , \quad k = 1, 2, \ldots, n$$

with

$$1 \leq b_1 \leq e_1 < b_2 \leq e_2 < \ldots < b_n \leq e_n \leq z$$

Lagrange Equation

$$F' = F + 2 \sum_{k=1}^{n} \lambda_k \left(\sum_{i=b_k}^{e_k} y_i - T_k \right)$$

I. Taking $\frac{\partial F'}{\partial y_i}$, i = 1, 2, ..., z :

$$i = 1 \quad \frac{\partial F'}{\partial y_1} = \frac{2}{x_1} \quad (A_2 - A_1)$$

$$i = z \quad \frac{\partial F'}{\partial y_z} = \frac{2}{x_z} \quad (A_{z-1} - A_z)$$

$\forall i \ni i \varepsilon \left[b_k \quad e_k\right] \quad k = 1, 2, \ldots, n$

$$\frac{\partial F'}{\partial y_i} = \frac{2}{x_i} \left(A_{i+1} - 2A_i + A_{i-1}\right) + 2\lambda_k$$

$\forall$ other i :

$$\frac{\partial F'}{\partial y_i} = \frac{2}{x_i} \left(A_{i+1} - 2A_i + A_{i-1}\right)$$

II. Setting $\frac{\partial F'}{\partial y_i} = 0 \ \forall_i$, and multiplying through by $\frac{x_i}{2}$,

and taking successive sums of the form:

$$\sum_{j=1}^{z} \frac{\partial F'}{\partial y_j} = 0 \ , \quad \sum_{j=1}^{z-1} \frac{\partial F'}{\partial y_j} \ , \ \ldots \quad \sum_{j=1}^{1} \frac{\partial F'}{\partial y_j} = 0$$

We get the following system of equations:

Let $\alpha_i = \mathrm{MAX}\ \{j : i > e_j\} \quad (e_o = 0)$

$$\beta_i = \begin{cases} j \text{ if } i\varepsilon \left[b_j \ , \ e_j\right] \\ 0 \text{ otherwise} \end{cases}$$

then

A. $\displaystyle\sum_{k=1}^{n} \lambda_k \sum_{i=b_k}^{e_k} x_i = 0$

B. $\displaystyle A_i = A_{i-1} - \sum_{k=1}^{\alpha_i} \lambda_k \sum_{j=b_k}^{e_k} x_j - \lambda_{\beta_i} \sum_{p=b_{\beta_i}}^{i-1} x_p$ with $\lambda_o = 0$

III. Solving the system of equations in II.B for A_i in terms of λ_i & C we get:

$$A_i = C - \sum_{k=1}^{\alpha_i} \lambda_k \sum_{j=b_k}^{e_k} (i - j)\, x_j$$

$$- \lambda_{\beta_i} \sum_{j=b_{\beta_i}}^{i-1} (i - j)\, x_j$$

or,

$$y_i = x_i C - x_i \sum_{k=1}^{\alpha_i} \lambda_k \sum_{j=b_k}^{e_k} (i - j)\, x_j$$

$$- x_i \lambda_{\beta_i} \sum_{j=b_{\beta_i}}^{i-1} (i - j)\, x_j \qquad (*)$$

IV. Form the system of equations

$$A: \quad \sum_{k=1}^{n} \lambda_k \sum_{i=b_k}^{e_k} x_i = 0$$

and from III:

B: $\forall k$, $k = 1, 2, \ldots, n$

$$T_k = C \sum_{i=b_k}^{e_k} x_i - \sum_{r=1}^{k} \lambda_r \sum_{i=b_k}^{e_k} \left[\sum_{p=b_r}^{\mathrm{MIN}\,(e_r\,,\; i-1)} (i-p)\, x_p \right]$$

These are n + 1 equations with n + 1 unknowns:

$\lambda_1 - \lambda_n$ and C. Solve this system and substitute into (*) in III for y_i .

APPENDIX C

Derivation of Solution to the Benchmarking Problem: Trend Revision

8/12/82
Based on work by
Dr. Bev Causey - 7/81
Revised by MTrager - 9/14/81

DERIVATION OF SOLUTION TO THE BENCHMARKING PROBLEM

TREND REVISION

I. Problem: Minimize the function

$$F_T = \sum_{i=1}^{z-1} \left(\frac{y_{i+1}}{y_i} - r_i \right)^2$$

where $r_i = x_{i+1} / x_i$

and y_i = revised series for period i

x_i = original series for period i

subject to the constraints:

$$\sum_{i=b_k}^{e_k} y_i = T_k \quad k = 1, 2, \ldots, n$$

with $1 = b_1 \leq e_1 < b_2 \leq e_2 < \ldots < b_n \leq e_n = z$

II. We already have the solution to the problem which minimizes the function

$$F_R = \sum_{i=1}^{z-1} \left(\frac{y_{i+1}}{x_{i+1}} - \frac{y_i}{x_i} \right)^2$$

subject to the same constraints.

Call this solution $y_i^{(o)}$ $\forall i$, $i = 1, 2, \ldots, z$

Evaluate F_T using solution to F_R $\left(\text{i.e., } y^{(o)}\right)$.

Set $F^{(o)} = F_T\left(y^{(o)}\right)$.

III. We will attempt to minimize F_T subject to the constraints using the method of steepest descent beginning with $y^{(o)}$ as the initial solution. This method is iterative with a stopping rule of the form $F_T\left(y^{(j+1)}\right)/F_T\left(y^{(j)}\right)<10^{-6}$ where j represents the iteration number.

Consider the function F_T^*

$$F_T^* = \sum_{i=1}^{z-1}\left(\frac{y_{i+1} + td_{i+1}}{y_i + td_i} - r_i\right)^2$$

where t is a scalar and

d is a direction vector

A. Now, $$\left.\frac{dF_T^*}{dt}\right|_{t=0} = \sum_{i=1}^{z} a_i d_i$$

with $$a_1 = -g_1 \frac{y_2}{y_1}$$

$$a_i = g_{i-1} - g_i \frac{y_{i+1}}{y_i}, \quad i = 2, \ldots, z-1$$

$$a_z = g_{z-1}$$

and $$g_i = \left(\frac{y_{i+1}}{y_i} - r_i\right) \Big/ y_i$$

B. To solve for the d vector:

minimize $$\sum_{i=1}^{z} a_i d_i$$

subject to: $$\sum_{i=1}^{z} d_i^2 = 1$$

and $$\sum_{i=b_k}^{e_k} d_i = 0, \quad k = 1, 2, \ldots, n$$

(This latter constraint will ensure that the resulting solution will continue to meet the desired set of original constraints.)

1. Lagrange equation:

$$P = \sum_{i=1}^{z} a_i d_i + \frac{\lambda^*}{2} \left(\sum_{i=1}^{z} d_i^2 - 1 \right) + \sum_{k=1}^{n} \lambda_k \sum_{i=b_k}^{e_k} d_i$$

2. $$\frac{\partial P}{\partial d_i} = \left\{ \begin{array}{ll} a_i + \lambda^* d_i & i \notin \left[b_k, e_k\right] \\ a_i + \lambda^* d_i + \lambda_k & i \varepsilon \left[b_k, e_k\right] \end{array} \right\} = 0$$

3. Now, with $n_k = e_k - b_k + 1$

$$\sum_{i=b_k}^{e_k} \frac{\partial P}{\partial d_i} = 0 = \sum_{i=b_k}^{e_k} a_i + \lambda^* \sum_{i=b_k}^{e_k} d_i + n_k \lambda_k$$

$$= \sum_{i=b_k}^{e_k} a_i + n_k \lambda_k = 0$$

$$\text{or} \quad \lambda_k = \frac{-\sum_{i=b_k}^{e_k} a_i}{n_k} = -\bar{a}_k$$

4. From 2.:

$$d_i = -a_i / \lambda^* \qquad i \notin \left[b_k , e_k\right]$$

$$\text{and,} \quad d_i = \frac{-a_i - \lambda_k}{\lambda^*} \qquad i \varepsilon \left[b_k , e_k\right]$$

$$\text{or,} \quad d_i = \frac{-a_i + \bar{a}_k}{\lambda^*} \qquad i \varepsilon \left[b_k , e_k\right]$$

5. Want $\sum_{i=1}^{z} d_i^2 = 1$

$$\text{or,} \sum_{i \in [b_k,\ e_k]} \left(-a_i / \lambda^*\right)^2 + \sum_{k=1}^{n} \sum_{i=b_k}^{e_k} \left(\frac{-a_i + \bar{a}_k}{\lambda^*}\right)^2 = 1$$

$$\text{thus, } \lambda^* = \left[\sum_{i \notin [b_k,\ e_k]} a_i^2 + \sum_{k=1}^{n} \sum_{i=b_k}^{e_k} \left(-a_i + \bar{a}_k\right)^2\right]^{\frac{1}{2}}$$

6. Using the result from 5. in 4. yields the desired direction vector.

7. To show $\sum_{i=1}^{z} a_i d_i \leq 0$:

From 4.,

$$d_i = \frac{-a_i}{\lambda^*} \qquad i \notin [b_k,\ e_k]$$

$$d_i = \frac{-a_i + \bar{a}_k}{\lambda^*} \qquad i \in [b_k,\ e_k] \quad ,$$

Since $\lambda^* > 0$, we can take λ^* to be 1 without effect on the proof. (If $\lambda^* = 0$, the solution to F_T^* is found and we need go no further.)

$$\sum_{i=1}^{z} a_i d_i = -\sum_{i \notin [b_k,\ e_k]} a_i^2 - \sum_{k=1}^{n} \sum_{i=b_k}^{e_k} a_i \left(a_i - \bar{a}_k\right)$$

$$= -\sum_{i \notin [b_k,\ e_k]} a_i^2 - \sum_{k=1}^{n} \left[\sum_{i=b_k}^{e_k} a_i^2 - \bar{a}_k \sum_{i=b_k}^{e_k} a_i\right]$$

$$= -\sum_{i \notin [b_k,\ e_k]} a_i^2 \quad - \sum_{k=1}^{n} \left[\sum_{i=b_k}^{e_k} a_i^2 - n_k\, \bar{a}_k^2 \right]$$

$$= -\sum_{i \notin [b_k,\ e_k]} a_i^2 \quad - \sum_{k=1}^{n} \sum_{i=b_k}^{e_k} \left(a_i - \bar{a}_k \right)^2$$

Therefore, at $t = 0$, $\dfrac{dF_T^*}{dt} = \sum_{i=1}^{z} a_i d_i \leq 0$

Therefore, F_T^* is decreasing at $t = 0$ and ≥ 0 everywhere. If $F_T^{*\prime}$ at $t = 0$ were 0, we would be finished; with the solution to F_R being the solution minimizing F_T.

If $F_T^{*\prime}$ is not zero at $t = 0$, we wish to find $t \ni F_T^{*\prime}(t) = 0$ (using the d - vector determined above).

D. To solve for the $t \ni F_T^{*\prime}(t) = 0$:

1. First note that we require

$$y_i + td_i > 0 \ \forall i$$

$$\text{or} \quad t \in \left[\max_{d_i > 0} -y_i/d_i \ , \ \min_{d_i < 0} -y_i/d_i \right]$$

However, since F_T^* is decreasing at $t = 0$ we need not consider values of $t < 0$.

Therefore, the feasible set of values of t lie in the range $\left[0,\ \min -y_i/d_i \right]$

2. Consider the Taylor expansion of $F_T^{*'}$ about a point x :

$$F_T^{*'}(t) = 0 = F_T^{*'}(x) + F_T^{*''}(x)\,(t-x)$$

$$+ \frac{F_T^{*'''}(x)}{2}(t - x)^2 + \ldots \qquad (4)$$

where, $F_T^{*'}(x) = 2\sum_{i=1}^{z-1}\left(\frac{y_{i+1} + xd_{i+1}}{y_i + xd_i} - r_i\right)\frac{h_i}{(y_i + xd_i)^2}$

$$F_T^{*''}(x) = 2\sum_{i=1}^{z-1}\left[\frac{h_i^2}{(y_i + xd_i)^4} - \frac{2h_i d_i}{(y_i + xd_i)^3}\left(\frac{y_{i+1} + xd_{i+1}}{y_i + xd_i} - r_i\right)\right]$$

$$F_T^{*'''}(x) = 2\sum_{i=1}^{z-1}\frac{6d_i}{(y_i + xd_i)}\left[\frac{h_i d_i}{(y_i + xd_i)^3}\left(\frac{y_{i+1} + xd_{i+1}}{y_i + xd_i} - r_i\right) - \frac{h_i^2}{(y_i + xd_i)^4}\right]$$

and, $h_i = d_{i+1}\, y_i - d_i y_{i+1}$

a. set $\ell = 0$ and $x^{(0)} = 0$

b. evaluate $F_T^{*'}\left(x^{(\ell)}\right)$, $F_T^{*''}\left(x^{(\ell)}\right)$, $F_T^{*'''}\left(x^{(\ell)}\right)$

c. solve (4) for $\left(t - x^{(\ell)}\right)$ using quadratic equation and consider the smallest nonimaginary root in absolute value - call it D.

d. then, $t = x^{(\ell)} + D/C$, $C \geq 1.0$

e. if $t/x^{(\ell)} < \varepsilon$, $\varepsilon > 0$ go to section E.

f. set $\ell = \ell + 1$ and $x^{(\ell)} = t$

g. go to b. and repeat the process

Note 1: This methodology will not yield a valid solution if any of the following occur:

1. $t > \min_{d_i<0} -y/d_i$ or $t < 0$

2. $F_T^{*'}(t) > 0$

3. $F_T^{*''}(t) < 0$

Note 2: Empirically, it appears that setting $C = 1$ and ε to any number such that a.-e. are performed only once, results in the fastest convergence.

E. Set $j = j + 1$

Define $y_i^{(j)} = y_i^{(j-1)} + td_i \; \forall i$

Evaluate F_T^* using $y_i^{(j)}$'s and call this value $F_T^{(j)}$.

If $F_T^{(j)} / F_T^{(j-1)} > 10^{-6}$ and j is less than some maximum number of iterations, repeat the entire iterative process beginning at B.5 using $y_i^{(j)}$'s as just computed.

