TERM STRUCTURE AND PROJECTED INTEREST RATES

Section 8.A - Treasury, LIBOR, and FHLB Advance Term Structures and Implied Forward Rates

The NPV Model uses several interest rates to discount cash flows or to estimate future levels of interest cash flows to be paid or received. Term structures (i.e., spot yield curves) and implied forward interest rates are calculated for three types of rates: Treasury, LIBOR, and FHLB Advances. These calculations are based on the assumptions that the term structures are arbitrage-free and that expected future interest rates equal implied forward rates.

The calculations for each of the three rates are performed in five steps. First, par (constant maturity) yields are estimated for maturities ranging from 1 to 360 months based on end-of-quarter market quotes. Second, spot (zero-coupon) rates are calculated from the par yields calculated in the first step. Third, spot rates for the eight alternate interest rate scenarios are calculated based on the spot rates from the second step and the interest rate shock appropriate to each scenario. Fourth, 1-month implied forward rates are calculated for each type of rate and interest rate scenario. Fifth, par yields are calculated for the alternate interest rate scenarios from the spot rates produced in the third step.

Step 1: Estimate Par Yields for Base-Case Scenario

A par bond is one whose market price is par (i.e., its fair market value equals its face value). The yield of a par bond is equal to the bond s coupon and is known as the par yield for that particular maturity. Par bonds do not exist for every possible maturity, indeed at any given time only a small minority of bonds are priced at par. To construct a yield curve that contains par yields for all maturities, therefore, par yields must be estimated for most maturities. To estimate the par yield curve for the base case scenario, quotes are obtained for several maturities of par bonds at the quarter-end. Par yields for those maturities for which market quotes are not available are estimated using a statistical technique known as splining. This estimation method is similar to drawing a continuous line, with no kinks, through a set of points that exhibit some pattern.[1] Quotes for Treasury par yields are obtained from Federal Reserve Statistical Release G.13; LIBOR quotes are obtained from Bloomberg, L.P.; and FHLB Advance rates are averages of the rates at the 12 Federal Home Loan Banks, published weekly by the Federal Housing Finance Board.

Step 2: Derive Spot Rates for Base-Case Scenario

The spot rate curve, also known as the zero-coupon curve and the term structure of interest rates, is derived next using the par yields estimated in step 1.[2] The equation below shows how the 12-month spot rate (s_{12}) can be derived from the 6-month (p_6) and 12-month (p_{12}) par yields:

$$1 = \frac{p_{12}}{(1+p_6)} + \frac{1+p_{12}}{(1+s_{12})^2}$$

Next, the 18-month spot rate (s_{18}) is calculated based on the 6-month par rate (p_6), the 12-month spot rate (s_{12}), and the 18-month par quote (p_{18}), using the equation below.

1 The particular technique that is used is know as the Variable Knot B-Spline Least Square Approximation method. For details on this estimation method, see Carl de Boor, *A Practical Guide to Splines,* Springer-Verlag, New York, 1978.

2 Details of the derivation of spot rates from par rates are well known. See, for example, Steven L. Allen and Arnold D. Kleinstein, *Valuing Fixed-Income Investments & Derivative Securities,* New York Institute of Finance, New York, 1993; and Frank J. Fabozzi and T. Dessa Fabozzi, eds., *The Handbook of Fixed Income Securities,* Irwin, New York, 1995.

$$1 = \frac{p_{18}}{\left(1+p_6\right)} + \frac{p_{18}}{\left(1+s_{12}\right)^2} + \frac{1+p_{18}}{\left(1+s_{18}\right)^3}$$

This bootstrap derivation of the spot rates continues until all the desired maturities for the spot rate are obtained.[3]

Step 3: Calculate Spot Rates for Alternate Rate Scenarios

The spot rates for the eight alternate interest rate scenarios are obtained by adding ±100, ±200, ±300 basis points to the base-case spot rates.

