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QUASIPARTICLE AGGREGATION IN THE FRACTIONAL QUANTUM HALL EFFECT

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Abstract

Quasiparticles in the Fractional Quantum Hall Effect behave qualitatively like electrons confined to the lowest landau level, and can do everything electrons can do, including condense into "second generation" Fractional Quantum Hall ground states. I review in this paper the reasoning leading to variational wavefunctions for ground state and quasiparticles in the 1/3 effect. I then show how two-quasiparticle eigenstates are uniquely determined from symmetry, and how this leads in a natural way to variational wavefunctions for composite states which have the correct densities (2/5, 2/7, ...). I show in the process that the boson, anyon and fermion representations for the quasiparticles used by Haldane, Halperin and me are all equivalent. I demonstrate a simple way to derive Halperin's multiple-valued quasiparticle wavefunction from the correct single-valued electron wavefunction.

In this paper I shall try to summarize and clarify what we know about the condensation of quasiparticles in the Fractional Quantum Hall effect into the hierarchy of stable states first suggested by Haldane.[1] Quasiparticles are acknowledged to be similar to electrons in the lowest landau level, yet they condense at densities different from those expected of electrons. Haldane has called these quasiparticles bosons, I have called them fermions,[2] and Halperin[3] has called them "anyons" obeying "fractional statistics." Who is right? What physically causes the quasiparticles to pack at the densities they do? Can quasiparticle motion really be understood by analogy with electron motion? I shall try to show in this paper, by supplying missing logical steps, that, in fact, everyone is right. Quasiparticles are like electrons in that their separations are quantized because of angular momentum conservation, but different from electrons in that the quantized separations that occur are compatible with "fractional statistics" as Halperin has asserted. Quasiparticles admit of both a boson and a fermion description, as is also the case for electrons in the lowest landau level. Multiplying together pair quasiparticle wavefunctions to make a variational wavefunction for the composite state, as was done with electrons to make the 1/3 state, leads to a class of wavefunctions for the composite suggested by Halperin and by me. One of these can be shown to be equivalent to Halperin's multi-valued "fractional statistics" wavefunction, which is known to describe a liquid at the appropriate density and to have a low energy. The existence of legitimate variational wavefunctions for these states is important because it is the physical basis for "angular momentum counting," which I shall explain in detail. This method is more powerful on a sphere than it is in a planar geometry because the charge density of an eigenstate of angular momentum is automatically uniform on a sphere. Angular momentum counting can work on a plane only if the state is known in advance to be uniform.

The Fractional Quantum Hall effect occurs in a two-dimensional electron gas subjected to a magnetic field. This system may be described by the idealized Hamiltonian

$$\mathcal{H} = \sum_j^N \left[\frac{1}{2m_e} \left| \frac{\hbar}{i}\vec{\nabla}_j - \frac{e}{c}\vec{A}_j \right|^2 + V(z_j) \right] + \sum_{j<k}^N \frac{e^2}{|z_j - z_k|} \quad , \tag{1}$$

where z_j is the location of the j^{th} electron expressed as a complex number, and where $V(z)$ is the potential generated by a uniform neutralizing background of density σ, in the manner

$$V(z) = -\sigma e^2 \int_{\text{sample}} \frac{d^2 z'}{|z - z'|} \quad , \tag{2}$$

In the absence of coulomb interactions, and in symmetric gauge

$$\vec{A} = \frac{H_0}{2}\left[y\hat{x} - x\hat{y} \right] \quad , \tag{3}$$

the single body eigenstates may be written

$$\psi_{m,n} = e^{\frac{1}{4}|z|^2} \left(\frac{\partial}{\partial z^*}\right)^m \left(\frac{\partial}{\partial z}\right)^n e^{-\frac{1}{2}|z|^2} \quad , \tag{4}$$