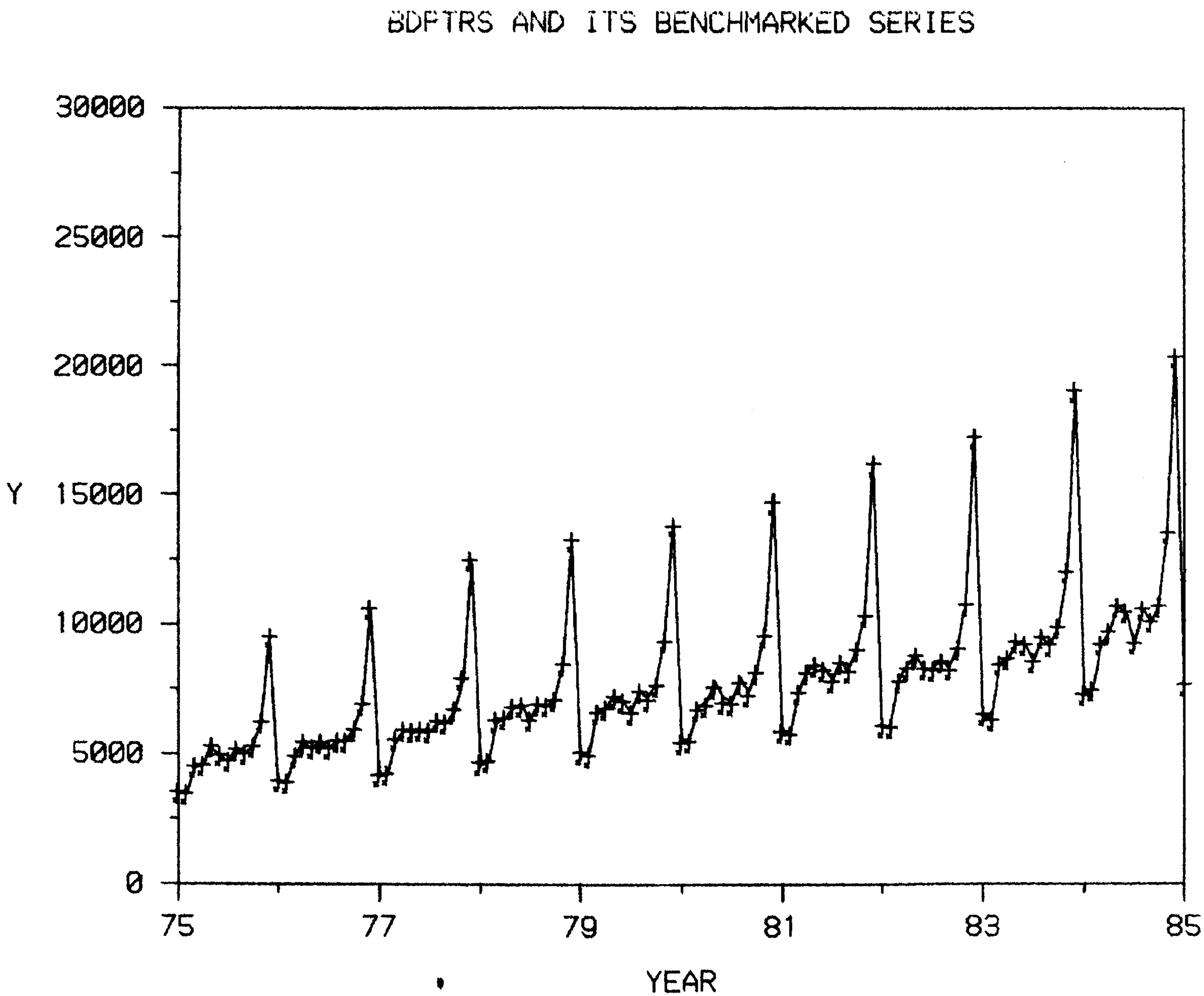
Otherwise, we are finished.

List of Figures

Figures 1 to 9. Original Series and UA and SA Method Benchmarked Series. The solid line represents the original series, the dot, '.', represents the UA benchmarked series, and the cross, '+', represents the SA benchmarked series. Note how close the two benchmark series are to each other even when the revisions from the original series are large.

Figures 10 to 18. UA and SA Method Percent Benchmark Revisions. The solid line represents the UA method percent benchmark revisions, $\{L_t^u\}$, and the cross, 'x', represents the SA percent benchmark revisions, $\{L_t^s\}$. The solid reference line at zero represents no revision at that time point. Points above the reference line are points where the benchmarked series are revised upward from the original series. Notice, the magnitude of the differences between the methods, i.e. the difference between the UA method line and the SA method x's.

Figures 19 to 27. Total Percent Benchmark Revisions and Their Percent Benchmark Revisions. The solid line is the total percent benchmark revision for each annual benchmark. The straight solid reference line at zero represents no revision. Above the reference line the benchmarked series is revised upward from the original series. The vertical reference lines show the breaks between the benchmark periods. The percent benchmark revisions are the percentage of the total benchmark revision occurring at that time point. Within a benchmark period these will add up to 100%. The percent revisions for the UA method are represented by the dots, '.', and the SA method is represented by the crosses, '+'. Notice that even though the UA and the SA methods are designed to smooth the change between the adjacent benchmarks sometimes there are abrupt jumps at the time points next to the benchmark boundaries, these jumps don't occur when the revisions are plotted as a percentage of the original series (Figures 9 to 18).


BDFTRS AND ITS BENCHMARKED SERIES
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20000
Y
15000
10000
5000
0
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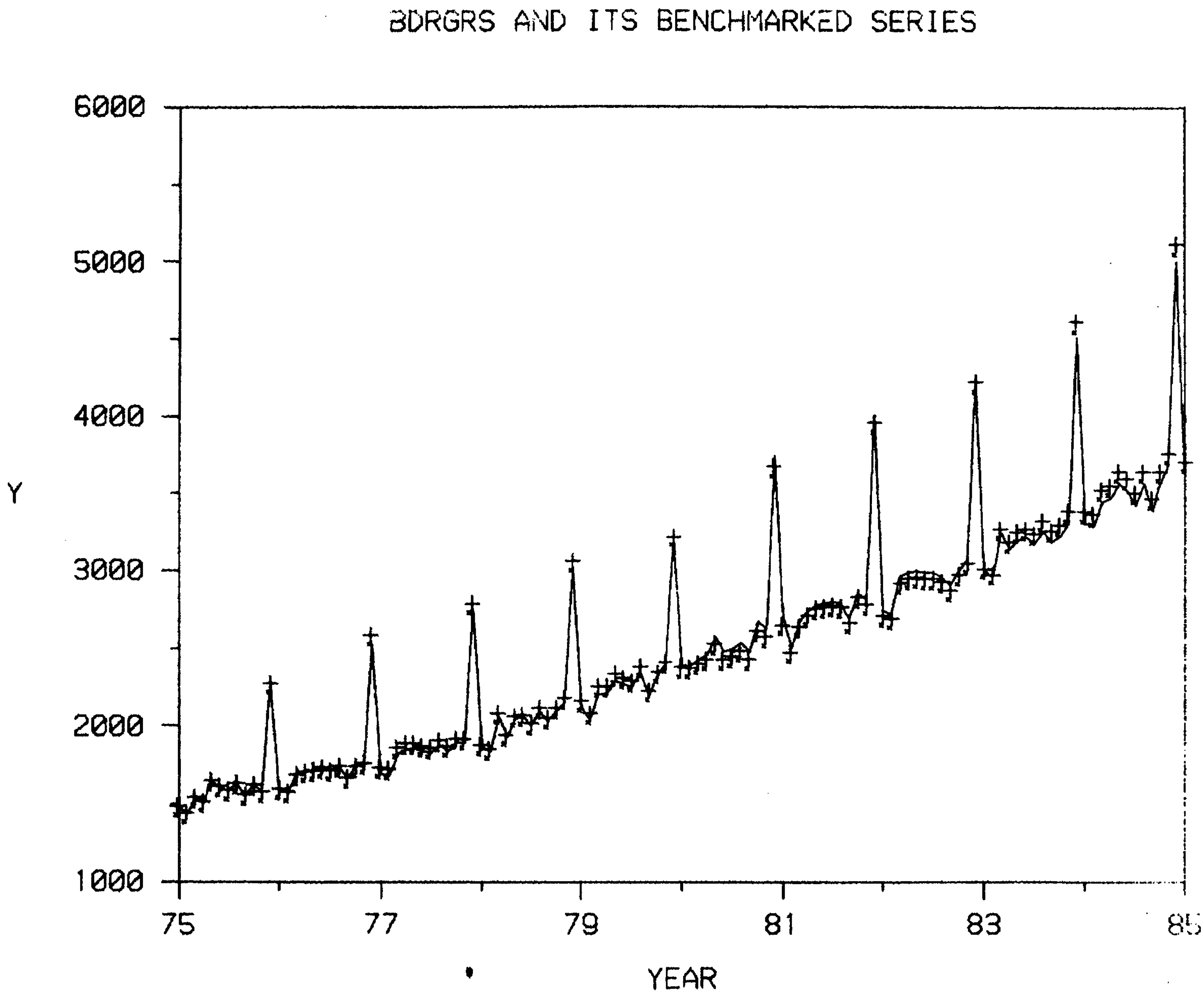
77
79
81
83
85
YEAR

Figure 1.


BDRGRS AND ITS BENCHMARKED SERIES
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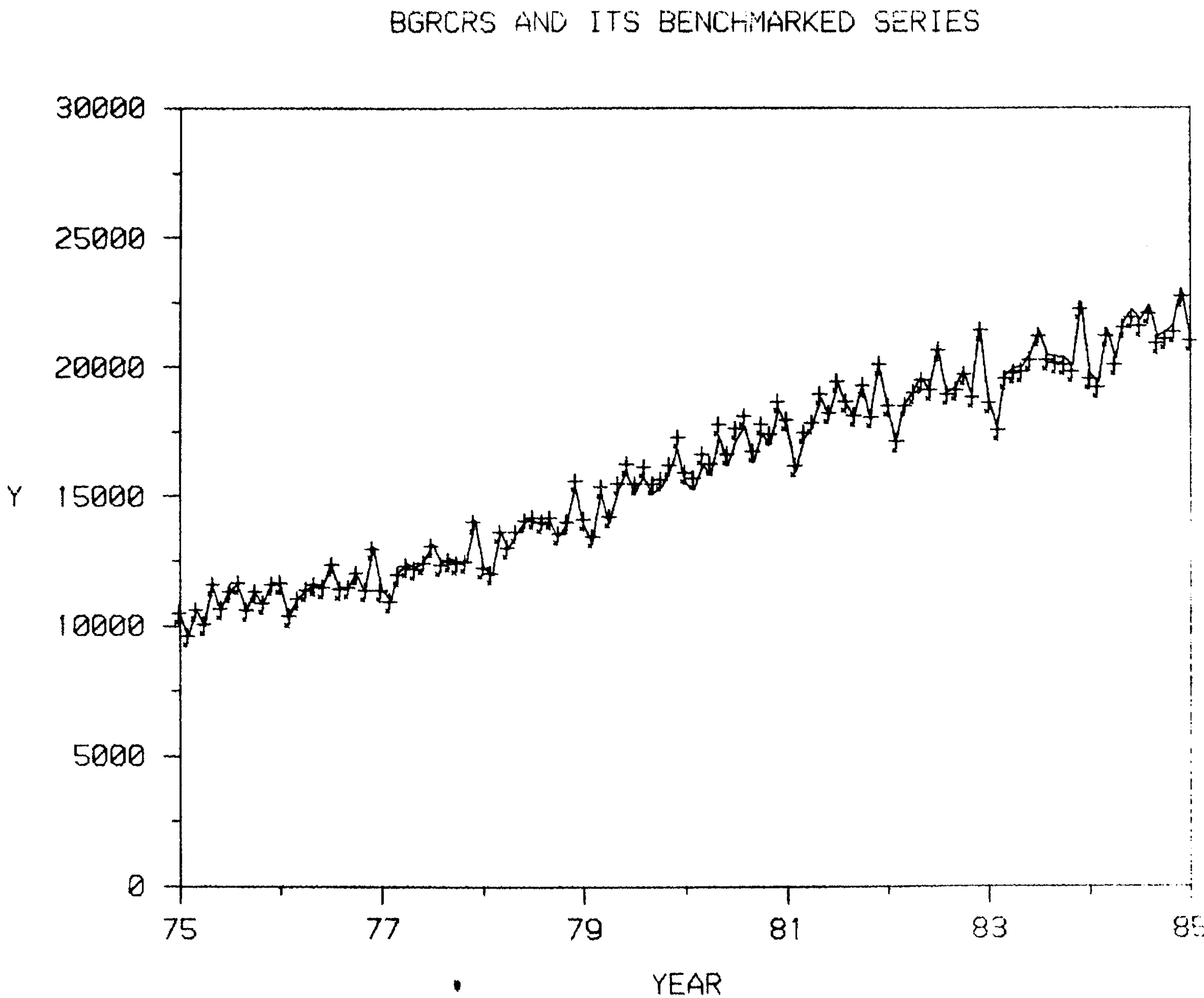
3000
2000
1000
75
77
79
81
83
85
YEAR

Figure 2.


BGRCRS AND ITS BENCHMARKED SERIES
30000
25000
20000
15000
10000
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0
Y
75
77
79
81
83
85
YEAR

Figure 3.

