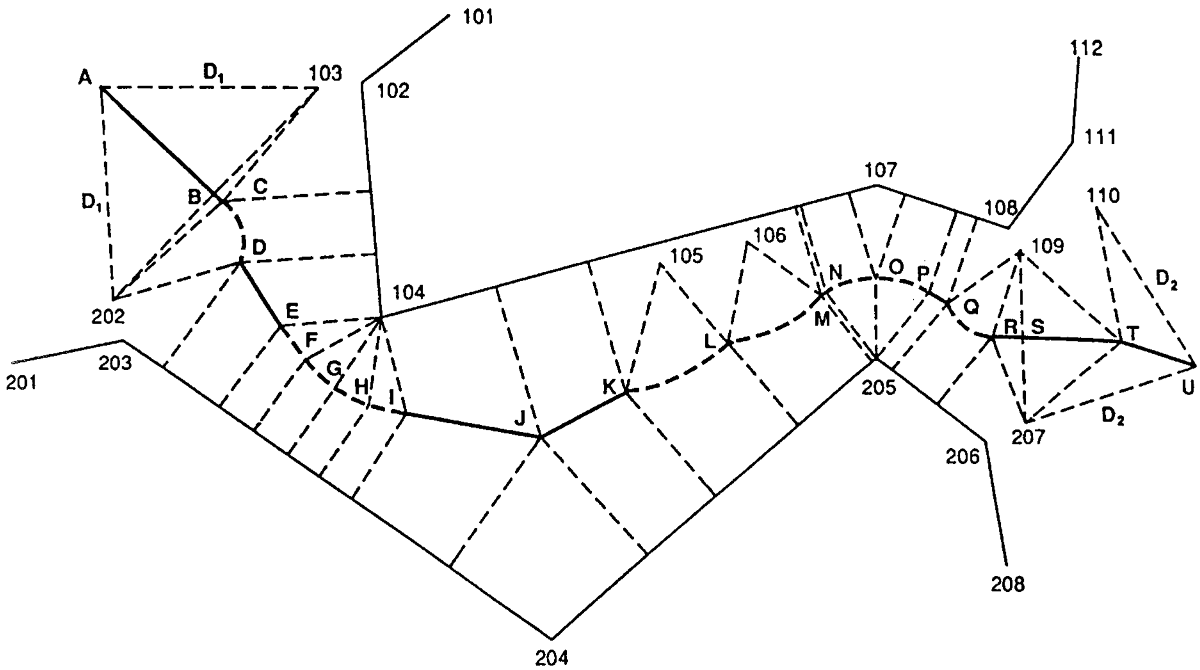
OCS Report
MMS 99-0045

Three-Dimensional Equidistant Line Computational Techniques for Determining the Location of Offshore Boundaries



U.S. Department of the Interior
Minerals Management Service
Mapping and Boundary Branch

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Three-Dimensional Equidistant Line Computational Techniques for Determining the Location of Offshore Boundaries

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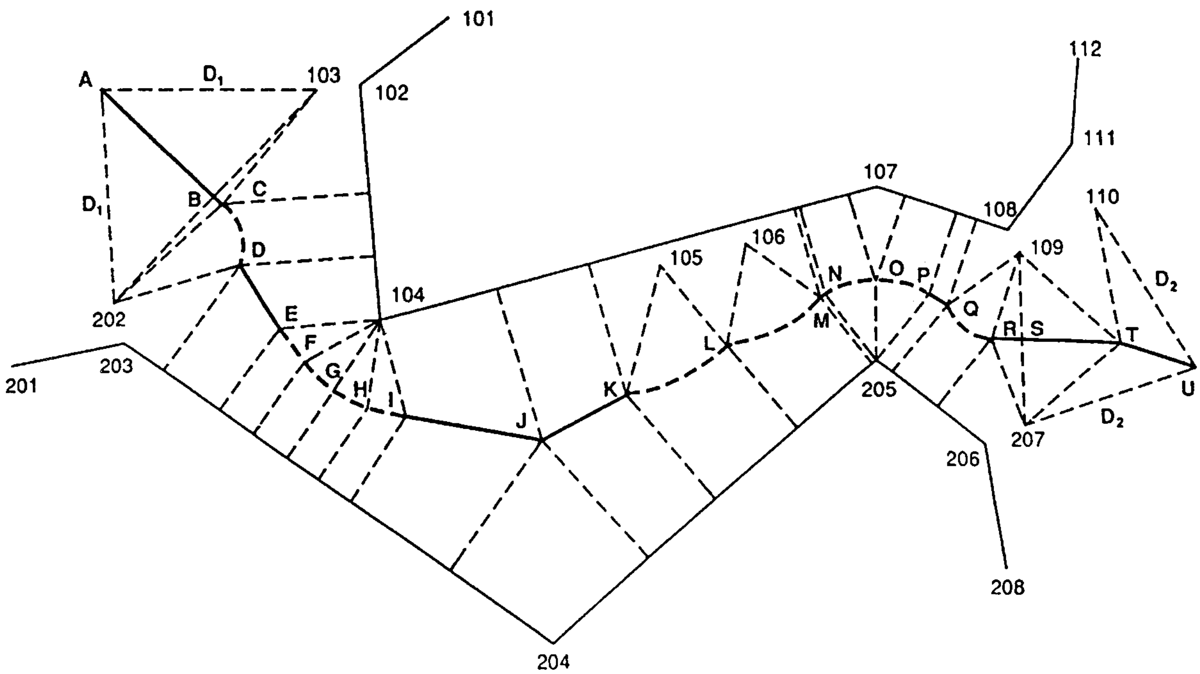

U.S. Department of the Interior
Minerals Management Service
Mapping and Boundary Branch

1999

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Abstract

Accurate locations of the offshore boundaries of the United States are needed by the U.S. Department of the Interior Minerals Management Service (MMS) for minerals management purposes on the Outer Continental Shelf (OCS).

Offshore boundary lines are (1) equidistant lines that are located at the midpoints between two coastlines or (2) boundary lines that are located by projecting a coastline seaward and more or less parallel to itself.

Equidistant lines, which include median lines and lateral lines, consist of an unbroken series of intersecting curved line segments and "straight" line segments, every point of which is equidistant from the closest points on opposite coastlines.

Three-dimensional geodetic computational techniques and mathematical equations are presented. These techniques and equations have been developed for computing the locations of and for preparing complete and accurate descriptions of the equidistant lines that form the offshore boundaries of the United States for minerals management purposes.

Introduction

The following is a description of three-dimensional computational techniques including mathematical equations that have been developed for determining the locations of and for preparing complete and accurate descriptions of the equidistant lines, which in many areas form the offshore boundaries of the United States.

Definitions

Listed below are definitions of terms that are used in describing the computational processes:

Median Lines, Lateral Lines, and Equidistant Lines - Geometric median lines, lateral lines, and equidistant lines consist of a continuous unbroken series of intersecting curved line segments and/or "straight line" segments, every point of which is equidistant from the closest points on opposite coastlines. In the case of a median line, the opposite coastlines are more or less parallel with each other, similar to the opposite banks of a river, and the median line is located at midpoint between the two coastlines. In the case of a lateral line, one coastline is actually a continuation of the other. The "opposite" coastlines are the same coastline. They are adjacent or side-by-side and the lateral line extends seaward following a course that is more or less perpendicular to the coastline. Lateral lines and median lines can both be described as equidistant lines.

Baseline - The normal baseline is the line of mean low water, or mean lower low water where applicable, along that portion of the seacoast that is in direct contact with the open sea. Baselines also include closing lines across the mouths of rivers and bays.

For computational purposes, a baseline is a mathematical approximation of the location of the (lower) low water line. It is represented by (1) a series of points only and (2) points connected by short straight line segments. The baseline points usually represent prominent points along irregular stretches of coastline. The straight line segments of the baseline represent smoother, straighter stretches of coastline and closing lines.

The locations of all baseline points are defined by their geographic coordinates (i.e., latitude and longitude), which are determined by surveys or by digitizing from nautical charts or from specially prepared large scale charts.

When baseline points and lines have been agreed upon by all interested parties, their coordinates are treated thereafter as though they are exact.

The Computational Baseline

For computational purposes, the baseline is defined as a mathematical approximation of the mean (lower) low water line and is represented on charts by a series of (1) points only and (2) points connected by straight line segments. The locations of all baseline points are obtained from best available information and are represented by their geographic coordinates.

For computational purposes, some of the baseline points that represent the coastline and which do not influence the location of an equidistant line are eliminated. The baseline for computational purposes is therefore represented by a subset of the fully defined baseline. All baseline points that might influence the equidistant line are treated as though they do influence the equidistant line. A conservative computational baseline subset will usually consist of discontinuous but otherwise unaltered portions of the baseline.

A subset of the baseline, which is to be used for an equidistant line between two coastlines separated by a distance of several hundred miles, might include only a small portion of the baseline points needed to represent the actual coastline.

A Semi-graphical Approach

The procedures outlined for computing the locations and types of equidistant line segments are a multi-step process that can best be described as semi-graphical or semi-analytical. First, a plot of the baseline points and lines is prepared at a workable map scale. Second, the approximate locations and types of equidistant line segments are developed using semi-graphical techniques. Then, the exact coordinates of equidistant angle points, endpoints, points of intersection and other associated points are computed using the equations presented in the Appendix. A computer program is used to compute all needed coordinates after the most appropriate baseline/equidistant line computational options have been identified.

Baseline/equidistant Line Combinations

The most frequently encountered baseline/equidistant line combinations are illustrated in figure 1 and are described throughout the text. The descriptions, figure 1 diagrams, and equations are linked by computational-option abbreviations, e.g. [P1L2].

Following are descriptions and illustrations of several different types of baselines, and descriptions of the geometric relationships between baselines and equidistant lines. Also included are descriptions and diagrams of the most commonly encountered baseline/equidistant line conditions that affect the shape and location of the equidistant line, which must be recognized and identified for computational purposes. In addition, analytical procedures and mathematical equations are presented that can be used to compute the coordinates of all points needed to determine the location of an equidistant line and to prepare an accurate and complete description of an equidistant line.

Types of Coastlines

Two different types of equidistant line/baseline relationships are considered: (1) equidistant lines associated with baselines defined by isolated baseline points or clusters of baseline points only and (2) equidistant lines associated with baselines defined by both straight line segments and by isolated points and/or clusters of points. In the case of a baseline defined by points only, the equidistant line consists of a set of intersecting straight line segments only. In the case of a baseline defined by straight line segments and by points, the equidistant line consists of a set of intersecting straight line segments and/or curved line segments.

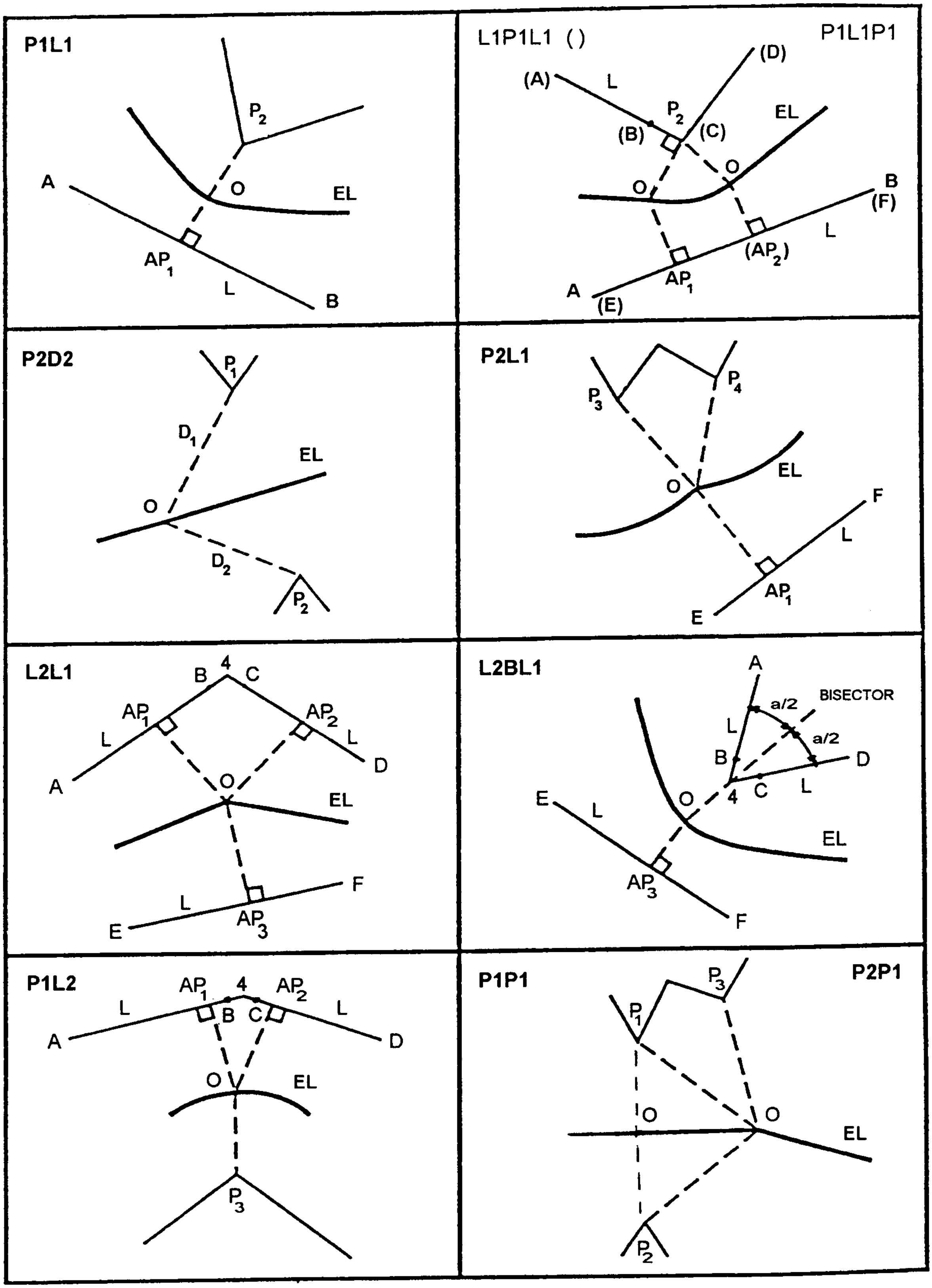


Figure 1. Most frequently encountered baseline conditions with equidistant line (EL) angle points, O, and associated points, AP, plus corresponding computational-option abbreviations.

Baseline Defined by Straight Line Segments and by Salient Points -If a baseline is defined by straight line segments only or by a combination of straight line segments and salient points, the equidistant line will consist of an unbroken set of intersecting straight line segments and/or curved line segments. A baseline, defined by straight line segments only and the associated equidistant line, exists within the channel shown in figure 2A between equidistant line points D and K. A baseline, defined by a combination of straight line segments and salient points and the associated equidistant line, exists within the same channel between equidistant line points K and M.

Equidistant line direction changes occur abruptly as angle points at the point of intersection of two straight line segments (point J, fig. 2A) at the point of intersection of a non-tangent curved line segment and a straight line segment (point D, fig. 2A) and at the point of intersection of two non-tangent curved line segments, (point L, fig. 2A). Direction changes can also occur gradually along curved line segments (line segment EFGHI, fig. 2A) at the point of intersection of a straight line segment that is tangent to a curved line segment (point E, fig. 2A) or at the point of intersection of two tangent curved line segments.

Straight equidistant line segments exist where an equidistant line passes between two straight baseline line segments, i.e., where all points of the equidistant line are equidistant from two straight baseline line segments on opposite baselines (segment IJ, fig 2A). Straight equidistant line segments also exist where an equidistant line passes between two opposite baseline angle points that are convex toward the equidistant line, or where an equidistant line passes between two baseline salient points (segment ABC, fig. 2A).

Curved equidistant line segments are usually parabolic; all points of which are equidistant from a baseline salient point or from a baseline angle point, which is convex toward the equidistant line on one baseline and a straight line segment on the opposite baseline (segment EFGHI, fig. 2A).

Equidistant line curves can be simple parabolic curves consisting of independent curved line segments bounded by straight line segments such as segment EFGHI, or can be relatively complex curves consisting of two or more intersecting curved line segments (NOP, fig. 2A).

When the baseline is defined by straight line segments only or by a combination of straight line segments and salient points, the location of the equidistant line can be defined accurately in terms of the coordinates of the equidistant line end points, the coordinates of equidistant line angle points, and the coordinates of equidistant line straight line segment end points, supplemented by the coordinates of a small number of additional points-on-curves, which are needed to define uniquely the shapes and locations of curved equidistant line segments.

