Calculation of Lift

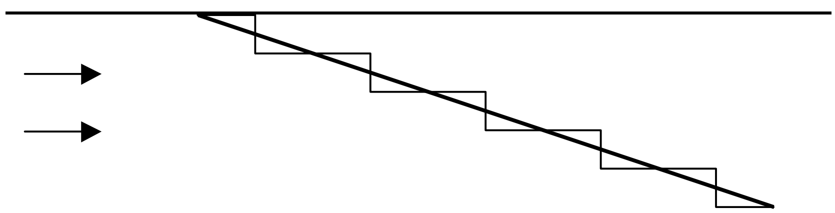
The lift force can be calculated by determining the pressure difference between the top surface and the bottom surface of the ramp boom.

It is convenient to use the dimensionless pressure, which can be obtained by choosing the density of water r_{water} , the upstream velocity U as the dimensional independent set.

$$p = \frac{\tilde{p}}{\frac{1}{2} r_{water} U^2}$$

where p is the dimensionless pressure, and $\tilde{p}$ is the dimension pressure.

We can use the stages to simulate the ramp boom, and calculate the dimensionless pressure of the ramp boom top surface p_1 and that of the bottom surface p_2 of every stage.



When the upstream velocity U =3 knots=5ft/sec, and the length of ramp boom L=6ft, and the angle is 15 degree, we apply 27 stages to simulate the ramp boom, and p_1 and p_2 can be calculated by CFD method. Below is the results of every stage's p_1 and p_2:

p_1: 0.3132, 0.3131, 0.3131, 0.3131, 0.3130, 0.3130, 0.3128, 0.3128, 0.3127, 0.3126, 0.3124, 0.3121, 0.3119, 0.3116, 0.3114, 0.3113, 0.3113, 0.3112, 0.3112, 0.3110, 0.3108, 0.3108, 0.3104, 0.3103, 0.3101, 0.3101, 0.3096

p_2: 1.2286, 1.0425, 1.0654, 1.0809, 1.1325, 1.0385, 1.1050, 1.1034, 1.0414, 0.9030, 1.0515, 1.0132, 0.9242, 0.9661, 0.9864, 0.9292, 0.8122, 0.8416, 0.8117, 0.7903, 0.9016, 0.8600, 0.7890, 0.6576, 0.7942, 0.7500, 0.6700

