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EYEBALL OPTICS OF NATURAL WATERS: SECCHI DISK SCIENCE

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ABSTRACT. The Secchi disk is used to visually measure the clarity of natural waters such as lakes and seas. It is usually a white disk of 30 cm diameter that is lowered into the water until it disappears from sight. The depth of disappearance is called the *Secchi depth* of the medium, and is a measure of the amount of organic and inorganic materials in the water along the path of sight. This is a technique that was systematically studied among others by the Italian physicist Angelo Secchi in 1865. It is, amazingly, still in use today by environmentalists and laid-back professors of marine and lacustrine biology, wishing to give an easily made and visualized index of the trophic state of natural hydrosols. It is one of the few instruments remaining in the armory of modern science for which the visual sense of the human operator is an integral part of the measurement loop.

This review examines the physiological and physical basis of the Secchi disk procedure. The theory of the white disk is detailed in the hope that the truth about its subjective shortcomings, once revealed, will set the inveterate Secchi disker free. On the other hand, for those die-hard Secchi opticians who will persist in its use, the subjective disadvantages of the technique are balanced against the advantages of its simple and inexpensive operation. In particular, the bases for updating the Secchi disk parameters are derived. However, this is not to be construed as a blanket endorsement of the use of the disk. The only legitimate use of the disk is as a visual gauge of the clarity of natural waters. It is not to be unthinkingly linked with the apparent and inherent optical properties of such media; it is not to be used as a substitute for objective measurements of the optical properties of natural hydrosols.

Were the great physicist Secchi a witness today, he would probably be delighted by the great advances in the spectroscopic study of natural hydrosols, using modern electronic devices. One can then imagine his reaction to the incongruity of the sight of his disk in all solemnity being lowered into a sea or a lake by someone wearing a quartz-driven, radio/television, computer-chip wristwatch, while speaking into the watch's microphone in satellite communication with his general purpose real-time data-processing computer at his home laboratory half a planet away.

1. INTRODUCTION: EIGHT LAWS OF THE DISK

About one hundred and twenty years ago, in Italy, in the spring of 1865, the Commander of the papal fleet asked a prominent physicist of the day to help document the transparency of the nearby coastal waters of the Mediterranean. The research was to fulfill the tactical needs of the fleet and answer various scientific questions raised by the Commander. So began physicist Angelo Secchi's celebrated optical experiments aboard *l'Immacolata Concezione* in which he explored the phenomenon of tethered white disks descending into and ultimately disappearing in watery depths.*

It was a delightful exercise for both the learned scientist and the Commander to lower the disks, first one (the larger) and then the other (the smaller) on the sunny and then on the shady side of the ship, now in calm water, then in heaving seas. Even the sailors delighted in clambering up and down the *Concezione*'s rigging and jumping into launches so as to view the disappearing disks either from mast height or with their faces virtually pressed onto the water surface. This was something even they could understand and enjoy. As they watched the disks descend, umbrellas and hats were used to shade the patch of sea surface through which they peered.

After two weeks of work a yellow colored disk and a brown colored disk also were devised to see the effects of differently colored surfaces on the depths of the disappearing disks. Cloudy skies and bright sunny skies with the sun at various altitudes were the settings of the above activities. Seven experiments in all were conducted over a six week period; and eventually all the data were accumulated. Then the sailors went on to more prosaic duties,

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* The spellings in the English translation of Secchi (1866) follow those of the O.N.I. document cited in the references. Incidentally, about half the footnotes below and some new text have been added to clarify or expand the first two drafts of this report. These changes were in response to reviewers who were kind enough to send me their comments.

and Secchi went home to mull over his results. And that's when trouble set in: he began to think.

As he turned the results before his mind's eye Secchi began to realize that there were hidden depths not only for the disks, but for the mind trying to rationally follow them into the sea. The works of the optical sages of the time (Bouguer, Lambert, and Arago) were consulted, only to see them disagree on mutual points of interest and only to witness silences on the more profound questions raised by Secchi's experiments. In the course of time, Secchi gave up trying to give precise quantitative, and universal expression to his findings. For posterity, he recorded his conclusions as part of Commander Cialdi's reports of the experiments on board the research ship *l'Immacolata Concezione*. A close reading of Secchi's text suggests some generalizations we may expect the disappearing depth of the submerged disk to obey. We formulate and compile some of the empirical laws of the submerged white disk here, as they will form qualitative validity tests for the binocle of eyeball optics, to be presented below. Each law is preceded implicitly by the proviso: "with the present factor within prescribed bounds and all other factors held fixed."

These factors are: (i) the diameter of the disk; (ii) spectral reflectance of the submerged disk; (iii) altitude of the sun above the horizon, (iv) optical state of the sea surface (variance of wave slopes); (v) reflected radiance of sun and sky in the sea surface; (vi) height of observer above the water surface; (vii) shadows and sunlight along the submerged path of sight, (viii) the amount of plankton below the surface. We may now state Secchi's laws for an observer in air looking vertically downward through the water surface toward the submerged tethered disk:

I. The depth of disappearance of the disk varies directly with its diameter.

II. The depth of disappearance of the disk varies directly with its reflectance.

III. The depth of disappearance of the disk varies directly with the altitude of the sun.

IV. The depth of disappearance of the disk varies inversely with the optical state of the sea surface.

V. The depth of disappearance of the disk varies inversely with the relative amount of reflected radiance of sun and sky in the sea surface compared to that transmitted upward from below the surface.

VI. The depth of disappearance of the disk varies inversely with the immediate height of the observer above the sea surface.

VII. The depth of disappearance of the disk is larger if the water path of sight between disk and observer is more shadowed; it is smaller if the water path of sight beyond the disk from the observer is more shadowed.

VIII. The depth of disappearance of the disk varies inversely as the amount of plankton below the surface.

While it is nice to have the laws of the white disk so crisply stated and neatly arrayed before us, don't lose sight of the horrible import of the first seven laws: the depth of disappearance of the disk is affected by many more things in the sea other than the plankton below the surface.

The first six laws are those that may be, after some thought, stated by any alert observer of nature, and even a land lubber with a retentive memory. The seventh law is more subtle, requiring considerable thought on the matter, and reflection on past experiences of seeing distant objects on foggy days or underwater objects while swimming in dirty pools. Law VIII is what brings us all together in this paper, hoping eventually to extract some quantitative guidance from modern forms of Secchi's qualitative statement which in his report (Secchi, 1866) he phrases as follows:

> We do not wish to pass over a circumstance which is pointed out to us that in these months [April and May] the sea is more transparent than in summer, perhaps owing to the lesser amount of animalcules and other organisms which grow there in the summer season.

All of these laws, including a ninth law, below, can be given quantitative expression by means of the binocle of eyeball optics in Eq. (8), and will be rationalized at appropriate points of our discussions.* For the moment we pause to lay the groundwork for Eq. (8) in the form of a slightly mythologized reminiscence of some relevant past events witnessed by the present author.

* In a condensed version of this essay, the laws were partly reformulated and a new one added, along with a mathematical statement for each. cf. Preisendorfer (1986).

2. THERE'S MANY A SLIP TWIXT THE CUP AND THE LIP

Here is a relatively painless introduction to the physical basis of the Secchi disk depth reading.

On a fine spring afternoon over twenty years ago, just after an early supper, I stepped up out of the wardroom onto the deck of the research vessel *Dayton Brown*, which was anchored off the west coast of one of the Coronado Islands a dozen or so miles into the Pacific, southwest of San Diego, California. The sea at this point was limpid, deep, and homogeneous under a partly cloudy sky. The surface of the sea was untroubled by wind or swell, and the sun was about 30 degrees above the thin white band of the horizon. Some sea lions were barking as they flopped about in the warm sun on the guano-encrusted rocks of the island shoreline. How appropriate, I thought, to be finishing up some research in hydrologic optics just at this point in time. It was just 4:00 pm on 20 April 1965, the centennial of Secchi's first optical experiment in water clarity using his two submersible disks, one a fixed white clay disk of 43 cm diameter, the other a white-painted sail cloth stretched on an iron ring of 60 cm diameter. I recalled how these were first lowered over the sunny side of the papal steam sloop, the *l'Immacolata Concezione*, into the blue, clear waters of the Tyrrhenian Sea, under partly cloudy skies, just at 4:00 pm of that day 100 years ago. The smaller and larger disks were observed to disappear at 16.5 meters and 24.5 meters, respectively.

My reverie of that moment in the past was disturbed by what I thought was the high-pitched excited voice of a young man (perhaps it was the call of a sea gull passing overhead?). As I turned toward the voice, still deep in my Mediterranean reverie, I thought I saw two men, with their backs to me, leaning over the bow rail, gazing down into the mosaic of sunlit and ship-shadowed water. I recognized the younger person as a former student of the older man. The young fellow, a callow theorist, had recently come out from an eastern college with his mentor, a seasoned experimenter, to do some work on Oceanographic Optics. What I experienced in the next five minutes of conversation between these two apparitions seemed to be echoes of memories formed while I was on Professor Duntley's Diamond Island research station in Lake Winnipesaukee, New Hampshire, fifteen years before. I recount the conversation here, as it may give an amusing entré into the subject of Secchi disk 'science'. The scene begins when one of the two men at the bow rail had just accidentally dropped an empty white glass coffee mug over the side of the ship.

<u>Theorist</u>: There you go again, being careless with your design of experiments. You didn't even note the sun altitude or what filter you were using.

<u>Experimenter</u>: I had an irresistible urge to see what would happen if I dropped it in.

<u>Th.</u> Good heavens man! Why the experiment? Have you forgotten Archimedes Law? On theoretical grounds, I predict that the mug will sink!

<u>Ex.</u> (Recovering from the accident) Look--it's turning bilious as it sinks deeper. What an interesting transformation of shades and hues. For an instant there the mug even had a halo and seemed to faintly light up the water around it. It looks like it's down 10 meters and I can still see it quite clearly!

<u>Th.</u> (Peering down over the railing) It must have reached terminal velocity by now and is surely sinking according to Stokes' law. (Looking at his

watch, then a pause) At the sound of the tone it will be exactly 20 meters.

Ex. (Ignoring the other's babbling) There it goes. I lost track of it. There's no doubt about it, this is pretty clear water.

Th. What're the alpha and the kay for this water? Did you measure them again this morning?

Ex. They're the same as yesterday. The alpha's about a tenth per meter and the kay is about fifty thousandths per meter, both in the green. What are you doing?

Th. (Jotting something on a piece of paper so that the other can see it) I'll bet I can connect the mug's depth of disappearance with the alpha and kay of this water.

Ex. (Smiling wearily to himself, and then with a sigh): Here we go again. Take it easy, Einstein, my calculus is buried under a ton of barnacles.

Th. We really don't need it. Didn't you explain to me how it's known that the light level generally goes down exponentially with depth in deep homogeneous water like this? I can use this fact to figure out how much light gets to the mug at each depth z. It would be (writing on the paper) $H_o \exp[-Kz]$, correct?

Ex. Yes, and let's say that H_o is the irradiance of green light on a horizontal surface just below the surface and K is the kay for this water, namely, 0.050 m^{-1}. So you can figure out the irradiance on a horizontal surface at depth z. (Then feigning puzzlement) Where does that get you?

Th. Why, this lets you compute the inherent radiance of the mug at depth z, if you know its reflectance.

Ex. Do you know it?

Th. No, but let's just call the mug's reflectance "R" and say it's for green light. Then (writing again) $R(H_o\exp[-Kz])$ would give an estimate of the radiance reflected upward by the mug.

Ex. Hmm---Yes, but that's its *inherent* radiance down at depth z. Here we are on deck.

Th. I see what you mean. So we need the *apparent* radiance of the mug. But that'll mean knowing the path radiance generated by scattered light between us and the mug including even the aureole around the mug produced by its reflected light, not to mention the effect of the air-water surface. All that's pretty hard to come by isn't it?

Ex. Quite. (Then, taking off his glasses and polishing them thoughtfully.) But if you remember what I told you the other day about radiance *differences*...

Th. Radiance *differences*? Oh, of course! They are transmitted exactly according to the exponential law $\exp[-\alpha r]$ for beamed light along a path of length r, where for this water, $\alpha = 0.100\ m^{-1}$. Let's see, the radiance difference in this case will be between the inherent radiance of the mug at depth z and the inherent radiance of the background water at the same depth. Such a difference is easy to figure.

Ex. Is it? Again you don't know the reflectance of the water at the depth of the mug. At least I haven't measured it yet for this place.

Th. That's O.K. Let's call the reflectance of the water "R_∞". It could not be much different from 0.02 for all depths. I was looking over some of your old reports and review articles yesterday. Everywhere you measured R_∞ you got something around 0.02 for green light, even in some deep clear lakes and ponds, *n'est ce pas*?

Ex. (Gritting his teeth) Perhaps so. Just a few surprises left there. Well, where are you leading me next with your paper and pencil?

Th. The average radiance of the water background at depth z is simply R_∞ times the downward irradiance at that depth. That is, we would have $R_\infty(H_o\exp[-Kz])$. Right?

Ex. Yes, except for a factor of π--but it'll cancel out anyway in the end. So don't worry about it now.

Th. (Looking up surprised) Say--how do you know that? Have you worked all this out before?

Ex. (With a straight face, looking out at the horizon) Not exactly. On with it--what is your next step?

Th. Well here is the radiance difference between the mug and the sea at depth z:

$$H_oRe^{-Kz} - H_oR_\infty e^{-Kz}$$

Ex. And then?

Th. And then at long last I can use the radiance difference law. That is I multiply this difference by exp[-αz] to transmit it up to just below the surface--where it'll be what we will actually see if we went there. Thus:

$$\left[H_oRe^{-Kz} - H_oR_\infty e^{-Kz}\right]e^{-\alpha z}$$

Ex. Can you simplify this mess?

Th. Sure, like this:

$$H_o(R-R_\infty)e^{-(\alpha+K)z}$$

Ex. Also I don't like to bother with absolute light levels. Can you take care of that, too?

Th. Yes, I suppose. Why not divide the whole thing by the amount of reflected radiance from the sea just below the surface? Like this:

$$\frac{H_o(R-R_\infty)e^{-(\alpha+K)z}}{H_oR_\infty}$$

<u>Ex.</u> That'll work fine. Now, what have you got for all your trouble?

<u>Th.</u> (A pause, and then) Why this looks like it could be a kind of contrast reduction formula...yes, it is...just let $H_o(R-R_\infty)/H_oR_\infty$ or simply $(R-R_\infty)/R_\infty$ be the inherent contrast C_o of the mug against its background. It looks like this contrast is independent of the depth of the mug. That's fantastic! Is that right?

<u>Ex.</u> (Blanching) Yes, go on...

<u>Th.</u> So if the apparent contrast of the mug at depth z as seen from just below the surface is C_z, then it looks like we have

$$C_z = C_o e^{-(\alpha+K)z} \quad (1)$$

<u>Ex.</u> (A little startled at the equation's quick appearance from an unexpected line of argument) Would you know how to use something like that?

<u>Th.</u> (After a while) Well, if we can agree that the mug has sampled enough of the medium between us and it when C_z/C_o is some small number, maybe like 1/50, and if we measure the z for such a ratio, then we can compute the corresponding α+K. It's true we couldn't find α and K separately this way, but the sum is probably still a good index of water clarity over the sampled path. We may even consider finding the value of C_z at which our eyes see the mug disappear.

<u>Ex.</u> (In mock anger) Incredible! Do you know what you've just done, boy?

<u>Th.</u> (Somewhat aghast) No, sir. But I do know that we haven't allowed for

the surface effects yet nor perturbative effects of the mug on the surrounding light field. Is something wrong?

<u>Ex.</u> No, it's just that throughout this discussion I've seen several old friends in a new light. You did well. Now, you run along below and get me a fresh mug of coffee. And on the way back drop into the ship's library. I want to show you something in Sec. 1.4 of 'Hydrologic Optics'.

In the course of time, the student learned how to derive the contrast reduction formula (1) in orthodox ways, and to make everything about it rigorous and true. But the heart of the grown-up theory, particularly in its applications to light and life in natural waters, is still (1), and on occasion, (6), below.

3. NOTHING THAT IS SEEN IS SEEN AT ONCE IN ITS ENTIRETY (THEOREM 1, OPTICS OF EUCLID)*

With the conversation of the Secchi disk derivation still in our ears, it may be well to append a few important caveats. Not everything about the disappearing disk has yet been revealed.

The derivation leading to (1) is not completely rigorous: First, the descending mug was assumed not to disturb the light field through which it fell. The experimenter's comment about the aureole gives the lie to this assumption.

Next, the eyes of master and student were responding photopically, while the volume attenuation coefficient α and the diffuse attenuation coefficient K of the sea below them had been measured spectrally, i.e., at individual wavelengths. There are two ways to get around this problem of mismatched radiometric and photometric quantities. Equation (1) is correctly used, e.g., if each observer on ship board wears well-mounted, broad band glass filters that pass only the near green (or near yellow, or near blue, etc.) photons, returning from the mug and for which wavelength bands α and K have been measured. Alternatively, α and K can be measured or computed photopically from their spectral values using standard luminosity curves; then the two observers could have used their bare eyes to watch the sinking mug disappear at the 'true' depth. One measures α and K, e.g., photopically by placing over the flux measuring collector a light filter, having the same spectral sensitivity as the human eye under photopic (daylight) conditions. Or then again, one can measure the light field spectrally and then photopically filter it numerically.

