THE IRA PROPOSAL CONTAINED IN S. 1682:
EFFECTS ON LONG-TERM REVENUES AND ON INCENTIVES FOR SAVING

Staff Memorandum
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The Congress of the United States
Congressional Budget Office

This memorandum discusses the incentive and revenue effects of Individual Retirement Accounts (IRAs), and in particular, of the proposal contained in S. 1682, introduced on September 27, 1989 by Senator Bentsen. The proposal would modify current law by allowing a deduction of 50 percent of the otherwise nondeductible portion of a contribution made to an IRA. It also would waive the additional 10 percent income tax penalty for early IRA withdrawals if the withdrawals were used to either purchase a first home or pay higher education expenses. This memorandum addresses the likely long-term effect of this proposal on revenue and the deficit as well as the potential effect of the proposal on national saving.

Revenue Effects

The proposal would lose revenue relative to the CBO baseline (current tax law) over the 1990-1994 period. The Joint Committee on Taxation (JCT) has estimated that it would lose revenue in each of these years. Like all IRAs, the short-term revenue effects of the proposal do not fully indicate the long-term effects.

The proposal would continue to lose revenue each year after 1994, although over time the revenue forgone on early contributions to IRA accounts would be partially recouped as a portion of withdrawals from IRAs were included in income. CBO and JCT cannot estimate precise revenue effects of the proposal beyond 1994, however, because of the absence of a revenue baseline as well as other constraints mentioned below.

Comparison with Other Types of IRAs

Over a single taxpayer's lifetime, the 50 percent deductible IRA reduces tax liability in present value by more than a nondeductible IRA but less than a fully deductible or a backloaded IRA (see Box).[1] The 50 percent deductible IRA, thus provides a larger incentive to participate than a nondeductible IRA. But, because the tax subsidy is smaller, the 50 percent deductible IRA provides a smaller incentive to participate than a fully deductible or a backloaded IRA.

The backloaded IRA is included in this comparison because the Congressional debate has focused on this type of IRA as well. A recent example of a backloaded IRA is the IRA Plus, introduced by Senator Packwood and cosponsored by Senator Roth and others on October 19.[2]

Effects on National Saving

The effect of the proposal on personal saving and, ultimately, national saving is indeterminate. For the proposal to increase national saving, it must raise private saving by more than it reduces net federal revenues. Some economists believe that the fully

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[1] A backloaded IRA does not allow a deduction for IRA contributions, but does not tax withdrawals. In terms of present value, it provides the same tax reduction as a fully deductible IRA.

[2] This IRA, contained in S. 1771, has other distinguishing characteristics not addressed here. See CBO Staff Memorandum, "The IRA Plus Proposal Contained in S. 1771: Effects on Long-Term Revenues and on Incentives for Saving", October 25, 1989.

Types of IRAs

Deductible IRA

Contributions are fully deductible and taxes on earnings are deferred until withdrawn. At withdrawal, all earnings and contributions are included in taxable income. Use of these IRAs is currently restricted to those without an employer pension or with AGI below allowable limits.

Nondeductible IRA

Contributions are not deductible, although taxes on earnings are deferred until withdrawn. Nondeductible IRAs are universally available, but the sum of contributions to deductible and nondeductible accounts may not exceed the overall limit in a year.

50% Deductible IRA

Half of contributions would be deductible and taxes on earnings would be deferred until withdrawn. At withdrawal, earnings and the portion of contributions deducted would be included in income. A 50 percent deductible IRA has been proposed by Senator Bentsen in S. 1682.

Backloaded IRA

Contributions would not be deductible and withdrawals would be tax free. A backloaded IRA was proposed by Senator Roth and included in S. 1771.

In addition to the above characteristics, all IRAs impose an additional 10 percent tax on unqualified withdrawals. Qualified withdrawals under current law are largely limited to withdrawals after age 59½. S. 1682 would also allow withdrawals without penalty for the first purchase of a principal residence and higher education expenses. In addition to these, S. 1771 would allow withdrawals from backloaded IRAs for catastrophic medical expenses, but would limit all qualified pre-retirement withdrawals to 25 percent of the account.

