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A STUDY OF PRE-ADJUSTMENT TRANSFORMATIONS

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Views expressed are solely those of the authors.

1. Introduction

In a recent study Runyan (1983) showed the improvements of concurrent seasonal adjustment over the method of using yearly projected seasonal factors for seasonal adjustment. Twenty-seven economic time series (see Appendix) from the manufacturer's survey of value of shipments (VS), total inventories (TI), and unfilled orders (UO) were examined using X-11 and assuming a multiplicative model. Examination of plots of the series as well as subject-matter knowledge of the series led to questions concerning the validity of a multiplicative model--which corresponds to the natural logarithm transformation.

This study uses a modelling approach for estimating appropriate transformations for the series used in Runyan's concurrent seasonal adjustment study and examines whether there is substantial improvement in performing the concurrent X-11 seasonal adjustment on an optimally transformed series. We limit our attention to Box-Cox transformations of the form

$$z_t^{(\lambda)} = \begin{cases} (z_t^{\lambda}-1)/\lambda & \lambda \neq 0 \\ \ln z_t & \lambda = 0 \end{cases} .$$

The X-11 program allows for the option of a multiplicative or additive adjustment, corresponding to values of λ equal to 0 or 1, respectively. However, series may be transformed prior to the application of the X-11 seasonal adjustment.

The behavior of many of these series changes over time. Changes may be due to a variety of factors--such as new definitions of the quantity being measured or changes in public policy. As the structure of the series varies, so may the optional transformation. Therefore, it may be necessary to exclude early portions of the data and model the transformation on more recent observations that accurately portray the present structure of the series. It must be noted, though, that enough data must be retained to preserve any information about the transformation.

Therefore, the first phase of this study addresses two main questions: What is the proper transformation parameter of each series? and, How sensitive is the estimate of the parameter to the choice of the series segment used for its determination? The second phase investigates the performance of the X-11 seasonal adjustment procedure performed concurrently with the estimated optimal transformation parameter.

2. Visual Evaluation

As a preliminary step, plots of the 27 series were examined visually over the entire 14-year span from January, 1968 through December, 1981. The series were crudely categorized as appearing to warrant either an additive or multiplicative model. In addition, points of abrupt change in the nature of the series were noted. Of the 27 series, six were tentatively categorized by inspection as additive and twelve as multiplicative. The remaining nine were too difficult to categorize visually. Ten of the 27 series were uniform in behavior over the entire 14-year span. The remaining 17 series showed abrupt changes in behavior, with eleven needing a cutoff before 1974 and six after. A summary of the visual inspection is given in Table 1. An analytic approach to determining the transformation was then taken and its results were compared with this empirical evaluation.

3. Analytical Investigation

The first step of the analytical investigation was to estimate the transformation parameter λ for each series via a maximum likelihood procedure. A general purpose seasonal ARIMA model of the form $(6,1,0)x(0,1,1)_{12}$ (in the Box-Jenkins notation) was fit to the λ-transformed values of each series for various λ's over a reasonable range, from -1 to +2 in increments of 0.5. Then this model was estimated via exact maximum likelihood. Multiplying this maximum likelihood value for the model of the λ-transformed data by the Jacobian of the Box-Cox transformation, we obtain a likelihood function for the original (untransformed) data as a function of λ. A plot of the likelihood versus λ indicates the shape of the function, and the value of λ that maximizes the likelihood was interpolated from the graph. An example is shown in Figure 1.

This procedure was followed for both the full series and the abridged series (i.e., from the cutoff date to the present), and the two estimates of λ compared. Any substantial discrepancies between λ for the full series and λ for the abridged series indicate that the nature of the series has changed, and that including the entire data set in the estimation of the transformation yields a value of λ inappropriate for current data.

3.1 Abridged series

For the abridged series, experimentation suggested January 1974 as the best cutoff date. No later date could be considered since seven or fewer years of data would probably be insufficient to estimate the general purpose model.

Judging also from the empirical evaluation (Table 1), this cutoff date seemed appropriate for most series. The abridged series thus spanned January, 1974 through December, 1981.

The likelihood-based estimates of the appropriate transformation for the abridged series, λ_{1974}, are shown graphically in Figure 2, along with confidence intervals. The 100 $(1-\alpha)\%$ confidence interval for λ is defined as

$$\{\lambda : L_{max}(\lambda) > L_{max}(\lambda) - (1/2)\chi_1^2(\alpha)\} ,$$

where L_{max} represents the value of the maximum likelihood function.

Three of the 27 series (S69U0, D37VS, S65VS) could not be modelled with the general purpose model. The parameter estimation procedure was not converging. Rather than attempt to individually model them, it was decided to remove these series from the analysis.

For the remaining 24 series, both 95% and 99% confidence intervals are shown in Figure 2. If the confidence interval for λ does not contain 0, we conclude that a multiplicative transformation is inappropriate for the data, and likewise for the additive transformation if 1 is not included. Based on the 95% confidence interval, neither the additive transformation (λ = 1) nor the multiplicative transformation (λ = 0) is appropriate for seven series. For the remaining series, the 95% confidence interval for four series excludes the multiplicative model, and for ten series excludes the additive model. Using the 99% confidence intervals, there is only one series (S42VS) for which neither the multiplicative nor the additive model is a candidate.

For S64TI and S50UO, $\lambda = 0$ (the multiplicative model) is excluded, while for eleven other series, $\lambda = 1$ (the additive model) is excluded. The choice of significance level will thus determine whether the additive or multiplicative model (or both) are rejected. For the purposes of the remainder of the study, we chose a significance level of 5%, to allow more series to be studied for which there appeared to be a preference for one value of $\lambda \varepsilon \{0,1\}$ over the other. Regardless of the significance level chosen, the analytical investigation of the abridged series suggested that the multiplicative model ($\lambda = 0$) is not the optimum transformation for the current data in every series.

