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USING THE KALMAN SMOOTHER TO ADJUST FOR MOVING TRADING DAY

by

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ABSTRACT: A procedure which utilizes the Kalman filter and smoother to adjust monthly time series for a moving trading day effect is examined. Simulated time series are used to compare this procedure to one which assumes a constant trading day effect. The Kalman procedure is shown to adjust these simulated series very well, and gives substantially better adjustments than a constant trading day procedure when a moving trading day effect is present in the data.

I. Introduction

Bell and Hillmer (1983) define trading day variation as "the variation in a monthly time series that is due to the changing number of times each day of the week occurs in a month." This variation is of interest to analysts in the seasonal adjustment of economic time series, and much work has been done in the identification and removal of trading day variation. Young (1965) described how the CENSUS X-11 program removed trading day effects. Cleveland and Devlin (1982) discusses new methods of identifying and removing trading day effects from monthly time series, using spectral analysis, power transformations and robust regression. Bell and Hillmer (1983) discuss the fitting of trading day components in ARIMA models.

However, all of these papers assume that the trading day effect is constant throughout the series, i.e., the same relationship between the different days of the week exists for each monthly observation. This paper will use a trading day procedure which utilizes the Kalman filter and Kalman smoother to track a changing trading day effect. Gersch and Kitagawa (1982) have already used these methods to model time series with trends and seasonals. The Kalman filter procedure will be tested on simulated monthly time series and will be compared to a trading day adjustment procedure which assumes a constant trading day effect.

II. Description of Data Generation

Since this is a first look at this procedure, we examined a simple situation: data with no trend or seasonal components, just a trading day component and an irregular term. We generated seven series in this manner: two with constant trading day effects and five with moving trading day effects. These series were ten years long, all simulating the period between January 1970 and December 1979. The series were constructed as follows:

$$Y(t) = TD(t) + e(t)$$

where $Y(t)$ is the value of the series at month t, $TD(t)$ is the trading day component at month t, and $e(t)$ is an error term, normally distributed with mean zero and nonzero variance.

The trading day component was generated as follows: let

$d(t)$ = number of days in month t

$d_1(t)$ = number of Mondays in month t

$d_2(t)$ = number of Tuesdays in month t

.
.
.

$d_7(t)$ = number of Sundays in month t.

Then, for i = 1,...,6, let

$$H_i(t) = \frac{d_i(t) - d_7(t)}{d(t)} \quad \text{, for each month t.}$$

We then use as our trading day component

$$TD(t) = \sum_{i=1}^{6} \alpha_i(t) H_i(t)$$

where $\alpha_i(t)$ are our trading day coefficients which define the relationship between the days of the week. The reason why we define our trading day component in this way stems from another representation of the trading day component,

$$TD(t) = \sum_{i=1}^{7} \frac{d_i(t)\beta_i}{d(t)}, \qquad \text{where } \sum_{i=1}^{7} \beta i = 0.$$

$$\text{Now } TD(t) = \sum_{i=1}^{6} \alpha_i(t) H_i(t)$$

$$= \sum_{i=1}^{6} \alpha_i(t) \frac{d_i(t)}{d(t)} - \left[\sum_{i=1}^{6} \alpha_i(t)\right]\left[\frac{d_7(t)}{d(t)}\right]$$

If we let $\beta_7(t) = -\sum_{i=1}^{6} \alpha_i(t)$, $\beta_i(t) = \alpha_i(t)$ for $i = 1,\ldots,6$,

we see that the two representations are equivalent. We use the first because it has better computational qualities.

Table 1 lists all of the series along with information on the nature of the trading day component and the variance of the irregular.

III. The Kalman Filter and Smoother

The Kalman filter and smoother are instrumental in getting estimates for the trading day coefficients defined in section II. In this section, we will set up the Kalman filter equations used to extract the trading day coefficients and define the Kalman smoother used to obtain the final estimates.

In general, the Kalman state equation can be defined as

$$X(t)=F(t-1)X(t-1) + G(t-1)U(t-1)$$

where $X(t)$ = state vector at time t

$F(t)$ = state transition matrix at time t

$G(t)$ = input matrix at time t

$U(t)$ = state innovation at time t

In this case, $X(t) = (\alpha_1(t), \alpha_2(t), \ldots, \alpha_6(t))$, or our trading day coefficients at time t. We now need to decide the structure for our state equation. For the purposes of this study, we used a first difference model, or

$$X(t) = X(t-1) + U(t-1).$$

This means $F(t) = I_{6x6}$, or the 6x6 identity matrix. G(t) is also set equal to the 6x6 identity matrix.

Now, let us do the same for the Kalman measurement equation. Let

$$Y(t) = H(t)X(t) + V(t)$$

where Y(t) = observation at time t

H(t) = output matrix at time t

X(t) = state vector at time t

V(t) = observation innovation at time t.

Here, $H(t) = (H_1(t), H_2(t), \ldots, H_6(t))$, with $H_i(t)$ defined as in section II.

Since U(t) is assumed normally distributed with mean zero and variance-covariance matrix Q and V(t) is assumed distributed normal with mean zero and variance R, we must decide on values for Q and R. We will hold Q and R constant over time. Here, we will introduce a hyperparameter, λ. This λ

will represent the ratio between the variance of the state equation versus the variance of the measurement equation. We will set $Q = \lambda I_{6x6}$ and R=1. It can be shown through application of the Kalman filter equations that if P_0 (our initial estimate of the variance of the state vector), Q(t) and R(t) (remember, Q and R are time invariant) are multiplied by a scale factor, say r_0, then K_t, the Kalman gain matrix at time t, is unchanged and $\hat{P}_t(+)$ and $\hat{P}_t(-)$, the updated and projected estimates of the covariance matrix of the state conditioned on the observations at time t, are altered by $r_0\hat{P}_t(+)$ and $r_0\hat{P}_t(-)$, respectively. If P_0 is large enough, say infinite, or close to it, r_0P_0 will not be much different in its effect from P_0. So, we can find r_0 such that $r_0R = 1$ and $r_0Q = \lambda I_{6x6}$. Since our values for the Kalman gain matrix is unchanged, we will get the same updated estimates for the state matrix.

The Kalman filter was run for different values of λ, and the adjustment for each of these values were compared and evaluated. More will be discussed on this matter later in the paper.

In setting up P_0 and $\hat{X}_0$, we have already mentioned that P_0 should be set at a very high value, or in this case $P_0 = 10^5 \; I_{6x6}$. We do this to reflect our uncertainty in how our estimate of the signal will behave in the adjustment process. In view of the results, this seems to be an appropriate method for choosing P_0. As for $\hat{X}_0$, the initial state estimate, we used a regression model to derive estimates for $\hat{X}_0$. We fit the model

$$Y(t) = B_1H_1(t) + B_2H_2(t) + \ldots + B_6H_6(t) + e(t)$$

on the first thirty-six data points. We had another option: use the actual value for the trading day coefficients used in generating the first observation. Since this information is available to us, we used both the estimated and true values of $\hat{X}_0$ and compared the results. It was found the adjustment using the regression estimated $\hat{X}_0$ differed negligibly from the adjustment using the true value of $\hat{X}_0$.

A program utilizing IMSL subroutines was then used to perform the signal extraction for the model shown earlier. After getting updated versions of the state estimates for each time t, these estimates were then smoothed using the Kalman smoothing algorithm provided by Ansley and Kohn (1982).

The Kalman smoothed estimate for the state vector, denoted as $\hat{X}(t|n)$ for time t, is given by

$$\hat{X}(t|n) = \hat{X}_t(+) + A_t(\hat{X}(t+1|n) - \hat{X}_{t+1}(-))$$

where $A_t = \hat{P}_t(+)F_{t+1}^T \ \hat{P}_{t+1}(-)^{-1}$

$= \hat{P}_t(+)\hat{P}_{t+1}(-)^{-1}$, since $F_t = I$ in this case.

Earlier in the paper we showed if P_0, Q and R are multiplied by a scale factor r_0 the values calculated for $\hat{P}_t(+)$ and $\hat{P}_t(-)$ are changed to $r_0\hat{P}_t(+)$ and $r_0\hat{P}_t(-)$. Say we have an r_0 such that $r_0R=1$, $r_0Q= I_{6x6}$, and P_0 is large enough such that r_0R_0 will not be much different in its effect on the process from P_0. How does this affect A_t? Let $P_t'(+) = r_0\hat{P}_t(+)$ and $P_t'(-) = r_0\hat{P}_t(-)$. The value of A_t for a Kalman filter with values P_0, Q, and R is

$$A_t = \hat{P}_t(+)F_{t+1}^T\hat{P}_{t+1}(-)^{-1}.$$

The value for A_t for a Kalman filter with values r_0P_0, r_0Q and r_0R is

$$\begin{aligned} A_t' &= P_t'(+)F_{t+1}^T P_{t+1}'(-)^{-1} \\ &= r_0\hat{P}_t(+)F_{t+1}^T(r_0\hat{P}_{t+1}(-))^{-1} \\ &= \frac{r_0\hat{P}_t(+)F_{t+1}^T}{r_0\hat{P}_{t+1}(-)} = \frac{\hat{P}_t(+)F_{t+1}^T}{\hat{P}_{t+1}(-)} = A_t. \end{aligned}$$

So our value for A_t is unchanged by multiplying P_0, Q and R by a scale factor; therefore, our smoothed values are going to be the same.

So, for this example, we first set $\hat{X}(120|120) = \hat{X}_{120}(+)$, then work backwards, using the equations above recursively.

After the state estimates are smoothed, they are used to calculate an estimate of the trading day component. We set

$$\hat{TD}(t) = H(t)\hat{X}(t|n).$$

We then get our trading day adjusted series, Y(t), by

$$\hat{Y}(t) = Y(t) - \hat{TD}(t).$$

IV. Constant Trading Day Adjustment

A constant trading day adjustment method was used on these series as well. The regression equation

$$Y(t) = B_1H_1(t) + B_2H_2(t) + \ldots + B_6H_6(t) + e(t) \quad (t = 1,\ldots,120)$$

was used to get trading day coefficients $B_1,\ldots,B_6$. These coefficients were used to get an estimate for the trading day component

$$\hat{TD}(t) = \sum_{i=1}^{6} B_iH_i(t) \quad ,$$

and the trading day adjusted series is

$$\hat{Y}(t) = Y(t) - \hat{TD}(t), \text{ as above.}$$

While this is a crude procedure, it is analogous to the trading day procedure found in CENSUS X-11 and other seasonal adjustment packages. This method will be referred to as the constant adjustment method for the remainder of this paper.

V. Methods of Analysis

For each of the seven data sets listed on Table 1, the Kalman adjustment method described in section III was run for 11 different values of λ, based on powers of four (i.e., $\lambda = 4^i$, $i = -5,\dots,5$). It was hoped that a procedure utilizing the maximum likelihood function would help in finding an appropriate λ; however, for all the series examined the estimate of the log likelihood function increased monotonically as λ increased. In a variable trading day situation, $\lambda > 1$ means the trading day coefficients are likely to be noisier than the irregular, so we would like $\lambda \leqslant 1$. Since the log likelihood procedure prefers values significantly greater than 1, it is clear we need to look at another method for finding an appropriate λ.

Since these series are simulated, we know the true value of the trading day adjusted series. This is e(t), the irregular generated in section II. We can then compare this value with Y(t), the adjusted value of Y(t) obtained by the Kalman filter for some λ. By doing this, not only can we compare the performance of the Kalman adjustment method for different values of λ, but for different values of $\hat{X}_0$ and other adjustment methods as well.

We will define two statistics to measure this property. Let

$$\text{EMSQ} = \sqrt{\sum_{i=1}^{120} \frac{(e(i) - \hat{Y}(i))^2}{120}} \qquad \text{and}$$

$$\text{EMAD} = \sum_{i=1}^{120} \left| \frac{(e(i) - \hat{Y}(i))}{120} \right| .$$

The standard deviation from the true value (EMSQ) and the absolute deviation from the true value (EMAD) will be used to compare different trading day adjustment methods for each series. A method or setting with the smallest EMSQ or EMAD will be judged to be the best.

Another way of examining the adequacy of these methods is graphs. We provide four graphs for each of the seven series. One plots the Kalman adjusted value of $\hat{Y}(t)$ and the true irregular $e(t)$ over time. Here, we get a visual picture of how good the Kalman adjustment method is adjusting the series. The constant adjusted value of $\hat{Y}(t)$ was plotted against the Kalman $\hat{Y}(t)$ and $e(t)$, both over time. These graphs give dramatic evidence for the use of the Kalman filter. Also, a plot of $Y(t)$ and the Kalman $\hat{Y}(t)$ is done versus time.

VI. Analysis and Conclusions

Table 2 contains a summary of the EMSQ and EMAD statistics for both the constant adjustment method and for the Kalman adjustment method performed for an optimal value of λ, meaning the value of λ which gives the lowest EMSQ statistic. We can see from this table that the difference between the Kalman adjustment method performed with the true value of X_0 and the adjustment performed with a regression estimate of X_0 is negligible for all seven series. This is very encouraging, for it shows that the adjustment will not be affected by the value of $\hat{X}_0$ as long as it is reasonable. Therefore, graphs of the adjustment performed with the estimated X_0 are omitted from this paper as they are, for all intents and purposes, precisely the same as those using the true value of $\hat{X}_0$. Table 3 contains the true and estimated X_0 for all seven series.

For our first two series, SIMA1A1 and SIM2A1A1, we have trading day coefficients which remain constant throughout the length of the series. The optimal value of λ in both of these series is 0.000977. This is reassuring, for since there is no variation in the trading day coefficients, Q must equal zero. We would expect the constant adjustment method would work well here, and it does. It is also encouraging to note that the Kalman adjustment method does just as well as the constant adjustment method for λ very small. Note, too, how the variance of the irregular effects the value of our EMSQ and EMAD statistics; the series with the smaller variance has lower EMSQ and EMAD values.

Series SIMA101 is a series in which the trading day components change radically over the course of the series. We can see a wide disparity in the two methods, with the constant adjustment method doing considerably worse than the Kalman adjustment method. It does particularly poorly at the beginning and the end of the series, a pattern we will see in other series.

Series SIMA202 and SIMA303 are similar in that the same general pattern is maintained throughout, but the amplitude of the trading day component changes, increasing for SIMA202, decreasing for SIMA303. This is readily apparent from examining the graphs of the two series. The same pattern emerges here as in SIMA101: the constant adjustment method is much less accurate than the Kalman adjustment method, and the graphs for the constant method for both series shows the constant adjustment method does poorly at the beginning and end of the two series.

In series SIMA404 and SIMA505, an attempt was made to simulate more complicated situations. In SIMA404, the trading day coefficients maintain the same basic pattern, but the amplitude of the trading day effect changes, at first decreasing, then increasing, as shown by the graph of the series. Here, Kalman adjustment again outperforms constant adjustment, but the constant adjustment method is now performing poorly over the whole series, not in specific sections of it as before.

Finally, SIMA505 contains a level shift in the trading day component, i.e., the coefficients are constant for a period of time, then change gradually until reaching a certain point in time, then remaining constant from this point on to the end of the series. By examining the graphs for this series, we can see the Kalman adjustment method does not perform as well here as it did in the past. However, it convincingly outperforms constant adjustment. Again, as in SIMA404, the constant adjustment performs poorly over the entire series.

VII. Areas for Further Study

I have shown in this paper that Kalman filtering and smoothing are effective in removing trading day variation generated by moving trading day, and shown there is a severe penalty paid by the analyst for assuming such series have constant coefficients. Much more work is needed. Some possible areas for further study are listed below:

1) Work must be done to see how these methods perform on series where trend and seasonality are present. This would include working with simulated and real data.

2) A criterion must be established for selecting λ that is reliable, for we will not be able to rely on EMSQ and EMAD statistics when working with real series.

3) This routine should be expanded to include holiday effects, not discussed in this paper.

4) Some work must be done on identifying moving trading day variation. This is a minor consideration as long as the Kalman adjustment method continues to do well on constant trading day effects.