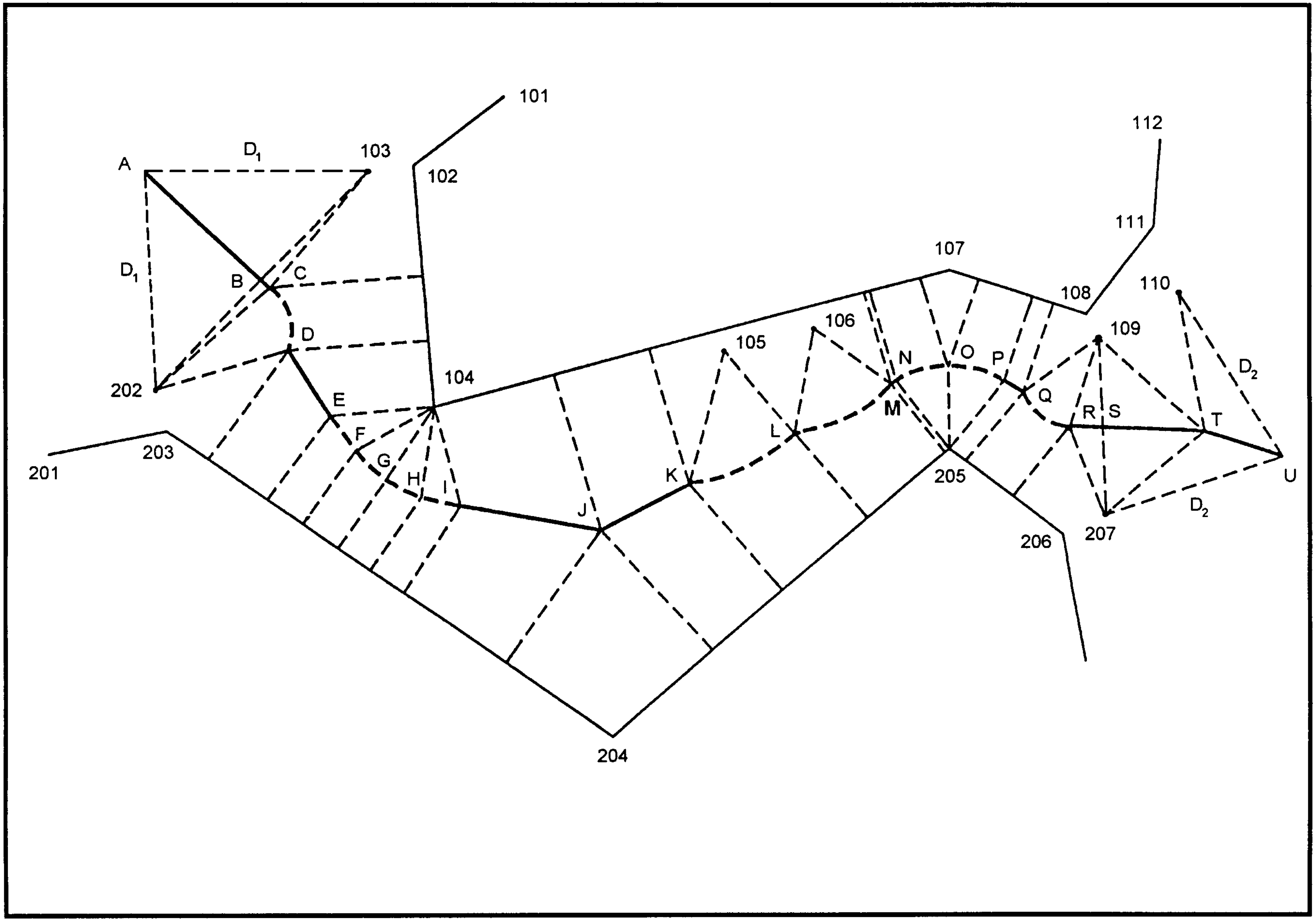


Figure 2A. Equidistant line associated with a baseline defined by straight line segments and by salient points.

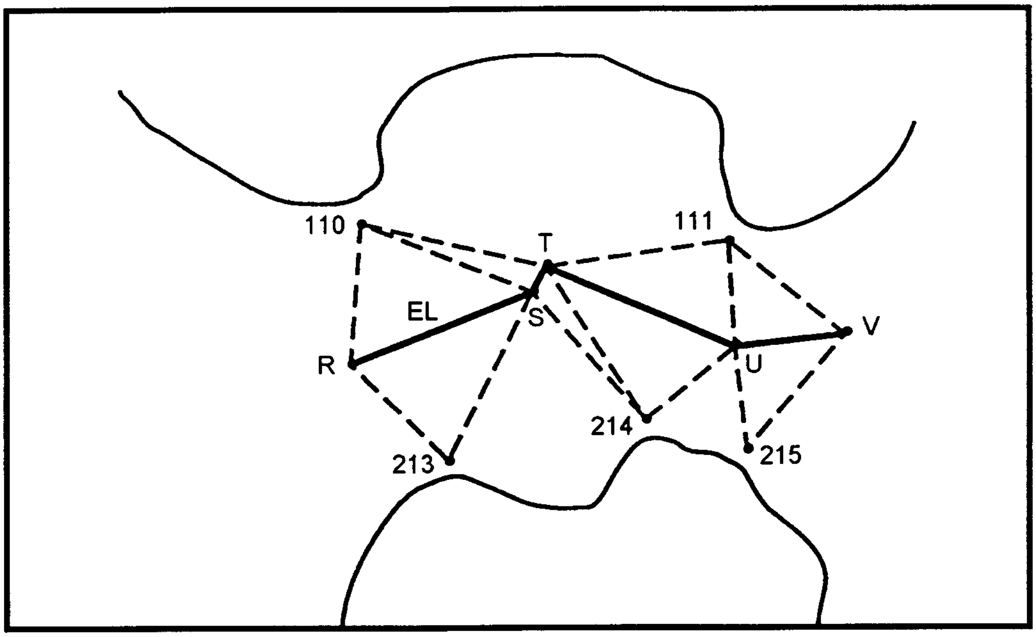


Figure 2B. Equidistant line associated with a baseline defined by salient points only.

Baselines Defined By Salient Points - If a baseline is defined only by the coordinates of prominently located points, such as large rocks or boulders scattered along or near the water's edge of an irregular coastline, for example, the baseline is described as being defined by salient points. In such cases, the equidistant line consists of an unbroken set of intersecting straight line segments only. A baseline of this type and the associated equidistant line are shown in figure 2B.

Straight line segments of the equidistant line, such as segments RS and ST in figure 2B, represent a portion of the equidistant line every point of which is equidistant from two salient points only (points 110 and 213 in the case of segment RS), one point on each opposite baseline.

Equidistant line direction changes occur abruptly as angle points at the points of intersection of adjoining straight line segments as occurs at point S in figure 2B at the point of intersection of segments RS and ST.

The location of this type of median line can be defined accurately and completely in terms of the coordinates of equidistant line angle points and end points only.

Equidistant Line Defining Points and Associated Points

Equidistant line defining points are points on an equidistant line, the coordinates of which are needed to accurately define the location of the equidistant line. Defining points include equidistant line beginning and ending points, straight line segment end points, curved line segment end points, points of intersection of adjoining curved and/or straight line segments, equidistant line angle points, and points on curves.

Equidistant line associated points are points on baseline straight line segments or non-defining points on the equidistant line, the coordinates of which are needed for computations associated with the determination of the coordinates of equidistant line defining points or for computations needed to relate the location of the equidistant line to the locations of nearby boundary lines, points on the coastline, etc.

Computing the Locations of Equidistant Line End Points

The points of beginning or ending of an equidistant line are usually points of intersection between the equidistant line and an offshore boundary line. The coordinates of the end points are determined by computing the coordinates of the points of intersection of the equidistant line and the limiting boundary lines.

Auxiliary End Points - In order to compute the coordinates of the end points, it is sometimes helpful to compute the coordinates of one or more associated points on the equidistant line. If these points are located near the probable location of the end points, they can be used to isolate a localized segment of the equidistant line in the vicinity of the boundary line, which will assist in computing the coordinates of the end points. The non-defining end point of, and points on, localized equidistant line segments are referred to as auxiliary end points.

Baselines Defined by Salient Points Only (Option P2D2) - In the case of baselines defined by salient points only, an auxiliary end point would be a point on a straight line segment of the equidistant line that is: (1) equidistant from two salient points that are located on opposite baselines and (2) that is also located a chosen distance from the two salient points. Point A in figure 2A, for example, is an auxiliary end point that is located a distance D from the two baseline points 103 and 202. One or more points A might be computed in the vicinity of a projected offshore boundary line (by varying the distance D) to assist in computing the coordinates of the point of intersection of the equidistant line with the offshore boundary line.

The coordinates of equidistant line auxiliary end points, such as A, can be determined utilizing the equations for point O presented in the Computational Option P2D2 section of the Appendix.

Also, in the case of a projected offshore boundary line, the actual equidistant line endpoint can sometimes be determined in a similar manner if (1) the distance D is equal to the offshore boundary projection distance; (2) the offshore boundary line at the point of intersection with the equidistant line consists of arc segments; and (3) the salient points, which determine the location of the equidistant line, also determine the location of the offshore boundary line at the point of intersection.

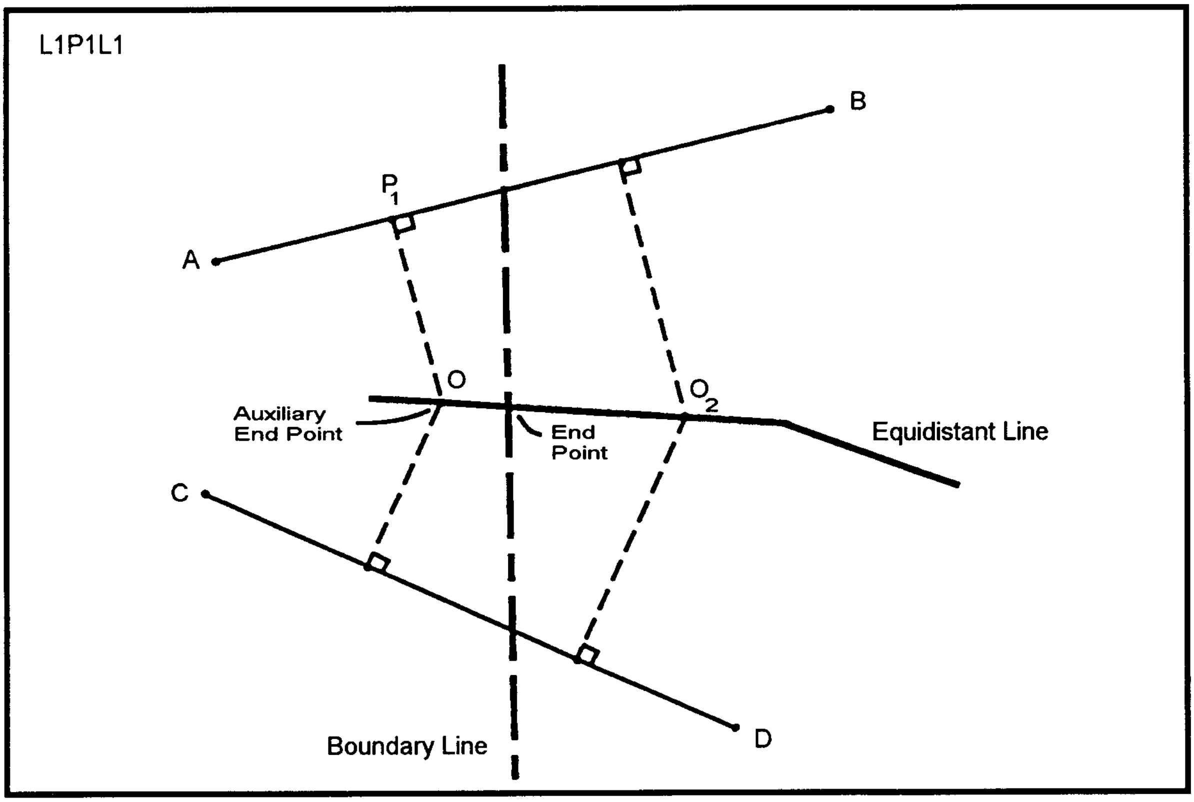


Figure 3. End point and auxiliary end point on a straight line segment of an equidistant line associated with a baseline defined by straight line segments only.

Baselines Defined by Straight Line Segments Only, or by a Combination of Straight Line Segments and Salient Points (Options L1P1L1 and P1L1P1) - In the case of baselines defined by straight line segments only, where the equidistant line passes between two straight baseline segments on opposite baselines, an auxiliary end point would lie on a straight line segment of the equidistant line and would be equidistant from (1) the straight baseline segment on one baseline and (2) a selected point, having known coordinates, which lies on the straight baseline segment on the opposite baseline. Point O in figure 3, for example, is an auxiliary end point that is equidistant from straight baseline segment CD on one baseline and from point P_1 on the straight baseline segment AB on the opposite baseline.

One or more points O might be computed in the vicinity of an offshore boundary line (by varying the location of point P) to assist in computing the coordinates of the point of intersection of the equidistant line with the boundary line.

The coordinates of equidistant line auxiliary end points such as O can be determined utilizing the equations for point O presented in the Computational Option L1P1L1 section of the Appendix.

In the case of an equidistant line that passes between a straight baseline segment on one baseline and either a salient point or a baseline angle point that is convex toward the equidistant line on the opposite baseline, the equidistant line would be curved.

When the equidistant line endpoint is the point of intersection of a curved line segment of the equidistant line with an offshore boundary line, three or more auxiliary end points on the equidistant line curve in the vicinity of the boundary line would provide sufficient information to define the location of a localized segment of the equidistant line curve. This would assist in computing the coordinates of the point of intersection of the equidistant line with the boundary line.

In this case an auxiliary end point would lie on the curved line segment of the equidistant line and would be equidistant from (1) a selected point on the straight baseline segment on one baseline; and (2) the salient point or convex angle point on the opposite baseline. Point O in figure 4, for example, is an auxiliary end point that is equidistant from point P_2 on the straight baseline segment CD on one baseline and from point P_1 on the opposite baseline.

Three or more points O computed in the vicinity of a boundary line (by varying the location of point P_2) would assist in computing the coordinates of the point of intersection of the equidistant line with the boundary line.

The coordinates of equidistant line auxiliary end points such as O can be determined utilizing the equations for point O presented in the Computational Option P1L1P1 section of the Appendix.

To be sure that the end point of an equidistant line is located accurately in relation to the baseline, the baseline should be extended a sufficient distance beyond the desired end of the equidistant line. This is because the equidistant line location can be influenced by baseline points or line segments that might be located some distance away from the end point of the equidistant line. Therefore, baseline salient points should be selected and/or straight line segments of the baseline should be extended well beyond points of intersection with a boundary line.

Equidistant Line Direction Changes

Equidistant line direction changes occur (1) at the point of intersection of adjoining straight equidistant line segments; (2) at the point of intersection of adjoining curved equidistant line segments with straight equidistant line segments that are either tangent or not tangent to one another; and (3) at the point of intersection of adjoining curved line segments that are either tangent or not tangent to one another.

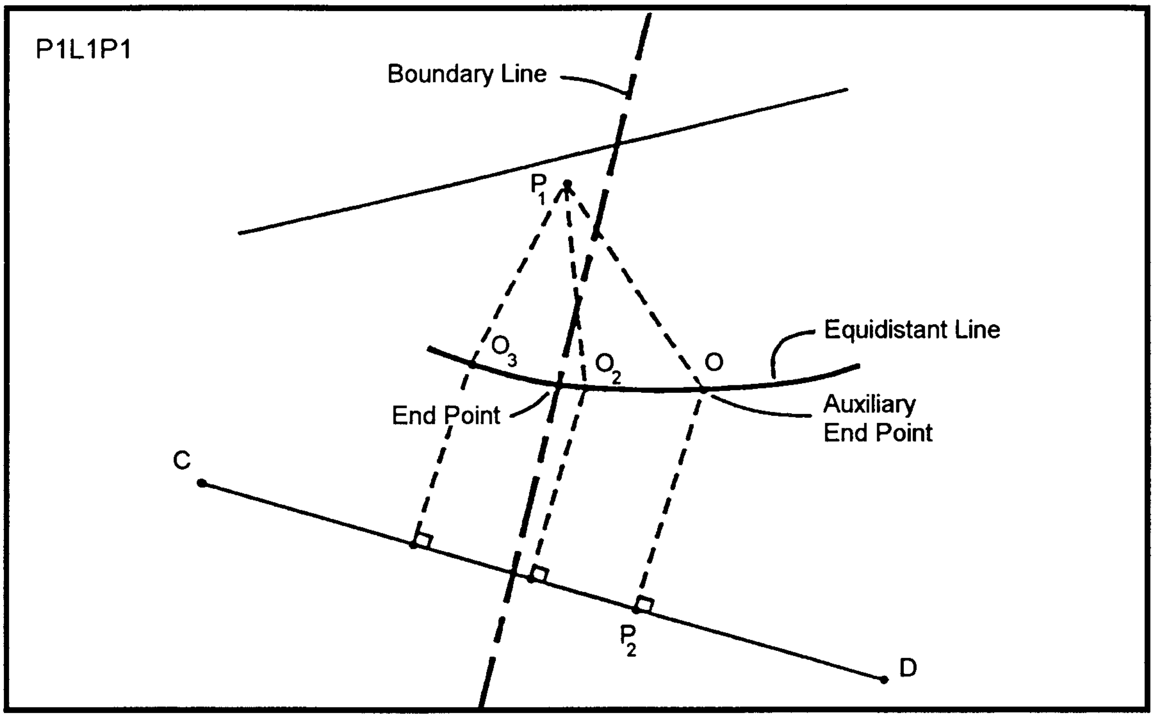


Figure 4. End point and auxiliary end points on a curved line segment of an equidistant line associated with a baseline defined by straight line segments and by salient points.

