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EXCITONS IN THE FRACTIONAL QUANTUM HALL EFFECT*

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Quasiparticles of charge 1/m in the Fractional Quantum Hall Effect form excitons, which are collective excitations physically similar to the transverse magnetoplasma oscillations of a Wigner crystal. I propose a variational exciton wavefunction which shows explicitly that the magnetic length is effectively longer for quasiparticles than for electons. I use this wavefunction to estimate the dispersion relation of these excitons and the matrix elements to generate them optically out of the ground state. I then use these quantities to describe a type of nonlinear conductivity which may occur in these systems when they are relatively clean.

The Fractional Quantum Hall Effect is due to the condensation of a two-dimensional electron gas in a strong magnetic field H_0 into a "new state of matter," described approximately by the ground state wavefunction

$$|m\rangle = \prod_{j<k}^{N} (z_j - z_k)^m \, e^{-1/4 \sum_{\ell}^{N} |z_\ell|^2} \quad (1)$$

where m is an odd integer, z_j is the location of the j^{th} electron expressed as a complex number, and the magnetic length $a_0 = (\hbar c/eH_0)^{1/2}$ is taken to be 1. The elementary excitations of this ground state are quasiparticles of charge 1/m represented approximately by the wavefunctions[1,2]

$$S_{z_A}|m\rangle = e^{-1/4 \sum_{\ell}^{N} |z_\ell|^2} \prod_{i}^{N} (z_i - z_A) \prod_{j<k}^{N} (z_j - z_k)^m \quad (2)$$

and

$$S_{z_B}^{\dagger}|m\rangle = e^{-1/4 \sum_{\ell}^{N} |z_\ell|^2} \prod_{i}^{N} \left(2\frac{\partial}{\partial z_i} - z_B^{*}\right) \prod_{j<k}^{N} (z_j - z_k)^m \quad (3)$$

for a quasihole residing at z_A or a quasielectron residing at z_B. In this paper I discuss bound states of two quasiparticles of opposite sign. Excitons play the role in the Fractional Quantum Hall Effect of the low-lying "transverse" phonons of a Wigner crystal in a strong magnetic field. An energy gap to make them is necessary for the electrons to conduct with no resistive loss.[3]

The Hamiltonian for this system may be written

$$\mathcal{H} = \sum_{j}^{N} \left[\frac{1}{2m_e} \left| \frac{\hbar}{i}\vec{\nabla}_j - \frac{e}{c}\vec{A}_j \right|^2 + V(z_j) \right] + \sum_{j<k}^{N} \frac{e^2}{|z_j - z_k|} \quad . \quad (4)$$

where j and k index the N electrons and V is the potential generated by a uniform neutralizing background. The electrons are cold and confined to the lowest Landau level. In symmetric gauge, when the vector potential is

$$\vec{A} = \frac{H_0}{2}\left[y\hat{x} - x\hat{y}\right] \quad .$$

noninteracting electrons occupy degenerate states of the form $z^k \exp(-|z|^2/4)$. I consider the high field limit ($\hbar\omega_c > e^2/a_0$, with $\omega_c = eH_0/m_e c$), when mixing of other states into the many-body wavefunction is unimportant.

The two-quasiparticle problem is soluble because it is physically equivalent to the problem[4]

$$\mathcal{H} = \frac{1}{2m_e}\left|\frac{\hbar}{i}\vec{\nabla}_1 - \frac{e}{c}\vec{A}_1\right|^2 + \frac{1}{2m_e}\left|\frac{\hbar}{i}\vec{\nabla}_2 + \frac{e}{c}\vec{A}_2\right|^2 - \frac{e^2}{|z_1 - z_2|} \quad . \quad (5)$$

but for the substitution $e \rightarrow e/m$. In the high field limit the eigenstates are