5) Different models can be used in the state equation. This study concentrated on the first difference model, or

$$X(t) = X(t-1) + e(t).$$

Some experimentation should be tried to see if there are some other types of models for this situation; for example, a second difference model of

$$X(t) = 2X(t-1)-X(t-2) + e(t)$$

could be tried for some series.

FOOTNOTE: The author has written preliminary software to perform the Kalman filter and smoother routines described in this paper. They involve the use of IMSL matrix multiplication and inversion subroutines. Interested parties are urged to contact the author for further information.

REFERENCES

Ansley, C. F. and Kohn, R. (1982), "A Geometrical Derivation of the Fixed Interval Smoothing Algorithm," Biometrika, 69, 2, pp. 486-487.

Bell, W. R. and Hillmer, S. C. (1983), "Modeling Time Series with Calendar Variation," to appear in Journal of the American Statistical Association.

Cleveland, W. S. and Devlin, S. J. (1982), "Calendar Effects in Monthly Time Series: Modeling and Adjustment," Journal of the American Statistical Association, 77, pp. 520-528.

Gelb, A., ed. (1974), Applied Optimal Estimation, Cambridge: The M.I.T. Press.

Gersch, W. and Kitagawa, G. (1983), "The Prediction of Time Series with Trends and Seasonalities," to appear in Journal of Business and Economic Statistics.

Young, A. H. (1965), "Estimating Trading-Day Variation in Monthly Economic Time Series," Technical Paper 12, U.S. Bureau of the Census.

TABLE 1: DESCRIPTION OF SERIES

SIMA1A1 :	Constant TD coefficient (1,1,1,1,1,-2.5) $V(e_t) = 0.02$
SIM2A1A1:	Constant TD coefficient (1,1,1,1,1,-2.5) $V(e_t) = 0.005$
SIMA101 :	TD coefficients change gradually from (1,1,1,1,1,-2.5) to (-2,-2,-2,1,2,2) $V(e_t) = 0.02$
SIMA202 :	TD coefficients change gradually from (0.5,0.5,0.5,0.5,0.5, -1.25) to (1,1,1,1,1,-2.5) $V(e_t) = 0.005$
SIMA303 :	TD coefficients change gradually from (-2,-2,-2,1,2,2) to (-1,-1,-1,0.5,1,1) $V(e_t) = 0.0075$
SIMA404 :	TD coefficients change gradually from (-2,-2,-2,1,2,2) to (-1,-1,-1,0.5,1,1) between observation 1 and 60; TD coefficients change gradually from (-1,-1,-1,0.5,1,1) to (-2,-2,-2,1,2,2) between observations 61 and 120 $V(e_t) = 0.0075$
SIMA505 :	TD coefficient constant (1,1,1,1,1,-2.5) for observations 1 to 36; TD coefficients gradually change from (1,1,1,1,1,-2.5) to (1.5,1.5,1.5,1.5,-2,-2)for observations 37 to 84; TD coefficients constant (1.5,1.5,1.5,1.5,-2,-2) for observations 85 to 120. $V(e_t) = 0.0075$

TABLE 2 : SUMMARY OF EMSQ AND EMAD

Series	TD Adjustment	EMSQ	EMAD
SIMA1A1	Constant	0.023454	0.017907
"	TX, λ = 0.000977	0.023474	0.017895
"	EX, λ = 0.000977	0.023476	0.017897
SIM2A1A1	Constant	0.011727	0.008954
"	TX, λ = 0.000977	0.011737	0.008948
"	EX, λ = 0.000977	0.011738	0.008949
SIMA101	Constant	0.176282	0.133321
"	TX, λ = 1.0	0.054572	0.041833
"	EX, λ = 1.0	0.054575	0.041835
SIMA202	Constant	0.048305	0.038974
"	TX, λ = 0.25	0.026314	0.020281
"	EX, λ = 0.25	0.026315	0.020282
SIMA303	Constant	0.057447	0.043426
"	TX, λ = 1.0	0.023662	0.018423
"	EX, λ = 1.0	0.023664	0.018425
SIMA404	Constant	0.057823	0.043932
"	TX, λ = 1.0	0.026595	0.020426
"	EX, λ = 1.0	0.026597	0.020428
SIMA505	Constant	0.079120	0.067264
"	TX, λ = 1.0	0.030006	0.023369
"	EX, λ = 1.0	0.030007	0.023370

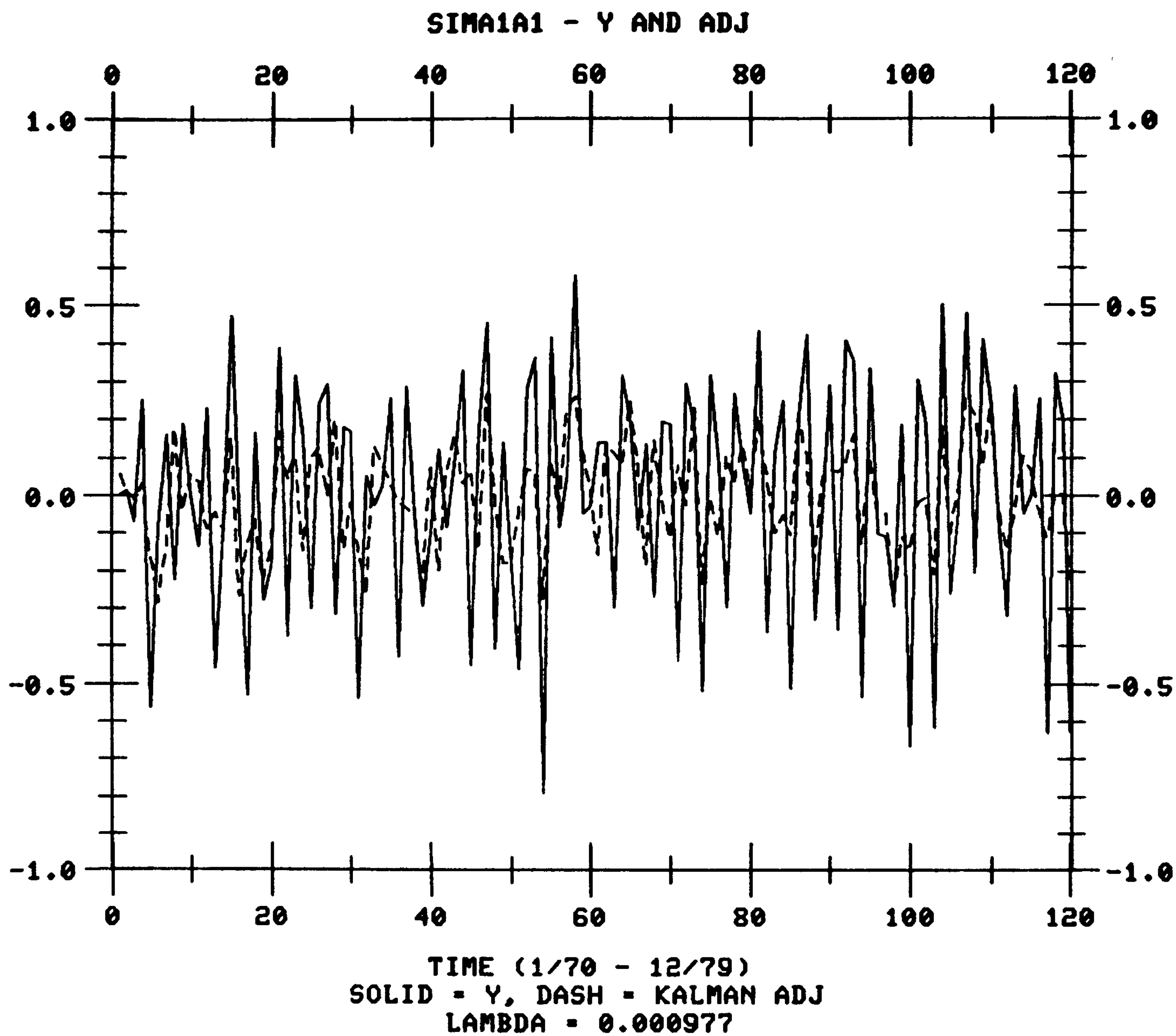
Note: TX = true value of $\hat{X}_0$ used to initialize Kalman filter

EX = regression estimate of $\hat{X}_0$ used to initialize Kalman filter

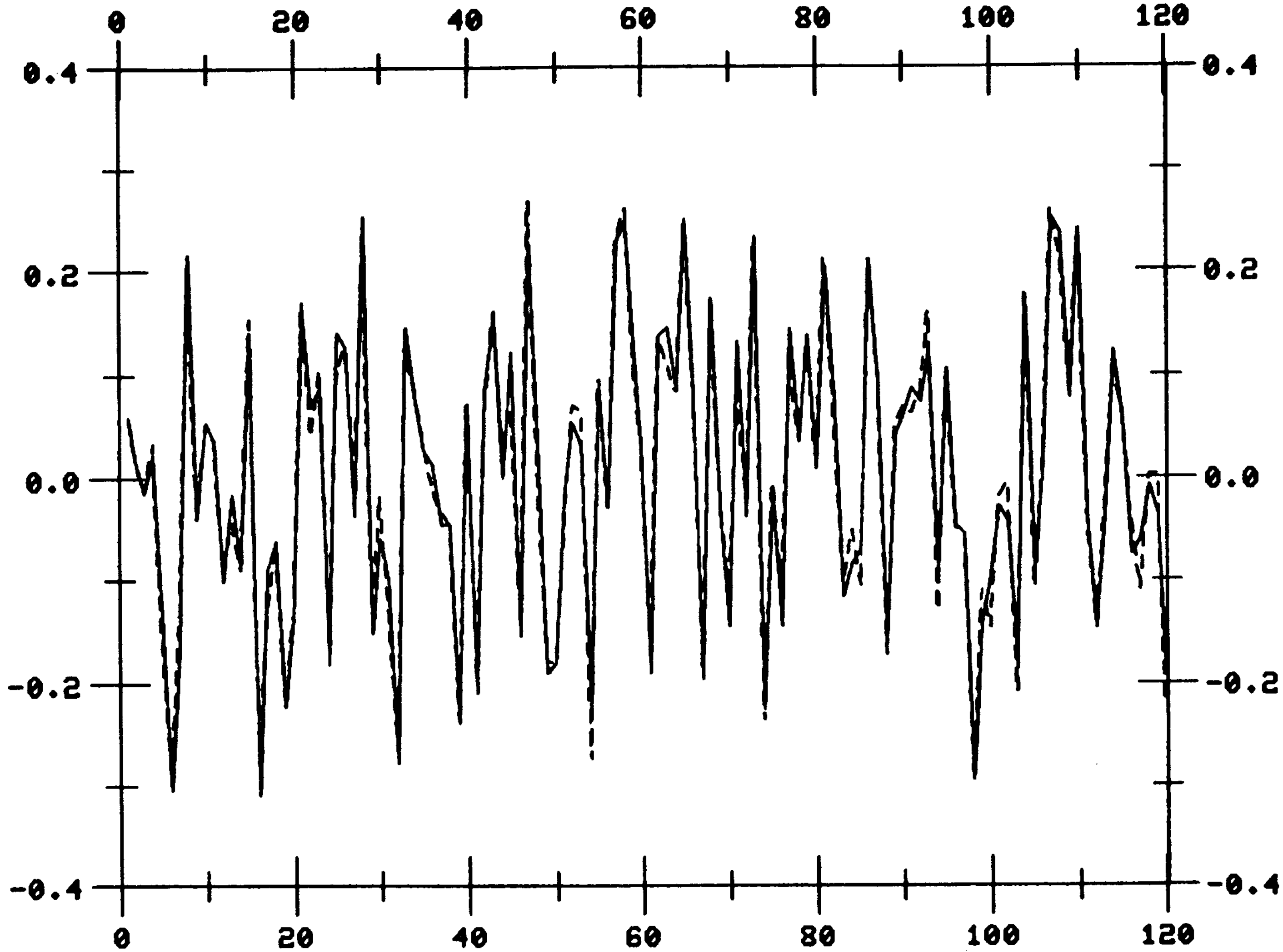
λ = optimal value of λ for each series

TABLE 3 : TRUE AND ESTIMATED $\hat{X}_0$

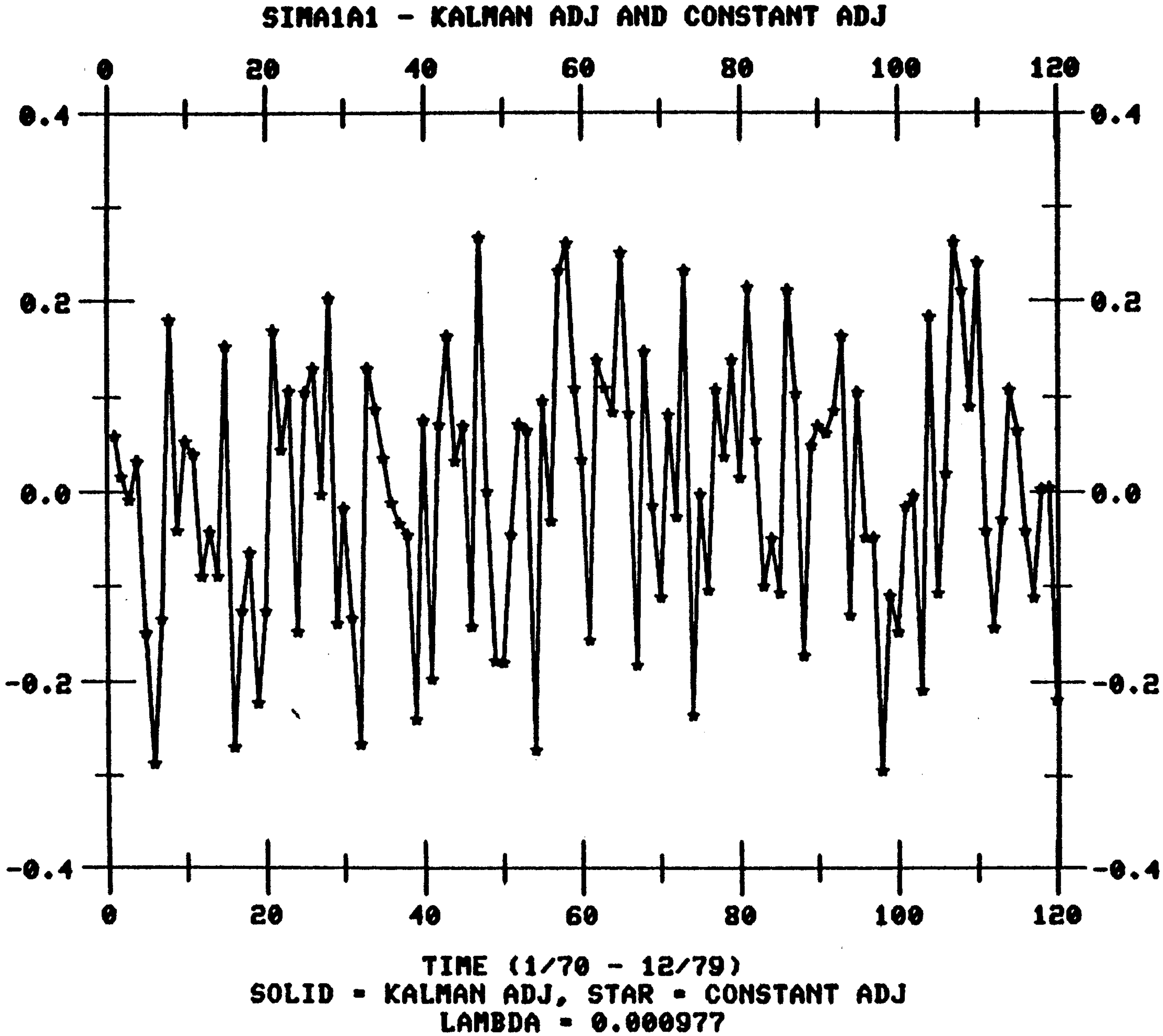
SIMA1A1	True $\hat{X}_0$ -	(1,1,1,1,1,-2.5)
	Est. $\hat{X}_0$ -	(0.851155, 0.697567, 1.442001, 0.595883, 0.156201,-1.405065)
SIM2A1A1	True $\hat{X}_0$ -	(1,1,1,1,1,-2.5)
	Est. $\hat{X}_0$ -	(0.925577, 0.848782, 1.220999, 0.797943, 0.578101,-1.952532)
SIMA101	True $\hat{X}_0$ -	(1,1,1,1,1,-2.5)
	Est. $\hat{X}_0$ -	(0.343387, 0.244778, 1.122321, 0.374926, 0.349021,-0.705228)
SIMA202	True $\hat{X}_0$ -	(0.5,0.5,0.5,0.5,0.5,-1.25)
	Est. $\hat{X}_0$ -	(0.666052, 0.472127, 0.544350, 0.783871, 0.413662,-1.804310)
SIMA303	True $\hat{X}_0$ -	(-2,-2,-2,1,2,2)
	Est. $\hat{X}_0$ -	(-1.767749, -2.370435, -1.378529, 0.887023, 1.696764, 1.747226)
SIMA404	True $\hat{X}_0$ -	(-2,-2,-2,1,2,2)
	Est. $\hat{X}_0$ -	(-1.609216, -2.209012, -1.278722, 0.894362, 1.500150, 1.632996)
SIMA505	True $\hat{X}_0$ -	(1,1,1,1,1,-2.5)
	Est. $\hat{X}_0$ -	(1.076364, 0.470828, 1.523333, 0.879800, 0.890105, -2.640447)