* Little did Euclid know what a 'universal truth' he proved with the aid of a few lines drawn on a triangle (cf. Burton, 1945). As we shall see, Euclid's chutzpah is reincarnated on a smaller scale, but quite regularly, by certain practitioners of eyeball optics.

The preferred alternative to these two ways to use (1) is the former (wear glasses) although that may require heroic efforts in practice.* The other alternative (photopic observations and photopic data collection) can be shown to be inappropriate in applications to problems of aquatic biology requiring special radiometric (spectral) measurements. Having stated my preference, the matter will not be labored further. In what follows, it will be assumed (except where noted) that the former (wear glasses) alternative will be followed. The notation and concept formation, therefore, will be predominantly in radiometric form i.e., we will write and mean 'N' for radiance, write and mean 'H' for irradiance, rather than 'B' for luminance, and 'E' for illuminance. (For an extensive development of radiometric and photometric concepts, see H.O., Vol. II, Ch. 2).†

For an account of how to obtain photopic α and K values in practice, see Tyler (1968). Another example of the correct use of photopically evaluated α and K readings may be found in Holmes (1970). These two papers still stand

* This advice to the reader may turn out to be as well-meaning and as impractical as Arago's to Secchi as the latter prepared for his experiments on the *l'Immacolata Concezione*. Arago, in his writings, advised the use of a polarizer to diminish the reflected light from the sea surface, and thereby the observer may better discern the 'true' depth of disappearance of the disk (cf. law V, above). Unfortunately, the polarizer was ineffective and only succeeded in restricting the field of vision and blocking half of the photons returning from the disk. Secchi was, accordingly, not at all impressed by Arago's suggestions. However, in the present case, if the observer can look through spectrally adjustable glasses, eyeball optics can take a quantum leap ahead (pun aside), as we shall see, below. Incidentally 'quantum leap' here means a *discontinuous* (but not necessarily large) change.

† In the subsequent version of this essay (Preisendorfer, 1986) I systematically explore the other main alternative, namely the use of *photometric* measurements and observations underlying the Secchi disk theory. It is interesting to note that the topographies of the main formulas are identical in both the photometric and radiometric contexts. The change, however, is there and it is profound: photometric Secchi disk theory works exclusively with *apparent* optical properties; and *inherent* optical properties are alien to the theory of the disk, when the latter is phrased photometrically.

like twin beacons in the sea of confusion about the correct use of (1) in practice. However, even these two beacons now need a little polishing of their lenses, and this will be done gently with soft cloth at various points of the discussion, below.

We come next to another improvement of the above shipboard derivation of (1). To begin, we have to face the problem of seeing through the surface, something neither Tyler nor Holmes satisfactorily discussed, even though this problem had been resolved by Duntley and the author as early as 1950 on Diamond Island, N.H. (Duntley and Preisendorfer, 1952). By including the surface effects, we can strengthen (1) so as to have it reproduce Secchi disk law V quantitatively, and more importantly, to keep Secchi diskers from becoming too undisciplined in their quest for universal truths.

4. DISKS SEEN THROUGH A SURFACE DARKLY

We now consider the effects on Secchi disk visibility of the reflective and refractive effects of the air-water surface.

Anyone who has gazed through a quiet swimming pool surface preparatory to stepping in, may have been at some time startled at the discrepancy between the apparent depth of the first step and its experienced depth a moment later: a submerged step seemingly 12 cm below the surface is actually 16 cm below the surface and the four meter end of the pool appears to be only three meters below the surface.* This refractive phenomenon through the static surface makes the angular subtense of the Secchi disk appear about 25% larger than it would have, should the observer have placed his head, with eyes open, below the water surface. In view of what is about to be revealed, this dunking of one's visual apparatus during an optical experiment is not at all an impractical expedient.

When the air-water surface is in rapid motion this magnification effect, although present in modified form, is masked by an erratically distorted moving image of the submerged Secchi disk. The apparent contrast of the disk's center against the immediate background of the watery depths, as seen by an observer in air, is markedly reduced by the rapid alternate interleavings of apparent background radiances and apparent disk radiances along the fixed line of sight, resulting in a time-averaged and reduced apparent contrast of the disk.

Let us consider a water surface, freshly crinkled by capillary waves produced by a passing breeze. It can be shown (H.O., Vol. VI, p. 256) that the contrast transmittance factor $\overline{\mathcal{T}}_o$, by which the apparent contrast C_z of the

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* This example alone should warn the reader that the waters into which he is about to step are deeper than they at first appear (cf. Euclid's 1st optical theorem).

center of the disk against its background is reduced by this time-averaged refraction effect, is:

$$\bar{\mathcal{T}}_o = 1-\exp[-\tan^2\phi/2\sigma^2] \tag{2}$$

where $\phi = 4\phi'$, and ϕ' is the angular subtense (to first order, in radians) of the disk's radius as seen from just below the still surface (so the refractive effect in the swimming pool example above is included in ϕ). Moreover, σ^2 is the variance of the slopes of the capillary wave surfaces (the *optical state* of the surface in law IV). We can estimate σ^2 by the formula (cf. H.O., Vol. VI, p. 149, where for now σ_u^2 and σ_c^2 have been averaged):

$$\sigma^2 = w\,U\ , \quad w = 2.54 \times 10^{-3}\ \text{sec}\cdot\text{m}^{-1} \tag{3}$$

and where U is wind speed in $\text{m}\cdot\text{sec}^{-1}$ (measured at 'mast height', i.e., 19 m above sea level). The main fact we should note here is that when $U = 0$ it follows that $\bar{\mathcal{T}}_o = 1$, and that $\bar{\mathcal{T}}_o$ decreases toward 0, and does so rapidly, as U increases from 0.

As an example of (2), consider Secchi's smaller disk of diameter 43 cm. This disappeared in calm water at 16.5 m depth. Hence we have a disk radius of $r = 21.5$ cm and $z_{SD} = 1650$ cm. The half-angular subtense of the disk at this depth is small so we have approximately $\phi' = r/z_{SD} = 21.5/1650 = 0.0130$ radians. The refractive effect of the water is accounted for by setting $\phi = 4\phi' = 0.0521$ radians. A fresh breeze springs up of $U = 1\ \text{m}\cdot\text{sec}^{-1}$, so that $\sigma^2 = 2.54 \times 10^{-3}$. Thus $\tan^2\phi/2\sigma^2 = 0.533$, whence by (2), the contrast transmittance for the wind ruffled surface is $\bar{\mathcal{T}}_o \equiv 0.420$. This reduction of contrast is relatively large and it means that, had the breeze sprung up as

Secchi was lowering the sail-cloth disk, it would have disappeared at a somewhat smaller depth than 16.5 m. Interested readers shall be able to compute that depth when we have arrived at the direct ocle (9), below.

The second main contrast transmittance factor is $\mathcal{T}_o$ and this is essentially associated with the reflected sky light in the water surface. To numerically evaluate $\mathcal{T}_o$, even for the still air water surface, requires a full solution of the radiative transfer equation for the hydrosol (cf. H.O., Vol. VI, p. 43, Eqs. (20), (21)). However, for our present discussion we may write out the formula for $\mathcal{T}_o$ simply, and correctly, as follows. Let N_o be the vertically downward radiance incident on the surface from the sky or sun and let $N_o r$ be the vertically upward reflected radiance from the surface alone. Here r is Fresnel's reflectance for normal incidence on a water surface. Let N^o be the vertically upward radiance of the hydrosol incident on the underside of the surface. Then $N^o t/m^2$ is the vertically upward transmitted radiance through the surface where $t = 1-r$ and m is the index of refraction of the water. The still-water contrast transmittance factor is then given by

$$\mathcal{T}_o = \frac{N^o t}{N^o t + N_o r\, m^2} \tag{4}$$

When Secchi placed his face right down onto the water he blocked out the sky light from directly above his head thereby removing N_o, but leaving the $N^o t$ term in (4) essentially intact. Hence, he effectively set N_o to zero, so that $\mathcal{T}_o$ became 1. An umbrella placed overhead during a disk observation would also work well to reduce $\mathcal{T}_o$ to 1. If this is not done, then $\mathcal{T}_o$ is less than 1 and a moment's thought would show that the disk will disappear at a shallower depth. It turns out that for some reasonably clear hydrosols viewed with green filters, N^o is on the order 0.02 N_o, while $r = 0.02$ and $t = 1-r = 0.98$, and finally, m = 4/3. Hence in such cases $\mathcal{T}_0 \approx 0.360$ which is a hefty loss of contrast. So it behooves one to make it standard practice to eliminate the downward flowing sky radiance N_0 when doing eyeball optics in any natural hydrosol. In view of our numerical example of $\bar{\bar{\mathcal{T}}}_0$, it also behooves one to do eyeball optics in calm water when U is essentially zero.

In summary, then, good Secchi opticians will use black umbrellas in becalmed row boats, when peering over the side at the descending white disk. Enterprising Secchi diskers just may go onto construct rowboats with hooded cabins and glass bottoms (cf. H.O., Vol. I, p. 46).

When one cannot control the factors $\bar{\bar{\mathcal{T}}}_0$ and $\mathcal{T}_0$, then they combine multiplicatively to become*

$$\mathcal{T} = \bar{\bar{\mathcal{T}}}_0 \mathcal{T}_0 \tag{5}$$

and the basic *contrast reduction formula* (1) must be written as

$$\boxed{C_z = \mathcal{T} \; C_0 \; e^{-(\alpha+K)z} \; .} \tag{6}$$

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* Equation (5) of course is not exact; but it provides a useful rule of thumb for the time-averaged contrast transmittance of a moving air water surface under a luminous sky. The correct numerical size of the factor $\mathcal{T}$ can be obtained only by completely solving the radiative transfer problem for a hydrosol with a wind-blown random air water surface. See for example (H.O., Vol. VI, pp. 210-260). The machinery for such a task has now been assembled for numerical work, and may be found in Preisendorfer and Mobley (1986). Even this work falls short of being truly general. The perturbation on the light field by Secchi disks of finite diameters leads to a full three dimensional radiative transfer problem in the hydrosol. A technique that would lead to solutions in this case is developed in Preisendorfer and Stephens (1984). But I think, and all will agree, that using such a theory to gild the Secchi disk lily, would be stepping beyond the sensible use of one's time and energy. There are now simpler and relatively exact procedures in hydrologic optics that supply the optical needs of lake and marine biologists.

This is the formula on which we can base Secchi disk theory for a homogeneous hydrosol (i.e., with depth independent attenuation coefficients, α, and K) in which a Secchi disk at depth z is viewed from just above the surface along a vertical downward path of sight. Here C_o is the inherent contrast of the disk (as seen at the disk) at an arbitrary depth z. A good approximation to C_o is given by

$$C_o = (R - R_\infty)/R_\infty \tag{7}$$

where R is the spectral reflectance of the submerged disk (at some wavelength) and R_∞ is the depth-independent irradiance reflectance of the optically infinitely deep hydrosol at the same wavelength. Note that R must be the *submerged* reflectance of the disk. A way of finding this number in the laboratory by simply wetting the disk is given in (H.O. Vol. I, p. 171). We will return to (6) in sec. 9, where we give it still another boost toward generality (cf. (40)). It will turn out, that (6) is already in its best form for all practical purposes.

5. THE BINOCLE OF EYEBALL OPTICS AND ITS DIRECT OCLE

We come now to the hub of Secchi disk territory from which we will at once branch out towards two very basic formulas of the field: the direct and inverse ocles of eyeball optics.

Let us write the basic contrast reduction formula (6) in the form:

$$\text{[binocle]} \qquad (\alpha+K)z = \ell n[\mathcal{T}C_o/C_z] \qquad (8)$$

We shall call this version of (6) the *binocle* of eyeball optics for the reason that, like its material counterparts, the binoculars or field glasses, it may be looked through from either end. The manufacturer of a real binocular expects its user to look through the small end so as to see objects magnified. When (8) is used this way, one can find, as its own inventors have intended, the depth z_{SD} of disappearance of the Secchi disk by means of the so-called* *direct ocle*:

$$\text{[direct ocle]} \qquad z_{SD} = \frac{\ell n[\mathcal{T}C_o/C_T]}{\alpha+K} \equiv \frac{\Gamma}{\alpha+K} \qquad (9)$$

Here $\Gamma \equiv \ell n[\mathcal{T}C_o/C_T]$ forms the heart of the ocle and is called the *coupling constant*.

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* There is no such noun as 'ocle', although I wish there was. In Webster's New International Dictionary, 2d Ed. unabridged, G.C. Merriam Co., Springfield, Mass. (1950), the closest word to my 'ocle' is the noun 'ocular', which is a humorous name for the eye. Unfortunately this 'ocular' could be confused with the adjective, 'ocular'. I wanted a noun, short of the pompous Latin 'oculus', and preferably an amusing one, to denote the mathematical counterpart to an eyeball. With the reader's indulgence, if only in the confines of this report, I will neologize with 'ocle.' Incidentally, 'binocle' does exist; see the above dictionary. It is a perfect name for (8), in view of the latter's double mode of use.

Notice that C_z has become $C_T = (B-B_o)/B_o$, the threshold (or liminal) contrast of a disk of luminance B (in foot lamberts) as perceived by a normal human eye against a background of luminance B_o. The disk is imagined to descend until C_z becomes equal to the ascending C_T. We shall now look into this in more detail.

To go from (8) to (9) requires more than mathematics; one must combine the physics of a hydrosol and disk (via α, K, and C_o) with the psychophysiology of the human eye-brain system (via C_T). In a pioneering paper, Blackwell (1946) reported experimental determinations of C_T in terms of the adaptation luminance B_o (called "brightness" in those days) of the eye and the angular subtense ψ of the diameter of a luminous circular target. About 450,000 observations (out of over two million) by 19 trained observers were analyzed to compile the threshold tables and diagrams in Blackwell's report. The contrast C_T of a disk is *threshold* if an observer, on repeated attempts under identical conditions to decide that the disk is seen, is correct 50% of the time.

A. Blackwell's Threshold Contrast Laws

Among other things, Blackwell determined two basic rules concerning C_T. We may formulate the two basic results of threshold contrast C_T in a way that is parallel to Secchi's laws. Two factors are involved: (i) the adaptation luminance B_o of the eye, and (ii) the angular subtense ψ of the visual stimulus, i.e., the angular diameter of the disk. With all other factors fixed and within their prescribed limits, we can say that:

A. Threshold contrast varies inversely with angular subtense of the disk.
B. Threshold contrast varies inversely with adaptation luminance.

Some typical values of C_T are reproduced in Table I, which is extracted from Table VIII of Blackwell* (1946):

Table I

Samples of Threshold Contrast C_T
(1 fL = 3.426 lumen·m^{-2}·sr^{-1})

Angular Subtense ψ (min arc)	Adaptation luminance B_o (foot lambert)		
	1000	1	10^{-1}
360.00	0.0027	0.0033	0.0053
55.20	0.0028	0.0037	0.0074
9.68	0.0046	0.0089	0.0213

The values in Table I may be placed in context by noting that the disks of the sun and moon subtend about 30 minutes of arc. Secchi's 43 cm disk subtended at disappearance depth 16.5 m about 45 minutes of arc, as seen from just below the surface. The luminance level on a day with highly variable clouds can vary from 3000 to 300 fL (≡ foot lamberts). Twilight has an adaptation luminance level of about 1 fL. A clear night with a full moon has an adaptation level of about 10^{-1} fL. The reader will have noticed that we are working with (English-system) photometric units (see H.O., Vol. I, p. 20, 26) while the more objective way to do eyeball optics is spectrally with filters, as already noted. Strictly, then, to use the direct ocle, (9), with C_T values from Table I, α and K should be determined photometrically.

The exact determination of z_{SD} in (9) by way of (6) is a tricky business: as z increases in (6), C_z on the left decreases, while C_T increases, as shown in Table I (since ψ is decreasing). At some depth the

* For some recent investigations of threshold contrast, see Wyszecki and Stiles (1982). The classic work of Blackwell (1946) of course still stands.

descending C_z will meet the ascending C_T to give birth to the direct ocle (9). Convenient nomographic solutions of (6) for z_{SD} in the case of underwater Secchi disk work (so that we have $\mathcal{T} = 1$) may be found in (H.O., Vol. I, Sec. 1.9). When using the nomographs, one determines the so-called *sighting range* v. For reasons discussed there (and briefly below) the required z_{SD} is given by* $z_{SD} = 2v$. The nomographic approach to visibility through a turbid medium was first explored by Duntley (1948).

B. Rational Bases for the Eight Laws of the Secchi Disk

The direct ocle (9) gives the quantitative basis for the eight laws of the submerged disk. For example, law I follows from (9) and Blackwell's rule A: By rule A, increasing the disk diameter, holding all else fixed, will decrease C_T. Now recall that the logarithm of a number grows with the number (albeit, for numbers larger than 1, at a slower rate). Since C_T sits in the denominator of the logarithm's argument, it follows that z_{SD} increases as the angular size ψ of the disk increases.

Each law may be checked out by similar simple reasoning. Some of the checks may need a little bit more knowledge than in the case of I. In the case of law III, for example, it will be helpful to know (see (71), below) two additional facts: (a) as the sun altitude increases, the mean path distance D_- of photons descending through a water layer of unit depth decreases, and

* There are two complete sets of nomographs in vol. I of H.O., namely the low-clarity and the high clarity nomographs prepared for photometric observations of the disk. The *high clarity* nomographs are in Fig. 1.85 and Figs. 1.89-1.97 with the range of α-Kcosθ being 0.01 ft^{-1} to 0.14 ft^{-1}. The *low clarity* nomographs are in Fig. 1.84 and Figs. 1.98-1.106, with the range of α-Kcosθ being 0.10 ft^{-1} to 1.40 ft^{-1} (rather than 0.01 ft^{-1} to 0.14 ft^{-1}, as drawn). Some further comments on Secchi depth z_{SD} vs. sighting range v will be made in the discussion of the inverse ocle (10), below. Users of these nomographs should observe their associated caveats.