with the magnetic length $a_0 = \sqrt{\frac{\hbar c}{eH_0}}$ set to 1. The energy of this state is $(n+1/2)\hbar\omega_c$, where $\omega_c = eH_0/mc$ is the cyclotron frequency. The manifold of states with the same n is the n^{th} landau level. I shall consider only the case when the lowest landau level is partially occupied and when the coulomb interactions are too weak to significantly mix landau levels ($e^2/a_0 < \hbar\omega_c$). When this is the case, the ground state and low energy excitations of the system are to a good approximation comprised of single-body wavefunctions in the lowest landau level solely. The most general such wavefunction takes the form

$$\Psi(z_1, \ldots, z_N) = P(z_1, \ldots, z_N) \prod_{j<k}^N (z_j - z_k)\, e^{-\frac{1}{4}\sum_\ell^N |z_\ell|^2} \quad , \tag{5}$$

where P is a symmetric polynomial. My wavefunction for the 1/3 state is one of a series of the form

$$|m\rangle = \prod_{j<k}^N (z_j - z_k)^m\, e^{-1/4 \sum_\ell^N |z_\ell|^2} \quad , \tag{6}$$

where m is an odd integer. These are the only functions which (1) lie in the lowest landau level, (2) are eigenstates of angular momentum and (3) are a product over pairs of some function of the difference coordinate. The restriction to functions of this type is motivated partially by our experience from liquid Helium, but also by the observation that

$$\chi_m = (z_1 - z_2)^m e^{-\frac{1}{4}(|z_1|^2 + |z_2|^2)} \quad , \tag{7}$$

is the only 2-particle wavefunction in the lowest landau level which has internal angular momentum m. In order to construct a wavefunction analogous to $|m\rangle$ for quasiparticles, it is necessary to construct and understand two-quasiparticle wavefunctions analogous to χ_m. I shall do this using quasiparticle creation operators, defined in the manner

$$S_{z_A}|m\rangle = e^{-1/4 \sum_{\ell}^{N} |z_\ell|^2} \prod_i^N (z_i - z_A) \prod_{j<k}^{N} (z_j - z_k)^m \quad , \tag{8}$$

and

$$S_{z_B}^{\dagger}|m\rangle = e^{-1/4 \sum_{\ell}^{N} |z_\ell|^2} \prod_i^N \left(2\frac{\partial}{\partial z_i} - z_B^*\right) \prod_{j<k}^{N} (z_j - z_k)^m \quad , \tag{9}$$

for a quasihole or quasielectron, respectively, residing at z_0. These operators approximate the action on the system of a thought experiment in which the system is pierced at location z_0 with an infinitely thin magnetic solenoid, and through this solenoid is adibatically passed a flux quantum hc/e. This procedure maps the exact ground state onto an exact excited state of the many-body Hamiltonian. The operators slide the ground state over so as to pile up excess charge ± e/m at z_0. That they do this most easily seen by interpreting the square of the wavefunction as the probability distribution function of a classical plasma, in the manner

$$\langle m|S_{z_0}^{\dagger} S_{z_0}|m\rangle = \int \cdots \int e^{-\beta\Phi'} d^2z_1 \ldots d^2z_N \quad , \tag{10}$$

where $\beta = 1/m$ and

$$\Phi' = -2m^2 \sum_{j<k}^{N} \ln|z_j - z_k| + \frac{m}{2} \sum_{\ell}^{N} |z_\ell|^2 - 2m \sum_i^N \ln|z_i - z_0| \quad . \tag{11}$$

Φ' is the potential energy of particles of "charge" m repelling one another with logarithmic interactions, the natural "coulomb" interaction in two dimensions, being attracted via the same "coulomb" interaction by a background of "charge" density $\sigma_1 = (2\pi a_0^2)^{-1}$, and being repelled by a "charge" 1 particle located at z_0. Since this plasma must be locally neutral, electrons distributed themselves uniformly at density $\sigma_m = \sigma_1/m$, except within a Debye length $(a_0/\sqrt{2})$ of z_0, where screening charge -1/m electrons accumulates. Similar reasoning works for the quasielectrons, except that the accumulated charge is +1/m electrons. The energy $S_{z_0}|m\rangle$ or $S_{z_0}^{\dagger}|m\rangle$ does not depend on z_0, so long as z_0 resides inside the sample, so that any linear combination of these is also an eigenstate. In particular, the elementary symmetric polynomials S_k defined by

$$\prod_i^N (z_i - z_0) = \sum_{k=0}^{N} S_k(z_1, \ldots, z_N)\, z_0^k \quad , \tag{12}$$

generate quasiparticles in angular momentum states.