2

deductible IRAs that were widely available to taxpayers from 1982 through 1986 have increased national saving. Other economists disagree; the effect on saving remains a matter of debate. If fully deductible IRAs increased net saving, then a 50 percent deductible IRA would be expected to have a similar though smaller effect. However, a net increase in saving cannot be counted on. First, individuals have not been found to be very responsive to higher rates of return on savings. Second, in many cases, the proposal would provide a tax benefit for private saving that would take place without the IRA. Third, by allowing withdrawals from IRAs for purposes other than retirement, the proposal would not strictly segregate funds for use in retirement. Therefore, contributions to IRAs may simply replace other savings.

IRAs UNDER CURRENT LAW

Under current law, deductible IRA contributions of up to $2,000 are allowed to workers who are not covered by an employer pension plan or have adjusted gross incomes (AGIs) of less than $25,000 if single or $40,000 if married and filing jointly. For AGIs that exceed these limits by up to $10,000, ceilings on contributions are phased down to zero. Contributions to IRAs for an eligible worker and a spouse not in the labor force can total $2,250.

Individuals not entitled to the maximum deductible contribution may make nondeductible contributions. Nondeductible contributions are limited to the difference between allowable deductible contributions and $2,000. For example, a worker with pension coverage who files a single return with an AGI of $30,000 could deduct an IRA contribution of $1,000 and make an additional nondeductible IRA contribution of up to $1,000. If the worker's AGI were $35,000 or more, he or she could make a contribution of up to $2,000, but could not deduct any of the contribution.

IRA earnings are not taxed until funds are withdrawn. At that time, earnings and all deductible contributions are included in income. Withdrawals before age 59½ are generally subject to an additional 10 percent income tax.

PROPOSED CHANGE IN IRAs UNDER S. 1682

Under the proposal in S. 1682, individuals would be allowed a 50 percent deduction for IRA contributions that would be nondeductible under current law. The amounts of fully deductible contributions allowed under current law would remain unchanged.

In the above example of a worker with $30,000 in AGI, he or she could deduct a $1,000 contribution in full and deduct 50 percent of additional contributions up to $1,000. If the worker contributed the full $2,000 to an IRA, he or she could deduct $1,500, whereas under current law the worker could deduct just $1,000. If the worker's AGI were $35,000 or more, he or she could deduct 50 percent of up to $2,000 in IRA contributions, for a maximum deduction of $1,000. Under current law, the IRA contribution would not be deductible.

3

The proposal would also allow certain qualifying withdrawals without penalty to purchase a first home or pay expenses for higher education. Withdrawals for these purposes could be made from any IRA balances, not only those stemming from the proposed 50 percent deductible contributions.

As under current law, IRA earnings would not be taxed until funds were withdrawn and, at that time, earnings and the portion of contributions that had been deducted would be included in income. Also, as under current law, nonqualifying withdrawals would be subject to the additional 10 percent income tax.

Withdrawals for first-time home purchases and for higher education expenses would be allowed without penalty beginning January 1, 1990, while the 50 percent deduction on IRA contributions would be allowed beginning January 1, 1991.

BACKLOADED IRAs

The recent debate about IRAs has included consideration of another type of IRA, referred to here as a backloaded IRA. Contributions to a backloaded IRA, like those to the present nondeductible IRA, would not be deductible, but unlike the nondeductible IRA, earnings accumulated in a backloaded IRA account would be fully exempt from tax on withdrawal in retirement (or, for other qualifying pre-retirement purposes). The IRA Plus, proposed by Senator Packwood and Senator Roth and contained in S. 1771, is a backloaded IRA.

EFFECTS OF THE PROPOSAL ON REVENUES

The Joint Committee on Taxation has estimated that the 50 percent deductible IRA will lose tax revenue in each of the next five years, resulting in a cumulative revenue loss of $12.7 billion over the 1990-1994 period. The revenue loss is made up of several components. The largest is the loss from the 50 percent deduction on contributions, both those that would have been made under current law and new contributions stimulated by the proposal. Additional losses arise because the taxation of earnings on the new contributions would be deferred, but those losses are small in the initial years under the proposal. The losses would be partially offset by taxes on increased withdrawals induced by the liberalized withdrawal provisions.