(b) $K = D_{-}[a+\overline{b}]$ where a and $\overline{b}$ are inherent optical constants of the medium, to be defined below in Sec. 9. Thus, as the sun rises, K in (9) decreases, and so z_{SD} increases.

In the case of law VI, note that as one ascends indefinitely from the surface, the term $N_o r$ in (4), which was blotted out by the observer's head or umbrella, now creeps back into play.

As for law VII, the interested reader will find some relevant quantitative observations in (H.O., Vol. V, p. 173).

C. The Ninth Law of the Secchi Disk

Secchi did not consider the effect of adaptation luminance on his depth readings. Had he done just a few experiments with the same disk and same body of water at noon and at sunset, on a heavily overcast day, all other conditions being the same, he may have lucked out and have been able to give us enough data so that we could go on to state also the ninth law of the disk:

IX. The depth of disappearance of the disk varies directly with adaptation luminance.

This of course now follows at once from Blackwell's rule B when used in the direct ocle (9).

From laws IX and VII, and some other observations above, we may conclude that: *good Secchi opticians will standardize their results by using black umbrellas over becalmed glass-bottomed row boats at local noon on clear sunny days.*

6. THE INVERSE OCLE

Return now to the binocle (8) and imagine a good Secchi disker taking a reading z_{SD} of the disappearing submerged disk. The observer has been careful to wait for a calm moment (so that $\mathcal{T} = 1$), knows the R of the disk, and has placed faith in the fact that $R_\infty = 0.02$. If the observer's eyes are near normal, then C_T and hence $\Gamma = \ln[\mathcal{T}C_o/C_T]$ in (8) are calculable, and we thereby obtain the *inverse ocle* of eyeball optics:

$$\text{[Inverse ocle]} \qquad \alpha+K = \frac{\ln[\mathcal{T}C_o/C_T]}{z_{SD}} = \frac{\Gamma}{z_{SD}} \tag{10}$$

whose main role in life is to estimate the *sum* $\alpha+K$ of the optical constants α and K of the medium from the depth z_{SD} of disappearance of the Secchi disk.

The distinction between the direct and inverse ocles is clear: the direct ocle (9) *calculates* z_{SD}, while the inverse ocle (10) uses an *observed* z_{SD}. In the direct case, knowledge of the optical constants α and K of the medium are known from previous measurements using calibrated electronic instruments along with prior objective measurements of the reflectances R, R_∞ of disk and hydrosol needed in (7), as well as wind speed U in (3) and zenith and nadir radiances N_o, N^o as defined in (4). Now, if all these measurements have been made (except for α, K in the homogeneous medium) then the inverse ocle (10) will yield a value of $\alpha+K$ that is consistent with all these known laws of the human eye and the modern theory of hydrologic optics. Thus eyeball optics and hydrologic optics will join hands through the binocle (8) under these ideal conditions. The hands are gripped tightly or loosely, depending in turn on our grasp of Γ.

A. Determining the Coupling Constant Γ

In the coupling constant Γ of the inverse ocle (10), there are two factors that are more difficult to control than the others. These are the threshold contrast C_T of the observer and the deep water reflectance R_∞ of the hydrosol. It is assumed, in other words, that the submerged reflectance R of the disk has been carefully measured (photopically or spectrally) and that precautions have been taken to assure that the contrast transmittance $\mathcal{T}$ of the surface is essentially unity. The two relatively uncontrollable factors C_T (via people's eyes) and R_∞ (via hydrosol clarity) can in principle vary over wide ranges, with Γ following accordingly. Fortunately (for once something is in our favor) the logarithm function's values vary sluggishly relative to those of its argument. Thus great variations of C_T and R_∞ are seen through the Γ^{-1} end of the binocle as small variations (cf. Eq. 11, below). The following table gives, for $\mathcal{T} = 1$, a range of $\Gamma = \ln[C_0/C_T]$ values for a selected range of C_T and R_∞ values* when the submerged reflectance† R is 0.85.

* We really cannot choose R_∞ independently of K. It can be shown (see (64) below) that, essentially, $R_\infty = \bar{b}/3a$; and therefore R_∞ is itself involved in the ingredients of $K = D_-[a+\bar{b}] = D_-a[1+3R_\infty]$; and it is α+K that is being sought by users of the inverse ocle (10)! So, in fixing R_∞ we are in effect fixing part of the very thing, namely K, that we are seeking through the Secchi disk! Fortunately, this muddle is mitigated by the insensitivity of Γ to changes in R_∞. See discussion of (11), below.

† The ranges of numbers in this table were suggested, in a personal communication, by R.W. Austin of the Visibility Laboratory of the University of California in San Diego.

Table II.

Values of $\Gamma = \ln[C_o/C_T]$ where $C_o = (R-R_\infty)/R_\infty$ with $R = 0.85$.

C_T \ R_∞	0.015	0.02	0.03	0.05	0.07	0.10
0.005	9.32	9.02	8.61	8.07	7.71	7.31
0.010	8.63	8.33	7.90	7.38	7.02	6.62
0.020	7.93	7.64	7.22	6.69	6.32	5.93

The three values of C_T in Table II are roughly in accord with the small angular subtense of $\psi = 9.68$ minutes of arc for a standard Secchi disk in Table I under the three adaptation luminances B_o there. A 'standard' Secchi disk diameter tends nowadays to be around 30 cm (≈1 foot).

B. Secchi Depth vs. Sighting Range

A rough rule of thumb for Secchi opticians can be fashioned from an appropriate choice of Γ from Table II. For example, choose $\Gamma = 8$, and consider $\alpha+K = 8/z_{SD}$ (but don't quote me on this choice of Γ; just use it, or something else from Table II, and good luck). The factor $\Gamma = 8$ is in rough accord with the estimates of *sighting range* v being $z_{SD}/2$ for which the nomographs in H.O., Vol. I, have been constructed (see the nomograph footnote, above, and H.O., Vol. I, pp. 194-195 for a discussion of v.) The distinction between vertical sighting range v and Secchi depth z_{SD} rests in the surprise (or warning) factor: if one intently watches a visual target disappear into the misty distance one can follow it out to perhaps twice the distance at which one would spot it if one is making a random search not knowing where it is. The visibility nomographs in Vol. I, H.O. were made for vertical sighting range v given by $v = 4/(\alpha+K)$. Thus psychophysiology swims into view in eyeball optics, and forces us to make a distinction between sighting range v and Secchi depth z_{SD} in order to use the nomographs in H.O., Vol. I.

Holmes (1970) finds Γ values of around 8.9 to 9.4. He also finds a value of C_T from independent data in the field. His C_T is on the order of 0.0014 with a standard deviation of 0.0013. That he got anything like the value 0.0014 is quite impressive in view of Table I. It is impressive in the sense that 0.0014 is off only by a factor of 2 to 4 under bright daylight conditions. Moreover, Holmes was working *backwards* through (10), i.e., from z_{SD} to C_T. This direction is the sensitive one to traverse (cf. (11), below). Holmes' value of 0.0014 is seemingly low in view of the rugged field conditions under which eyes and brain were doing their work in his experiments. In a recent reexamination of this matter, Højerslev (1986) finds a somewhat more believable value of $C_T = 0.0070 \pm 0.0003$ for photopic vision. Højerslev (1986) believes Holmes' C_T determination was affected by the presence of bottom luminance.

C. A Case of Paleo-Eyeball Optics

Let us now consider an application of the inverse ocle (10) and try to infer the clarity of some natural waters that existed 120 years ago. We thus will indulge in a bit of paleo-eyeball optics. Consider Secchi's first experiment with his 43 cm diameter disk that disappeared at 16.5 m (54.0 ft). We shall ignore the water surface effects by setting $\mathcal{T} = 1$ in (10). The disk had an area of 1.56 ft². The adaptation luminance, judging from Secchi's description of the day, is estimated to have had been about 1000 foot lamberts, and the disk is assumed to have had a submerged photopic reflectance of $R = 0.85$. Moreover the photopic R_∞ for the Tyrrhenian Sea on that day and at that spot is assumed to have been $R_\infty = 0.02$. From Fig. 1.98, Vol. I, H.O., with sighting range being $v = z_{SD}/2 = 27$ ft, we find $\alpha+K = 0.300\ ft^{-1} = 0.984\ m^{-1}$. This value is not unreasonable for coastal waters near a large city (cf. H.O., Vol. I, p. 137). The *l'Immacolata Concezione* was anchored at the time four miles off the coast of the great Italian city of Civitavecchia.

It is tempting to try to find α and K individually from the datum $\alpha+K = 0.984\ m^{-1}$. Using the rule of thumb (H.O., Vol. I, p. 195) that $K = 0.4\alpha$, one finds $\alpha = 0.702\ m^{-1}$ and $K = 0.282\ m^{-1}$. These are not unreasonable values; but of course they are mere guesses. It should be emphasized that the rule $K = 0.4\alpha$ is just another rule of thumb on which good research should not be based. Using it for eyeballpark estimates, and idle curiosity, as just indulged in, yes. But basing on it important ecological conclusions? Definitely not.

D. The Well-Documented Secchi Disk Experiment

Before leaving this example, we can use it to illustrate law IX of the disk, and to make an important point about the nine laws assembled above. Suppose Secchi had also made a measurement later that first day of his experiment, say, near twilight, when the adaptation luminance was on the order of 0.1 foot lambert. What may have been the observed z_{SD} value then? From Fig. 1.102 in Vol. I, H.O., under such lighting conditions, we find the sighting range v for a natural hydrosol with $\alpha+K = 0.300\ ft^{-1}$ to be 26.5 ft, whence $z_{SD} = 2v = 53$ ft. Comparing this with the observed $z_{SD} = 54$ ft, noted above, we see that this is about a 2% decrease in depth of disappearance. Now, had Secchi actually determined the $z_{SD} = 53$ ft (= 16.1 m) on that day at twilight, and recorded it for posterity, we would then, all other factors remaining the same, have been able to check his work and consistently deduce once again from this shallower reading and the twilight adaptive luminance level that $\alpha+K = 0.300\ \text{ft}^{-1} = 0.984\ \text{m}^{-1}$. In other words, law IX assures us that, although adaptation luminance may change from locale to locale and time to time, we can account for its effect on z_{SD} (all other factors fixed) if we take the trouble to document that luminance, even if roughly. This may be done e.g., in recording merely the time of day and noting the sky condition (cloud coverage). Then we could later use H.O., Vol. I, Fig. 1.2 to estimate the adaptation luminance.

All the other eight laws of the disk make the same kind of assurance as law IX: *if you record or carefully estimate the factors comprising Γ, in a series of experiments, then the inverse ocle (10) will in turn give you estimates of $\alpha+K$ that are independent of the variations in the factors of the nine laws at that point in the medium during that series of experiments.*

E. The Sensitivity Formula for the Binocle

We may succinctly summarize the quantitative aspects of these observations and the nine laws themselves in the following sensitivity analysis of the binocle which is obtained by taking the relative differentials of both sides of (8):

$$\frac{\delta z}{z} + \frac{\delta(\alpha+K)}{\alpha+K} = \frac{\delta\Gamma}{\Gamma} = \frac{[\delta\bar{\mathcal{T}}_o/\bar{\mathcal{T}}_o + \delta\mathcal{T}_o/\mathcal{T}_o + \delta C_o/C_o - \delta C_z/C_z]}{\Gamma} \tag{11}$$

This form of the binocle is helpful in gauging the relative changes in any of the factors, knowing the changes of the other factors. It applies equally well to either ocle (9), (10).

As an illustration of the use of this relation in the context of the direct ocle (9), let us reexamine the preceding example of the effect on z_{SD}

of a change in adaptation luminance. Hence we suppose only B_o changes, and that C_o, $\mathcal{T} = \bar{\mathcal{T}}_o \mathcal{T}_o$, α, and K remain fixed. Then in (11) we may replace z by z_{SD}, and C_z by C_T to find:

$$\frac{\delta z_{SD}}{z_{SD}} = - \frac{\delta C_T / C_T}{\Gamma} \tag{12}$$

Now to find $\delta C_T/C_T$ in this case we use Table II of Blackwell (1946) which shows how log C_T changes with adaptation luminance B_o. We use the 55.5 minute of arc column there because it's closest to the case of our 43 minute of arc disk. In going from 1000 to 0.1 foot lamberts or even from 100 to 0.1, the Table II and Fig. 10 in Blackwell (1946) show that log C_T changes essentially from -2.041 to about -1.858. Hence $\delta \log C_T = \delta C_T/C_T \cong 0.183$. With $\Gamma = 8$, we find from (12) that $\delta z_{SD} \cong -0.183/8 = -0.022$, a 2% change, which checks with the preceding nomographical estimate.

As a final example of the use of the sensitivity equation (11) we consider the quantitative aspect of Secchi Disk law I. Recall that Secchi found, in going from the 43 cm diameter disk to the 60 cm diameter disk, that the depth of disappearance z_{SD} went from 16.5 m to 24.5 m. This amounts to about a 50% increase in depth. Holding all other factors fixed in (11) and changing only the diameter of the disk, we come again to (12). Next, at the disappearance depth z_{SD} for the smaller disk we introduce a change $\delta\psi$ in the disk subtense that is needed to go to the larger disk. As a result C_T will undergo a change δC_T. From Table VIII, Blackwell (1946), in the 1000 foot lambert column, we can find $\delta C_T/C_T$. Just to get an idea of how much of a relative change $\delta C_T/C_T$ we have here, we go from the 18.2 to the 55.2 angular subtenses (which then certainly will give a larger $\delta z_{SD}/z_{SD}$ than for the Secchi experiment). We find that $\delta C_T/C_T = \delta \log C_T = (-2.547)$ -

(-2.460) = -0.087. Choosing $\Gamma = 8$, we find from (11) that $\delta z_{SD}/z_{SD} = 0.011$. Hence we would expect the 60 cm disk to descend to a depth only about 1% deeper than the 16.5 m of the 43 cm disk, i.e., to about 16.7 m. Secchi's recorded depth of 24.5 m for the 60 cm disk accordingly seems inordinately large. Blackwell's Table VIII and the associated Fig. 16 indicate that, in this size range of target, $\delta C_T/C_T$ is quite small. The nomographs in H.O., Vol. I, built on Blackwell's results, restate this fact graphically and very clearly. Finally, the physics behind the binocle (8) stands firm. In this case, then, venerable Père Secchi's data must give way. One or the other of Secchi's readings must be in error. Holmes (1970) reports some experiments that corroborate our findings. Thus while law I is in principle *qualitatively* correct, for disk sizes in the range used in eyeball optics, the direct quantitative dependence of z_{SD} on disk diameter is quite weak.

We now may summarize some of our work so far with the sensitivity formula (11).

Standing back and looking at (11) and noting that the coupling constant Γ is on the order of 8 or 9, we see the important fact that $\delta z_{SD}/z_{SD}$, i.e., the relative changes in z_{SD} (with α+K fixed) is about an order of magnitude smaller than any of the individual changes $\delta\mathcal{T}/\mathcal{T}$ (= $\delta\mathcal{T}_o/\mathcal{T}_o + \delta\bar{\mathcal{T}}_o/\bar{\mathcal{T}}_o$) or $\delta C_o/C_o$ or $\delta C_z/C_z$, when all other changes are zero. Conversely, the relative change $\delta(\alpha+K)/(\alpha+K)$ (with z_{SD} fixed) also is an order of magnitude less than each of the individual relative changes that can occur in $\mathcal{T}$, C_o or C_z. *Finally, and perhaps most importantly for work with the binocle equation (8), the sensitivity relation (11) says that z_{SD} and α+K are equivalent measures of each other for fixed coupling constant Γ:* errors in estimating z_{SD} are directly picked up in the estimate of α+K by the inverse ocle (10); and conversely, uncertainty in α+K is picked up by variability of the same magnitude in z_{SD} via the direct ocle (9).

7. WIRING THE INVERSE OCLE INTO THE WORLD

Caution to all who enter here: 'Improper use of the white disk may be harmful to your professional career.'

The utility of the Secchi disk technique increases the more we can relate its readings to other optical parameters in a natural hydrosol. In this section we consider the matter, wherein the inverse ocle is wired in various ways into the hydrosols of the world, so as to be able to tell us about the basic attenuation coefficients of a hydrosol, the euphotic depth, and so on. But first we have to get something clear.

A. An Ocle Is Not an Oracle

During the inverse ocle's history of applications to the various optical problems of biologists studying light and life in lakes and seas there have been attempts to turn the ocle into an oracle, something with the supernatural ability to reveal the innermost optical workings of the watery parts of our world. The recounting of the tale of the old mug and the sea above was designed to reveal the humble origins of the binocle and its offspring the direct and inverse ocles (9) and (10), and particularly to apprise anyone of the difficulty in making the inverse ocle behave* like an oracle. To bring this point home, we next examine a few typical cases of an overworked ocle, balanced by some cases of a well-worked ocle. Throughout this discussion we should keep in mind this salient point: the visually derived information z_{SD} about the medium can only pertain to other visually derived or visually meaningful properties of a lake or sea. To do more than this, i.e., to infer

* Anyone trying to do so is welcome. For example, one way of doing this is the following. It is well known that the Egyptian god of light and life is Ra. Therefore, by infusing an 'ocle' with the essence of 'ra', we obtain an 'oracle'!

sizes of strictly instrumentally derivable properties, would be logically indefensible, and unscientific.