I wish now to determine the two-quasiparticle eigenstates analogous to χ_m in Eq. (7). To do this, for quasiholes, I shall project the Hamiltonian onto the set of states of the form $S_{z_A} S_{z_B}|m\rangle$ and then diagonalize this projected Hamiltonian. I first need to calculate the normalization integral

$$\langle m|S_{z_B}^{\dagger} S_{z_A}^{\dagger} S_{z_A} S_{z_B}|m\rangle = \int \cdots \int \prod_{j<k}^{N} |z_j - z_k|^{2m}$$

$$\times \prod_i^N |z_i - z_A|^2 |z_i - z_B|^2 \, e^{-\frac{1}{2}\sum_\ell^N |z_\ell|^2} \, d^2z_1 \ldots d^2z_N \quad . \tag{13}$$

This is not difficult because the integrand is the probability distribution function of a classical plasma $e^{-\beta\Phi''}$, with $\beta = 1/m$ and

$$\Phi'' = -2m^2 \sum_{j<k}^N \ell n|z_j - z_k| + \frac{m}{2}\sum_\ell^N |z_\ell|^2 - 2m \sum_i^N \left[\ell n|z_i - z_A| + \ell n|z_i - z_B|\right] . \tag{14}$$

If one were to add to this potential energy a term

$$\Delta\Phi = \frac{1}{2}(|z_A|^2 + |z_B|^2) - 2\ \ell n|z_A - z_B| \quad , \tag{15}$$

to account for the interaction of the "charge" 1 particles at z_A and z_B with the background and with each other, then Eq. (13) would be the partition function of a plasma with two of its particles held fixed. Up to an unimportant constant, this is just the probability to find these particles at z_A and z_B if they are allowed to roam around in the plasma. Thus, we have

$$\langle m|S_{z_B}^\dagger S_{z_A}^\dagger S_{z_A} S_{z_B}|m\rangle = C \, \frac{e^{\frac{-1}{2m}(|z_A|^2 + |z_B|^2)}}{|z_A - z_B|^{2/m}} \, g_{22}(|z_A - z_B|)$$

$$= C \, e^{\frac{-1}{2m}(|z_A|^2 + |z_B|^2)} F\left[|z_A - z_B|^2\right] \quad , \tag{16}$$

where g_{22} is the radial distribution function for particles of "charge" 1 and C is a constant. I have performed hypernetted chain calculations for g_{22} and have found F to be approximately fit by the formula

$$F\left[|z|^2\right] \cong \frac{1}{4\pi m} \int \frac{e^{-\frac{1}{4}|z - z'|^2}}{|z'|^{2/3}} \, d^2z' \quad . \tag{17}$$

For our purposes it is important only that F falls off asymptotically as $|z_A - z_B|^{-2/m}$ and contains no large fourier components. In order to make the translational invariance of this problem apparent I shall take as a basis set the wavefunctions

$$|z_A, z_B\rangle = e^{-\frac{1}{4m}(|z_A|^2 + |z_B|^2)} S_{z_A} S_{z_B} |m\rangle \quad . \tag{18}$$

One sees by inspection that the overlap matrix $\langle z_A', z_B' | z_A, z_B\rangle$ is analytic in the variables z_A, z_B, z_A^*, and z_B^*, and thus is determined uniquely by analytic continuation of the normalization integral to be

$$\langle z_{A'}, z_{B'} | z_A, z_B \rangle = C\, e^{-\frac{1}{4m}(|z_A|^2+|z_B|^2+|z_{A'}|^2+|z_{B'}|^2)}$$

$$\times\, e^{\frac{1}{2m}(z_{A'}^* \cdot z_A + z_{B'}^* \cdot z_B)} F\Big[(z_A - z_B)(z_{A'}^* - z_{B'}^*)\Big] \quad . \tag{19}$$