SHORT-TERM VERSUS LONG-TERM REVENUE EFFECTS OF IRAs

The short-term revenue losses from introducing different types of IRAs can give a misleading picture of their long-term revenue effects. Short-term revenue losses from deductible IRAs tend to *overstate* the long-term revenue loss whereas the short-term revenue losses from backloaded IRAs tend to *understate* the long-term revenue loss.

4

Over the full life of an IRA—which may be 30 years or more—a backloaded and a fully deductible IRA have identical revenue effects in terms of present value as long as the taxpayer's marginal tax rate remains constant. A 50 percent deductible IRA would cause a smaller revenue loss because the initial tax subsidy is only one-half that of the fully deductible IRA. The nondeductible IRA causes the smallest revenue loss because both contributions and withdrawals are taxable; the only tax benefit is the deferral of tax on accrued earnings.

Figure 1. Discounted Cumulative Revenue Losses
From a $1,000 After-Tax IRA Contribution
(Four Types of IRAs)
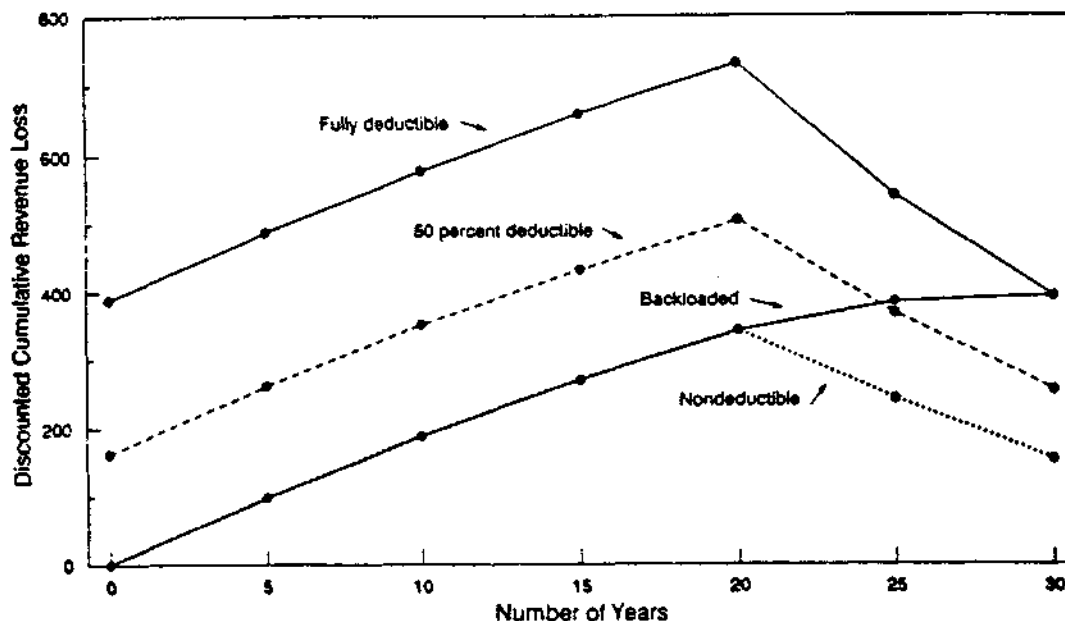
These points are illustrated in Figure 1, which shows how cumulative revenue losses (measured in terms of present value) change over time for a fully deductible IRA, a nondeductible IRA, a backloaded IRA, and a 50 percent deductible IRA. In this example, contributions are assumed to be made in year 0 and funds are withdrawn as an annuity from year 21 through 30.[3] In the example, the taxpayer is in the 28 percent tax bracket throughout the entire period, and all forms of savings pay 8 percent, which is also assumed to be the government's discount rate.

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[3] The Appendix explains the computations underlying Figure 1.