Baseline Defined By Salient Points Only

Points of Intersection of Straight Line Segments (Option P2P1) - In the case of a baseline defined by salient points only where all equidistant line segments are straight line segments the equidistant line angle points are points of intersection of adjoining straight equidistant line segments and are equidistant from three salient points. One pair of salient points determines the location of one straight equidistant line segment, a second pair of salient points determines the location of the intersecting straight equidistant line segment, and one of each of the two pairs of salient points is the same point (i.e., one salient point is common to both pairs). Angle point T in figure 5, for example, is the point of intersection of equidistant line segment ST (which is equidistant from salient points P_2 and P_3) and equidistant line segment TU (which is equidistant from salient points P_1 and P_3). In this case, point P_3 is the common salient point, and angle point T is equidistant from all three salient points P_1, P_2, and P_3.

The coordinates of equidistant line angle points such as T can be determined utilizing the equations for point O presented in the Computational Option P2P1 section of the Appendix.

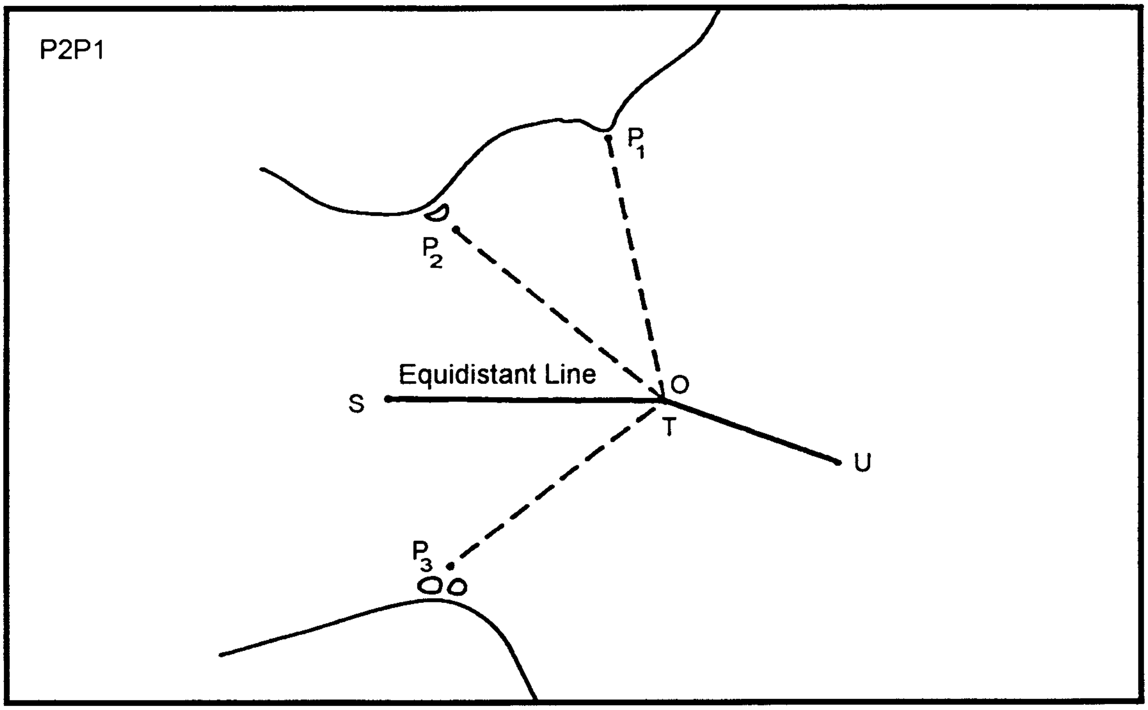


Figure 5. Equidistant line angle point associated with a baseline defined by salient points only.

Baseline Defined by Straight line Segments Only

Points of Intersection of Straight line Segments (Option L2L1) - In the case of baselines defined by straight line segments only, straight line segments of the equidistant line exist where the location of the equidistant line is determined by two straight baseline segments, one on each opposite baseline. Equidistant line direction changes occur at angle points when the equidistant line passes between (1) two straight baseline segments connected by a baseline angle point, which is concave, toward the equidistant line on one baseline and (2) one straight baseline segment on the opposite baseline. At an equidistant line angle point of this type, the angle point is equidistant from all three of the straight baseline segments. Angle point J, figure 6, for example, is the point of intersection of equidistant line segment IJ (which is equidistant from straight baseline segments EF and AB) with straight equidistant line segment JK (which is equidistant from straight baseline segments EF and CD). In this case, angle point J is equidistant from all three straight baseline segments AB, CD, and EF. (Baseline points B and C might or might not be the same point).

The coordinates of equidistant line angle points such as J can be determined utilizing the equations for point O presented in the Computational Option L2L1 section of the Appendix.

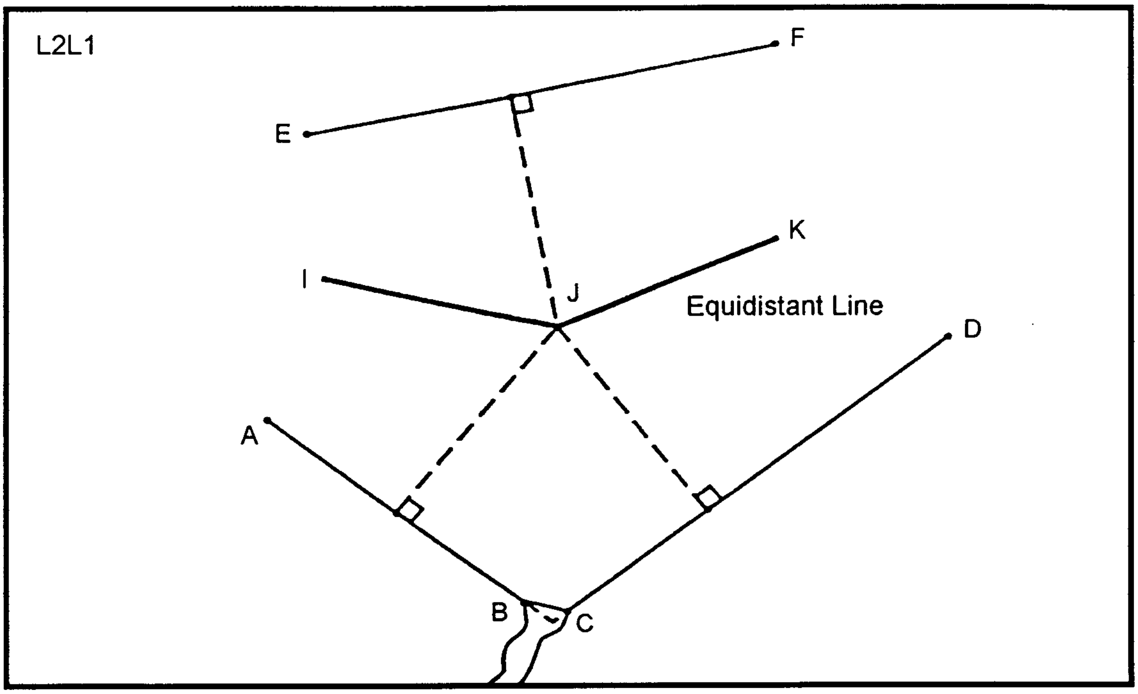


Figure 6. Equidistant line angle point associated with a baseline defined by straight line segments only.

Points of Intersection of a Straight line Segment and a Curved Line Segment (Option L1P1L1) **-** In the case of baseline defined by straight line segments only, curved line segments of the equidistant line exist where the location of the equidistant line is determined by a baseline angle point that is (1) convex toward the equidistant line on one baseline and (2) by a straight baseline segment on the opposite baseline. Straight line segments of the equidistant line exist where the location of the equidistant line is determined by two straight baseline segments, one on each opposite baseline. Equidistant line direction changes occur gradually along a parabolic curve when a straight line segment of the equidistant line intersects a curved line segment of the equidistant line at a point of tangency. This occurs when the locations of both a curved line segment of the equidistant line and a straight line segment of the equidistant line are determined by a common straight baseline segment. In this case, the point of intersection (i.e., point of tangency) is equidistant from the common straight baseline segment, the straight baseline segment on the opposite baseline, and the angle point. Point F, figure 7, for example, is the point of intersection (point of tangency) of the straight equidistant line segment EF (which is equidistant from straight baseline segments AB and CD) with the curved median line segment FG (which is equidistant from baseline point P_t and from the common straight baseline segment CD). (Baseline point B and angle point P_t might or might not be the same point).

The coordinates of equidistant line points of intersection such as F can be determined utilizing the equations for point O presented in the Computational Option L1P1L1 section of the Appendix.

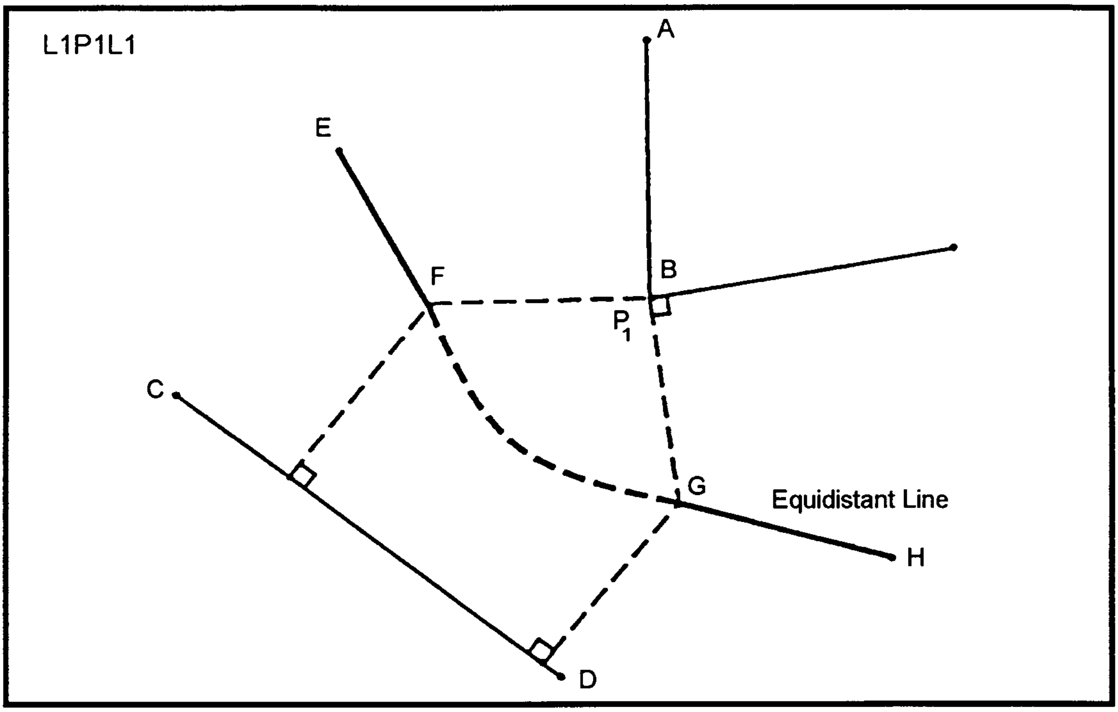


Figure 7. Point of intersection of a curved equidistant line segment with a straight equidistant line segment at a point of tangency associated with a baseline defined by straight line segments only.

Points of Intersection of curved Line Segments (Option P1L2) - In the case of baselines defined by straight baseline segments only, curved line segments of the equidistant line exist where the location of the equidistant line is determined by a baseline angle point that is convex toward the equidistant line on one baseline and by a straight baseline segment on the opposite baseline. Equidistant line direction changes occur abruptly when curved equidistant line segments intersect adjoining curved equidistant line segments that are not tangent to one another. This occurs when the locations of two adjoining curved line segments of the equidistant line are determined by a common baseline angle point that is convex toward the equidistant line on one baseline and by two different straight baseline segments connected by a baseline angle point, which is concave toward the equidistant line on the opposite baseline. The common convex angle point and the two straight baseline segments determine the locations of the two adjoining curved equidistant line segments. At an equidistant line angle point of this type, the angle point is equidistant from (1) the common convex baseline angle point and (2) from both of the two straight baseline segments. Angle point O, figure 8, for example, is the point of intersection of curved equidistant line segment NO (which is equidistant from baseline angle point P_3 and from straight baseline segment AB) with curved equidistant line segment OP (which is equidistant from the common convex baseline angle point P_3 and from straight baseline segment CD). In this case angle point 0 is equidistant from both straight baseline segments AB and CD and from the common baseline angle point P_3. (Baseline points B and C might or might not be the same point).

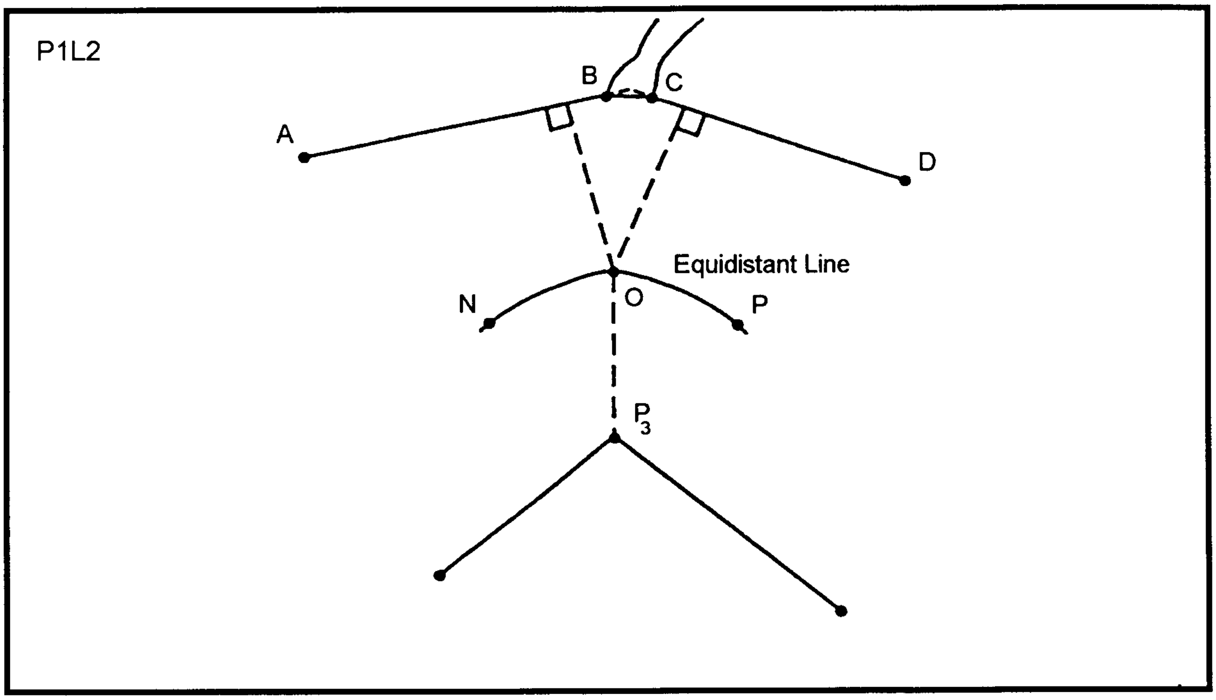


Figure 8. Point of intersection of non-tangent curved equidistant line segments associated with a baseline defined by straight line segments only.

The coordinates of equidistant line angle points such as point O can be determined utilizing the equations for point O presented in the Computational Option P1L2 section of the Appendix.