Step 4: Calculate Implied Forward Rates for All Seven Scenarios

The 1-month implied forward rates are the 1-month interest rates that are expected to prevail in the future, and they are calculated based on the spot rates. For example, the rate on a 1-month security in month 24 is calculated based on the 24- and 25-month spot rates, as follows[4]:

$$TB1_{24} = \frac{\left(1+s_{25}\right)^{25}}{\left(1+s_{24}\right)^{24}} - 1$$

where: TB_{24} = implied forward 1-month rate, 24 months in future (in monthly, decimal form)
s_t = t-month spot rate (in monthly, decimal form)

Example: Derivation of Implied Forward Rates

Assume the 24- and 25-month spot rates are equal to 4 and 4.05%, respectively, on a bond-equivalent basis. To apply the above formula, first convert the spot rates to monthly form:

$$S_{24} = \left(1 + \frac{0.04}{2}\right)^{\frac{1}{6}} - 1 = 0.00331 \text{ and}$$

$$S_{25} = \left(1 + \frac{0.0405}{2}\right)^{\frac{1}{6}} - 1 = 0.00335$$

Replacing these values in the above formula results in the implied forward $TB1_{24}$ = 0.00433, which is 5.25% (i.e., 5.25 = [(1 + 0.00433)^6 - 1] ·200) on a bond-equivalent basis.

3 The rates in the above equations are in semi-annual, decimal form.

4 Descriptions of the derivation of par and implied-forward rates from spot rates are provided in the references cited in footnote 2, above.

Once the series of implied forward 1-month rates has been derived, as described above, they can be combined to provide expected future rates of any maturity in any future period. For example, several areas of the NPV Model require projections of future 1-year Treasury rates. Using the series of 1-month implied forward Treasury rates, the implied forward 1-year rates are calculated as follows:

$$TB12_t = \left[(1 + TB1_t)(1 + TB1_{t+1})\ldots(1 + TB1_{t+11})\right]^{1/12} - 1$$

where: $TB12_t$ = implied forward 1-year Treasury bill rate in month t (in monthly, decimal form)

$TB1_t$ = implied forward 1-month Treasury bill rate in month t (in monthly, decimal form)

Step 5: Calculate Par Yields for the Alternate Scenarios

Par yields for the eight alternate interest rate scenarios are calculated from the spot rates for those scenarios using the relationship between the two types of rates that is described in Step 2 above.

Section 8.B - Retail and Secondary-Market CD Term Structures and Projected Future Rates

The estimated term structures and projected future rates for retail and secondary-market CDs are calculated in two steps. First, the 3-month future deposit rates are projected based on statistically-derived equations of the relationship between each of the two deposits rates and the 3-month implied forward Treasury rates. Second, 1- to 360-month spot (zero-coupon) rates are derived from the rates calculated in the first step. The NPV Model uses projected retail CD rates to estimate future coupons of variable-rate retail CDs and of fixed-rate retail CDs that are rolled over at maturity. Projected secondary-market CD rates are used to do the same for brokered CDs and also to estimate future interest rates for demand deposits. Secondary-market spot CD rates are used to discount deposit cash flows.

Step 1: Project Future Rates

The projected future **retail CD rates** for the seven interest rate scenarios are calculated using an empirically-derived equation that relates the current CD rate to the lagged CD rate and the 3-month Treasury bill rate (see Equation 8.B.1 below). The projected future retail CD rates adjust only partially, and with a lag, to changes in Treasury bill rates. The adjustment is symmetrical; i.e., the increase in the CD rate following an increase in the Treasury bill rate is equal to the decline in the CD rate following a comparable decline in the Treasury-bill rate. (This is in contrast to the way interest rates on various demand deposits adjust to changes in market rates.)

Equation 8.B.1 - Retail CD Rate Equation

$$CD_t^s = -0.29203 + 0.34313\left(TB3_t^s - TB3_{t-1}^s\right) + 0.615437\ CD_{t-1}^s + 0.42367\ TB3_{t-1}^s$$

where: CD_t^s = 3-month retail CD rate in month t and interest rate scenario s (in annual percentage form)

$TB3_t^s$ = implied forward 3-month Treasury bill rate in month t and interest rate scenario s (in annual, percentage form)

The above formula is used to calculate the projected rates for months 1 through 360. The rates for month 0, $TB3_0$ and CD_0, are the same in all interest rate scenarios and are obtained from Federal Reserve Statistical Release G.13 and *Bank Rate Monitor* (the Thrift National Index), respectively.