5

The revenue losses are based on the assumption that the taxpayer would contribute $1,000 to a fully taxable savings account if IRAs were not available. A contribution of $1,389 to a deductible IRA has the same out-of-pocket cost as the $1,000 contribution to a fully taxable account because the IRA contribution reduces taxes by $389 (28 percent of $1,389). Since the backloaded IRA and the nondeductible IRA do not receive an immediate tax deduction, a $1,000 contribution to these accounts has the same out-of-pocket cost as a $1,000 contribution to the fully taxable savings account. Under the 50 percent deductible IRA, the taxpayer could contribute $1,163, because this contribution reduces taxes by $163 (28 percent of 50 percent of $1,163).

The fully deductible IRA and the 50 percent deductible IRA initially lose revenue because of the tax deduction, as shown in Figure 1. Over time, the cumulative discounted revenue costs of all four types of IRAs increase because tax liability on the accruing interest is deferred.[4] However, because withdrawals from the fully deductible IRA and the 50 percent deductible IRA are taxable, the cumulative revenue losses for these IRAs fall when the taxpayer withdraws these funds in years 21 through 30. Upon withdrawal of the entire IRA balance, the cumulative revenue loss from the deductible IRA is equal in present value to the revenue loss from the backloaded IRA. In other words, the fully deductible and the backloaded IRAs are equivalent: they have the same revenue loss in terms of present value, and the taxpayer ends up with the same amount of income in retirement.

The revenue loss from the 50 percent deductible IRA is lower because the immediate tax deduction was smaller. Not surprisingly, the lowest revenue loss occurs in the case of the nondeductible IRA.

As mentioned above, the illustration assumes that funds would have been invested in a fully taxable savings account in the absence of an IRA. However, taxpayers can avoid full taxation in several ways. For example, appreciating assets are taxed only when a gain is realized and are thus subject to lower effective tax rates. Furthermore, college savings bonds are equivalent to backloaded IRAs if funds are withdrawn to pay for college expenses. If non-IRA savings are not fully taxed, the revenue losses attributed to each type of IRA would be smaller, but the qualitative conclusions would remain the same.

The tax rate on withdrawals may differ from the tax rate on contributions. If the tax rate on withdrawals is lower, the present value of the revenue loss from a backloaded IRA will be lower than the comparable revenue loss from a fully deductible IRA. The opposite is true if a higher tax rate applies to withdrawals than to contributions. Estimating the net revenue effect of an IRA, therefore, depends partly on assumptions about the future tax rates of participants compared with their current rates.

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[4] The cumulative discounted revenue cost at year T is the sum of all revenue losses from time period 0 to time period T, discounted back to time period 0 at a discount rate of 8 percent.

It should also be noted that the difference between short-term and long-term revenue effects of different types of IRAs is largely a transitional problem. In the very long term, after several generations would have had access to a new type of IRA, the annual revenue loss would combine the effects of younger taxpayers making contributions and older taxpayers making withdrawals. In this "steady state", the annual revenue loss would be indicative of long-term revenue losses. In other words, the annual revenue losses would be approximately equal under a fully deductible and a backloaded IRA.[5] Losses would be smaller under a 50 percent deductible and a nondeductible IRA.

THE EFFECTS OF CONTRIBUTION LIMITS ON LONG-TERM REVENUE LOSS

In comparing IRAs, it must be recognized that similar contribution limits affect different types of IRAs in different ways. An equal dollar limit on the contribution to each type of IRA would result in an unequal amount of revenue forgone from taxpayers who contribute the dollar limit. The reason is that each dollar contributed to a backloaded IRA is out of *after-tax* income, while a dollar contributed to a fully or 50 percent deductible IRA is essentially *pre-tax* income. As a result, a backloaded IRA allows a greater amount of retirement savings to be sheltered from tax, thereby conferring greater tax benefits over the life of the IRA than does a fully deductible IRA with the same dollar limit on contributions.