B. Secchi Depth and Euphotic Depth

Consider, for example, the case of a biologist trying to use the Secchi depth z_{SD} to compute the depth z_{eu} of the euphotic zone, or in general embarking on the futile task of finding the downward irradiance H_z ($watt \cdot m^{-2} \cdot nm^{-1}$) at arbitrary depths z in a natural hydrosol from knowledge of z_{SD} alone. The significance of the presence of α in the inverse ocle (10) has obviously eluded such a person. However, someone understanding the meaning of α in (10) will realize that if one wishes to compute downward irradiance H_z using

$$H_z = H_o \exp[-Kz] \tag{13}$$

one must make what amounts to an independent optical measurement of α in the hydrosol and then subtract α from α+K yielded up by the inverse ocle (10) in order to find K for use in (13). (This is assuming, as always, that the z_{SD} reading was done photometrically or with filtered eyeglasses, as the case may be.) Thus one must go to the hydrosol armed not only with a Secchi disk, but, say, with a transmissometer, a device that shoots a narrow beam of monochromatic (or photopically prepared) light of inherent radiance N_o ($watt \cdot m^{-2} \cdot sr^{-1} \cdot nm^{-1}$) across a short stretch r of homogeneous path and measures the directly transmitted (i.e., unabsorbed and unscattered residual) radiance N_r^o arriving at the end of that path. Then since N_o and N_r^o are connected by

$$N_r^o = N_o \exp[-\alpha r] \equiv N_o T_r^o \tag{14}$$

one can estimate the volume (beam) attenuation coefficient α by means of

$$\alpha = r^{-1} \ln[N^o/N_r^o] = -r^{-1} \ln T_r^o \tag{15}$$

If r is measured in meters, then α has units of m^{-1}. The quantity T_r^o is an extremely important optical quantity, the beam transmittance for a path of length r.

Once α is in hand, one can estimate the downward irradiance H_z at any depth z from z_{SD}. For example suppose we agree that the *euphotic depth* z_{eu} is the depth at which H_{eu}/H_o is some fixed fraction, like 0.01, e.g. Then by definition

$$H_{eu}/H_o = \exp[-Kz_{eu}]$$

and so

$$z_{eu} = K^{-1} \ln[H_o/H_{eu}] \tag{16a}$$

(We note parenthetically, that if H_o/H_{eu} is set to be 100, then $z_{eu} = 4.6/K$, i.e., the euphotic depth is 4.6 diffuse attenuation lengths K^{-1}.)

Finally, in our quest for a formula for euphotic depth z_{eu}, solve (10) for K in terms of α, Γ and z_{SD}. Then using this K so found, in the preceding equation (16a) we have, on rearranging, the expression for the euphotic depth:

$$z_{eu} = \frac{z_{SD} \ln[H_o/H_{eu}]}{\Gamma - \alpha z_{SD}} \tag{16b}$$

For practical work, one may use some Γ from Table II above, say $\Gamma = 8$, and set $\ln[H_o/H_{eu}] = 4.60$ (for $H_o/H_{eu} = 100$). Thus z_{eu} may be characterized in terms of z_{SD} and the beam attenuation coefficient α. Having derived formula (16b), I disown it, for reasons that, if not clear by now, will continue to evolve as the discussions proceed.

C. A Happy Union of Hydrologic and Eyeball Optics

I can just picture a clear-headed marine biologist with a strong physics background coming on the preceding discussion culminating in (16b), say a fresh graduate student who has not had exposure to any Secchi disk shenanigans, and one who has been charged by her thesis advisor with the task of finding the depth z_{eu} of the euphotic zone in some region of the sea. I can hear her snort at (16b) and go right for (13), rent a well-calibrated flat-plate irradiance meter (with perhaps a dozen built-in wavelength bands), and do a downward sweep with the meter to get a multi-colored depth profile of H_z with z. If she's lucky the medium will be homogeneous,* the depth profile on semilog paper will be a straight line, and she will then have a single K value for the medium at each of (say) a dozen wavelengths determined by using:

$$K = z^{-1} \ln[H_o/H_z]. \tag{17}$$

The euphotic depth z_{eu} will then fall right out of (16a), for whatever defining ratio H_o/H_{eu} is chosen.

* The student may be even luckier to encounter a wiggly $\ln[H_z/H_o]$ vs. z plot packed with information about absorption layers in the sea, but more of that later (sec. 9).

On the way to do a trial run of her experiment the student was joined by her Professor, an old Secchi disk dunker with a white disk tucked under his arm and a pair of broad-band 555 nm eyeglasses in his breast pocket, supplied by the student. Readings of z_{SD} and H_z were taken simultaneously that day off the end of the pier at their Institute. After reduction of their respective findings (K via (17) by the student; α+K via (10) with Γ = 8 by the Professor) it was decided that they may as well subtract one 555 nm result from the other and list the α reading along with the single green K and α+K readings. In retrospect they agreed that this is one of the more sensible ways to wire the inverse ocle (10) into the physical world. In a later experiment, the student light heartedly supplied the professor with some rose colored (600 nm) glasses. He wasn't receptive to this at all (for his state of mind matched the 555 nm glasses, as he contemplated her research approach), and a week later she bought herself a pair of 450 nm glasses to match her mood and walked around the lab with them on. The old boy still didn't see the humor in it. (She got her degree, anyway.)

D. Some Forms of Secchi Disker Madness

After a Secchi reading z_{SD} has been obtained and the inverse ocle (10) yields up α+K, some Secchi diskers who have been left to themselves succumb to a certain form of madness in which they attempt with bare hands to rend asunder α and K from the sum α+K that God hath made. I momentarily fell into this madness a while back in this narrative when I 'deduced' α and K from Secchi's 1865 readings of z_{SD}, using the rule of thumb K = 0.4α.

A lesser form of madness occurs when α and K are extracted from their union α+K by means of some empirical relation drawn among α+K, α, or K. Holmes (1970, Eq. (4)) for example, did some work along this line for natural waters in the Santa Barbara California area, being conscious of the delicacy of the task. Of course I wouldn't dare use his regression of α on α+K for the waters of Lake Washington outside my lab without a thorough preliminary check. In doing so, I would have to generate my own regression by first taking measurements with a transmissometer and an irradiance meter. But if I am going to go so far as to build or rent or buy these instruments, dip them in, and then reduce the data, then, like the graduate student mentioned above, I will really not need the Secchi disk, except perhaps for sentimental reasons.

This is the point of view advocated by Tyler (1968). Holmes (1970), a careful student of hydrologic optics, while aware of Tyler's advice, still wanted to find the statistical connections between α and α+K. So be it. Holmes' quantitative findings on law I of the disk are adequate, and his independent determinations of C_T are noteworthy (if somewhat questionable). However, when the regression between beam transmittance for a unit length path $T_1^o = \exp[-\alpha]$ and z_{SD} was produced, the good fit notwithstanding, the result was only of provincial interest; it could not be regarded as universal.

E. Three Basic Secchi Measures of Optical Depth

After Tyler and Holmes, the subsequent work on combining Secchi disk readings with objective measurements of α and K and other optical properties seemed to proliferate. There was considerable interest for example in the dimensionless quantity $\Gamma \equiv (\alpha+K)z_{SD}$ and its statistical relation to other such quantities as $\Gamma_\alpha \equiv \alpha z_{SD}$ and $\Gamma_K \equiv K z_{SD}$. Each of these quantities is a form of *optical depth*, Γ_α being the most commonly used. Since α and K have units of m^{-1}, we may interpret α^{-1} and K^{-1} as attenuation lengths over which residual radiance and diffuse irradiance, respectively, decay to 37% (= 100 e^{-1}%) of their initial values. It follows that Γ_α is the *beam optical depth* (in the sense of α) of the disappearing Secchi disk; and Γ_K is the *diffuse optical depth* (in the sense of K) of the same disappearing Secchi disk. Suggestions in the literature were made to adopt one or the other of these optical depths as characterizing natural waters.

A set of suggestions along these lines for example was made by Gordon and Wouters (1978). For one thing, they suggest that αz_{SD} is less variable than $(\alpha+K)z_{SD}$. This suggestion can be examined as follows. By means of the direct ocle (9) we can write αz_{SD} as:

$$\alpha z_{SD} = \frac{\Gamma}{1+(K/\alpha)} \equiv \Gamma_\alpha \qquad (18a)$$

Now, it has been our experience (H.O., Vol. I, pp. 136-137) that K/α is a relatively stable parameter of water clarity. (But of course it is not a universal constant, just as we have seen above that Γ is not a universal constant.) There is even a strong sanction for isolating and idolizing the ratio K/α, a sanction that arises from a certain basis in theory. (See, e.g. H.O., Vol. V, Eq. (19), p. 238, and p. 252. See also (81), below). If the Gordon-Wouters suggestion about αz_{SD} is true, then it will be of interest to see whether Kz_{SD} (also recommended by Gordon and Wouters (1978), and others) may be less variable than $(\alpha+K)z_{SD}$. For we can find from the direct ocle (9) that

$$Kz_{SD} = \frac{\Gamma}{1+(\alpha/K)} \equiv \Gamma_K \qquad (18b)$$

From (18a), for example, we see that the question of the relative variability of αz_{SD} and $(\alpha+K)z_{SD}$ can be investigated if the joint probability density function of Γ, α, and K can be determined. This may be done by means of carefully designed experiments that are either Monte Carlo or real. A variational analysis of Γ, Γ_α, and Γ_K along the lines of (11) can then be made, and the matter of relative variability settled. One important outcome of such research will hopefully be confidence intervals (or at least scatter diagrams of α vs. z_{SD} or K vs. z_{SD}) for α and K when estimated by z_{SD} and (18a,b).

If in (18a) we use $\Gamma = 8$ and $K/\alpha = 0.40$ from the above cited H.O. reference, then $\alpha z_{SD} = 5.7$. If instead we use $\Gamma = 9$, then $\alpha z_{SD} = 6.4$, which is in broad agreement with current folklore on the subject. See, e.g., Højerslev (1986) for a set of experimental results leading to $\alpha z_{SD} = 6.0$ under a wide variety of conditions. With this relation and an observation of z_{SD}, one can then estimate α. From the scatter diagram of Højerslev (1986, Fig. 5) one can even estimate the confidence interval for α. This is rather large for the smaller z_{SD} values, and so the relation is not too useful in the range of z_{SD} from 3 m to 5 m.

A similar service is performed by (18b) in the form $Kz_{SD} = 2.3$, in which $\Gamma = 8$ and $\alpha/K = 2.5$. From this we have at once an estimate of the ratio H_{SD}/H_o, i.e., the relative irradiance at Secchi depth. Thus by (13)

$$H_{SD}/H_o = \exp[-Kz_{SD}] = \exp[-\Gamma_K] = 0.10 \qquad (18c)$$

Of course, other choices of Γ and α/K are up for grabs to make the right sides of (18a,b,c) come out 'right' in the optical medium momentarily at hand.

F. The Responsibilities of Hydrologic Opticians

I believe it is a responsibility particularly of the physicists in the hydrologic optics field to continually and gently guide their biological brethren* into the paths of conceptual righteousness by giving them logically sound, simple, and rugged tools with which to pursue their biological studies. That guidance cannot be achieved by proliferating basically non-viable *numerical* chimeras such as those just derived from (18a,b).

It should be emphasized here that I am not averse to deriving relations such as (18a,b); they are admittedly rather beautiful and, in the context of their derivation, exact. Moreover, some good experimental work on them has been done, as noted. What I object to is fixing forever, without qualification, the *numerical size* of the right sides of (18a,b), or making claims about the permanent relative sizes and relative variability of apparent optical properties, claims which are not based on general principles (conservation of energy, e.g.) or thorough physical experiments.†

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* Some reviewers of the first draft of this manuscript felt that this statement tended to talk down to aquatic biologists. In a sense, this is correct. But there is a hidden symmetry here. Let us turn the situation around. You're a physicist who has to look up some procedures for culturing and assaying phytoplankton *in situ* so as to set up a realistic σ-meter experiment. You look up the aquatic biologic literature and find conflicting advice on seeding, propagation, flushing and irradiation for phytoplankton procedures. The more you read the less certain you are on how to proceed. It is all empirical, all cookbookery; there are no guiding principles. The aquatic biologists know how to do it; they have the guiding principles in their heads; they've spent years perfecting the best procedures; but no one has thought it necessary to write a guide for their poor physical brethren.

† For an example of a statement about α and K that is based on general physical principles, we have $K(z,-) \le \alpha(z,-)$; cf. H.O., Vol. V, p. 119. Here $\alpha(z,-) \equiv \alpha(z)D(z,-)$. For notation, and further examples of physically-based exact formulas see sec. 9, below, particularly (51), (52), and (61). The inequality in (74), e.g., follows directly from $K(z,-) \le \alpha(z,-)$.

Even when qualifications have been made the carefully stated conditions under which the chimera was originally made are in danger of eventually being lost in the cascading citations of references by subsequent users; and a mite is preserved in amber, forever. This is what must be guarded against; this is done by always preceding the rule of thumb by its premise.

G. Good Secchi Disk Science vs. Numerology

If the preceding advice is not acted upon by both physicists and aquatic biologists then there is the possibility of having a numerological cult springing up in our midst whose sole occupation is to interbreed rules of thumb and to endlessly spawn algorismic progeny that would be the envy of a modern day DNA mechanic. For example, it has probably already occurred to some readers to place the 'universal law' (18a) for αz_{SD} into the 'universal law' (16b) for z_{eu} to produce the new 'universal law' for finding euphotic depths from Secchi depths:

$$z_{eu} = \gamma \, z_{SD} \tag{19}$$

where

$$\gamma \equiv \Gamma_K^{-1} \, \ln[H_o/H_{eu}] \tag{20}$$

Using $K/\alpha = 0.4$ as above along with $\Gamma = 8$ and $H_o/H_{eu} = 100$ we find

$$z_{eu} = 2.0 \, z_{SD} \tag{21}$$

This numerical chimera tells us that euphotic depths are expected to be about twice the Secchi depths in each hydrosol in the world. Some gullible students of eyeball optics will adopt this rule at once and use it uncritically, and pass it on to *their* students. Others at once (and rightly so) will take issue with (21) because I didn't use their carefully devised value of Γ or K/α to produce the rule of thumb that z_{eu} is actually three times z_{SD}, or whatever, in their part of the world. And thus starts a new round of letters and papers each expecting some locally measured connection between α and K to be of universal significance and of planet-wide applicability. All the while the fact has been lost sight of that K is an apparent (not an inherent) optical property which could be either photopically or spectrally defined, and that the Secchi version Γ of optical depth (recall (10)) is packed to the rafters with physiological and meteorological ephemera. In sum, then, (19) is a logically sound, simple and rugged tool which, when reduced to numerical form, must be applied and interpreted each time in a local sense, not a universal sense.

H. When to Use the White Disk

From our discussions above, one may conclude that the only reasonable use of a Secchi disk reading is as *a simple visual index of the clarity of a natural body of water*: for example, a lake, an estuary, a coastal zone, or the open sea. An excellent example of such a use may be found in Arnone (1985), which gives maps of Secchi depths around about 50% of the world's coastlines. Some linkages between α, K and z_{SD} are mentioned. We have learned now to regard such linkages with great caution. The charts are based on a survey of about 96,000 Secchi depths and are intended for a quick impression of water clarity and perhaps a basis for more detailed optical surveys leading to α, K, and perhaps some day even σ (the volume scattering function). Four main oceanic regions of the world were surveyed. Each region has a four-season sequence of Secchi depth charts plus an annual mean chart.

Hence there are 20 charts in all. The depths are color-coded into six depth ranges ranging from brown (z_{SD} < 5 m) to purple (z_{SD} > 25 m). Aquatic biologists would do well to emulate this atlas in their own domain. And remember, an ocle is not an oracle: take the charts on face value. Do you want α or K in addition? Then make the appropriate measurement.

8. CHLOROPHYL ASSAYS USING EYEBALL OPTICS AND HYDROLOGIC OPTICS

Aquatic biologists studying life processes in lakes and seas are recurrently concerned with determining chlorophyll-a (and other) concentrations. One simple index of chlorophyll-a concentration in a natural hydrosol is the Secchi depth reading z_{SD}, because the reading is easily and cheaply made. In order to achieve such a simple chlorophyll index of the medium, a statistical link must first be set up between the z_{SD} reading (in m) and the concentration c (in $mg \cdot m^{-3}$) of the chlorophyll-a. In what follows this linking exercise will be gone through four different ways. The attentive reader will experience a sea change of philosophy from eyeball optics to hydrologic optics as he progresses through the sequence. The intent of the sequence is to take the inveterate Secchi disker by the hand and firmly put him up on more solid ground by the time method D has been explained.* It should be noted at the outset that the overall intent of this section is to lay the groundwork for the eventual abandonment of Secchi disk science.