Baseline Defined by Straight line Segments and by Salient Points

Points of Intersection of a Straight Equidistant Line Segment and a Curved Equidistant Line Segment (Options P1L2 and P2L1) - In the case of baselines defined by straight line segments and by salient points, curved line segments of the equidistant line exist where the location of the equidistant line is determined by a baseline salient point on one baseline and by a straight baseline segment on the opposite baseline. Straight line segments of the equidistant line exist where the location of the equidistant line is determined by two straight baseline segments, one on each opposite baseline. Equidistant line direction changes occur abruptly when a straight line segment of the equidistant line intersects a curved line segment of the equidistant line (at a point that might not be a point of tangency). In this case, the point of intersection is equidistant from (1) the salient point and (2) from both of the straight baseline segments. Angle point G, figure 9, for example, is the point of intersection of curved equidistant line segment FG (which is equidistant from baseline salient point P_3 and from straight baseline segment CD) with straight equidistant line segment GH (which is equidistant

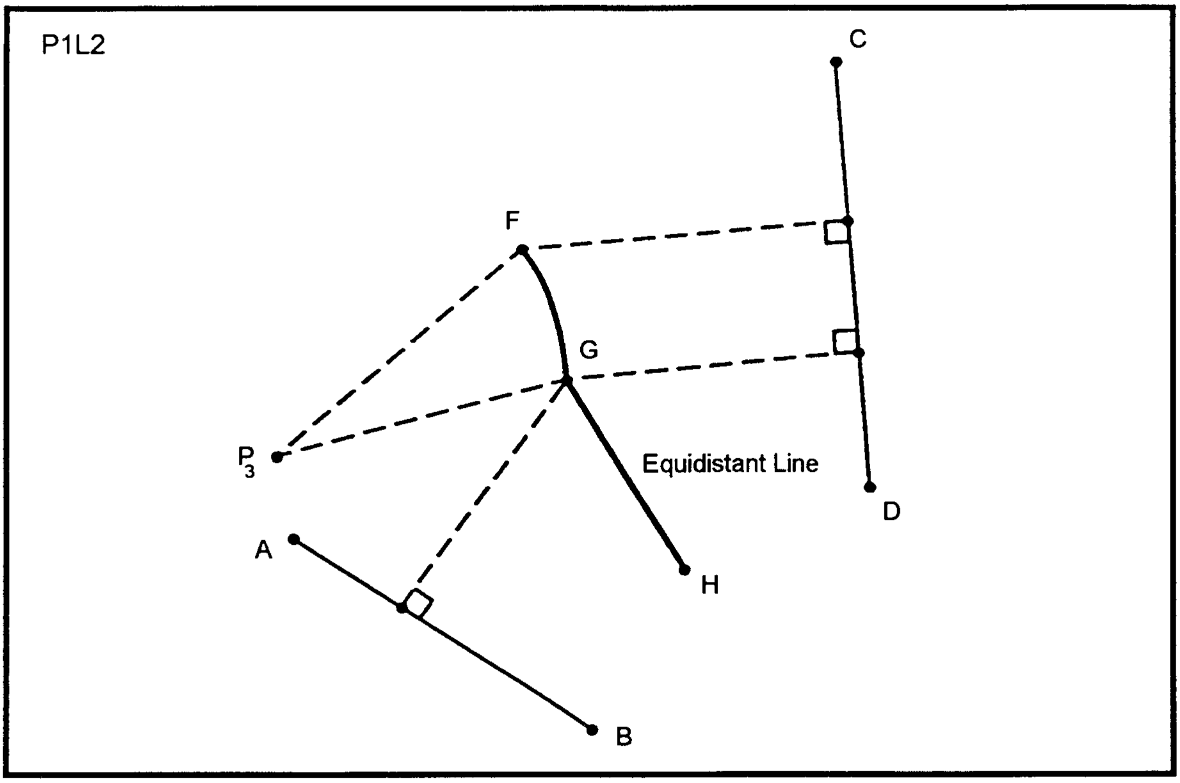


Figure 9. Point of intersection of a curved equidistant line segment and a straight equidistant line segment (which might not be tangent) associated with a baseline defined by straight line segments and by salient points.

from straight baseline segments AB and CD). In this case, angle point G is equidistant from both straight baseline segments AB and CD and from baseline salient point P_3.

The coordinates of equidistant line angle points such as G can be determined utilizing the equations for point O presented in the Computational Option P1L2 section of the Appendix.

Also, in the case of baselines defined by straight line segments and by salient points, straight line segments of the equidistant line will exist wherever the equidistant line passes between two salient points (i.e., where the location of the equidistant line is determined by two salient points that are located on opposite baselines). In this case, the point of intersection of the straight line segment and the curved line segment is equidistant from (1) the two salient points and (2) from the straight baseline segment, which determines the location of the curved line segment of the equidistant line. Angle point Q, figure 10, for example, is the point of intersection of straight equidistant line segment PQ (which is equidistant from salient points P_2 and P_3) with curved equidistant line segment QR (which is equidistant from salient point P_3 and from straight baseline segment AB). In this case point Q is equidistant from the two salient points P_2 and P_3 and from the straight baseline segment AB.

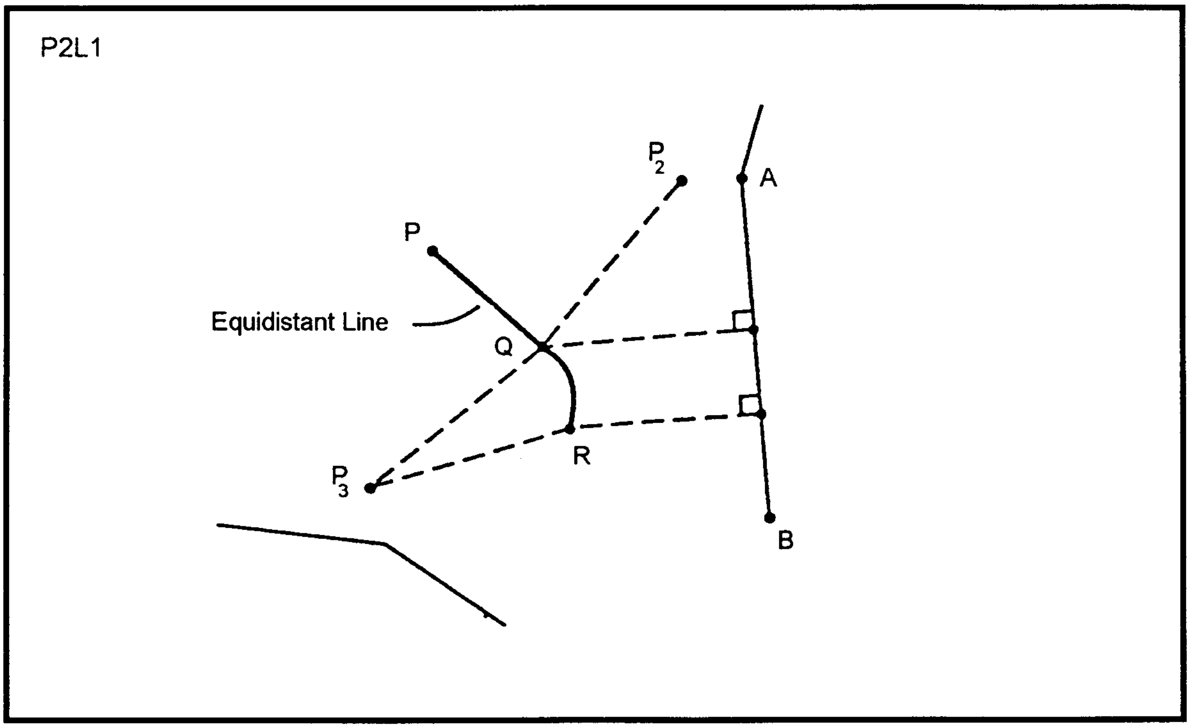


Figure 10. Point of intersection of a curved equidistant line segment and a straight equidistant line segment (which might not be tangent) associated with a baseline defined by straight line segments and by salient points.

The coordinates of equidistant line angle points such as Q can be determined utilizing the equations for point O presented in the Computation Option P2L1 section of the Appendix.

Point of Intersection of Curved Line Segments (Option P2L1) - In the case of baselines defined by straight line segments and by salient points, curved line segments of the equidistant line exist where the location of the equidistant line is determined by a salient point on one baseline and by a straight baseline segment on the opposite baseline. Equidistant line direction changes occur abruptly when curved equidistant line segments intersect adjoining curved equidistant line segments that are not tangent to one another. This occurs when the locations of two adjoining curved line segments of the equidistant line are determined by (1) a common straight baseline segment on one baseline and (2) by two different baseline salient points on the opposite baseline. The straight baseline segment and one salient point determine the location of one of the curved equidistant line segments, and the same straight baseline segment and the other salient point determine the location of the second curved equidistant line segment. At an equidistant line angle point of this type, the angle point is equidistant from (1) the common straight baseline segment and (2) from both of the two baseline salient points. Angle point L, figure 11, for example, is the point of intersection of curved equidistant line segment KL (which is equidistant from baseline salient point P_2 and from straight baseline segment AB) with

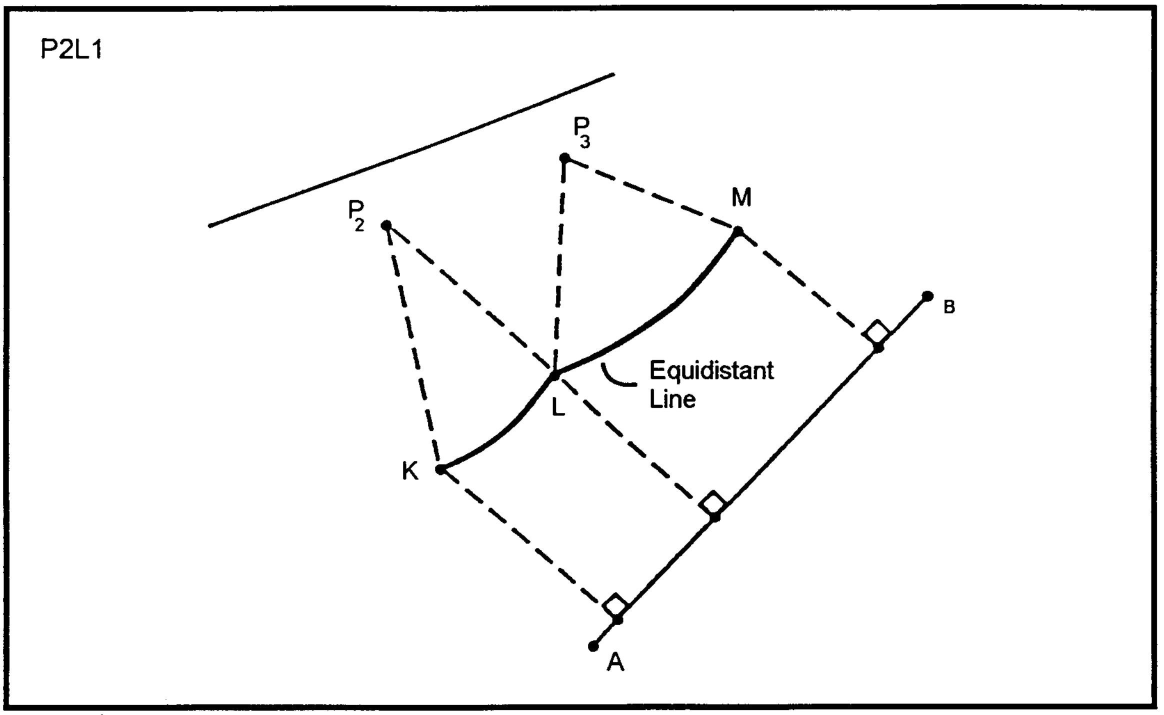


Figure 11. Point of intersection of non-tangent curved equidistant line segments associated with a baseline defined by straight line segments and by salient points.

curved equidistant line segment LM (which is equidistant from baseline salient point P_3 and from the common straight baseline segment AB). In this case, angle point L is equidistant from both salient points P_2 and P_3 and from the common straight baseline segment AB.

The coordinates of equidistant line angle points such as L can be determined utilizing the equations for point O presented in the Computational Option P2L1 section of the Appendix.

Points on Curves

In order to accurately define the shape and location of a curved line segment of an equidistant line, the coordinates of one or more points on the curved line segment of the equidistant line are needed in addition to the coordinates of the end points. Since an equidistant line curve is equidistant from a baseline salient point or from a convex baseline angle point on one baseline and from a straight baseline segment on the opposite baseline, the convex angle point or salient point plus several other selected points located on the straight baseline segment are utilized to determine the locations of points on the equidistant line curve, the coordinates of which can be computed accurately and conveniently.

One convenient choice for the location of a point-on-curve includes the point on the curve that lies on the line that is perpendicular to the straight baseline segment on one baseline and that also passes through the salient point or the convex baseline angle point on the opposite baseline. Point D, figure 12, is a point-on-curve of this type.

The coordinates of equidistant line points-on-curves such as point D can be determined using the equations for point O presented in the Computational Option P1L1 section of the Appendix.

Another convenient choice is the point on the curve that lies on the bisector of the convex baseline angle point. Point C, figure 12, is a point-on-curve of this type.

The coordinates of equidistant line points-on-curves such as point C can be determined using the equations for point O presented in the Computational Option L2BL1 section of the Appendix.

Other choices might correspond to points on the straight baseline segment that are at the mid-point or quarter points of the portion of the straight baseline segment that lies between those points that correspond to the end points of the equidistant line curved line segment. Points B and E, figure 12, are points-on-curve that correspond to the quarter points B1 and E1 of the straight baseline segment that lies between straight baseline segment points A1 and F1. A1 and F1 correspond to the endpoints A and F of the curved line segment of the equidistant line. (Points A1 through F1, fig. 12, are points on the straight baseline segment from which perpendiculars pass through corresponding points-on-curve A through F).

The coordinates of equidistant line points-on-curves such as B and E can be determined using the equations for point O presented in the Computational Option P1L1P1 section of the Appendix. The coordinates of the curved line end points such as A and F can also be determined using the equations presented in the appropriate Computational Option section of the appendix. (For example, in the case of a straight line/curved line point of tangency as shown in figure 12, the coordinates of end points such as A and F can be computed using the equations for point O presented in the Computational Option L1P1L1 of the Appendix).

Midpoints

The coordinates of midpoints are needed for many purposes. The coordinates of midpoints and quarter points (such as points B, and E, fig. 12) are needed to compute the coordinates of points on curves, for example.

The coordinates of a midpoint can be determined using the equations for point O presented in the Computational Option P1P1 section of the Appendix.

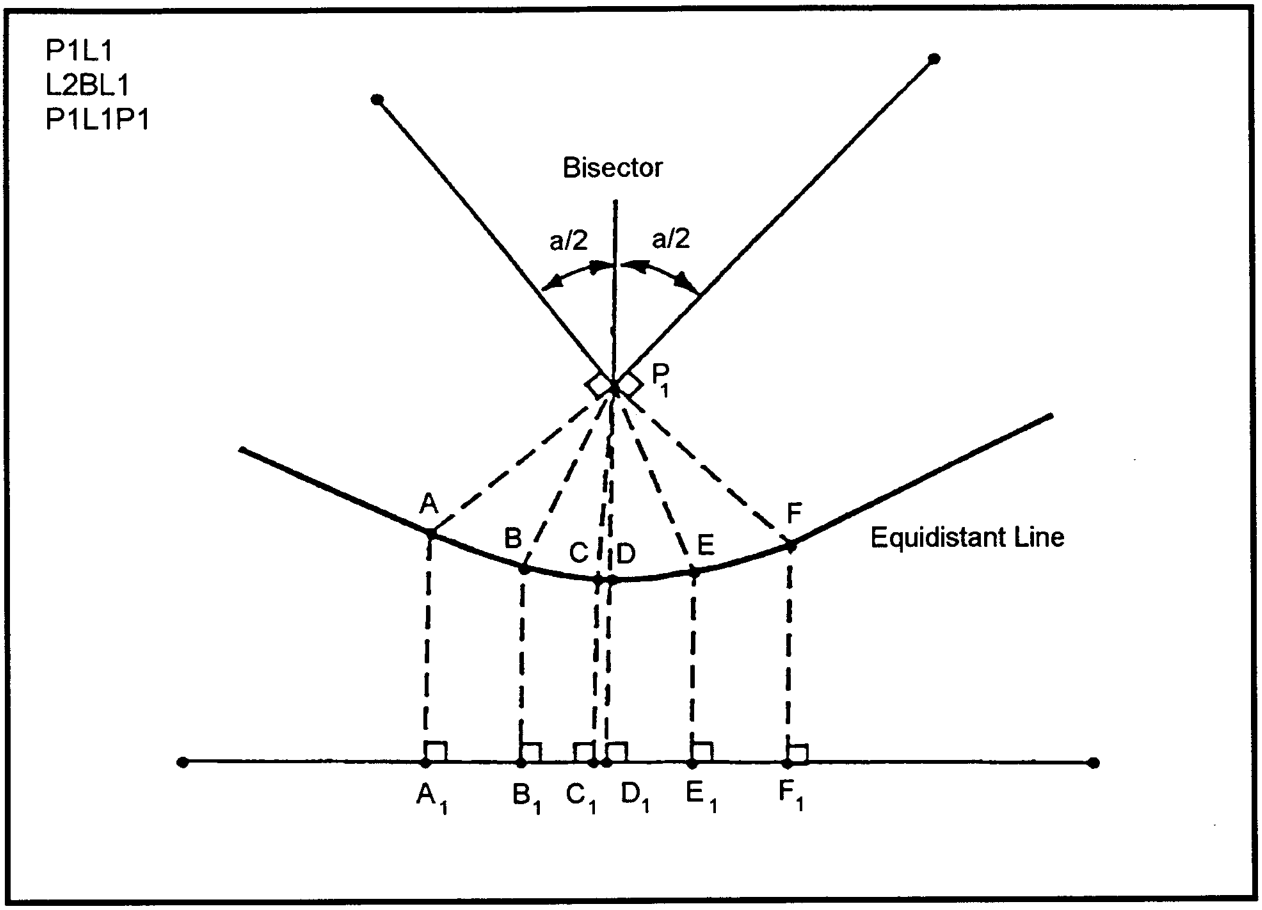


Figure 12. Defining points on a curved equidistant line segment and associated points on the straight baseline segment.

Identifying the Correct Computational Option and Correct Multiple Solution

To be sure that the correct coordinate computational equations are utilized, the geometric baseline/equidistant line computational option must be identified correctly. (In a preceding paragraph, for example, it was mentioned that the coordinates of the particular type of curved line end points associated with a point of tangency such as points A and F should be computed using the Computational Option L1P1L1 equations).

Similar appearing baseline/equidistant line conditions might be represented mathematically by significantly different equations. For example, the curved line end points F, figure 7, and Q, figure 10, might appear to be the same (i.e., a straight line/curved line type of point of intersection); but the two are actually different geometrically, and the coordinates of the two end points are computed using different sets of equations (L1P1L1 and P2L1).