3.2 Comparison of Visual and Analytical Investigations

We are now able to check whether the analytical investigation coincides with our initial visual inspection. In only eight of the 24 series which were modelled and the transformation parameter estimated did our guess as to the appropriate value of λ based on visual inspection agree with the maximum likelihood estimate. Three series were correctly identified as following a multiplicative model, one was correctly chosen to follow an additive model, and four were correctly identified as lying somewhere between the two ($0<\lambda<1$). Hence, the optimal transformation parameter may not always be apparent by just visually examining the series.

3.3 Full Series

The next logical step would be to repeat the exact likelihood estimation procedures for all full-length series. The confidence intervals for the full series could be compared to the confidence intervals for the abridged series for evidence of overlapping. Then the questions of what portion of the data should be used in estimating the transformation parameter could be answered.

The large number of series being examined, however, is ill-suited for an in-depth analysis of this sort. A subset of the original group of series was selected for further study based on two qualifications: 1) the series chosen were uniform in behavior over the entire data span, and 2) the 95% confidence interval for the transformation parameter did not include 0. The reason for the latter criterion is the goal of comparing alternative transformations to the presently-used multiplicative option of X-11 (here considered to be equivalent to the natural logarithm, or $\lambda = 0$). Using the estimates of λ from the abridged series as a guide (Figure 2), there were six series for which λ was estimated over the full data span: S94TI, S30VS, S22UO, S52UO, S42VS and S50UO.

The transformation parameter estimate for each of the six full series was obtained by fitting the general purpose model. In addition, the series were individually modelled, utilizing an outlier detection/correction procedure developed by Bell (1982). Then, the transformation parameter was once again estimated using the maximum likelihood procedure described earlier but using the individual model which was fit to that series, rather than the general purpose model.

3.4 Comparison of λ_{full} and $\lambda_{abridged}$

The results are summarized in Table 2. It is interesting to note that the parameter estimate λ changed with the move from abridged series to full series but did not vary much as the model was altered. It may not be crucial, therefore, to have the precisely correct individual model to accurately estimate the transformation parameter. In future studies, therefore, greater emphasis should probably be placed on the issue of cutoff dates and on using portions of the data to estimate the transformation, than on detailed modelling.

With one exception, the change in λ when the full series was considered, although significant in some cases, was not extreme. For series S50U0, however, $\lambda_{abridged} = 1.1$ must be compared with $\lambda_{full} = +0.2$. Upon examining the graph of this series (Figure 5), it seemed that the series did change its behavior somewhere around the beginning of 1972 (two years before the start of the abridged series). We decided to continue using $\lambda = 1.1$ for S50U0 since the next stage of investigation--concurrent analysis--involved the latter part of the series where the value of $\lambda = 1.1$ was more appropriate. An additional year's worth of data was obtained so a concurrent seasonal adjustment analysis also could be performed on S50U0 ranging from 1972-1982, thereby avoiding the initial inconsistent behavior.

Also from Table 2 it is seen that for two series, S22U0 and S30VS, the full series transformation parameter estimate equals 0. We therefore chose not to include them in further analysis, since the natural logarithm (i.e., multiplicative model) was appropriate over the full series and, as mentioned, we are more interested in examining seasonal adjustments of non-multiplicative transformations of series. Four full-length series--S94TI, S50U0, S52U0, and S42VS--remained to be studied. In addition S50U0 was examined over the range 1972-1982. Hereafter this series will be referred to as S50U0*.

4. Concurrent Adjustment Analysis on Transformed ($\lambda \neq 0$) vs. Logged ($\lambda = 0$) Series

Concurrent adjustment includes data up through the current month in the calculation of the current month's seasonal adjustment factor. This is in contrast to the projected adjustment, in which a seasonal adjustment is performed each December on all data then available, at which time the seasonal components are projected for the next twelve months. The concurrent adjustment analysis using the X-11 program requires 7 years of data prior to the experimental period to "warm up", and 3-5 years beyond the experimental timeframe to permit the use of symmetrical filters so that "final" seasonal adjustments can be obtained. This constrained us to data sets which began prior to 1971. Thus, although the issue of cutting off an early portion of a series is important and timely, it cannot at present be evaluated in terms of its effect on concurrent adjustment. Therefore, we limited our attention to the effect on concurrent seasonal adjustment of optimally transforming the entire series.

The concurrent adjustment analysis was performed as follows. First, the series were transformed using the full-length transformation parameters described in Section 3.3. The resulting series were seasonally adjusted using X-11 with the additive option. The concurrent and projected seasonal components were extracted for each month of the years 1975 through 1978. This timeframe was selected because a final seasonal component was available for each month in this period. The final seasonal component is defined to be the value obtained when symmetrical filters can be applied and thus the addition of more data will not cause any further revisions in that month's seasonal component.

Before the analysis was begun, all data values required for the procedure were transformed back to their original scale. The results can therefore be directly compared with those obtained from the multiplicative adjustment. To evaluate the relative improvement offered by concurrent adjustment, several summary measures were computed. The Mean Absolute Error in Level (MAEL), measures differences in level while the Mean Absolute Month-to-Month Percentage Error (MAMM) measures the month-to-month rates of change. In addition several revision summary measures were calculated (Findley and Monsell 1984). The Cumulative Percent Revision (CPREV) is a measure of how the seasonally adjusted value for a given month has fluctuated over its revisions history. The Total Revision (TOTREV) is a measure of how the initial value varies from the final value. The "Convergence" Ratio (CONRAT) is a measure of how quickly the initial seasonally adjusted value approaches the final value. The summary measures are defined as follows:

$$\text{MAEL} = \frac{1}{n} \sum_{t=1}^{n} |x_t - X_t|$$

$$\text{MAMM} = \frac{1}{n-1} \sum_{t=1}^{n-1} \frac{x_{t+1}}{x_t} - \frac{X_{t+1}}{X_t}$$

$$\text{CPREV}_t = \sum_{i=0}^{N-1} |X_{t,i+1} - X_{t,i}| / X_{t,0}$$

Note: If 3x9 seasonal filters are not used (i.e. $N \neq 60$), then CPREV_t is adjusted to account for fewer or greater number of terms in the sum.