We begin with perhaps the most naive linking process and progress in three steps to more sophisticated ways of linking Secchi depth z_{SD} and concentration c. To keep the essential ideas always before us I will adopt in each case perhaps the simplest natural hydrosol model imaginable: a mixture of two types of molecules: chlorophyll molecules (encased in living plankton skeletons) and the complementary hydrosol soup (the non chlorophyll part of the hydrosol) consisting of water and all other kinds of molecules. At the

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* I sent out a first draft of the present section to several colleagues for comment. The comments were conflicting. One group thought that I was on dangerous ground here because, by giving so much attention to Methods A and B below, my intent may be misinterpreted as a sanction of these methods. Thus Methods A and B should be expunged and C and D played up. The other group thought Methods C and D were out of place in a Secchi disk discussion and should be removed, and Methods A and B should be retained and improved. Section 8 thus stands as first written.

close of the discussion, in par. E, below, it will be clear how to make Model D more realistic and use it as a method of determining the desired chlorophyll concentration.

A. Chlorophyll Assay Via Naive Eyeball Optics

Basic assumptions:

(i) Medium is homogeneous. Surface effects are such that $\mathcal{T} = 1$ (cf. (5))

(ii) Diffuse attenuation coefficient K (in m^{-1}, preferably in spectral form) of medium and Secchi depth readings z_{SD} (in m) are related by $Kz_{SD} = \Gamma_K$ (cf. (18b)). A choice of Γ_K is $\Gamma_K = 2.3$, based, e.g., on the naive rules of thumb, $\Gamma = 8$, $K/\alpha = 0.4$.

(iii) Diffuse attenuation coefficient K modeled by $K = K_o + cK_c$. Here c (in $mg \cdot m^{-3}$) is the concentration of chlorophyll-a particles assumed uniformly distributed throughout the medium. K_c (in $m^2 \cdot mg^{-1}$) is the diffuse attenuation *cross section* (or *specific* diffuse attenuation) of the chlorophyll-bearing particles, while K_o (in m^{-1}) is the diffuse attenuation coefficient of the non-chlorophyll part of the hydrosol.

The *training stage* of the statistical model $K = K_o + cK_c$ consists in first taking a set of measurements of Secchi depths z_{SD} and concentrations c in the medium. Call these readings '$\hat{z}_{SD}$' and '$\hat{c}$'. Then from (ii) the $\hat{z}_{SD}$ readings are converted to estimates $\hat{K}$ of K ($\hat{K} \equiv \Gamma_K/\hat{z}_{SD}$). Form the regression model, with these $\hat{c}$ and $\hat{K}$ samples, and find by standard linear regression procedures the least squares estimates $\hat{K}_o$, $\hat{K}_c$ of K_o and K_c. The resultant *trained model* is then $K = \hat{K}_o + c\hat{K}_c$.

The *applicational stage* of the statistical model occurs when a fresh Secchi depth reading z'_{SD} is made in order to estimate the associated chlorophyll concentration c'. From (ii) we first find the associated $K' = \Gamma_K/z'_{SD}$. From the trained model we then find $c' = (K'-\hat{K}_o)/\hat{K}_c$. This applicational stage may be repeated as often as required. Update the trained model regularly and adjust $\hat{K}_o$ and $\hat{K}_c$ accordingly.

B. Chlorophyll Assay via Sophisticated Eyeball Optics

Basic Assumptions:

(i) Medium is homogeneous. Surface effects are such that $\mathcal{T} = 1$ (cf. (5)).

(ii) Diffuse attenuation coefficient K (in m^{-1}) of medium and Secchi depth readings z_{SD} (in m) are related by $Kz_{SD} = \Gamma/[1+(\alpha/K)] \equiv \Gamma_K$ (cf. (18b)).

(iii) Diffuse attenuation coefficient K modeled by $K = K_o + cK_c$ (definitions as in method A).

The *training stage* of the statistical model $K = K_o + cK_c$ consists in first taking a set of measurements of $\hat{z}_{SD}$ and $\hat{c}$ as in method A, but now in addition downward irradiance measurements H_z are taken over a set of depths z. From knowledge of disk size, submerged disk reflectance R, and $R_\infty = 0.02$, find $\hat{C}_o$ via (7) and $\hat{C}_T$ via Table I (by interpolation), whence the *sophisticated estimate* $\hat{\Gamma} = \ln[\hat{C}_o/\hat{C}_T]$ of Γ. From $\hat{z}_{SD}$, $\hat{\Gamma}$ and the inverse ocle (10) find the estimate of $\alpha+K$, which for now may be denoted by '$(\alpha+K)\hat{}$'. From H_z readings for several depths z, and (17) find $\hat{K}$. From this and $(\alpha+K)\hat{}$ find $\hat{\alpha} \equiv (\alpha+K)\hat{} - \hat{K}$. Then obtain the sophisticated estimate $\hat{\Gamma}_K = \hat{\Gamma}/[1+(\hat{\alpha}/\hat{K})]$ of Γ_K. In this way we obtain the locally defined $\hat{\Gamma}_K$ and the desired collection of $\hat{c}$ and $\hat{K}$ pairs. The trained model $K = \hat{K}_o + c\hat{K}_c$ is then obtained exactly as in method A above.

The *applicational stage* occurs when a fresh Secchi depth reading z'_{SD} is made. Then one finds $K' = \hat{\Gamma}_K / z'_{SD}$. From the trained model one then finds the desired $c' = [K' - \hat{K}_o]\hat{K}_c$.

C. Chlorophyll Assay via Irradiance Meter

Basic Assumptions

(i) Medium is arbitrarily stratified in depth z.

(ii) Spectral diffuse attenuation function $K = K(z,\lambda)$ for downward irradiance of wavelength λ is modeled by $K(z,\lambda) = c_o(z)K_o(\lambda) + c(z)K_c(\lambda)$. Here $c_o(z)$ and $c(z)$ (both in $mg \cdot m^{-3}$) are concentrations as in method A, and K_o, K_c are the respective diffuse attenuation cross sections (in $m^2 \cdot mg^{-1}$).

The *training stage* of the statistical model consists in first making a sweep of downward spectral irradiance values $H(z,\lambda)$ $watt \cdot m^{-2} \cdot nm^{-1}$ over a range of depths z at a fixed pair of wavelengths $\lambda = \lambda_1, \lambda_2$. This yields estimates $\hat{K}(z,\lambda)$ of $K(z,\lambda)$ via

$$\hat{K}(z,\lambda) \equiv -H^{-1}(z,\lambda)[H(z+\tfrac{1}{2}\Delta z,\lambda) - H(z-\tfrac{1}{2}\Delta z,\lambda)]/\Delta z \tag{22}$$

at small depth increments Δz around selected depths z for the two wavelengths $\lambda = \lambda_1, \lambda_2$. Concurrently, estimates $\hat{c}(z)$ of chlorophyll-a concentration and non-chlorophyll concentration $\hat{c}_o(z)$ are made at the same depths z. Then the spectral functions $K_o(\lambda)$, $K_c(\lambda)$ are estimated at $\lambda = \lambda_1, \lambda_2$ by linear least square regression techniques in which the number m of z-depths exceeds 4 (the the more z measurements the better). The linear regression equations are

$$\hat{K}(z,\lambda_j) = \hat{c}_o(z)K_o(\lambda_j) + \hat{c}(z)\, K_c(\lambda_j) \tag{23}$$
$$z = z_1,\ldots,z_m,\; j = 1,2$$

Here we have m equations in four unknowns $K_o(\lambda_j)$, $K_c(\lambda_j)$, $j = 1,2$. Let $\hat{K}_o(\lambda_j)$ and $\hat{K}_c(\lambda_j)$, $j = 1,2$, be the least squares estimated values arising from the system (23).

The *application stage* occurs when a fresh sweep of $H(z,\lambda)$ over the depth range of z is made at wavelengths $\lambda = \lambda_1,\lambda_2$, resulting in values $H'(z,\lambda)$. The purpose of the swap is to find $c_o(z)$ and $c(z)$ at depths z not necessarily those used in the training stage. If $K'(z,\lambda_j)$, $j = 1,2$ are the new K-values found from $H'(z,\lambda)$, as in (22), then at each z of interest the associated $c_o'(z)$ and $c'(z)$ values are given by simultaneous solutions of the pair of equations:

$$K'(z,\lambda_j) = c_o'(z)\, \hat{K}_o(\lambda_j) + c'(z)\, \hat{K}_c(\lambda_j)\,, \tag{24}$$
$$j = 1,2$$

D. Chlorophyll Assay via Beam Transmittance Meter

Basic Assumptions:

(i) Medium is arbitrarily stratified in depth z.

(ii) Spectral volume attenuation function $\alpha(z,\lambda)$ (in m^{-1}) for each wavelength λ is modeled by $\alpha(z,\lambda) = c_o(z)\alpha_o(\lambda) + c(z)\alpha_c(\lambda)$, with $c_o(z)$, $c(z)$ as in part A. Here $\alpha_o(\lambda)$ and $\alpha_c(\lambda)$ (in $m^2 \cdot mg^{-1}$) are the beam attenuation cross sections of the non-chlorophyll-a and chlorophyll-a components of the hydrosol.

The *training stage* consists in first lowering the beam transmittance meter, i.e., a transmissometer (recall (14)) and taking readings at two wavelengths $\lambda = \lambda_1, \lambda_2$ at a selected set of depths z. This yields estimates $\hat{\alpha}(z,\lambda)$ of $\alpha(z,\lambda)$ via (15). Concurrently, estimates $\hat{c}_o(z)$ $\hat{c}(z)$ of non-chlorophyll-a and chlorophyll-a concentrations are made at the same depths z. Then the spectral functions $\alpha_o(\lambda)$, $\alpha_c(\lambda)$ at each $\lambda = \lambda_1, \lambda_2$ are estimated using the same least square linear regression technique as in part C:

$$\hat{\alpha}(z,\lambda_j) = \hat{c}_o(z)\, \alpha_o(\lambda_j) + \hat{c}(z)\, \alpha_c(\lambda_j) \qquad (25)$$
$$z = z_1, \ldots, z_m \quad , \quad j = 1,2$$

Let the estimates be $\hat{\alpha}_o(\lambda_j)$, $\hat{\alpha}_c(\lambda_j)$, $j = 1,2$.

The *application stage* occurs when a fresh set of $\alpha(z,\lambda)$ readings, say $\alpha'(z,\lambda)$, $\lambda = \lambda_1, \lambda_2$, is obtained at depths z not necessarily those used in the training stage. Then the associated concentrations $c_o'(z)$, $c'(z)$ are given by solutions of the pair of simultaneous equations:

$$\alpha'(z,\lambda_j) = c_o'(z)\hat{\alpha}_o(\lambda_j) + c'(z)\hat{\alpha}_c(\lambda_j) \qquad (26)$$
$$j = 1,2$$

E. Discussion of the Chlorophyll Assay Problem

The procedure in par. A strips to its essence the Secchi disk procedure for assaying the chlorophyll concentration c in a homogeneous natural hydrosol. The basic idea is to make a series of measurements of chlorophyll concentration c and the diffuse attenuation coefficient K and then derive values of the coefficients K_o and K_c in the model for K. This trains or educates the model for K as to the sizes of the attenuation coefficient K_o and attenuation cross section K_c for the material comprising the hydrosol. The main point to note here is that values of K needed for the training exercise are supplied by the Secchi disk depths z_{SD} via the particular form (18b) of the inverse ocle (10). Moreover, once the model has been trained then the chlorophyll concentration c can be inferred from the model using new observed values of the diffuse attenuation coefficient K, once again supplied by Secchi disk readings z_{SD} via (18b). Hence the K-model in par. A is both built and operated solely through Secchi disk readings; a feature which endows the method with the simplicity and elegance we have come to associate with eyeball optics.

F. All is Not Well in Method A

The main unsatisfactory feature of the procedure in A is that we had to take a precooked value of Γ_K in order to convert z_{SD} into K. This unsatisfactory part of the procedure can be partially removed if we are willing to include an interloper from hydrologic optics, namely an irradiance meter. This is what is done in method B. From the point of view of the Secchi disker, the inclusion of the irradiance measurements is only temporary in order that Γ_K can be estimated for the hydrosol under study. Once the K model has been trained using the estimated Γ_K, the subsequent applications of the K-model proceed using only Secchi readings to estimate K. This estimated Γ_K will eventually have to be updated when lighting, hydrosol, and operator materially change.

However, once the foot of hydrologic optics is in the door of eyeball optics, the reader should see that a radical transformation in conceptual character of the Secchi-disk based chlorophyll assay problem occurs. As shown in method C, the Secchi disk can be laid aside and the assay can be done completely with irradiance measurements. Moreover, we obtain as a dividend diffuse attenuation functions $K_o(\lambda)$, $K_c(\lambda)$ of both the chlorophyll and non-chlorophyll components of the hydrosol, and at two wavelengths λ to boot.

The main unsatisfactory feature of method C's procedure is the use of the *apparent* optical property $K(z,\lambda)$: its magnitude is often markedly dependent on the directional disposition of sunlight and skylight in the medium (as is abundantly clear when one has learned how to read (71), below). To remedy this defect we convert over to the *inherent* optical property $\alpha(z,\lambda)$ in the method D procedure. This procedure is relatively satisfactory in that it is simple, rugged, and conceptually correct. Unfortunately for Secchi diskers, it needs a transmissometer instead of a Secchi disk.

G. Towards Standard Optical Indexing of Natural Hydrosols

Perhaps some day method D will become a useful way to *optically index* the trophic states of lakes and seas when submersible, remotely operated multiwavelength transmissometers doing real-time data analysis on programmable chips are as simple to operate and nearly as cheap as a Secchi disk. The concentrations $c(z)$ and $c_o(z)$ of the chlorophyll and non-chlorophyll components could then be mapped by vertical and horizontal contour maps, something just as easily visualized and far more pertinent than contours of Secchi depth readings.

The procedure in method A is in essence that suggested by Carlson (1977) who made an ill-fated attempt to produce a simple eyeball optical index of the trophic state of lakes. I have simply sorted out the essential pieces in his method needed to establish the statistical link between optics and biology and have enumerated them as basic assumptions (i), (ii), (iii) in par. A. This attempt to clarify the logical structure of Carlson's procedure should not be construed as an endorsement of his procedure or of method A. It was correctly pointed out by Lorenzen (1980) that Carlson underplayed the size, variability and importance of the non-chlorophyll part K_0 of the model of K. This view was taken up and supported by Megard *et al.* (1980) who went on to use the full K-model $K = K_0 + cK_c$ with K_0 and K_c estimated and studied in several lakes.

For those potential users of the Secchi disk procedure who are still not convinced of its inherent shortcomings after reading the Carlson, Lorenzen, and Megard *et al.* papers, I would ask that they reflect on the following. Anyone who observes on a regular basis lakes, estuaries, catchments and other inland or nearly land-locked waters, will find over time, seemingly random and certainly relatively large changes in the optical properties of these media. Secchi depths will fluctuate, α and K readings will vary markedly, but these changes will often have nothing to do with the phytoplankton level changes. After a long, dry period, for example, a river or lake may be visually clear, i.e., it may have a large z_{SD}, low α and low K readings because the suspended materials may have settled out. Then a rain or wind storm--or both--occur with massive increases in water flows, along with the deposit, stirring and mixing of sediments and erosion materials. As a result, in the space of a day the K, α readings could rise by an order of magnitude and z_{SD} decrease in corresponding fashion; and none of this has anything to do with a change in the trophic state of the water body. An unwary Secchi disker surveying the scene in the storm aftermath may be completely misled. Carlson's trophic index should never have been accepted as a possible indicator of biologic activity, and should never be used.

Megard *et al.* (1980) are particularly careful to emphasize the need for continued recalibration of the Secchi disk optical system which they rightly view as the indissoluble union of *Secchi disk and observer and lake*.

Therefore if measurements of chlorophyll concentration c are made along with a set of Secchi disk readings in *that* lake by a single observer, the same one whose data trained the K-model in the par. B set up, then there should in principle be a regression plane of K vs. c_o and c, of optimum fit. But just think of the ridiculous picture such activity brings to mind: an ancient device continually recalibrated by fine, precision equipment using modern electronic concepts, just so the ancient disk can be used as a visual clarity index of the water.

Here are some further observations on the futility of the Secchi disk procedure. The optimal fit alluded to above need not mean a perfect fit, for the basic K-model may be inadequate to sort out the optical effects of different separate molecular species or combinations of such species making up the hydrosol. For example it may be indicated by direct observation that one must allow other molecular species into the K-model, such as phaeophytins, dissolved (yellow) substances, or still other forms of chlorophyll-bearing plankton--for example, the Anabaena or Aphanizomenon mentioned by Edmondson (1980); or phosphorous, as in Carlson (1980); and so on. Thus the diffuse attenuation K-model may have to evolve from its simple form $K = K_o + cK_c$ in methods A, B to its general form

$$K = c_o K_o + c_1 K_1 + \cdots + c_\ell K_\ell \qquad (27a)$$

or, in view of the discussion of par. D, preferably to the general form of the beam attenuation function:

$$\alpha = c_o \alpha_o + c_1 \alpha_1 + \cdots + c_\ell \alpha_\ell \qquad (27b)$$

where ℓ may be on the order of from 1 to 10.

That such an extension of the K-model is necessary for accurate optical assays of natural hydrosol biota is indicated repeatedly in the comments by Megard *et al.* (1980) and by certain observations of Carlson (1980) in his reply to the comments by these workers. The procedure in method D, having models already of the form (27) (for $\ell = 1$) is only nominally changed by this generalization of the α model.

