Bulletin of the Seismological Society of America, Vol. 93, No. 5, pp. 2017–2024, October 2003

Analog-to-Digital Conversion as a Source of Drifts in Displacements Derived from Digital Recordings of Ground Acceleration

by David M. Boore

Abstract Displacements obtained from double integration of digitally recorded ground accelerations often show drifts much larger than those expected for the true ground displacements. These drifts might be due to many things, including dynamic elastic ground tilt, inelastic ground deformation, hysteresis in the instruments, and cross feed due to misalignment of nominally orthogonal sensors. This article shows that even if those effects were not present, the analog-to-digital conversion (ADC) process can produce apparent "pulses" and offsets in the acceleration baseline if the ground motion is slowly varying compared with the quantization level of the digitization. Such slowly varying signals can be produced by constant offsets that do not coincide with a quantization level and by near- and intermediate-field terms in the wave field radiated from earthquakes. Double integration of these apparent pulses and offsets leads to drifts in the displacements similar to those found in processing real recordings. These effects decrease in importance as the resolution of the ADC process increases.

Introduction

Long-period drifts are pervasive in displacements obtained by double integration of ground accelerations obtained from digital recorders deployed to capture ground motion from earthquakes (e.g., Chiu, 1997; Boore *et al.*, 2002; Wang *et al.*, 2003). These drifts are usually interpreted as being due to small shifts in the baseline of the recorded acceleration and have been attributed to many things, including mechanical or electrical hysteresis in the transducer (Iwan *et al.*, 1985; Shakal and Petersen, 2001), ground tilt (Bradner and Reichle, 1973; Graizer, 1989; Trifunac and Todorovska, 2001), or cross-axis effects due to misalignment of sensors (Wong and Trifunac, 1977; Todorovska, 1998). In this article I consider another possible cause of baseline offsets: analog-to-digital conversion (ADC). A brief investigation of this was given by Chiu (1997), who referred to it as the "insufficient-resolution error."

That ADC might be a source of baseline offsets first occured to me during a study of several colocated recordings of the 1999 Chi-Chi earthquake at Hualien station (see Wang *et al.*, 2003). One of the recordings (HWA) was obtained on a Geotech A-800 instrument, a predecessor of the A-900 instruments on which most of the recordings of the earthquake were obtained. The displacement time series derived from the A-800 record shows pronounced trends on all three components, with the trends on two of the components (UD, NS) being roughly linear with a slope near 3 cm/sec. The piezoelectric transducers used in the A-800 have a low-cut frequency of 0.02 Hz, and the signal was filtered with a 0.1-Hz low-cut analog filter before it entered the ADC; the presence of long-period energy in a signal that had been filtered twice to remove long periods implies that the baseline offset producing the drift in displacement most likely occurred during or after the ADC process. (The A-900 transducer and recorder do not have low-cut filters.)

The Effect of Random Quantization Error on Ground Displacement Derived from Accelerations

If the acceleration time series varies rapidly enough and with large enough amplitude, then the error in ADC can be modeled as random, uncorrelated white noise distributed uniformly between $\pm 0.5Q$, where the quanta, Q, is the number of centimeters per square second per digital count (also known as the least significant bit value). The quanta Q is related to the full-scale range $\pm Y$ and the number of bits N used in the ADC by the equation

$$Q = 2Y/2^N \quad (1)$$

The standard deviation of the error is

$$\sigma_a = Q/\sqrt{12} \quad (2)$$

(Scherbaum, 2001). The quantity of concern in this article is the error in displacements derived by double integration of an acceleration time series containing random, uncorrelated noise. Integrating a random series is a random-walk process, and double integration might be termed a double-random walk. The final displacement will be a random variable with zero mean but nonzero standard deviation, and the final displacement of any realization will almost certainly be nonzero. The standard deviation of the final displacement ($\sigma_{d_{\mathrm{end}}}$) depends on the duration of the time series (T) and the sampling interval (Δt) according to the following equation:

$$\sigma_{d_{\mathrm{end}}} = \left(\frac{T^3 \Delta t}{3}\right)^{1/2} \sigma_{\mathrm{a}} \tag{3}$$

(Boore *et al.*, 2002; see Schiff and Bogdanoff [1967] for a similar equation and Graizer [1979], who performed a numerical simulation of this process). For the Chi-Chi earthquake data, the $\pm 1g$, 12-bit A-800 instrument and the $\pm 2g$, 16-bit A-900 instrument have Q values of 0.48 and 0.060 cm/sec^2, respectively, and the sampling interval is 0.005 sec. (Note that in this article I sometimes loosely refer to these as "12-bit" and "16-bit" ADC, although the Y value for the 16-bit ADC is twice that of the 12-bit ADC.) With these values, using equations (2) and (3) gives $\sigma_{d_{\mathrm{end}}}$ of 5.7 and 0.7 cm for the A-800 and A-900 instruments, respectively, after integration of 100 sec. These are much smaller than the drifts seen on many of the records (e.g., Boore, 2001; Wang *et al.*, 2003), suggesting that another source of the drifts in the displacements must exist. If the digitization error is not uncorrelated random noise, however, then equation (3) is not applicable. To study this situation I used simulations of the acceleration ground motion, as discussed in the next section.