$$\mathrm{TOTREV}_t = |X_{t,N} - X_{t,0}|/X_{t,N}$$

$$\mathrm{CONRAT}_t = \sum_{i=0}^{N-1} \beta^{N-1-i} \frac{X_{t,i} - X_{t,N}}{X_{t,N}} \quad \sum_{i=j}^{N-1} \beta^{i}$$

where x_t = the first published estimated seasonal adjustment from one of the modes of adjustment $t=1,2,\ldots,n$

X_t = the final estimated seasonal adjustment for month t $t=1,2,\ldots n$

$X_{t,i}^{*}$ = seasonally adjusted value of month t when i months of data beyond month t are available $t=1,\ldots,n$ $i=1,\ldots,N$

$X_{t,0}$ = concurrent value for month t $t=1,\ldots,n$

$X_{t,N}$ = final value for month t $t=1,\ldots,n$

n = number of observations in experimental period

N = number of months until a final seasonally adjusted value is obtained

β = constant, $0<\beta<1$. Here $\beta = .962226 = \sqrt[N/2]{0.5}$

Table 3 summarizes the results for the differences in level and month-to-month changes. The column entries under "ratio" are ratios of the value of the measure using concurrent adjustment divided by that for projected adjustment. Comparing the ratios for the measures of level and month-to-month change, we find the maximum likelihood transformation improved the ratios in two series (S94TI, S50UO*), made no difference in the ratios in one series (S52UO), and was worse than the logarithm's ratios for two series (S50UO, S42VS). The result for S50UO is not surprising considering the transformation analysis was performed with the transformation parameter λ = 1.1, as suggested by the maximum likelihood estimation based on data from 1974 on, yet our investigation determined the estimate of λ for the full series should be +0.2. The logarithm (where $\lambda = 0$) is actually closer to the correct transformation value than the transformation value obtained by maximum likelihood. Therefore, the full S50U0 series was excluded from further study. The shortened S50U0* where the transformation parameter $\lambda = 1.1$ is correct was included in further investigations along with the three remaining full series: S94TI, S42VS, S52U0.

To compare the quality of the seasonal adjustment of the transformed and logged series, the values of the measures of level and month-to-month change for concurrent adjustment and the values of these measures for projected adjustment were examined. Smaller values of the measures indicate a better job of estimating the final seasonally adjusted values.

The values of the measures (Table 3) for S94TI and S50U0* with the maximum likelihood transformation were always smaller than their counterparts with the logarithm transformation for both modes of seasonal adjustment (projected and concurrent). For series S52U0 and S42VS, however, the logarithm almost always produced smaller values of the measures for both modes of seasonal adjustment. The type of transformation that increased the improvement of concurrent adjustment over projected adjustment also improved the quality of the seasonal adjustment.

Table 4 summarizes the revision measures for the maximum likelihood and the log transformed series. In agreement with the previous discussion, the revision measures indicate that the log transformed series has a better revision pattern than the maximum likelihood transformed series for S52U0 and S42VS. The revision measures were also consistent with other measures in revealing that for

S50UO* the maximum likelihood transformed series behaved better than the log-transformed series. With series S94TI however, previous measures have tended to favor the maximum likelihood transformation but the revision measures show this method is no different or perhaps a little worse than the log-transformed method with regard to revisions.

5. Summary of Concurrent Analysis

In summary, to date, out of four series studied under the concurrent adjustment procedure, only for shortened series S50UO* did the maximum likelihood transformation improve the quality of the concurrent adjustment. The transformation could be described as better or worse for S94TI depending on which measures are examined. The transformation was actually worse for the concurrent adjustment of S42VS and S52UO. Why is the multiplicative model giving better results in situations where the confidence interval for the transformation parameter does not even include $\lambda = 0$? There are several plausible explanations.

The concurrent adjustment analysis examines a section of the series in the middle portion of the observed data. If the series behaves differently in the experimental timeframe than outside (due to recessions, strikes, etc.), the quality of the seasonal adjustment in the experimental period may not be indicative of the quality of the seasonal adjustment on the series as a whole. Therefore, although the transformation seems to reduce the quality of the seasonal adjustment based on results from the experimental period, in fact, the transformation may actually improve the adjustment at the most recent portion of the data, which is most relevant. However, based on graphs of each transformed series as a whole and in the experimental period only, there seem to be no outliers or odd behavior in the experimental periods of these four series.

Even if there is no unusual behavior, the optimum transformation may vary over time. As we have already seen, the transformation estimated for series from 1974 on often can sometimes differ drastically from the value of the transformation parameter estimated over the entire data span. Thus, even if we use the optimum transformation parameter value based on the entire series, this value may be totally inadequate for the portion of data in the experimental timeframe. By examining graphs of the maximum likelihood transformed series in the experimental timeframe and comparing them to graphs of the natural logarithm of each series in the experimental timeframe (see Figures 3 and 4), one finds that for series S52UO and S42VS the logarithm seems to stabilize the variation in the series. Thus, within the experimental timeframe, the multiplicative model does seem more appropriate as suggested by our analysis. In contrast, for series S94TI the graphs suggest the maximum likelihood transformation chosen better stabilizes the variation for the experimental timeframe. The variation is more similar throughout the series with the maximum likelihood transformation. It is not obvious from the graphs of S50UO (full) and S50UO* (shortened) which transformation is more appropriate.

If the user is most interested in the seasonal adjustment of the most recent observations, the transformation parameter value for this portion of the data may indeed differ from that for the experimental period. Therefore, a reasonable procedure may be to use just the most recent data to estimate the transformation parameter. Just how much data to include in the estimation may depend on the estimation procedure and the nature of the series.

Finally, it must be remembered that χ^2-distribution used to obtain the confidence intervals is an approximation to the true distribution of λ, and may in fact be a poor approximation.

6. Comparison of MLE of λ with transformations selected by SABL

Cleveland et al. (1981) have suggested an alternative procedure for estimating λ, the transformation parameter. Their method chooses the value for λ that minimizes the lowest degree interaction between the trend and seasonal components of the additive decomposition model. Estimation of this transformation parameter using the SABL procedures for each of the four full series and S50U0* yields the results shown in the chart below.