The average p_1=0.3117

The average p_2=0.9329

Then the average dimensionless pressure difference

$\Delta p = p_2 - p_1$=0.9329-0.3117 =0.6213

So, the lift force

$$F_c = \Delta\tilde{p} \cdot A = \frac{1}{2} r_{water} U^2 \Delta p A$$

Its vertical component $F_{cv} = F_c \cos 20°$

Calculations for Ramp Boom Inclination Angle of 15^o

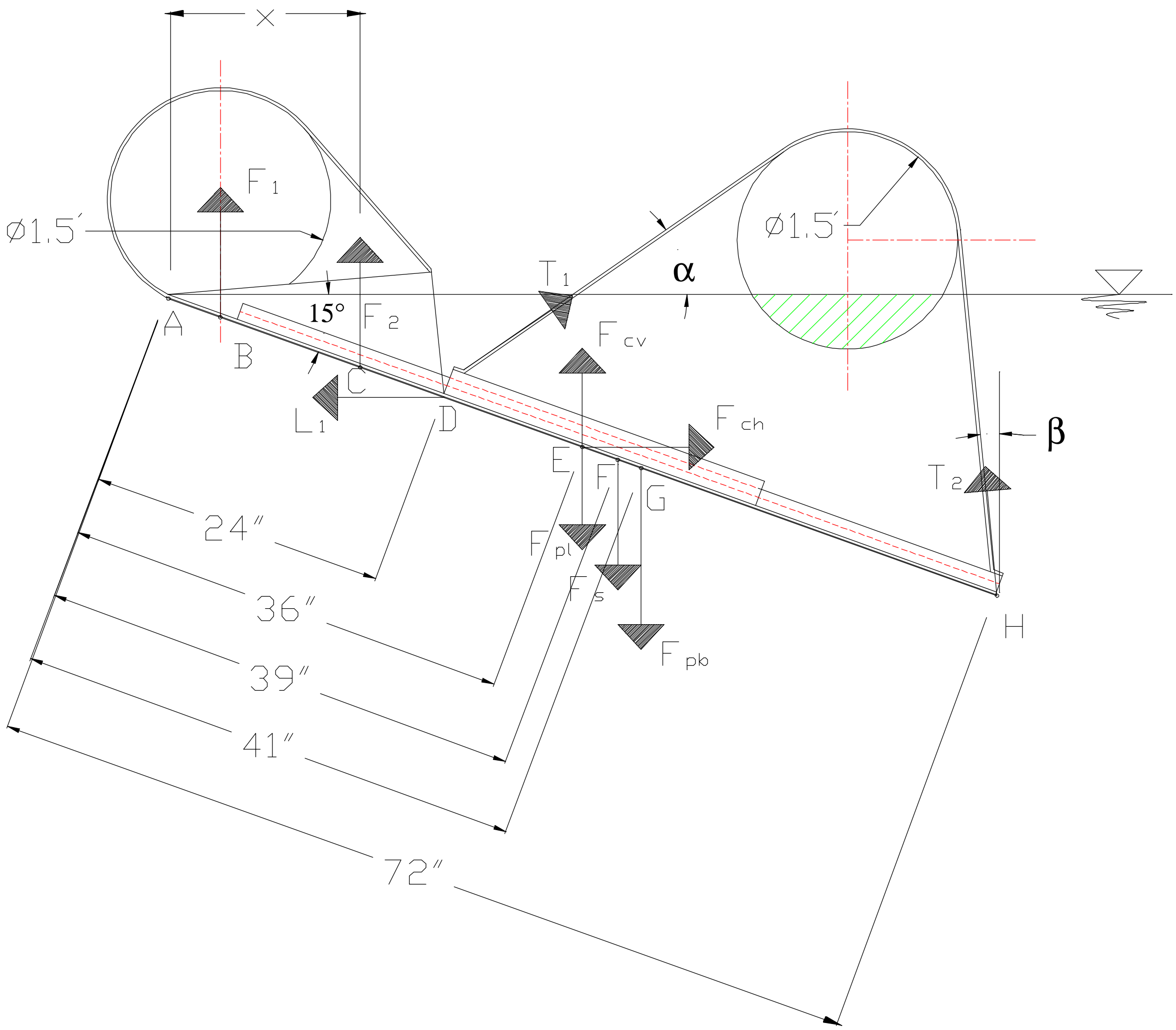


Figure 1

In figure 1, x stands for the horizontal distance between the centroid of syntactic foam block and the forward-most point of the ramp boom.

Known parameters from the original design are;

F_{pb} = $130.6^{\#}$

F_{pl} = $1.5^{\#}$

F_s = $3.4^{\#}$

F_1 = 0

Calculation of F_{cv}

The equation for the normal lift on a fixed inclined plate submerged in a fluid, moving with a velocity of 3 knots is :

$F_c = (1/2).\rho.\ V^2\ .\ C_L\ .\ A$ where $\rho = (64^{\#}/ft^3) / (32.2 ft/sec^2) = 1.99$ slugs/ft

$V = 3$ kts $= 5$ ft/sec

$C_L = 0.62$ for a 15^0 angle

$A = 1' \times 6' = 6\ ft^2$

$F_c = (1/2)[(64^{\#}/ft^3)/(32.2ft/sec^2)](5ft/sec)^2(0.62)(6ft^2)$

$F_c = 92.42^{\#}$ Normal to the plane of the Ramp Boom

Breaking this normal force into its vertical and horizontal components for a unit Ramp Boom width of 1 ft yields :

$F_{cv} = 92.42^{\#} (\cos 15^0) = 89.27^{\#}$ = Vertical lift component

$F_{ch} = 92.42^{\#} (\sin 15^0) = 23.92^{\#}$ = Horizontal lift component

Summing the horizontal forces from the Ramp Boom free-body diagram;

L_1 = Tensile force required to restrain the boom / unit length

$L_1 = F_{ch} = 23.92^{\#}/ft$

Therefore to restrain a 16′-6” wide prototype section of the Ramp Boom at a 3 kts current ;