The Effect of the ADC Process on Simulated Ground Motions

To investigate the possible role of ADC in producing drifts in displacement, I simulated ground accelerations and "digitized" these simulations by using a Fortran "floor" function that seems to be the equivalent of the ADC used in the A-800 and A-900 instruments (Crystal Semiconductor Corporation, 2001; J. Kerr, written comm., 2002). I consider the motion without the ADC process to be the equivalent of analog motion, although obviously it is represented using a finite word length in the computer.

In the first version of this article I used the stochastic method (Boore, 2003a) to generate a suite of simulated ground accelerations at 84 km from an **M** 7.6 earthquake. (This is the situation for the Hualien station that recorded the 1999 Chi-Chi earthquake.) I adjusted the site amplifications to get rough agreement with the observed response spectrum from the east–west component of record HWA019. Double integration of the simulated accelerations showed large, nearly linear drifts in displacements, larger for the 12-bit than for the 16-bit ADC. Close inspection of expanded plots of the various time series showed that these drifts were associated with slowly varying pre-event motions produced by the acausal filtering used in the simulation program (SMSIM, Boore, [2000, 2003b]). (The procedure does not demand that the real and imaginary parts of the spectrum of ground motion have the proper relation to guarantee causality.) If the amplitude of these motions varies slowly relative to the Q value, the digitized signal resulting from the ADC can have a positive or negative bias, depending on what portion of the record is used to determine the mean that is removed from the whole record before double integration. This pulse-like bias leads to linear trends in displacement.

Although the slowly varying, acausal motions in the simulations are clearly unphysical, ramplike behavior can occur in real ground motions (due to intermediate- and near-field terms), and the results of the simulation might be meaningful. But to overcome any objections related to the unphysical nature of the simulations, I redid the study using simulations from a full-wave theory program (COMPSYN, Spudich and Xu [2003]). I simulated the ground accelerations at distances of 10 and 128 km from a finite vertical strikeslip fault 30 km long, extending from 5 to 9 km in depth. The crustal model was that used by Ji *et al.* (2002) to simulate the 1999 Hector Mine, California earthquake. The fault–station geometry for the 128-km distance was chosen as a rough simulation of the motions at station 513 (see Fig. 1 in Boore *et al.* [2002]), and the distance of 10 km was chosen to provide a ground motion with a significant residual displacement. The calculations were limited to frequencies less than 1 Hz, but for purposes described later, the motions were interpolated to about 100 samples per second. The simulated velocities were differentiated and integrated to produce simulated accelerations and displacements. The accelerations were filtered with a 25-Hz high-cut filter to remove some high-frequency ripple; the ADC study was done using the filtered accelerations. No low-cut filtering was done in the simulations. The waveforms for the horizontal component of motion parallel to the fault strike are shown in Figure 1. As expected, the waveforms are causal, the ground motion at 10 km has a large residual displacement, and the motion at 118 km shows a long-period trend between the initial P and S arrivals, because of near- and intermediate-field terms in the simulations. Unlike the stochastic method simulations, there is no randomness in the calculations (at least none explicitly included); the complexity of the acceleration waveforms is due to reverberations in the layered model (no inelastic attenuation was included in the calculations). Realizing that in real recordings the instruments are rarely so perfectly adjusted that the "zero" level is actually zero, I added a random constant dc offset to the whole acceleration time series. Guided by real data, the offset was chosen from samples of random noise distributed uniformly between -15 cm/sec^2 and $+15$ cm/sec^2. The actual range is not important, however, because the critical quantity is the fractional part of a quanta; similar results would have been obtained if the range of offsets had been only one quanta. This procedure was repeated 99 times, yielding a suite of 99 "analog" accelerations. Each of these was then digitized as described earlier, and the displacements were calculated by

