

# A Scheme of Moist Convective Adjustment

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**ABSTRACT**—A new method of moist convective adjustment is presented. A large-scale state of the atmosphere is assumed to be in a thermodynamically critical condition when a hypothetical cloud element can develop. As a result of the convective adjustment, the atmosphere is altered to a new state that is marginal or unfavorable to the occurrence of a free moist convection. The numerical scheme of the adjustment is described in detail.

## 1. INTRODUCTION

How to incorporate the effect of the ensemble cumulus convection into a grid-scale thermohydrodynamical system is one of the important but underdeveloped problems in the modeling of the large-scale atmospheric circulation.

In one type of parameterization of cumulus convection, the hypothesis of penetrative convection is used to formulate the mechanism of the conditional instability of the second kind (e.g., Ooyama 1964). Later, Ooyama (1971) stated the philosophy upon which a more sophisticated parameterization of this kind should be developed; he stressed that cloud physics must be embodied in the parameterization and that the statistics concerning cloud ensembles must be known.

The so-called convective adjustment is another type of parameterization (Manabe et al. 1965). In this method, it is assumed that there exists a critical state for the large-scale thermodynamical field. When the field tends to become unstable, it is adjusted, under some constraints, to a new stable or mild state. This process of adjustment is presumed to be a substitute for the convective process. The problems in adjustment methods are, therefore, (1) determination of a critical state of the large-scale field or the condition for the convection to occur, and (2) formulation of a scheme by which the large-scale field is altered to a new state.

In the adjustment methods proposed so far, the critical relative humidity has to be prescribed to determine the condition of adjustment (e.g., Miyakoda et al. 1969). In this paper, we attempt to remove this requirement despite the complication of the adjustment process. In the new scheme, a large-scale field of temperature and mixing ratio is adjusted if the condition for the development of a hypothetical cloud element is met. The adjusted environment is considered to be neutral for the convection of such an element.

In section 2, we derive the condition for free moist convection. In section 3, we explain how to adjust a large-scale field to a neutral state. The detailed computation scheme is described and some examples are presented in sections 4 and 5, respectively.

## 2. CONDITION FOR FREE MOIST CONVECTION

### Temperature Change of a Rising Cloud Element

First, we derive an expression for the temperature change of a rising cloud element in terms of the two large-scale field quantities, temperature,  $T$ , and mixing ratio,  $r$ . Let us consider a mass integral of the sum of potential energy and enthalpy of a cloud element. Such a mass integral will be conserved if a system is closed, the process is moist-adiabatic, and the contribution of kinetic energy to the energy integral is negligible. When a system is open and environmental air is entrained into a cloud, the change in the mass integral is given by

$$d[m(c_p T + gz + Lr)]_c = dm(c_p T + gz + Lr)_e \quad (1)$$

where  $m$  is the mass of moist air,  $c_p$  is specific heat at constant pressure,  $g$  is the acceleration of gravity,  $z$  is height,  $L$  is the latent heat of condensation, and  $r$  is the mixing ratio of water vapor. Subscripts  $c$  and  $e$  are used to indicate a quantity for the cloud element and for the environment, respectively.

By definition,  $r_c$  is the saturation mixing ratio at  $T_c$ . It is approximately equal to  $\epsilon e_s/p_c$  where  $\epsilon=0.622$ ,  $p_c$  and  $e_s$  are respectively the saturation air and vapor pressures at  $T_c$ . We assume that  $p_c=p_e (=p)$  at a given height and that the hydrostatic relation holds in the environment. Defining the rate of entrainment by  $E=(1/m)(dm/dz)$ , we rewrite eq (1) as

$$\left(c_p + L \frac{\epsilon de_s}{p dT}\right) \left(\frac{dT}{dz}\right)_c + L \frac{\epsilon e_s}{p} \frac{g}{RT_e} + g = E[c_p(T_e - T_c) + L(r_e - r_c)]. \quad (2)$$

This is the formula derived by Stommel (1947).

If there is no entrainment (i.e.,  $E=0$ ), we obtain the moist adiabatic lapse rate,

$$\gamma_m = -\left(\frac{dT}{dz}\right)_c = \frac{g \left(1 + \frac{L \epsilon e_s}{p R T_e}\right)}{c_p \left(1 + \frac{L \epsilon}{p c_p} \frac{de_s}{dT}\right)}. \quad (3)$$

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Since the right side of eq (2) is generally negative, the temperature change of a cloud element [i.e.,  $-(dT/dz)_c$ ] becomes larger than  $\gamma_m$  when the effect of entrainment is taken into consideration.