BGMRRI AND ITS BENCHMARKED SERIES

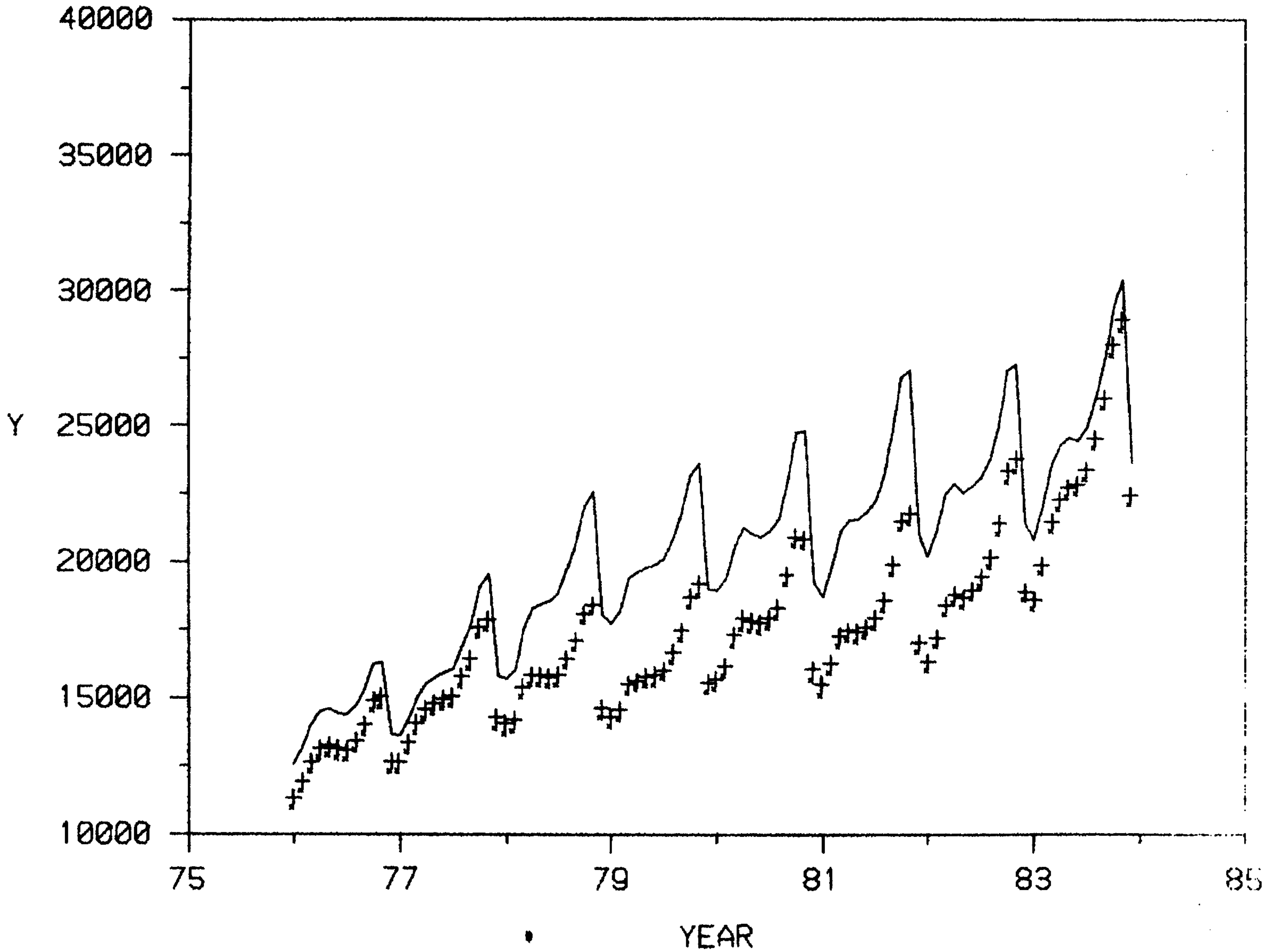
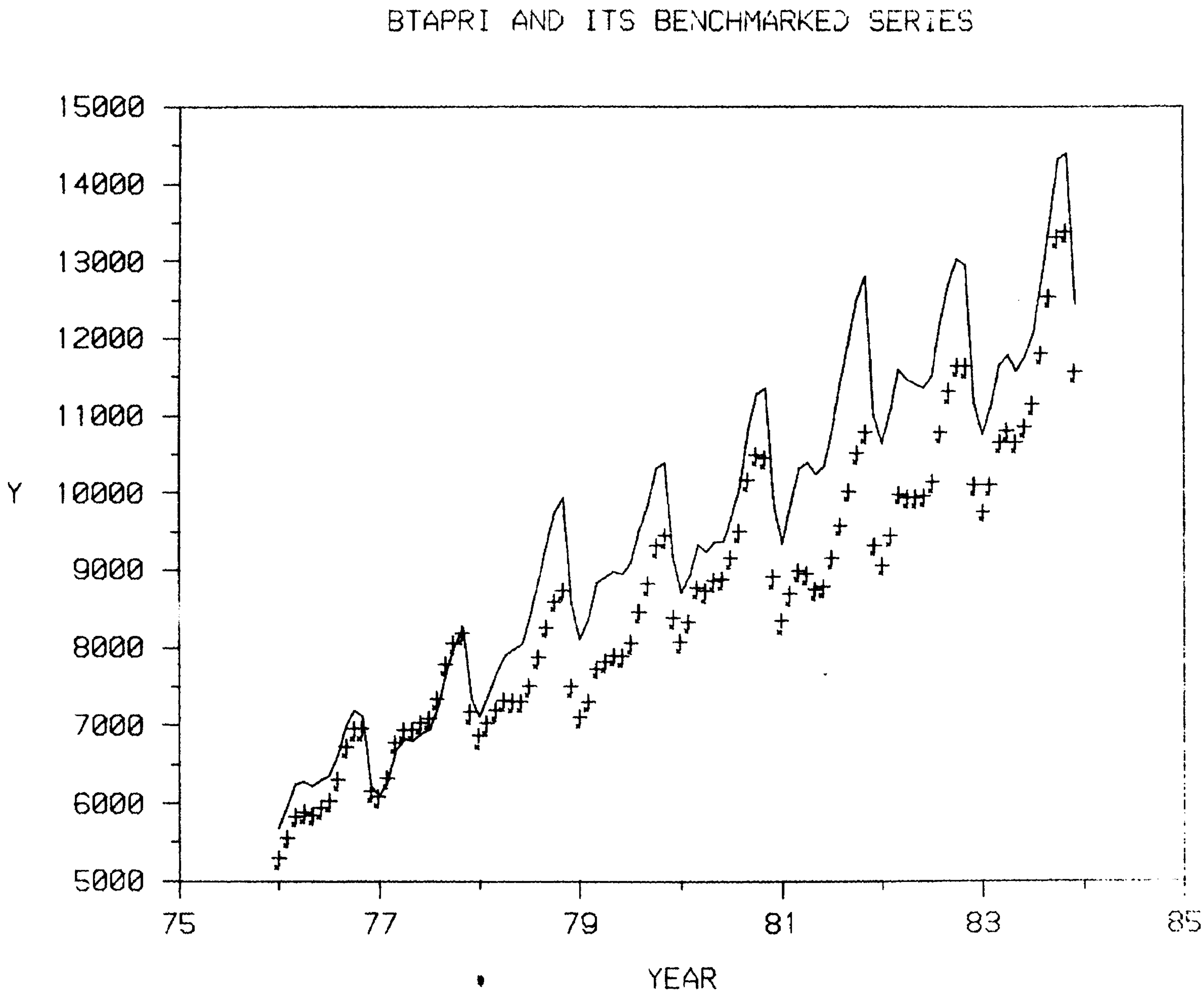


Figure 4.


BTAPRI AND ITS BENCHMARKED SERIES
15000
14000
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12000
11000
10000
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8000
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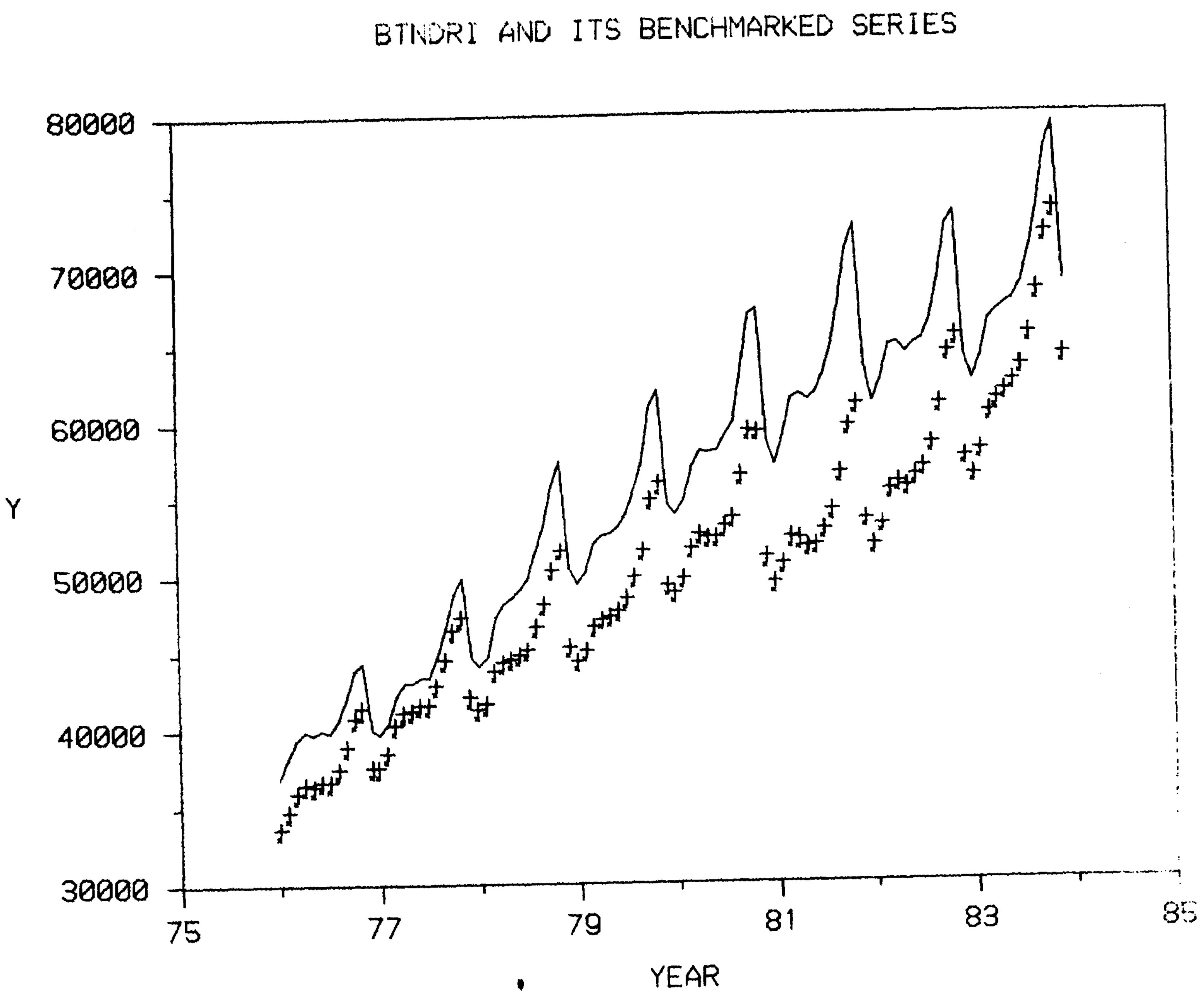
77
79
81
83
85
YEAR

Figure 5.


BTNDRI AND ITS BENCHMARKED SERIES
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85
YEAR

Figure 6.

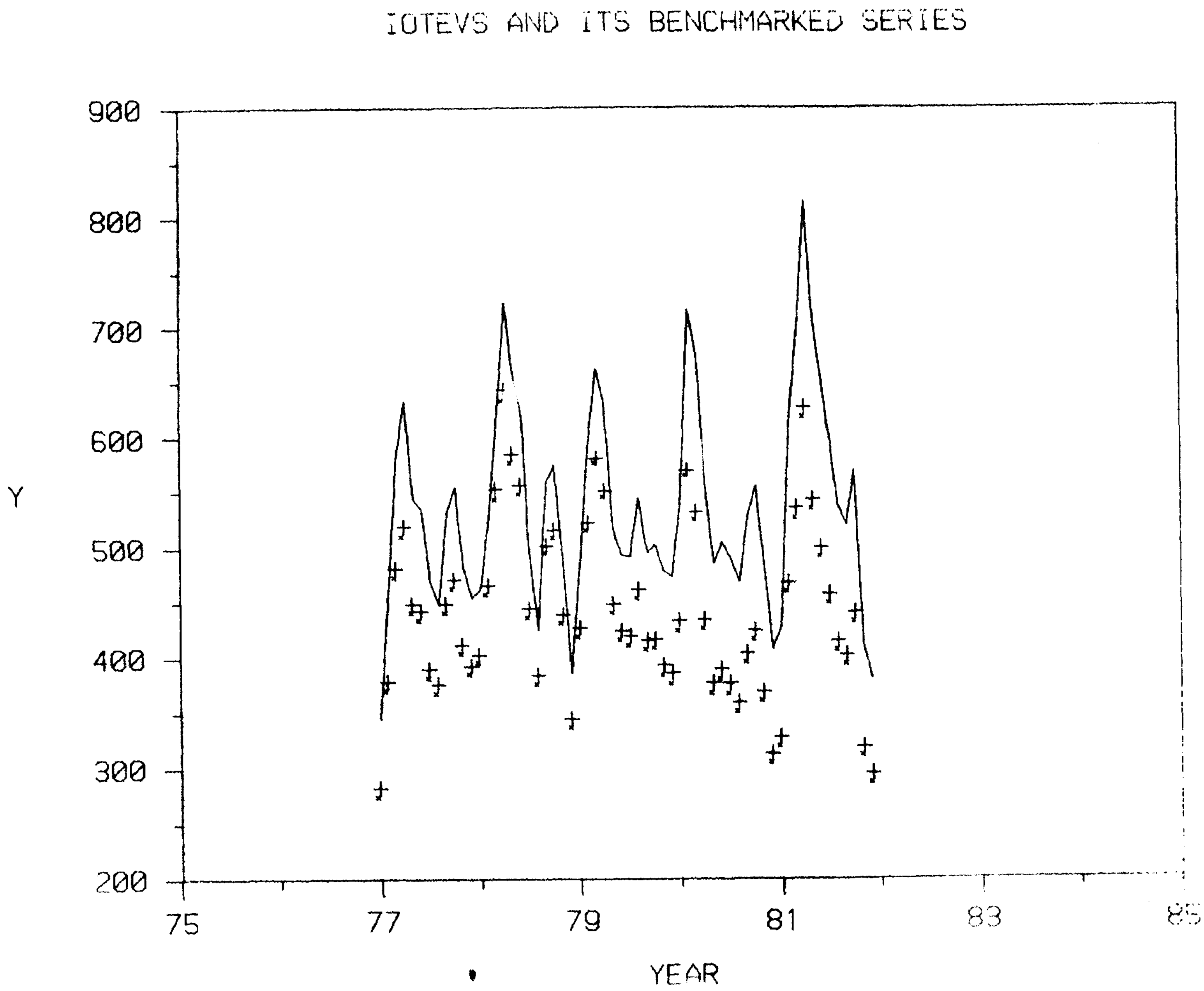

IOTEVS AND ITS BENCHMARKED SERIES
900
800
700
600
500
400
300
200
Y
75
77
79
81
83
85
YEAR

Figure 7.