When using the expanded method D, the experimenter should anticipate some initial technical problems with the linear regression techniques needed to determine the $\hat{\alpha}_j$, $j = 0,\ldots,\ell$ estimates, in (25), e.g. Some practice will be needed to find linearly independent sets of $\hat{c}_o(z)$ and $\hat{c}(z)$ readings for the least squares determinations of $\alpha_o(\lambda_j)$ and $\alpha_c(\lambda_j)$. Also, the solution of the linear equation systems for the c_j' in (26) will depend on the spectral curves of the $\hat{\alpha}_j$ being sufficiently linearly independent. However, this is not the place to dwell on these details; they are merely technical and very likely can be overcome.

Of course the procedures hardest hit by going to (27a or b) will be the procedures in A and B, and that is the hope behind this entire discussion. Taking the cue from methods C and D (in which $\ell = 1$ in (27)) one would now in methods A and B have to do measurements at $\ell+1$ wavelengths λ_j using filtered eyeglasses, and determine z_{SD} (λ_j), $j = 1,\ldots,\ell+1$. Γ_K would now be wavelength dependent! Moreover, the $\ell+1$ molecular and substance species would have to be isolatable and their concentrations c_j, $j = 0,\ldots,\ell$ would have to be estimated in preparation for the training period. Indeed, the unique identifiability and collectibility of a molecular species or substance is an essential criterion that the substance must pass before being admitted a seat in the models of (27), for we need to estimate its concentration c_j prior to training the model. This is another technical problem that surely can be overcome. But it should not be used in methods A, B, or C; only in method D or its equivalents.

Perhaps by now the reader will be sufficiently wary of the basic intractability of the Secchi disk procedure.

H. Resistance in the Ranks?

I anticipate resistance by Secchi disk biologists to the adoption of the general models of (27) in the preceding four procedures, certainly a resistance to changes in A and B. Moreover, they are constitutionally unable to shift to method C or D. So be it. But times are a-changing. Better optical equipment for surveying natural hydrosols is continually coming along; multichannel simultaneous measurements are now routine in some biological circles (cf., e.g., Smith *et al.*, 1984); and computer software (if the data are friendly) already makes short work of the required linear regression and equation solving activities in (25), (26), for example. Eyeball optics is now beginning to fade into the back field as the race toward objective environmental documentation goes on. Perhaps eyeball optics will still poke happily along out in various watery pastures; but the remaining and larger part of the race is for those practicing hydrologic optics.

I. The Importance of Specific vs. Volume Attenuation Coefficients

A comment is in order on the usefulness of the beam attenuation cross sections (i.e., *specific* beam attenuation) α_0 and α_1, in the model $\alpha = c_0\alpha_0 + c_1,\alpha_1$. This will help our understanding of the relation between chlorophyll concentration distributions and Secchi disk readings.

The beam attenuation function α has units of m^{-1} and represents the amount dN of radiance N removed from a beam of photons by absorption and scattering as that beam of unit cross section area traverses a distance dr along its path (hence, as in (14), the defining statement of α is: $dN = -\alpha N dr$). This concept of beam attenuation as it stands is an *inherent optical property*, i.e., the amount of radiance removed from the beam within the unit volume is independent of the direction of irradiation of the volume in the defining statement* of α. Together with the volume scattering function σ, another inherent optical property, α is the foundation on which radiative transfer theory of natural hydrosols is based.

However, when the theory is to be applied to real hydrosols it is necessary, as in the above discussion of the chlorophyll assay problem, for example, to introduce the attenuation *cross section* functions (or *specific* attenuation functions) $\alpha_0, \alpha_1, \ldots, \alpha_\ell$ of α. Each α_j is associated with one of the molecular species or substances comprising an element of hydrosol volume, and together with the associated concentrations $c_0, c_1, \ldots, c_\ell$ these α_j define the molecular aspects of α. Since the units of α are m^{-1}, and the units of the c_j are $mg \cdot m^{-3}$, it follows that the units of the α_j are $m^2 \cdot mg^{-1}$, i.e., area per unit mass.

From a photon's point of view, there are some important cases where the cross section concept α_j is more fundamental than the volume concept† α. For as the photon enters a volume occupied by particles, the particles it

* This implies that random samplings of the particles and solvent molecules of the hydrosol's element of volume show no systematic patterns in directional or spatial distribution of these particles and molecules.

† This is not the case in the mathematical theory of the radiance distribution. There the volume concepts α and σ are all that are needed to solve the equation of transfer for the radiance distribution in a given optical medium.

encounters initially present their cross sections, not their volumes, to the photon. In highly absorbing media, the photon interacts mostly with the electrons stuck on or near the surface of the absorbing particles, rather than with those deep inside their volumes. If the absorbing particles are spherical, say, then their cross section areas presented to the photon stand in a fixed ratio to their total areas (cross section: πr^2; total area: $4\pi r^2$). Thus α_j in such cases represents a measure of the total area of a unit mass of chlorophyll-bearing plankton that is exposed to the photons entering the element of volume of hydrosol the plankton cells occupy.

This interpretation of the α_j helps us in part to understand the findings of Haffner and Evans (1974), namely that attenuation of light is more closely related to the surface area of hydrosol particles than to their volumes. Edmondson (1980) brings this point home to his students by having them compare the transparency of a flask of tapwater with a piece of chalk in it, to the transparency of the same volume of water after the piece of chalk has been ground up and dispersed in it. This phenomenon is in some measure related to the 'flattening effect' on absorption spectra of suspensions as compared to absorption spectra of solutions. See Duysens (1956).

J. Edmondson's Thought Experiment on Secchi Transparency

Edmondson (1980) goes on to use his experiment with the ground-up chalk in tapwater to understand why there may be large Secchi transparencies even in the abundant presence of certain plankton populations (such as Anabaena or Aphanizomenon). In the same note, to encourage his readers to gain insight into such a phenomenon, he poses an interesting thought experiment which under certain simplifying assumptions can be quantitatively analyzed in an intuitive manner: consider a deep homogeneous hydrosol of perfectly clear water with a suspension of spherical phytoplankton cells of concentration c_0 and beam attenuation cross section α_0, so that the volume attenuation coefficient of the suspension is $c_0\alpha_0 = \alpha'$. In order to make the thought experiment initially quite simple, we will postulate that the cells are mainly absorptive and of low transparency. In other words, the incident photons will interact only with cell surface electrons.

Now imagine two experiments: *Exp. 1*. The number of cells per unit volume of the hydrosol is doubled, with chlorophyll mass per cell and cell size the same. *Exp. 2*: The chlorophyll mass per cell is doubled, with the cell size the same and number of cells per unit volume the same. The question is: which change would affect the Secchi disk depth z_{SD} more? (Here it may be advisable to imagine use of filtered glasses that pass photons around 440 nm.) The intuitive feeling is that Exp. 1 would affect z_{SD} more. Using the notion of attenuation, absorption, and scattering *cross sections*, this conclusion, under the above simple assumption, can be verified and in fact be given in quantitative form. Suppose the Secchi depth in the original hydrosol is z'. Let the Secchi depths in the hydrosols of Exp. 1 and Exp. 2 be z_1, z_2, respectively. Then we will show that $z_1 = \frac{1}{2}z'$ and $z_2 = z'$.

In Exp. 1, it is clear that the new values c_1, α_1 of the concentration and attenuation cross section are $c_1 = 2c_0$ and $\alpha_1 = \alpha_0$, so that $c_1 a_1 = 2c_0\alpha_0 = 2\alpha'$, i.e., we double the volume attenuation coefficient in Exp. 1. In Exp. 2, we have concentration $c_2 = 2c_0$ and attenuation cross section $\alpha_2 = \frac{1}{2}\alpha_0$ (since the surface area of a cell is the same as before but now twice as much mass is packed in the cell). Hence $c_2 a_2 = c_0\alpha_0 = \alpha'$, i.e., the volume attenuation coefficient is unchanged in Exp. 2.

Before a conclusion about z_{SD} can be reached, the direct ocle (9) requires a similar analysis for the diffuse attenuation coefficient K. Now, as we shall see in Sec. 9, we can write K very nearly as: $K = D_{-}a[1+3R_{\infty}]$, where $R_{\infty} = \overline{b}/3a$, D_{-} is the distribution factor, and where a is the volume absorption coefficient and $\overline{b}$ the mean backscatter coefficient of the medium. We must then determine the effect on a, $\overline{b}$ and R_{∞} of the Experiments 1 and 2. As for the absorption coefficient in the original medium we have $a' = c_0 a_0$ in analogy to the case for the attenuation coefficient $\alpha' = c_0\alpha_0$. Similarly with the mean backscatter cross section $\overline{b}_0$ of the original medium, we can write $\overline{b}' = c_0\overline{b}_0$. In Exp. 1, for the same reasons as in the case of α_1 for the attenuation cross sections, we now find $a_1 = a_0$ for the absorption cross section; and similarly in Exp. 2, the absorption cross section $a_2 = \frac{1}{2}a_0$. Exactly similar conclusions hold for $\overline{b}$ (even if of small magnitude, by hypothesis) in the two experiments. Thus $\overline{b}_1 = \overline{b}_0$ and $\overline{b}_2 = \frac{1}{2}\overline{b}_0$. Hence $K_1 = 2K'$ and $K_2 = K'$ (for fixed D_{-}) where K' is the diffuse attenuation coefficient K of the original hydrosol. Finally, observe that R_{∞}, being a ratio of $\overline{b}$ to 3a, is unchanged by either experiment.

Thus, assuming the distribution factor D_{-} is unchanged by Exp. 1 or Exp. 2 (not precisely true, but close enough for the present discussion) we conclude from the direct ocle (9) that the Secchi depths z_1 and z_2 of the two experiments are given by $z_1 = \frac{1}{2}z'$ and $z_2 = z'$, as was to be shown.

As a dividend, we find from (18b) that Γ_K of the present simplistic medium is unchanged by either Exp. 1 or by Exp. 2. Hence by (18c) the irradiance H_{SD} at Secchi depth remains 10% of the surface irradiance H_0 in each experiment. By (21) the euphotic depth is halved in Exp. 1 and left unchanged in Exp. 2.

This thought experiment can be elaborated in several ways. The next level would be to decrease the transparency of the water body itself, in which the phytoplankton are suspended, by requiring it to have non zero volume attenuation and absorption coefficients. Another extension is to increase the transparency of the cell material, so that photons can penetrate and be absorbed within the volumes of the cells. Duysens (1956) gives a formula for the absorptance (fraction of photons absorbed entering the cell) of a spherical cell in terms of the concentration and specific absorption coefficient of the material of the cell. These extensions are left as exercises for the interested student. At some point, multiple scattering in and around the cell ensemble will become important. Then the thought experiment may no longer be appropriate, and resort should be made to the equation of transfer and the various methods of its solution.

K. Finding the Intrinsic Optical Properties

The preceding discussion of the Edmondson (1980) thought experiment is intended to illustrate the power of the attenuation cross section (or *specific* or *mass* attenuation) concept. The concept goes right to the molecular level in the hydrosol, and for that reason we can call the various α_j in (27b) the *intrinsic* attenuation properties of the medium.* In this way we can sharpen the usual two-class partitioning of all optical properties into inherent and apparent optical properties: The intrinsic optical properties tend to describe the inherent optical properties more on the molecular level rather

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* Some notice should be made, even if in passing, of the fundamental assumption of additivity of attenuation coefficients that has implicitly been made when working with intrinsic optical properties represented by the linear form (27b). This assumption, interestingly, can be studied using deductions from the basic theory outlined in sec. 9. One would make use of the criterion that linearity in (27b) holds for a layer of given depth when the reflectances of the union of any collection of subslabs comprising the layer add, and the transmittances multiply, to yield respectively the reflectances and transmittances of the given layer. For such numerical tests, the appropriate tools are e.g., (37)-(40) in Preisendorfer and Mobley (1984). The central issue here is the validity of Beer's Law. For some other approaches to studying this assumption of additivity, see Hardy and Young (1948), Duysens (1956), and Kirk (1976).

than on the volume level. Procedure D above gives us a way to experimentally infer the magnitudes of these intrinsic optical properties. Method D is readily adapted to work with the volume absorption function $a(y,\lambda)$ also. For some recent work on a linear model of the volume absorption function using specific absorption (cross sections) as functions of wavelength, see Prieur and Sathyendranath (1981).

9. A PRACTICAL PHYSICAL FOUNDATION FOR SECCHI DISK SCIENCE

As we have seen, Secchi disk science rests on the twin pillars of physiological optics and hydrologic optics, the former being characterized by the early and careful work of Blackwell (1946), the latter being the accumulated work over this century by all manners of scientists around the world (cf., e.g., the introductory comments in Preisendorfer and Mobley, 1984). I will summarize here the essentials of the physical basis of Secchi disk science. Fuller details may be found in the preceding reference and in H.O., Vol. I (Sec. 1.4, pp. 96-102), Vol. V (Sec. 9.2, 9.5), and Vol. VI (Sec. 13.9). For a different perspective on Secchi disk theory, see Levin (1980).

The present discussion will consist of two main parts. First the exact theoretical foundation of Secchi disk theory will be outlined, and then the more practical and useful basis for Secchi disk science will be presented. The practical basis is given in a form that will show how modern limnologists and oceanographers can wean themselves from eyeball optics and go on to hydrologic optics.

Remember that we have agreed to work with radiometric concepts (via broad-band colored glasses). It turns out that all the main formulas derived below go over, *mutatis mutandis*, to photometric form (N to B, H to E, etc.). However, the *interpretation* and *utility* of the two different sets of formulas are profoundly different, as will be clear from a study of Preisendorfer (1986).

A. The General Binocle of Eyeball Optics

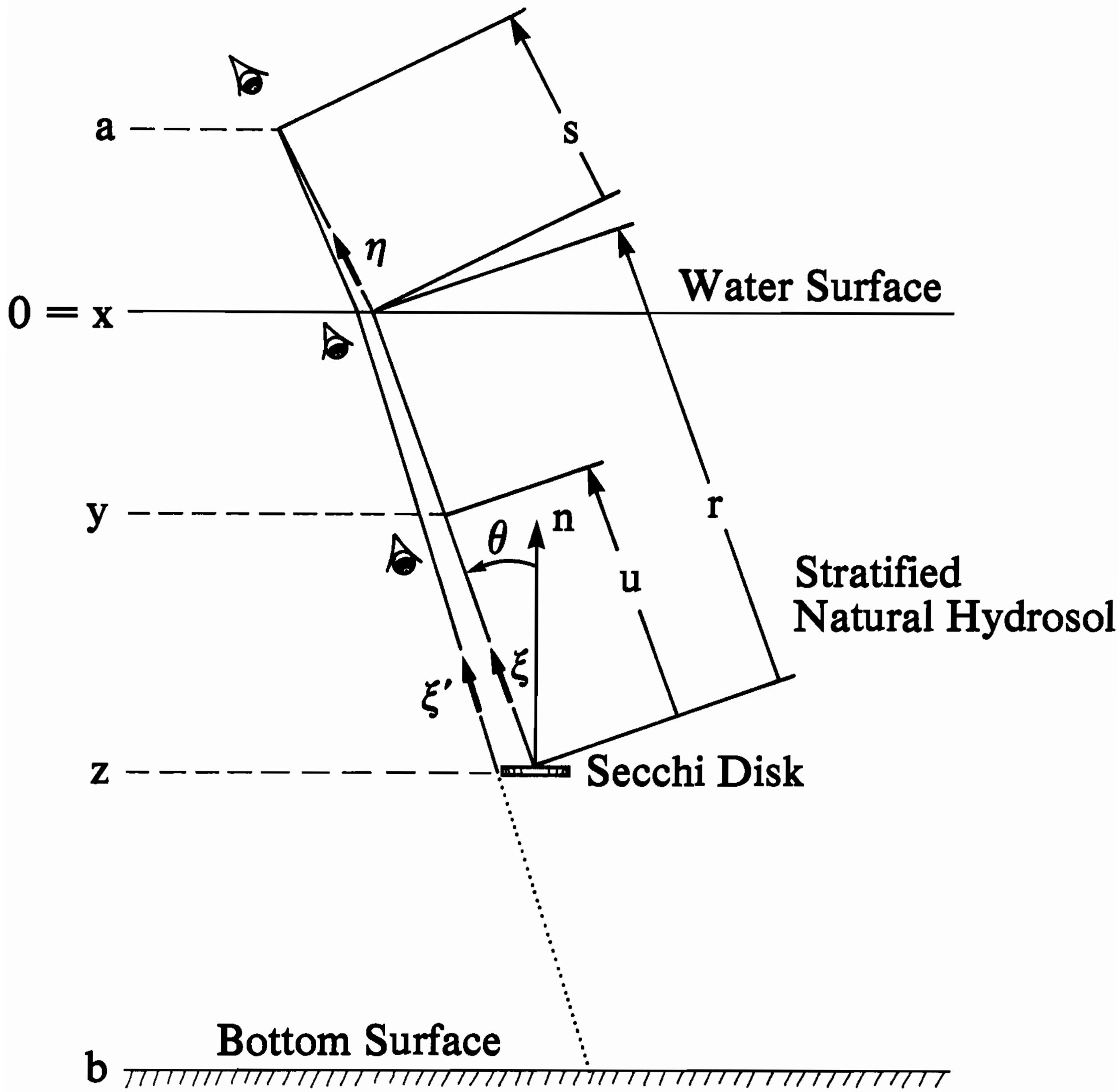
The notions of *contrast* of a visible object against its background and *transmission* of this contrast through the medium in which the object is viewed are the two Key Ideas behind Secchi disk science. In Fig. 1 an eyeball at level a looks through the air-water surface toward a submerged Secchi disk at level z. For most of the ensuing discussion x, the submerged depth of the observer's eyeball, can be at a finite distance below the surface. In the final stage, however, x will be 0 and just below the surface. The disk is suspended above a bottom at level b, where $a \le x \le y \le z \le b$ are distances measured vertically from some earth-fixed origin. The medium is arbitrarily stratified with depth y (in m).