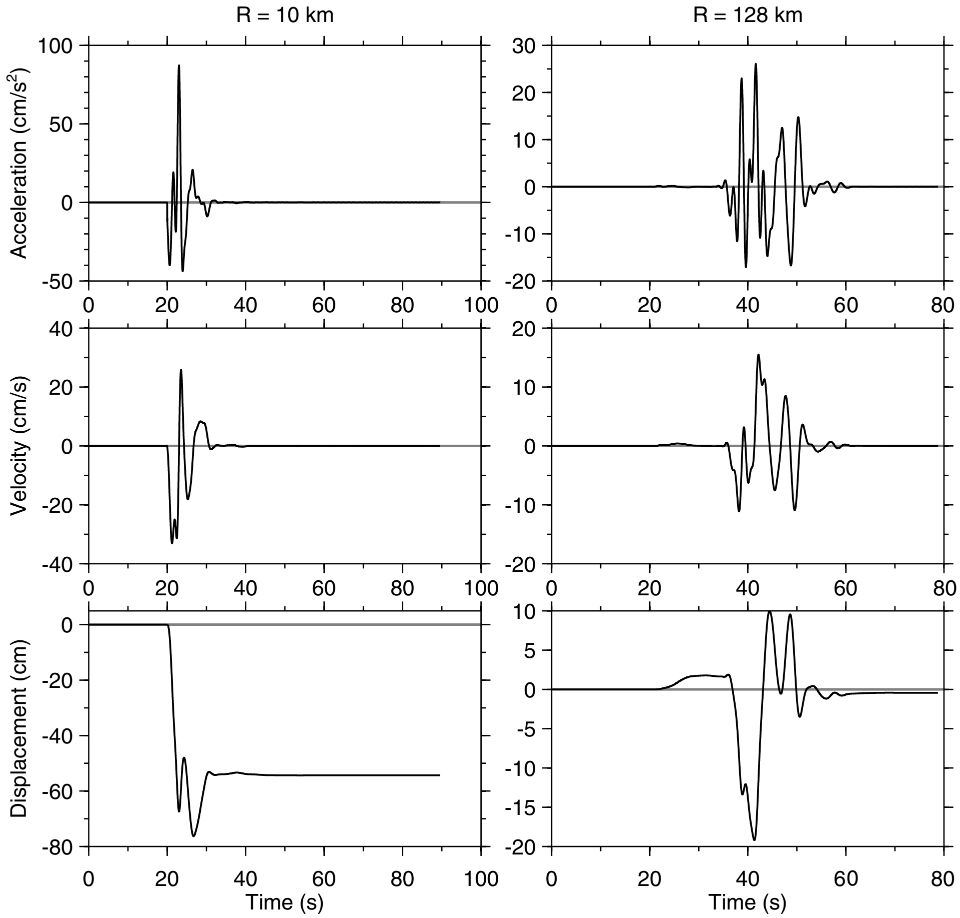

Figure 1. Fault-parallel acceleration, velocity, and displacement for simulations at two distances using full wave theory in a layered crust with an extended fault (see text), without digitization.

computing a mean of a portion of the acceleration, removing this mean from the whole acceleration time series, and then integrating twice.

The first 10 displacements for the two distances are shown in Figure 2. The mean was determined from the pre-event portion of each record. Clear trends exist in the displacements, which are entirely due to the ADC process. Because I used the same initial seed for the random number generator when processing the accelerations at the two distances, the random offsets are the same for the first, second, etc. traces for the two distances, and therefore the drifts for the two distances are qualitatively similar in their trends (up or down). To understand what the source of the drifts might be, I plotted at an expanded amplitude scale the difference between the digitized acceleration resulting from the 16-bit ADC and the analog acceleration (no ADC) and similar differences for the derived the velocity time series. I used the ninth of the suite of 99 motions, corresponding to the next-to-last displacement in the right panel of Figure 2. The difference plots are shown in Figure 3, along with a small segment of the acceleration (plotted at an expanded timescale to illustrate the digitizing) before and after digitizing and after removal of the pre-event mean from the whole record. Because no noise was added to the baseline, the corrected digitized acceleration is equal to zero for the portion used to determine the mean; differences between the analog and the digitized accelerations begin at the arrival of the *P* wave (third graph from the top in Fig. 3). What is particularly interesting is that the difference is not symmetrical about 0.0, but for the particular example shown in Figure 3, is biased to negative values by a fraction of a quanta (Q = 0.06 in this case). The bias in acceleration leads to a linear trend in velocity (until the end of the motion at about 60 sec, in this case), and this leads to the drift seen in the displacement (Fig. 2). Spot checking other cases leads to the same result. Apparently the ADC process applied to records containing slowly varying motions (the random dc offset is an extreme of such a motion) introduces a bias in the corrected acceleration time series that leads to drifts in the displacement.