This point is illustrated in Table 1, which shows the value of retirement savings for a taxpayer who saves $3,000 of after-tax income. Table 1 assumes that each type of IRA has the same limit of $2,000. If the taxpayer puts all of the money into a taxable savings account, he or she could make a lump-sum withdrawal of $9,195 in 20 years.[6] If the taxpayer makes the maximum contribution to a fully deductible IRA, he or she would receive a tax deduction worth $560 (28 percent of $2,000), allowing $1,560 in taxable savings in addition to the IRA. In 20 years, the taxpayer could retire with $11,493. If, instead, the taxpayer makes the $2,000 contribution to a backloaded IRA, he or she would have only $1,000 for taxable saving. Because the backloaded IRA is not taxed at withdrawal, however, the taxpayer could retire with $12,387. This amount is $894 more than in the case of the fully deductible IRA with the same $2,000 contribution limit.

Another way to look at the difference between the fully deductible IRA and the backloaded IRA is to note that the $560 of tax savings under the fully deductible IRA

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[5] This equivalence depends on simplifying assumptions. Differences in annual revenue effects could persist, but they are likely to be small.

[6] The assumption of a single lump-sum withdrawal is made to simplify computations. The qualitative conclusions of this section would not change if a more typical pattern of withdrawals were assumed.

TABLE 1. INITIAL CONTRIBUTION AND RETIREMENT SAVINGS UNDER FOUR IRA ALTERNATIVES
(In dollars, compared with fully taxable savings of $3,000)

| | Initial IRA Contributions and Savings | | | | Value at Retirement | | Present Value of Revenue Loss |
| | Pre-Tax | | | | | | |
	IRA	Other Savings	Initial Tax Savings	After-Tax Total	Current Value	Present Value	
Fully Taxable Savings	0	3,000	0	3,000	9,195	1,973	0
Fully Deductible IRA	2,000	1,560	560	3,000	11,494	2,466	493
Deductible IRA	2,000	1,280	280	3,000	10,915	2,342	369
Backloaded IRA	2,000	1,000	0	3,000	12,387	2,658	685
Nondeductible IRA	2,000	1,000	0	3,000	10,337	2,218	245

SOURCE: Congressional Budget Office computations.

NOTE: Computations assume that deposits accumulate for 20 years, at which time the entire balance is withdrawn. The accounts earn 8 percent interest and are discounted at the same rate. The tax rate is 28 percent in all years.

represents a deferral--not a forgiveness--of tax. The tax payment on withdrawal of the deductible IRA is simply a repayment, with interest, of the original $560 tax deduction. From this perspective, the benefit of the deductible IRA is that the remaining $1,440 in the IRA ($2,000 minus $560) accumulates earnings tax free. This same tax treatment applies under the backloaded IRA. But because the backloaded IRA allows an initial contribution of $2,000 of after-tax income, the amount of income that accrues tax free is higher than under the deductible IRA with the same dollar limit.

Nondeductible and partially deductible IRAs are affected by the contribution limit as well as the less generous tax treatment. A nondeductible IRA, such as that allowed under current law to taxpayers unable to make deductible contributions, would be similar to a backloaded IRA in that the full amount of the $2,000 contribution would represent otherwise fully taxable saving. Unlike a backloaded IRA, however, earnings of the nondeductible IRA would be taxable when withdrawn. As a result, the nondeductible IRA would allow the taxpayer to retire in 20 years with $10,337. Using the 50 percent deductible IRA, the taxpayer would have taxable saving of $1,280 in addition to the IRA because the $2,000 contribution effectively provides a tax rebate of $280 (28 percent of 50 percent of $2,000). In this case, the taxpayer would retire with $10,915.

As the example shows, the backloaded IRA provides the greatest benefit to the taxpayer for an equal dollar contribution. But this greater benefit corresponds to a larger tax subsidy. The revenue cost to the government of a backloaded IRA will be greater than that of a fully deductible IRA with an equal dollar limit on contributions. The increased future consumption for the individual simply represents the increased tax benefits that have been conferred. As shown in Table 1, the backloaded IRA results in retirement income that is greater in terms of present value than the deductible IRA by $192. This amount is the same as the difference in the present value of the revenue lost to the government.