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$$|z_0\rangle = \frac{1}{\sqrt{2\pi L}} e^{-\frac{1}{4}(|z_1|^2+|z_2|^2)}$$

$$e^{\frac{1}{2}z_1 z_2^*} \, e^{\frac{1}{2}(z_2^* z_0 - z_1 z_0^*)} \, e^{-\frac{1}{4}|z_0|^2} \quad . \qquad (6)$$

where L^2 is the sample area, for any translationally invariant interaction between the particles. For the case of a coulomb attraction, the eigenvalue is

$$\langle z_0|\mathcal{H}|z_0\rangle = \hbar\omega_c - \sqrt{\frac{\pi}{8}} \, e^{-\frac{1}{4}|z_0|^2} I_0\left[\tfrac{1}{4}|z_0|^2\right] \frac{e^2}{a_0} \quad . \qquad (7)$$

where I_0 is a modified Bessel function of the first kind. $|z_0\rangle$ describes particles displaced z_0 from one another and traveling with momentum $q=|z_0|$ ($a_0=1$) perpendicular to z_0. The exciton energy at q is the coulomb energy of two particles a distance q apart.

To solve the two quasiparticle problem I. make the "variational" assumption that the quasiexcitons are linear combinations of states of the form $S^{\dagger}_{z_B} S_{z_A} |m\rangle$. $S^{\dagger}_{z_B} S_{z_A} |m\rangle$ describes electrons uniformly spread out to a density $(2\pi m)^{-1}$ except within a magnetic length of z_A or z_B, where the density is depressed or enhanced so as to accumulate an excess charge $\pm$ 1/m. $S^{\dagger}_{z_B}$ and S_{z_A} do not commute. I do not use the combination $S_{z_A} S^{\dagger}_{z_B}$ or higher-order combinations such as $S_{z_C} S^{\dagger}_{z_B} S_{z_A}$, because this extra variational freedom would unduely complicate the calculation. A highly accurate calculation would need to include all combinations of S's.

The Hamiltonian projected onto the set of states of this form is diagonalized by the wavefunction

$$|z_0\rangle = \frac{1}{\sqrt{2\pi m L}} \iint e^{-\frac{1}{2m}(|z_A|^2+|z_B|^2)} \, e^{\frac{1}{2m} z_A^* z_B}$$

$$\times \, e^{\frac{1}{2m}(z_B z_0^* - z_A^* z_0)} \, e^{-\frac{1}{4m}|z_0|^2} \, S^{\dagger}_{z_B} S_{z_A} |m\rangle \, d^2 z_A \, d^2 z_B \quad . \qquad (8)$$

The energy eigenvalue is given by

$$\frac{\langle z_0|\mathcal{H}|z_0\rangle}{\langle z_0|z_0\rangle} = \frac{E_0(|z_0|)}{F_0(|z_0|)} \quad . \qquad (9)$$

where F_0 and E_0 are related to the diagonal matrix elements of overlap

$$\langle m|S^{\dagger}_{z_A} S_{z_B} S^{\dagger}_{z_B} S_{z_A}|m\rangle = \langle m|m\rangle \, e^{2\beta\left[U(mN+1)-U(mN)\right]} \, e^{-(\beta\mu^+_{ex} + \beta\mu^-_{ex}+1)}$$

$$\times \, e^{\frac{1}{2m}(|z_A|^2+|z_B|^2)} \, F(|z_A - z_B|) \quad . \qquad (10)$$

and energy

$$\frac{\langle m|S^{\dagger}_{z_A} S_{z_B} \mathcal{H} S^{\dagger}_{z_B} S_{z_A}|m\rangle}{\langle m|S^{\dagger}_{z_A} S_{z_B} S^{\dagger}_{z_B} S_{z_A}|m\rangle} = \frac{E(|z_A - z_B|)}{F(|z_A - z_B|)} \quad . \qquad (11)$$

by the smoothing operation

$$E(|z_0|) = \frac{1}{2\pi m} \int e^{-\frac{1}{2m}|z-z_0|^2} E_0(|z|) \, d^2 z \quad . \qquad (12)$$