Consider first the case where the eye is below the surface at level x. The *apparent radiance* $N_r(x,\xi)$ of the disk at x produced by photons streaming along direction ξ of the path of length r may generally be written as (H.O., Vol. V, p. 177):

$$N_r(x,\xi) = N_r^o(x,\xi) + N_r^*(x,\xi) \qquad (\text{watt}\cdot\text{m}^{-2}\cdot\text{sr}^{-1}\cdot\text{nm}^{-1}) \tag{28}$$

Here $N_r^o(x,\xi)$ is the *directly transmitted* (or *residual*) radiance over the distance r from the disk. This radiance is carried from the disk to the eye by photons not having experienced scattering or absorption. $N_r^*(x,\xi)$ is the *path radiance* generated by scattering of the environment's photons at each point of the path into the direction ξ and thence directly transmitted to the eye. Now at each level y the radiance of the directly transmitted photons, from level z moving along direction ξ, decays according to the law:

$$\frac{dN_u^o(y,\xi)}{du} \equiv -\alpha(y)\, N_u^o(y,\xi) \tag{29}$$



Geometry of the Secchi Disk Sighting

Figure 1.

where u is positive distance (in m) along the path to the eye at level y from the fixed depth z of origin of the photons. Equation (29) essentially defines the beam attenuation function $\alpha(y)$. From this we have

$$N_r^o(x,\xi) = N_o(z,\xi) \exp[-\int_0^r \alpha(y)du] \tag{30}$$
$$\equiv N_o(z,\xi)\ T_r^o(z,\xi)$$

where

$$y = x+(r-u)\cos\theta$$

and where θ is the angle of inclination of ξ from the vertical.

Now, by analogy to $N_u^o(y,\xi)$, the apparent radiance $N_u(y,\xi)$ of the disk (Fig. 1) as seen by the eye at depth y along a path of length u, has the general decay law:*

$$\frac{dN_u(y,\xi)}{dy} \equiv - K(y,\xi)\ N_u(y,\xi) \tag{31}$$

Since $N_u(y,\xi)$ is measurable at each y and ξ of the stratified medium, (31) serves simply as a definition of $K(y,\xi)$ in that medium. The minus sign is a convenient convention; it is designed to make $K(y,\xi)$, for arbitrary ξ, eventually positive as y increases in a *general deep* medium (without a disk or bottom to locally perturb the field). In analogy to (30), we find from (31),

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* cf. H.O., Vol. III, p. 14; and H.O., Vol. V, p. 177 for general discussions of radiance K functions and apparent radiance. The cited discussion in the latter volume shows that the subscript u in (31) is removable without changing the validity or meaning of (31). Strictly, however, we should be using three dimensional notation where y is not a depth but an ordered triple (y_1,y_2,y_3) locating the place where the radiance $N_u(y,\xi)$ is being observed by the eyeball. If such changes are made the derivation proceeds once again to the same conclusion (39).

the formula for the apparent radiance $N_r(x,\xi)$ of the disk, given the inherent radiance $N_o(z,\xi)$ of the disk and $K(y,\xi)$ along the path:

$$N_r(x,\xi) = N_o(z,\xi) \exp[-\int_x^z K(y,\xi)dy] \tag{32a}$$

$$\equiv N_o(z,\xi)\ T_r(z,\xi)$$

Setting $y = x+(r-u)\cos\theta$, this may be written as

$$N_r(x,\xi) = N_o(z,\xi) \exp[\cos\theta \int_0^r K(y,\xi)du] \tag{32b}$$

Next, imagine an eye-brain system at level x to alternate its attention between direction ξ from the center of the disk and direction ξ' from the immediate background of the disk (see Fig. 1). The apparent radiances it perceives are represented respectively by

$$N_r(x,\xi) = N_r^o(x,\xi) + N_r^*(x,\xi) \tag{33a}$$

$$N_r(x,\xi') = N_r^o(x,\xi') + N_r^*(x,\xi') \tag{33b}$$

Now it is intuitively clear that, if ξ and ξ' are nearly coincident, we would have essentially that

$$N_r^*(x,\xi) = N_r^*(x,\xi') \tag{34a}$$

and

$$T_r^o(z,\xi) = T_r^o(z,\xi') \tag{34b}$$

Two paths having this property form part of a *regular neighborhood* of paths (cf. H.O., Vol. V, p. 166). It may rigorously be shown that in a general natural hydrosol one always can find such pairs of paths under all viewing conditions. (For this, the equation of transfer, or an equivalent form of it, is needed.) This fact is crucial to the next step involving the apparent and inherent contrasts of the disk.

Now (see Fig. 1) the *inherent contrast* of the disk against the background is that experienced by the eye at level z:

$$C_o(z,\xi) = \frac{N_o(z,\xi) - N_o(z,\xi')}{N_o(z,\xi')} \tag{35}$$

The *apparent contrast* of the disk against its background is that experienced by the eye at level x a distance r from the disk:

$$C_r(x,\xi) = \frac{N_r(x,\xi) - N_r(x,\xi')}{N_r(x,\xi')} \tag{36}$$

With the help of (30), (32a), (33) and (34), the apparent contrast (36) may be written as

$$C_r(x,\xi) = \frac{N_o(x,\xi) - N_o(x,\xi')}{N_o(z,\xi')} \cdot \frac{T_r^o(z,\xi)}{T_r(z,\xi)} \tag{37}$$

Using (30), (32b), and the definition of inherent contrast, (37) reduces to

$$C_r(x,\xi) = C_o(z,\xi)\ \exp\{-\int_0^r [\alpha(y) + K(y,\xi)\ \cos\theta]du\} \tag{38}$$

This is the basic contrast transmittance law for a generally stratified natural hydrosol. The *contrast transmittance*

$$\mathcal{T}_r(z,\xi) \equiv \exp\{-\int_0^r [\alpha(y) + K(y,\xi)\ \cos\theta]du\} \tag{39}$$

shows how the inherent contrast $C_o(z,\xi)$ is transmitted from the disk's depth z along the direction ξ a distance r to become the apparent contrast of the disk at level x of the eye.

The notion of contrast transmittance, as used in (39), is general enough to include the possibility of light fields perturbed by large and even self-luminous Secchi disks (cf. H.O., Vol. III, p. 15; Vol. V, p. 169). In (39) $\mathcal{T}_r(z,\xi)$ in principle includes not only the effects of lateral inhomogeneities of the hydrosol vertically stratified in any way with depth y but also includes the average surface lighting effects, and lighting effects of the presence of the bottom at level b (Fig. 1). In order to point this up, (39) should be written in full three dimensional notation, rather than the one-dimensional notation for a stratified medium (see footnote to (31)).

Next, let $\mathcal{T}_s(x,\xi)$ be the contrast transmittance of the composite path of sight starting at x (just below the surface) along direction (unit vector) ξ through the air water surface *and* the path along η through the atmosphere a distance s from level x to level a. Then, since contrast transmittances of contiguous paths multiply together to form the contrast transmittance of their union (H.O., Vol. I, pp. 93-96), we have finally

$$C_{r+s}(a,\eta) = C_o(z,\xi)\ \mathcal{T}_s(x,\xi)\ \mathcal{T}_r(z,\xi)\ ,$$

which, by (39), may be written:

$$\boxed{\begin{array}{l} C_{r+s}(a,\eta) = C_o(z,\xi)\ \mathcal{T}_s(x,\xi)\ \exp\{-\int_0^r [\alpha(y) + K(y,\xi)\cos\theta]du\} \\ y = x+(r-u)\cos\theta \end{array}} \qquad (40)$$

This is the desired general form of (6), for a general stratified natural hydrosol and stratified atmosphere above it.

The general version of the binocle (8) for inhomogeneous media can be derived from (40), and takes the following form. We set $s = 0$, so that the eyeball is just above the surface. Thus we have $a = x = 0$ (but with a just above and x just below the surface). Next, set $\xi = \eta = n$ (Fig. 1), i.e., we set $\theta = 0$ for the vertical path of sight of length $r = z$. Then

$$\boxed{\int_0^z [\alpha(y) + K(y,n)]dy = \ln[\mathcal{T} C_o / C_z]} \qquad (41)$$

where for brevity we have set

$$\mathcal{T} \equiv \mathcal{T}_o(x,n), \quad C_o \equiv C_o(z,n), \quad C_z \equiv C_z(a,n)$$

As it stands, (41) is the *general binocle of eyeball optics*. It stands there in splendid isolation, beautiful and useless, mainly because, in a general light field, there is no simple behavior of $K(y,n)$ with y. There is, however, one important class of inhomogeneous natural hydrosols for which (41) reduces essentially to the binocle (8), and that is the class of optically deep separable media (H.O., Vol. IV, p. 145). A natural hydrosol is *separable* if (i) the ratio of the total volume scattering function $s(y)$ to the volume attenuation function $\alpha(y)$, i.e., $\omega(y) \equiv s(y)/\alpha(y)$ is independent of depth, and (ii) the volume scattering function $\sigma(y;\xi';\xi)$ can be written in the form $\sigma(y;\xi';\xi) = s(y)p(\xi';\xi)$. It is readily shown that in separable media, the dimensionless backscatter ratio $\beta(y) = \overline{b}(y)/\alpha(y)$ is also independent of depth (for further definitions see par. B, below). In this way the volume scattering function σ is split, or *separated*, into a part (the volume total scattering function $s(y)$) that depends only on location and a part (the *normalized phase function* $p(\xi';\xi)$) that depends only on directions ξ',ξ. For our present purposes, separability implies that, with $\omega(y) \equiv \omega$ and $\beta(y) \equiv \beta$ being independent of depth, the ratio $\varepsilon(y) \equiv K(y)/\alpha(y) = D_-(y)[1-\omega+\beta]$ (cf. (81)) will depend on depth only through the distribution function $D_-(y)$. Now, for any fixed sun and sky lighting patterns, $D_-(y)$ varies relatively little with depth (cf., e.g., H.O., Vol. V, p. 189). We shall adopt this assumption of depth independence of $D_-(y)$ here. Then $K(y)/\alpha(y)$ is independent of y, and of magnitude, say, ε. Thus ε is known once D_-, ω and β in a medium are known. Indeed, in most natural hydrosols (cf. (81), below) we can neglect β in the expression for ε and write $\varepsilon = D_-[1-\omega]$. Hence, knowing D_- and ω will allow rough estimates of ε.

With the preceding preliminaries in mind we imagine that (41) is applied to a separable natural hydrosol. Next, observe that the integral in (41) generally can be written as:

$$\int_0^z \alpha(y)[1 + \frac{K(y,n)}{\alpha(y)}]dy$$

Now it is an interesting fact that in deep separable optical media, $K(y,n)$ is essentially equal to $K(y,-) \equiv K(y)$ at all depths y (see, e.g., H.O., Vol. V, p. 230, Fig. 10.14, particularly the $\mu = 1.00$ curve, which is for $K(y,n)$). Hence $K(y,n)/\alpha(y) = K(y)/\alpha(y) = \varepsilon$. Accordingly, in the above integral we can replace $K(y,n)/\alpha(y)$ by ε. Thus (41) becomes [binocle] $$\boxed{\bar{\alpha}(1+\varepsilon)z = (\bar{\alpha}+\bar{K})z = \ln[\mathcal{T}C_o/C_z]} \tag{42}$$

where $$\varepsilon \equiv K(y)/\alpha(y) = \bar{K}/\bar{\alpha} \tag{43}$$

and

$$\bar{\alpha} \equiv z^{-1} \int_0^z \alpha(y)dy \tag{44}$$

$$\bar{K} \equiv z^{-1} \int_0^z K(y)dy \tag{45}$$

In this way we obtain the *binocle of eyeball optics* for *separable natural hydrosols*, i.e., the generalization of (8) to media in which $K(y)/\alpha(y)$ is independent of depth y. A practical test for when (42) is applicable is to simply measure the diffuse attenuation function $K(y,-)$ and volume attenuation function $\alpha(y)$, and to form the ratios $K(y,-)/\alpha(y)$ at various depths. But then we come back to the old refrain of, 'why-don't-you-just-continue-to-work-with-irradiance-collectors-and-beam-transmissometers-and-forget-eyeball-optics?' Good question.

We take the final step toward the ocles by returning to (42) and replacing C_z by the threshold contrast C_T and hence z by z_{SD}. As a consequence we obtain (cf. (9)) the

[direct ocle for separable media] $$\boxed{z_{SD} = \frac{\ln[\mathcal{T}C_o/C_T]}{\bar{\alpha}(1+\varepsilon)} = \frac{\Gamma}{(\bar{\alpha}+\bar{K})} = \frac{\bar{\Gamma}_\alpha}{\bar{\alpha}}} \tag{46}$$

and (cf. (10)) the

[inverse ocle for separable media) $$\boxed{\bar{\alpha} = \frac{\Gamma}{z_{SD}(1+\varepsilon)} = \frac{\bar{\Gamma}_\alpha}{z_{SD}}} \tag{47}$$

where (cf 18a):

$$\bar{\Gamma}_\alpha \equiv \frac{\Gamma}{1+\bar{K}/\bar{\alpha}} \tag{47a}$$

The salient difference between the old and new ocles is the presence of the diffuse-beam attenuation ratio ε which must be known before the new inverse ocle (47) can be used. As noted above, if the scattering-attenuation ratio ω and distribution factor D_- are known, then ε may be estimated by* $\varepsilon = D_-[1-\omega]$. It is interesting to observe that with ε known, the inverse ocle (47) allows a direct estimate of the average volume attenuation coefficient $\bar{\alpha}$ of the medium and by (43) we have also the average diffuse attenuation coefficient $\bar{K}$. In other words the separation of $\alpha+K$ into α and K is rigorously possible in separable natural hydrosols. The price for this of course is having to measure or estimate the ratio $\varepsilon = K(y)/\alpha(y)$; which is our old friend in disguise: $K(y) = \varepsilon\, \alpha(y)$, that is, ε is just a reincarnation of the rule of thumb relating K and α that played such a key part in the discussions before and after equations (18a,b)).

In practice, when applying the inverse ocle (47) the procedure is essentially that of the homogeneous case. Now we automatically find $\bar{\alpha}$ and $\bar{K}$ instead of α and K. We will continue to be good Secchi opticians and try to make $\mathcal{T} = 1$ in Γ. Ambient lighting conditions and disk size should be routinely noted so that C_T can be estimated from Blackwell's (1946) tables. And finally, C_o, the inherent contrast of the disk against the background, when nothing more is known about it, is estimated as usual using the so-called 'R_∞ correction' (cf. H.O., Vol. I, p. 172). This amounts to the ploy used in the derivation of (6) on board the *Dayton Brown*, i.e., determining C_o in (35) by setting the background radiance

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* It should be observed that for larger depths ($\sim 20\alpha^{-1}$) in separable media the ratio $K(y)/\alpha(y) = \varepsilon$ depends essentially on only the shape of the volume scattering function $\sigma(y;\xi';\xi)$ of the medium (cf. H.O., Vol. V, p. 252). The alternative replacement of $\sigma(y;\xi';\xi)$ by the more easily measured D_- thus raises our estimate of the practical value of the present rule of thumb for ε. If β is also available (cf. (74)) this additional information about the shape of σ may be incorporated in the estimate of ε via (81).

$N_o(z,\xi') = H(z,-)R_\infty/\pi$, by setting the disk radiance $N_o(z,\xi) = H(z,-)R/\pi$ for both ξ' and ξ directed vertically upward, and finally by noting that $R_\infty \approx 0.02 << R \approx 1$. Under these conditions

$$C_o(z,\xi) \approx R/R_\infty \tag{48}$$

Therefore C_o in (48) can be estimated simply as 50R if nothing further is known about the medium. Otherwise, one can use $R_\infty = \bar{b}/3a = \beta/3(1-\omega)$ if ω and β of the separable medium are known or estimable (cf. (76) and (77)).

B. The Two Flow Model of the Light Field

We turn now to a very practically oriented approach to the light field in natural hydrosols, an approach that on the one hand serves to lay the foundations for our derivations and remarks in the body of the text above, and on the other, serves to prepare for more objective ways than the Secchi disk of documenting light fields for biological purposes.