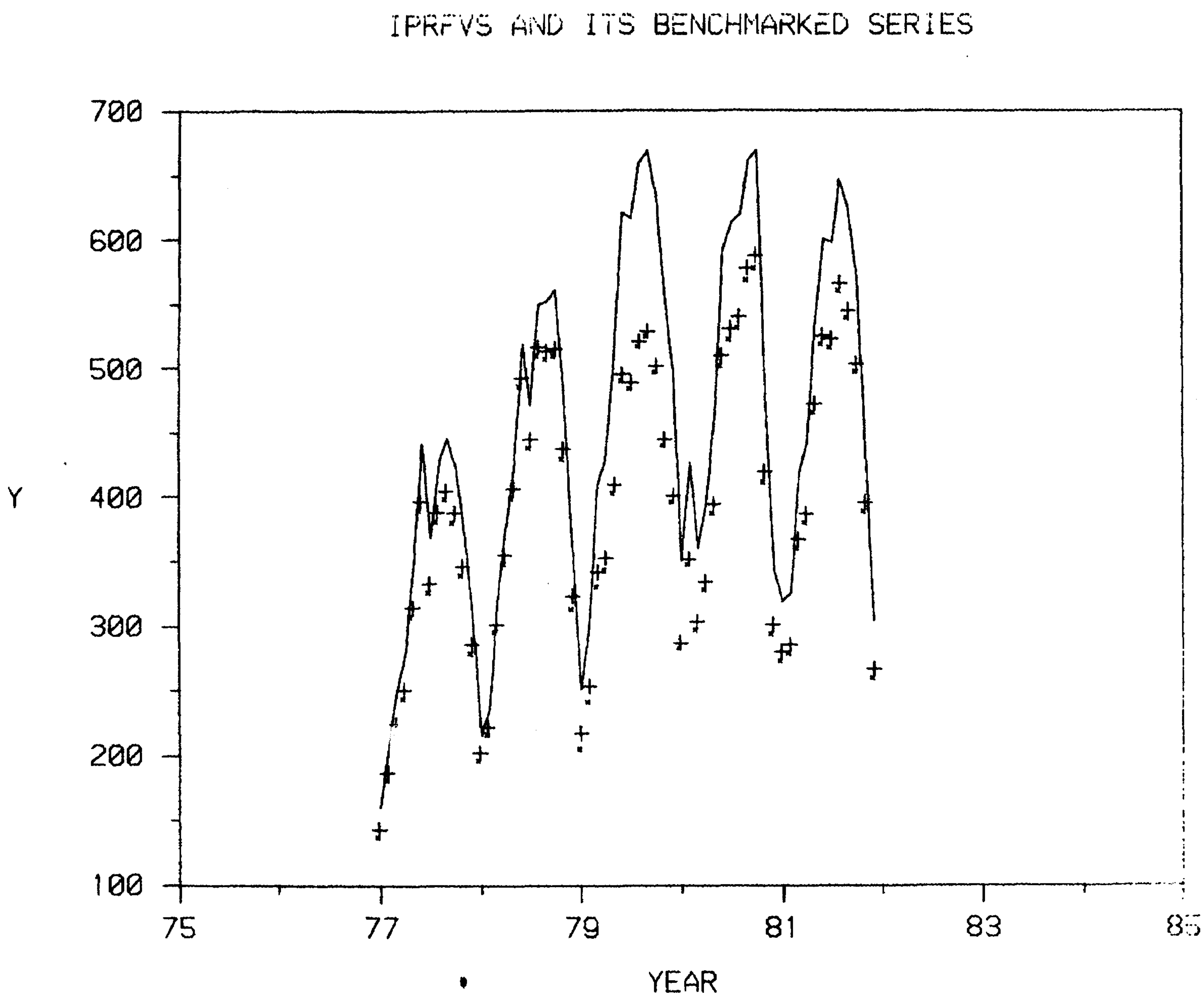
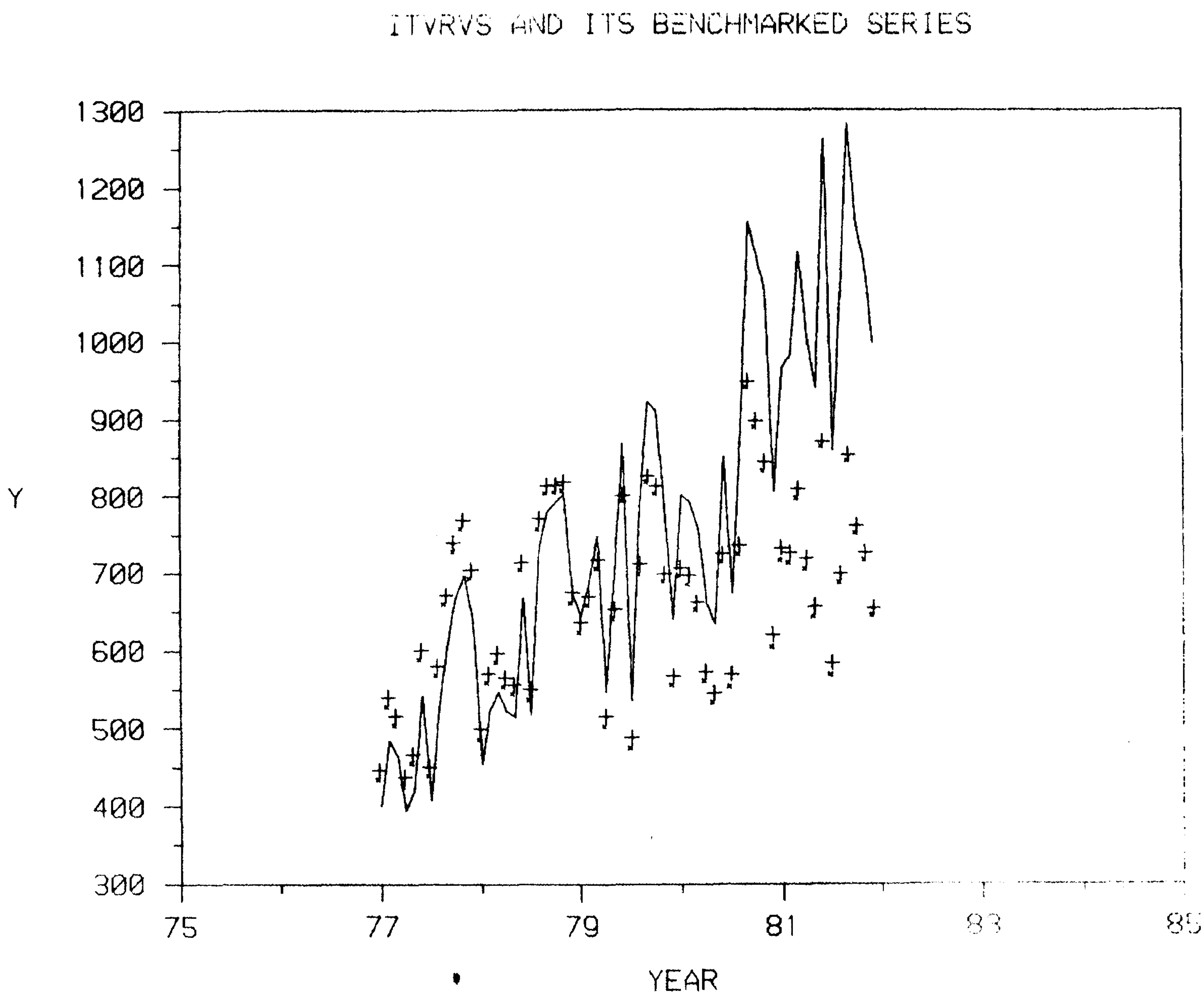

IPRFVS AND ITS BENCHMARKED SERIES
Y
700
600
500
400
300
200
100
75
77
79
81
83
85
YEAR

Figure 8.

Figure 9.


ITVRVS AND ITS BENCHMARKED SERIES
1300
1200
1100
1000
900
800
700
600
500
400
300
Y
75
77
79
81
83
85
YEAR

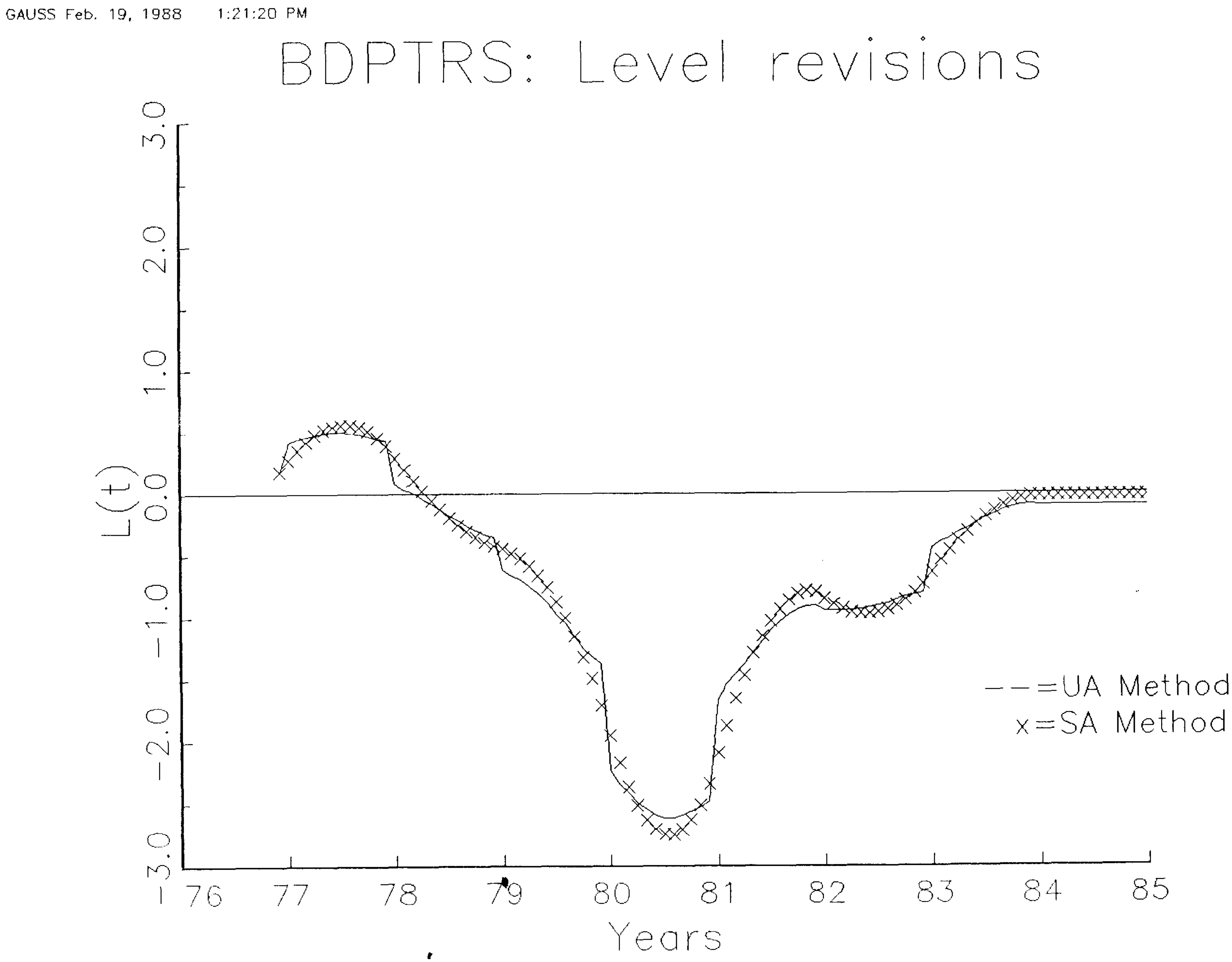
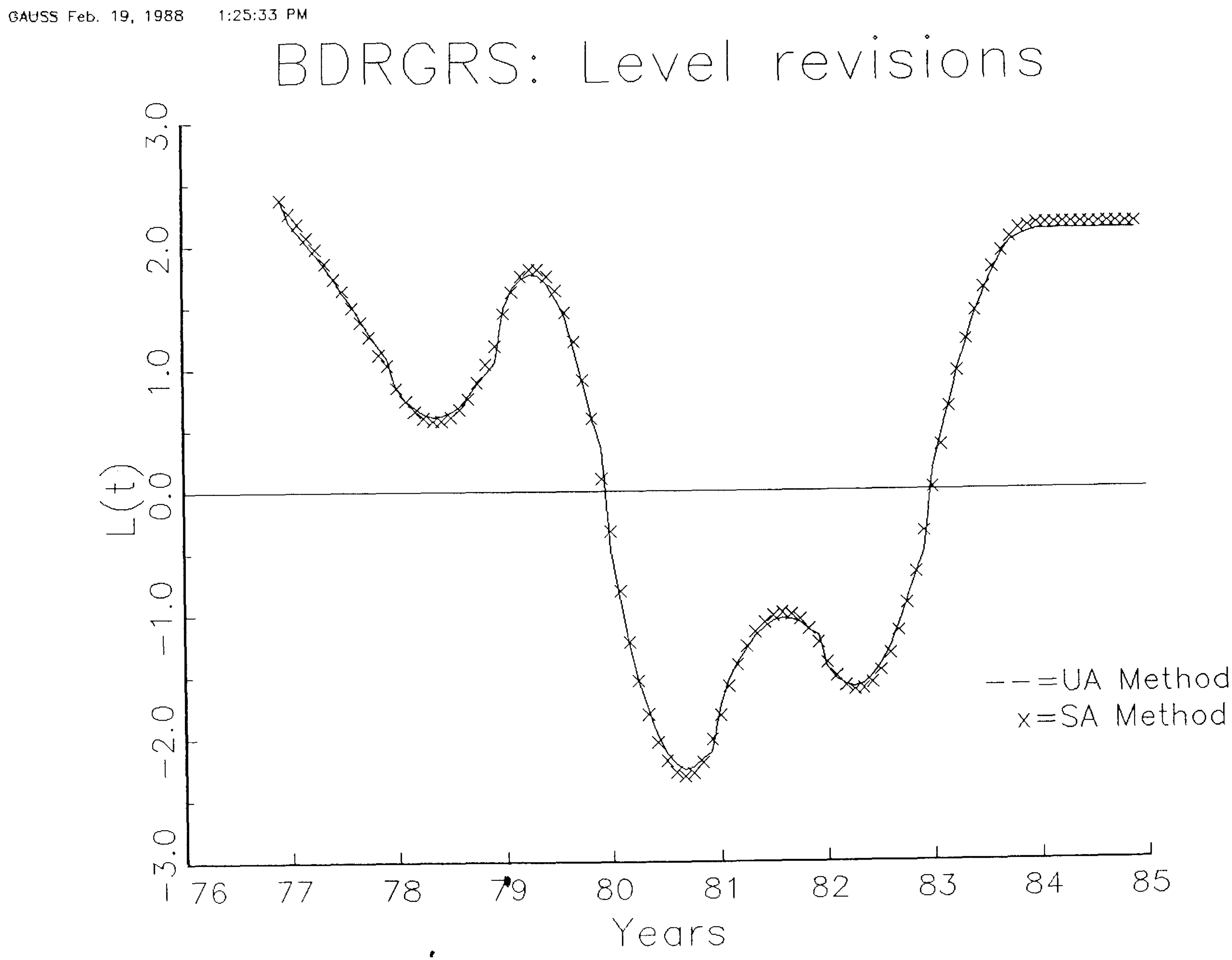
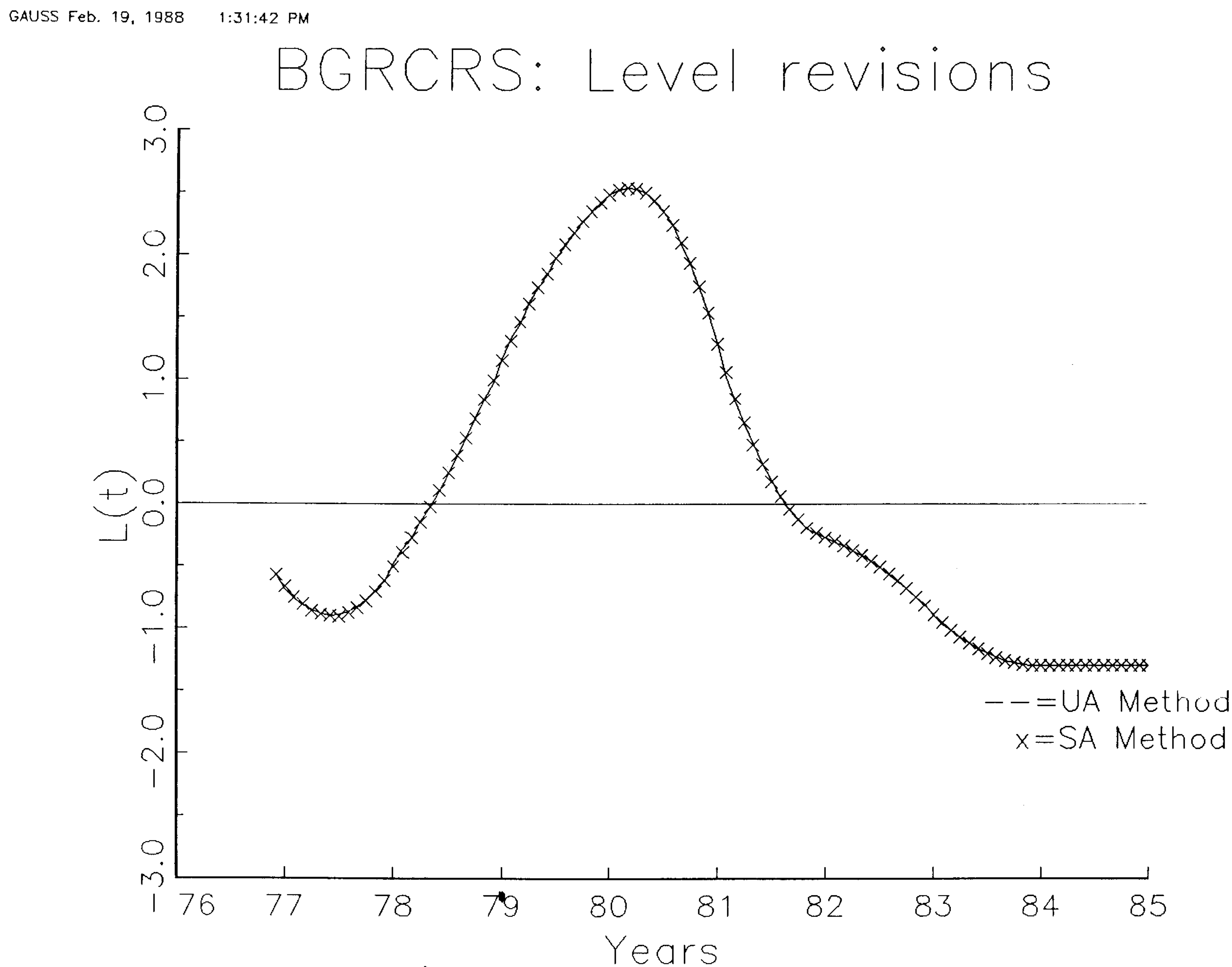

GAUSS Feb. 19, 1988 1:21:20 PM
BDPTRS: Level revisions
L(t)
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
76
77
78
79
80
81
82
83
84
85
Years
--=UA Method
x=SA Method

Figure 10.


