NOTES AND CORRESPONDENCE

Ultra-Long Baroclinic Waves and Jupiter's Great Red Spot

By Gareth P. Williams

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, New Jersey 08540

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1. Introduction

The main features of Jupiter's general circulation can be reproduced in solutions to Phillips' (1956) two-level, quasi-geostrophic β plane model, when it is integrated with the appropriate planetary parameter values (Williams, 1975). While the large-scale bands of multiple jet-streams can be explained in terms of the concepts of two-dimensional (quasi-barotropic) turbulence (Williams, 1978), the large-scale eddies comparable to the Great Red Spot depend critically on the baroclinic aspects of the flow for their existence (Williams, 1979). The correlation be-

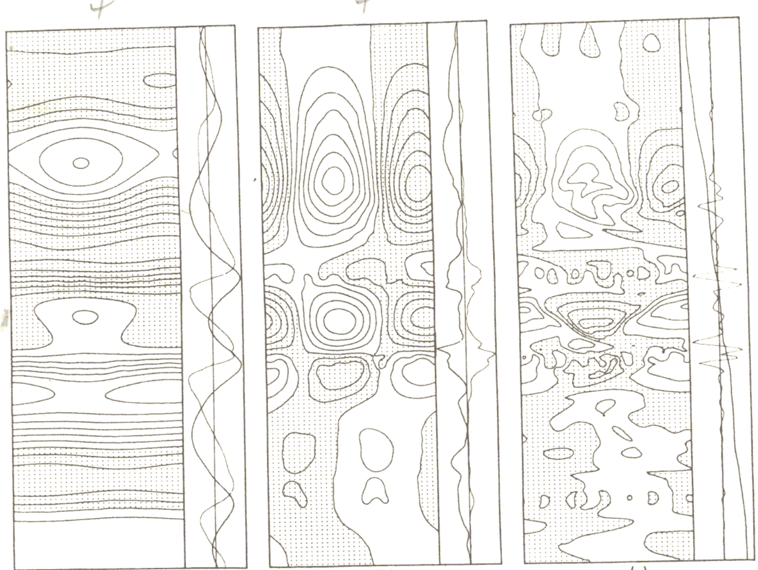

Fig. 1 Basic fields and zonal mean profiles of a solution to the two-level quasi-geostrophic β plane model (Phillips, 1956), integrated for Jovian parameters (from Williams 1979). Fields are upper level stream function, eddy stream function and eddy temperature in (a)–(c) respectively. Periodic boundary conditions on abscissa $x=0$, X and impermeability conditions on the ordinate (latitudinal) $y=0$, Y. Fields denoted by contour increment, *e.g.*, $\Delta\psi$, with zero and negative values shaded by a selection of gridpoints. On right hand side, latitudinal profiles of mean fields are drawn between asterisked values, with zero line at center and heavy line listed first:
(a) $\Delta\psi=150\,\mathrm{km^2s^{-1}}$, $U^*=160\,\mathrm{ms^{-1}}$, $\zeta^*=4\times10^{-5}\,\mathrm{s^{-1}}$,
(b) $\Delta\psi'=30\,\mathrm{km^2s^{-1}}$, $T^*_{2y}{}'=0.7°\mathrm{K}$, $|w'|^*=8\times10^{-6}\,\mathrm{kms^{-1}}$,
(c) $\Delta\hat{\psi}'=8\,\mathrm{km^2s^{-1}}$, $T^*=30°\mathrm{K}$, $w^*=7\times10^{-7}\,\mathrm{kms^{-1}}$,
($T_{2y}{}'$ denotes temperature difference between grid points). Parameter values are $X=35\times10^3$ km., $Y=110\times10^3$ km., $f_0=2.5\times10^{-4}\,\mathrm{s^{-1}}$, $\beta=0.36\times10^{-8}\,\mathrm{s^{-1}\,km^{-1}}$, $\gamma^2=25\,\mathrm{s^2\,km^{-2}}$.

tween the geopotential and temperature fields and the presence of vertical shear indicate that, in this model, the Great Red Spot corresponds to the warm anticyclonic core of a neutral baroclinic wave, Fig. 1. The wave lies between the prevailing easterly and westerly winds.

A deficiency in the neutral wave view of the Great Red Spot lies in its inability to explain the longitudinal scale of the object; the numerical gyres always have the same dimension as the integration domain. This suggests that the quasi-geostrophic equation can describe the Great Red Spot but that the two-level approximation excludes the scale-selection mechanism. The reason for this becomes apparent when we consider that the scale of the object places it within the domain of the ultra-long baroclinic waves, Fig. 2, and that the two-level model filters out these modes or approximates them as neutral waves. Thus, to interpret the Great Red Spot properly