The effect of ADC on the displacements can be sensitive to the portion of record from which the mean is determined. This is shown in Figure 4, in which the displacements are compared at the two distances when the mean was deter-

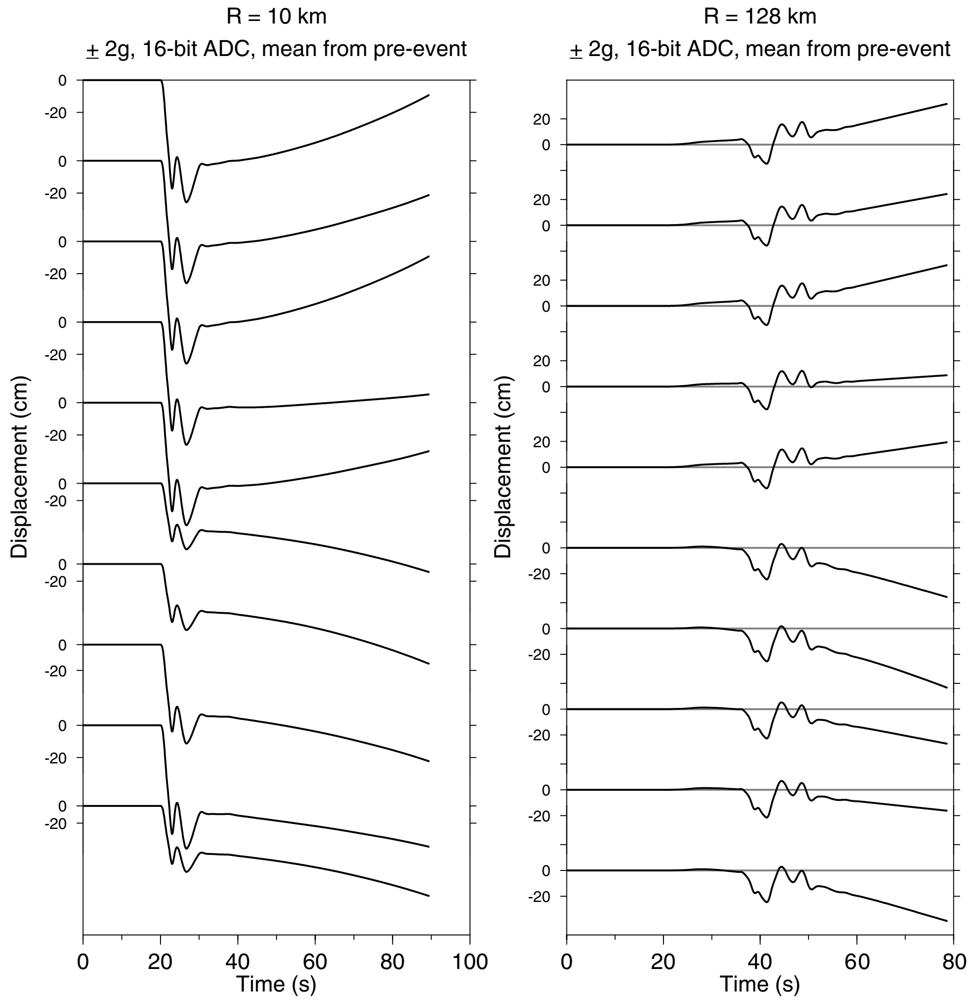

Figure 2. The sensitivity of the displacements obtained from ADC of simulated accelerations at distances of 10 and 128 km to different, randomly determined constant offsets. The offsets were added to each simulated trace before digitizing (see text), and the mean from the pre-event of each digitized trace was removed from the whole record before double integration to obtain the displacement time series. The ADC was for $\pm 2g$, 16 bits, with a quanta of 0.06 cm/sec^2/count. (This ADC corresponds to the ADC used in the A-900 instruments that recorded the 1999 Chi-Chi, Taiwan earthquake.)

mined from the pre-event portion of the record and from the whole record. As will be shown later, the displacements obtained using the overall mean are better, in general, than those from the pre-event portion for the motions studied in this article. This would not be the case, however, if a shift in baseline occurred during the shaking. (All examples in this article have a constant, but random, baseline shift.) Such changes in the baseline seem to be pervasive in many digital recordings (e.g., those from the 1999 Chi-Chi, Taiwan mainshock; Wang *et al.*, 2003). The result of such a change in baseline is such that removal of the mean determined from the whole record will produce displacements in the pre-event portion that will have trends similar to, but often larger than, the pre-event trends shown in Figure 4.