GAUSS Feb. 19, 1988 1:25:33 PM
BDRGRS: Level revisions
L(t)
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
76
77
78
79
80
81
82
83
84
85
Years
- - =UA Method
x=SA Method
Figure 11.



GAUSS Feb. 19, 1988 1:31:42 PM
BGRCRS: Level revisions
L(t)
3.0
2.0
1.0
0.0
-1.0
-2.0
-3.0
76
77
78
79
80
81
82
83
84
85
Years
--=UA Method
x=SA Method

Figure 12.

GAUSS Feb. 19, 1988 2:11:50 PM

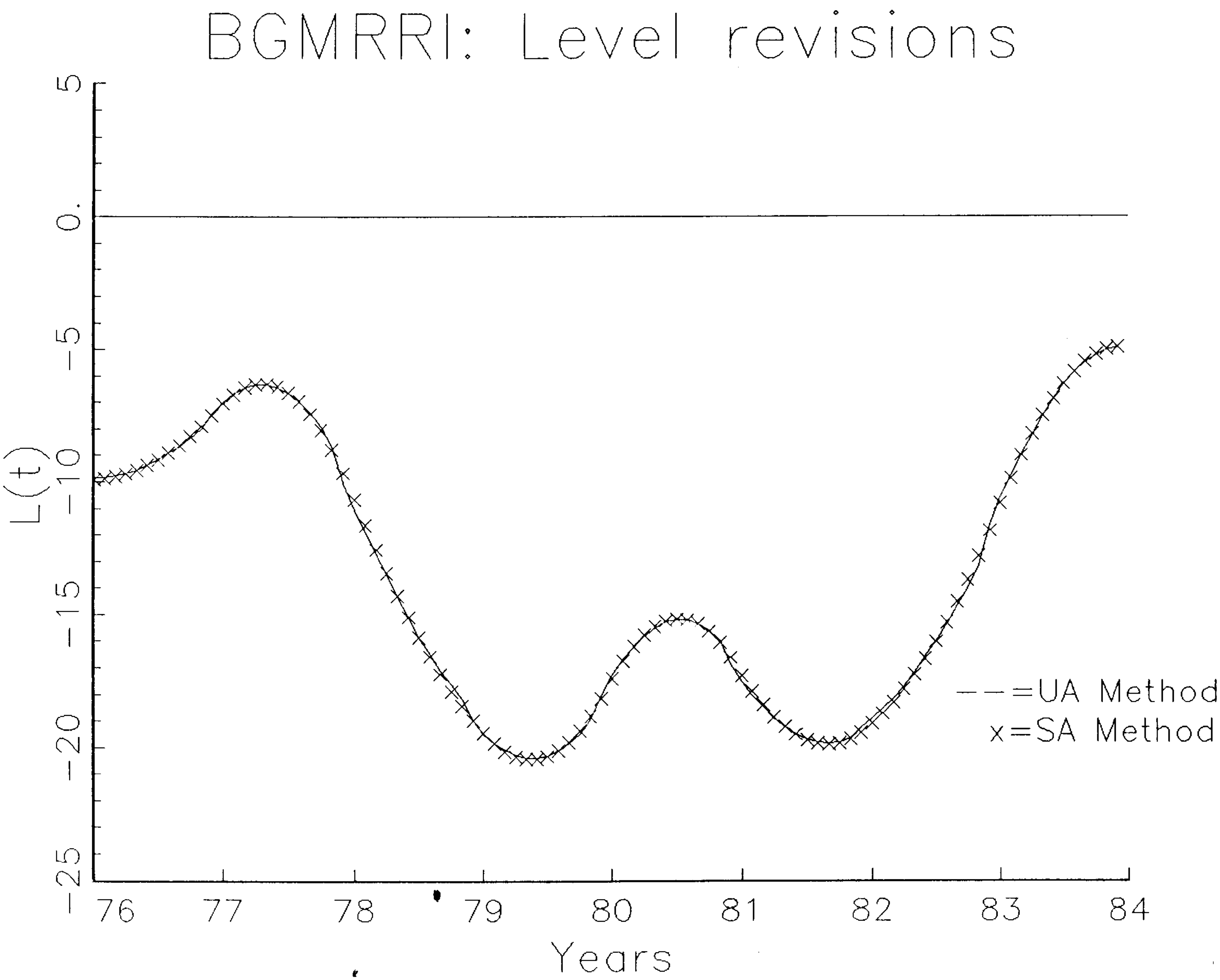


Figure 13.

GAUSS Feb. 19, 1988 3:06:24 PM

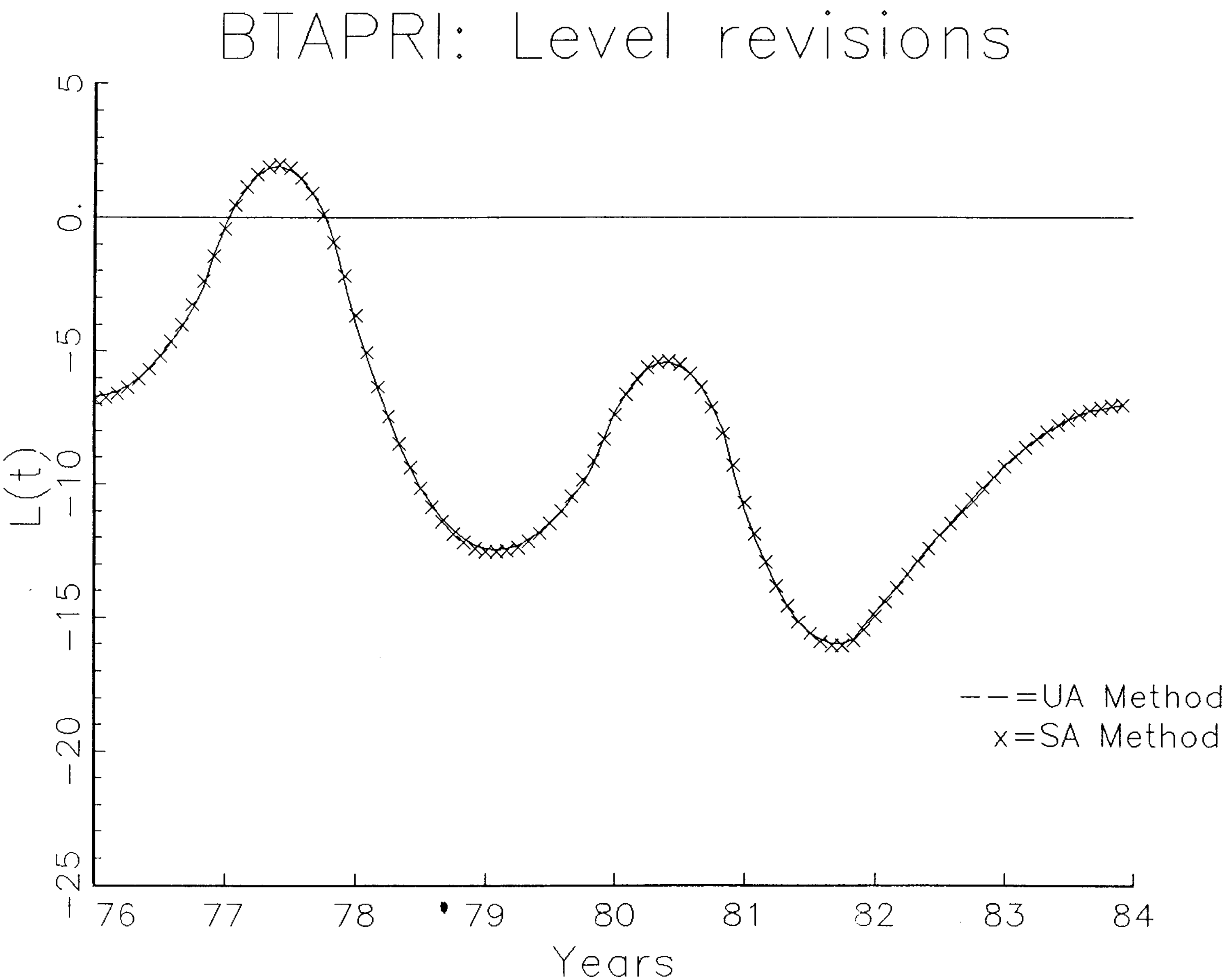


Figure 14.

GAUSS Feb. 19, 1988 3:10:07 PM

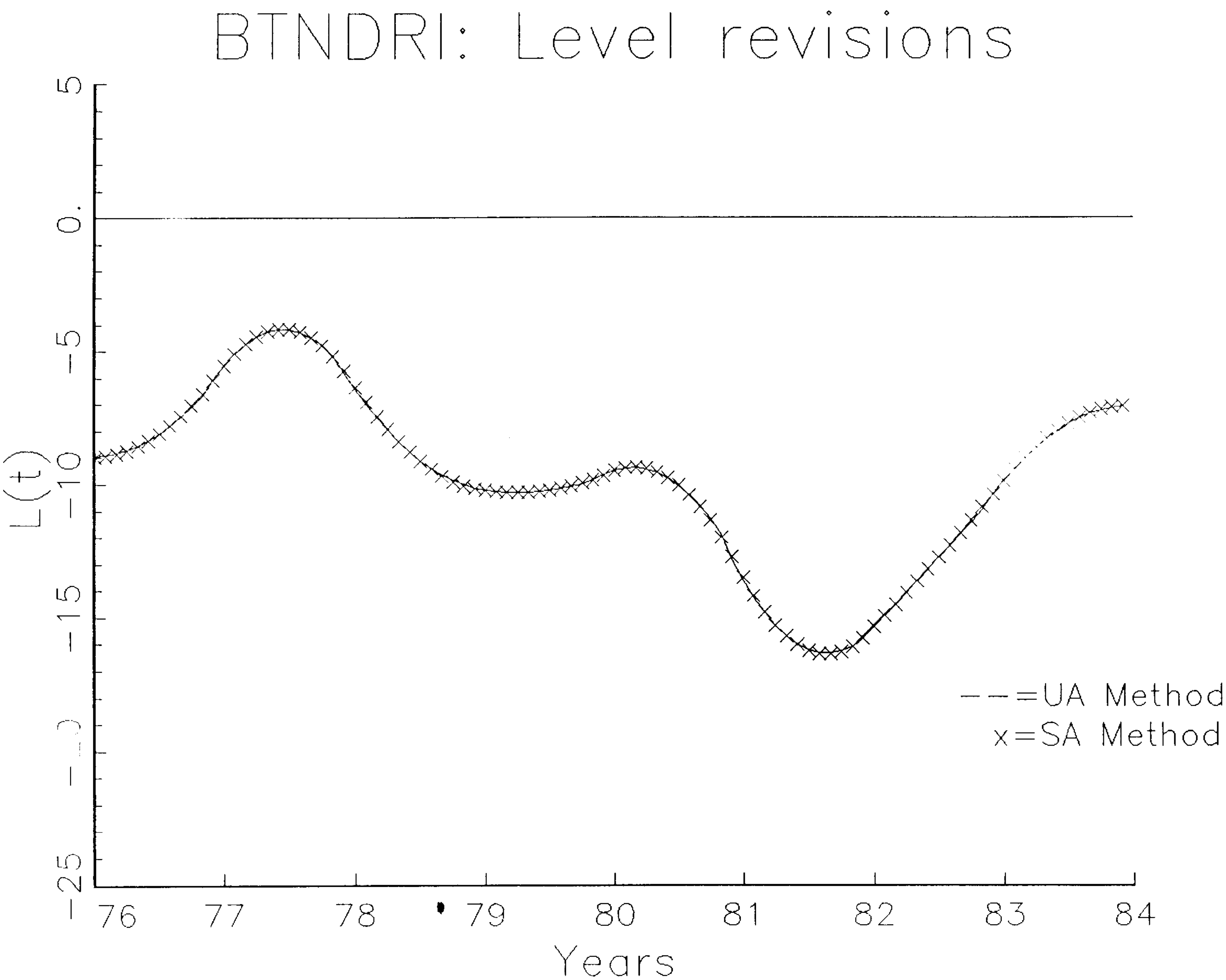


Figure 15.