Similar reasoning applied to matrix elements of energy leads to

$$\langle z_{A'}, z_{B'} | \mathcal{H} | z_A, z_B \rangle = C\, e^{-\frac{1}{4m}(|z_A|^2+|z_B|^2+|z_{A'}|^2+|z_{B'}|^2)}$$

$$\times\, e^{\frac{1}{2m}(z_{A'}^* \cdot z_A + z_{B'}^* \cdot z_B)} E\Big[(z_A - z_B)(z_{A'}^* - z_{B'}^*)\Big] \quad , \tag{20}$$

where E is fit roughly by the formula

$$E\Big[|z|^2\Big] \cong \frac{1}{4\pi m} \int \frac{e^{-\frac{1}{4m}|z-z'|^2}}{|z'|^{2/m}} \left[\frac{(e/m)^2}{|z'|}\right] d^2z' \quad , \tag{21}$$

with the ground state energy taken to be 0. Matrices of this form are diagonalized by the states

$$|n\rangle = \iint e^{-\frac{1}{4m}(|z_A|^2+|z_B|^2)} (z_A^* - z_B^*)^n |z_A, z_B\rangle \; d^2z_A \; d^2z_B \quad , \tag{22}$$

where n is an <u>even</u> integer. $|n\rangle$ is analogous to χ_{n+1}, and is in fact its electron-hole conjugate when m=1. The integer is even because we are using the bose representation for the quasiparticles. S_{z_A} and S_{z_B} obviously commute. It would need to be odd had we chosen as our basis the states $(z_A^*-z_B)|z_A,z_B\rangle$, which form the fermi representation. The state $(z_A-z_B)|z_A,z_B\rangle$ is the electron-hole conjugate of the two-electron wavefunction

$$\psi(z_1, z_2) = \varphi_{z_A}(z_1)\varphi_{z_B}(z_2) - \varphi_{z_B}(z_1)\varphi_{z_A}(z_2) \quad , \tag{23}$$

with

$$\varphi_{z_A}(z) = e^{-\frac{1}{4}|z|^2}\, e^{\frac{1}{2}zz_A^*}\, e^{-\frac{1}{4}|z_A|^2} \quad , \tag{24}$$

when m=1. The energy eigenvalue for $|n\rangle$ is given by

$$\frac{\langle n|\mathcal{H}|n\rangle}{\langle n|n\rangle} = \frac{\langle E_0\rangle_n}{\langle F_0\rangle_n} \quad , \tag{25}$$

with

$$\langle F_0\rangle_n = (2\pi m)^2 \frac{\displaystyle\int F_0\left[|z|^2\right] |z|^{2n} e^{-\frac{1}{4m}|z|^2} d^2z}{\displaystyle\int |z|^{2n} e^{-\frac{1}{4m}|z|^2} d^2z} , \tag{26}$$

and similarly for $\langle E_0\rangle_n$, where F_0 is an "image enhanced" version of F, defined by

$$F\left[|z|^2\right] = \frac{1}{4\pi m} \int e^{-\frac{1}{4m}|z-z'|^2} F_0\left[|z'|^2\right] d^2z' . \tag{27}$$

Since this is an important result, I shall add some algebraic details. We wish to show that

$$\iint e^{-\frac{1}{4m}(|z_A|^2+|z_B|^2)} (z_A^*-z_B^*)^n \langle z_{A'}, z_{B'}|z_A, z_B\rangle d^2z_A\, d^2z_B$$

$$= \langle F_0\rangle_n (z_{A'}^*-z_{B'}^*)^n e^{-\frac{1}{4m}(|z_{A'}|^2+|z_{B'}|^2)} . \tag{28}$$

