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SRD Research Report Number: CENSUS/SRD/RR-88/05<br>DOCUMENTATION OF THE REDESIGN OF THE QUARTERLY PROPERTY TAX SURVEY<br>by<br>Carma R. Hogue<br>Statistical Research Division<br>Bureau of the Census<br>Room 3130, F.O.B. \#4<br>Washington, D.C. 20233 U.S.A.

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> Documentation of the Redesign of the Quarterly Property Tax Survey

## 1. Introduction

The Quarterly Property Tax Survey was started in the mid-1970's and was designed to obtain a national estimate of property tax revenue for the Quarterly Summary of Federal, State, and Local Tax Revenue. Most of the data for the report are gathered from State sources, but property tax collections must be obtained from local tax collecting agencies. The sampling unit for this survey is the county-area. County-areas are sometimes serviced by one tax-codlecting agency, but most county-areas contain several local agencies. When a county-area is selected for inclusion in the sample, all agencies within the county-area are included in the sample. The current sample contains 530 county-areas from a universe of 3132 county-areas as identified in the 1982 Census of Governments. These 530 county-areas contain approximately 7000 local agencies.

The sample design is described in section 2 . The estimation is discussed in section 3, and the variance estimation is discussed in section 4 . Considerations for future redesigns are given in section 5.

## 2. Sample Design

This sample is a stratified random sample of county-areas. All countyareas with a 1980 population of at least 200,000 and a 1982 property tax value of over $\$ 60$ million were included in the sample with certainty. Certainty units accounted for 71 percent of the total 1982 property tax revenue. Eight other strata based on population were used for the selection of noncertainty units. The sample design is expected to yield a coefficient of variation of
.01 on a national total of tax revenue. The sample was allocated to the strata based on 1982 property tax revenue. A total of 530 county-areas was selected from the universe of 3132 county-areas. Table 1 summarizes the sample allocation. Of the 257 certainty units, 42 had populations under 200,000 but property tax revenues of more than $\$ 60$ million.

Table 1. Sample Allocation

| Stratum | Definition (Population) | $\mathrm{N}_{\mathrm{h}}$ | ${ }^{n} h$ | Random Start | Take Every |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Certainty | 257 | 257 | 1.0000 | 1.0000 |
| 2 | 100,000-199,999 | 158 | 42 | 3.5909 | 3.7619 |
| - 3 | 70,000-99,999 | 167 | 37 | 3.3855 | 4.5135 |
| 4 | 50,000-69,999 | 208 | 32 | 1.7736 | 6.5000 |
| 5 | 40,000-49,999 | 165 | 20 | 6.8562 | 8.2500 |
| 6 | 30,000-39,999 | 285 | 29 | 2.6816 | 9.8276 |
| 7 | 20,000-29,999 | 400 | 36 | 10.6060 | 11.1111 |
| 8 | 10,000-19,999 | 741 | 49 | 14.4350 | 15.1224 |
| 9 | < 10,000 | 751 | $28$ | 16.7660 | 26.8214 |
|  |  | 3132 | 530 |  |  |

Within each stratum, the file was ordered by decreasing property tax revenue; a systematic sample was drawn from each stratum using the random starts and take rates given in Table 1.

## 3. Estimation

Each quarter, property tax totals are published for several certainty counties for 12 months ending with the current quarter and for 12 months ending the same quarter a year ago. The data are not subject to sampling error. National totals of local property tax revenue are given for each quarter during the last five years and for 12 months ending with each quarter for the past five years. These estimates are subject to sampling error. Text statements make comparisons between quarters and between totals for 12 months ending with certain quarters. Section 3.1 will give equations for the
quarterly estimates; section 3.2 will contain estimators for 12 months ending with the current quarter; section 3.3 will contain estimates of change between quarters; and section 3.4 will contain estimators of change between totals for 12 months ending with the current quarter. Section 4.1-4.4 will contain variance estimates for the estimates given in sections 3.1-3.4.