Let us now estimate the quantities  $(T_e - T_c)$  and  $(r_e - r_c)$  on the right side of eq (2). We shall consider a cloud that may start to develop at height  $z$ . Assuming that  $(T_e - T_c)$  and  $(r_e - r_c)$  vanish at the cloud base, we estimate the mean values of these quantities for an arbitrary finite distance,  $\Delta z$ , above the base as follows:

$$T_e - T_c = \frac{\Delta z}{2} \left[ \left( \frac{\partial T}{\partial z} \right)_e - \left( \frac{dT}{dz} \right)_c \right] \quad (4)$$

and

$$r_e - r_c = r_e - (r_s)_e + (r_s)_e - r_c \simeq r_e - (r_s)_e + \frac{\epsilon}{p} \frac{de_s}{dT} (T_e - T_c). \quad (5)$$

In eq (5),  $(r_s)_e$  is the saturation mixing ratio at  $T_e$ . From eq (2) through (5), we obtain the formula that gives the lapse rate of a cloud element with entrainment; that is,

$$\left( \frac{dT}{dz} \right)_c = -\gamma_m + \frac{\frac{E\Delta z}{2}}{1 + \frac{E\Delta z}{2}} \left[ \left( \frac{\partial T}{\partial z} \right)_e + \gamma_m \right] + \frac{L}{c_p + \frac{L\epsilon}{p} \frac{de_s}{dT}} \frac{E}{1 + \frac{E\Delta z}{2}} [r_e - (r_s)_e]. \quad (6)$$

Although an arbitrary factor  $\Delta z$  is involved in eq (6), it does not affect a criterion for the occurrence of moist convection.

### Criterion of Free Moist Convection

Free moist convection will start to develop when a cloud element becomes buoyant at a height slightly above the cloud base. This condition is given by  $T_c - T_e > 0$ . We see from eq (4) that this is equivalent to

$$\left( \frac{dT}{dz} \right)_c > \left( \frac{\partial T}{\partial z} \right)_e. \quad (7)$$

Using eq (6), we can rewrite expression (7) as

$$-\left( \frac{dT}{dz} \right)_c - \gamma_m > \frac{L}{c_p + \frac{L\epsilon}{p} \frac{de_s}{dT}} [(r_s)_e - r_e] E. \quad (8)$$

Note that the arbitrary quantity  $\Delta z$  disappeared. The quantity  $de_s/dT$  is equal to  $L\epsilon_s/RT^2$  by the Clausius-Clapeyron equation.

When  $T_e$  is used for calculating  $\gamma_m$  and  $de_s/dT$ , expression (8) gives the condition for free convection in terms of the environmental quantities except for  $E$ . Note that a condition is more favorable for moist convection if (1) the stratification is conditionally more unstable, (2) the relative humidity is higher, and (3) entrainment into a cloud element is less.

### Entrainment Into a Cloud Element

To use expression (8) as a criterion for moist convective adjustment, we must specify the fractional entrainment rate for a cloud element. In an entraining jet model or in a bubble model, the entrainment rate is assumed to be proportional to the velocity of a rising element. This hypothesis yields the following relation (e.g., Squires and Turner 1962, Simpson and Wiggert 1969):

$$E = \frac{0.2}{D} \quad (9)$$

where  $D$  is the radius of a plume or a bubble. As mentioned by Simpson (1971), the range of the validity of this relationship cannot be specified at present. However, Simpson cited several evidences that support, to first-order accuracy, this relation.

When air is completely dry, the possibility of moist convection should vanish. In the present scheme, this requires that  $E$  in expression (8) becomes infinite when  $r_e$  approaches zero. For this reason, we determine  $D$  from

$$D = D_0 \left[ \frac{r_e}{(r_s)_e} \right]^{1/2} \quad (10)$$

where  $D_0$  is a characteristic size of the cloud in the 100-percent relative humidity environment. A functional form of eq (10) is chosen so that the dependency of  $D$  on  $r_e$  is weak when the relative humidity is high.

With expression (8) and eq (9) and (10), we can check the possibility of free moist convection in a given environmental field.

