Calculations for the Current Equilibrium Position

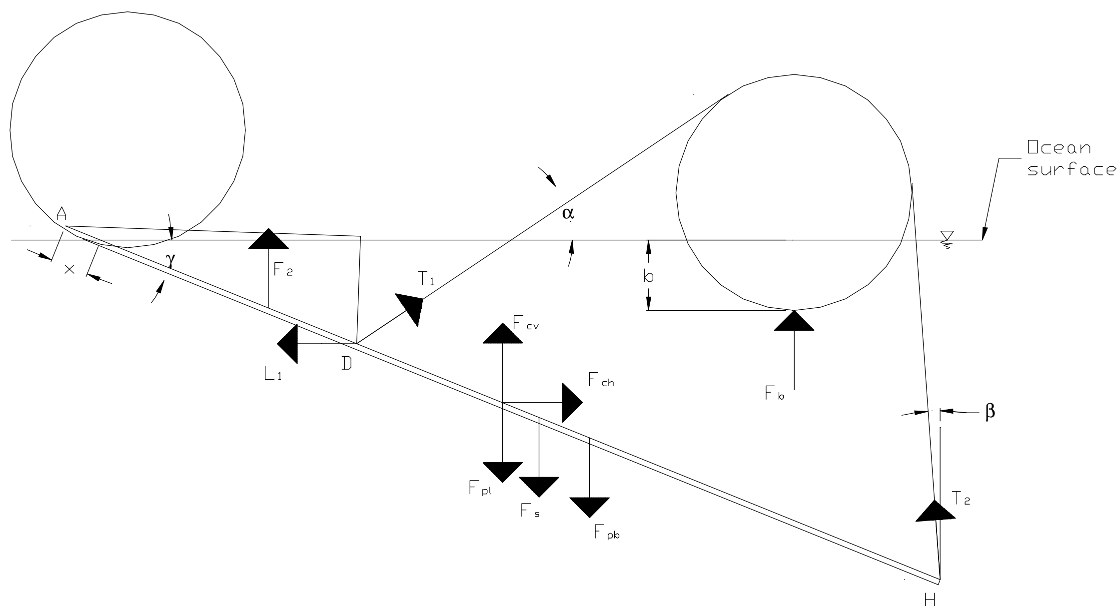

Figure 1

The towing point is changed from point A to point D, this change introduces an unbalanced moment on the Ramp Boom. It can easily be seen that the system can only balance this moment with an increase in the aft pontoon buoyancy. However, since the unbalanced moment is due to a horizontal force L_1, this additional buoyancy will be realized by a decrease in the buoyancy of the foam block, in order to satisfy the vertical force equilibrium. The system's anticipated equilibrium position is given in fig. 1.

In the figure;

F_2 = buoyant force of foam block in the new equilibrium position.
F_b = buoyant force of aft pontoon in the new equilibrium position.
x = the non-submerged portion of the foam block.
γ = the inclination angle of the Ramp Boom wrt ocean surface.

Known parameters;

F_{cv}	$= 98^{\#}$	(Vertical lift component)
F_{ch}	$= 35.7^{\#}$	(Horizontal lift component)
L_1	$= 35.7^{\#}$	(Tensile force to restrain Ramp Boom)
F_{pl}	$= 1.5^{\#}$	(In-water weight of Ramp Boom sheet)
F_{pb}	$= 130.6^{\#}$	(In-water weight of lead ballast)
F_s	$= 3.4^{\#}$	(In-water weight of aluminum stiffeners)
$F_{2,old}$	$= 15.7^{\#}$	(Buoyancy of foam block in original equil. position)
$F_{b,old}$	$= 21.8^{\#}$	(Buoyancy of aft pontoon in original equil. position)

Since the vertical force equilibrium has to be satisfied in the new equilibrium position;

$F_{2,old} + F_{b,old} = F_2 + F_b$

$F_2 + F_b = 15.7 + 21.8 = 37.5^{\#}$

From geometry of the foam block;

By approximating the area of submerged portion of the foam block, below equation is obtained (good for small change of inclination angle)

$$F_2 = (1.006)(64-30)^{\#} / ft^3 \left(\frac{1}{2}\right)(24-x)^2 \sin\gamma \cos\gamma \left(\frac{30''(1\,ft)}{(144 in^2/ft^2)(42'')}\right)$$

The rope lengths are constant;

$$34'' = \frac{(24-x)\sin\gamma + (9-b) + 9\cos\alpha}{\sin\alpha}$$

$$30.2'' = \frac{(72-x)\sin\gamma + (9-b) + 9\sin\beta}{\cos\beta}$$

From equilibrium of aft pontoon;