Letting $\Delta = z_A-z_B$ and $\Delta' = z_A'-z_B'$, and making use of the identity

$$\int z^n e^{(zz_A^*+z^*z_B)} e^{-|z|^2} d^2z = \pi\, z_B^n\, e^{z_A^*z_B} , \tag{29}$$

we have

$$\iint e^{-\frac{1}{4m}(|z_A|^2+|z_B|^2)} (z_A^*-z_B^*)^n \langle z_{A'}, z_{B'}|z_A, z_B\rangle d^2z_A\, d^2z_B$$

$$= \frac{1}{4} e^{-\frac{1}{4m}(|z_{A'}|^2+|z_{B'}|^2)} \iint e^{-\frac{1}{4m}|\Delta|^2} e^{\frac{1}{4m}(\Delta z_0^*+\Delta'^* z_0)}$$

$$\times (\Delta^*)^n e^{-\frac{1}{4m}|z_C|^2} F_0\left[|z_0|^2\right] d^2z_0\, d^2\Delta$$

$$= (\Delta'^*)^n e^{-\frac{1}{4m}(|z_{A'}|^2+|z_{B'}|^2)}$$

$$\times \frac{\pi m}{(4m)^n n!} \int |z_0|^{2n} e^{-\frac{1}{4m}|z_0|^2} F_0\left[|z_0|^2\right] d^2 z_0 \quad . \tag{30}$$

Like the two-electron state χ_{n+1}, the two-quasiparticle state does not depend on the repulsive potential between quasiparticles. It is also not basis dependent. If we solve the problem in the fermi representation, the overlap matrix becomes

$$\langle z_{A'}, z_{B'} | (z_{A'}^* - z_{B'}^*)(z_A - z_B) | z_A, z_B \rangle = C\, e^{-\frac{1}{4m}(|z_A|^2 + |z_B|^2 + |z_{A'}|^2 + |z_{B'}|^2)}$$

$$\times\, e^{\frac{1}{2m}(z_{A'}^* z_A + z_{B'}^* z_B)} F^f\left[(z_A - z_B)(z_{A'}^* - z_{B'}^*)\right] \quad , \tag{31}$$

with

$$F^f\left[|z|^2\right] = |z|^2 F\left[|z|^2\right] \quad , \tag{32}$$

and is diagonalized by wavefunctions of the form

$$|n+1\rangle = \iint e^{-\frac{1}{4m}(|z_A|^2 + |z_B|^2)} (z_A^* - z_B^*)^{n+1} (z_A - z_B) | z_A, z_B \rangle\, d^2 z_A\, d^2 z_B , \tag{33}$$

with n+1 <u>odd</u>. However, this is the same state as $|n\rangle$, since for any function g,

$$\iint e^{-\frac{1}{4m}(|z_A|^2 + |z_B|^2)} (z_A^* - z_B^*)^n\, g(|z_A - z_B|^2) | z_A, z_B \rangle\, d^2 z_A\, d^2 z_B$$

$$\left[\frac{\pi m}{2^n} \int e^{-\frac{1}{4m}|\Delta|^2} |\Delta|^{2n} g(|\Delta|^2)\, d^2\Delta \right] \sum_{k=0}^{\ell} (-1)^k S_k S_{\ell-k} |m\rangle \quad , \tag{34}$$

with S_k defined per Eq. (12). This is extremely important. The actual wavefunction generated by an expression of the form of Eq. (34) is *unaffected by any factor in the integrand not carrying angular momentum*. It is also unaffected by the length scale transformation

$$e^{-\frac{1}{2m}(|z_A|^2 + |z_B|^2)} \rightarrow e^{-\alpha(|z_A|^2 + |z_B|^2)} \quad , \tag{35}$$

with α any positive number. What is affected are the normalization and formal expression for the energy, which in the case of the fermion representation is