### 3. ADJUSTMENT

The effects of the ensemble cumulus convection on the horizontally averaged temperature and mixing ratio are represented as apparent heat and moisture sources (e.g., Yanai 1964, 1971). For the case of the pseudo-adiabatic process, they may be written as

$$\frac{\partial T}{\partial t} + \dots = \frac{L}{c_p} c - \frac{\partial}{\rho \partial z} \left[ M_c (T_c - T_e) \right] \quad (11)$$

and

$$\frac{\partial r}{\partial t} + \dots = -c - \frac{\partial}{\rho \partial z} \left[ M_c (r_c - r_e) \right].$$

In eq (11),  $c$  is the rate of condensation per unit mass of dry air. The quantity  $M_c$  denotes the total mass flux in the cloud. The transport in the cloud (i.e.,  $M_c T_c$  and  $M_c r_c$ ) and that in the environment by the compensating downward flux (i.e.,  $-M_c T_e$  and  $-M_c r_e$ ) are combined to make the last term in eq (11), where  $(T_c - T_e)$  and  $(r_c - r_e)$  are the excess temperature and the excess mixing ratio in the cloud, respectively. Determination of  $M_c c$ ,  $(T_c - T_e)$ , and  $(r_c - r_e)$  is a main subject of the study on the parameterization of the convection processes (Ooyama 1969, 1971; Arakawa 1971, Yanai 1971).

In the present scheme, however, the fluxes of the excess temperature and mixing ratio are not evaluated explicitly

from the cloud mass flux but are treated kinematically, based on some speculations on the moist convection.

The cumulus convection takes place when a large-scale thermodynamical state is favorable for it. Such a state may be established through the processes such as the convergence of moisture, the destabilization due to the large-scale vertical motion, the vertical diffusion of enthalpy, and the radiative transfer of heat. While the convection proceeds, it alters the large-scale thermodynamical field by the effect expressed in the right side of eq (11). A characteristic of convection is its tendency to neutralize the favorable condition for itself that existed initially or may have been developed later through its own process. Therefore, the convection ceases eventually if the feedback effect of the changing thermodynamical state on the large-scale dynamics is ignored. The convective adjustment is complete at this point. This feedback is taken at the next marching step of the integration.

In applying the preceding idea to the multilayer atmospheric model, we modify the horizontally averaged temperature and mixing ratio iteratively. At each iteration, the following formula, which is analogous to eq (11), is used to change  $T$  and  $r$ :

$$T_k \rightarrow T_k + q_k - \frac{(F_T)_{k-1/2} - (F_T)_{k+1/2}}{m_k} \quad (12)$$

and

$$r_k \rightarrow r_k - w_k - \frac{(F_r)_{k-1/2} - (F_r)_{k+1/2}}{m_k}.$$

The suffix  $k$  is the layer or level index and takes a larger value for a lower level. In expression (12),  $w_k$  is the amount of condensation per unit mass of dry air that causes the temperature change  $q_k$ . The relation between  $w_k$  and  $q_k$  is

$$w_k = \frac{c_p}{L} q_k. \quad (13)$$

The quantities  $F_T$  and  $F_r$  denote the total convective flux of excess temperature and mixing ratio, respectively, across the interfaces of the layers in a single iteration. The air mass per unit area in a layer is expressed by  $m$ . Accordingly, the last term in expression (12) corresponds to the second term on the right side of eq (11).

The problem is how to estimate  $q$ ,  $F_T$ , and  $F_r$ . As mentioned earlier, the overall effect of the cumulus convection is to stabilize a large-scale thermodynamical state. Perhaps, this is achieved mostly by the decrease of latent energy in the lower layers due to condensation and the warming of the upper layers due to the flux of the excess temperature by which a part of the released latent heat is effectively transported upward. We presume that this kind of process takes place even locally in each iteration; that is, when a state between the two neighboring levels is favorable for the free moist convection, the condensation occurs at the lower level and the heat and water vapor are transported from the lower to the upper level. This would lead to local neutralization if those processes could be isolated. This hypothesis is made from the consideration

that convection is primarily a process to destroy a pre-existing unstable state, although the resulting heat flux may establish a new unstable state for the next iteration. It is expected that the integral of such local processes over whole iterations brings about the before-mentioned overall stabilization of the thermodynamical state. We will derive the equation for  $q$ ,  $F_T$ , and  $F_r$  from the preceding hypothesis.