Also, different appearing baseline/equidistant line conditions might be represented mathematically by the same equations. For example, the coordinates of the points of intersection O, figure 8, and G, figure 9, which appear to be different, are both computed using the same set of equations (P1L2). Therefore, in determining the computational option to apply, the underlying geometric baseline/equidistant line relationship must be identified.

To assist in matching baseline/equidistant line conditions with corresponding computational options, the baseline/equidistance line conditions that have been described and illustrated separately are summarized graphically in figure 1. Also, the computational option abbreviations accompany the first line of each description, the description illustrations in the appendix, and the corresponding figure 1 diagram.

In addition, valid multiple solutions are obtained for the computed coordinates of many points. In some cases, the desired coordinates can be identified relatively easily; but in other cases, the preferred solution is not obvious, and some additional analysis is required to make the proper selection.

Because of the necessity for selecting the correct computational option (based occasionally upon subtle differences) and for identifying the correct solution (which might sometimes require considerable judgement), it is difficult to fully automate scientific computations of this type safely. For this reason, the existing computer programs have been designed to depend upon human interaction.

Legal Description of an Equidistant Line

An equidistant line, which is a single, continuous unbroken line, is described by a sequence of points of intersection having known geographic coordinates that are connected by identified "straight line" segments and/or curved line segments. The phrase "BEGINNING AT" is used to describe the beginning point of the line. Thereafter, when the equidistant line follows the path of a straight line segment the phrase "BY STRAIGHT LINE TO" is printed, and the geographic coordinates of the point of intersection with the next segment in sequence are listed. When the equidistant line follows the path of a curved line segment, the phrase, "BY CURVED LINE TO" is printed, and the geographic coordinates of the point of intersection are specified.

Locating an Equidistant Line Within a Block /Grid System

For minerals management purposes, the OCS has been subdivided into a planar network of square or rectangular blocks bounded by grid lines uniformly spaced in both X and Y directions. The grid system adopted for use on the OCS, when adjacent to areas previously leased by a coastal State, is frequently the State Plane Coordinate System of the State. In most other cases, the Universal Transverse Mercator (UTM) coordinate system has been adopted.

The location of an equidistant line is determined without considering the block/grid system. However, the two must be compatible if they are to be related. A common grid scale factor and plane coordinate system are necessary.

If there is compatibility, the location of the equidistant line on the grid can be determined by computing the coordinates of the points of intersection of the equidistant line with the lease block grid lines and by determining the grid coordinates of other needed equidistant line defining points and associated points. Because the equidistant line is defined geodetically, an equation in terms of grid coordinates must be derived to represent the equidistant line in the plane coordinate system.

Polynomial equations are usually used for this purpose and can be derived in the following manner. The geographic coordinates of a relatively large number of points spaced at regular intervals along the equidistant line are computed, and these coordinates are transformed into the plane coordinate system of the leasing area. Employing curve fitting procedures, a polynomial equation is derived that will closely approximate the offshore boundary line on the grid. In all subsequent cartographic computations, the equidistant line is represented by the polynomial equation.

Summary

Three-dimensional equidistant line computational techniques and mathematical equations are described, which have been developed for computing the locations of and for preparing complete and accurate descriptions of the equidistant lines that form the offshore boundaries of the United States for minerals leasing purposes.

A baseline is an approximation of a coastline that can be represented mathematically. Baselines are sometimes defined by meander traverses only, and are sometimes defined by prominently located salient points only, or by combinations of meander traverses and salient points.

A geometric median line, lateral line, or equidistant line consists of a series of intersecting curved line segments and/or straight line segments, every point of which is equidistant from the closest point on opposite baselines, as measured along an arc over the earth's ellipsoidal surface..

The shape and location of an equidistant line is determined by the shape and location of the baseline, and the type of equidistant line is determined by the type of baseline.

A semi-analytical process is utilized for computing the locations of equidistant lines in which baseline plots are examined, baseline/equidistant line conditions and corresponding computational options are identified, and the coordinates of all points needed to determine the accurate location of and to prepare a description of an equidistant line are determined.

Descriptions and illustrations of the most commonly encountered baseline/equidistant line conditions plus the corresponding mathematical equations that are used to compute the coordinates of all needed equidistant line points have been presented.

References

Ball, W.E., 1971, Offshore Computations, Proceedings of the 1971 ACSM Annual Convention.

—1980, Geometric Median Line Computations, Proceedings of the 1980 ACSM Fall Convention.

—1993, Planar Median (Equidistant) Line Computations for Narrow Channels, Proceedings of the 1993 ACSM Annual Convention.

—1997a, Three-Dimensional Coastline Projection Techniques for Determining the Locations of Offshore Boundaries, Proceedings of the 1997 ACSM Annual Convention.

—1997b, Three-Dimensional Equidistant Line Computational Techniques for Determining the Locations of Offshore Boundaries, Proceedings of the 1997 ACSM Annual Convention.

Two Non-linear Equations Having Two Unknown Variables

An iterative procedure can be used to obtain the values of two variables when two non-linear equations have been obtained and when estimated values of the two unknown variables can be determined. For example, if the two equations $F_{11}(y, z)$ and $F_{21}(y, z)$ exist where

$$F_{11} = d_1 z^2 + d_2 z + d_3 zy + d_4 y + d_5 y^2 + d_6 = 0 ,$$

$$F_{21} = e_1 z^2 + e_2 z + e_3 zy + e_4 y + e_5 y^2 + e_6 = 0 ,$$

and estimated values of y and z have been determined; then improved estimates y' and z' can be obtained using the equations

$$y' = y + dy ,$$

and

$$z' = z + dz ,$$

where

$$dz = \frac{\left(F_{11}F_{22} - F_{12}F_{21}\right)}{\left(F_{12}F_{23} - F_{13}F_{22}\right)} ,$$

and

$$dy = \frac{\left(-F_{11} - F_{13}dz\right)}{F_{12}} ,$$

in which

$$F_{12} = 2d_5 y + d_3 z + d_4 ,$$

$$F_{13} = 2d_1 z + d_3 y + d_2 ,$$

$$F_{22} = 2e_5 y + e_3 y + e_4 y ,$$

and

$$F_{23} = 2e_1 z + e_3 y + e_2 .$$

Similarly, new improved estimates

$$y'' = y' + dy' ,$$

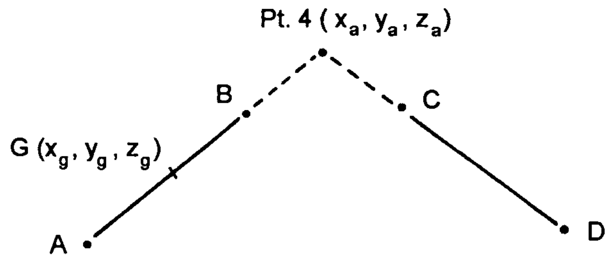
and

$$z'' = z' + dz'$$

are obtained by substituting y' for y and z' for z in the preceding equations. The entire procedure is performed repeatedly until the values of dy and dz become very small, approaching zero. The final estimates of y and z are then accepted as approximations of the true values of y and z.

Point of Intersection

The geocentric coordinates of point 4, (x_a, y_a, z_a), the point at which the straight baseline segment



AB intersects the straight baseline segment CD can be obtained using the equations

$$z_4 = \frac{(M_1 z_a - M_2 z_d + x_D - x_a)}{(M_1 - M_2)},$$

$$x_4 = M_1(z_4 - z_a) + x_a,$$

$$y_4 = \frac{[M_3(z_4 - z_a) + y_a + M_4(z_4 - z_d) + y_d]}{2},$$

in which

$$M_1 = \frac{(x_b - x_a)}{(z_b - z_a)},$$

$$M_2 = \frac{(x_c - x_d)}{(z_c - z_d)},$$

$$M_3 = \frac{(y_b - y_a)}{(z_b - z_a)},$$

and

$$M_4 = \frac{(y_c - y_d)}{(z_c - z_d)}.$$

The geocentric coordinates of point G, (x_g, y_g, z_g) ,which is a point on the straight baseline AB where the distance from point G to point4 is equal to the distance from point D to point 4 can be obtained from

$$x_g = a_1 dG + x_4 ,$$

$$y_g = a_2 dG + y_4 ,$$

and

$$z_g = a_3 dG + z_4 ,$$

in which

$$a_1 = x_a - x_4 ,$$

$$a_2 = y_a - y_4 ,$$

$$a_3 = z_a - z_4 ,$$

$$c_1 = x_d - x_4 ,$$

$$c_2 = y_d - y_4 ,$$

$$c_3 = z_d - z_4 ,$$

$$dA = \left(a_1^2 + a_2^2 + a_3^2\right)^{\frac{1}{2}} ,$$

$$dD = \left(c_1^2 + c_2^2 + c_3^2\right)^{\frac{1}{2}} ,$$

and

$$dG = \frac{dD}{dA} .$$

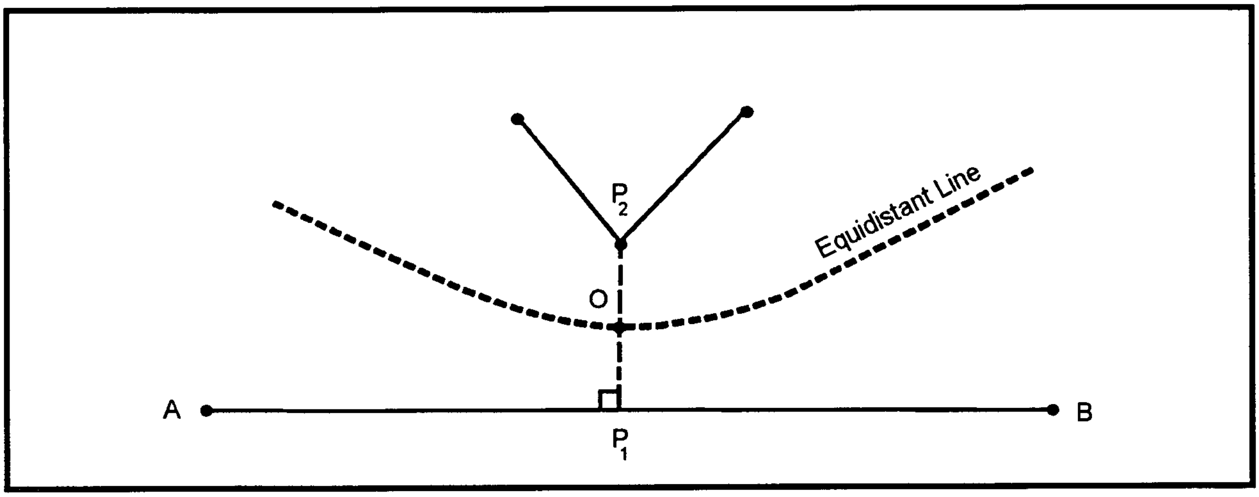
Notation

a = length of ellipsoidal semi-major axis

b = length of ellipsoidal semi-minor axis

$Pt.4(x_4,\ y_4,\ z_4)$	=	the point at which the straight baseline segment AB intersects the straight baseline segment CD
$Pt.G(x_g,\ y_g,\ z_g)$	=	a point on the straight baseline segment AB where the distance from Pt. G to Pt. 4 is equal to the distance from Pt. D to Pt. 4.

Computational Option P1L1



Given: The geographic coordinates (φ, λ) of point A (φ_a, λ_a), point B (φ_b, λ_b), and point P_2 (φ_2, λ_2).

Compute: The geographic coordinates (φ, λ) of point P_1 (φ_1, λ_1) and equidistant point O (φ_o, λ_o).

$$Az_0^2 + Bz_0 + C = 0 \quad (1)$$

$$x_0 = B_2 - M_2 z_0 \quad (2)$$

$$y_0 = M_1 z_0 + B_1 \quad (3)$$

$$E = x_b - x_a \quad (4)$$

$$F = y_b - y_a \quad (5)$$

$$G = z_b - z_a \quad (6)$$

$$H = Ex_2 + Fy_2 + Gz_2 \quad (7)$$

$$M_3 = \frac{E}{G} \quad (8)$$

$$M_4 = \frac{F}{G} \quad (9)$$

$$B_3 = x_a - M_3 z_a \quad (10)$$

$$B_4 = y_a - M_4 z_a \quad (11)$$

$$z_1 = \frac{(H - EB_3 - FB_4)}{(EM_3 + FM_4 + G)} \quad (12)$$

$$x_1 = M_3 z_1 + B_3 \quad (13)$$

$$y_1 = M_4 z_1 + B_4 \quad (14)$$

$$Q = x_2 - x_1 \quad (15)$$

$$R = y_2 - y_1 \quad (16)$$

$$S = z_2 - z_1 \quad (17)$$

$$T = \frac{1}{2}\left[\left(x_2^2 + y_2^2 + z_2^2 \right) - \left(x_1^2 + y_1^2 + z_1^2 \right) \right] \quad (18)$$

$$M_1 = \frac{(GQ - SE)}{(RE - FQ)} \tag{19}$$

$$M_2 = \frac{(FM_1 + G)}{E} \tag{20}$$

$$B_1 - \frac{(TE - HQ)}{(RE - FQ)} \tag{21}$$

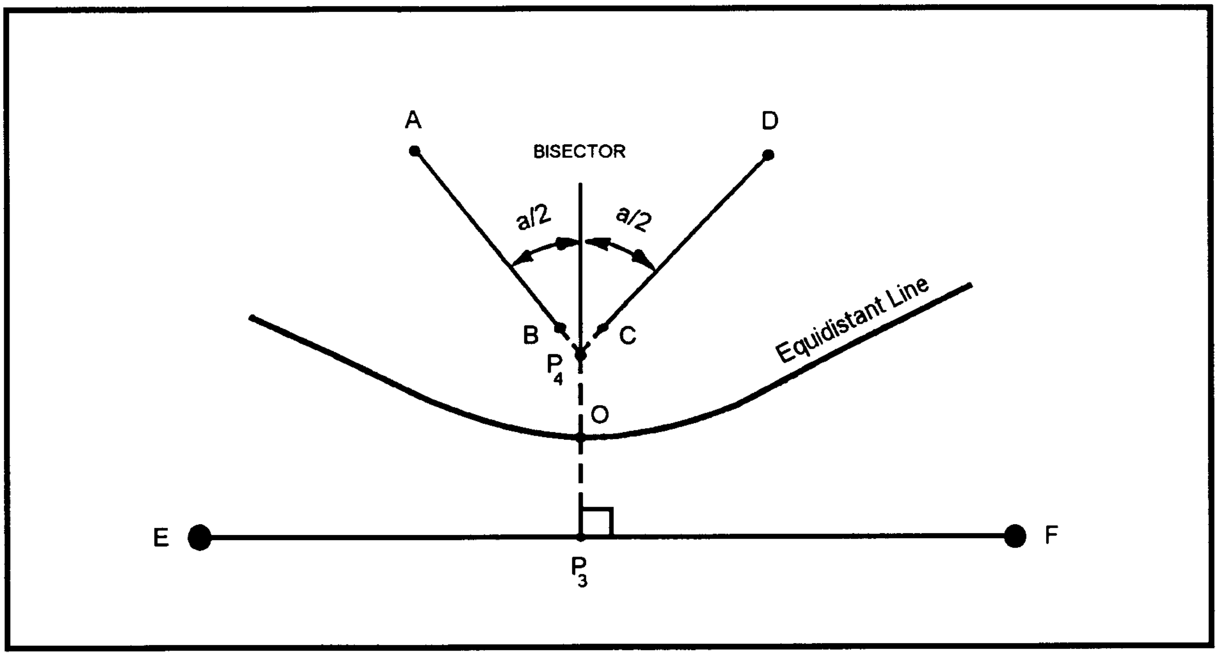
$$B_2 = \frac{(H - FB_1)}{E} \tag{22}$$

$$A = M_1^2 + M_2^2 + \frac{a^2}{b^2} \tag{23}$$

$$B = 2(M_1B_1 - M_2B_2) \tag{24}$$

$$C = B_1^2 + B_2^2 - a^2 \tag{25}$$

Computational Option L2BL1



Given: The geographic coordinates (φ,λ) of point A (φ_a, λ_a), point B (φ_b, λ_b), point C (φ_c, λ_c), point D (φ_d, λ_d), point E (φ_e, λ_e), and point F (φ_f, λ_f) . (Points B, C, and P_4 can be the same point.)