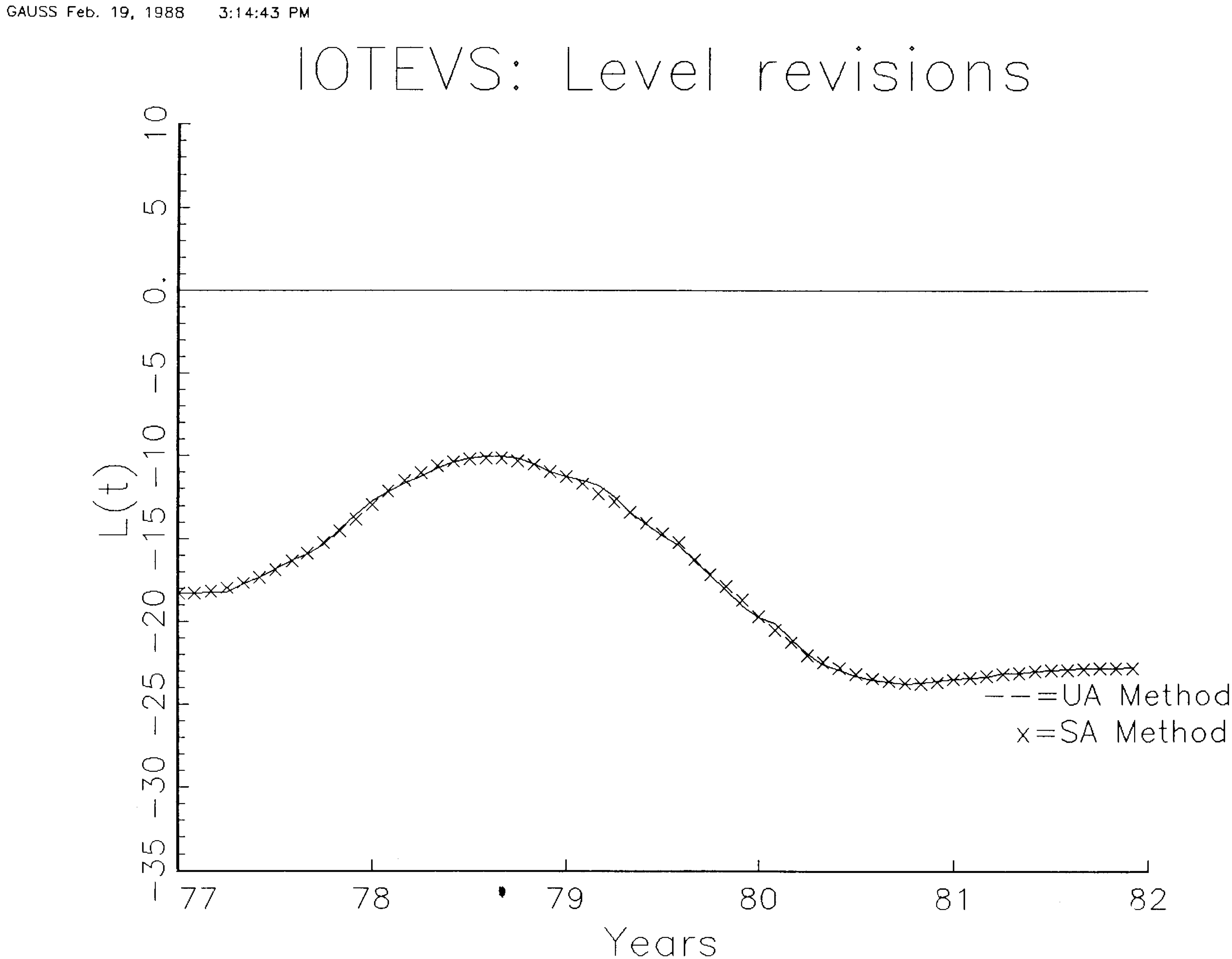

GAUSS Feb. 19, 1988 3:14:43 PM
IOTEVS: Level revisions
L(t)
10
5
0.
−5
−10
−15
−20
−25
−30
−35
77
78
79
80
81
82
Years
− − =UA Method
x=SA Method

Figure 16.

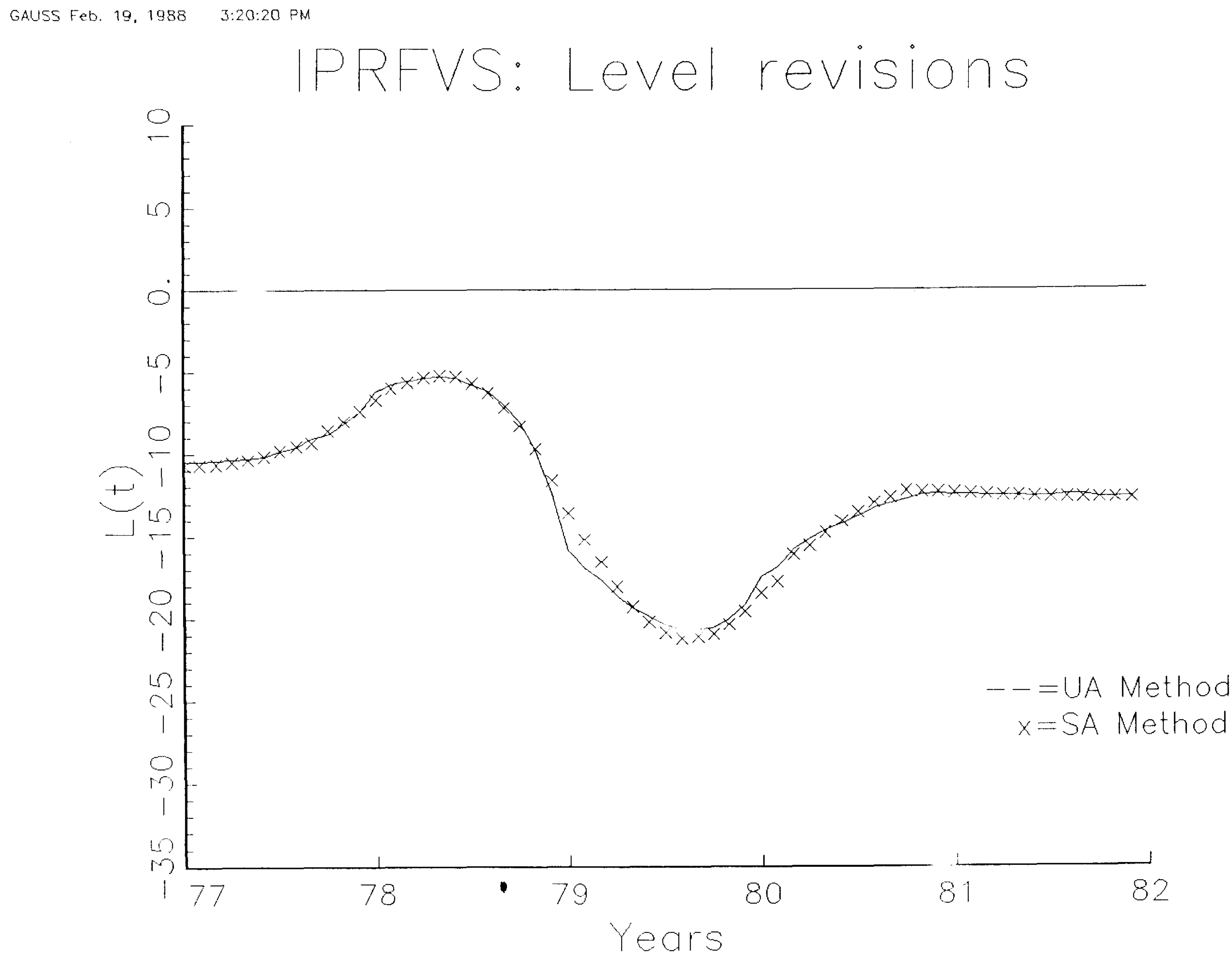

GAUSS Feb. 19, 1988 3:20:20 PM
IPRFVS: Level revisions
L(t)
10
5
0.
-5
-10
-15
-20
-25
-30
-35
77
78
79
80
81
82
Years
– – =UA Method
x=SA Method

Figure 17.

GAUSS Feb. 19, 1988 3:24:31 PM

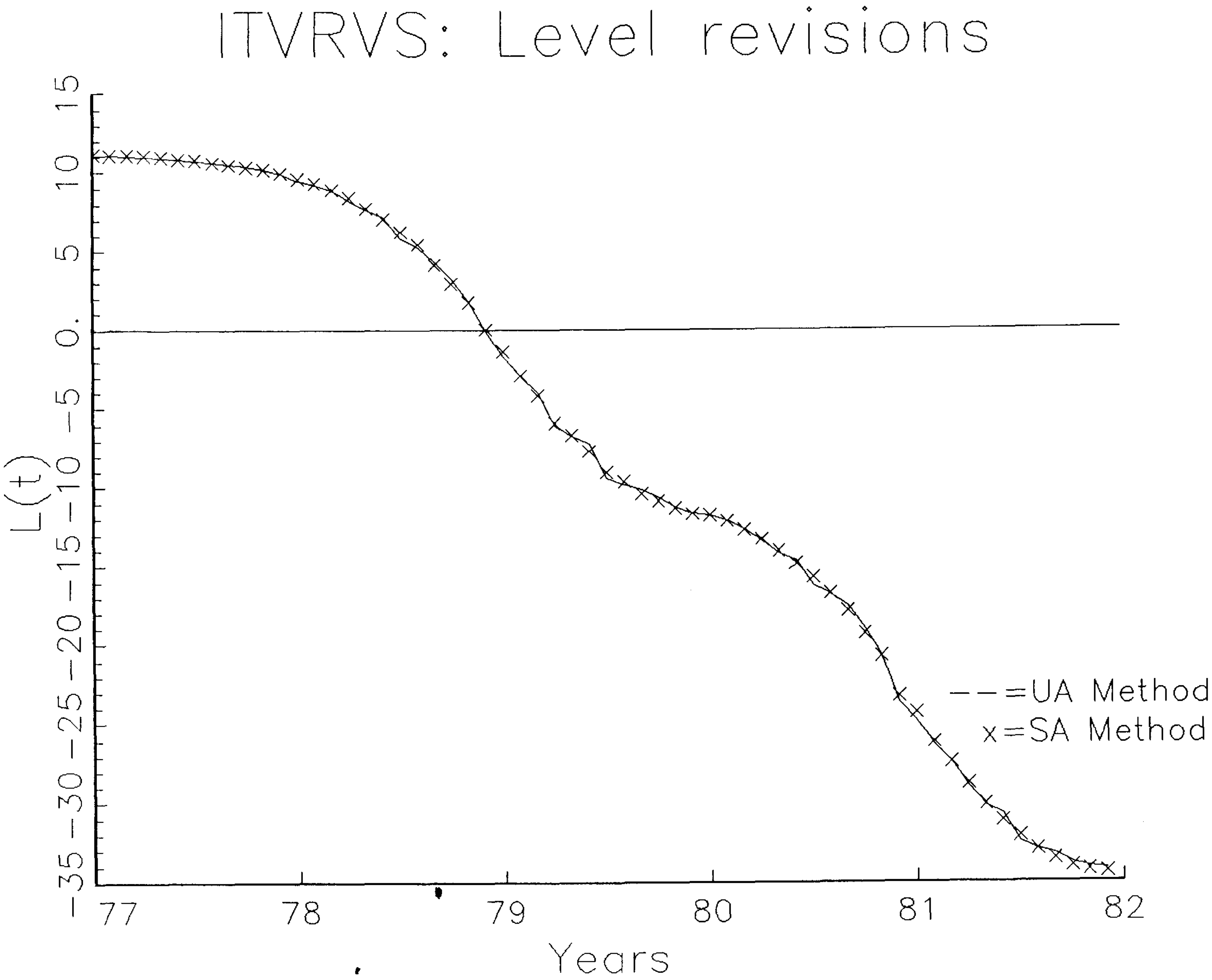


Figure 18.

BDPTRS % REVISION FOR BOTH METHODS

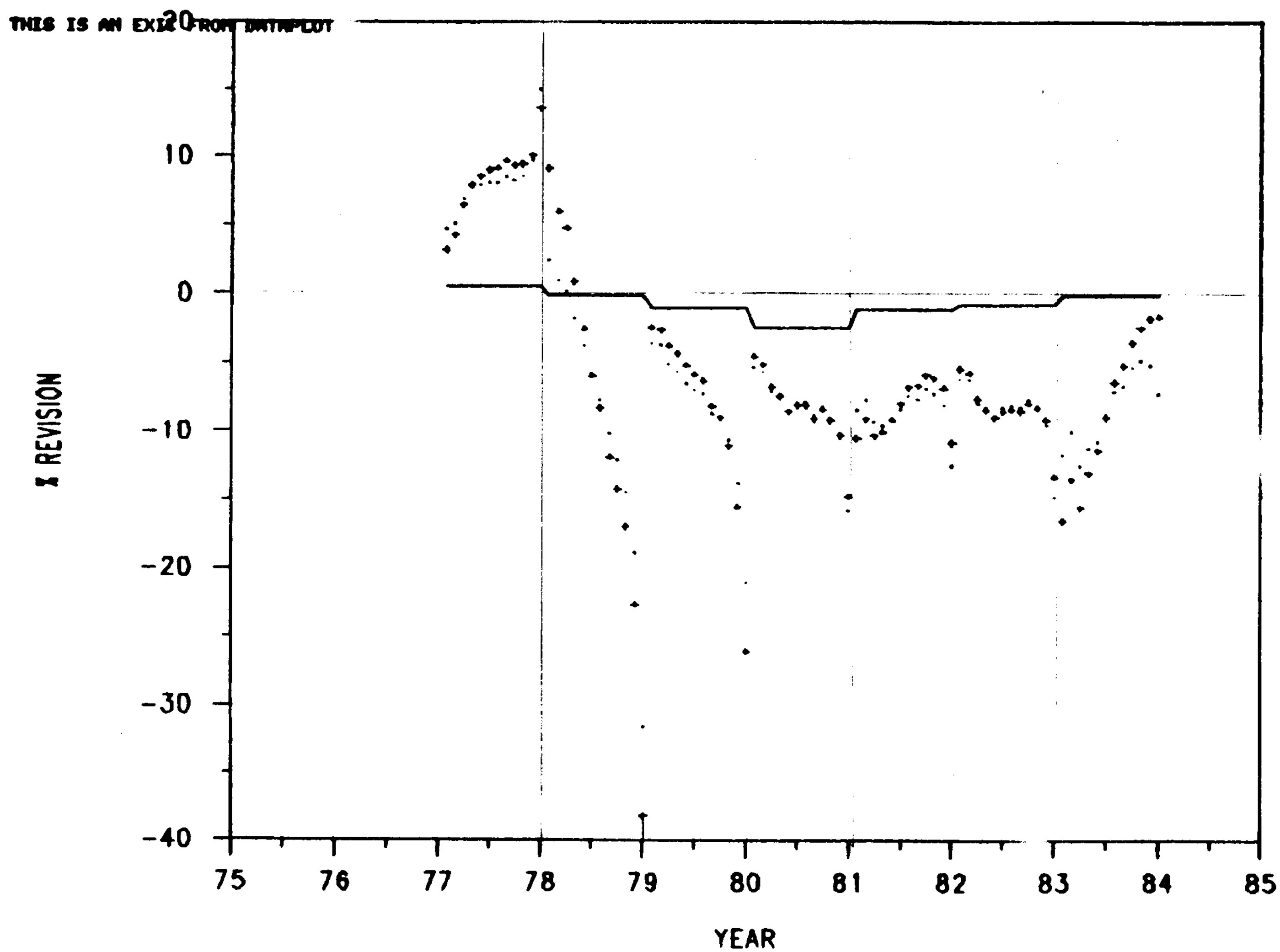


Figure 19.