### 3.1 Quarterly Estimates of Total

A ratio estimator to 1982 tax revenue is used for estimating quarterly totals. Equation (1) gives the ratio estimator and equation (la) gives the simple unbiased estimate.

$$
\begin{equation*}
x_{a}^{\prime \prime}=x_{a c}+\sum_{h=1}^{8} r_{a h} Y_{h} \tag{1}
\end{equation*}
$$

$$
\begin{equation*}
x_{y_{a}^{\prime}}^{\prime}=x_{a c}+\sum_{h=1}^{8} \frac{N_{h}}{n_{h}} \sum_{i=1}^{n_{h}} x_{a h i} \tag{1a}
\end{equation*}
$$

where $X_{a c}$ is the total property tax revenue for the certainty counties $r_{a h}=\frac{x_{a h}^{\prime}}{y_{h}^{\prime}}=\left(\frac{N_{h}}{n_{h}} \sum_{i=1}^{n_{h}} x_{a h i}\right) /\left(\frac{n_{h}}{n_{h}} \sum_{i=1}^{n_{h}} y_{h i}\right)=\sum_{i=1}^{n_{h}} x_{a h i} / \sum_{i=1}^{n_{h}} y_{h i}$
$\mathrm{x}_{\mathrm{ah}}$ is the property tax collection for noncertainty county area i
in stratum h for quarter a ;
$Y_{\text {hi }}$ is the 1982 census tax revenue for county area $i$ in stratum
$h$;
$Y_{h}$ is the 1982 census tax revenue for all county areas in stratum $h$;
$n_{h}$ is the sample number of county areas in stratum $h$; and
$N_{h}$ is the total number of county areas in stratum $h$.

### 3.2 Total for Twelve Months Ending with Current Quarter

The ratio estimate of total for the 12 months ending with the current quarter should be calculated as in equation (2). The unbiased is calculated as in equation (2a).

$$
\begin{align*}
& x^{\prime \prime}=\sum_{a=1}^{4} x_{a}^{\prime \prime}  \tag{2}\\
& x_{U}^{\prime \prime}=\sum_{a=1}^{4} x_{U a}^{\prime \prime} \tag{2a}
\end{align*}
$$

where $x_{a}^{\prime \prime}$ is calculated as in equation (1) with the sum being over the four quarters of interest.

- XUá is similarly defined in equation (la).


### 3.3 Changes Between Quarters

If changes between the total tax revenue for this quarter and for the same quarter a year ago are desired, equation (3) should be used to estimate the percentage change.

$$
\begin{equation*}
\hat{D}_{a}=100\left(x_{a}^{\prime \prime}-z_{a}^{\prime \prime}\right) / z_{a}^{\prime \prime} \tag{3}
\end{equation*}
$$

where $x_{a}^{\prime \prime}$ is calculated for the current quarter using equation (1), and $z_{a}^{\prime \prime}$ ' is the calculated total for the same quarter 12 months ago.

### 3.4 Changes in 12-Month Totals Between Quarters

Let $x^{\prime \prime}$ be an estimate of total revenue for the 12 months ending with the current quarter. Let $z^{\prime \prime}$ be the same estimate for 12 months ending 4 quarters ago. The estimate of change (in percent) between these totals is calculated as follows:
$\bar{D}=\left(\frac{x^{\prime \prime}-z^{\prime \prime}}{z^{\prime \prime}}\right) 100$

## 4. Variance Estimation

The following sections give variance estimates for the estimators given in section 3. All calculations should be done in double precision.

Estimates of variances of quarterly totals should be calculated quarterly, if possible, from the beginning of the survey. Estimates of annual aggregates cannot be calculated until the new sample has been in for 4 quarters. (This is because variance estimates were not calculated in the old sample.) Estimates of change between quarters cannot be done until 5 quarters of data from the new sample are available. (These are the estimates that will be used in testing statements of increases or decreases in property tax revenue.) Finally, eight quarters of data are needed to estimate variances of changes between annual aggregates.

Printouts should contain ratio and simple unbiased estimates and their estimated variances, standard errors, and coefficients of variation.