Force Required = $(L_1)(16.5ft) = 394.68^{\#}$

Calculation of F_2

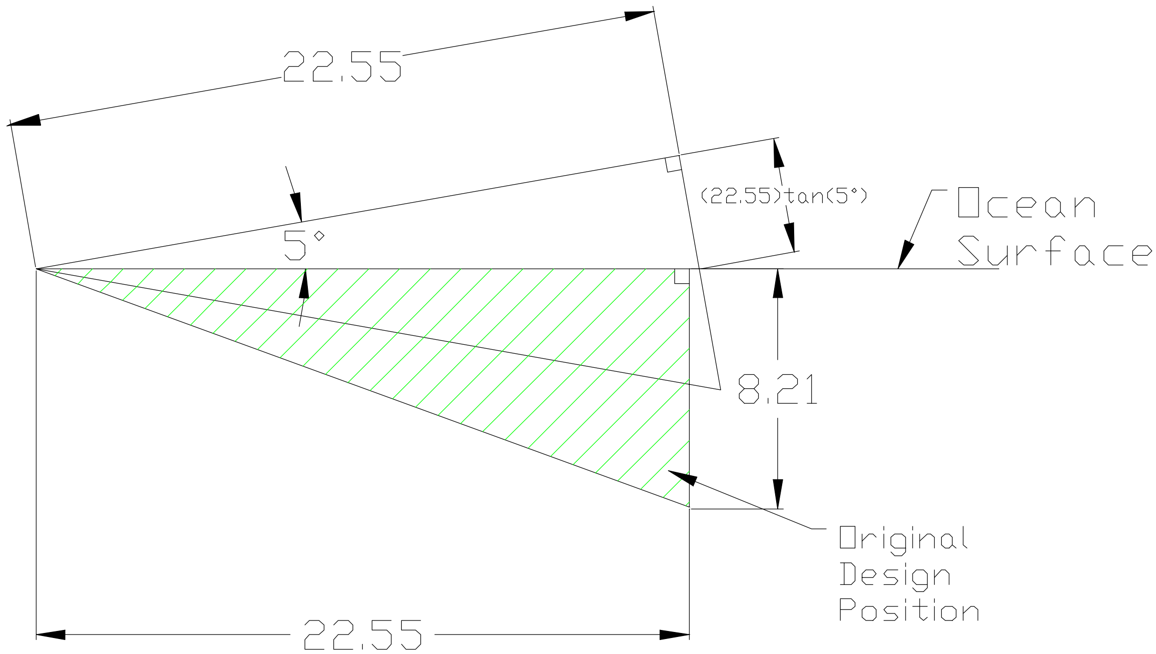


Figure 2

Referring to figure 2 ;

Total foam block area = (1/2)(22.55")(8.21") = 92.57 in^2

Total submerged area with 15^o inclination =92.57 – (1/2)$(22.55")^2[\tan(5^o)]$
=70.356 in^2

$$F_2 = (64 - 30)^{\#} / ft^3 \left[(70.356 in^2)(1 ft)\left(\frac{1 ft^2}{144 in^2} \right) \right]$$

$$F_2 = 16.61^{\#}$$

Averaging this buoyant force over a 42" overall width to account a 30" wide block section and a 12" wide free - flooded section ;

$$F_2 = \frac{(16.61^{\#})(30")}{42"} = 11.86^{\#}$$

Note : The centroid of the foam block on the horizontal will change only 0.2" due to the inclination difference and this difference will be neglected.