BDRGRS % REVISION FOR BOTH METHODS

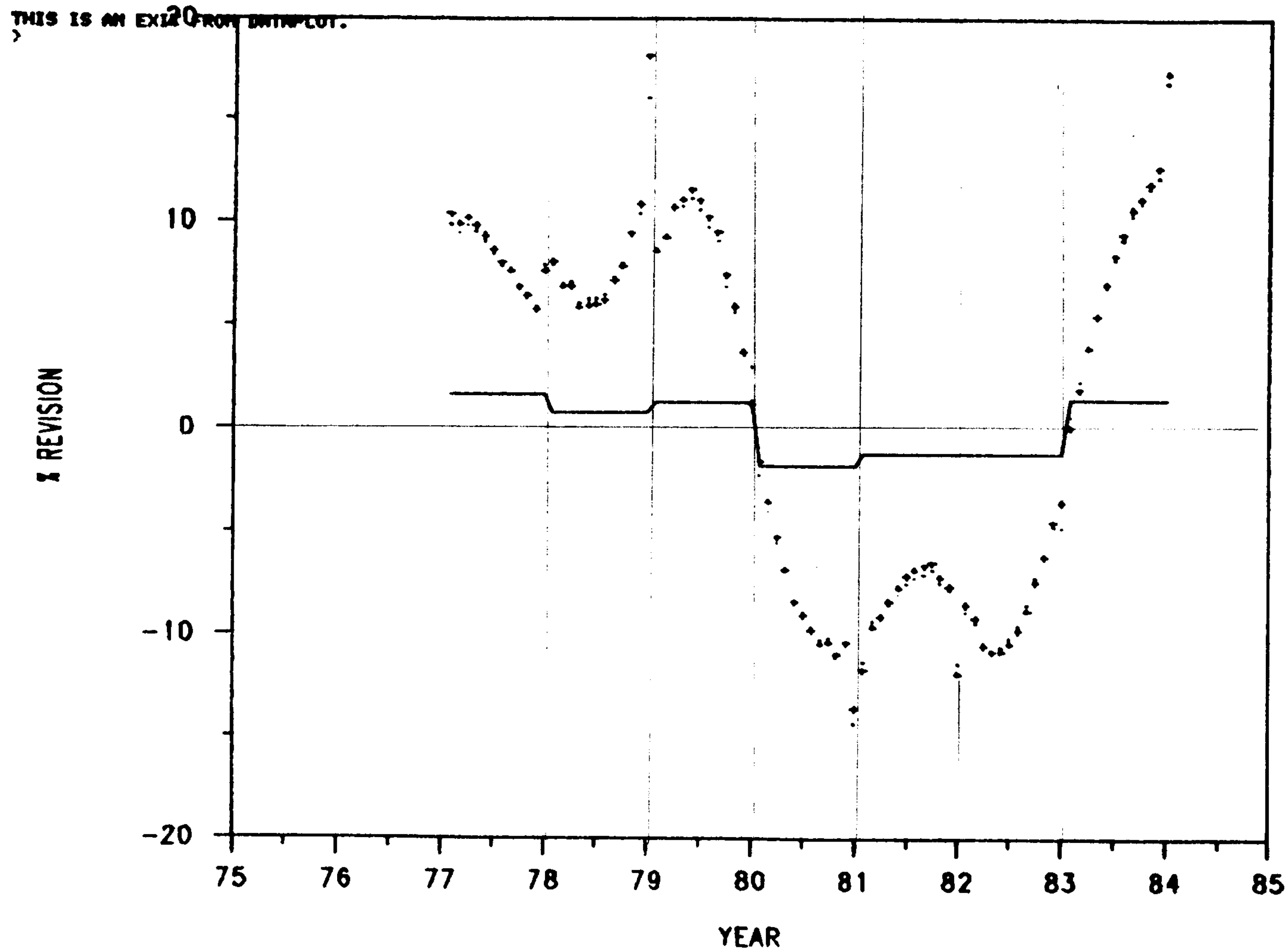


Figure 20.

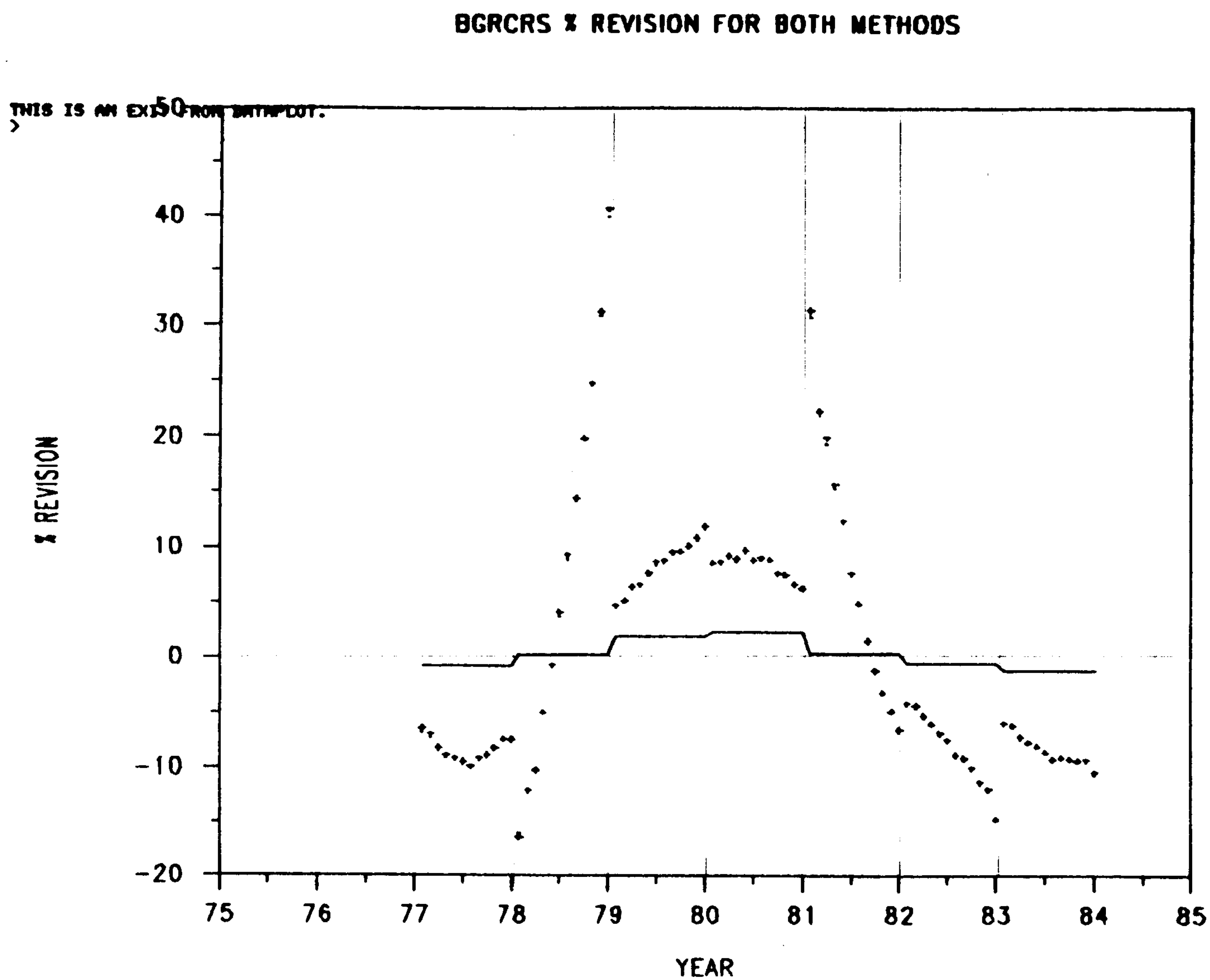

BGRCRS % REVISION FOR BOTH METHODS
THIS IS AN EXIT FROM DATAPLOT.
% REVISION
50
40
30
20
10
0
-10
-20
75
76
77
78
79
80
81
82
83
84
85
YEAR

Figure 21.

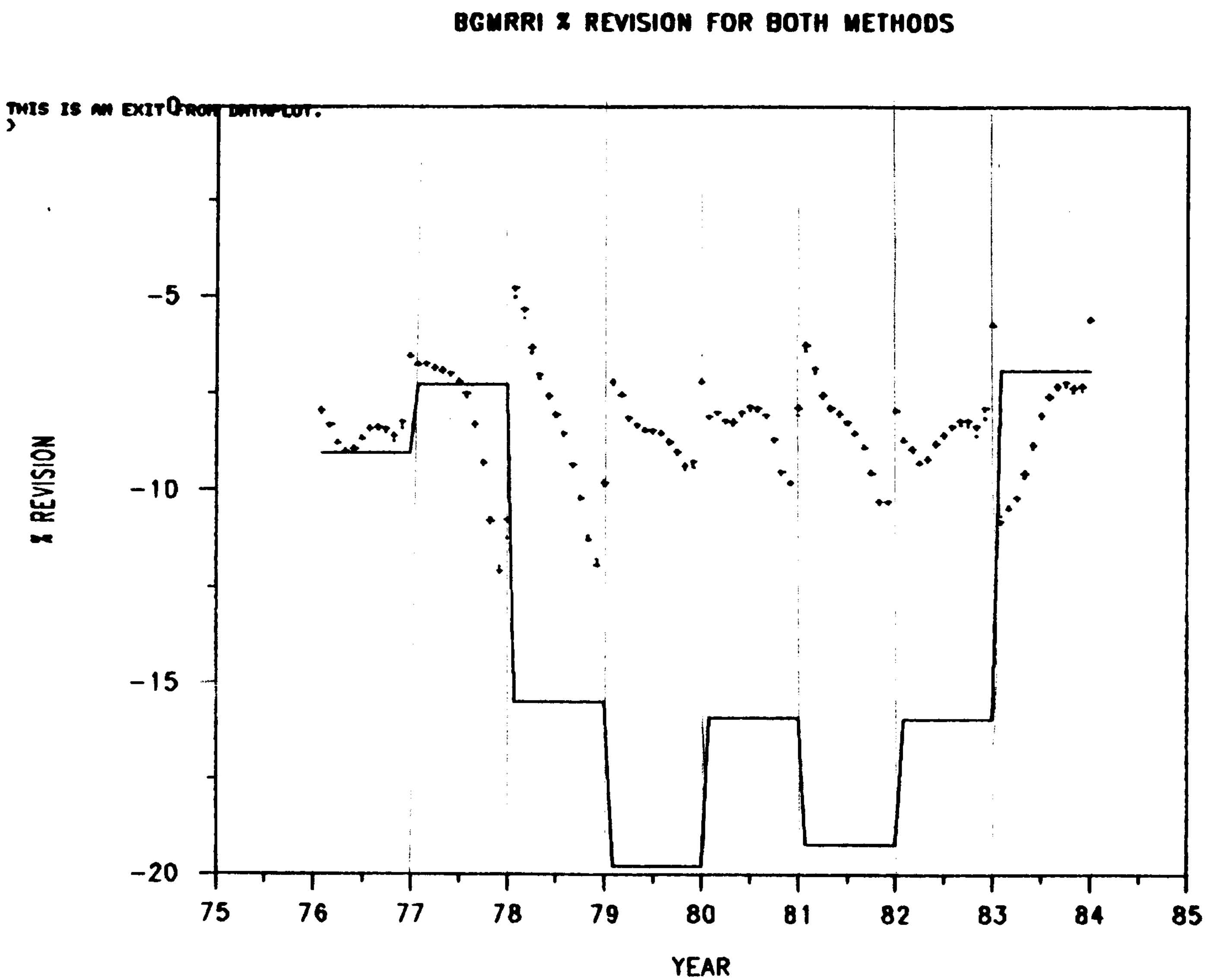

BGMRRI % REVISION FOR BOTH METHODS
THIS IS AN EXIT FROM DATAPLOT.
% REVISION
0
-5
-10
-15
-20
75
76
77
78
79
80
81
82
83
84
85
YEAR

Figure 22.

BTAPRI % REVISION FOR BOTH METHODS

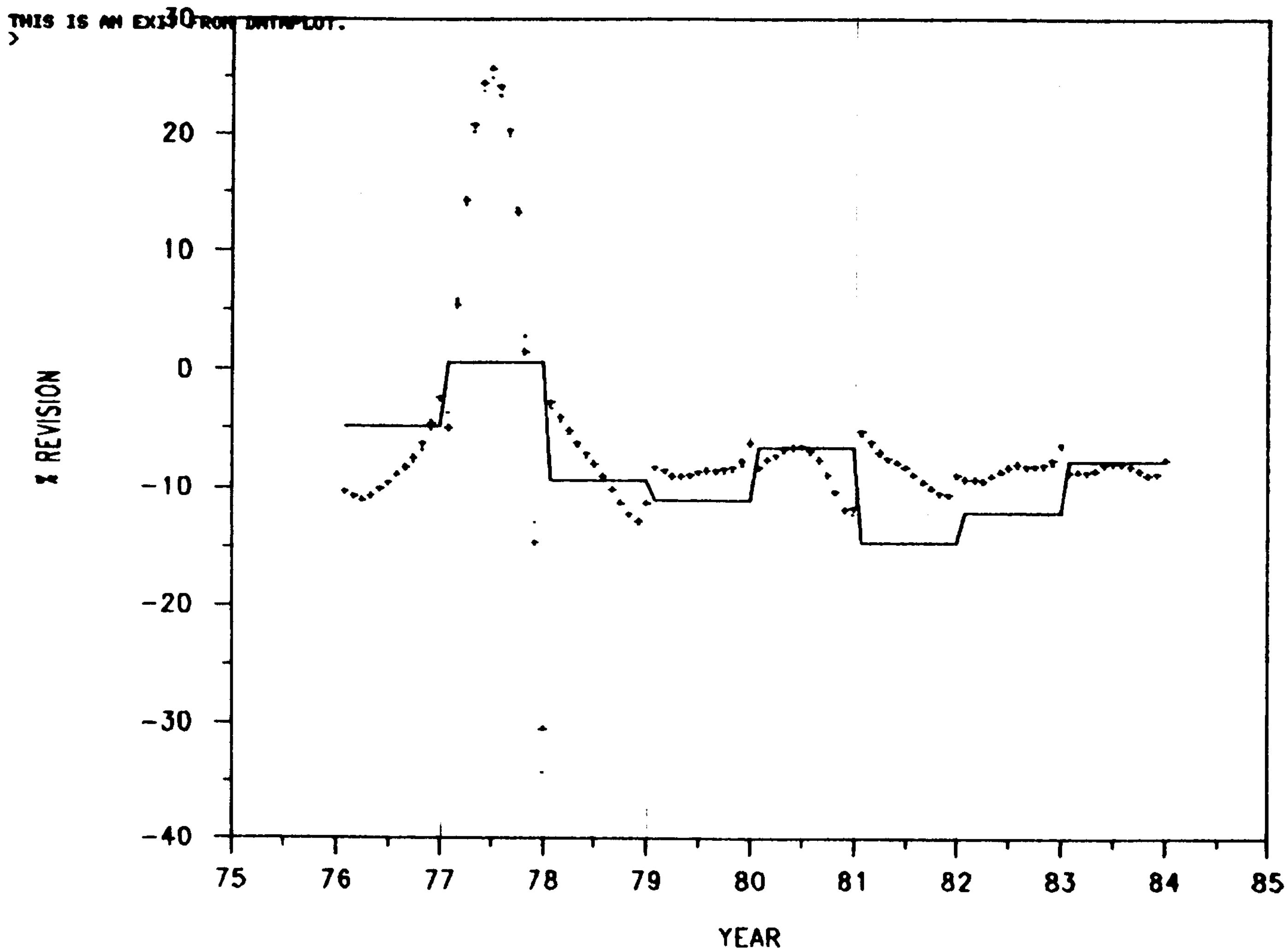


Figure 23.