### 4.1 Estimates of Quarterly Total

An estimate of the variance of the estimate given in equation (1) is given in equation (5).

$$
\begin{equation*}
s_{x_{a}^{\prime}}^{2}=\sum_{h=1}^{8} N_{h}\left(\frac{N_{h}}{n_{h}}-1\right) s_{x_{a h}^{\prime}}^{2} \tag{5}
\end{equation*}
$$

where

$$
\begin{aligned}
s_{x_{a h}}^{2} & =s_{x_{a h}}^{2}+r_{a h}^{2} s_{y_{a h}}^{2}-2 r_{a h} s_{x_{a h}} y_{a h} \\
\text { where } s_{x_{a h}}^{2} & =\left[\sum_{i=1}^{n_{h}} x_{a h i}^{2}-\left(\sum_{i=1}^{n_{h}} x_{a h i}\right)^{2} / n_{h}\right] /\left(n_{h}-1\right) \\
s_{y_{a h}}^{2} & \text { is similarly defined for the } y \text {-variable. }
\end{aligned}
$$

$$
s_{x_{a h} y_{a h}}=\left[\sum_{i=1}^{n} x_{a h i} y_{a h i}-\left(\sum_{i=1}^{n_{h}} x_{a h i}\right)\left(\sum_{i=1}^{n h} y_{a h i}\right) / n_{h}\right] /\left(n_{h}-1\right)
$$

The estimated variance of the unbiased estimate in equation (la) is given in equation (5a).

$$
\begin{equation*}
s_{x_{\text {Ua }}^{\prime}}^{2}=\sum_{h=1}^{8} N_{h}\left(\frac{N_{h}}{n_{h}}-1\right) s_{x_{a h}^{2}}^{2} \tag{5a}
\end{equation*}
$$

### 4.2 Estimates of 12-Month Totals

- The estimated variance of the estimate given in equation (2) is given in equation (6).

$$
\begin{equation*}
s_{x^{\prime \prime}}^{2}=\sum_{a=1}^{4} s_{x_{a}^{\prime \prime}}^{2}+2 \sum_{a=2}^{4} \sum_{b=1}^{a-1} s_{x_{a}^{\prime \prime}, x_{b}^{\prime \prime}} \tag{6}
\end{equation*}
$$

where $s_{a}^{2}{ }_{a}^{\prime \prime}$ is defined in equation (5), and

$$
\begin{align*}
s_{x_{a}^{\prime}}, x_{b}^{\prime \prime} & =\sum_{h=1}^{8} N_{h}\left(\frac{N_{h}}{n_{h}}-1\right) s_{x_{a h}^{\prime} x_{b h}^{\prime}}  \tag{6a}\\
s_{x_{a h}^{\prime} x_{b h}^{\prime}} & =\left[\sum_{i=1}^{n_{h}^{h}} x_{a h i} x_{b h i}+r_{a h} r_{b h} \sum_{i=1}^{n_{h}^{h}} y_{h i}^{2}-r_{a h} \sum_{i=1}^{n_{h}^{h}} x_{b h i} y_{h i}\right. \\
& \left.-r_{b h} \sum_{i=1}^{n_{n}^{h}} x_{a h i} y_{h i}\right] /\left(n_{h}-1\right)
\end{align*}
$$

All other variables have been defined previously.
The estimated variance of the simple unbiased estimate given equation (2a) is calculated in a similar manner.

$$
\begin{equation*}
s_{X_{U}^{\prime}}^{2}=\sum_{a=1}^{4} s_{X_{U a}^{\prime \prime}}^{2}+2 \sum_{a=2}^{4} \sum_{b=1}^{a-1} s_{X_{U a}^{\prime}, x_{U b}^{\prime}} \tag{6a}
\end{equation*}
$$

where

$$
\begin{aligned}
& s_{x_{U a}^{\prime}}^{2} \text { is defined in equation }(5 a) \text {, and } \\
& s_{x_{U a}^{\prime}, x_{U b}^{\prime}}=\sum_{h=1}^{8} N_{h}\left(\frac{N_{h}}{n_{h}}-1\right) s_{x_{a h} x_{b h}} \\
& s_{x_{a h} x_{b h}}=\left[\sum_{i=1}^{n h} x_{a h i} x_{b h i}-\left(\sum_{i=1}^{n} x_{a h i}\right)\left(\sum_{i=1}^{n} x_{b h i}\right) / n_{h}\right] /\left(n_{h}-1\right)
\end{aligned}
$$

