NGS Geodetic Toolkit, Part 6: Geoid Tools

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Numerous geodetic tools are available from the NGS Tool-kit, which can be found on the Internet at *www.ngs. noaa.gov/TOOLS/*. Included are tools for interactively finding values from the G99SSS, GEOID99, and DEFLEC99 models.
A brief overview of geoids and deflections of the vertical is necessary to understand their limitations before discussing these NGS models. Once the underlying concepts and models are explained, the graphic user interfaces are covered.

Overview

The first and most basic concept to understand is that the geoid is one of an infinite number of geopotential surfaces that surround the earth. These geopotential surfaces are an expression of the gravitational attraction of masses that comprise the earth. They have the same geopotential value everywhere on a given surface, no matter how irregular that surface might be. To best describe these geopotential surfaces, an analogy can be made to a topographic map.
On a map, an isocontour line represents the same height above a datum for different locations. No matter how the line curves around, the height value is the same all along it. The line will not cross any other line although it may come close. The geopotential surfaces similarly ring the earth and grow progressively lower in geopotential value the higher up you go (incidentally, the change in geopotential is how orthometric height is measured). The geoid is selected as that geopotential surface that best approximates global Mean Sea Level (MSL). GEOID99 is shown in Figure 1 with contour lines to highlight this principal.
To be able to relate to the geoid, it is next necessary to refer to an ellipsoid of rotation. This is essentially a spheroid where the two of the three axes are the same length. For the earth, the polar axis is actually about 21 km shorter than the two equatorial axes. This ellipsoid model provides a first order estimate of the size and shape of the earth but does not contain the shorter wavelength features, which represent how the geoid surface undulates about the ellipsoid surface.
These geoid undulations or heights average near zero globally (*i.e.*, there are as many

areas of the geoid surface above the ellipsoid as below it), range from +85 m to -107 m, and have a RMS difference of 30 m. The continental United States lies entirely within a region where the geoid is actually below the ellipsoid. This is why geoid model heights (*e.g.*, GEOID99, G99SSS) are negative for this area. A depiction of this relationship is given in Figure 2.

Note the equation given in Figure 2 and given again below in equation (1). Geoid heights (N) provide a reliable and efficient mechanism for transforming easily obtainable GPS-derived heights (h) to more desirable orthometric heights (H), which are estimates of heights above MSL:

$$h = H + N$$

Deflections of the vertical (DoV) are closely related to geoid undulations, because they both represent deviations of the geoid from the ellipsoid. The plumb bob at a station is oriented "down" along the normal to the geopotential surface (i.e., perpendicular to it) at the observation point. A normal to the ellipsoid surface also provides an estimate of "where" down is located. However, since the geoid surface is more irregular than the ellipsoid surface, it is quite likely that the normals to both surfaces will be different. The difference between the two is the deflection of the vertical for a location. In Figure 2, the normals to both the geopotential and ellipsoid surfaces are shown. These are exaggerated for effect-normally, DoVs are usually less than 10 arc-seconds in magnitude.

NGS Models

With that background, the NGS models are now presented for understanding. Both G99SSS and GEOID99 provide estimates of the geoid undulations or heights, which provide the offset between the ellipsoid and geoid for a given location. G99SSS is generated from gravity observations, terrain data, and a global reference model. It does not reference the NAD 83 ellipsoidal datum but instead references an ITRF 1996/GRS-80 ellipsoid, which is very similar to WGS-84. It also does not reference NAVD 88. So use of G99SSS to transform from WGS-84 to NAVD 88 is not possible, although the values will be close (within a meter or so). The long wavelength portion of the G99SSS is derived from EGM96, a global geoid model produced by NIMA, which references the WGS-84 ellipsoid. G99SSS adds additional information to the short wavelength geoid field from the gravity and terrain data. Therefore, G99SSS can provide a reasonable transformation between WGS-84 and an improved EGM96 surface. If such a transformation is desired, then the G99SSS interactive computation is available using the interface shown in Figure 3. The nature and use of the interfaces for G99SSS, GEOID99, and DEFLEC99 are much the same and will be discussed in the next section.

Since GPS satellites operate in an ellipsoidal reference frame, locations derived from these satellites are likewise expressed about an ellipsoidal datum such as NAD 83. However, NAVD 88 provides the recognized vertical datum for the United States and best expresses height changes with respect to MSL. While G99SSS provides excellent spatial coverage (a value every arc-minute), it does not reference the NAVD 88 datum. To generate a model that meets requirements, further information must be included. GPS-derived ellipsoidal heights on leveled bench marks (GPSBMs) provide height information in the respective NAD 83 and NAVD 88 datums. The distribution of the points used to make the GEOID99 model are given in Figure 4. The spatial coverage of the 6169 points increases in density from regions such as Rocky Mountains to the New England region. However, these are still point values and cannot provide geoid height estimates everywhere nationally even in the regions of densest coverage.