$$\frac{\langle n+1|\mathcal{H}|n+1\rangle}{\langle n+1|n+1\rangle} = \frac{\langle E_0^f\rangle_{n+1}}{\langle F_0^f\rangle_{n+1}} \quad , \tag{36}$$

with

$$E^f\Big[\,|z|^2\Big] = |z|^2\; E\Big[\,|z|^2\Big] \quad . \tag{37}$$

The energies in Eq. (36) and Eq. (25) are identical, despite their involving different moments, because there is a $z_0 \to 0$ core contribution to the energy which is strongly enhanced by the sharpening procedure Eq. (27). In injudicious choice of basis, therefore, can generate expressions for the charge density and energy which are formally correct but technically unmanageable. For the case m=1, the "optimal" representation is obviously Eq. (33), although accurate calculations can also be done starting from Eq. (22). When $m \neq 1$, neither is optimal, but both are acceptable. For large m, it is easier to perform accurate calculations using the boson representation.

The generalization of this pair wavefunction to a composite state containing M quasiparticles with "pair" quantum number n is

$$|n\rangle = \int \cdots \int \prod_{j<k}^{M} (\eta_j^* - \eta_k^*)^n \prod_i^M S_{\eta_i} |m\rangle \; e^{-\frac{1}{2m}\sum_\ell^M |\eta_\ell|^2} \, d^2\eta_1 \; \ldots \; d^2\eta_M \quad , \tag{38}$$

for the bose representation and

$$|n+1\rangle = \int \cdots \int \prod_{j<k}^{M} (\eta_j^* - \eta_k^*)^{n+1} (\eta_j - \eta_k) \prod_i^M S_{\eta_i} |m\rangle$$

$$\times \quad e^{-\frac{1}{2m}\sum_\ell^M |\eta_\ell|^2} \, d^2\eta_1 \; \ldots \; d^2\eta_M \quad , \tag{39}$$

for the fermi representation. Up to length scale changes of the form of Eq. (35), these are the wavefunctions proposed by Halperin[3] and by me[2] for the 2/7 state. As opposed to the case of two quasiparticles, the factor $|\eta_j - \eta_k|^2$ in Eq. (39) makes these wavefunctions slightly different. Both states are normalized in a way analogous to the procedure in Eq. (30). For the bose wavefunction, the diagonal overlap matrix elements

$$F\Big[\eta_1, \; \ldots \; , \eta_M\Big] = e^{-\frac{1}{2m}\sum_\ell^M |\eta_\ell|^2} \langle m| \prod_i^M S_{\eta_i}^\dagger \prod_j^M S_{\eta_j} |m\rangle \quad , \tag{40}$$

which behave asymptotically as

$$F\Big[\eta_1, \; \ldots \; , \eta_M\Big] \cong \prod_{j<k}^{M} |\eta_j - \eta_k|^{-2/m} \quad , \tag{41}$$

image-enhanced in the manner

$$F\Big[\eta_1, \; \ldots \; , \eta_M\Big] = \frac{1}{(2\pi m)^M} \int \cdots \int e^{-\frac{1}{2m}\sum_\ell^M |\eta_\ell - \eta_\ell'|^2}$$

$$\times F_0\left[\eta_1', \ \ldots \ ,\eta_M'\right] d^2\eta_1' \ \ldots \ d^2\eta_M' \quad , \tag{42}$$

lead to the exact result

$$\langle n|n\rangle = (2\pi m)^M \int\cdots\int e^{-\frac{1}{2m}\sum_{\ell}^{M}|\eta_\ell|^2} \prod_{j<k}^{M} |\eta_j-\eta_k|^{2n}$$

$$F_0\left[\eta_1 \ \ldots \ \eta_M\right] d^2\eta_1 \ \ldots \ d^2\eta_M \quad . \tag{43}$$