Let us suppose that a state of the temperature and mixing ratio of a layer  $k$  is, when examined under the condition of the layer  $k-1$ , favorable for the occurrence of free moist convection; that is, the condition given in expression (8) obtained in the preceding section holds between the two levels considered. Since  $T_e$  and  $r_e$  are almost equal to the horizontal mean temperature  $T$  and mixing ratio  $r$ , respectively, it takes the form

$$\frac{T_k - T_{k-1} - \gamma_m > H_k (\bar{r}_s - \bar{r}) E_k \quad (14)$$

where

$$H_k = \left( \frac{L}{c_p + \frac{L\epsilon}{p} \frac{de_s}{dT}} \right)_k$$

In expression (14),  $\Delta h$  is the distance between the two levels; that is,  $\Delta h = (\phi_{k-1} - \phi_k)/g$  and  $\phi = gz$ . The subscript  $k$  indicates that  $T_k$  and/or  $r_k$  are used to estimate the subscripted term, and the overbar denotes the average for the two levels, weighted by the mass in each layer.  $E$  is estimated with  $r_k$  to insure that there is no convection when the layer  $k$  is dry. Considering the effect of condensation in the layer  $k$  and that of the upward fluxes  $F_T$  and  $F_r$  across the interface  $k-1/2$ , we define  $T^*$  and  $r^*$  as

$$T_k^* = T_k + q_k - \frac{(F_T)_{k-1/2}}{m_k},$$

$$r_k^* = r_k - w_k - \frac{(F_r)_{k-1/2}}{m_k},$$

$$T_{k-1}^* = T_{k-1} + \frac{(F_T)_{k-1/2}}{m_{k-1}},$$

and

$$r_{k-1}^* = r_{k-1} + \frac{(F_r)_{k-1/2}}{m_{k-1}}. \quad (15)$$

Note that  $w_k$  is related to  $q_k$  by eq (13). According to the hypothesis,  $T^*$  and  $r^*$  should represent a neutralized state. Replacing  $T$  and  $r$  in expression (14) with  $T^*$  and  $r^*$  and equating both sides, we obtain the expression for a neutral condition for free moist convection,

$$\frac{T_k^* - T_{k-1}^* - \gamma_m^* = H_k^* (\bar{r}_s^* - \bar{r}^*) E_k^*, \quad (16)$$

where the asterisk indicates a quantity that is a function of  $T^*$  and/or  $r^*$ . By virtue of eq (15), eq (16) gives a relation among  $q_k$ ,  $(F_T)_{k-1/2}$ , and  $(F_r)_{k-1/2}$ .

The flux  $F_T$  depends on the mass flux in the cloud and the excess temperature, as seen in the last term of eq (11). The mass flux, and hence  $F_T$  is probably large when the rate of condensation is large. Therefore, we relate  $(F_T)_{k-1/2}$

kinematically to  $q_k$  by means of a form of diffusion. Using the notations  $K_c$  and  $\delta t$  for the diffusion coefficient and the duration of the diffusion, respectively, we write

$$(F_T)_{k-1/2} = \rho K_c \frac{q_k}{z_{k-1} - z_k} \delta t.$$

For the diffusion process with a characteristic scale  $z_c$  and time  $t_c$ , the coefficient,  $K_c$ , is given by  $z_c^2/4t_c$  (Taylor 1915, Brunt 1952). We estimate  $\delta t$  by  $(z_{k-1} - z_k)/(z_c/t_c)$ . Then,  $(F_T)_{k-1/2}$  becomes proportional to  $q_k$  with the proportionality coefficient  $\mu = \rho z_c/4$ . For the case of convection with  $z_c$  of the order of several kilometers,  $\mu$  is of the order of  $10^2$  in cgs units. Assume that a certain value is assigned to  $\mu$ . When  $\mu q_k$  is substituted for  $(F_T)_{k-1/2}$  in eq (15),  $(T^* - T)$  at levels  $k$  and  $k-1$  become  $(1 - \mu/m_k)q_k$  and  $(\mu/m_k)(m_k/m_{k-1})q_k$ , respectively. It is seen that  $(T_k^* - T_k)$  is negative if  $\mu/m_k$  is greater than unity. This situation should not result from a diffusion-type process. The heat flux may stop if a large fraction, say 90 percent, of the released latent heat is transported into the upper layer; therefore, we reduce  $\mu/m_k$  to 0.9 when it exceeds this value. For stabilizing a state, however,  $(T_{k-1}^* - T_{k-1})$  should not be less than  $(T_k^* - T_k)$ . If it is less than  $(T_k^* - T_k)$ ,  $\mu/m_k$  is reset at  $1/(1 + m_k/m_{k-1})$ , which makes the temperature changes at two levels equal. Consequently, defining the proportionality coefficient,  $\lambda$ , as