$F_b = T_2\cos\beta + T_1\sin\alpha$

$T_1\cos\alpha = T_2\sin\beta$

Relation of aft pontoon buoyancy and depth of submersion (b);

$$F_b = \frac{64}{144}\left|\left\langle \frac{(81\pi)\left\{2\left[\sin^{-1}\left(\frac{2\sqrt{18b-b^2}}{18}\right)\right]\right\}}{360}\right\rangle - \frac{1}{2}\left[2\sqrt{18b-b^2}\right](9-b)\right|$$

From moment equilibrium of Ramp Boom around point A (fig. 1);

$$f = \tan g - \frac{\left(2013.2 - 72T_2 \cos b - 24T_1 \sin a - 16.71F_2\right)}{\left(428.4 + 24T_1 \cos a - 72T_2 \sin b\right)}$$

The equations above cannot be solved simultaneously, therefore noting that;

$\gamma = 20^o + \Delta$

$\beta = 6.1^o - \Delta$

$\alpha = 36.1^o - \Delta$

(Δ is the change in angle of inclination of Ramp boom from 20^o, it is positive in clockwise direction. 20, 36.1 and 6.1 are the γ, α, β of the original design respectively. The above three equations are valid since due to taped straps the system will behave as a rigid body.)

Assign $\gamma = 22^o$;

$\alpha = 34.1$ $\beta = 4.1$

Choose b = 5” , the following can be obtained corresponding to b = 5”,

x = 3.94” $F_2 = 11.86^\#$ $F_b = 25.64^\#$ $T_1 = 2.117^\#$ $T_2 = 24.52^\#$ f = +113

Choose b = 4.7” , the following can be obtained corresponding to b = 4.7”,

x = 2.21” $F_2 = 13.99^\#$ $F_b = 23.51^\#$ $T_1 = 1.941^\#$ $T_2 = 22.48^\#$ $f \cong 0$

Therefore the system will be in equilibrium consistent with the above particulars. The anticipated equilibrium condition may differ from the calculated one due to the initial assumption of constant lift. The actual procedure is iteration of the results with changing inclination angle. However that approach requires a large amount of effort. The accuracy in the current technique is well enough to predict the equilibrium position of the system.

Static Condition of Current Setup

The lift force (F_c) and the tensile force to restrain the Ramp Boom (L_1) no longer exist in the static condition (refer to fig. 1);

Total downward forces = $F_{pl} + F_s + F_{pb} = 1.5 + 3.4 + 130.6 = 135.5^{\#}$
Total buoyancy of one pontoon;

$$\text{Total buoyancy} = p\,(9")^2(1\,ft)\left(\frac{1\,ft}{144\,in^2}\right)\left(64^{\#}/\,ft^3\right) = 113.1^{\#}$$

Assuming total submersion of foam block which provides $15.7^{\#}$ upward buoyancy;
Net downward forces = $135.5 - 15.7 = 119.8^{\#}$

It can be seen that total submersion of both aft pontoon and foam block is not enough to balance the weights of lead ballast, ramp boom sheet and aluminum stiffeners, therefore the forward pontoon should also be submerged. The static equilibrium position is given in figure 1 below;