EFFECTS OF THE PROPOSAL ON NATIONAL SAVING

To raise national saving, a 50 percent deductible IRA must raise private saving by more than it reduces net federal revenues. While some economists believe that fully deductible IRAs raise national saving, the effect of such IRAs on saving is still a matter of debate.

Some economists believe that deductible IRAs have raised saving because they allow saving to earn a tax-free rate of return. More recent analyses have also focused on the up-front tax deduction provided by deductible IRAs as a potentially important factor in motivating individuals to save more.

A 50 percent deductible IRA would allow saving in IRA accounts to earn a partially tax-free rate of return and would provide some up-front tax incentive. Allowing 50 percent of IRA contributions to be deducted, therefore, would provide incentives that are similar to, though smaller than, those provided by a fully deductible IRA. Thus, if fully deductible IRAs were successful in encouraging private saving when they were widely available, one might expect 50 percent deductible IRAs to have a similar, though smaller, positive effect on national saving.

Neither the fully nor the 50 percent deductible IRAs, however, can be counted on to raise savings. The Joint Committee on Taxation estimates that in 1990 about one-half of the tax benefits from allowing a 50 percent deduction for those taxpayers not currently allowed to make deductible IRA contributions would be received by taxpayers with adjusted gross incomes in excess of $75,000. Such relatively affluent taxpayers are likely to save more than the $2,000 dollar limit. The higher after-tax rate of return available on funds contributed to deductible IRAs will not affect the marginal return to new saving by such taxpayers. Even those for whom the higher rate of return represents a new incentive to save more may not be willing to sacrifice much current consumption for the higher return on savings. Most empirical studies of personal saving show it to be largely unresponsive to increased rates of return.

Allowing access to IRA balances before retirement makes IRAs more attractive as a form of saving. But it is unclear whether this would raise total private savings. Taxpayers would be more willing to place savings in IRA accounts if the accounts could be used to finance higher education or the purchase of a first home. But if households

would have saved in any event for such purposes, new funds deposited in IRAs would substitute for existing savings. Moreover, fewer dollars would need to be saved for such future expenses if the savings could be placed in a tax-preferred saving account such as an IRA. As a result, allowing access to IRA balances before retirement may make IRAs more attractive, but at the expense of saving in other accounts.

APPENDIX
Computation of the Revenue Loss
From Equal After-Tax IRA Contributions

This appendix describes the assumptions and methodology underlying the calculations of revenue loss discussed in the memo and summarized in Figure 1.

Assumptions

These are the assumptions underlying the example. *Note that these assumptions are only illustrative and do not necessarily reflect the assumptions made for an actual revenue estimate.*

Parameters

- Interest rate = 8%.

   All savings and IRA accounts earn a constant rate of interest of 8%. (This is approximately the government's borrowing rate on one-year Treasury bills.)

- The government's discount rate = 8%.

   This number is used for computing present values.

- Taxpayer's marginal tax rate = 28%.

   This rate is assumed to remain constant over the taxpayer's life.

Timing of contribution and withdrawal

- A single contribution with an after-tax cost of $1,000 is made at the end of year 0.

- The account is withdrawn in equal portions (after tax) at the end of years 21 through 30. I.e., the IRA or savings account is withdrawn as a fixed annuity.

Methodology

1. The baseline savings account

The basis for comparison is a fully taxable savings account. It is assumed that the taxpayer would deposit $1,000 into this account at the end of year zero, accrue the after-tax interest over 20 years, and withdraw equal after-tax amounts at the end of years 21 through 30. The balance in the savings account at the end of years 1 through 20 is determined as follows:

$$S_i = 1000(1 + r(1-t))^i , \qquad i = 1,2,...,20.$$

11

TABLE A.1. DATA UNDERLYING FIGURE 1: IRA Balance and Revenue Loss by Type of IRA

Types of IRA

Year	Backloaded IRA Balance	Backloaded Revenue Loss	Fully Deductible IRA Balance	Fully Deductible Revenue Loss	50% Deductible IRA Balance	50% Deductible Revenue Loss	Nondeductible IRA Balance	Nondeductible Revenue Loss	Taxable Savings Account Balance	Taxable Savings Tax
0	$1,000	$0	$1,389	$389	$1,163	$163	$1,000	$0	$1,000	$0
1	$1,080	$22	$1,500	$22	$1,256	$22	$1,080	$22	$1,058	$22
2	$1,166	$24	$1,620	$24	$1,356	$24	$1,166	$24	$1,119	$24
3	$1,260	$25	$1,750	$25	$1,465	$25	$1,260	$25	$1,183	$25
4	$1,360	$26	$1,890	$26	$1,582	$26	$1,360	$26	$1,251	$26
5	$1,469	$28	$2,041	$28	$1,709	$28	$1,469	$28	$1,323	$28
6	$1,587	$30	$2,204	$30	$1,845	$30	$1,587	$30	$1,399	$30
7	$1,714	$31	$2,380	$31	$1,993	$31	$1,714	$31	$1,480	$31
8	$1,851	$33	$2,571	$33	$2,152	$33	$1,851	$33	$1,565	$33
9	$1,999	$35	$2,776	$35	$2,324	$35	$1,999	$35	$1,655	$35
10	$2,159	$37	$2,999	$37	$2,510	$37	$2,159	$37	$1,751	$37
11	$2,332	$39	$3,238	$39	$2,711	$39	$2,332	$39	$1,852	$39
12	$2,518	$41	$3,497	$41	$2,928	$41	$2,518	$41	$1,958	$41
13	$2,720	$44	$3,777	$44	$3,162	$44	$2,720	$44	$2,071	$44
14	$2,937	$46	$4,079	$46	$3,415	$46	$2,937	$46	$2,190	$46
15	$3,172	$49	$4,406	$49	$3,689	$49	$3,172	$49	$2,316	$49
16	$3,426	$52	$4,758	$52	$3,984	$52	$3,426	$52	$2,450	$52
17	$3,700	$55	$5,139	$55	$4,302	$55	$3,700	$55	$2,591	$55
18	$3,996	$58	$5,550	$58	$4,647	$58	$3,996	$58	$2,740	$58
19	$4,316	$61	$5,994	$61	$5,018	$61	$4,316	$61	$2,898	$61
20	$4,661	$65	$6,474	$65	$5,420	$65	$4,661	$65	$3,065	$65
21	$4,339	$69	$6,027	($201)	$5,046	($141)	$4,339	($98)	$2,830	$69
22	$3,992	$63	$5,544	($207)	$4,642	($146)	$3,992	($103)	$2,581	$63
23	$3,616	$58	$5,023	($212)	$4,205	($152)	$3,616	($109)	$2,318	$58
24	$3,211	$52	$4,460	($218)	$3,734	($158)	$3,211	($115)	$2,040	$52
25	$2,773	$46	$3,852	($224)	$3,225	($164)	$2,773	($121)	$1,746	$46
26	$2,301	$39	$3,195	($231)	$2,675	($171)	$2,301	($127)	$1,434	$39
27	$1,790	$32	$2,486	($238)	$2,082	($178)	$1,790	($134)	$1,105	$32
28	$1,239	$25	$1,720	($245)	$1,440	($185)	$1,239	($142)	$757	$25
29	$643	$17	$893	($253)	$748	($193)	$643	($150)	$389	$17
30	$0	$9	$0	($261)	($0)	($201)	$0	($158)	$0	$9
Present Value	$407		$407		$268		$168			$407

Annuity Amount

Pre-Tax	$695	$965	$808	$695	varies
Exclusion %	100.00%	0.00%	7.20%	14.40%	varies
Tax	$0	$270	$210	$166	$412
After-Tax	$695	$595	$598	$528	

where S_i is the balance in the savings account at the end of year i, r is the interest rate (8%) and t is the marginal tax rate (28%).

If S_{20} is the amount in the account at the end of year 20, the after-tax value of the annuity is as follows:

$$A_s = \frac{S_{20}\, r\, (1-t)}{1 - (1+r(1-t))^{-10}} \quad .$$

This annuity, plus tax, will deplete the account by the end of year 30.