	λ SABL	λ ML
S94TI	-1.0	-0.5
S52U0	-1.0	0.3
S42VS	0.25	0.5
S50U0	-0.5	0.2
S50U0*	-0.5	1.1

The SABL estimate is significantly different from the maximum likelihood estimate (in the sense that λ_{SABL} is not contained in the interval [$\lambda_{ML} \pm$ 2 S.E.])

for S50U0, S50U0*, and S52U0. The two methods never agree exactly. The concurrent adjustment analysis was repeated on the five series using the SABL estimates of the transformation (Table 3). The quality of the seasonal adjustment was compared to that of the series transformed with the maximum likelihood estimates.

SABL produced the best estimate of λ for series S42VS but overall, the SABL procedure ranked third behind log transformation and the maximum likelihood procedure in the quality of the seasonal adjustment (see Table 5). There was no clear-cut winner among the three procedures. The maximum likelihood transformation performed best for S94TI and S50U0*. The log transformation performed best for S52U0 and S50U0. Hence, the SABL methodology does not offer a solution to the problem of consistently producing the optimum transformation parameter for seasonal adjustment with X-11 methodology.

7. Conclusions

Although the results of this study indicate that a transformation associated with a non-multiplicative adjustment enhanced the seasonal adjustment completely for S50U0*, marginally for S94TI, and not at all for the other two series, the effectiveness of Box-Cox transformations should not be ruled out. In the series where the transformation hurt the quality of the seasonal adjustment, further investigation revealed the maximum likelihood estimate of λ used was not appropriate for the span of data considered in the experimental timeframe.

When the proper transformation was used, the quality of the seasonal adjustment did improve. The difficulty lies in determining the correct transformation parameter for the time period of interest and the proper portion of data over which to estimate it. Further studies should concentrate on these issues.

In summary, the maximum likelihood transformation parameter is sensitive to the amount of data used for the estimation but is not as sensitive to the specific model chosen. The log transformation is not appropriate for all series, as suspected. No single type of transformation performs consistently better. Unless the appropriate transformation is invariant over different portions of data, the maximum likelihood procedure will not necessarily choose the best estimate for current data.

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Appendix: Industry Series Used in the Transformation Study

Code Name	Title
N45TI	Total Inventories: Nondefense Shipbuilding and Military Tank Vehicles
D37TI	Total Inventories: Defense Communication Equipment
N43VS	Value of Shipments: Nondefense Complete Aircraft
D37VS	Value of Shipments: Defense Communication Equipment
S94TI	Total Inventories: Leather, Industrial Products and Cut Stock
S20TI	Total Inventories: Other Fabricated Metal Products
S36TI	Total Inventories: Radio and TV
S64TI	Total Inventories: All Other Foods
S92TI	Total Inventories: Tires and Tubes
S48TI	Total Inventories: Scientific and Engineering Equipment
S16TI	Total Inventories: Metal Cans, Barrels, and Drums
S69UO	Unfilled Orders: Floor Covering Mills
S52UO	Unfilled Orders: Miscellaneous Personal Goods
S74UO	Unfilled Orders: Pulp and Paperboard Mills, Except Building
S50UO	Unfilled Orders: Photographic Goods
S22UO	Unfilled Orders: Internal Combustion Engines
S48UO	Unfilled Orders: Scientific and Engineering Instruments
S46UO	Unfilled Orders: Railroad Equipment
S38UO	Unfilled Orders: Electronic Components
S65VS	Value of Shipments: Tobacco Maufacturers
S21VS	Value of Shipments: Steam Engines and Turbines
S86VS	Value of Shipments: Agricultural Chemicals
S42VS	Value of Shipments: Motor Vehicle Assembly Operations
S18VS	Value of Shipments: Building Materials and Wire Products
S90VS	Value of Shipments: Other Petroleum Products
S30VS	Value of Shipments: Office and Computing Machines
S11VS	Value of Shipments: Blast Furnaces, Steel Mills

Table 1. Visual Evaluation of Appropriate Transformation

Series	Initial Transformation (by inspection)	Portion of data to Include
N45TI	multiplicative	1975^+
D37TI	not sure	1972^+ or 1977^+
N43VS	multiplicative	full series
D37VS	multiplicative	1975^+
S94TI	multiplicative	1974^+
S20TI	additive	full series
S36TI	not sure	1970^+
S64TI	additive	full series
S92TI	multiplicative	1974^+
S48TI	additive	full series
S16TI	multiplicative	1974^+
S69UO	multiplicative	1973^+
S52UO	multiplicative	1974^+
S74UO	not sure	1973^+
S50UO	multiplicative	1976^+
S22UO	not sure	1975^+
S48UO	additive	1975^+
S46UO	additive	1972^+
S38UO	additive	1972^+
S65VS	multiplicative	full series
S21VS	not sure	full series
S86VS	not sure	full series
S42VS	multiplicative	1974^+
S18VS	not sure	full series
S90VS	not sure	full series
S30VS	multiplicative	full series
S11VS	not sure	1974^+

Table 2. Results of the Individual Modelling of the Series

Series	$\lambda_{abridged}$ from general purpose model	λ_{full} from general purpose model	λ_{full} from individual model	Individual model
S22UO	+0.2	0.0	0.0	$(0,1,2)x(0,1,0)_{12}$ + outliers
S94TI	-0.8	-0.5	-0.5	$(6,1,0)x(0,1,1)_{12}$ + outliers
S50UO	+1.1	+0.2	+0.2	$(3,1,0)x(0,1,1)_{12}$ + outliers
S52UO	+0.5	+0.3	+0.3	$(1,1,0)x(0,1,1)_{12}$ + outliers
S42VS	+0.45	+0.7	+0.5	$(0,1,1)x(0,1,1)_{12}$ + outliers
S30VS	-0.35	0.0	0.0	$(5,1,0)x(0,1,1)_{12}$ + outliers
S50UO*	+1.1	+1.0	+1.1	$(3,1,0)x(0,1,1)_{12}$ + outliers

$(6,1,0)x(0,1,1)_{12}$

Table 3. Summary of Performance of Each Type Transformation.