SIMA1A1 - Y AND ADJ
0
20
40
60
80
100
120
1.0
0.5
0.0
-0.5
-1.0
TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 0.000977

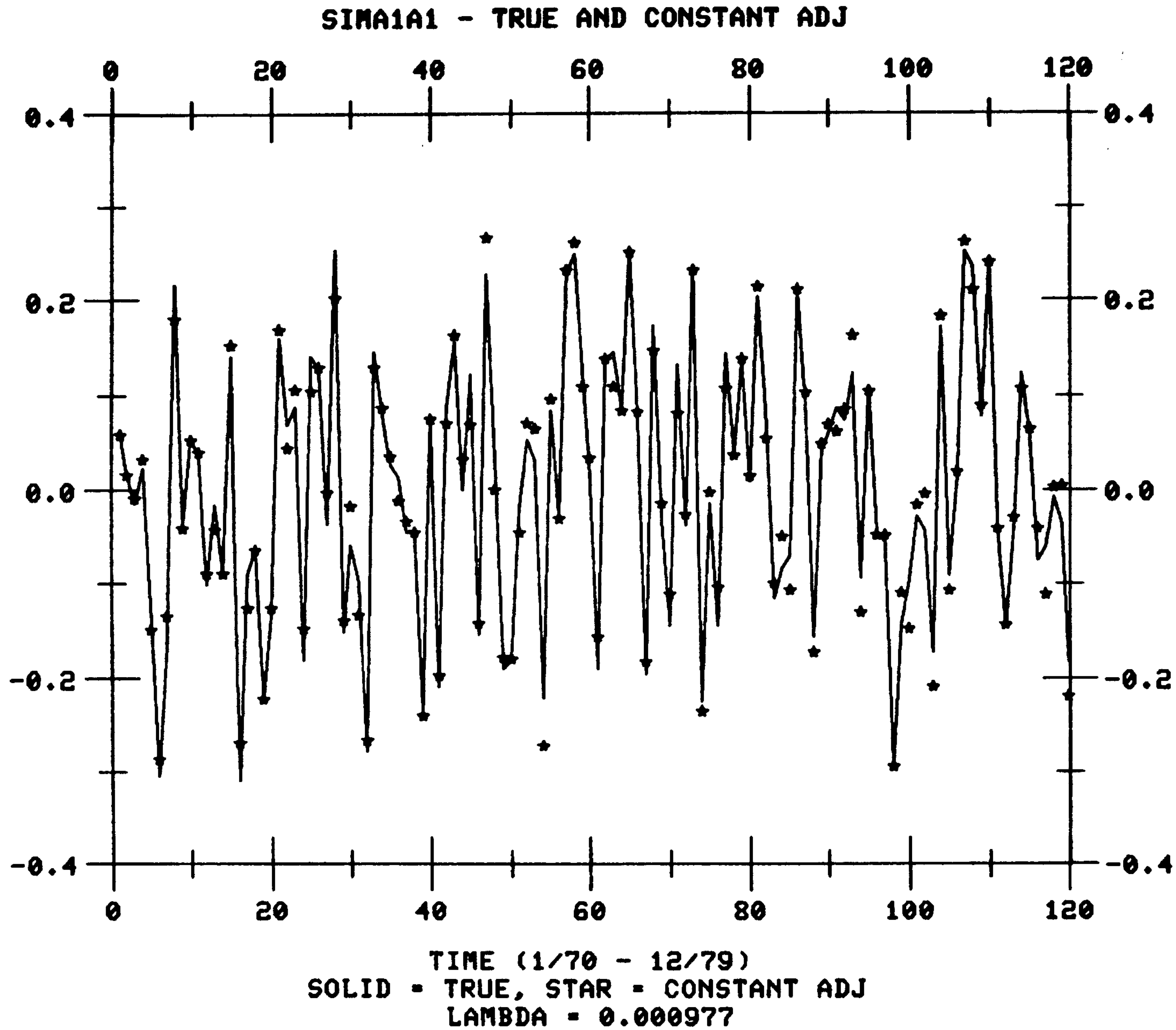
SIMA1A1 - TRUE AND KALMAN ADJ

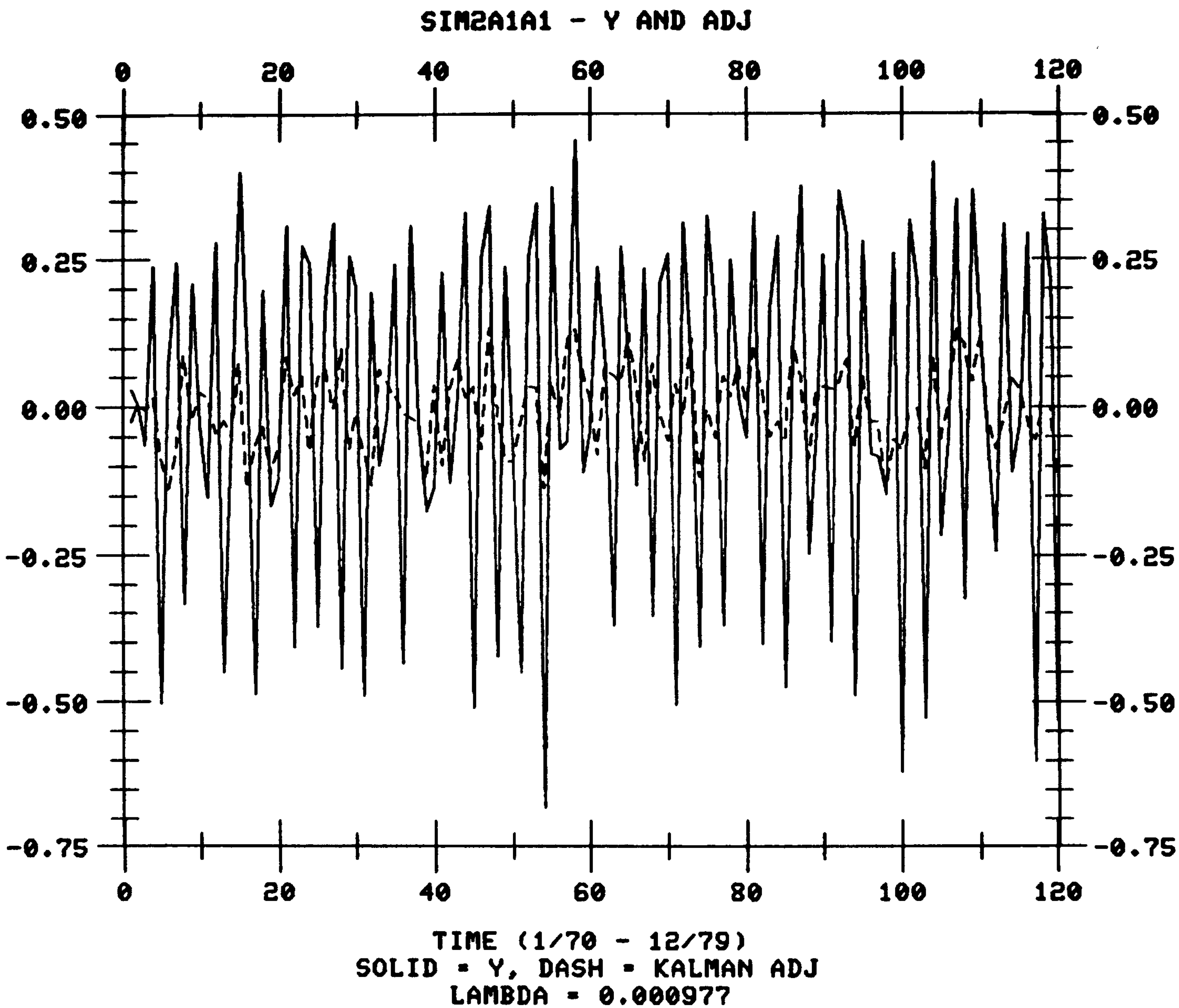


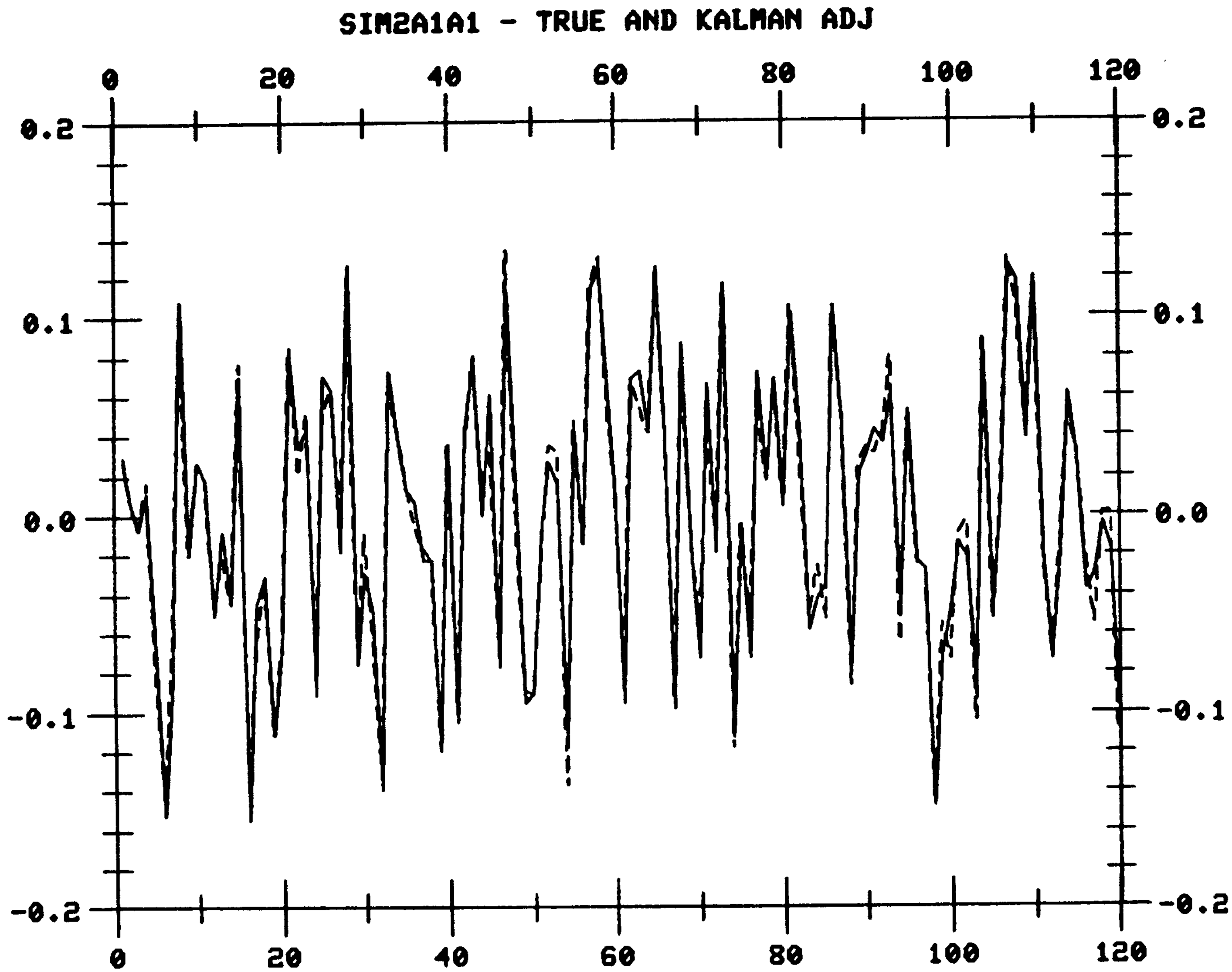
TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 0.000977



SIMA1A1 - KALMAN ADJ AND CONSTANT ADJ
0
20
40
60
80
100
120
0.4
0.2
0.0
-0.2
-0.4
TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 0.000977

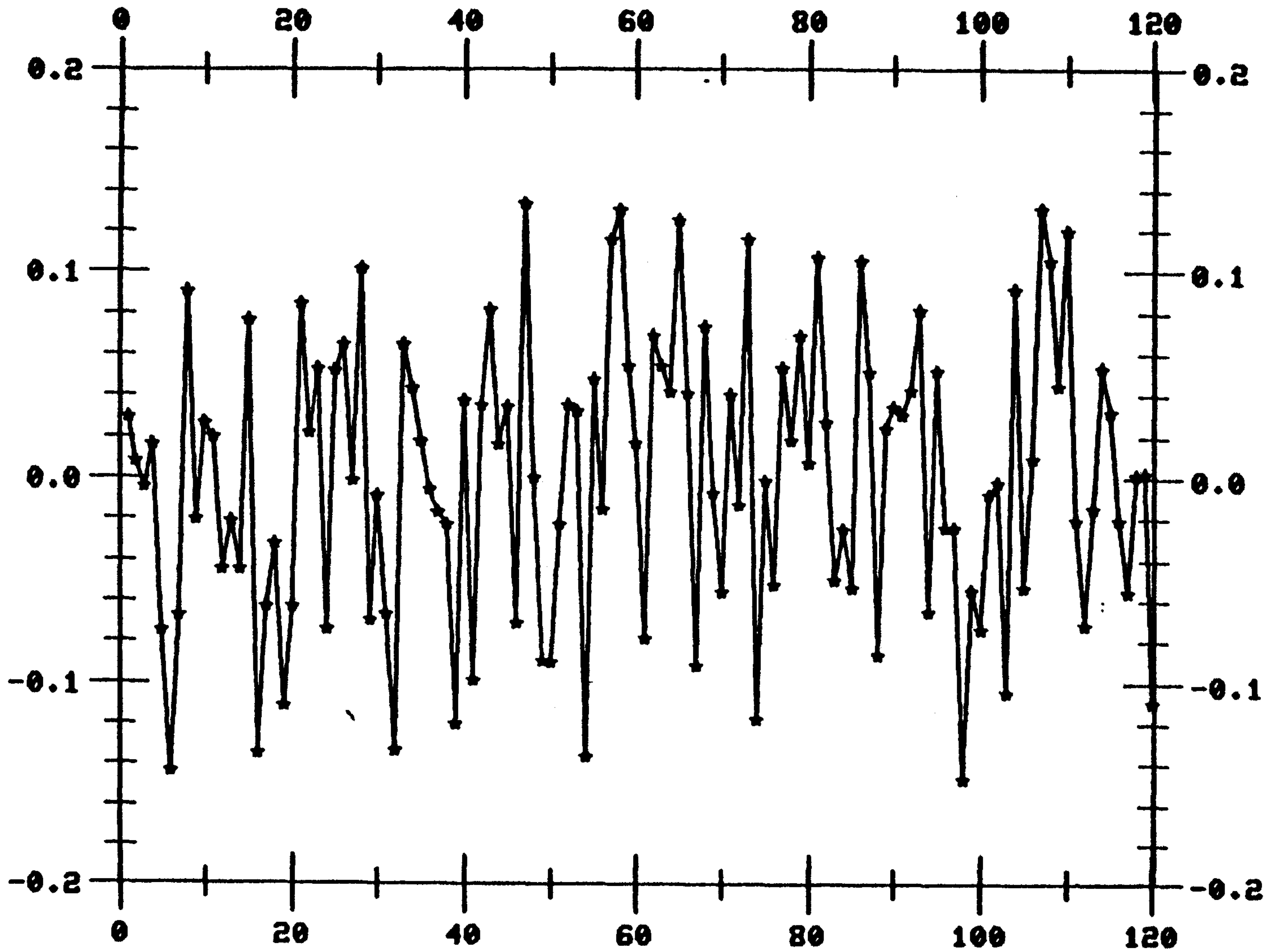

SIMA1A1 - TRUE AND CONSTANT ADJ
0
20
40
60
80
100
120
0.4
0.2
0.0
-0.2
-0.4
TIME (1/70 - 12/79)
SOLID = TRUE, STAR = CONSTANT ADJ
LAMBDA = 0.000977