The problems that can arise from applying ADC to slowly varying signals has long been recognized in audio digital signal processing (e.g., Smith, 1997; Pohlmann, 2000; Watkinson, 2001), where the result is described as undesirable low-frequency distortion. A common way of reducing such effects is to add a small amount of random noise to the signal before ADC; this process is known as "dithering." (The A-800 and A-900 instruments have not used dithering in the ADC; J. Kerr, written comm., 2002.) An example of dithering is shown in Figure 5. The upper graph shows the undigitized signal before and after addition of Gaussian noise with a standard deviation equal to 2/3 Q. The middle graph shows the two signals after ADC, and the bottom graph shows the resulting displacements. The displacement for the dithered signal is much closer to the true displacement (Fig. 2) than that from the undithered signal, and as shown later, for a given resolution of the ADC, this holds in general.

The severity of the drifts in displacements is clearly a

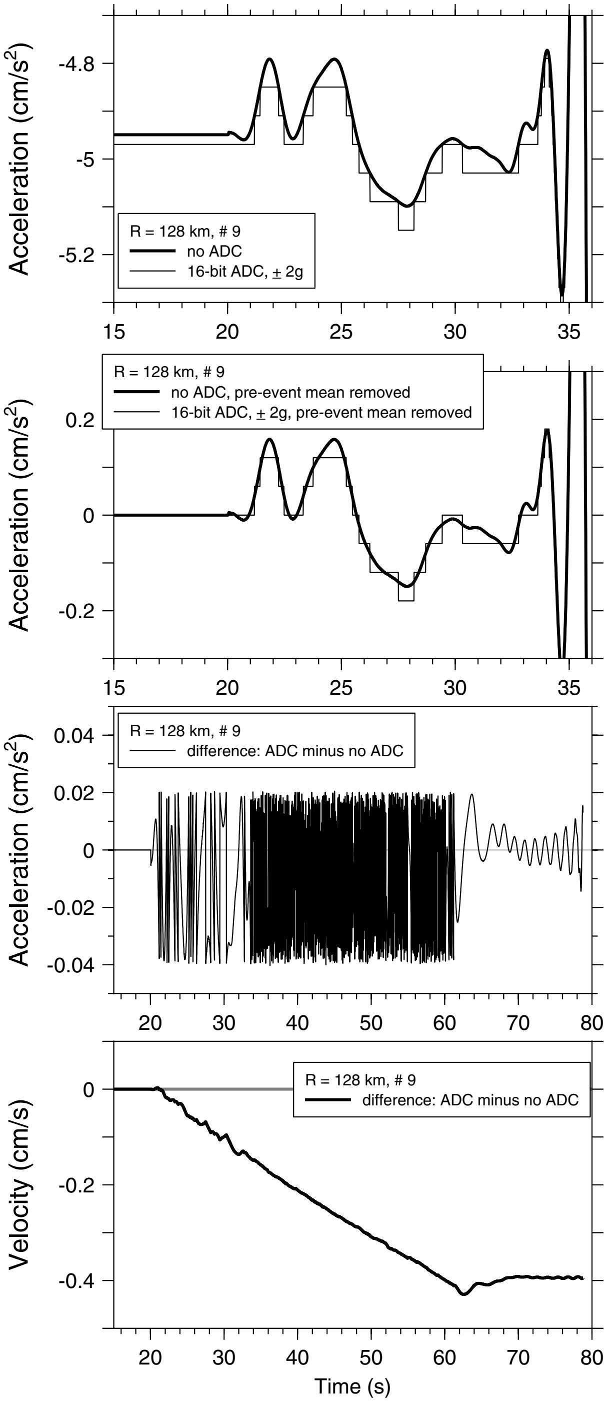

Figure 3. Cause of drift in displacement for a particular example. The top graph shows the unprocessed simulated and digitized acceleration; the second graph shows the top traces after removing the mean determined from the pre-event portion of the traces in the top graph; the third graph shows the difference of the mean-removed digitized and simulated accelerations; the integration of the difference acceleration yields the curve shown in the bottom graph. A second integration results in the linear drift in displacement seen in the second-to-last trace of the right-hand panel in Figure 2.