Compute: The geographic coordinates (φ,λ) of point P_3 (φ_3, λ_3) and equidistant point O (φ_o, λ_o).

$$d_1 y_o^2 + d_2 z_o^2 + d_3 y_o z_o + d_4 y_o + d_5 z_o + d_6 = 0 \qquad (1)$$

$$e_1 y_o^2 + e_2 z_o^2 + e_3 y_o z_o + e_4 y_o + e_5 z_o + e_6 = 0 \qquad (2)$$

$$x_o = \frac{(D - B y_o - C z_o)}{A} \qquad (3)$$

$$x_2 = M_6 z_2 + B_6 \qquad (4)$$

$$y_2 = M_7 z_2 + B_7 \qquad (5)$$

$$z_2 = B_1 y_o + B_2 z_o + B_3 \qquad (6)$$

$$A = x_g - x_d \qquad (7)$$

$$B = y_g - y_d \qquad (8)$$

$$C = z_g - z_d \qquad (9)$$

$$D = A x_4 + B y_4 + C z_4 \qquad (10)$$

$$E = x_f - x_e \qquad (11)$$

$$F = y_f - y_e \qquad (12)$$

$$G = z_f - z_e \qquad (13)$$

$$H = x_4^2 + y_4^2 + z_4^2 \qquad (14)$$

$$M_6 = \frac{E}{G} \qquad (15)$$

$$M_7 = \frac{F}{G} \qquad (16)$$

$$B_6 = x_e - M_6 z_e \qquad (17)$$

$$B_7 = y_e - M_7 z_e \qquad (18)$$

$$P = EM_6 + FM_7 + G \qquad (19)$$

$$Q = EB_6 + FB_7 \qquad (20)$$

$$B_1 = \frac{\left(F - \frac{BE}{A}\right)}{P} \qquad (21)$$

$$B_2 = \frac{\left(G - \frac{CE}{A}\right)}{P} \qquad (22)$$

$$B_3 = \frac{\left(\frac{DE}{A} - Q\right)}{P} \qquad (23)$$

$$B_4 = M_6 B_3 + B_6 \qquad (24)$$

$$B_5 = M_7 B_3 + B_7 \qquad (25)$$

$$A_{11} = -M_6 B_1 \qquad (26)$$

$$A_{12} = -M_6 B_2 \qquad (27)$$

$$A_{13} = x_4 - M_6 B_3 - B_6 \qquad (28)$$

$$A_{21} = -M_7 B_1 \qquad (29)$$

$$A_{22} = -M_7 B_2 \qquad (30)$$

$$A_{23} = y_4 - M_7 B_3 - B_7 \qquad (31)$$

$$A_{31} = -B_1 \qquad (32)$$

$$A_{32} = -B_2 \qquad (33)$$

$$A_{33} = z_4 - B_3 \qquad (34)$$

$$C_1 = M_6^2 + M_7^2 + 1.0 \qquad (35)$$

$$C_2 = M_6 B_4 + M_7 B_5 + B_3 \qquad (36)$$

$$C_3 = B_4^2 + B_5^2 + B_3^2 \qquad (37)$$

$$d_1 = A_{21}A - A_{11}B + \frac{AC_1B_1^2}{2} \qquad (38)$$

$$d_2 = A_{32}A - A_{12}C + \frac{AC_1B_2^2}{2} \qquad (39)$$

$$d_3 = A_{22}A + A_{31}A - A_{11}C - A_{12}B + AC_1B_1B_2 \qquad (40)$$

$$d_4 = A_{23}A + A_{11}D - A_{13}B + AC_2B_1 \qquad (41)$$

$$d_5 = A_{33}A + A_{12}D - A_{13}C + AC_2B_2 \qquad (42)$$

$$d_6 = A_{13}D + \frac{A(C_3 - H)}{2} \qquad (43)$$

$$e_1 = A^2 + B^2 \qquad (44)$$

$$e_2 = C^2 + \frac{A^2 a^2}{b^2} \tag{45}$$

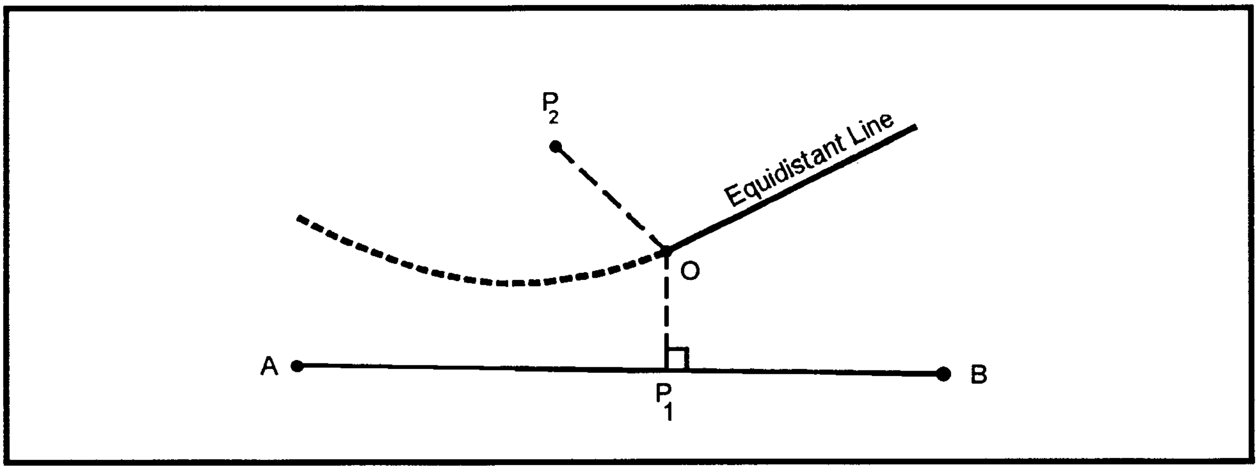
$$e_3 = 2BC \tag{46}$$

$$e_4 = -2BD \tag{47}$$

$$e_5 = -2CD \tag{48}$$

$$e_6 = D^2 - A^2 a^2 \tag{49}$$

Computational Option P1L1P1



Given: The geographic coordinates (φ, λ) of point A (φ_a, λ_a), point B (φ_b, λ_b), point P_1 (φ_1, λ_1) and point P_2 (φ_2, λ_2).

Compute: The geographic coordinates (φ, λ) of equidistant point O (φ_o, λ_o).

$$Az_0^2 + Bz_0 + C = 0 \quad (1)$$

$$x_0 = B_2 - M_2 z_0 \quad (2)$$

$$y_0 = M_1 z_0 + B_1 \quad (3)$$

$$E = x_b - x_a \quad (4)$$

$$F = y_b - y_a \quad (5)$$

$$G = z_b - z_a \quad (6)$$

$$H = Ex_1 + Fy_1 + Gz_1 \quad (7)$$

$$Q = x_2 - x_1 \quad (8)$$

$$R = y_2 - y_1 \quad (9)$$

$$S = z_2 - z_1 \quad (10)$$

$$T = \frac{1}{2}\left[\left(x_2^2 + y_2^2 + z_2^2\right) - \left(x_1^2 + y_1^2 + z_1^2\right) \right] \quad (11)$$

$$M_1 = \frac{(GQ - SE)}{(RE - FQ)} \quad (12)$$

$$M_2 = \frac{(FM_1 + G)}{E} \quad (13)$$

$$B_1 = \frac{(TE - HQ)}{(RE - FQ)} \quad (14)$$

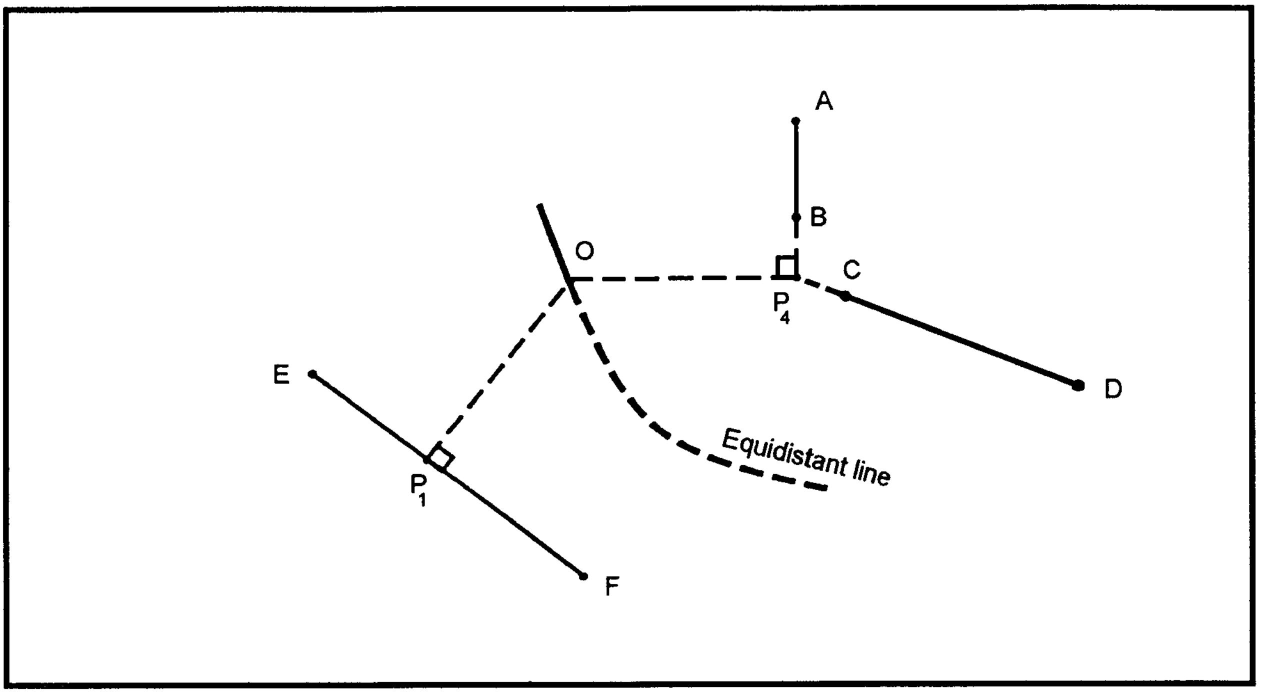
$$B_2 = \frac{(H - FB_1)}{E} \quad (15)$$

$$A = M_1^2 + M_2^2 + \frac{a^2}{b^2} \quad (16)$$

$$B = 2(M_1 B_1 - M_2 B_2) \quad (17)$$

$$C = B_1^2 + B_2^2 - a^2 \quad (18)$$

Computational Option L1P1L1



Given: The geographic coordinates (φ, λ) of point A (φ_a, λ_a), point B (φ_b, λ_b), point C (φ_c, λ_c), point D (φ_d, λ_d), point E (φ_e, λ_e), and point F (φ_f, λ_f) . (Points B, C, and P_4 can be the same point.)

Compute: The geographic coordinates (φ, λ) of point P_1 (φ_1, λ_1) and equidistant point O (φ_o, λ_o).

$$d_1 y_o^2 + d_2 z_o^2 + d_3 y_o z_o - d_4 y_o - d_5 z_o + d_6 = 0 \qquad (1)$$

$$e_1 y_o^2 + e_2 z_o^2 + e_3 y_o z_o - e_4 y_o - e_5 z_o + e_6 = 0 \qquad (2)$$

$$x_o = \frac{(D - B y_o - C z_o)}{A} \qquad (3)$$

$$z_2 = L y_o + M z_o + N \qquad (4)$$

$$x_2 = M_1 z_2 + B_1 \qquad (5)$$

$$y_2 = M_2 z_2 + B_2 \qquad (6)$$

$$M_1 = \frac{(x_f - x_e)}{(z_f - z_e)} \qquad (7)$$

$$M_2 = \frac{(y_f - y_e)}{(z_f - z_e)} \qquad (8)$$

$$B_1 = x_f - M_1 z_f \qquad (9)$$

$$B_2 = y_f - M_2 z_f \qquad (10)$$

$$E = x_f - x_e \qquad (11)$$

$$F = y_f - y_e \qquad (12)$$

$$G = z_f - z_e \quad (13)$$

$$J = EM_1 + FM_2 + G \quad (14)$$

$$K = EB_1 + FB_2 \quad (15)$$

$$A = x_b - x_a \quad (16)$$

$$B = y_b - y_a \quad (17)$$

$$C = z_b - z_a \quad (18)$$

$$D = Ax_1 + By_1 + Cz_1 \quad (19)$$

$$L = \frac{(AF - EB)}{AJ} \quad (20)$$

$$M = \frac{(AG - EC)}{AJ} \quad (21)$$

$$N = \frac{\left(\frac{DE}{A} - K\right)}{J} \quad (22)$$

$$d_1 = \frac{B^2}{A^2} + 1.0 \quad (23)$$

$$d_2 = \frac{C^2}{A^2} + \frac{a^2}{b^2} \quad (24)$$

$$d_3 = \frac{2BC}{A^2} \quad (25)$$

$$d_4 = \frac{2BD}{A^2} \quad (26)$$

$$d_5 = \frac{2CD}{A^2} \quad (27)$$

$$d_6 = \frac{D^2}{A^2} - a^2 \quad (28)$$

$$e_1 = L\left[2\left(M_2 - \frac{M_1 B}{A}\right) - L\left(M_1^2 + M_2^2 + 1.0\right)\right] \quad (29)$$

$$e_2 = M\left[2\left(1.0 - \frac{M_1 C}{A}\right) - M\left(M_1^2 + M_2^2 + 1.0\right)\right] \quad (30)$$

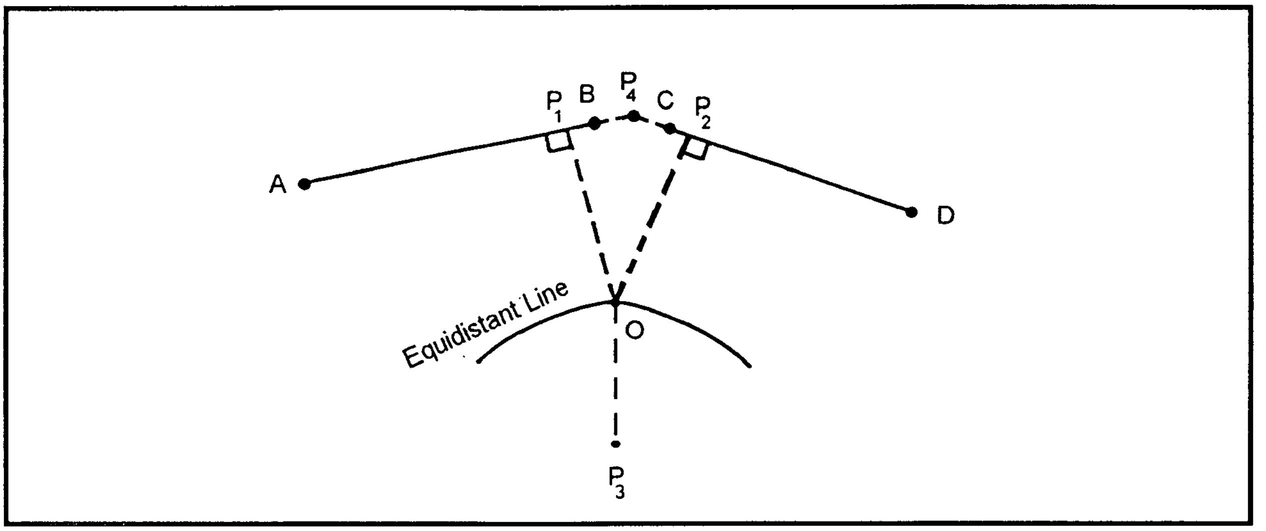
$$e_3 = 2\left\{M\left[M_2 - \frac{M_1 B}{A} - L\left(M_1^2 + M_2^2 + 1.0\right)\right] + L\left(1.0 - \frac{M_1 C}{A}\right)\right\} \quad (31)$$

$$e_4 = 2\left\{L\left[\frac{M_1 D}{A} - M_1\left(M_1 N + B_1\right) - M_2\left(M_2 N + B_2\right) - N\right] - \frac{B}{A}\left(M_1 N + B_1 - x_1\right) + M_2 N + B_2 - y_1\right\} \quad (32)$$

$$e_5 = 2\left\{\left[\frac{M_1MD - C(M_1N + B_1) + Cx_1}{A}\right] + N - z_1 - M\left[N + M_1(M_1N + B_1) + M_2(M_2N + B_2)\right]\right\} \tag{33}$$

$$e_6 = \frac{2D(M_1N + B_1 - x_1)}{A} + x_1^2 + y_1^2 + z_1^2 - N^2 - (M_1N + B_1)^2 - (M_2N + B_2)^2 \tag{34}$$

Computational Option P1L2



Given: The geographic coordinates (ϕ,λ) of point A (ϕ_a, λ_a), point B (ϕ_b, λ_b), point C (ϕ_c, λ_c), point D (ϕ_d, λ_d), and point P_3 (ϕ_3, λ_3). (Points B, C, and P_4 can be the same point.)