The normalization integral [Eq. (10)] is the partition function of a plasma.[1,2] I evaluate it using the 3-component hypernetted chain equations[2,5]

$$g_{ij}(|z|) = \exp\left[-\beta v_{ij}(|z|) + h_{ij}(|z|) - c_{ij}(|z|)\right] \qquad (13a)$$

$$h_{ij}(|z|) = g_{ij}(|z|) - 1 \quad . \qquad (13b)$$

$$h_{ij}(|z|) = c_{ij}(|z|)$$

$$+ \sum_k \sigma_k \int c_{ik}(|z'|) \, h_{ij}(|z-z'|) \, d^2 z' \quad . \qquad (13c)$$

where $\exp(-\beta v_{11}) = |z|^{2m}$, $\exp(-\beta v_{12}) = |z|^2$, $\exp(-\beta v_{13}) = |z|^2 - 2$, $\exp(-\beta v_{23}) = |z|^{2/m}$, $\sigma_1 = (2\pi)^{-1}$, and $\sigma_2 = \sigma_3 = \sigma_3 = \sigma_1/N$. In particular

$$F(|z|) = \frac{g_{23}(|z|)}{|z|^{2/m}} \quad . \qquad (14)$$

The function U in Eq. (10) is a "coulomb" energy given by

$$U(Q) = Q^2\left[\frac{1}{2}\ln(2Q) - \frac{3}{4}\right] \quad . \tag{15}$$

The quantitities μ_{ex}^{+} and μ_{ex}^{-} are the quasihole and quasielectron excess chemical potentials, given in the hypernetted chain approximation[6] by

$$\beta\mu_{ex}^{+} = \frac{1}{2\pi m}\int\Big\{-h_{12}(|z|) + \ln\left[g_{12}(|z|)\right]$$

$$+ \frac{1}{2}\ln\left[\frac{g_{12}(|z|)}{|z|^2}\right] h_{12}(|z|)\Big\}\, d^2z \quad . \tag{16}$$

and

$$\beta\mu_{ex}^{-} = \lim_{R\to\infty}\Bigg[\frac{1}{2\pi m}\int_{|z|<R}\Big\{-h_{13}(|z|) + \ln\left[\left(\frac{|z|^2}{|z|^2-2}\right) g_{13}(|z|)\right]$$

$$+ \frac{1}{2}\ln\left[\frac{g_{12}(|z|)}{|z|^2-2}\right] h_{13}(|z|)\Big\}\, d^2z - \frac{2}{m}\ln(R)\Bigg] \quad . \tag{17}$$

I evaluate the energy by adding the quasiparticle charge densities and inserting the sum into the formulas appropriate for calculating the quasihole creation energy. I have verified numerically that the superposition principle is valid when the particles are coalesced, and it is trivially valid when they are far apart. This energy limits at large separations to my expression[2] for the energy to create a particle-hole pair at infinity and at small separations to the energy to make a composite particle, as calculated with the formali m used for the quasielectron. This latter point is important because the procedure is ad-hoc and may be inaccurate.[7] The quasielectron charge density is defined by

$$g_{13}^{\prime}(|z|) = e^{-\frac{1}{2}|z|^2}\left(2\frac{\partial}{\partial z}\right)\left(2\frac{\partial}{\partial z^*}\right)\left[\frac{g_{13}(|z|)}{|z|^2-2}\, e^{\frac{1}{2}|z|^2}\right] \quad . \tag{18}$$

with $|z| = \sqrt{2m}\, x$ and with fourier transforms defined in the manner

$$\hat{h}(k) = \int_0^\infty J_0(kx)\, h(x)\, x\, dx \quad . \tag{19}$$

where J_0 is an ordinary Bessel function of the first kind. I calculate the change to h_{11} due to the presence of a quasielectron-quasihole pair separated by x_0 in the manner

$$\delta\hat{h}_{11} = \left[1+2\hat{h}_{11}\right]^2 \delta\hat{c}_{11}$$

$$+ \frac{2}{N}\left[\hat{h}_{12}^2 + \hat{h}_{13}^{\prime 2} - 2\hat{h}_{12}\hat{h}_{13}^{\prime} J_0(kx_0)\right] \quad . \tag{20}$$

and

$$\delta c_{11} = \left[\frac{h_{11}}{1+h_{11}}\right]\delta h_{11} \quad . \tag{21}$$