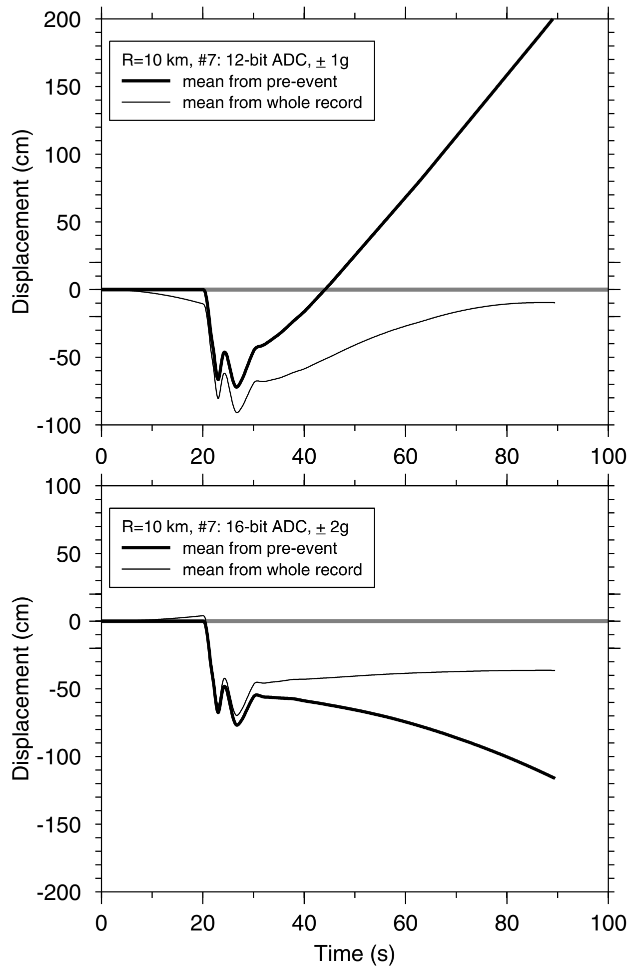

Figure 4. The effect of the length of record used to determine the means removed from the acceleration traces before double integration. The acceleration traces were derived from ADC of the simulation at 10 km, for a particular randomly chosen constant offset. The top and bottom graphs show the results using the types of ADC in the A-800 and A-900 instruments that recorded the 1999 Chi-Chi, Taiwan earthquakes.

function of the resolution of the ADC. One particular example of this is shown in Figure 6. Note that the 24-bit ADC leads to a displacement time series essentially identical with the true motion, and that the dithered 12-bit ADC is in general better than the undithered 16-bit ADC. (Processing of recent recordings on 24-bit systems from the 1999 Chi-Chi, Taiwan, and the 1999 Hector Mine, California, earthquakes shows that displacements from those recordings are contaminated by large drifts [Boore *et al.*, 2002; Wang *et al.*, 2003], so something other than the ADC effect discussed in this article is present in those recordings.)

Up to this point, examples have been shown for individual realizations of the random process. In Figure 7 I show the results of all 99 realizations for the simulation at a distance of 10 km. The graphs show the value of the displacement for each realization at the end of the record (d_{end}), for

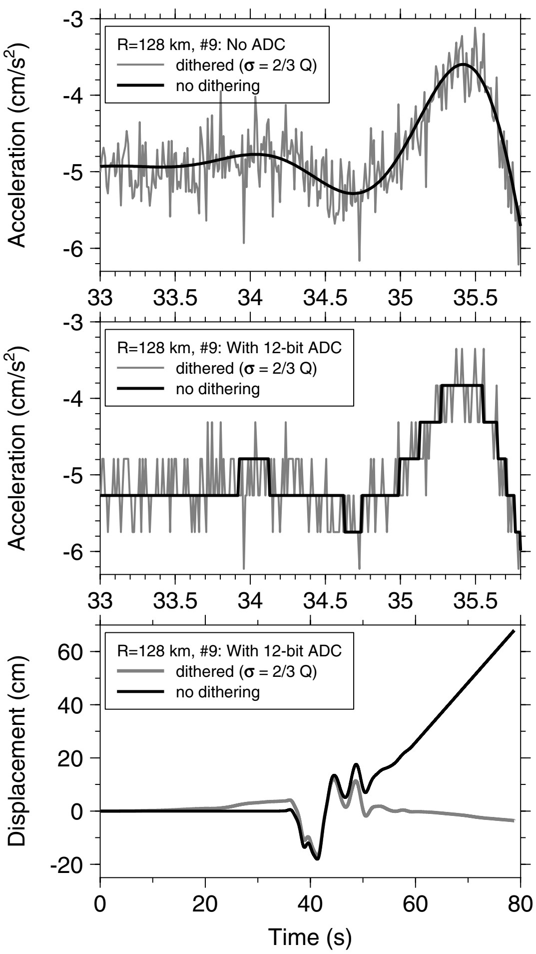

Figure 5. The effect of dithering on displacement. The top graph shows the simulated acceleration, with a random constant offset, before and after dithering (with noise from a normal distribution with a standard deviation of 2/3 Q); the middle graph shows the same traces as in the top graph after ADC. Both the top and middle graphs are plotted at a greatly expanded time-scale. The bottom graph shows the displacements obtained by double integration of the nondithered and dithered accelerations.