For the fermion representation we have the similar exact result

$$\langle n+1|n+1\rangle = (2\pi m)^M \int\cdots\int e^{-\frac{1}{2m}\sum_{\ell}^{M}|\eta_\ell|^2} \prod_{j<k}^{M} |\eta_j-\eta_k|^{2n+2}$$

$$\times F_0^f\left[\eta_1, \ \ldots \ ,\eta_M\right] d^2\eta_1 \ \ldots \ d^2\eta_M \quad , \tag{44}$$

where F_0^f is the image-enhanced version of F^f, with

$$F^f = \prod_{j<k}^{M} |\eta_j-\eta_k|^2 \ F \quad . \tag{45}$$

However, Eq. (43) and Eq. (44) are technically infeasible to evaluate because both F and F^f are pathological near the surface $\eta_1=\eta_2= \ldots =\eta_M$: F has an enormous maximum and F^f has a correspondingly deep minimum. These functions behave increasingly uncontrollably under image enhancement as the number of quasiparticles is increased. As opposed to the case of two quasiparticles, neither the bose nor the fermi representation is adequate for performing reliable calculations. We are led instead to the wavefunction

$$|n + \tfrac{1}{m}\rangle = \int\cdots\int \prod_{j<k}^{M} (\eta_j^*-\eta_k^*)^n \ |\eta_j-\eta_k|^{2/m} \prod_{i}^{M} S_{\eta_i} |m\rangle$$

$$\times e^{-\frac{1}{2m}\sum_{\ell}^{M}|\eta_\ell|^2} d^2\eta_1 \ \ldots \ d^2\eta_M \quad , \tag{46}$$

which is the "fractional" representation. There appears to be no exact result for this wavefunction analogous to Eq. (43) and Eq. (44), so I shall interpret it as an interpolation.

I define

$$\overline{F} = \prod_{j<k}^{M} |\eta_j-\eta_k|^{2/m} \ F \quad , \tag{47}$$

and then image-enhance as well as possible

$$\overline{F} \cong \frac{1}{(2\pi m)^M} \int \cdots \int e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell - \eta'_\ell|^2} \, \overline{F}_0 \, d^2\eta'_1 \ldots d^2\eta'_M \quad , \tag{48}$$

to obtain

$$\langle n + \tfrac{1}{m} | n + \tfrac{1}{m} \rangle \propto \int \cdots \int \prod_{j<k}^{M} |\eta_j - \eta_k|^{2n+2/m} \, \overline{F}_0 \, e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \, d^2\eta_1 \ldots d^2\eta_M$$

$$\cong \int \cdots \int \prod_{j<k}^{M} |\eta_j - \eta_k|^{2n+2/m} \, e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \, d^2\eta_1 \ldots d^2\eta_M \quad . \tag{49}$$

The integrand of Eq. (49) is the square of the multiply-valued wavefunction Halperin assumed describes quasiparticle condensation. It can legitimately be interpreted as the probability to find the quasiparticles at locations $\eta_1, \ldots, \eta_M$. Thus quasiparticles distribute themselves like a plasma of "charge" n+1/m particles in a background "charge" density $1/2\pi m$. This leads to an actual electron density of $\frac{1}{2m}\left[\frac{n}{1+nm}\right] = \frac{1}{2\pi}\left[\frac{2}{7}\right]$ when m=3 and n=2.

The wavefunction analogous to Eq. (38) for quasielectrons is

$$|n\rangle = \int \cdots \int \prod_{j<k}^{M} (\eta_j - \eta_k)^n \prod_{i}^{M} S^\dagger_{\eta_i} |m\rangle \, e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \, d^2\eta_1 \ldots d^2\eta_M \quad . \tag{50}$$