SIM2A1A1 - Y AND ADJ
0
20
40
60
80
100
120
0.50
0.25
0.00
-0.25
-0.50
-0.75
TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 0.000977



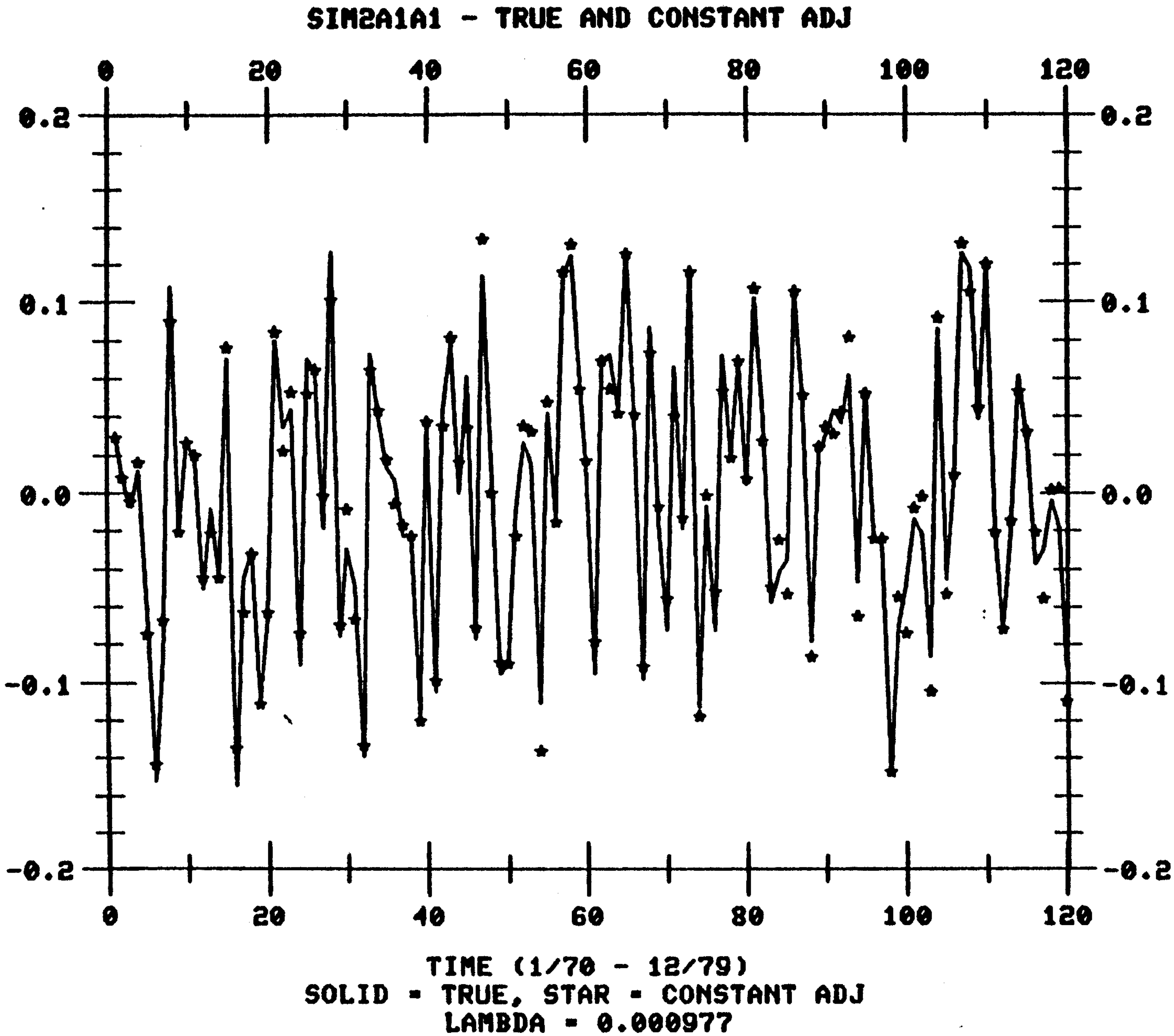
TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 0.000977

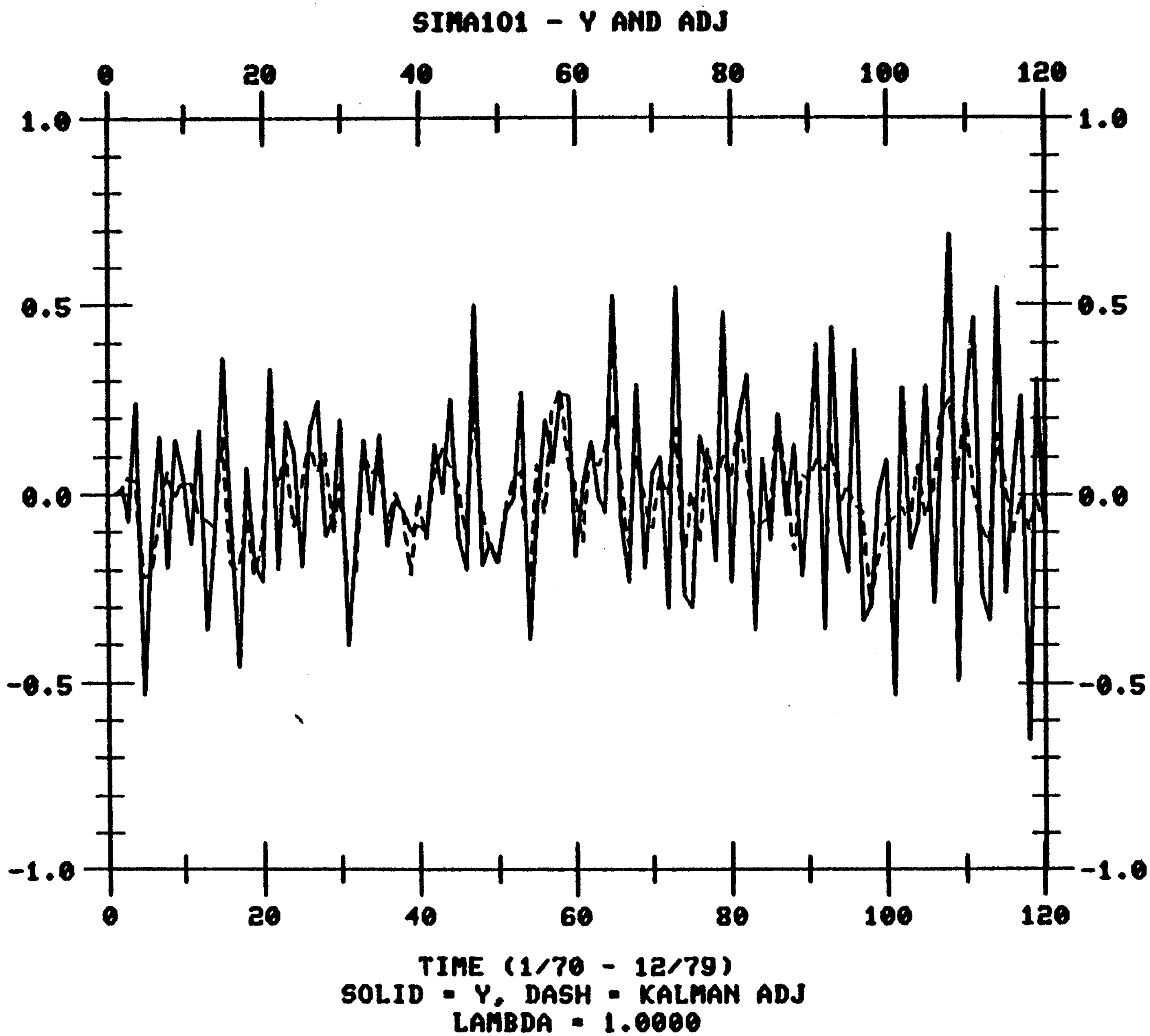
SIM2A1A1 - KALMAN ADJ AND CONSTANT ADJ

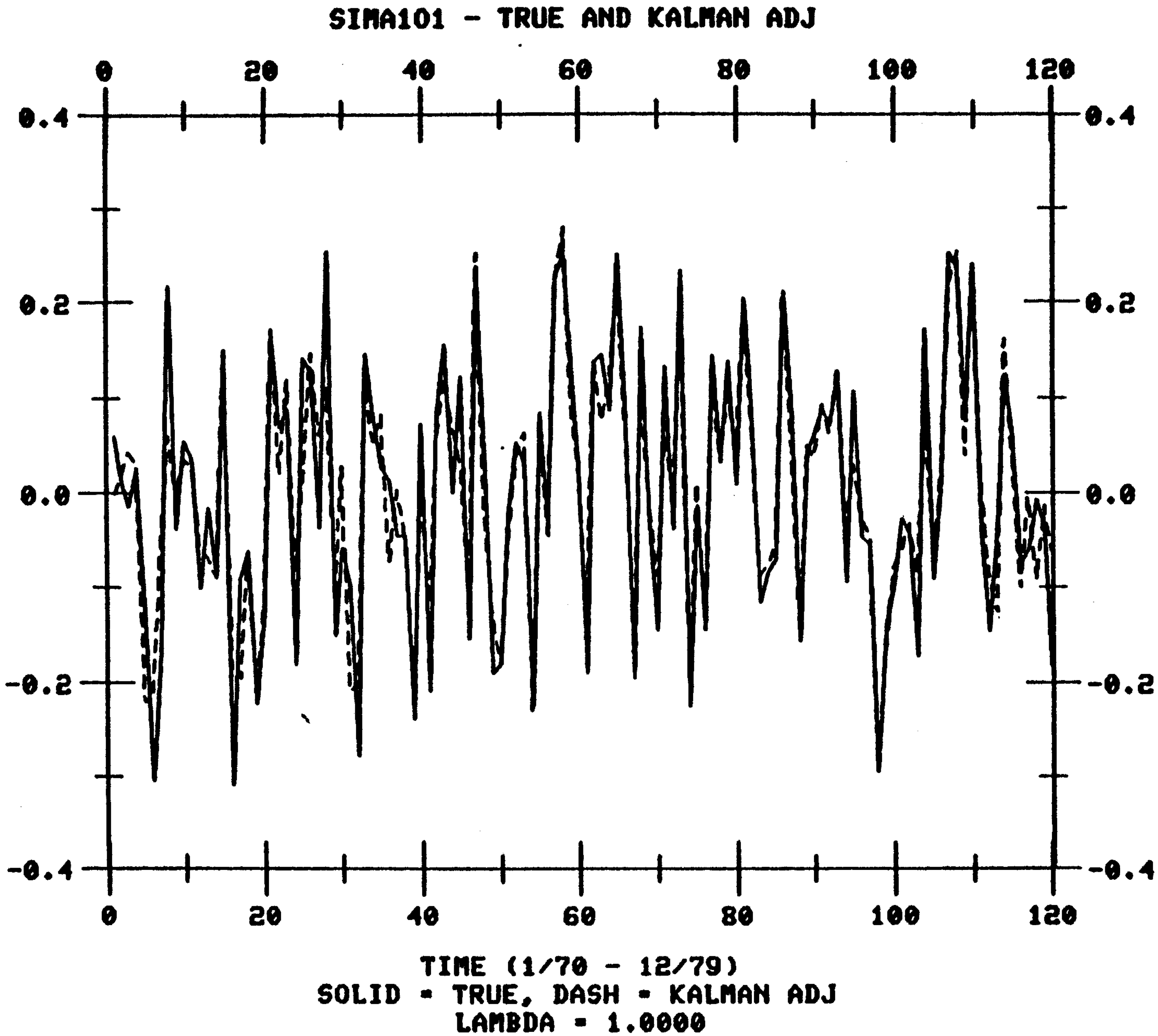


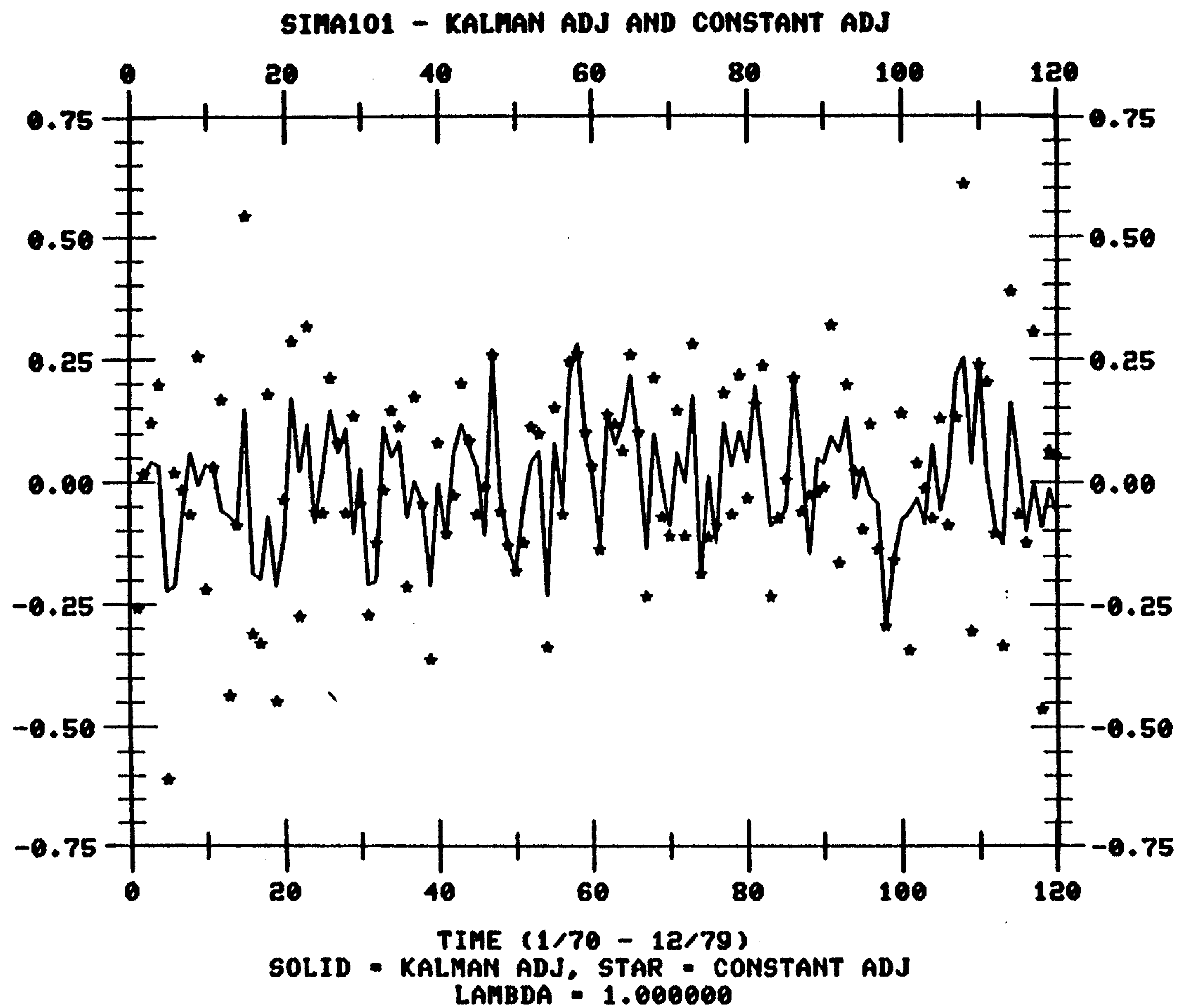
TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 0.000977