Summation of the vertical forces on the Ramp Boom ;

$F_b = 130.6 + 3.4 + 1.5 - 89.27 - 11.86 = 34.37^{\#}$
where F_b = Buoyant force of aft pontoon that is : $F_b = T_1(\sin\alpha) + T_2(\cos\beta)$

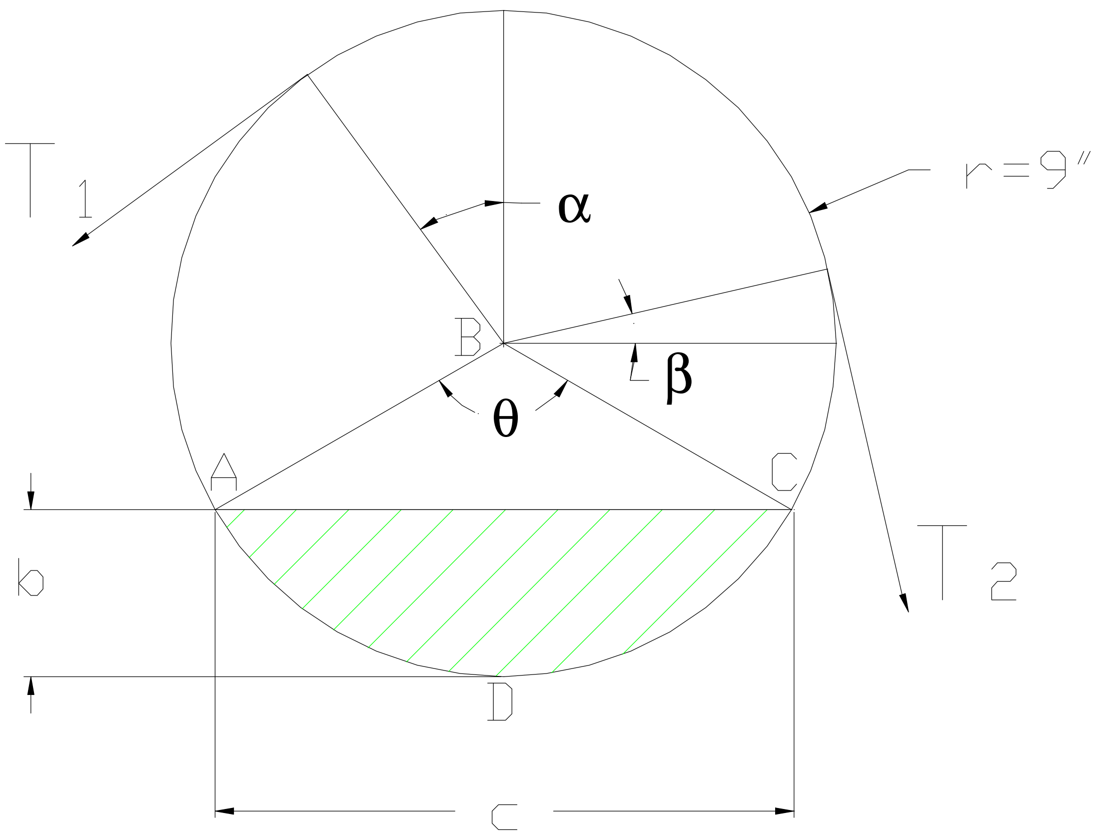


Figure 3

The depth of submersion is calculated as follows;

A = Area of segment ADC

$$A=\frac{34.37^{\#}}{(1\,ft)\left(\frac{1\,ft^{2}}{144in^{2}}\right)(64^{\#}/ft^{3})}=77.33in^{2}$$

$$\text{Area of sector ABCD} = \text{A} + \text{Area of } \Delta\text{ABC}$$

$$=77.33+\frac{1}{2}c(9-b)=p\,(9)^{2}\left(\frac{q}{360}\right)$$

$$77.33+\frac{1}{2}c.(9-b)=81p\left(\frac{q}{360}\right) \qquad \text{(Eqn. 1)}$$

$$c=(2)(9)\sin\left(\frac{q}{2}\right) \qquad \text{(Eqn. 2)}$$

$$c=2\sqrt{(2)(9)b-b^{2}} \qquad \text{(Eqn. 3)}$$

$$\text{Rewriting Eqn. 2: } q=2\left[\sin^{-1}\left(\frac{c}{18}\right)\right] \qquad \text{(Eqn. 4)}$$

Substitute Eqn. 3 into Eqn. 4;

$$q=2\left[\sin^{-1}\left(\frac{2\sqrt{18b-b^{2}}}{18}\right)\right] \qquad \text{(Eqn. 5)}$$