Compute: The geographic coordinates (ϕ,λ) of point P_1 (ϕ_1, λ_1) point P_2 (ϕ_2, λ_2) and equidistant point O (ϕ_o, λ_o).

$$d_1 y_o^2 + d_2 z_o^2 + d_3 y_o z_o + d_4 y_o + d_5 z_o + d_6 = 0 \qquad (1)$$

$$e_1 y_o^2 + e_2 z_o^2 + e_3 y_o z_o + e_4 y_o + e_5 z_o + e_6 = 0 \qquad (2)$$

$$x_o = \frac{(D - By_o - Cz_o)}{A} \qquad (3)$$

$$z_3 = B_{21} y_o + B_{22} z_o + B_{23} \qquad (4)$$

$$z_4 = B_{31} y_o + B_{32} z_o + B_{33} \qquad (5)$$

$$x_3 = M_2 z_3 + B_2 \qquad (6)$$

$$x_4 = M_3 z_4 + B_3 \qquad (7)$$

$$y_3 = M_5 z_3 + B_5 \qquad (8)$$

$$y_4 = M_6 z_4 + B_6 \qquad (9)$$

$$A = x_g - x_d \qquad (10)$$

$$B = y_g - y_d \qquad (11)$$

$$C = z_g - z_d \qquad (12)$$

$$D = Ax_4 + By_4 + Cz_4 \qquad (13)$$

$$H = x_4 - x_a \qquad (14)$$

$$J = y_4 - y_a \qquad (15)$$

$$K = z_4 - z_a \qquad (16)$$

$$L = x_4 - x_d \quad (17)$$

$$M = y_4 - y_d \quad (18)$$

$$N = z_4 - z_d \quad (19)$$

$$M_2 = \frac{H}{K} \quad (20)$$

$$M_3 = \frac{L}{N} \quad (21)$$

$$M_5 = \frac{J}{K} \quad (22)$$

$$M_6 = \frac{M}{N} \quad (23)$$

$$B_2 = x_a - M_2 z_a \quad (24)$$

$$B_3 = x_d - M_3 z_d \quad (25)$$

$$B_5 = y_a - M_5 z_a \quad (26)$$

$$B_6 = y_d - M_6 z_d \quad (27)$$

$$R = HM_2 + JM_5 + K \quad (28)$$

$$S = HB_2 + JB_5 \quad (29)$$

$$T = LM_3 + MM_6 + N \quad (30)$$

$$U = LB_3 + MB_6 \quad (31)$$

$$B_{21} = \frac{\left(J - \frac{BH}{A}\right)}{R} \quad (32)$$

$$B_{22} = \frac{\left(K - \frac{HC}{A}\right)}{R} \quad (33)$$

$$B_{23} = \frac{\left(\frac{DH}{A} - S\right)}{R} \quad (34)$$

$$B_{31} = \frac{\left(M - \frac{LB}{A}\right)}{T} \quad (35)$$

$$B_{32} = \frac{\left(N - \frac{LC}{A}\right)}{T} \quad (36)$$

$$B_{33} = \frac{\left(\frac{LD}{A} - U\right)}{T} \quad (37)$$

$$A_{11} = M_2 B_{21} \quad (38)$$

$$A_{12} = M_2 B_{22} \quad (39)$$

$$A_{13} = M_2 B_{23} + B_2 - x_3 \quad (40)$$

$$A_{21} = M_5 B_{21} \quad (41)$$

$$A_{22} = M_5 B_{22} \quad (42)$$

$$A_{23} = M_5 B_{23} + B_5 - y_3 \quad (43)$$

$$A_{31} = B_{21} \quad (44)$$

$$A_{32} = B_{22} \quad (45)$$

$$A_{33} = B_{23} - z_3 \quad (46)$$

$$B_7 = M_2 B_{23} + B_2 \quad (47)$$

$$B_8 = M_5 B_{23} + B_5 \quad (48)$$

$$C_1 = M_2^2 + M_5^2 + 1.0 \quad (49)$$

$$C_2 = M_2 B_7 + M_5 B_8 + B_{23} \quad (50)$$

$$C_3 = B_7^2 + B_8^2 + B_{23}^2 \quad (51)$$

$$d_1 = A_{21} A - A_{11} B - \frac{A C_1 B_{21}^2}{2} \quad (52)$$

$$d_2 = A_{32} A - A_{12} C - \frac{A C_1 B_{22}^2}{2} \quad (53)$$

$$d_3 = A(A_{22} + A_{31}) - A_{11} C - A_{12} B - A C_1 B_{21} B_{22} \quad (54)$$

$$d_4 = A_{23} A + A_{11} D - A_{13} B - A C_2 B_{21} \quad (55)$$

$$d_5 = A_{33} A + A_{12} D - A_{13} C - A C_2 B_{22} \quad (56)$$

$$d_6 = A_{13} D - \frac{A C_3}{2} + \frac{A\left(x_3^2 + y_3^2 + z_3^2\right)}{2} \quad (57)$$

$$e_1 = A^2 + B^2 \quad (58)$$

$$e_2 = C^2 + \frac{A^2 a^2}{b^2} \quad (59)$$

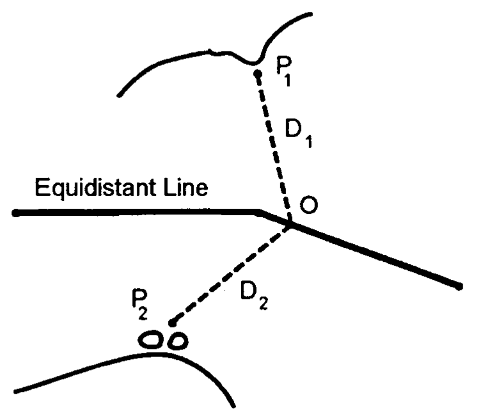
$$e_3 = 2BC \quad (60)$$

$$e_4 = -2BD \quad (61)$$

$$e_5 = -2CD \quad (62)$$

$$e_6 = D^2 - A^2 a^2 \quad (63)$$

Computational Option P2D2



Given: The geographic coordinates (φ, λ) of point $P_1(\varphi_1, \lambda_1)$, point $P_2(\varphi_2, \lambda_2)$ and distances D_1 and D_2 (D_1 and D_2 can be different.)

Compute: The geographic coordinates of equidistant point $O(\varphi_o, \lambda_o)$.

$$c_1 z_0^2 + c_2 x_0^2 + c_3 x_0 z_0 + c_4 x_0 + c_5 z_0 + c_6 = 0 \quad (1)$$

$$b_1 z_0^2 + b_2 x_0^2 + b_3 x_0 z_0 + b_4 x_0 + b_5 z_0 + b_6 = 0 \quad (2)$$

$$y_o = A + Bx_0 + Cz_0 \quad (3)$$

$$A_1 = x_2 - x_1 \quad (4)$$

$$A_2 = y_2 - y_1 \quad (5)$$

$$A_3 = z_2 - z_1 \quad (6)$$

$$A_4 = \frac{\left(D_1^2 - D_2^2 + x_2^2 + y_2^2 + z_2^2 - A_5\right)}{2} \quad (7)$$

$$A_5 = x_1^2 + y_1^2 + z_1^2 \quad (8)$$

$$A = \frac{A_4}{A_2} \quad (9)$$

$$B = -\frac{A_1}{A_2} \quad (10)$$

$$C = -\frac{A_3}{A_2} \quad (11)$$

$$b_1 = C^2 + 1.0 \quad (12)$$

$$b_2 = B^2 + 1.0 \quad (13)$$

$$b_3 = 2BC \quad (14)$$

$$b_4 = 2\left(AB - By_1 - x_1\right) \quad (15)$$

$$b_5 = 2(AC - Cy_1 - z_1) \quad (16)$$

$$b_6 = A(A - 2y_1) + A_5 - D_1^2 \quad (17)$$

$$c_1 = C^2 + \frac{a^2}{b^2} \quad (18)$$

$$c_2 = b_2 \quad (19)$$

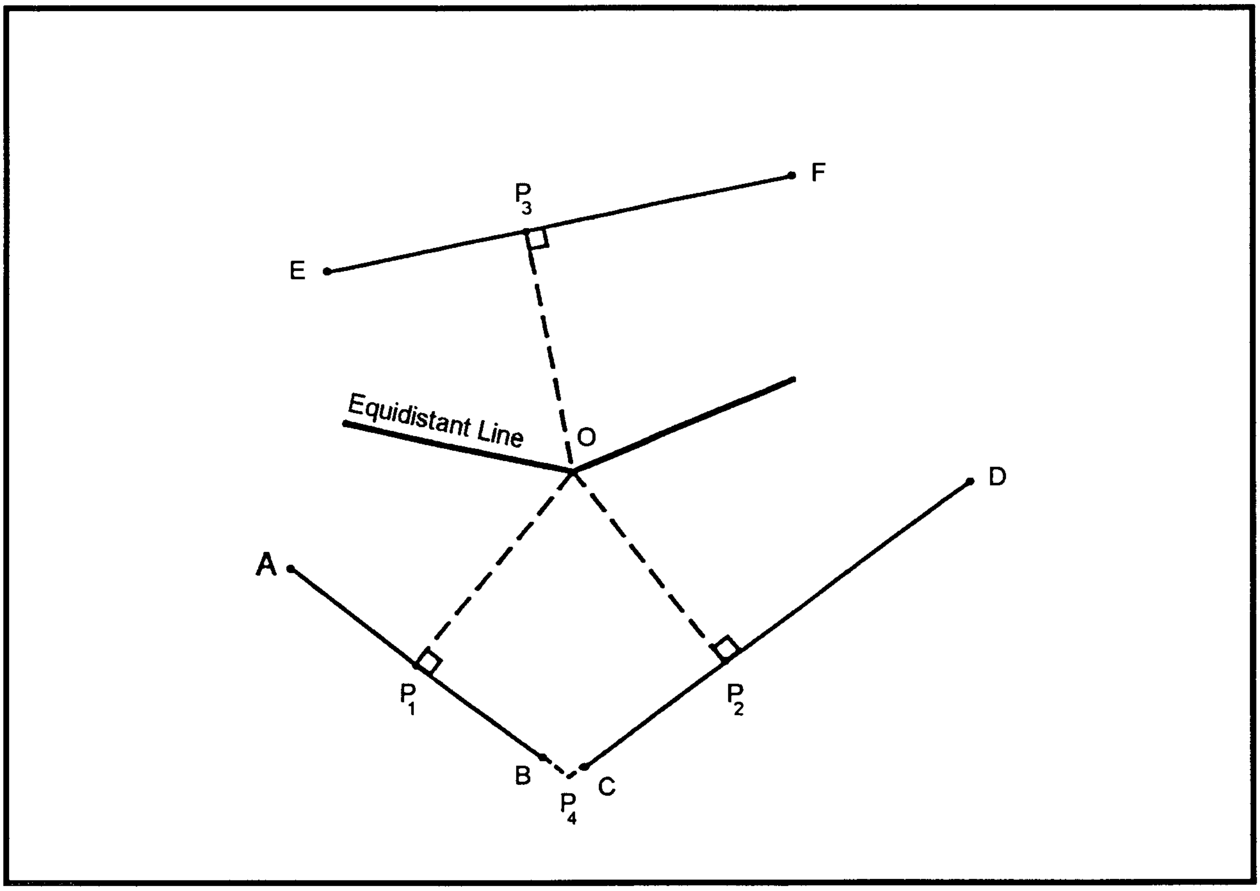
$$c_3 = b_3 \quad (20)$$

$$c_4 = 2AB \quad (21)$$

$$c_5 = 2AC \quad (22)$$

$$c_6 = A^2 - a^2 \quad (23)$$

Computational Option L2L1



Given: The geographic coordinates (φ,λ) of point A (φ_a, λ_a), point B (φ_b, λ_b), point C (φ_c, λ_c), point D (φ_d, λ_d), point E (φ_e, λ_e), and point F (φ_f, λ_f) . (Points B, C, and P_4 can be the same point.)

Compute: The geographic coordinates (φ,λ) of point P_1 (φ_1, λ_1), point P_2 (φ_2, λ_2), point P_3 (φ_3, λ_3), and equidistant point O (φ_o, λ_o).

$$d_1 y_o^2 + d_2 z_o^2 + d_3 y_o z_o + d_4 y_o + d_5 z_o + d_6 = 0 \quad (1)$$

$$e_1 y_o^2 + e_2 z_o^2 + e_3 y_o z_o + e_4 y_o + e_5 z_o + e_6 = 0 \quad (2)$$

$$x_o = \frac{(D - By_o - Cz_o)}{A} \quad (3)$$

$$z_2 = B_{11} y_o + B_{12} z_o + B_{13} \quad (4)$$

$$z_3 = B_{21} y_o + B_{22} z_o + B_{23} \quad (5)$$

$$z_4 = B_{31} y_o + B_{32} z_o + B_{33} \quad (6)$$

$$x_2 = M_1 z_2 + B_1 \quad (7)$$

$$x_3 = M_2 z_3 + B_2 \quad (8)$$

$$x_4 = M_3 z_4 + B_3 \quad (9)$$

$$y_2 = M_4 z_2 + B_4 \quad (10)$$

$$y_3 = M_5 z_3 + B_5 \quad (11)$$

$$y_4 = M_6 z_4 + B_6 \quad (12)$$

$$A = x_g - x_d \quad (13)$$

$$B = y_g - y_d \quad (14)$$

$$C = z_g - z_d \quad (15)$$

$$D = Ax_4 + By_4 + Cz_4 \quad (16)$$

$$E = x_f - x_e \quad (17)$$

$$F = y_f - y_e \quad (18)$$

$$G = z_f - z_e \quad (19)$$

$$H = x_4 - x_a \quad (20)$$

$$J = y_4 - y_a \quad (21)$$

$$K = z_4 - z_a \quad (22)$$

$$L = x_4 - x_d \quad (23)$$

$$M = y_4 - y_d \quad (24)$$

$$N = z_4 - z_d \quad (25)$$

$$M_1 = \frac{E}{G} \quad (26)$$

$$M_2 = \frac{H}{K} \quad (27)$$

$$M_3 = \frac{L}{N} \quad (28)$$

$$M_4 = \frac{F}{G} \quad (29)$$

$$M_5 = \frac{J}{K} \quad (30)$$

$$M_6 = \frac{M}{N} \quad (31)$$

$$B_1 = x_e - M_1 z_e \quad (32)$$

$$B_2 = x_a - M_2 z_a \quad (33)$$

$$B_3 = x_d - M_3 z_d \quad (34)$$

$$B_4 = y_e - M_4 z_e \quad (35)$$

$$B_5 = y_a - M_5 z_a \quad (36)$$

$$B_6 = y_d - M_6 z_d \quad (37)$$

$$P = EM_1 + FM_4 + G \quad (38)$$

$$Q = EB_1 + FB_4 \quad (39)$$

$$R = HM_2 + JM_5 + K \quad (40)$$

$$S = HB_2 + JB_5 \quad (41)$$

$$T = LM_3 + MM_6 + N \quad (42)$$

$$U = LB_3 + MB_6 \quad (43)$$

$$B_{11} = \frac{\left(F - \frac{BE}{A}\right)}{P} \quad (44)$$

$$B_{12} = \frac{\left(G - \frac{CE}{A}\right)}{P} \quad (45)$$

$$B_{13} = \frac{\left(\frac{DE}{A} - Q\right)}{P} \quad (46)$$

$$B_{21} = \frac{\left(J - \frac{BH}{A}\right)}{R} \quad (47)$$

$$B_{22} = \frac{\left(K - \frac{CH}{A}\right)}{R} \quad (48)$$

$$B_{23} = \frac{\left(\frac{DH}{A} - S\right)}{R} \quad (49)$$

$$B_{31} = \frac{\left(M - \frac{BL}{A}\right)}{T} \quad (50)$$

$$B_{32} = \frac{\left(N - \frac{CL}{A}\right)}{T} \quad (51)$$

$$B_{33} = \frac{\left(\frac{DL}{A} - U\right)}{T} \quad (52)$$

$$A_{11} = M_2B_{21} - M_1B_{11} \quad (53)$$

$$A_{12} = M_2B_{22} - M_1B_{12} \quad (54)$$

$$A_{13} = M_2B_{23} - M_1B_{13} + B_2 - B_1 \quad (55)$$

$$A_{21} = M_5B_{21} - M_4B_{11} \quad (56)$$

$$A_{22} = M_5B_{22} - M_4B_{12} \quad (57)$$

$$A_{23} = M_5B_{23} - M_4B_{13} + B_5 - B_4 \quad (58)$$

$$A_{31} = B_{21} - B_{11} \quad (59)$$

$$A_{32} = B_{22} - B_{12} \quad (60)$$

$$A_{33} = B_{23} - B_{13} \quad (61)$$

$$B_7 = M_1B_{13} + B_1 \quad (62)$$

$$B_8 = M_2B_{23} + B_2 \quad (63)$$

$$B_9 = M_4B_{13} + B_4 \quad (64)$$

$$B_{10} = M_5B_{23} + B_5 \quad (65)$$

$$C_{11} = M_1^2 + M_4^2 + 1.0 \quad (66)$$

$$C_{12} = M_1B_7 + M_4B_9 + B_{13} \quad (67)$$

$$C_{13} = B_7^2 + B_9^2 + B_{13}^2 \quad (68)$$

$$C_{21} = M_2^2 + M_5^2 + 1.0 \quad (69)$$

$$C_{22} = M_2B_8 + M_5B_{10} + B_{23} \quad (70)$$

$$C_{23} = B_8^2 + B_{10}^2 + B_{23}^2 \quad (71)$$

$$d_1 = A_{21}A - A_{11}B + \frac{A\left(C_{11}B_{11}^2 - C_{21}B_{21}^2\right)}{2} \qquad (72)$$

$$d_2 = A_{32}A - A_{12}C + \frac{A\left(C_{11}B_{12}^2 - C_{21}B_{22}^2\right)}{2} \qquad (73)$$

$$d_3 = A_{22}A + A_{31}A - A_{11}C - A_{12}B + A\left(C_{11}B_{11}B_{12} - C_{21}B_{21}B_{22}\right) \qquad (74)$$

$$d_4 = A_{23}A + A_{11}D - A_{13}B - A\left(C_{12}B_{11} - C_{22}B_{21}\right) \qquad (75)$$

$$d_5 = A_{33}A + A_{12}D - A_{13}C + A\left(C_{12}B_{12} - C_{22}B_{22}\right) \qquad (76)$$