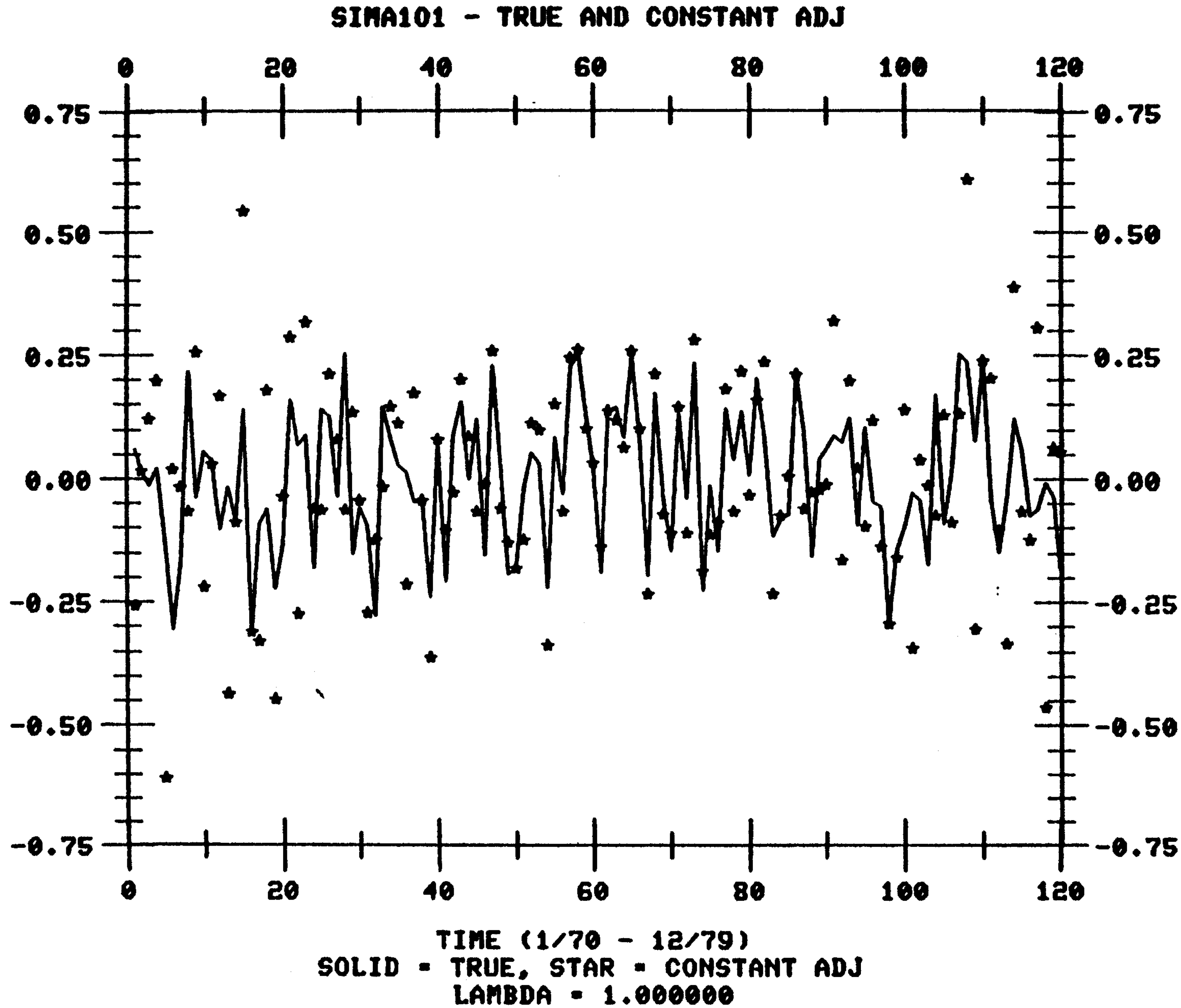
>@ADD


SIM2A1A1 - TRUE AND CONSTANT ADJ
0
20
40
60
80
100
120
0.2
0.1
0.0
-0.1
-0.2
TIME (1/70 - 12/79)
SOLID = TRUE, STAR = CONSTANT ADJ
LAMBDA = 0.000977

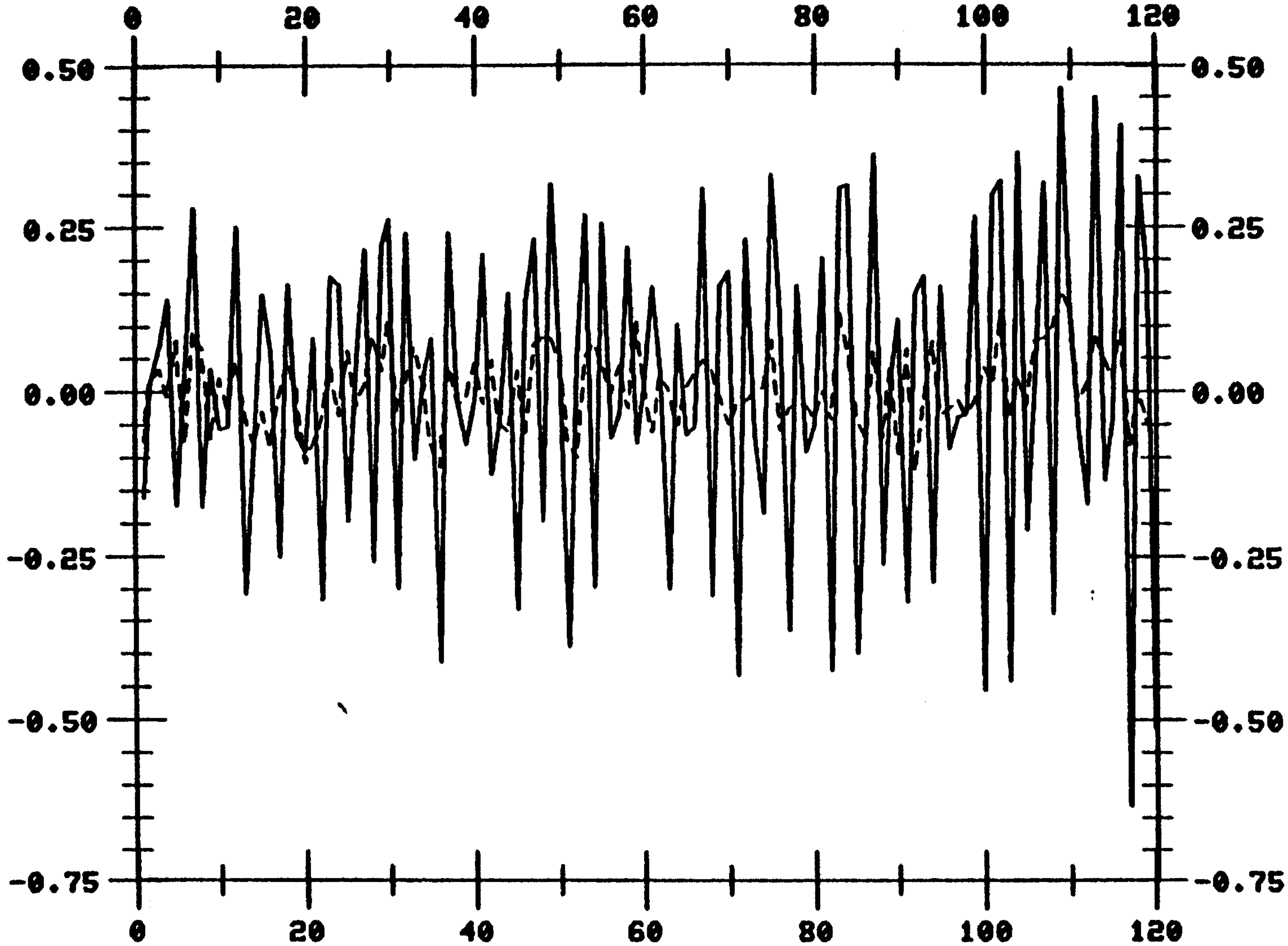

SIMA101 - Y AND ADJ
0
20
40
60
80
100
120
1.0
0.5
0.0
-0.5
-1.0
TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 1.0000


SIMA101 - TRUE AND KALMAN ADJ
0
20
40
60
80
100
120
0.4
0.2
0.0
-0.2
-0.4
TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 1.0000

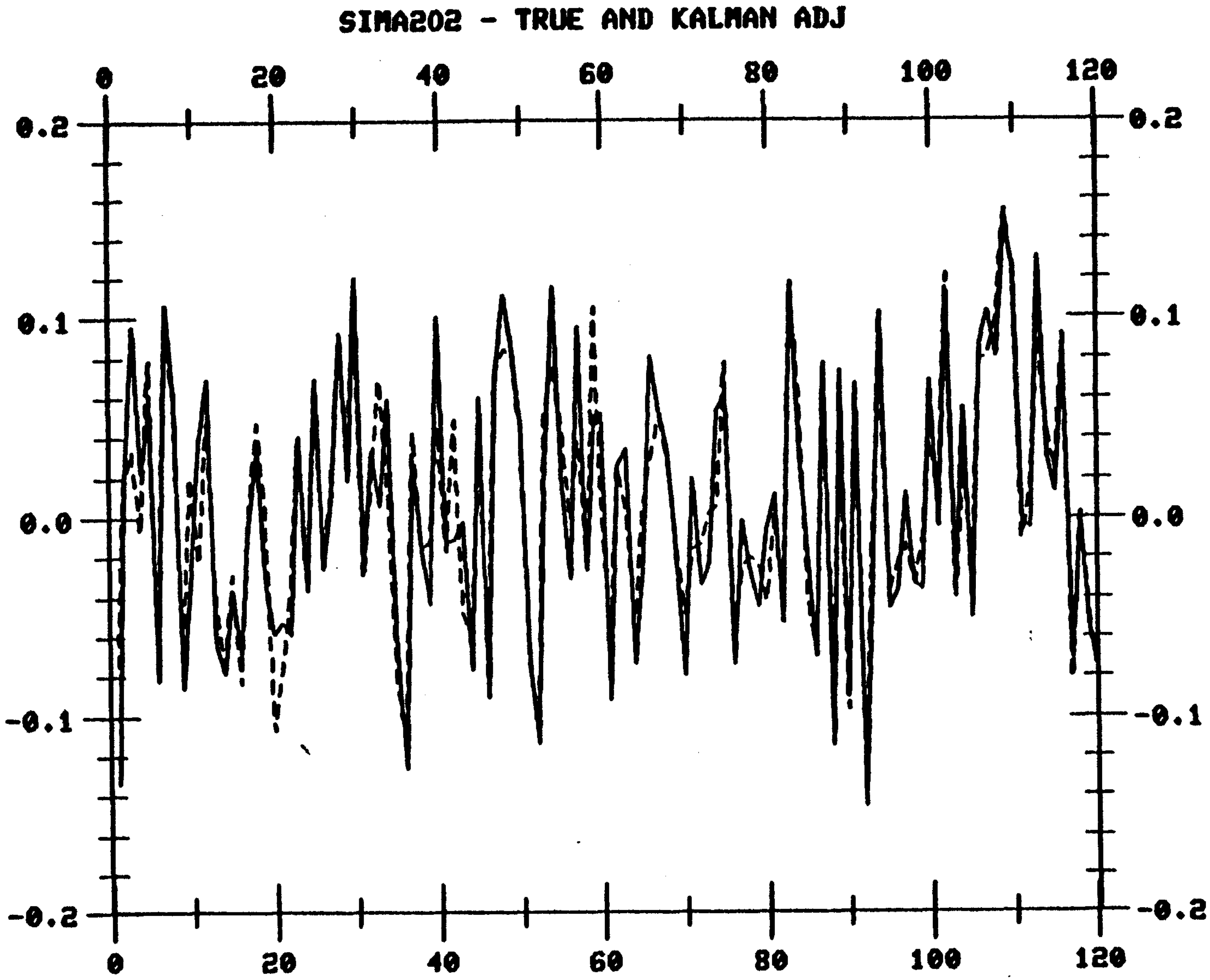

SIMA101 - KALMAN ADJ AND CONSTANT ADJ
0
20
40
60
80
100
120
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 1.000000


SIMA101 - TRUE AND CONSTANT ADJ
0
20
40
60
80
100
120
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
TIME (1/70 - 12/79)
SOLID = TRUE, STAR = CONSTANT ADJ
LAMBDA = 1.000000