Substitute Eqns. 3 & 5 into Eqn. 1;

$$\text{f(b)}=77.33+\left\{\frac{1}{2}\left[2\sqrt{18b-b^{2}}\right](9-b)\right\}-\frac{\left\langle(81p)\left\{2\left[\sin^{-1}\left(\frac{2\sqrt{18b-b^{2}}}{18}\right)\right]\right\}\right\rangle}{360}=0$$

Using simple one point iteration to find b;

b=4.5” f(b)= +27.58

b=7.0” f(b)= -14.2

b=6.18” f(b)= 0

Therefore

b=6.18”

c=17.09” (from Eqn. 3)

θ=143.5 deg. (from Eqn. 2)

Equations to solve are;

From geometry of the problem;

$$\frac{2.82+24\sin 15+9\cos a}{\tan a}+(2.82+72\sin 15+9\sin b)\tan b+9(\sin a+\cos b)$$
$$-48\cos 15=0$$

From equilibrium of the aft pontoon;

$$T_1\sin a+T_2\cos b=34.37^{\#}$$
$$T_1\cos a-T_2\sin b=0$$

From moment equilibrium of Ramp Boom (refer to figure 1);

$$f=T_1\sin a(24\cos 15)+T_1\cos a(24\sin 15)+T_2\cos b(72\cos 15)-T_2\sin b(72\sin 15)$$
$$-1878.75^{\#}in$$

Choose β = 6^o ; α = 28.84^o T_1 = 3.9$^{\#}$ T_2 = 32.67$^{\#}$ f = +382.09
Choose β = 10^o; α = 30.17^o T_1 = 6.36$^{\#}$ T_2 = 31.65$^{\#}$ f = +294.8
Choose β = 20^o; α = 34.55^o T_1 = 12.14$^{\#}$ T_2 = 29.25$^{\#}$ f = +68.10
Choose β = 24^o ; α = 36.97^o T_1 = 14.35$^{\#}$ T_2 = 28.18$^{\#}$ f = -30.67
Interpolating for the next iteration ;

$$b_{next}=24-\left[\frac{30.67}{(30.67+68.10)}(24-20)\right]=22.76^{o}$$

Therefore ;

Choose β = 22.76^o ; α = 36.18^o T_1 = 13.67$^{\#}$ T_2 = 28.52$^{\#}$ f = 0

Calculating the total rope length ;

$$x_1=\text{Length of slack side}=\frac{(24\sin 15)+(2.82)+(9\cos 36.18)}{\sin 36.18}=27.61''$$

$$x_2=\text{Length of tight side}=\frac{(24\sin 15)+(2.82)+(9\sin 22.76)}{\cos 22.76}+\frac{48\sin 15}{\cos 22.76}=27''$$

$$x_3=\text{Length of middle round section}=\frac{18p(36.18+90-22.76)}{360}=16.25''$$

$$\text{Total rope length}=x_1+x_2+x_3=70.9''$$

Therefore the total rope length is reduced by 12.1" when compared with the original design.

Modifications on the test setup :

1. The high tension (tight) side rope length will be shortened by 3.2". In the original design this length was 30.2".
2. The low tension (slack) side rope length will be shortened by 6.4". In the original design this length was 34".