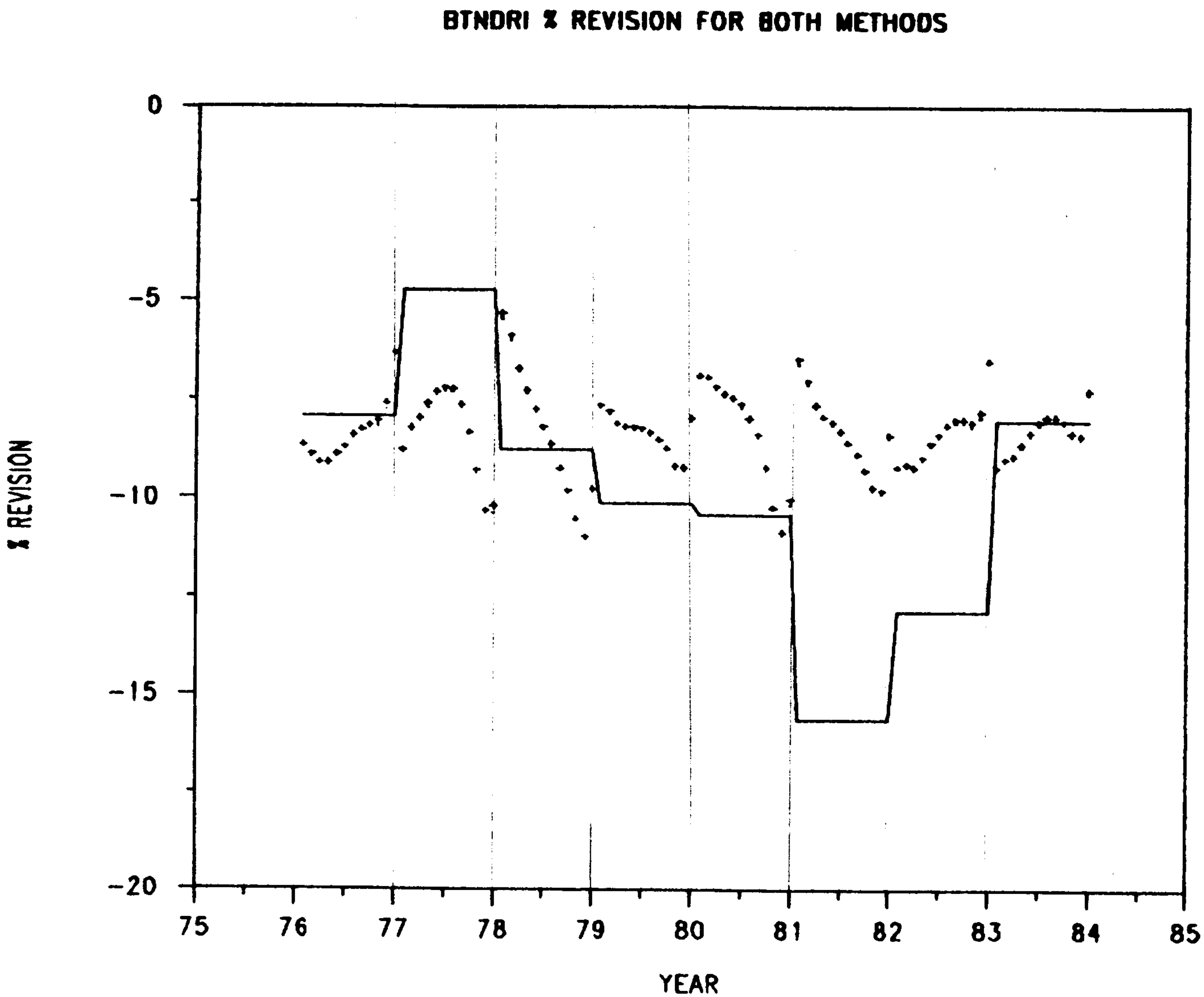

BTNDRI % REVISION FOR BOTH METHODS
% REVISION
0
-5
-10
-15
-20
75
76
77
78
79
80
81
82
83
84
85
YEAR

Figure 24.

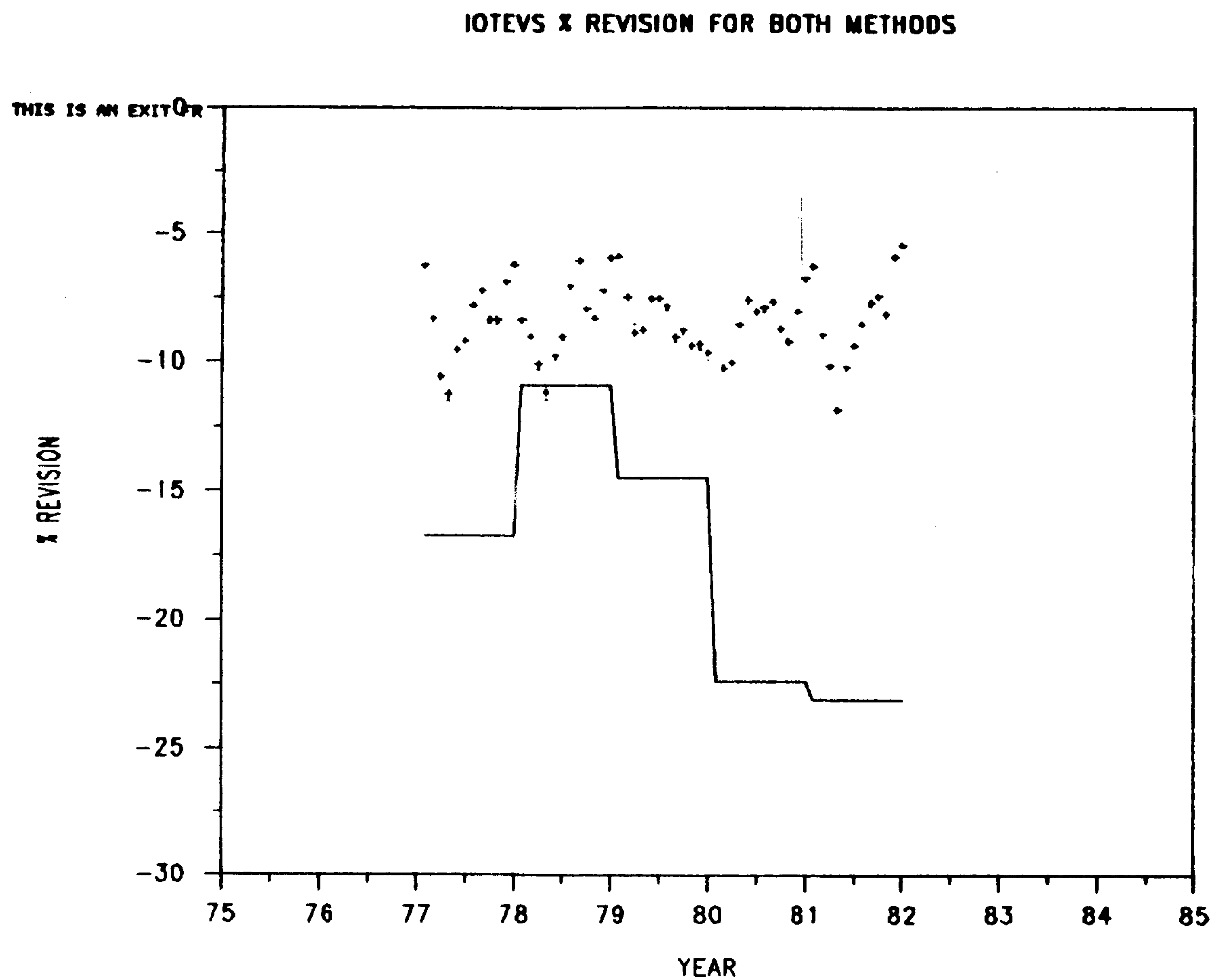

IOTEVS % REVISION FOR BOTH METHODS
THIS IS AN EXIT FR
% REVISION
0
-5
-10
-15
-20
-25
-30
75
76
77
78
79
80
81
82
83
84
85
YEAR

Figure 25.

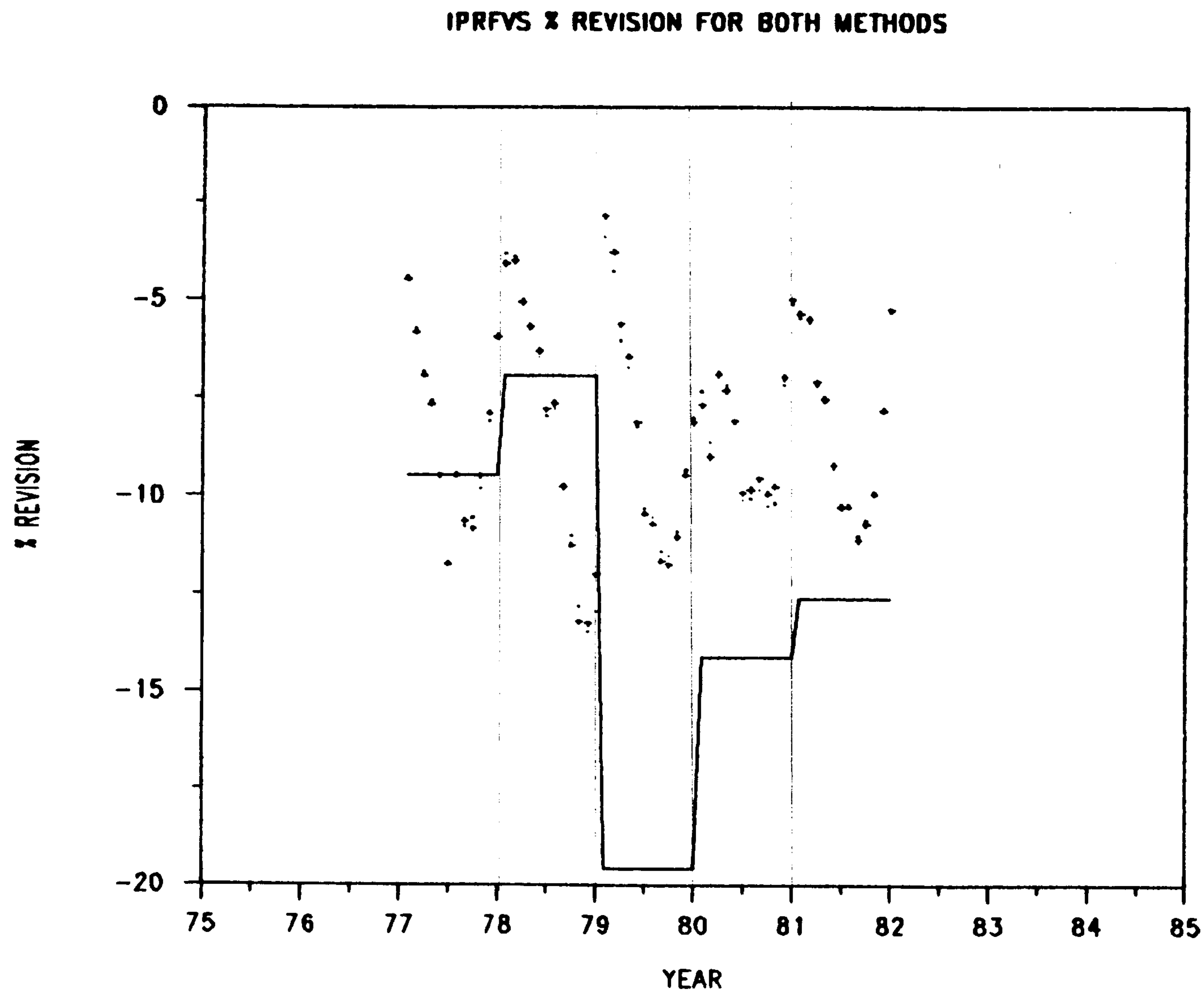

IPRFVS % REVISION FOR BOTH METHODS
% REVISION
0
-5
-10
-15
-20
75
76
77
78
79
80
81
82
83
84
85
YEAR

Figure 26.

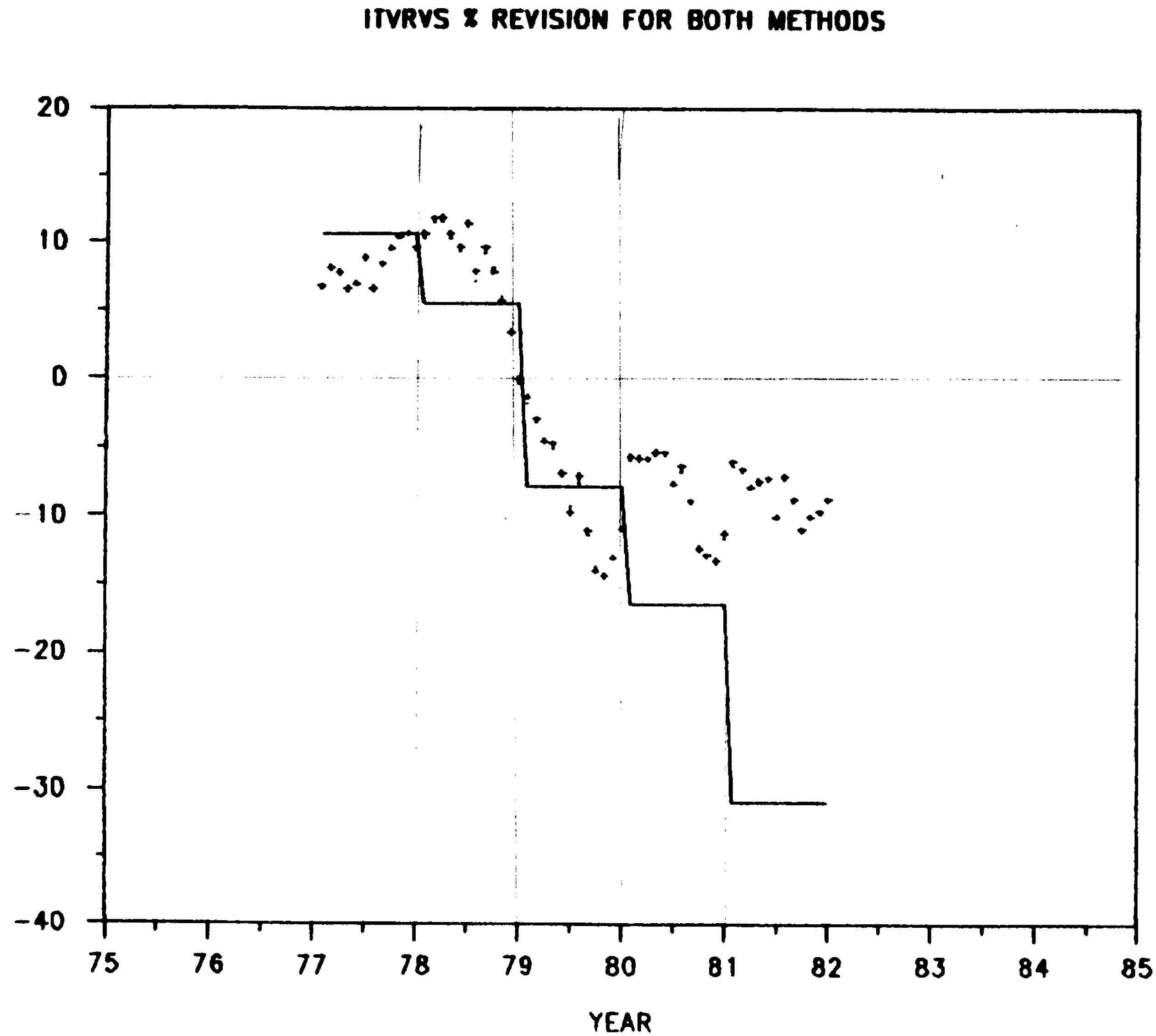

ITVRVS % REVISION FOR BOTH METHODS
% REVISION
20
10
0
-10
-20
-30
-40
75
76
77
78
79
80
81
82
83
84
85
YEAR

Figure 27.