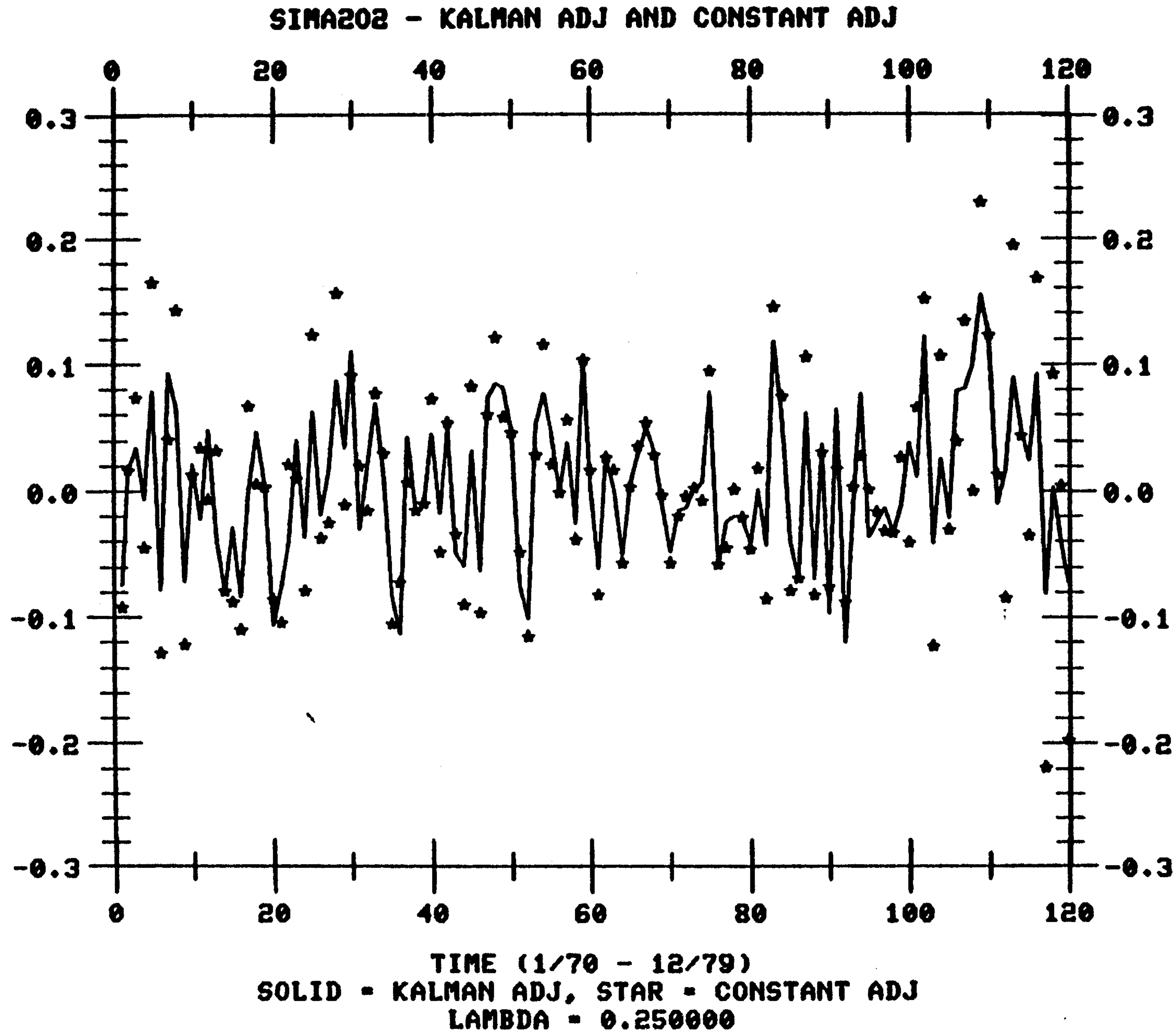
SIMA202 - Y AND ADJ

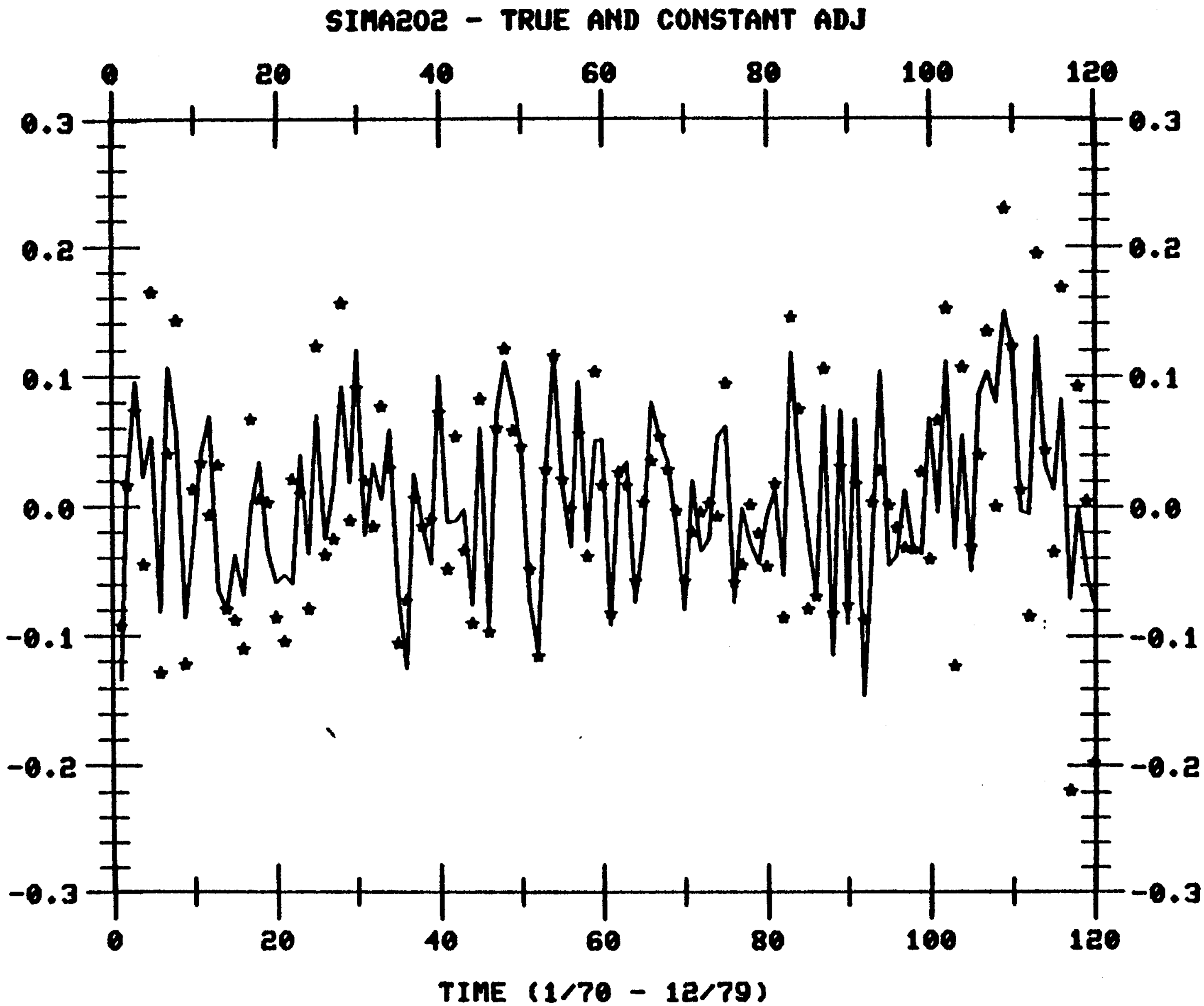


TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 0.2500

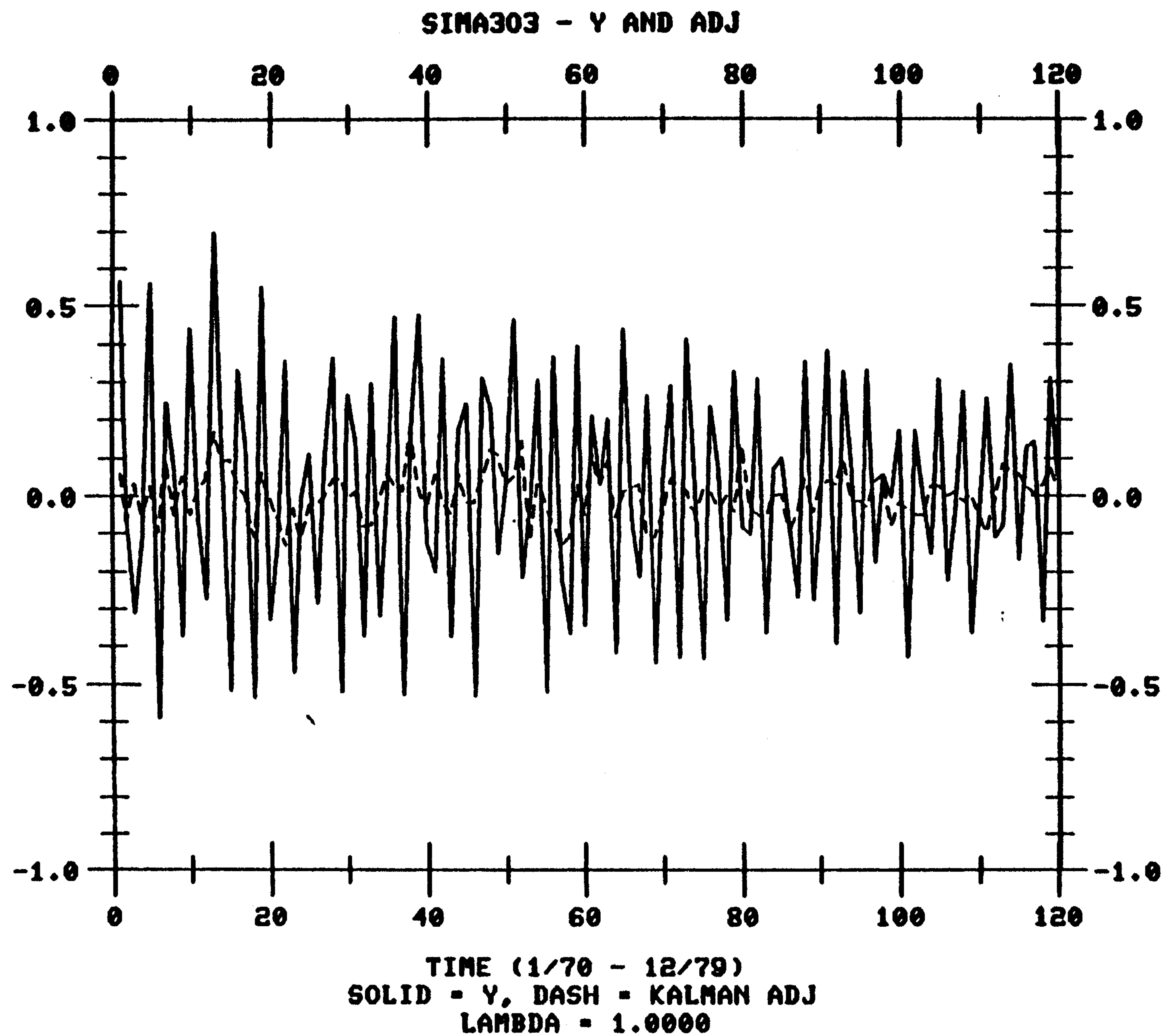


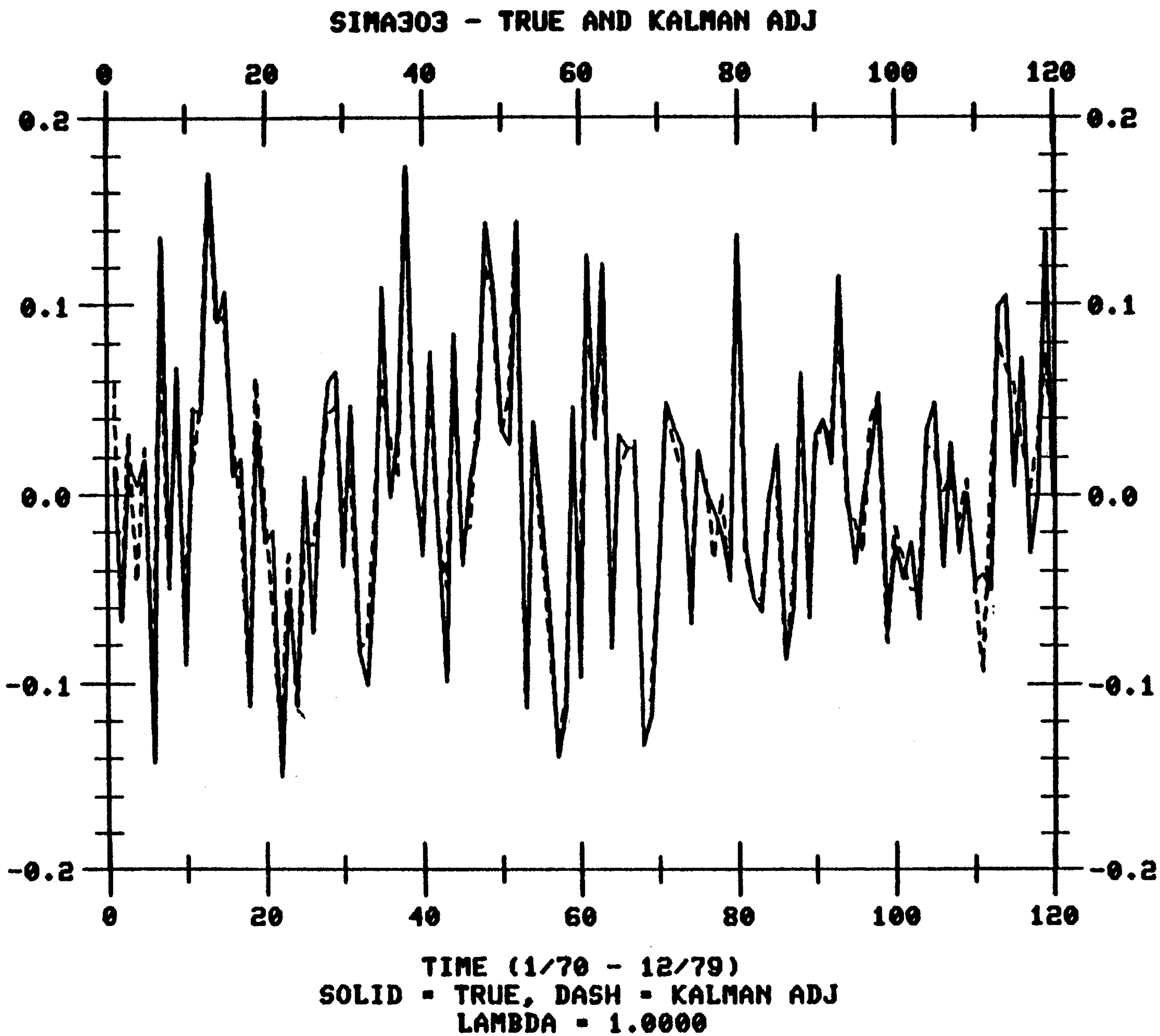
TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 0.2500


SIMA202 - KALMAN ADJ AND CONSTANT ADJ
0
20
40
60
80
100
120
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 0.250000

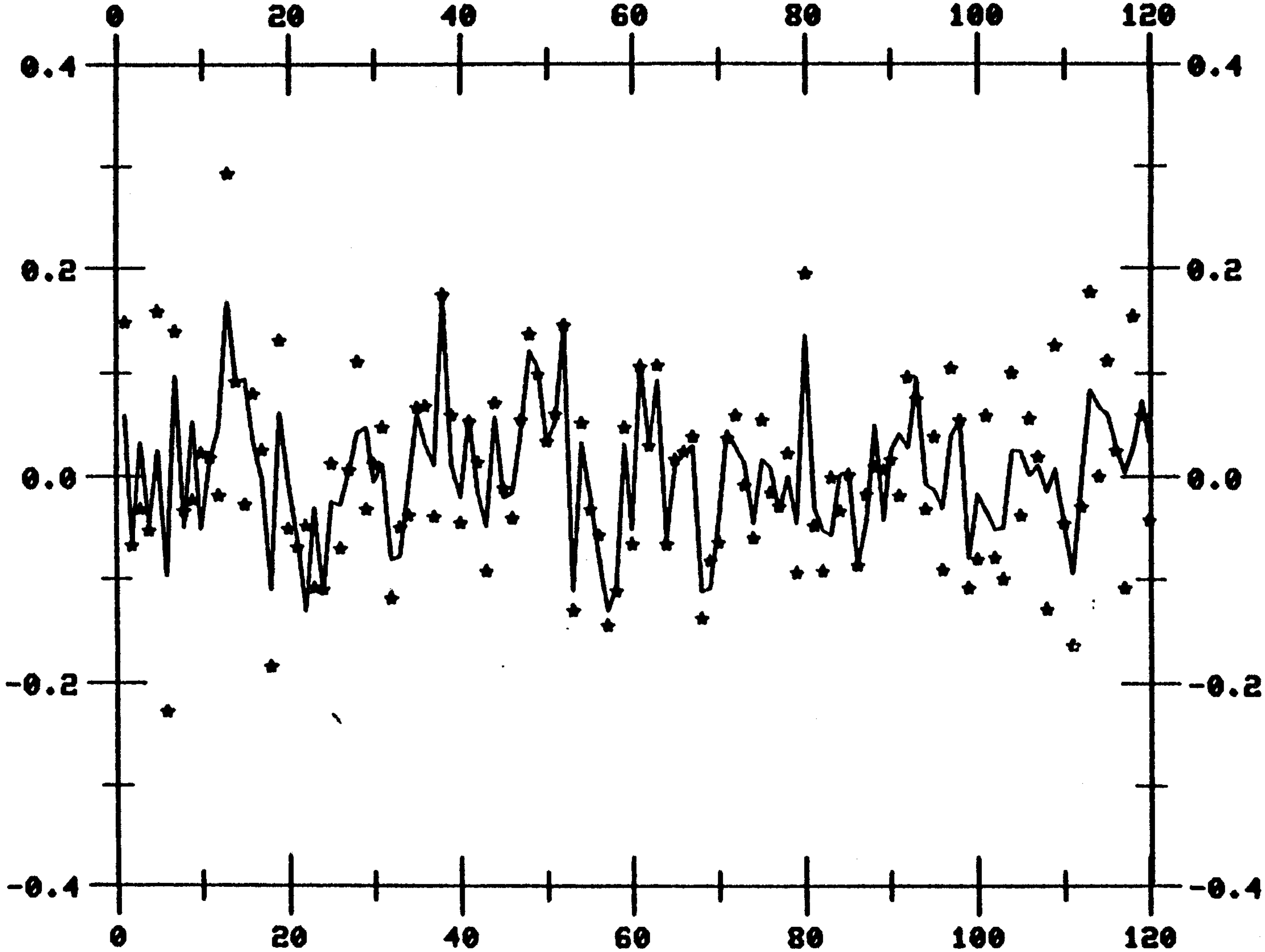


SOLID = TRUE, STAR = CONSTANT ADJ
LAMBDA = 0.250000


SIMA303 - Y AND ADJ
0
20
40
60
80
100
120
1.0
0.5
0.0
-0.5
-1.0
TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 1.0000