Arrangement with Regard to Strap Lengths

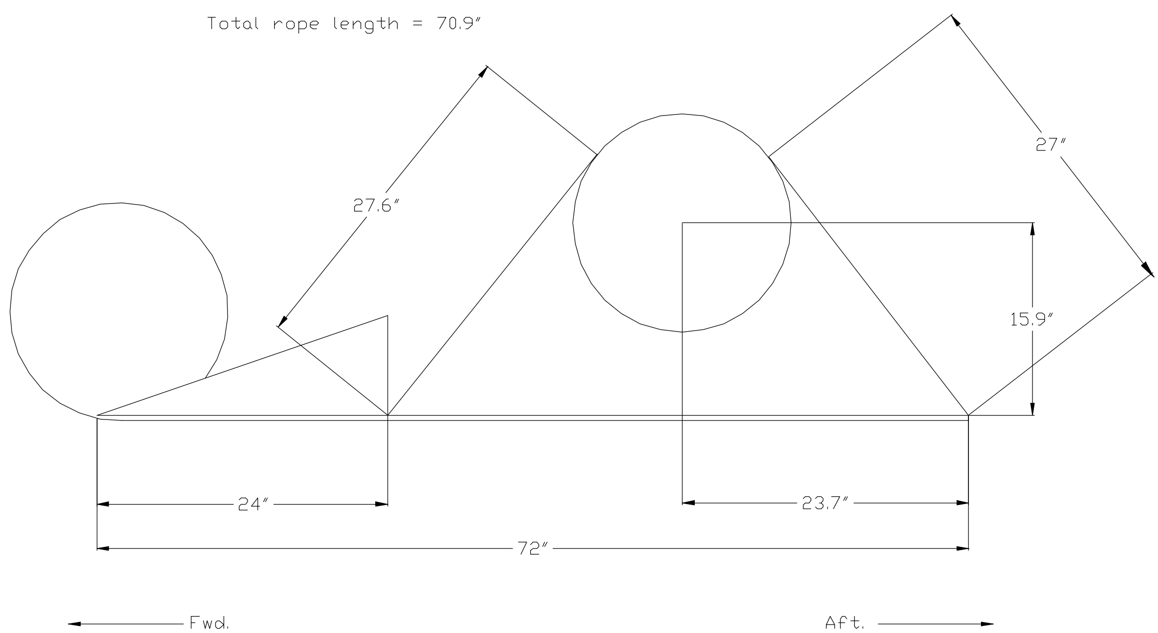


Figure 4

Note : An additional 56.55" of strap is necessary for each additional wrap.

Static Condition

The lift force (F_c) and the tensile force to restrain the Ramp Boom (L_1) no longer exist in the static condition (refer to fig. 1);

Total downward forces = $F_{pl} + F_s + F_{pb} = 1.5 + 3.4 + 130.6 = 135.5^{\#}$
Total buoyancy of one pontoon;

$$\text{Total buoyancy} = p\,(9'')^2 (1\,ft)\left(\frac{1\,ft}{144 in^2}\right)\left(64^{\#}/ft^3\right) = 113.1^{\#}$$

Assuming total submersion of foam block which provides $15.7^{\#}$ upward buoyancy;
Net downward forces = $135.5 - 15.7 = 119.8^{\#}$

It can be seen that total submersion of both aft pontoon and foam block is not enough to balance the weights of lead ballast, ramp boom sheet and aluminum stiffeners, therefore the forward pontoon should also be submerged. The static equilibrium position is given in figure 5 below;