$$d_6 = A_{13}D + \frac{A\left(C_{13} - C_{23}\right)}{2} \qquad (77)$$

$$e_1 = B^2 + A^2 \qquad (78)$$

$$e_2 = C^2 + \frac{A^2a^2}{b^2} \qquad (79)$$

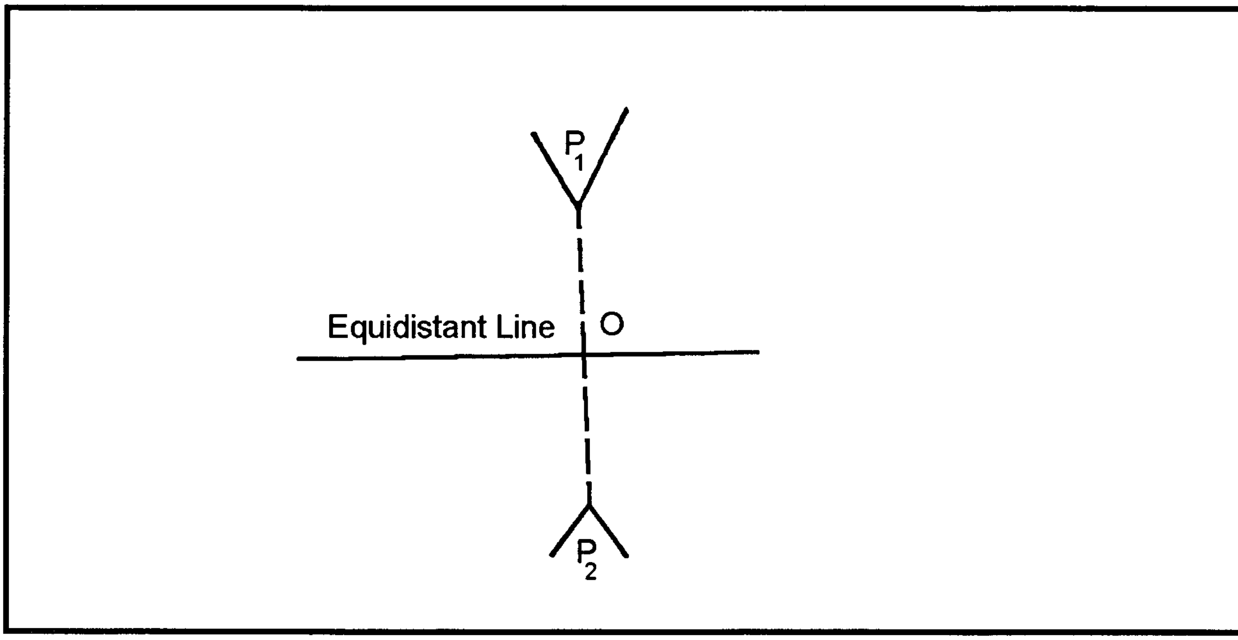
$$e_3 = 2BC \qquad (80)$$

$$e_4 = -2BD \qquad (81)$$

$$e_5 = -2CD \qquad (82)$$

$$e_6 = D^2 - A^2a^2 \qquad (83)$$

Computational Option P1P1



Given: The geographic coordinates (φ, λ) of point P_1 (φ_1, λ_1) and point P_2 (φ_2, λ_2).

Compute: The geographic coordinates (φ, λ) of equidistant point O (φ_o, λ_o).

$$Az_0^2 + Bz_0 + C = 0 \quad (1)$$

$$x_0 = B_2 - M_2 z_0 \quad (2)$$

$$y_0 = M_1 z_0 + B_1 \quad (3)$$

$$P = x_2 - x_1 \quad (4)$$

$$Q = y_2 - y_1 \quad (5)$$

$$R = z_2 - z_1 \quad (6)$$

$$S = \frac{1}{2}\left[\left(x_2^2 + y_2^2 + z_2^2\right) - \left(x_1^2 + y_1^2 + z_1^2\right)\right] \quad (7)$$

$$T = x_2 + x_1 \quad (8)$$

$$U = y_2 + y_1 \quad (9)$$

$$V = z_2 + z_1 \quad (10)$$

$$W = \frac{1}{2}(PT + QU + RV) \quad (11)$$

$$z_4 = \frac{W}{R} \quad (12)$$

$$E = y_1(z_2 - z_4) - y_2(z_1 - z_4) \quad (13)$$

$$F = x_2(z_1 - z_4) - x_1(z_2 - z_4) \quad (14)$$

$$G = x_1 y_2 - x_2 y_1 \quad (15)$$

$$H = Gz_4 \quad (16)$$

$$M_1 = \frac{(GP - RE)}{(QE - FP)} \quad (17)$$

$$M_2 = \frac{(FM_1 + G)}{E} \quad (18)$$

$$B_1 = \frac{(SE - HP)}{(QE - FP)} \quad (19)$$

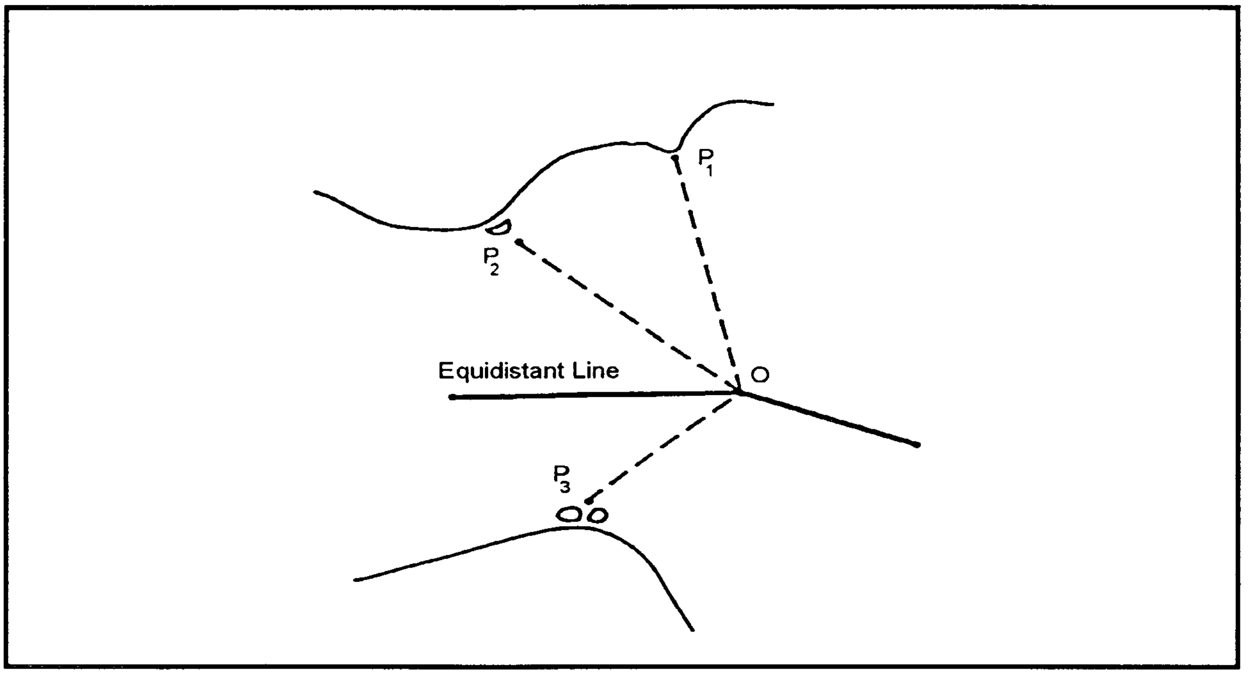
$$B_2 = \frac{(H - FB_1)}{E} \quad (20)$$

$$A = M_1^2 + M_2^2 + \frac{a^2}{b^2} \quad (21)$$

$$B = 2(B_1M_1 - B_2M_2) \quad (22)$$

$$C = B_1^2 + B_2^2 - a^2 \quad (23)$$

Computational Option P2P1



Given: The geographic coordinates (ϕ,λ) of point P_1 (ϕ_1, λ_1), point P_2 (ϕ_2, λ_2) and point P_3 (ϕ_3, λ_3).

Compute: The geographic coordinates (ϕ,λ) of equidistant point O (ϕ_o, λ_o).

$$Az_0^2 + Bz_0 + C = 0 \qquad (1)$$

$$x_0 = M_2 z_0 + B_2 \qquad (2)$$

$$y_0 = M_1 z_0 + B_1 \qquad (3)$$

$$A_0 = \frac{1}{2}\left[\left(x_2^2 + y_2^2 + z_2^2\right) - \left(x_1^2 + y_1^2 + z_1^2\right) \right] \qquad (4)$$

$$A_1 = \frac{(x_2 - x_1)}{A_0} \qquad (5)$$

$$A_2 = \frac{(y_2 - y_1)}{A_0} \qquad (6)$$

$$A_3 = \frac{(z_2 - z_1)}{A_0} \qquad (7)$$

$$C_0 = \frac{1}{2}\left[\left(x_3^2 + y_3^2 + z_3^2\right) - \left(x_1^2 + y_1^2 + z_1^2\right) \right] \qquad (8)$$

$$C_1 = \frac{(x_3 - x_1)}{C_0} \quad (9)$$

$$C_2 = \frac{(y_3 - y_1)}{C_0} \quad (10)$$

$$C_3 = \frac{(z_3 - z_1)}{C_0} \quad (11)$$

$$D = A_1C_2 - A_2C_1 \quad (12)$$

$$M_1 = \frac{(A_3C_1 - A_1C_3)}{D} \quad (13)$$

$$M_2 = \frac{(A_2C_3 - A_3C_2)}{D} \quad (14)$$

$$B_1 = \frac{(A_1 - C_1)}{D} \quad (15)$$

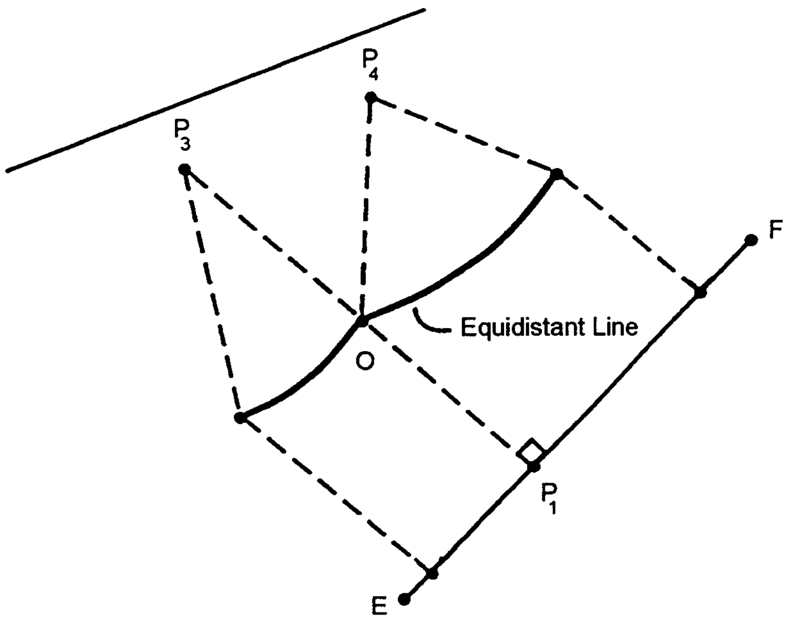
$$B_2 = \frac{(C_2 - A_2)}{D} \quad (16)$$

$$A = M_1^2 + M_2^2 + \frac{a^2}{b^2} \quad (17)$$

$$B = 2(M_1B_1 + M_2B_2) \quad (18)$$

$$C = B_1^2 + B_2^2 - a^2 \quad (19)$$

Computation Option P2L1



Given: The geographic coordinates (φ, λ) of point E (φ_e, λ_e), point F (φ_f, λ_f), point P_3 (φ_3, λ_3) and point P_4 (φ_4, λ_4) .

Compute: The geographic coordinates (φ,λ) of point P_1 (φ_1, λ_1) and equidistant point O (φ_o, λ_o).

$$d_1 y_o^2 + d_2 z_o^2 + d_3 y_o z_o + d_4 y_o + d_5 z_o + d_6 = 0 \qquad (1)$$

$$e_1 y_o^2 + e_2 z_o^2 + e_3 y_o z_o + e_4 y_o + e_5 z_o + e_6 = 0 \qquad (2)$$

$$x_c = \frac{(D - By_o - Cz_o)}{A} \qquad (3)$$

$$z_2 = \frac{\left(Ex_o + Fy_o + Gz_o - Q\right)}{P} \qquad (4)$$

$$x_2 = M_1 z_2 + B_1 \qquad (5)$$

$$y_2 = M_4 z_2 + B_4 \qquad (6)$$

$$A = 2(x_4 - x_3) \qquad (7)$$

$$B = 2(y_4 - y_3) \qquad (8)$$

$$C = 2(z_4 - z_3) \qquad (9)$$

$$H = x_3^2 + y_3^2 + z_3^2 \qquad (10)$$

$$D = \left[\left(x_4^2 + y_4^2 + z_4^2\right) - H\right] \quad (11)$$

$$E = x_f - x_e \quad (12)$$

$$F = y_f - y_e \quad (13)$$

$$G = z_f - z_e \quad (14)$$

$$M_1 = \frac{E}{G} \quad (15)$$

$$M_4 = \frac{F}{G} \quad (16)$$

$$B_1 = x_e - M_1 z_e \quad (17)$$

$$B_4 = y_e - M_4 z_e \quad (18)$$

$$P = EM_1 + FM_4 + G \quad (19)$$

$$Q = EB_1 + FB_4 \quad (20)$$

$$L = \frac{\left(\frac{F - BE}{A}\right)}{P} \quad (21)$$

$$M = \frac{\left(\frac{G - CE}{A}\right)}{P} \quad (22)$$

$$N = \frac{\left(\frac{DE}{A} - Q\right)}{P} \quad (23)$$

$$B_2 = M_1 N + B_1 \quad (24)$$

$$B_3 = M_4 N + B_4 \quad (25)$$

$$A_{11} = -M_1 L \quad (26)$$

$$A_{12} = -M_1 M \quad (27)$$

$$A_{13} = x_3 - M_1 N - B_1 \quad (28)$$

$$A_{21} = -M_4 L \quad (29)$$

$$A_{22} = -M_4 M \quad (30)$$

$$A_{23} = y_3 - M_4 N - B_4 \quad (31)$$

$$A_{31} = -L \quad (32)$$

$$A_{32} = -M \quad (33)$$

$$A_{33} = z_3 - N \quad (34)$$

$$C_1 = M_1^2 + M_4^2 + 1.0 \quad (35)$$

$$C_2 = M_1 B_2 + M_4 B_3 + N \quad (36)$$

$$C_3 = B_2^2 + B_3^2 + N^2 \quad (37)$$

$$d_1 = A_{21} A - A_{11} B + \frac{A\left(C_1 L^2\right)}{2} \quad (38)$$

$$d_2 = A_{32} A - A_{12} C + \frac{A\left(C_1 M^2\right)}{2} \quad (39)$$

$$d_3 = A_{22} A + A_{31} A - A_{11} C - A_{12} B + AC_1 LM \quad (40)$$

$$d_4 = A_{23} A + A_{11} D - A_{13} B + AC_2 L \quad (41)$$

$$d_5 = A_{33}A + A_{12}D - A_{13}C + AC_2M \quad (42)$$

$$d_6 = A_{13}D + \frac{A(C_3 - H)}{2} \quad (43)$$

$$e_1 = A^2 + B^2 \quad (44)$$

$$e_2 = C^2 + \frac{A^2a^2}{b^2} \quad (45)$$

$$e_3 = 2BC \quad (46)$$

$$e_4 = -2BD \quad (47)$$

$$e_5 = -2CD \quad (48)$$

$$e_6 = D^2 - A^2a^2 \quad (49)$$

The Department of the Interior Mission

As the Nation's principal conservation agency, the Department of the Interior has responsibility for most of our nationally owned public lands and natural resources. This includes fostering sound use of our land and water resources; protecting our fish, wildlife, and biological diversity; preserving the environmental and cultural values of our national parks and historical places; and providing for the enjoyment of life through outdoor recreation. The Department assesses our energy and mineral resources and works to ensure that their development is in the best interests of all our people by encouraging stewardship and citizen participation in their care. The Department also has a major responsibility for American Indian reservation communities and for people who live in island territories under U.S. administration.

The Minerals Management Service Mission

As a bureau of the Department of the Interior, the Minerals Management Service's (MMS) primary responsibilities are to manage the mineral resources located on the Nation's Outer Continental Shelf (OCS), collect revenue from the Federal OCS and onshore Federal and Indian lands, and distribute those revenues.

Moreover, in working to meet its responsibilities, the **Offshore Minerals Management Program** administers the OCS competitive leasing program and oversees the safe and environmentally sound exploration and production of our Nation's offshore natural gas, oil and other mineral resources. The MMS **Royalty Management Program** meets its responsibilities by ensuring the efficient, timely and accurate collection and disbursement of revenue from mineral leasing and production due to Indian tribes and allottees, States and the U.S. Treasury.

The MMS strives to fulfill its responsibilities through the general guiding principles of: (1) being responsive to the public's concerns and interests by maintaining a dialogue with all potentially affected parties and (2) carrying out its programs with an emphasis on working to enhance the quality of life for all Americans by lending MMS assistance and expertise to economic development and environmental protection.