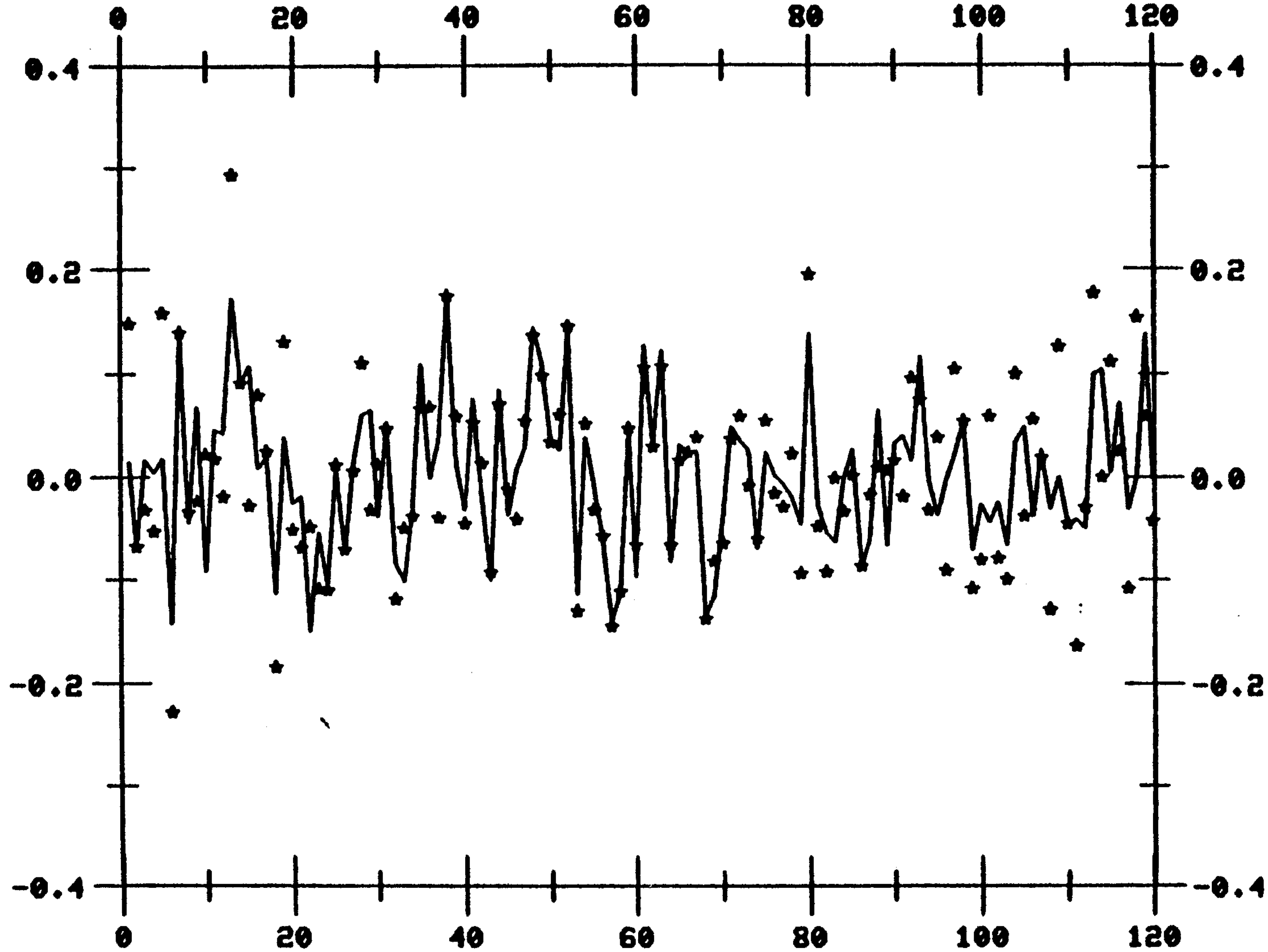

SIMA303 - TRUE AND KALMAN ADJ
0
20
40
60
80
100
120
0.2
0.1
0.0
-0.1
-0.2
TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 1.0000

SIMA303 - KALMAN ADJ AND CONSTANT ADJ

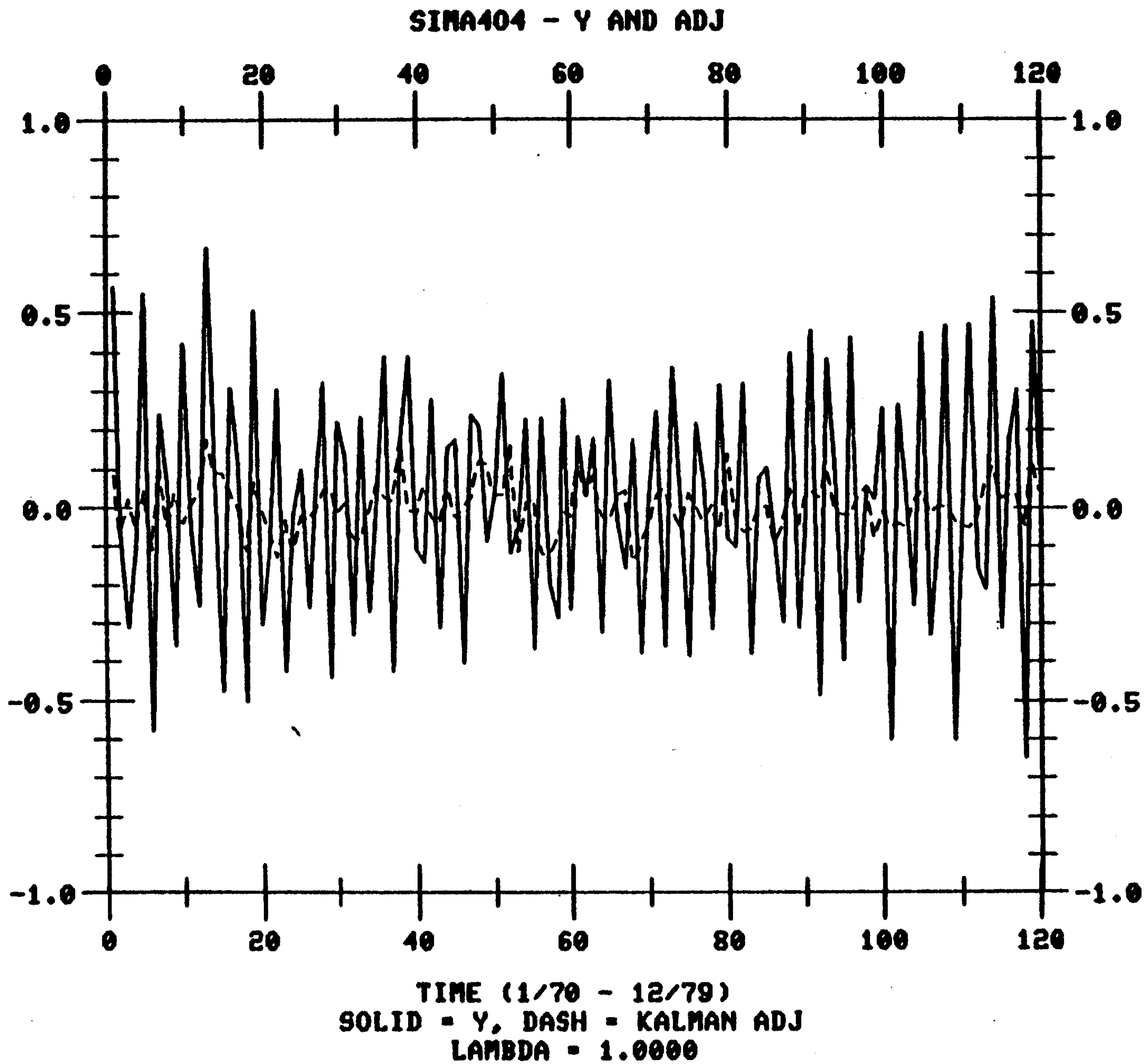


TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 1.000000

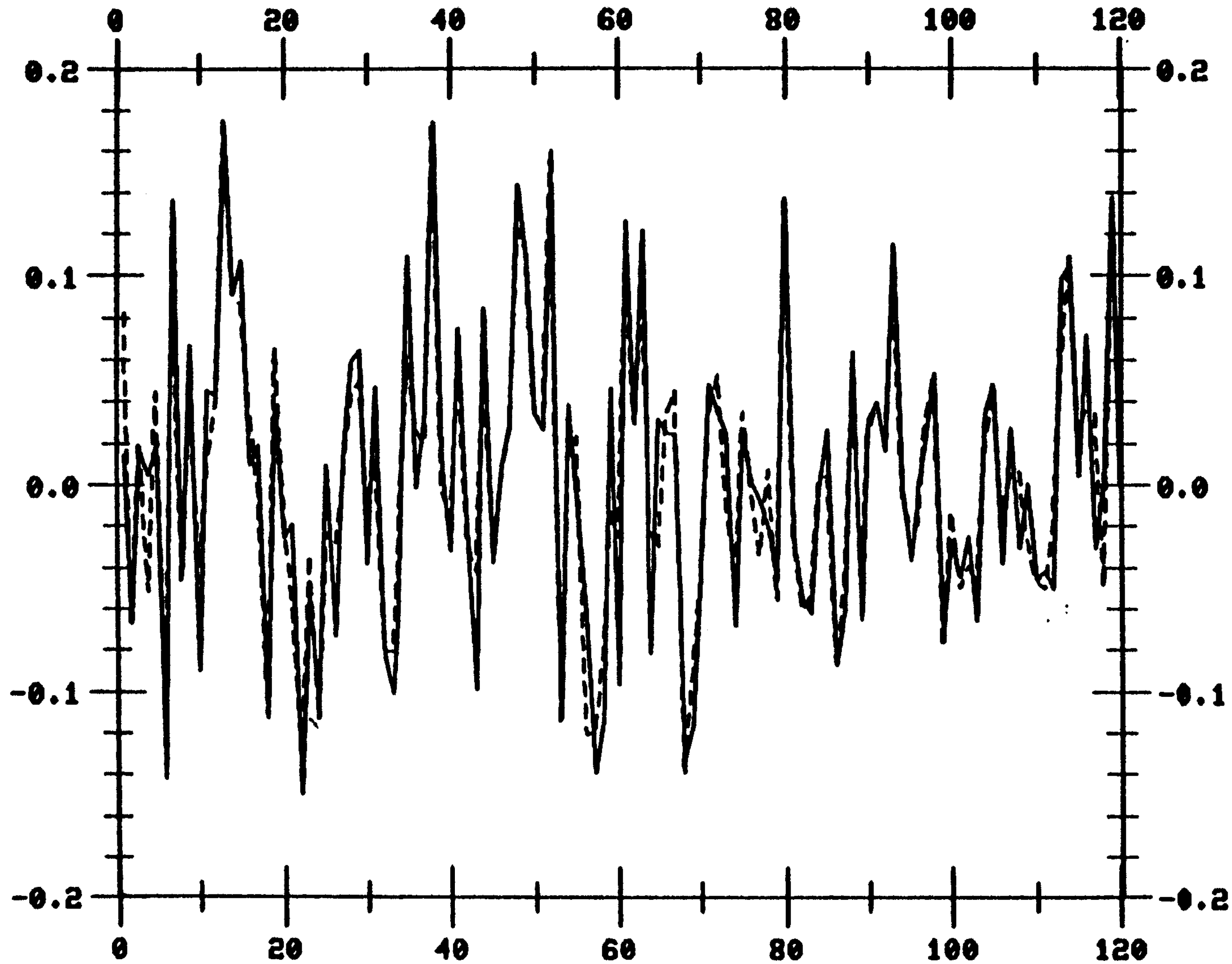
SIMA303 - TRUE AND CONSTANT ADJ



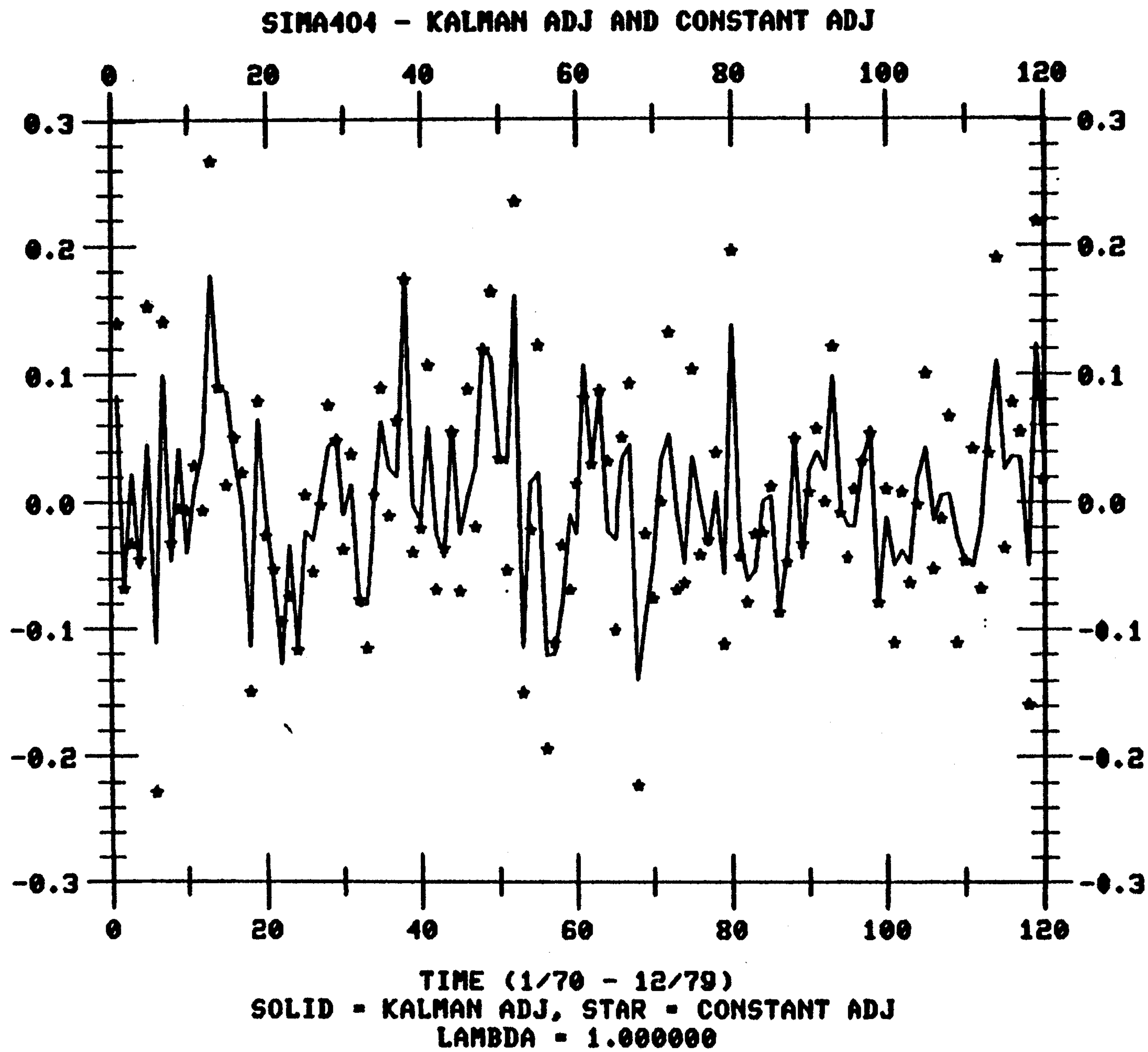
TIME (1/70 - 12/79)
SOLID = TRUE, STAR = CONSTANT ADJ
LAMBDA = 1.000000