several types of ADC and for different portions the records used to determine the mean. Note the different scales for the ordinate in each graph (the scales are the same for the bottom two graphs). The statistics corresponding to these scatter plots are contained in Table 1 for the simulations at 10 and 128 km. The statistical results are consistent with those discussed earlier for particular realizations: (1) the higher the resolution, the smaller the effect of ADC on the displacements; (2) using means from the whole record leads to smaller drifts than using a mean from the pre-event portion; (3) for a given resolution, dithering leads to much smaller drifts than for undithered signals (for the examples here, dithering led to smaller scatter than obtained by going from $\pm 1g$, 12-bit ADC to $\pm 2g$, 16-bit ADC, although the mean

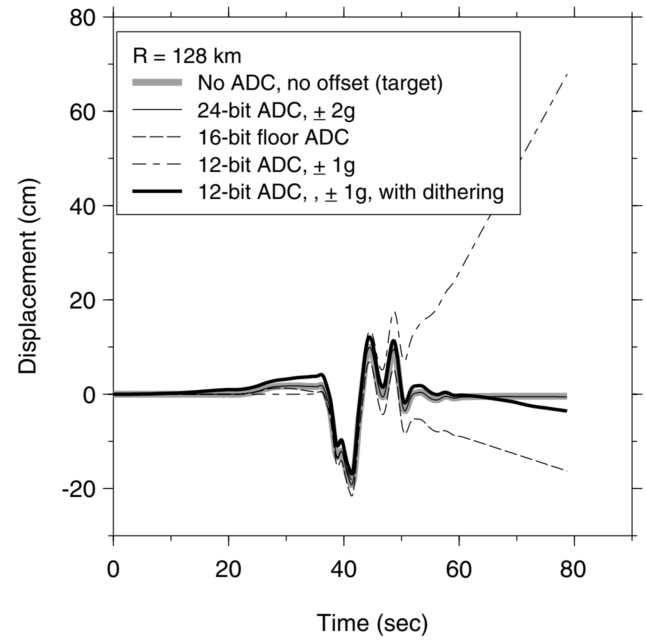

Figure 6. An example of the displacements obtained using different ADC.

of the final displacements for the 16-bit ADC was closer to the true mean).

Summary

The ADC process can produce apparent offsets in the baseline of acceleration time series and these offsets can lead to drifts in displacement similar to those seen in analysis of observed data. The offsets can occur because of shifts in the constant level of the voltage of the transducer or they can be associated with slowly varying signals, as are produced by near- and intermediate-field terms in the wave field radiated from earthquakes. Dithering is an effective way of reducing the effect of ADC. In conclusion, even without the many other sources of baseline offsets the ADC process alone can introduce significant drifts in displacements derived from digitally recorded accelerations.

Acknowledgments

I thank John Evans, Jim Kerr, Frank Scherbaum, Joe Steim, Chris Stephens, and Guoquan Wang for in-person and email conversations and Hung-Chie Chiu, John Evans, Vladimir Graizer, and Chris Stephens for reviewing the article. In particular, I thank John Evans for alerting me to the use of dithering in audio digital signal processing, and for making me realize that absolute zero is a fiction in electrical recording systems.