### 4.3 Estimates of Changes Between Quarters

In order to estimate the variance of the percentage change between quarters as given in equation (3), the following equation should be used:

$$
\begin{equation*}
=\quad s_{D_{4}^{-2}}^{2} \equiv\left(\frac{100 x_{a}^{\prime \prime} z^{2}}{z_{a}^{\prime \prime}}\right)^{s^{\prime}}\left[\frac{s_{a}^{\prime \prime}}{x_{a}^{\prime \prime \prime}}+\frac{s_{z_{a}^{\prime \prime}}^{2}}{z_{a}^{\prime 1^{\prime 2}}}-2 \frac{s_{x_{a}^{\prime \prime}} z_{a}^{\prime \prime}}{x_{a}^{\prime \prime} z_{a}^{\prime \prime}}\right] \tag{7}
\end{equation*}
$$

where $s_{x_{a}^{\prime \prime}}^{2}$ and $s_{z_{a}^{\prime}}^{2}$, are defined in equation (5);

$$
x_{a}^{\prime \prime} \text { and } z_{a}^{\prime \prime} \text { are defined in equation (1); and }
$$

$$
{ }^{s} x_{a}^{\prime \prime}, z_{a}^{\prime \prime} \text { is defined as in equation (6a) by substituting }
$$

$$
z_{a}^{\prime \prime} \text { for } x_{b}^{\prime \prime} \text { and } z_{a h} \text { for } x_{b h} \text {. }
$$

### 4.4 Changes in 12-Month Totals Between Quarters

An estimator of the variance of the percent change in 12-month totals between quarters, as given in equation (4), is given in equation (8).

$$
\begin{equation*}
s_{D}^{2} \doteq\left(\frac{100 x^{\prime \prime}}{z^{\prime \prime}}\right)^{2}\left[\frac{s_{x^{\prime \prime}}^{2}}{x^{\prime \prime 2}}+\frac{s^{2} z^{\prime \prime}}{z^{\prime \prime \prime}}-2 \frac{s_{x^{\prime \prime}}^{\prime \prime}, z^{\prime \prime}}{x^{\prime \prime} z^{\prime \prime}}\right] \tag{8}
\end{equation*}
$$

where $s_{x}^{2}$ ', and $s_{z}^{2}$, , are estimated using equation (6) ;

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x'' and z'' are estimated using equation (2); and
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$s_{x^{\prime}}, z^{\prime \prime}=\sum_{a=1}^{4} \sum_{b=1}^{4} s_{x_{a}^{\prime \prime}, z_{b}^{\prime \prime}}$
where $s_{a}^{\prime \prime}{ }^{\prime}, z_{b}^{\prime \prime}$ ' is defined above in section 4.3.

### 4.5 Other Estimates of Error

The estimated standard error of an estimate is easily derived from the estimated variance by taking the square root of the variance. The relative standard error (or coefficient of variation) is obtained by dividing the estimated standard error of an estimate by the estimate.

Ninety-percent confidence intervals about estimates of change may be derived by multiplying 1.645 times the estimated standard error and adding the result to and subtracting it from the estimate of change. These confidence intervals are used to test textual statements about increases or decreases in property tax revenue. If zero is included in the interval, the statement of an increase or decrease cannot be made.

## 5. Considerations for Future Redesigns

In the future this survey should be redesigned after each census. Most of the certainties will remain in the sample, but changes in the noncertainty units will be captured with a 5 -year redesign plan. For the next redesign, noncertainty strata should be assigned to cost strata so that areas that cost more for data collection (those with several tax-collecting agencies) will be sampled at a lower rate than counties with central tax collecting agencies. For the next redesign, perhaps 3 cost strata should be established, based on the number of agencies in the area and the difficulty of getting data from the
area. Within each cost stratum, population will be used for further stratification.

In this survey and future surveys, nonresponse should be monitored more closely. Procedures should include followup calls to all large county-area nonrespondents. At a minimum, small areas should receive a reminder card. More extensive efforts for the first quarter in sample should yield great rewards. The extensive followup will verify addresses and will serve to inform newcomers that the survey is important. Hard-core nonrespondents can be identified and suitable substitutes may be found for them. Imputation and "editing procedures should also be examined.