The spectral downward (-) and upward (+) irradiances $H(y,\pm)$ ($\mathrm{watt \cdot m^{-2} \cdot nm^{-1}}$) incident on a horizontal plane at depth y play essential roles in the study of Secchi disk visibility in a source-free non fluorescing natural hydrosol.* The basic law governing the changes of $H(y,\pm)$ with depth y (cf. H.O., Vol. V, p. 12) is:

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* The connection between radiometric and quantal units of light is often needed in the study of photosynthetically active radiation. The connection is as follows. Multiplying $H(y,\pm)$ by $\lambda/(h_o v)$ (in photons per joule) yields the numbers of photons of wavelength λ at depth y incident per sec per m^2 per nm. Here Planck's constant $h_o = 6.63 \times 10^{-34}$ joule sec, v is the speed of photons at their point of recording (usually $v = 3.00 \times 10^8$ m sec^{-1}), and λ is in nanometers (nm) (= 10^{-9} m). In general, if $n(y,\xi,\lambda)$ is the number of λ wavelength photons at depth y crossing a unit area normal to the direction ξ, per steradian per unit time per unit wavelength interval, then the radiance $N(y,\xi,\lambda) = (h_o v/\lambda)\, n(y,\xi,\lambda)$.

$$\mp \frac{dH(y,\pm)}{dy} = -[a(y,\pm) + b(y,\pm)]\ H(y,\pm) + b(y,\mp)\ H(y,\mp) \tag{49}$$

where one reads all upper signs together and all lower signs together. The depth rate of decay of $H(y,\pm)$ is given by the diffuse attenuation functions $K(y,\pm)$ whose definitions are

$$K(y,\pm) \equiv -H^{-1}(y,-)\ dH(y,\pm)/dy \tag{50}$$

From (49) the exact expressions for $K(y,\pm)$ are (cf. H.O., Vol. V, p. 117):

$$K(y,-) = a(y,-) + b(y,-) - b(y,+)\ R(y,-) \tag{51}$$

$$-K(y,+) = a(y,+) + b(y,+) - b(y,-)\ R(y,+) \tag{52}$$

The volume absorption function $a(y)$ and volume mean backscatter function $\bar{b}(y)$ are related to $a(y,\pm)$ and $b(y,\pm)$ by:

$$a(y,\pm) \equiv a(y)\ D(y,\pm) \tag{53}$$

$$b(y,\pm) \equiv \bar{b}(y)\ D(y,\pm) \tag{54}$$

and in like manner we may define the two-flow volume attenuation function:

$$\alpha(y,\pm) \equiv \alpha(y)\ D(y,\pm)\ . \tag{55}$$

Here $D(y,\pm)$ are the *distribution* functions giving the mean distance at level y traveled downward (-) or upward (+) by photons through a layer of unit depth in the hydrosol. A precise definition of $b(y,\pm)$ is given in H.O., Vol. V,

p. 11; see also Preisendorfer and Mobley (1984). Moreover, the downward irradiance reflectance of the medium R(y,-) is defined as:

$$R(y,-) \equiv H(y,+)/H(y,-) \ , \tag{56}$$

and we set

$$R(y,+) \equiv R^{-1}(y,-)$$

In practice the distribution functions $D(y,\pm)$ are determined by

$$D(y,\pm) = h(y,\pm)/H(y,\pm) \tag{57}$$

where $h(y,\pm)$ are the hemispherical scalar irradiances measured by spherical collectors exposed to the photons streaming in upward (+) or downward (-) directions. The $H(y,\pm)$ are measured analogously but by flat plate collectors.

C. Some Useful Special Cases of the Irradiance Field

The quantities $K(y,\pm)$, $R(y,\pm)$, and $D(y,\pm)$ along with the volume attenuation function $\alpha(y)$ are all that are needed for a practical* physical foundation for Secchi disk science and beyond. I will now illustrate this by deriving two approximate basic relations used several times in the preceding discussions, namely the estimate of the diffuse attenuation coefficient

* The complete theory of the Secchi disk, as noted earlier, involves the solution of the full radiative transfer equation in a three dimensional setting. While this may readily be done, all indications to date are that for the rugged, simple needs of applied biology in natural hydrosols, the model (6) is adequate. The theoretical framework in this section gives the basis of (6).

$$K \approx D_{-}a[1+3R_{\infty}] \tag{58}$$

and the estimate of the irradiance reflectance of the body of the natural hydrosol

$$R_{\infty} \approx \bar{b}/3a \tag{59}$$

These formulas are approximately valid around the middle of the visible spectrum in *infinitely deep* hydrosols that are either separable or *homogeneous*. As we have seen these are the most useful settings for Secchi disk dippings. In such settings K, R, and D are essentially independent of y (where y is geometric or optical depth). In this form we write R(y,-) as 'R_{∞}', K(y,±) as '$K_{\pm}$' and D(y,±) as '$D_{\pm}$'. Moreover, we set $K_{+} \approx K_{-} \equiv K$ (cf. H.O., Vol. V, p. 123). Consequently (51), (52) imply

$$K \equiv K_{\pm} = D_{-}[a+\bar{b}] \tag{60}$$

on dropping the product term in (51), which is relatively small (~2% of b(y,-) and ~0.2% of a(y,-)), over the visible spectrum in e.g., coastal and lake natural hydrosols. Here $D_{-} \approx 1.3$ and $D_{+} \approx 2.7$ for green light. Next, from (49), the following exact relation holds (cf., H.O., Vol. V, p. 118):

$$R(y,-) = \frac{K(y,-) - a(y,-)}{K(y,+) + a(y,+)} \tag{61}$$

With (60) being used to replace the K(y,±)'s, this may be approximated by

$$R(y,-) \simeq \frac{D_-\bar{b}}{[D_+ + D_-]a + D_-\bar{b}} \tag{62}$$

that is, by

$$R(y,-) \equiv R_\infty \equiv [\bar{b}/(3a+\bar{b})] \tag{63}$$

on using $D_+/D_- \cong 2$.

On dropping the $\bar{b}$ term in the denominator of (63) we obtain the rough and ready estimate for R_∞ in green light

$$R_\infty = \bar{b}/3a, \tag{64}$$

as was to be shown. Eq. (58) now follows from (60) and (64).

The form (62) may be needed in media that have absorption small compared to scattering, such as milky-appearing media. In most natural hydrosols, $\bar{b}$ is somewhat smaller than a, perhaps an order of magnitude smaller around green wavelengths.

D. Basic Optical Apparatus

We observe next that the optical apparatus needed to do the physical foundation work for Secchi disk science is the *transmissometer* for $\alpha(y)$ and a *quartet of irradiance collectors* (cf., H.O., Vol. VI, p. 325) that measures the corresponding irradiance quartet $[H(y,\pm), h(y,\pm)]$ at each depth y for a discrete set $\lambda_1,\ldots,\lambda_\ell$ of wavelengths. These readings will yield the K, R, D values directly, along with the diffuse attenuation ratio K/α (cf. (18)). Using the exact inverse procedure in Preisendorfer and Mobley (1984), one can recover the volume absorption function a(y) and volume mean backscatter function $\overline{b}(y)$ at all depths y. From the transmissometer measurement $\alpha(y)$ (= $a(y) + s(y)$, where $s(y)$ is the volume total scattering function), we can find $s(y)$ via $s(y) = \alpha(y) - a(y)$, using the recovered volume absorption $a(y)$.

From this point on, all quantities useful to present day Secchi disk science are known or computable, as we have seen repeatedly in the preceding sections. Using such measurements, the assumptions underlying eyeball optical procedures (as in A, B of Sec. 8) can be examined, and updated, if required, for each natural hydrosol of interest.

E. The Main Optical Properties of the Two-Flow Irradiance Model

We close by assembling the basic and derived optical properties which are recommended for optically characterizing natural hydrosols on the level where Secchi Disk Science operates and where close links exist between optics and biology. From the inverse procedure mentioned above, the irradiance quartet readings, and beam transmissometer readings, we have at each depth y the following sets of basic properties. They are written in depth-dependent form for that day when eyeball optics (which can work meaningfully only in homogeneous or separable media) has been superseded by hydrologic optics.

The *basic properties* are

$\alpha(y)$	volume (beam) attenuation function (m^{-1})	(65)
$a(y)$	volume absorption function (m^{-1})	(66)
$\overline{b}(y)$	volume mean backscatter function (m^{-1})	(67)
$D_{\pm}(y)$	distribution functions (dimensionless)	(68)

The *derived properties* obtained from these are:

$s(y)$	volume total scattering function ($s(y) = \alpha(y)-a(y)$)	(69)
$\overline{f}(y)$	volume mean forwardscatter function ($\overline{f}(y) = s(y)-\overline{b}(y)$)	(70)

$K_-(y)$ volume diffuse attenuation function

$$K_-(y) = D_-(y)[a(y) + \bar{b}(y)]) \; (\equiv K(y), \text{ where the minus subscript is dropped from } K_- \text{ when downward } (-) \text{ photon flow is understood}) \tag{71}$$

$R_-(y)$ irradiance reflectance ratio (dimensionless)

$$R_-(y) = D_-(y)\bar{b}(y)/[(D_+(y)+D_-(y))a(y)+D_-(y)\bar{b}(y)] \tag{72}$$

Three *dimensionless parameters* of a natural hydrosol are:

$$\omega(y) \equiv s(y)/\alpha(y) \qquad \text{(scattering-attenuation ratio)} \tag{73}$$

$$\beta(y) \equiv \bar{b}(y)/\alpha(y) \le \omega(y) \qquad \text{(backscatter ratio)} \tag{74}$$

$$\varepsilon(y) \equiv K_-(y)/\alpha(y) \qquad \text{(diffuse attenuation ratio)} \tag{75}$$

From $\omega(y)$,$\beta(y)$ and measurements of $\alpha(y)$ and $D_\pm(y)$, all of the above basic and derived quantities follow. Thus

$$a(y) = (1-\omega(y))\,\alpha(y) \tag{76}$$

$$\bar{b}(y) = \beta(y)\alpha(y) \tag{77}$$

$$s(y) = \omega(y)\alpha(y) \tag{78}$$

$$\bar{f}(y) = [\omega(y) - \beta(y)]\alpha(y) \tag{79}$$

$$K_-(y) = D_-(y)[1-\omega(y) + \beta(y)]\,\alpha(y) \; (\equiv K(y) \tag{80}$$

$$\varepsilon(y) = K_-(y)/\alpha(y) = D_-(y)[1-\omega(y) + \beta(y)] \tag{81}$$

$$R_-(y) \simeq [D_-(y)\beta(y)/[(D_+(y)+D_-(y))(1-\omega(y)) + D_-(y)\beta(y)] \tag{82}$$

We view D_+, D_-, ω and β as the basic dimensionless *dry parameters* of the optical medium. Add aqua (i.e., experimentally determined α) and one may completely reconstitute the medium for consumption, as shown above.

F. Toward Objective Optical Biology in Natural Hydrosols

It remains for optical biologists to measure and assemble the *specific* spectral counterparts to the basic optical properties. One does this by being guided by the *linear models* of these basic properties.* For example, one can adopt

$$\alpha(y,\lambda) = c_0(y)\,\alpha_0(\lambda) + c_1(y)\,\alpha_1(\lambda) + \cdots + c_\ell(y)\,\alpha_\ell(\lambda) = \sum_{j=0}^{\ell} c_j(y)\,\alpha_j(\lambda) \qquad (83)$$

and then characterize the medium optically by measuring, as described in the extended method D of sec. 8 above, the specific beam attenuation components $\alpha_j(\lambda)$ ($m^2 \cdot mg^{-1}$) of $\alpha(y,\lambda)$ and their concentrations $c_j(y)$ ($mg \cdot m^{-3}$).

The most complete optical characterization of a natural hydrosol in the present context requires the similar treatment of the models for volume absorption $a(y,\lambda)$ and volume backscatter $\overline{b}(y,\lambda)$:

$$a(y,\lambda) = \sum_{j=0}^{\ell} c_j(y)\,a_j(\lambda) \qquad (84)$$

$$\overline{b}(y,\lambda) = \sum_{j=0}^{\ell} c_j(y)\,\overline{b}_j(\lambda) \qquad (85)$$

in which the spectral behaviors $a_j(\lambda)$ and $\overline{b}_j(\lambda)$ are documented. This is because the most general optical activity of a medium involves both absorption and scattering mechanisms which are more or less independent.

* There may of course be alternate, independent ways of determining the specific optical properties and the associated concentrations. Here we remain with the linear model approach extended from Method D in sec. 8.

Thus a complete biological/optical characterization of a natural hydrosol would be obtained once and for all if the specific optical quantities $\alpha_j(\lambda)$, $a_j(\lambda)$, and $\bar{b}_j(\lambda)$, $j = 0,\ldots,\ell$ defined via (65), (66), (67) are determined over the spectrum for each set of molecular species which, at the current state of the art are conceptually and experimentally distinguishable. At the present time, the concensus appears to be that four such molecular species are of interest: water, soluble yellow substance, phytoplankton, and inanimate particulate matter. The two main tasks of biologic optics can then be addressed. First: When the concentrations $c_j(y)$ are determined, $j = 0,\ldots,\ell$, (at present, $\ell = 4$) the light field in the hydrosol is computable and chlorophyll growth can be predicted using chlorophyll/light-field dynamical models (cf. H.O., Vol. I, sec. 1.10). Second: After the aquatic biologist optically surveys the hydrosol and measures $\alpha(y,\lambda)$ and $a(y,\lambda)$, at various locations y and wavelengths λ, the concentrations c_j of the (at present four) molecular species can be estimated using the extended method D in sec. 8.

10. QUO VADIS, EYEBALL OPTICS?

Early one day, after writing the last section, I had the following dream just before awakening.

It was a fine, misty morning in late April 2065, and I was climbing a hiking trail that precipitously wound through the foothills high above Civitavecchia toward eutrophying Lake Bracciano. I could look down on a great cloud bank below me. The Tyrrhenian Sea lay calm beneath the morning clouds. I had on a pack laden with a sleeping bag, victuals, and a newly designed remotely operated irradiance-quartet kit which contained a folded-path 40-wavelength transmissometer monitored by real-time data processor chips. I was going to combine a hiking holiday with an optical survey of Lake Limne which lay astride a small tributary of Lake Bracciano. As I rounded a corner of the trail, there before me were two figures descending along the path. The sun behind them gave their mist-enshrouded outlines a brilliant aureole, but I thought I saw the familiar outlines of Professor Secchi clothed in the flowing robes of a Jesuit priest. He was supporting a frail and obviously ancient apparition consisting of what seemed like a head, spindly arms, and some legs intertwined with seaweed. The creature was heavily burdened with chains, bagsful of bloodshot eyeballs, and moldy disks of various colors. As the two figures passed, I ventured a quiet query of the stouter of the two figures, "Whither goest thou, Father Angelo?" "Down to the sea, there to send this Ancient One to its final rest." "And where would that be?" I asked. "In the aquamarine depths four miles off the coast of Civitavecchia, whence it first saw the light of day." While Secchi was answering, the Ancient One had slowly ambled on. For a few precious seconds, I showed the fascinated Shade the optical devices in my pack, and explained what they could do. He was amazed at the great advances that were made beyond the spectrographic tools he used when classifying stars by their spectra. Then our conversation was interrupted by the high-pitched excited voice of a young man who had materialized out of nowhere; his features and mannerisms were hauntingly familiar. The young fellow had the Ancient One by one of the creature's moldy white disks and was moving it rapidly down the trail amidst a rattling of chains. As they passed out of sight around a bend I heard the excited voice say, "I have a wonderful idea I want to tell you about." I heard the quiet voice of the Ancient One form a demur, but the excited, high-pitched voice persisted: "I think we can measure the transpectral volume scattering function using a combination of your disks. If we just take two of them and... and then turn them so... and put on a pair of filters here, and blacken this disk there and..." At this point, the voice faded into the distance. Secchi's Shade and I, looking in amazement at each other, stood transfixed as we tried to absorb the babblings of the young one's voice. Secchi shrugged and raised his palms in that slow southern Italian way and, smiling, placed a hand on my shoulder. "If only Cialdi and I had those optical marvels in your pack on that first day aboard *l'Immacolata Concezione*." Then he turned and scampered down the trail to catch up with the others. As the Shade neared the bend, he hesitated and, with a reassuring wink, passed from view into the mists.

11. BIOGRAPHICAL NOTE

Angelo Secchi was born in Reggio, Italy on 18 June 1818. From early youth he trained to become a Jesuit priest. He went on to become a lecturer in mathematics and physics at Collegio Romano in 1839. In 1848 Père Angelo Secchi was driven into exile for being a Jesuit. He went briefly to England and then for a year's stay at Georgetown University in Washington D.C. It was during his stay in America that his interest in solar physics was sparked. In 1850, Father Secchi was appointed Professor of Astronomy and director of the Gregonia University Observatory at his alma mater Collegio Romano. The set-up there was a researcher's paradise: Secchi had unlimited papal funds to build up the equipment at the Observatory, to staff it with assistants, and to publish his work. For 28 years he contributed prodigiously to the fields of terrestrial magnetism, meteorology and astronomy. With William Huggins, he was the first man to adapt spectroscopy to astronomy and he made the first spectroscopic survey of the stars in the sky. Secchi initiated the modern system of classification of stars by their spectra, based on his detailed study of 4000 stars. He summarized this in his great final work, *Le Soleil* (1875, 1877). Father Angelo Secchi died in Rome on 26 February 1878.

It is fitting, in the context of this review, that the main effort of Secchi's life was devoted to the study of the sun, the giver of light and life on earth. His work is inspiring in that he classified the sun and other stars by the spectral distribution of the photons they emit and scatter toward their worlds. In hydrologic optics we complete this link by classifying the life-harboring, light-suffused hydrosols of this planet by the spectral distributions of the solar photons they scatter and absorb.

It is ironic that, in the latest available English summaries of Secchi's life, there is no mention of his adventure with the white disk aboard *l'Immacolata Concezione* in the spring of 1865 (cf. Abbott, 1984; Williams, 1982). For some historical notes on experiments with the white disk, see Sauberer and Ruttner (1942). It seems that a very early white disk experiment was done in the Pacific by the Russian Kotzebue, in 1815. Quite a few Frenchmen, between 1815 and 1865, also had their turn at disk experiments.

12. ACKNOWLEDGMENTS

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