$$\lambda = \max \left[ \frac{1}{1 + \frac{m_k}{m_{k-1}}}, \min \left( \frac{\mu}{m_k}, 0.9 \right) \right] \quad (17)$$

where  $\max$  or  $\min(a, b)$  means the larger or the smaller value of the two arguments  $a$  and  $b$ , we obtain the following formula:

$$\frac{(F_T)_{k-1/2}}{m_k} = \lambda q_k. \quad (18)$$

Since  $\lambda$  is smaller than unity, cooling of a layer does not occur in the present scheme.

Next, let us consider the flux of moisture,  $F_r$ . We note that the ratio  $F_r/F_T$  is equivalent to a ratio between the flux terms of eq (11). It represents the ratio of the excess mixing ratio to the excess temperature in the cloud. The excess temperature at the interface  $k - 1/2$  is supposedly large when the temperature decrease outside the cloud from the level  $k$  to  $k-1$  is large. A similar situation probably holds for the mixing ratio. Accordingly, an approximation formula for  $F_r/F_T$  takes the form

$$\frac{(F_r)_{k-1/2}}{(F_T)_{k-1/2}} = \frac{r_k^* - r_{k-1}^*}{T_k^* - T_{k-1}^*}. \quad (19)$$

From numerical considerations, an implicit form is chosen in writing the above equation.

From eq (16), (18), and (19), we can now determine the three unknowns  $q_k$ ,  $(F_T)_{k-1/2}$ , and  $(F_r)_{k-1/2}$ . We examine a thermodynamical state for every set of two sequential levels. Wherever the condition given in expression (14) is satisfied, we estimate the condensation amount and the

heat and moisture fluxes for each set with the scheme derived above. The temperature and the mixing ratio are then modified according to expression (12). The modified field is checked for the further possibility of convection. The iteration is continued until the free moist convection disappears from whole layers. Through such iterative process, the unstable layers in the lower part of the air column may affect the upper portion.

The present scheme of convective adjustment is derived from the hypothetical speculation on the convective processes and formulated with kinematical form. It has to be tested in a numerical simulation model. An example given in section 5 shows a fairly good performance of the present scheme in a numerical simulation experiment of tropical cyclones.

## 4. COMPUTATION SCHEME

### Stability Check

In this section, we explain the computation scheme of moist convective adjustment, which was described in the preceding section. Instead of the height coordinate, let us adopt the  $\sigma$  coordinate system, where  $\sigma = p/p_s$  and  $p_s$  is the pressure at the earth's surface.

The procedure we take in the course of numerical integration is shown in figure 1. In a marching step, the prediction is done at first by taking the grid-scale dynamics and diffusion process into account. Then, we check the moisture field. If it is in a state of supersaturation,  $T$  and  $r$  are changed to  $T + \Delta T$  and  $r + \Delta r$ , respectively, under the condition that  $c_p \Delta T + L \Delta r = 0$  to make the relative humidity 100 percent after the adjustment.

Next, we check the stability. If it is absolutely unstable, it is adjusted to the dry adiabatic lapse rate,  $\gamma_d$ . The total potential energy must be conserved in this process.

A preliminary check for the free moist convection follows. Let us take the two levels labeled as  $k$  and  $k-1$ . If the stratification is absolutely stable, no adjustment is made. When it is conditionally unstable; that is,

$$\gamma_m < \frac{T_k - T_{k-1}}{\frac{\phi_{k-1} - \phi_k}{g}} \leq \gamma_d, \quad (20)$$

the condition for the free moist convection is examined. In expression (20),  $\phi$  is the geopotential of a constant  $\sigma$  surface.

### Condition for Adjustment

If the atmosphere is conditionally unstable, the possibility of free moist convection is checked by the criterion given in expression (14). When the  $\sigma$  coordinate system is used, the weight for each layer for evaluating the weighted average [indicated by an overbar in expression (14)] should be the  $\sigma$  thickness; that is,  $\delta_k \sigma = \sigma_{k+1/2} - \sigma_{k-1/2}$ . Expression (14) is rewritten as

$$T_k - T_{k-1} - \gamma_m \Delta h - H_k (\bar{r}_s - \bar{r}) E_k \Delta h > 0. \quad (21)$$

## Estimation of Condensation and Flux

When the condition given in expression (21) is satisfied, we utilize the scheme presented in the previous section to estimate the condensation amount at level  $k$  and the flux of heat and moisture across the interface  $k-1/2$ .