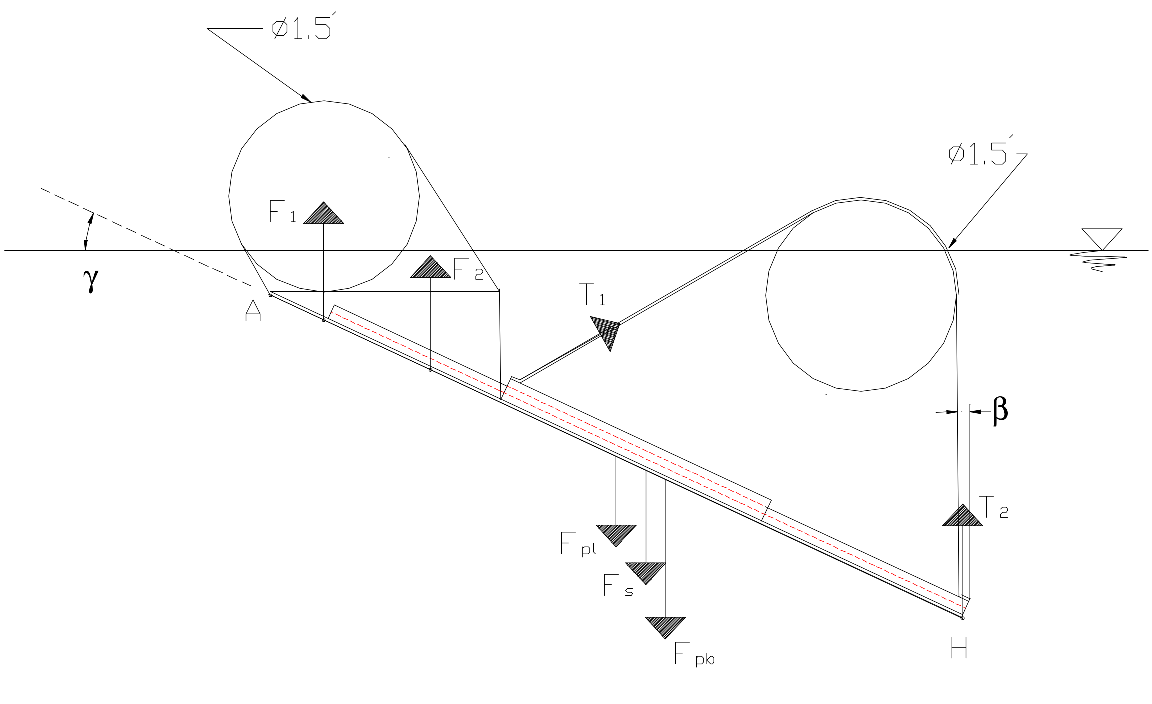

Figure 1

Since the straps are taped on the aft pontoon, the pontoon + straps are constrained to move as a rigid body, therefore we can write;

$\gamma \quad = \Delta + 20$

$\alpha \quad = 36.1 - \Delta$

$\beta \quad = 6.1 - \Delta$

where Δ is the change in the inclination angle of ramp boom (+, clockwise), 20, 36.1, 6.1 are γ, α, β values corresponding to equilibrium condition with 3 kts current, respectively.

The equations to solve are;

$F_2 = 15.7^{\#}$

$F_b = T_1\sin\alpha + T_2\cos\beta$ (Buoyancy of aft pontoon)

$F_1 + F_b = 119.8^{\#}$

$T_2\sin\beta = T_1\cos\alpha$

$T_1\sin\alpha(24\cos\gamma) + T_1\cos\alpha(24\sin\gamma) + T_2\cos\beta(72\cos\gamma) - T_2\sin\beta(72\sin\gamma) + F_1(4.8\cos\gamma) = 5278.853\cos\gamma$

Also from geometry of the equilibrium position;

$b_1 + 24\sin\gamma - 34\sin\alpha - 4.5\sin\Delta + 9\cos\alpha = b_2 - 9$

where b_1 and b_2 are depths of submersion of forward and after pontoons respectively. (34" is the length of the slack side strap).

$$F_b = \frac{64}{144}\left|\left\langle \frac{(81p)\left\{2\left[\sin^{-1}\left(\frac{2\sqrt{18b_2 - b_2^{\,2}}}{18}\right)\right]\right\}}{360} \right\rangle - \frac{1}{2}\left[2\sqrt{18b_2 - b_2^{\,2}}\right](9 - b_2)\right|$$

$$F_1 = \frac{64}{144}\left|\left\langle \frac{(81p)\left\{2\left[\sin^{-1}\left(\frac{2\sqrt{18b_1 - b_1^{\,2}}}{18}\right)\right]\right\}}{360} \right\rangle - \frac{1}{2}\left[2\sqrt{18b_1 - b_1^{\,2}}\right](9 - b_1)\right|$$

Iterating the above equations by an initial selection of γ ;

$\gamma = 24^o$ $F_1 = 47.83^\#$ $F_b = 71.97^\#$ $b_1 = 7.91''$ $b_2 = 10.94''$
$\rightarrow$ This iteration does not satisfy the geometry, it is required to $b_1\downarrow$, $b_2\uparrow$.

$\gamma = 22^o$ $F_1 = 45.90^\#$ $F_b = 73.90^\#$ $b_1 = 7.67''$ $b_2 = 11.20''$
$\rightarrow$ This iteration does not satisfy the geometry, it is required to $b_1\uparrow$, $b_2\downarrow$.

$\gamma = 20^o$ $F_1 = 43.93^\#$ $F_b = 75.87^\#$ $b_1 = 7.42''$ $b_2 = 11.45''$
$\rightarrow$ This iteration satisfies the geometry!

The aft pontoon will be submerged such that 64% of its diameter will be in water. The forward pontoon will be submerged such that 41% of its diameter will be in water. Note that these are the submersion depths for static condition.