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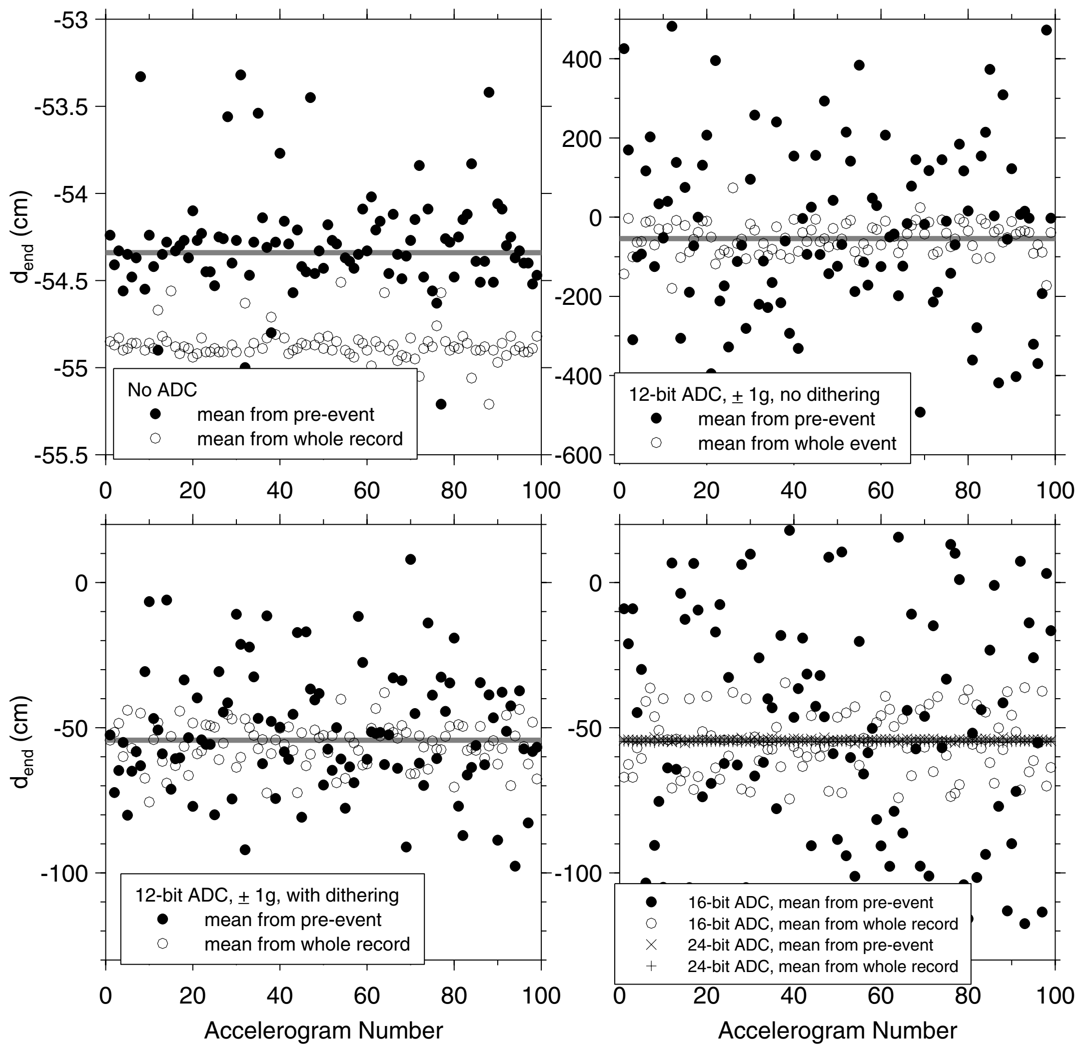

Figure 7. Values of final value of displacement (d_{end}) after 90 sec, from processing of 99 accelerograms formed by adding constant, randomly chosen constant offsets to the simulation at $R = 10$ km (see text). The upper-left plot is for no ADC, the upper-right plot is for 12-bit ADC with no dithering, the bottom-left plot is for 12-bit ADC with dithering, and the bottom-right plot is for 16-bit and 24-bit ADC (both undithered). Each plot contains the accelerograms processed with means determined from the whole record and from the pre-event portion of the record. The horizontal gray line in each graph is the exact value of d_{end}. Note the large differences in the ranges of the ordinate axes.

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Table 1
Mean and Standard Deviation of d_{end} from 99 Simulations of Random Constant Offset
(All Displacements in Centimeters)

R (km)	Mean from*	ADC†	d_{end}‡	$\langle d_{end} \rangle$§	$\sigma_{d_{end}}$	σ_{rw}‖
10	Pre	None	−54.3	−54.3	0.3	
10	Pre	12	−54.3	−29.7	217.2	6.6
10	Pre	12, dithered	−54.3	−51.3	21.1	15.4
10	Pre	16	−54.3	−52.3	40.3	0.8
10	Pre	24	−54.3	−54.3	0.4	0.003
10	Whole	None	−54.3	−54.9	0.1	
10	Whole	12	−54.3	−53.7	39.9	6.6
10	Whole	12, dithered	−54.3	−55.7	7.6	15.4
10	Whole	16	−54.3	−55.4	10.9	0.8
10	Whole	24	−54.3	−54.9	0.1	0.003
118	Pre	None	−0.4	−0.4	0.3	
118	Pre	12	−0.4	+19.2	189.0	5.5
118	Pre	12, dithered	−0.4	−3.5	16.5	12.7
118	Pre	16	−0.4	+0.9	29.2	0.7
118	Pre	24	−0.4	−0.4	0.3	0.003

*The portion of the acceleration used to determine the mean that was then subtracted from the whole acceleration trace before double integration is indicated by "pre" and "whole," where "pre" indicates pre-event mean (usually the first 19 sec, where the motion starts at about 20 sec), and "whole" indicates the mean of the whole record.

†Types of ADC as follows: 12 = $\pm 1g$, 12-bits; 16 = $\pm 2g$, 16-bits; 24 = $\pm 2g$, 24-bits. "dithered" = adding Gaussian noise with standard deviation 2/3 Q to acceleration trace before ADC.

‡Actual final displacement.

§Mean value from 99 simulations.

‖From random walk, equation (3).

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Manuscript received 6 December 2002.