I presently believe that this wavefunction is most suitable for performing calculations on the quasielectron condensate. My reasons are not rigorous, however, and more work needs to be done. The overlap integral for this wavefunction

$$F\left[\eta_1, \ldots, \eta_M\right] = e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \langle m | \prod_{i}^{M} S_{\eta_i} \prod_{j}^{M} S^\dagger_{\eta_j} |m\rangle \quad , \tag{51}$$

behaves asymptotically as Eq. (41) as may be seen by integrating it by parts to obtain

$$F = e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \int \cdots \int e^{-\frac{1}{2}\sum_{\alpha}^{N} |z_\alpha|^2} \prod_{\gamma<\delta}^{M} |z_\gamma - z_\delta|^{2m} \prod_{\beta}^{M} \prod_{i}^{N}$$

$$\times |z_\beta - \eta_i|^2 \left[1 - 4 \sum_{j \neq k}^{M} \frac{1}{(z_\beta - \eta_j)(z^*_\beta - \eta^*_k)} + \ldots \right] d^2z_1 \ldots d^2z_N \quad . \tag{52}$$

This is the partition function of a plasma in which some of the particles have short range repulsions in addition to "coulomb" forces. I believe that this function has no pathology near $\eta_1 = \eta_2 = \cdots = \eta_M$ because of the short range corrections. If this is so, then it is image enhanceable, and we may write meaningfully

$$\langle n | n \rangle \propto \int \cdots \int \prod_{j<k}^{M} |\eta_j - \eta_k|^{2n} \, F_0 \, e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \, d^2\eta_1 \ldots d^2\eta_M$$

$$\cong \int \cdots \int \prod_{j<k}^{M} |\eta_j - \eta_k|^{2n-2/m} \; e^{-\frac{1}{2m}\sum_{\ell}^{M} |\eta_\ell|^2} \; d^2\eta_1 \; \ldots \; d^2\eta_M \quad . \tag{53}$$

The integrand may again be interpreted as the probability to find the quasiparticles at locations $\eta_1, \cdots, \eta_M$, and one thus calculates the charge density of this state to be $1/2\pi[2/5]$.

It is important that the densities I obtain are compatible with those obtained by angular momentum counting. Wavefunctions of the form of Eq. (44), (45) or (47) increase the system's angular momentum by

$$\Delta L = MN - nM^2/2 \tag{54}$$

in the limit of large M and N. They do this by expelling charge from a dise of radius R, at the center of the sample and depositing it at the sample edge, so as to increase the sample radius to R_2. Wavefunctions of the form of Eq. (45) decrease the angular momentum by ΔL by adding to a disc of radius R, charge removed from the sample edge, thus decreasing its radius to R_2. If we assume that the excess or missing charge inside R_1 is uniformly $\delta\rho$, then we have three quantities to determine (R_1, R_2 and $\delta\rho$) and three equations:

$$\pi R_1^2 \; \delta\rho = \pm \; (M/m) \quad , \tag{55}$$

which relates the missing or excess charge to the number of quasiparticles,

$$\pi R_2^2 \; \left(\frac{1}{2\pi m}\right) + \pi R_1^2 \; \delta\rho = N \quad , \tag{56}$$

which equates the total charge before and after the addition of quasiparticles, and

$$\frac{\pi R_2^4}{4} \; \frac{1}{2\pi m} + \frac{\pi R_1^4}{4} \; \delta\rho = \frac{mN^2}{2} \pm \Delta L \quad , \tag{57}$$

which accounts for the angular momentum increase or decrease. (An electron at radius r carries angular momentum $r^2/2$). Solving these equations one obtains

$$\delta\rho = -\left(\frac{1}{2\pi m}\right) \left[\frac{1}{1 \pm nm}\right] \quad , \tag{58}$$

or a total charge density of $1/2\pi$ [$n/nm \pm 1$].

I remark finally that the generalization to elementary excitations of the composite state is straightforward and leads to wavefunctions such as

$$\int \cdots \int \prod_{i}^{M} (\eta_i^* - z_0^*) \prod_{j<k}^{M} (\eta_j^* - \eta_k^*)^n \; |\eta_j - \eta_k|^{2/m}$$

$$\times \prod_i^M S_{\eta_i} |m> e^{-\frac{1}{2m}\sum_\ell^M |\eta_\ell|^2} d^2\eta_1 \ldots d^2\eta_M \quad , \qquad (59)$$

for a quasihole excitation of the composite 2/7 state. This is a particle of charge 1/7, exactly as expected.

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