For converting the equations in section 3 to those for  $\sigma$  coordinate, we need to divide  $m$ ,  $F_T$ ,  $F_r$ , and  $\mu$  by  $p_s/g$ . Hence, if we reuse the symbols  $F_T$ ,  $F_r$ , and  $\mu$  for the divided quantities, respectively, the forms of the equations do not change except that  $m_k$  is replaced by  $\delta_k\sigma$ . A redefined parameter  $\mu$  is  $\rho z_c g/4p_s$ . It is dimensionless and of the order  $10^{-1}$  for  $\rho \approx 10^{-3} \text{g}\cdot\text{cm}^{-3}$  and  $z_c$  is of the order of several kilometers. Following the explanations in the preceding section, we determine  $\lambda$  from the equation that corresponds to eq (17); that is,

$$\lambda = \max \left[ \frac{1}{1 + \frac{\delta_k\sigma}{\delta_{k-1}\sigma}}, \min \left( \frac{\mu}{\delta_k\sigma}, 0.9 \right) \right]. \quad (22)$$

We now show how to obtain  $q_k$ . The following relations are derived from the formulas corresponding to eq (13), (18), (19), and (15):

$$\frac{(F_T)_{k-1/2}}{\delta_k\sigma} = \lambda q_k,$$

$$w_k = \frac{c_p q_k}{L},$$

and

$$\frac{(F_r)_{k-1/2}}{\delta_k\sigma} = \frac{A}{1+A} \frac{1}{1 + \frac{\delta_k\sigma}{\delta_{k-1}\sigma}} \left( r_k - r_{k-1} - \frac{c_p}{L} q_k \right)$$

where

$$A = \frac{\lambda \left( 1 + \frac{\delta_k\sigma}{\delta_{k-1}\sigma} \right) q_k}{T_k - T_{k-1} - \left[ \lambda \left( 1 + \frac{\delta_k\sigma}{\delta_{k-1}\sigma} \right) - 1 \right] q_k}.$$

Accordingly,  $T^*$  and  $r^*$ , which are similar to those defined in eq (15), can be expressed in terms of  $q_k$ . For example, we have

$$T_k^* = T_k + (1-\lambda)q_k$$

and

$$T_{k-1}^* = T_{k-1} + \lambda \left( \frac{\delta_k\sigma}{\delta_{k-1}\sigma} \right) q_k.$$

Substituting such expressions for  $T^*$  and  $r^*$  in expression (16), and noting that  $\bar{r}^* = \bar{r} - [\delta_k\sigma/(\delta_k\sigma + \delta_{k-1}\sigma)](c_p/L)q_k$ , we obtain the following equation:

$$\frac{(E_k^*)^{-1}(T_k - T_{k-1} - \gamma_m^* \Delta h^*) - H_k^* \Delta h^* (\bar{r}^* - \bar{r})}{\left[ \lambda \frac{\delta_k\sigma}{\delta_{k-1}\sigma} - (1-\lambda) \right] (E_k^*)^{-1} + H_k^* \Delta h^* \frac{c_p}{L} \frac{\delta_k\sigma}{\delta_k\sigma + \delta_{k-1}\sigma}} = q_k \quad (24)$$

where the asterisk indicates a quantity affected by  $q_k$ . The left side of eq (24) is a function of  $q_k$ .

Wegstein's iteration method, which is of the form  $f(x) = x$  where  $x = q_k$ , can be used for solving eq (24). We attach a superscript to  $x$  for denoting an iteration order. The