The energy above the ground state is then given by

$$\frac{E(|z_0|)}{F(|z_0|)} = \frac{N}{\sqrt{2m}}\left[\int_0^\infty \delta h_{11}(x)\, dx\right]\frac{e^2}{a_0} \quad . \tag{22}$$

The matrix elements of the electron density operator are given by

$$\langle m|\rho(z)|z_0\rangle = \frac{\langle m|m\rangle}{\sqrt{2\pi m L}}\, e^{\beta\left[U(mN+1)-U(mN)\right]}\, e^{-(\beta\mu_{ex}^{+}+\frac{1}{2})}\, e^{-\frac{1}{4m}|z_0|^2}$$

$$\times\, e^{\frac{1}{2m}(zz_0^*-z^*z_0)}\int e^{\frac{1}{2m}(z'z_0^*-z'^*z_0)}\, G(|z'|)\, d^2z' \quad . \tag{23}$$

where

$$G(|z|) = e^{-\frac{1}{2}|z|^2}\left(2\frac{\partial}{\partial z}\right)\left[\frac{g_{12}(|z|)}{z^*}\, e^{\frac{1}{2}|z|^2}\right] \quad . \tag{24}$$

I thus obtain for the exciton contribution to the density-density correlation function S(q)

$$S_{ex}(q) = e^{\beta\mu_{ex}^{-}-\beta\mu_{ex}^{+}}\, e^{\frac{1}{4}k_0^2}\, \frac{|\hat{G}(k_0)|^2}{F_0(x_0)} \quad . \tag{25}$$

evaluated at $k_0 = \sqrt{2m}\, qa_0$ and $x_0 = k_0/2$. The contribution to S(q) from all excitations within the lowest Landau level is given by the sum rule

$$S_{total}(q) = \left[2\hat{h}_{11}(k_0) + e^{-\frac{1}{4m}k_0^2}\right] \quad . \tag{26}$$

For transitions within the lowest Landau level, the transverse response is q^2 times the longitudinal one.

I have numerically evaluated E, F, and G for m=3 and have fit the results with the following analytic expressions:

$$E(x_0) = -0.025\, e^{-\frac{1}{2}x_0^2} \int_0^\infty e^{-\frac{1}{2}x^2}\, I_0(x x_0)\, x^{-2/3}\, dx + 0.036\, e^{-\frac{1}{2}x_0^2} - 0.101\, e^{-x_0^2} \quad (27)$$

$$F(x_0) = 0.55\, e^{-\frac{1}{2}x_0^2} \int_0^\infty e^{-\frac{1}{2}x^2}\, I_0(x x_0)\, x^{1/3}\, dx + 1.706\, e^{-x_0^2} \quad (28)$$

and

$$\tilde{G}(k_0) = -0.0046\, k_0^4\, e^{-\frac{1}{4}k_0^2} \quad (29)$$

Analytic fitting is necessary because numerical inversion of Eq. (12) cannot be done reliably. I calculate the chemical potentials $\beta\mu_{ex}^+$ and $\beta\mu_{ex}^-$ to be -0.094 and 0.221. The exciton dispersion curve and creation strength I obtain are shown in Figs. 1 and 2. Note the similarity of the dispersion curve to Eq. (7): at high momentum the exciton energy is the energy to create a particle-hole pair at infinity ($\Delta_{\text{quasielectrons}} + \Delta_{\text{quasiholes}} = .057\ e^2/a_0$) minus the coulomb binding energy of particles of charge 1/3 separated a