The projected future **secondary-market CD rates** for the seven interest rate scenarios are calculated using an empirically-derived equation which relates the current CD rate to the lagged CD rate and the 3-month Treasury bill rate (see Equation 8.B.2 below). Though structurally similar to the equation for retail CDs, this equation projects secondary-market CD rates that adjust almost completely, and with little lag, to changes in the Treasury-bill rate.

Equation 8.B.2 - Secondary-Market CD Rate Equation

$$SD_t^s = -0.147 + 0.6503\left(TB3_t^s - TB3_{t-1}^s\right) + 0.502957\ SCD_{t-1}^s + 0.5553\ TB3_{t-1}^s$$

where: SCD_t^s = 3-month secondary-market CD rate in month t and interest rate scenario s (in annual, percentage form)

$TB3_t^s$ = implied forward 3-month Treasury bill rate inmonth t and interest rate scenario s (in annual percentage form)

The equation above is used to project rates for months 1 through 360. The rates for month 0, $TB3_0$ and SCD_0, are the same in all interest rate scenarios and are obtained from Federal Reserve Statistical Release G.13.

The implied forward 3-month Treasury bill rates used in equations 8.B.1 and 8.B.2 are calculated based on the implied forward 1-month Treasury rates as follows:

$$TB3_t^s = \left\{ \left[\left(1 + TB1_t^s\right)\left(1 + TB1_{t+1}^s\right)\left(1 + TB1_{t+2}^s\right) \right]^{\frac{1}{3}} - 1 \right\}$$

where: $TB3_t^s$ = implied forward 3-month Treasury bill rate in month t and interest rate scenario s (in decimal form)

$TB1_t^s$ = implied forward 3-month Treasury bill rate in month t and interest rate scenario s (in decimal form)

Step 2: Estimate Spot Rates

For both types of deposit rates, spot rates are calculated based on projected future 1-month rates using Equation B.3 below.

Equation 8.B.3 - Spot Rate Equation

$$SPOT_t^s = \left[\prod_{i=1}^{t} \left(1 + FORWARD_t^s\right)^{\frac{1}{t}} - 1$$

where: $SPOT_t^s$ = estimated t-month spot rate in interest rate scenario s (in monthly, decimal form)

$FORWARD_t^s$ = projected future 1-month rate in month c and interest rate scenario s (in monthly, decimal form)

Section 8.C - Projected Future Prime and COFI Rates

The projected future prime rate and 11th District Cost-of-Funds Index are calculated using the statistically-derived relationships between each of the two rates and the 1-year Treasury bill rate.

Projected future prime rates are calculated using the following equation:

Equation 8.C.1 - Prime Rate Equation

$$PRIME_t^s = -0.008 + 0.372\left(TB12_t^s - TB12_{t-1}^s\right) + 0.667\left(TB12_{t-1}^s - TB12_{1-2}^s\right)$$

where: $PRIME_t^s$ = projected future prime rate in month t and interest rate scenario s (in annual, percentage form)

$TB12_t^s$ = implied forward 1-year Treasury bill rate in month t and interest rate scenario s (in annual, percentage form)

Projected future values of the 11th District Cost-of-Funds Index (COFI) are calculated using the following equation:

Equation 8.C.2 - 11th District COFI Equation

$$COFI_t^s = 0.9041 \cdot COFI_{t-1}^s + 0.0959 \cdot TB12_t^s$$

where: $COFI_t^s$ = projected future COFI in month t and interest rate scenario s (in annual, percentage form)

$TB12_t^s$ = implied forward 1-year Treasury bill rate in month t and interest rate scenario s (in annual, percentage form)

The implied forward 1-year Treasury bill rates used in equations 8.C.1 an 8.C.2 are calculated based on the implied forward 1-month Treasury rates as follows:

$$TB12_t^s = \left[\left(1+TB1_t^s\right)\left(1+TB1_{t+1}^s\right)\ldots\left(1+TB1_{t+11}^s\right)\right]^{1/12} - 1$$

where: $TB12_t^s$ = implied forward 1-year Treasury bill rate in month t and interest rate scenario s (in monthly, decimal form)

$TB1_t^s$ = implied forward 1-month Treasury bill rate in month t and interest rate scenario s (in monthly, decimal form)