Series	MAMM conc.	MAMM proj.	MAMM ratio (c/p)	MAEL conc.	MAEL proj.	MAEL ratio (c/p)
S94TI						
λ = -.5 ML	.0105	.0120	.8747	2.3063	2.4137	.9555
λ = 0 log	.0137	.0124	.9190	2.3797	2.4606	.9671
λ = -1.0 SABL	.0143	.0127	1.1249	2.3428	2.1888	1.0704
S52U0						
λ = 0.3 ML	.0493	.0543	.9082	52.379	57.241	.9151
λ = 0 log	.0468	.0510	.9184	47.052	52.327	.8992
λ = -1.0 SABL	.0529	.0554	.9546	47.867	52.333	.9147
S42VS*						
λ = 0.5 ML	.0313	.0332	.9431	134.40	142.80	.9411
λ = 0 log	.0252	.0336	.7501	108.55	132.03	.8222
λ = 0.25 SABL	.0259	.0319	.8098	102.36	125.68	.8542
S50U0						
λ = 1.1 ML	.0275	.0252	1.0931	5.7384	5.6290	1.0194
λ = 0 log	.0212	.0227	.9300	4.7995	5.9629	.8049
λ = -.5 SABL	.0196	.0241	.8132	5.6621	6.8217	.8300
S50U0*						
λ = 1.1 ML	.0102	.0133	.7724	3.4011	5.0472	.6739
λ = 0 log	.0134	.0163	.8182	4.7879	6.4254	.7448
λ = -.5 SABL	.0171	.0208	.8187	5.3292	9.1619	.5817

Table 4. Comparison of Revision Statistics Between the Transformed and Nontransformed Series.

		S94TI		S52U0		S42VS		S50U0*	
		log $\lambda = 0$	max. likeli. trans.	log $\lambda = 0$	max. likeli. trans.	log $\lambda = 0$	max. likeli. trans.	log $\lambda = 0$	max. likeli. trans.
CPREV	avg	.0655	.0691	.2056	.2291	.1474	.1507	.0845	.0634
	(sd)	(.0252)	(.0333)	(.1220)	(.1366)	(.0722)	(.0650)	(.0298)	(.0243)
	max	.1343	.1671	.8412	.9996	.3498	.3296	.1387	.1250
	min	.0258	.0224	.0851	.0770	.0647	.0773	.0499	.0420
TOTREV	avg	.0123	.0124	.0541	.0587	.0204	.0233	.0088	.0062
	(sd)	(.0089)	(.0087)	(.0484)	(.0464)	(.0172)	(.0218)	(.0068)	(.0041)
	max	.0387	.0381	.2400	.2198	.0816	.0926	.0236	.0131
	min	.0000	.0000	.0009	.0014	.0000	.0004	.0010	.0009
CONRAT	avg	.0045	.0043	.0211	.0228	.0088	.0100	.0044	.0029
	(sd)	(.0025)	(.0022)	(.0137)	(.0114)	(.0052)	(.0075)	(.0033)	(.0021)
	max	.0093	.0090	.0791	.0528	.0294	.0471	.0105	.0071
	min	.0008	.0007	.0017	.0012	.0022	.0019	.0012	.0009

Table 5. Rank of Each Type Transformation

Series		MAMM conc.	MAMM proj.	MAEL conc.	MAEL proj.	Overall (rank sum)
S94TI						
$\lambda = -.5$	ML	1	1	1	2	1
$\lambda = 0$	log	2	2	3	3	3
$\lambda = -1.0$	SABL	3	3	2	1	2
S52U0						
$\lambda = +.3$	ML	2	2	3	3	2.5
$\lambda = 0$	log	1	1	1	1	1
$\lambda = -1.0$	SABL	3	3	2	2	2.5
S42VS						
$\lambda = +.5$	ML	3	2	3	3	3
$\lambda = 0$	log	1	3	2	2	2
$\lambda = +.25$	SABL	2	1	1	1	1
S50U0						
$\lambda = 1.1$	ML	3	3	3	1	3
$\lambda = 0$	log	2	1	1	2	1
$\lambda = -.5$	SABL	1	2	2	3	2
S50U0*						
$\lambda = 1.1$	ML	1	1	1	1	1
$\lambda = 0$	log	2	2	2	2	2
$\lambda = -.5$	SABL	3	3	3	3	3