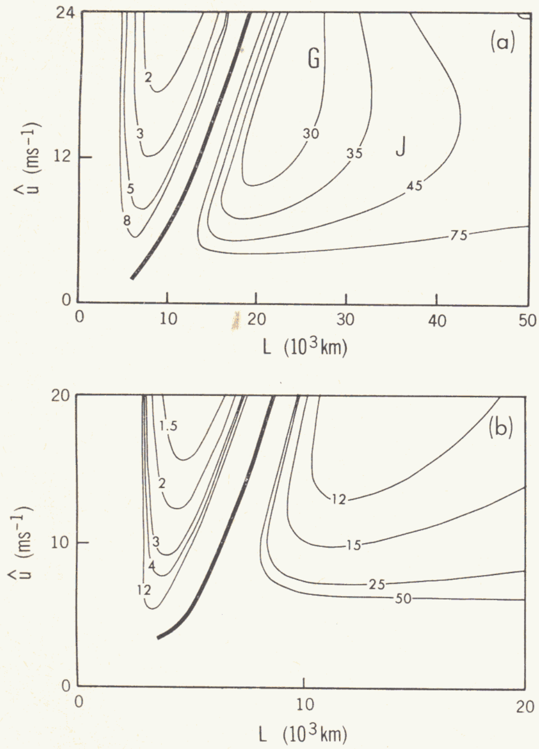

Fig. 2 Baroclinic instability growth rates, with doubling time in days, for the four-level model (after Hirota (1968)). Shown as function of length scale and the vertical shear $\hat{u}$ corresponding to that of the two-level model. (a) Jupiter parameters. J denotes location of numerical gyre, G the possible location of the Great Red Spot. (b) Earth parameters.

in terms of the wave concept requires a more complete vertical representation than that provided by the two-level model.

2. Jovian ultra-long waves

The representation of ultra-long baroclinic waves requires a quasi-geostrophic model with at least four levels (Hirota, 1968). For such an inviscid, adiabatic model, the geopotential and thermodynamical equations—when linearized for small perturbations about a basic zonal flow with constant shear and static stability—can be written as:

$$\frac{\partial}{\partial t}\nabla^2\phi_j+U_j\frac{\partial}{\partial x}\nabla^2\phi_j+\beta\frac{\partial}{\partial x}\phi_j =f^2\Delta p^{-1}(\omega_{j+1/2}-\omega_{j-1/2})\ , \quad (j=1,2,3,4) \qquad (1)$$

$$\frac{\partial}{\partial t}(\phi_{j+1}-\phi_j)+U_{j+1/2}\frac{\partial}{\partial x}(\phi_{j+1}-\phi_j) +\frac{\Lambda}{2}\Delta p\frac{\partial}{\partial x}(\phi_j+\phi_{j+1})=-S\Delta p\omega_{j+1/2}\ , \quad (j=1,2,3) \qquad (2)$$

where j denotes vertical pressure levels, measured from the top of the fluid—see Hirota (1968) for details. We adopt the standard notation for β plane flow:

- p = pressure
- ϕ = geopotential,
- ψ = streamfunction $(=\phi/f)$,
- ζ = vorticity,
- ω = vertical p-velocity,
- f = Coriolis frequency,
- β = Coriolis gradient,
- Δp = pressure difference between levels j and $j+1$,
- U = mean zonal current,
- Λ = vertical shear $(-\partial U/\partial p)$,
- S = static stability measure in 4-level model,
- γ^2 = inverse static stability parameter of 2-level model, $(=\theta_2[(\theta_1-\theta_3)(\phi_1-\phi_3)]^{-1}$ in a 2-level notation)

These equations can be solved as an eigenvalue problem for waves of the form $e^{ik(x-ct)}$, where k is the zonal wavenumber and c the complex wave speed. The resulting quartic equation for c, having real coefficients, can be solved numerically by the Newton-Raphson method. The following parameter values are used for Jupiter and a comparative earth case:

	JUPITER	EARTH	UNITS
f	2.5	1.0	$10^{-4}\,s^{-1}$
β	0.36	1.6	$10^{-8}\,s^{-1}\,km^{-1}$
γ^2	10	150	$s^2\,km^{-2}$

where $S\Delta p^2 = 1/4\gamma^2$ gives the relationship between the two models.

The resulting instability growth rate diagrams, Fig. 2, indicate the preferred scales for development of regular and ultra-long waves in a strongly stable atmosphere.† The preferred scale of the ultra-long waves increases with increasing S and for $\gamma^2 = 10\ s^2\ km^{-2}$ compares with the longitudinal size of the Great Red Spot (currently 28,000 km). Although there is no direct evidence as to whether Jupiter is strongly stable or not, the fact that the optimal length scales of *both* the regular and ultra-long baroclinic instabilities correspond to the size of the observed eddies and the Great Red Spot—for the same S value—suggests indirectly that the atmosphere has such a static-stability. (The regular-scale instability is visible in the eddies of Fig. 1c).

The weakness of the ultra-long wave instabilities—their growth rates are an order of magnitude less than those of the cyclone scale waves—explains their tenuous occurrence in planetary atmospheres. Such waves can exist, perhaps marginally, on Jupiter because of the absence of the strong surface dissipation that inhibits their terrestrial development and the presence of the weak internal dissipation associated with two-dimensional turbulence cascades. Nonetheless, the weak growth rates indicate that sufficient energy could be released to sustain a Red Spot phenomenon over a long period of time.

† For compatibility with the two-level model, the ordinate is expressed in terms of $\hat{u} = u_{1.5} - u_{3.5}$.

3. Further problems

Although a relationship between the Great Red Spot and ultra-long baroclinically unstable waves of the Hirota type may exist, many questions remain. In particular, because of the likelihood that dynamical activity on Jupiter decreases exponentially rather than linearly with depth and because the planet may not possess a solid surface, the effect of height varying static-stability and shear and of the $\omega = 0$ boundary condition on the ultra-long wave instability needs to be explored; such an analysis would also have oceanic application. In addition, all calculations need to be re-evaluated with the more accurate twenty-level model. Above all lies the question of the wave's solitary character.

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超波長のバロクリニック波と木星の巨大赤斑点について

Gareth P. Williams

地球流体力学研究所・ノア，アメリカ合衆国