Geoid height estimates at the bench marks from both these GPSBMs and interpolated from G99SSS are fairly close, but their differences have an overall bias of 52 cm with a standard deviation of 22 cm. Modeling the significant features in these differences provided a method for converting the G99SSS model into a more useful geoid product. A conversion surface for G99SSS was generated from residual values derived between G99SSS and GPSBMs according to:

$$e = h_{NAD83} - H_{NAVD88} - N_{G99SSS}$$

The most correlative parts of the residual values (e) are used to generate the conversion surface, which is then applied to G99SSS to generate GEOID99. This conversion surface also accounts for the switch to the NAD 83 datum from G99SSS to GEOID99. To transform from NAD 83 to NAVD 88 then only requires an estimate of the geoid height interpolated from the GEOID99 grid of values:

$$h^{GPS}_{NAD83} - N^{Interpolated}_{GEOID99} = H^{Estimated}_{NAVD88}$$

The accuracy of this GPS-leveled height is a function of both the GPS observation and GEOID99 for that location, and the combined GPS and GEOID99 error is about 9 cm at two standard deviations (about 95% confidence level) nationally. Errors in the GPS observation may be minimized by following the standards set by NGS to obtain heights to 2 cm accuracy. Surveyors using GEOID99 to conduct GPS-leveling should also be aware that while the *national* accuracy of GEOID99 is about 5.0 cm (2 standard deviations), these errors are also a function of location.

Look closely at Figure 4. Note the uneven distribution of points from one region to the next. Lack of data in a given area will result in interpolation over gaps and less accurate results. Additionally, localities with better coverage but less accurate data might have significantly worse results than implied by the national accuracy. An example would include most of the mountainous states in the West. Results near the national borders, where the solution of the conversion surface was poorly constrained, may also have significant errors. Finally, note that GPSBMs are only available in the conterminous United States. Although models are named "GEOID99" outside of CONUS, the models are basically G99SSS models. Geoids for Hawaii, Puerto Rico, the Virgin Islands, and Alaska do not refer to either NAD 83 or NAVD 88.

The NGS deflection model is DEFLEC99, which was derived from GEOID99 and available gravity and terrain data. Since geoid heights occur in the masses, the effect of these masses and associated terrain relief must be taken into account to define the related deflection of the vertical values. The models are provided in both the North-South (Xi) and East-West (Eta) directions. Based on comparison with available data, both models are accurate to about 2 arc-seconds (at 2 standard deviations).

Graphic User Interface

The graphic user interfaces for G99SSS, GEOID99, and DEFLEC99 all function in much the same manner, so a generic discussion of their use is given using the G99SSS interface shown in Figure 3. Note that these interfaces return the same data as the software downloaded from the Website, because they actually interface with the interpolation program and source grids. These programs could be downloaded from the respective web pages and return the same values, and the top level directory for them is located at *www.ngs.noaa.gov/GEOID/*. The interfaces are designed to get the user point values determined from the models. The input values can be either single points or from a file of points. They all operate the same except for the data that are returned.

The top of each page lists the coverage extents for the models. Note the caution that location coordinates must either be in NAD 83 or ITRF. The ITRF models represent solutions for the earth's geocenter of several different years. For example, the G99SSS model actually refers to ITRF 1996 (epoch 1997.0). While these models all are unique, the resulting horizontal location differences are at the meter-level when using them. This has a sub-centimeter impact on geoid height calculations because the GEOID99 data spacing is 1 arc-minute (about 1850 m). Hence, interpolating from data referencing NAD 83, any of the ITRF models, or WGS-84 will not impact geoid height calculations. However, the differences between NAD 27 and these models is several hundred meters and is sufficient enough to warrant first converting such data into the NAD 83 datum. Conversion from NAD 27 to NAD 83 can be accomplished using the NADCON programs also available in the Geodetic Tool Box.

The next section in all three interfaces is for input of an individual point. Different formats for the coordinates are accepted anoted (degrees-minutes-seconds, decimal degrees, degrees-decimal minutes, etc.). Note that while there is no default input style, the default longitudinal direction is assumed West. Most members of the U.S. surveying community adopt this standard, but toggling between buttons will enable the use of East longitude.

The final block on all data is a bit more complicated. This entails the input by file of a number of points. Because the process is less regulated by the interface, the format of the files has to be more particular. There are three possible input style: Free Format Type 1, Free Format Type 2, and NGS Blue Book. Explanations and examples of these formats are given next to each of the format descriptors.

The Free Format Type 1 expects no header, that the first 40 characters in each line do not include coordinate information, and that one of the formats described above are used. While dropping the leading '0' in '075' West longitude will not cause an error, tabbing out 40 spaces will cause a point to not be read. Again ensure that you toggle appropriately between West and East Longitudes.

Free Format Type 2 differs only in the arrangement of the data on each line. The location information must be in the first 32 characters with station name (*i.e.*, an ASCII string) in the last 23 characters of an 80 character line.

Finally, there is the NGS Blue Book Format using the *80* and *86* records, which are formats used to describe much of the available NGS data that is made publically available. Hence, their style of input is selected here as a choice for determining values from the NGS geoid models.

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