FIGURE 1. LIKELIHOOD FUNCTION & CONFIDENCE INTERVALS FOR S64TI

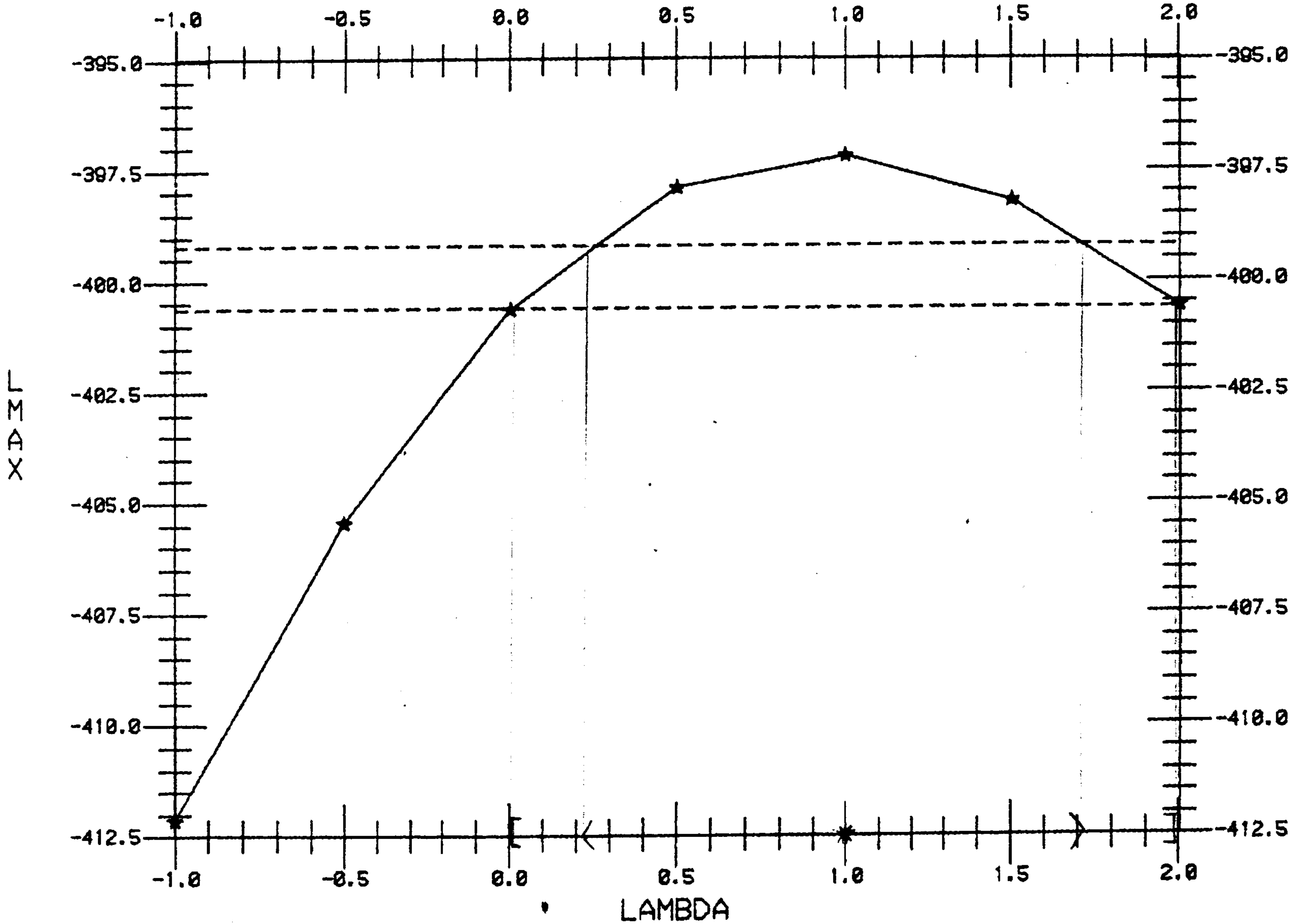


FIGURE 2. CONFIDENCE INTERVALS FOR ESTIMATES OF THE ABRIDGED SERIES

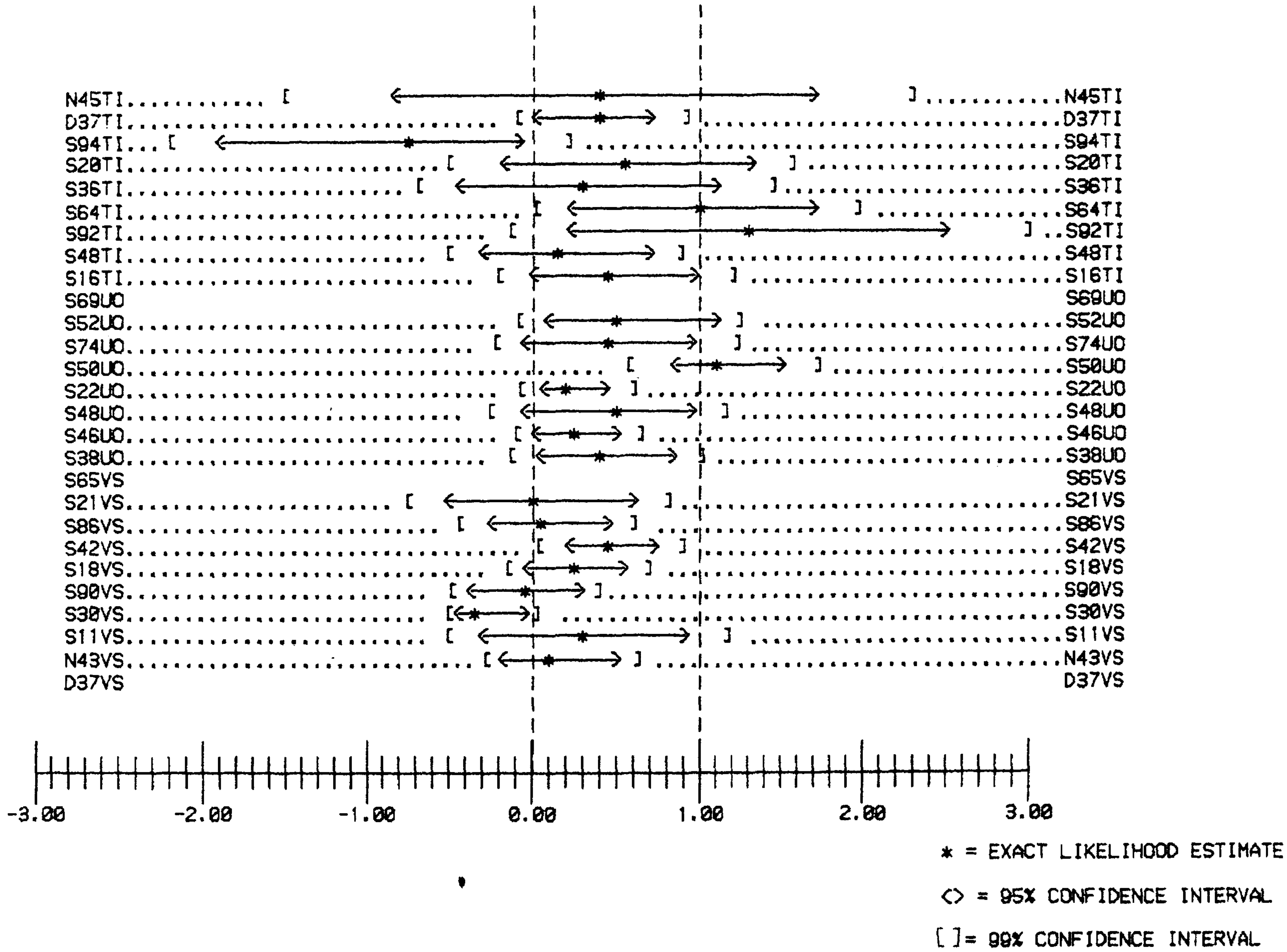


Figure 3. Graphs of Logged and Maximum Likelihood Transformed Series in Experimental Timeframe