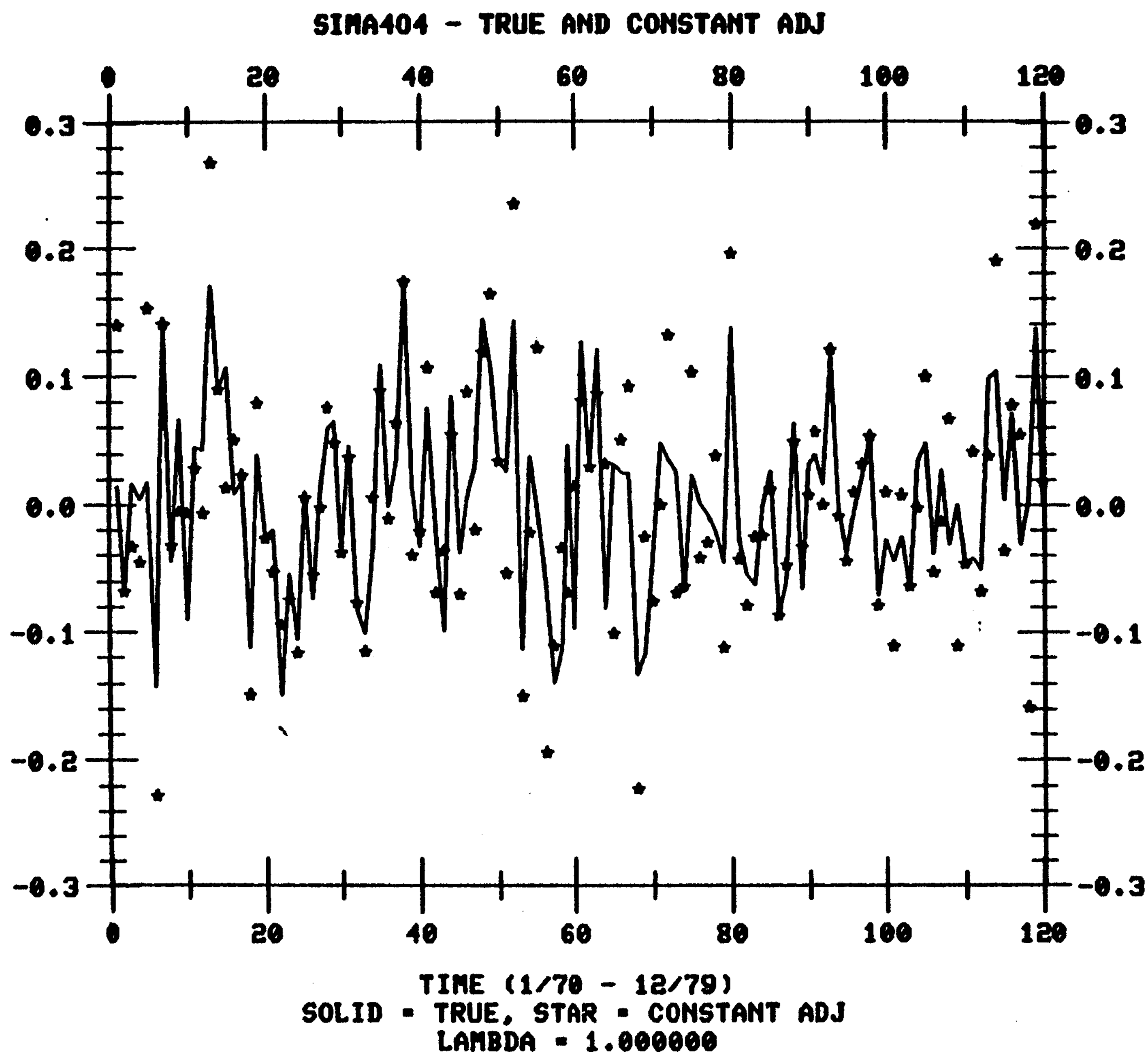

SIMA404 - Y AND ADJ
0
20
40
60
80
100
120
1.0
0.5
0.0
-0.5
-1.0
TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 1.0000

SIMA404 - TRUE AND KALMAN ADJ

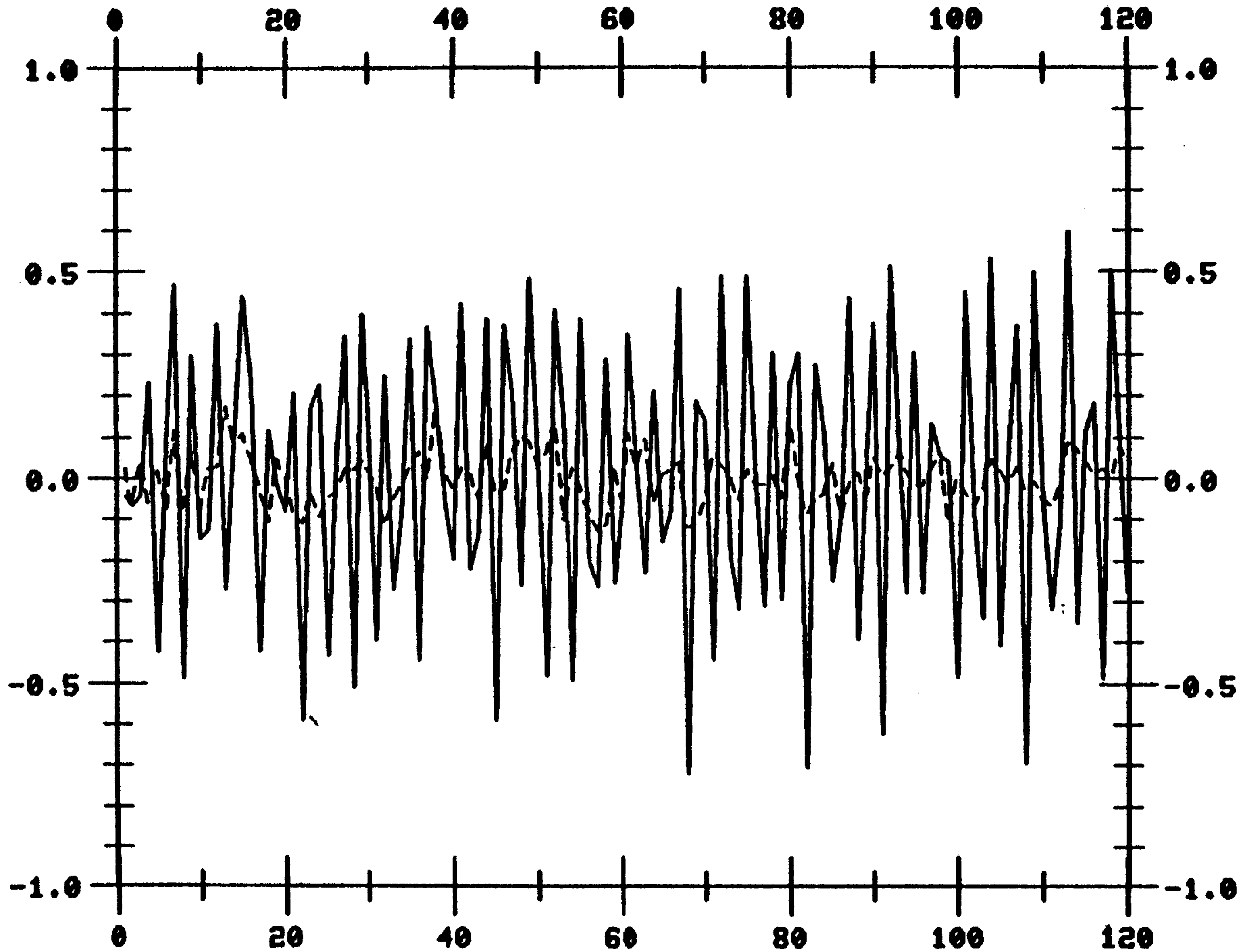


TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 1.0000


SIMA404 - KALMAN ADJ AND CONSTANT ADJ
0
20
40
60
80
100
120
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 1.000000

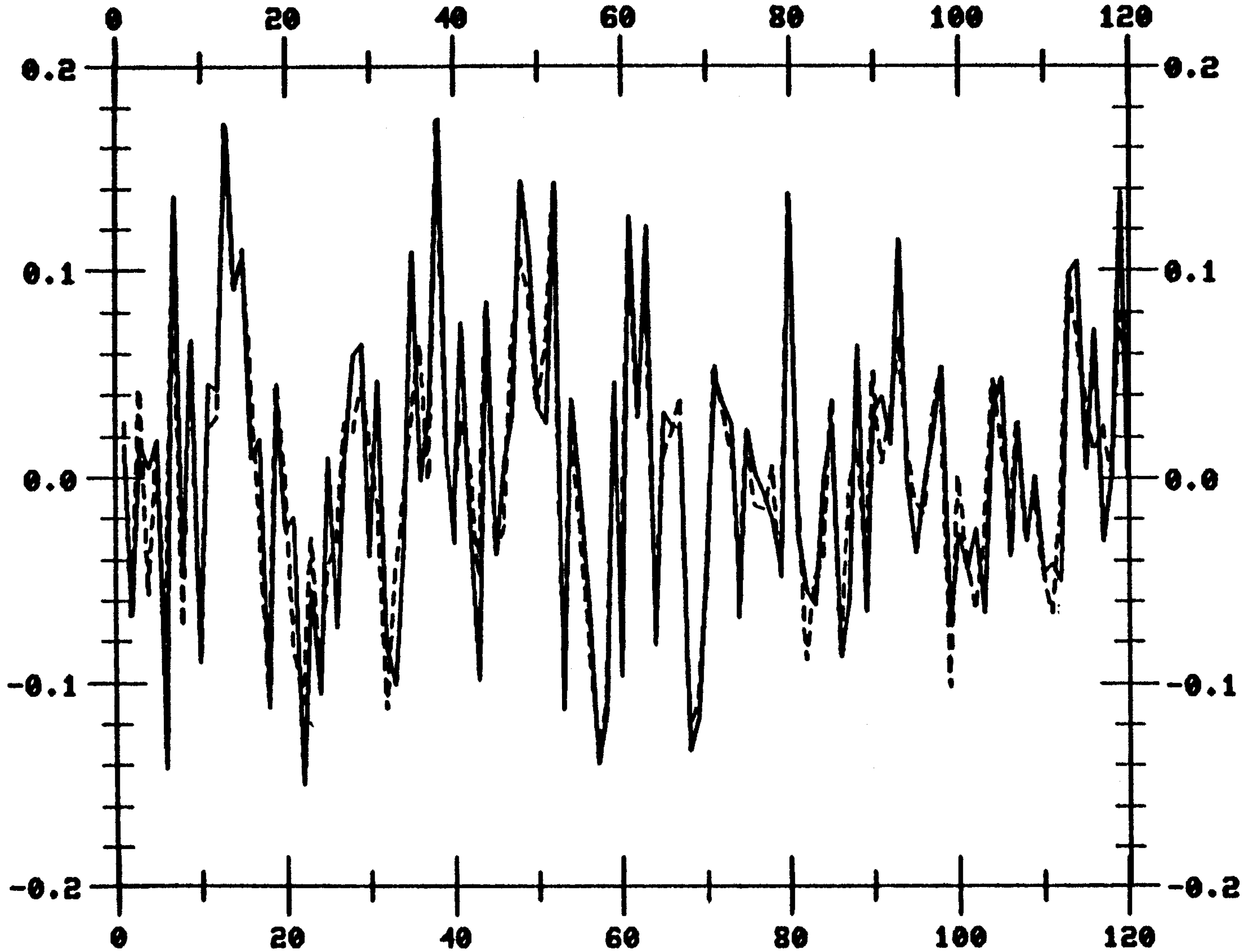

SIMA404 - TRUE AND CONSTANT ADJ
0
20
40
60
80
100
120
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
TIME (1/70 - 12/79)
SOLID = TRUE, STAR = CONSTANT ADJ
LAMBDA = 1.000000

SIMA505 - Y AND ADJ

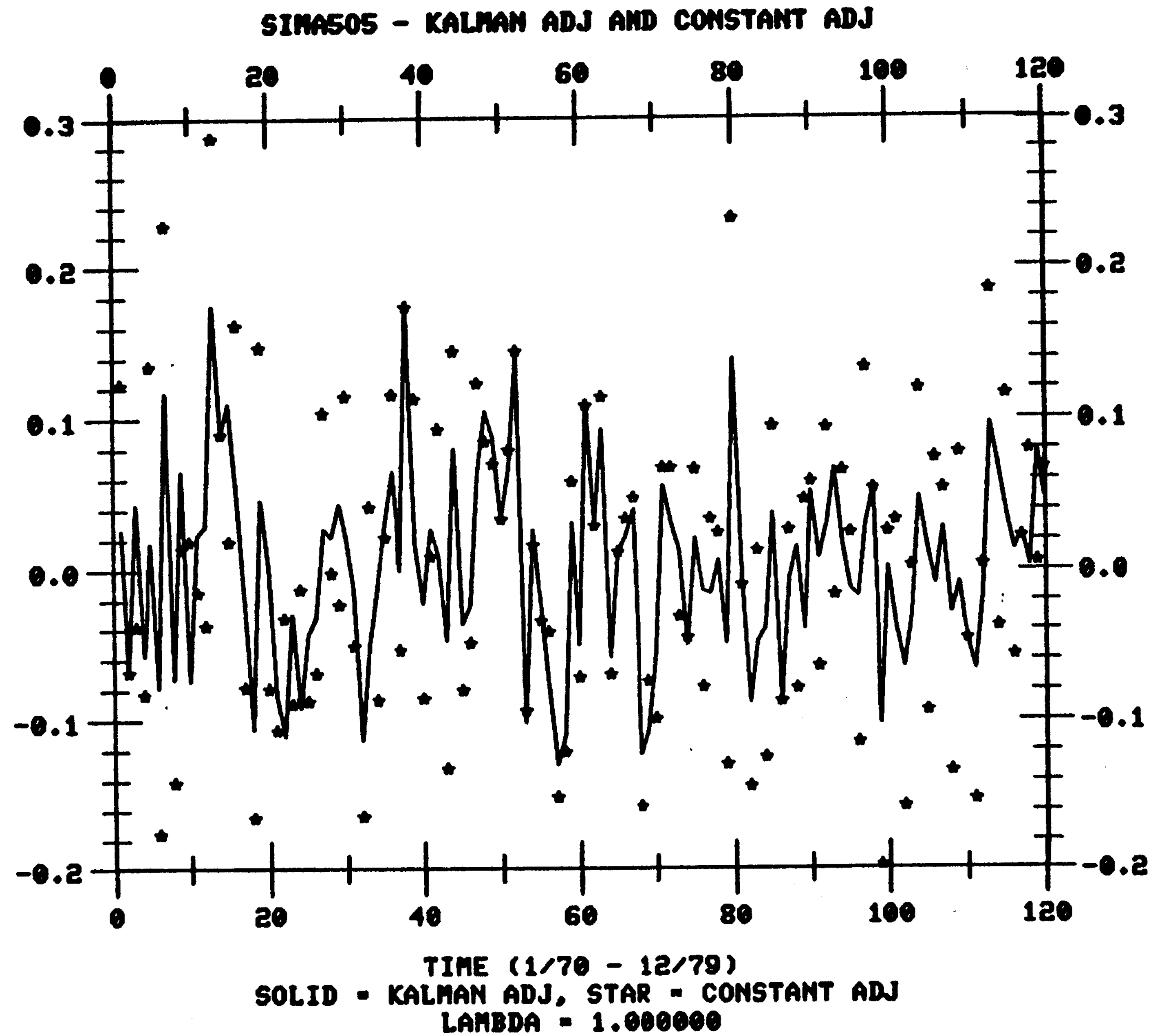


TIME (1/70 - 12/79)
SOLID = Y, DASH = KALMAN ADJ
LAMBDA = 1.0000

SIMA505 - TRUE AND KALMAN ADJ



TIME (1/70 - 12/79)
SOLID = TRUE, DASH = KALMAN ADJ
LAMBDA = 1.0000


SIMA505 - KALMAN ADJ AND CONSTANT ADJ
0
20
40
60
80
100
120
0.3
0.2
0.1
0.0
-0.1
-0.2
TIME (1/70 - 12/79)
SOLID = KALMAN ADJ, STAR = CONSTANT ADJ
LAMBDA = 1.000000