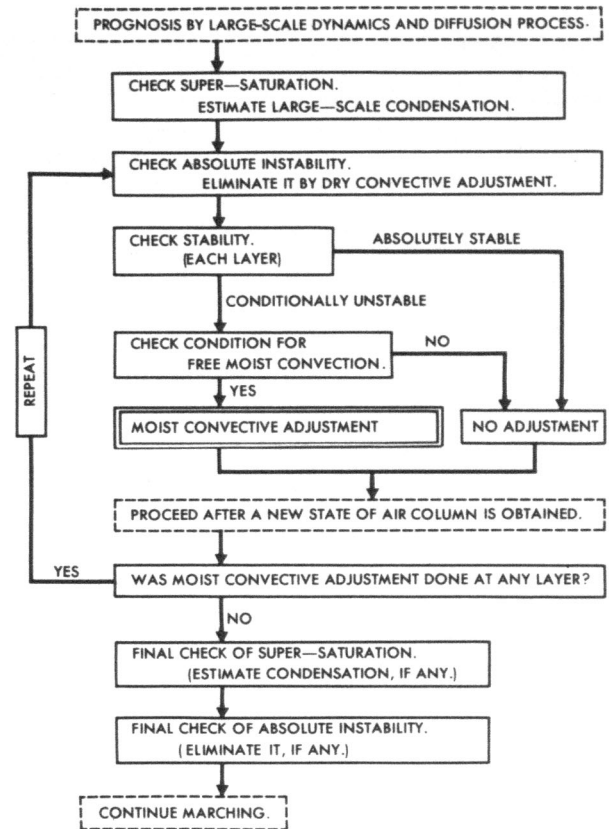


FIGURE 1.—Flow chart showing the process of moist convective adjustment.

formula of iteration is

$$x^{n+1} = \frac{f(x^n) - x^n Q}{1 - Q} \quad (25)$$

where

$$Q = \frac{f(x^n) - f(x^{n-1})}{x^n - x^{n-1}}.$$

In the present case, the iteration may start with  $x^0 = 0$ ,  $x^1 = f(x^0)/2$ . We repeat the iteration until  $|f(x^n) - x^n|$  becomes less than a small value, say  $0.05^\circ\text{K}$ . Once the final value for  $x$  (i.e.,  $q_k$ ), is obtained, the quantities  $w_k$ ,  $(F_T)_{k-1/2}$ , and  $(F_r)_{k-1/2}$ , are determined from eq (23).

### Modification of State

As mentioned in the previous section, we modify the initial temperature and mixing ratio after applying the preceding step to all sets of two sequential layers. The modified values of temperature and mixing ratio at level  $k$  are

$$T_k + q_k \frac{(F_T)_{k-1/2} - (F_T)_{k+1/2}}{\delta_k\sigma} \quad (26)$$

and

$$r_k - w_k \frac{(F_r)_{k-1/2} - (F_r)_{k+1/2}}{\delta_k\sigma},$$

respectively. These forms correspond to expression (12), which is written for the height coordinate system.

After computing a new state of air column, we check its stability and repeat a cycle of adjustment as shown in

TABLE 1.—Distribution of temperature ( $^{\circ}\text{K}$ ) and relative humidity (percent) used to test the adjustment scheme. The selected  $\sigma$  levels and the interfaces are also shown.

$k$	$\sigma_k$	$\sigma_{k+1/2}$	$T_k$	Relative humidity (%)		
				Case A, D	B	C
1	0.0306	0.0832	218.7	1	1	1
2	.120	.1731	200.3	20	20	20
3	.215	.2670	223.9	30	30	30
4	.335	.4203	247.4	35	35	35
5	.500	.5949	267.8	40	40	40
6	.665	.7434	281.1	45	45	45
7	.800	.8609	289.9	62	71	56
8	.895	.9304	294.9	76	87	68
9	.950	.9700	297.7	78	90	70
10	.977	.9841	299.1	79	91	71
11	.992	1.0000	299.8	80	92	72

figure 1 until the whole air column becomes neutral or stable for the free moist convection. We finally check and eliminate the absolute instability, if any, and proceed to the next marching step.

### 5. EXAMPLES

As an example of the use of the proposed scheme of moist convective adjustment, we apply it for the three different large-scale fields. The temperature distribution is common to the three cases. Values at the selected 11  $\sigma$  levels are tabulated in table 1, which represents approximately the annual mean distribution in the tropical latitudes. Table 1 also contains the distribution of relative humidity. Case A shows a typical distribution in the Tropics. Case B is different from case A in that the lowest five levels are very humid. The relative humidity at the lowest five levels is less in case C than in case A. The moist convective adjustment is performed for each case assuming that the surface pressure is 1000 mb. We chose  $\mu=0.15$  for this computation. The parameter  $D_0$  is specified as 0.5 km in cases A, B, and C. The computed changes of temperature and mixing ratio for each case are shown in figure 2. As we discussed in section 2, high humidity (case B in the present example) is a favorable condition for the free moist convection. It is seen that the change of state in case B is large, and the one for the relatively dry case (case C) is very small. We also mentioned in section 2 that another favorable condition for moist convection is a small rate of entrainment for a cloud. In case D, the large-scale field is the same as that for case A, but the entrainment rate is decreased by the use of a larger value (i.e., 1.0 km) for the parameter  $D_0$ . The result of the adjustment calculation is shown in figure 2. The modification of the field in this case is large and is made through a deep portion of an air column. This suggests that the specification of the parameter  $D_0$  should be done carefully in the adjustment scheme proposed in this paper.