Logarithm

Maximum Likelihood

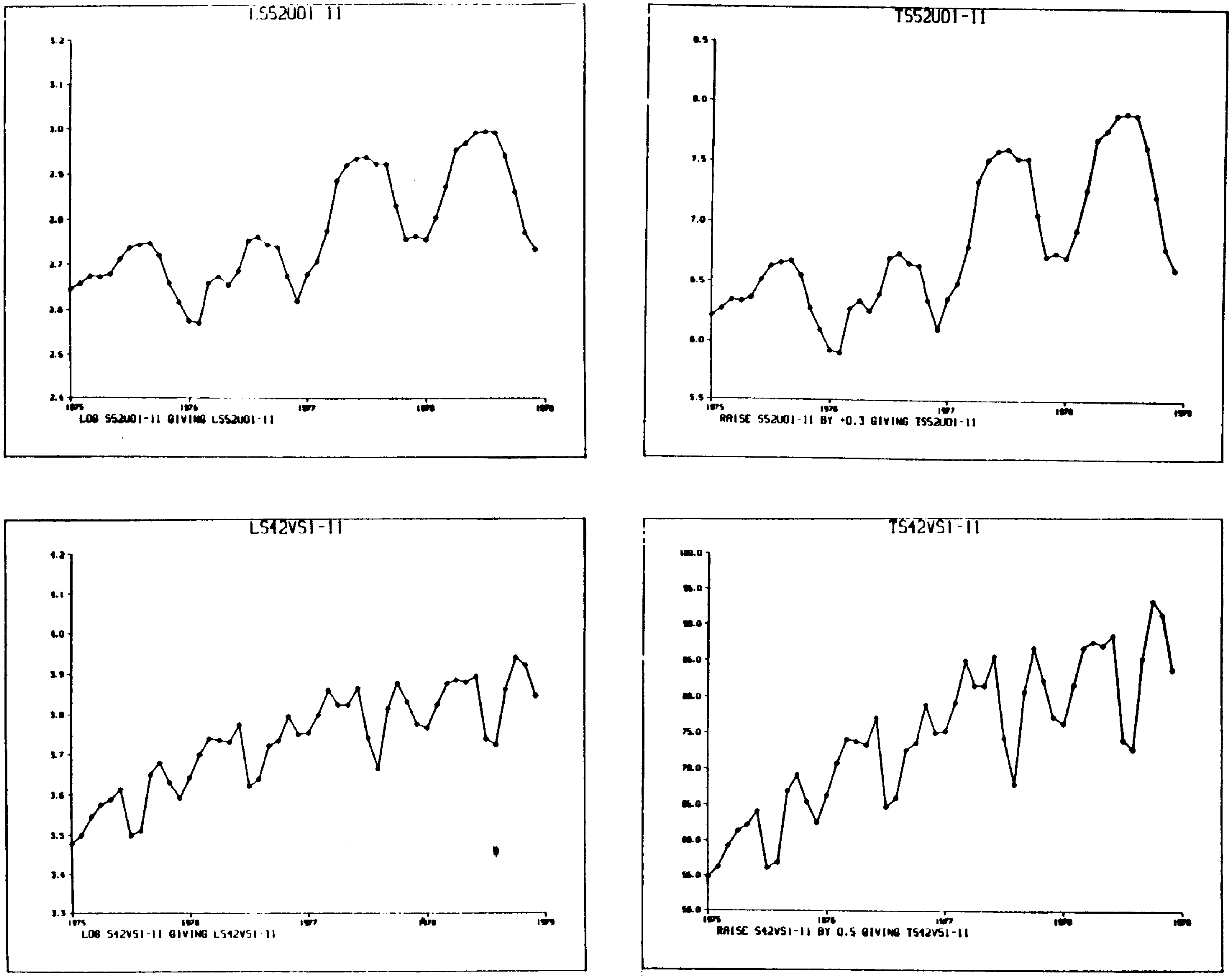


Figure 4. Graphs of Logged and Maximum Likelihood Transformed Series in Experimental Timeframe