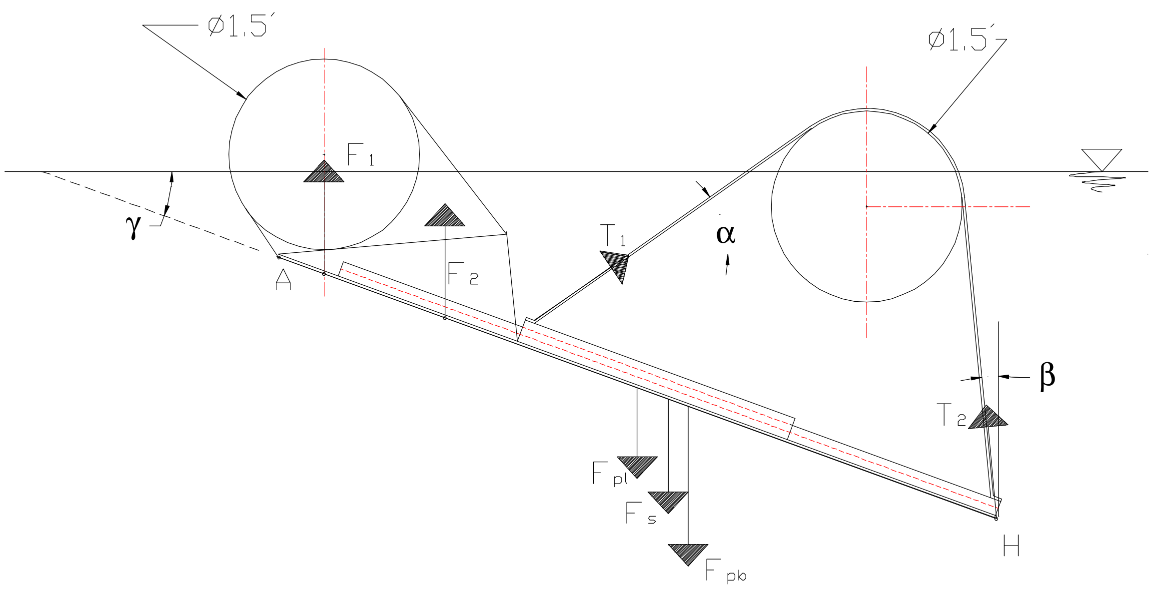


Figure 5

Since the straps are taped on the aft pontoon, the pontoon + straps are constrained to move as a rigid body, therefore we can write;

$\gamma \quad = \Delta + 15$

$\alpha \quad = 36.18 - \Delta$

$\beta \quad = 22.76 - \Delta$

where Δ is the change in the inclination angle of ramp boom (+, clockwise), 15, 36.18, 22.76 are γ, α, β values corresponding to equilibrium condition with 3 kts current, respectively.

The equations to solve are;

$F_2 = 15.7^{\#}$

$F_b = T_1\sin\alpha + T_2\cos\beta$ (Buoyancy of aft pontoon)

$F_1 + F_b = 119.8^{\#}$

$T_2\sin\beta = T_1\cos\alpha$

$T_1\sin\alpha(24\cos\gamma) + T_1\cos\alpha(24\sin\gamma) + T_2\cos\beta(72\cos\gamma) - T_2\sin\beta(72\sin\gamma) +$
$F_1(4.8\cos\gamma) = 5278.853 \cos\gamma$

Also from geometry of the equilibrium position;

$b_1 + 24\sin\gamma - 27.61\sin\alpha + 9\cos\alpha = b_2 - 9$

where b_1 and b_2 are depths of submersion of forward and after pontoons respectively. (27.61" is the length of the slack side strap which is calculated at page 7).

$$F_b = \frac{64}{144}\left| \left\langle \frac{(81p)\left\{2\left[\sin^{-1}\left(\frac{2\sqrt{18b_2 - b_2^{\,2}}}{18}\right)\right]\right\}}{360} \right\rangle - \frac{1}{2}\left[2\sqrt{18b_2 - b_2^{\,2}}\right](9 - b_2) \right|$$

$$F_1 = \frac{64}{144}\left| \left\langle \frac{(81p)\left\{2\left[\sin^{-1}\left(\frac{2\sqrt{18b_1 - b_1^{\,2}}}{18}\right)\right]\right\}}{360} \right\rangle - \frac{1}{2}\left[2\sqrt{18b_1 - b_1^{\,2}}\right](9 - b_1) \right|$$

Iterating the above equations by an initial selection of γ ;

$\gamma = 17^o$ $F_1 = 30.96^\#$ $F_b = 88.84^\#$ $b_1 = 5.73''$ $b_2 = 13.19''$
$\rightarrow$ This iteration does not satisfy the geometry, it is required to $b_1\downarrow$, $b_2\uparrow$.

$\gamma = 15.2^o$ $F_1 = 29.19^\#$ $F_b = 90.61^\#$ $b_1 = 5.49''$ $b_2 = 13.45''$
$\rightarrow$ This iteration does not satisfy the geometry, it is required to $b_1\uparrow$, $b_2\downarrow$.

$\gamma = 16.4^o$ $F_1 = 30.38^\#$ $F_b = 89.42^\#$ $b_1 = 5.65''$ $b_2 = 13.28''$
$\rightarrow$ This iteration satisfies the geometry!

Therefore the system will be in static equilibrium consistent with the above particulars. The aft pontoon will be submerged such that 74% of its diameter will be in water. The forward pontoon will be submerged such that 31% of its diameter will be in water. Note that these are the submersion depths for static condition. When the ramp boom is subject to a 3 kts current, the fwd pontoon will be completely out of water, whereas the aft pontoon will be submerged such that 34% of its diameter will be in water.