The balance in the account from year 21 through 30 solves the following recursive formula:

$$S_i = S_{i-1}(1 + r(1-t)) - A_s \quad , \quad i = 21,22,\ldots,30.$$

The annual tax revenue on the savings account is simply the product of the tax rate and the interest earned on the beginning-of-period balance:

$$T_i^s = trS_{i-1} \quad , \qquad\qquad i = 1,\ldots,30.$$

The annual balances and tax revenues from a savings account with an initial balance of \$1,000 are shown in the last two columns of Table A.1.

2. IRAs

The method of calculating revenue streams is similar for the four types of IRA. These IRAs may be characterized algebraically as functions of two parameters, denoted α and β. Let α be the fraction of IRA contributions that is deductible from income. Let β be the fraction of IRA earnings that is included in income when funds are withdrawn. The appropriate values of these parameters for each IRA are listed in Table A.2.

Table A.2. IRA Tax Parameters

Type of IRA	Deductible Percentage of Contributions (α)	Taxable Percentage of Earnings (β)
Backloaded	0%	0%
Fully deductible	100%	100%
50 percent deductible	50%	100%
Nondeductible	0%	100%

Based on these parameters, the initial tax payment (refund) due to the initial IRA contribution is denoted T_0, defined as follows:

$$T_0 = -\alpha I_0 t,$$

where I_0 is the initial IRA contribution.

The amount of the initial IRA contribution is such that the after-tax cost of the IRA is \$1,000. Algebraically, this means that

$$I_0 + T_0 = 1000 .$$

Substituting for T_0 from the earlier expression produces an expression for I_0 in terms of the tax rate and the parameter, α:

$$I_0 = \frac{1000}{1 - \alpha t} .$$

Because interest accrues tax-free in all IRAs, the expression for the IRA balance in year i, I_i, is as follows:

$$I_i = I_0 (1 + r)^i , \qquad\qquad i = 1, 2, \ldots, 20.$$

The IRA is withdrawn in equal portions over the remaining 10 years. The pre-tax value of the annuity is determined in a similar way to the after-tax annuity in the savings account case. For the IRA, the before tax interest rate is used. The expression for the pre-tax IRA annuity, A_I, is as follows:

$$A_I = \frac{I_{20}\, r}{1 - (1 + r)^{-10}} .$$

The IRA balance during the withdrawal period solves the following recursive formula:

$$I_i = I_{i-1} (1 + r) - A_I , \qquad\qquad i = 21, 22, \ldots, 30.$$

The taxation of withdrawals depends on the type of IRA. Withdrawals from backloaded IRAs are not taxed; withdrawals from fully deductible IRAs are fully taxed. Withdrawals from nondeductible and 50 percent deductible IRAs are partially taxed. For a non-backloaded IRA, the portion of withdrawals that is excluded from tax is the portion that was contributed from after-tax income. The excluded fraction is thus the nondeductible part of the original contribution, $(1-\alpha)I_0$, divided by the total withdrawal, $10A_I$, as follows:

14

$$E = \frac{(1 - \alpha)I_0}{10A_1} .$$

It follows that the tax on withdrawals is the included fraction of withdrawals, $(1-E)$, multiplied by the annual pre-tax withdrawal, A_1, and the tax rate, t. To generalize this expression to encompass backloaded IRAs, it is multiplied by β, the fraction of earnings that is taxable. The annual tax on withdrawals, T_w, is then:

$$T_w = (1 - E) A_1 t\beta .$$

The revenue loss from an IRA in this example is simply the difference between the taxes paid (or tax reduction in year 0) and the taxes that would have been paid on the taxable savings account. The formula for revenue losses is shown in Table A.3.

Table A.3. General Formula for Revenue Loss From Different IRAs

Years	Revenue Loss
0	$\alpha I_0 t$
1 through 20	T_i^*
21 through 30	$T_i^* - (1 - E)A_i t\beta$

The annual balances and revenue losses attributable to each kind of IRA based on the above formulae are displayed in Table A.1.

15