Logarithm

Maximum Likelihood

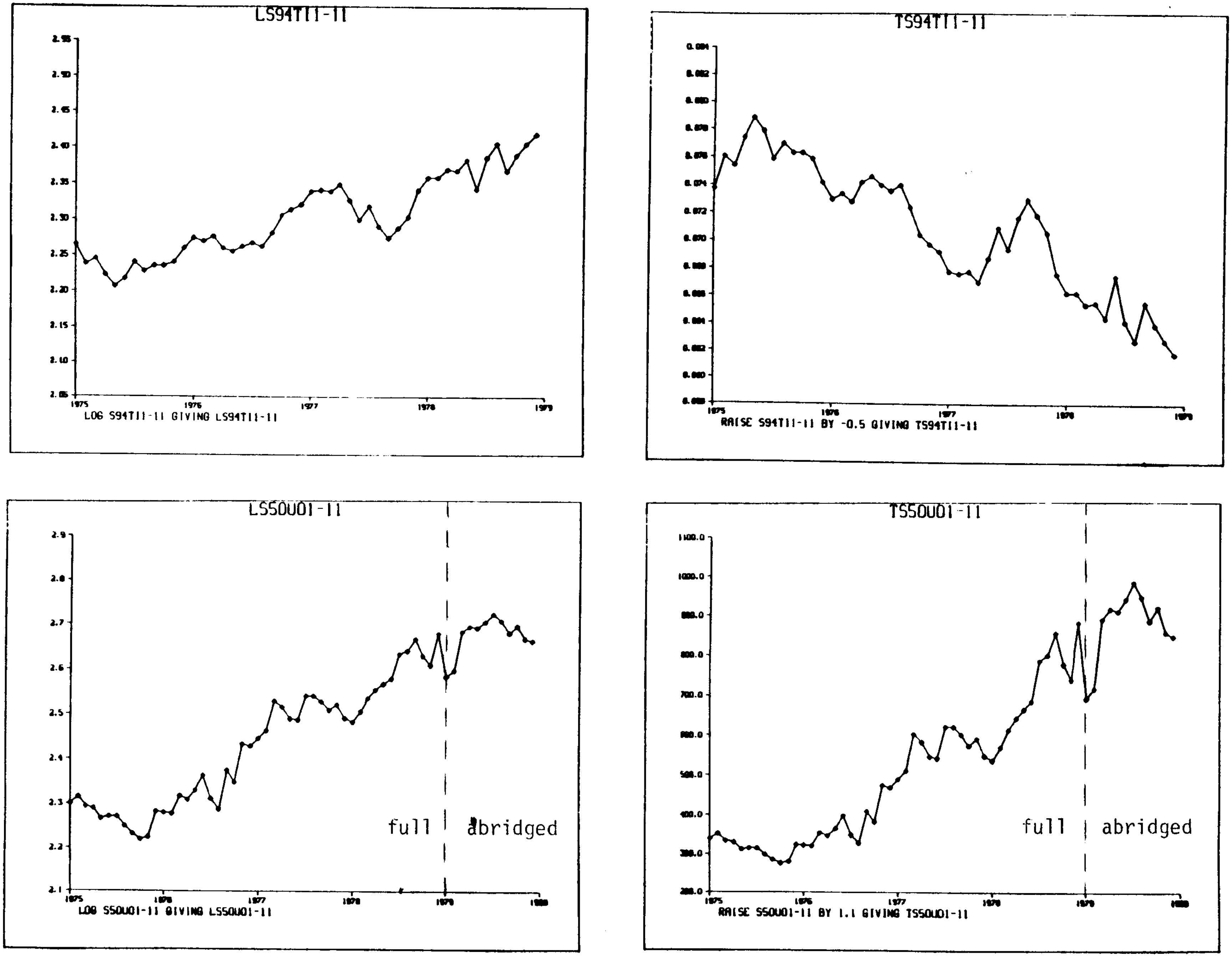
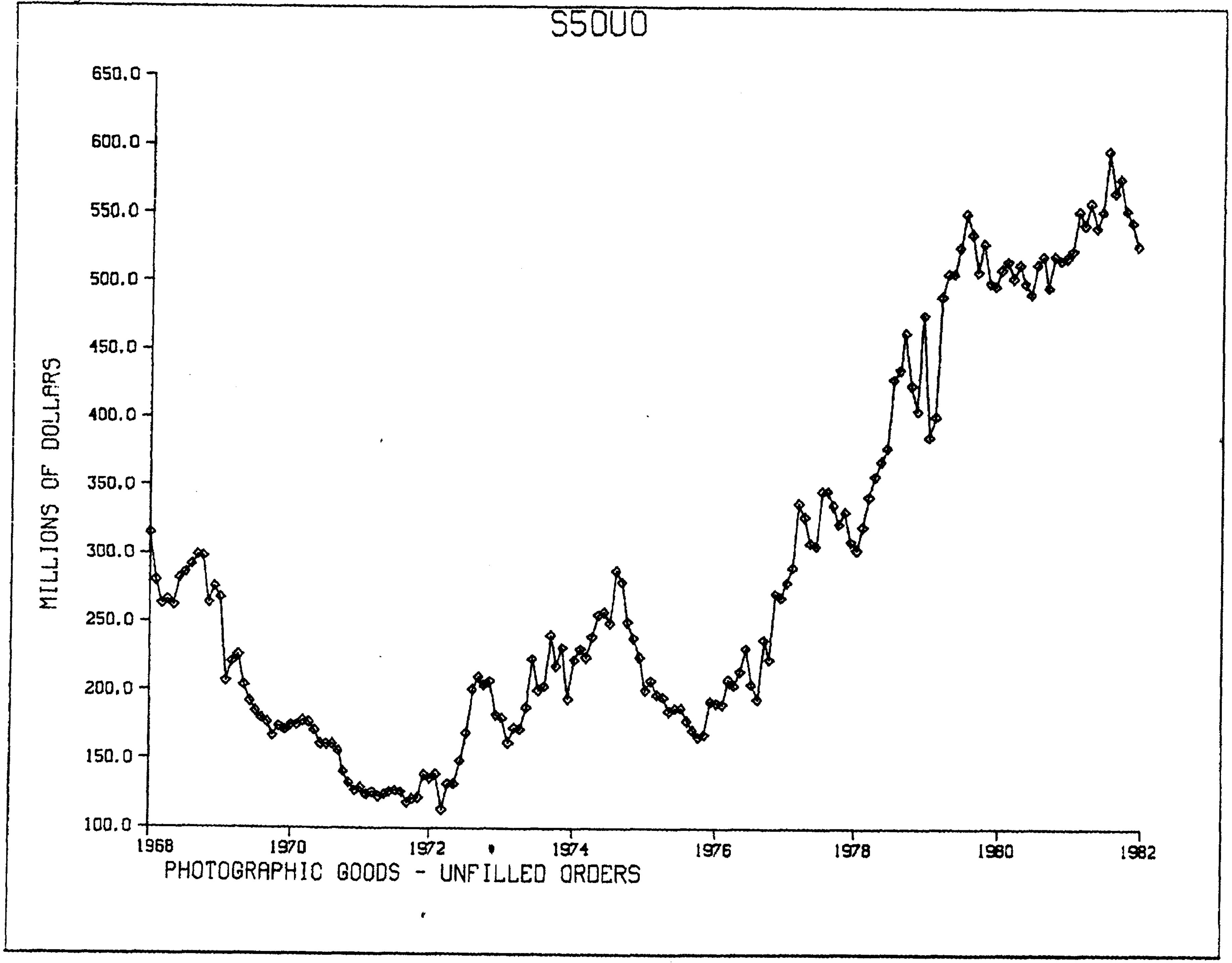


Figure 5.



S50UO
650.0
600.0
550.0
500.0
450.0
400.0
350.0
300.0
250.0
200.0
150.0
100.0
MILLIONS OF DOLLARS
1968
1970
1972
1974
1976
1978
1980
1982
PHOTOGRAPHIC GOODS - UNFILLED ORDERS