The present scheme has been adopted in the numerical modeling of the tropical cyclones at the Geophysical

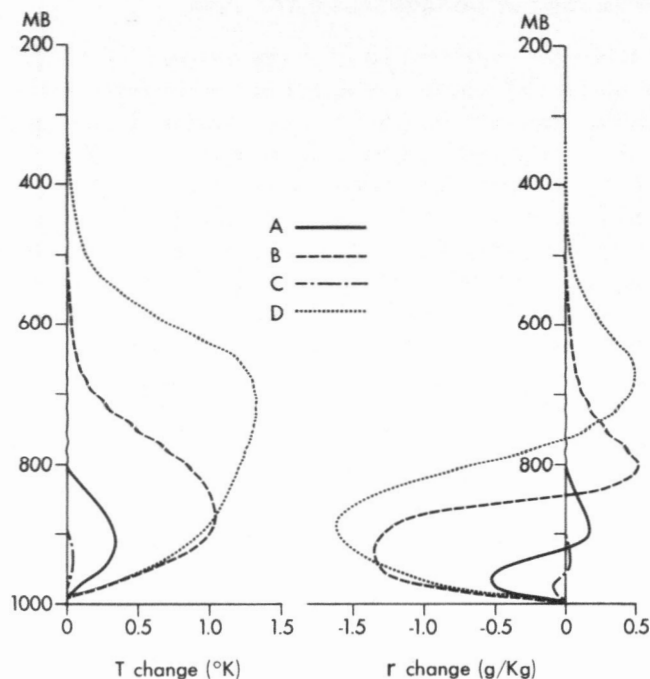


FIGURE 2.—Change of temperature ( $^{\circ}\text{K}$ ) and mixing ratio (g/kg) due to the moist convective adjustment. Initial distributions of temperature and relative humidity are tabulated in table 1.

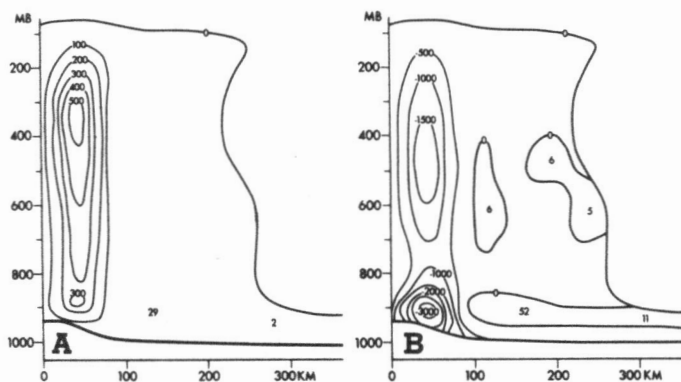


FIGURE 3.—Pressure-radius distribution of the 20-hr average of (A) heat source due to condensation and convective transport of heat ( $10^{-5}\text{K/s}$ ) and (B) moisture source ( $10^{-6}\text{g}\cdot\text{kg}^{-1}\cdot\text{s}^{-1}$ ) at the mature stage in the axisymmetric hurricane model.

Fluid Dynamics Laboratory, NOAA. The vertical resolution of the model is the same as the one shown in table 1. The parameters are taken to be  $\mu=0.15$  and  $D_0=0.5$  km. Starting from a weak vortex, an intense hurricane-like structure was developed in the model. The region of condensation was well organized at and above a convergence area in the low layers in the early stage of the integration. Apparently, the total effect of the condensation heating and the convective transport of heat established a solenoidal field to which the growth of the vortex was attributable. In the mature stage, the heating rate due to the above effect was large at the eye-wall region. Its distribution in the axisymmetric model is shown in figure 3A, where the ordinate is the pressure level and the abscissa is the distance from the center. The distribution of the apparent source

of mixing ratio is shown in figure 3B. A small positive area is due to the influx of moisture from below. The maximum heat source is located at a higher level than the maximum moisture sink as a result of the upward transport of enthalpy in the adjustment process.

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