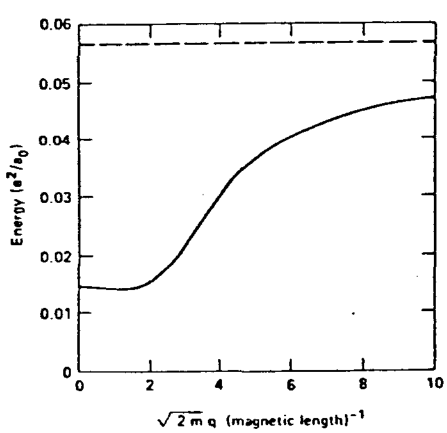

Fig. 1. Exciton dispersion curve. Dashed line is the energy to create a quasielectron-quasihole pair at infinity.

distance $|z_0| = 3q$. The $q \to 0$ exciton binding energy is less than that in Eq. (7) because the particle and hole cannot be brought together without strongly overlapping with the ground state. This overlap functions as a hard core repulsion.

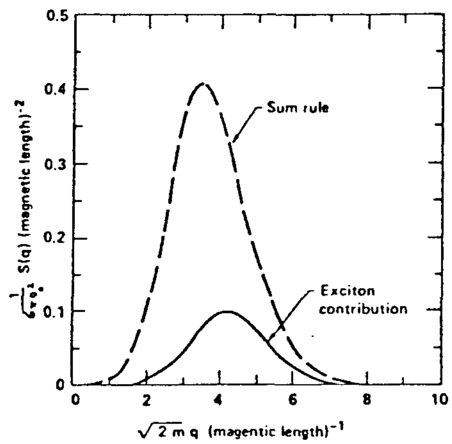

Fig. 2. Sum rule and single exciton contributions to the density-density correlation function.

The neutrality and perfect screening sum rules[8] on g_{12} cause my exciton contribution to S(q) to vanish at small q as q^8. The same sum rules applied to g_{11} lead to vanishing of $S_{total}(q)$ as q^4, with coefficient

$$\lim_{q \to 0} S_{total}(q) = (m-1)\,\frac{(qa_0)^4}{8} \quad (30)$$

determined from the compressibility sum rule. It is not clear whether the disparity at low momentum is due to multi-exciton creation processes or inadequate variational freedom in the exciton wavefunction.

Because of the numerical uncertainties in evaluating E, F, G, μ_{ex}^+ and μ_{ex}^-, as well as the noisy nature of the deconvolution step, the results presented here must be considered qualitative. The substantive content of this work is that a simple exciton wavefunction for a many-electron system exists, that the quasiexciton is larger than the ordinary exciton[4] because the quasiparticles behave kinematically as though they carry fractional charge, and that the creation of single excitons exhausts a significant fraction of S(q).

It is helpful in understanding the exciton physically to consider its similarity to a phonon in a Wigner crystal. This connection may be understood by substituting for $|m\rangle$

in Eq. (8) the Hartree-Fock charge density wave state $|W\rangle$, given by

$$|W\rangle = \frac{1}{\sqrt{N!}} \sum_{\sigma} \mathrm{sgn}(\sigma)\, \phi_{\sigma(1)}(z_1) \cdots \phi_{\sigma(N)}(z_N) \quad . \tag{31}$$

where σ is a permutation, $\mathrm{sgn}(\sigma)$ is its sign, and ϕ_j is a gaussian orbital centered at lattice site x_j. and given by

$$\phi_j(z) = \frac{1}{2\pi}\, e^{-\frac{1}{4}|z|^2}\, e^{\frac{1}{2}x_j^* z}\, e^{-\frac{1}{4}|x_j|^2} \quad . \tag{32}$$

One has

$$|z_0\rangle = \frac{1}{\sqrt{2\pi m}L} \iint e^{-\frac{1}{2m}(|z_A|^2+|z_B|^2)}\, e^{\frac{1}{2m}z_A^* z_B}$$

$$\times\, e^{\frac{1}{2m}(z_B z_0^* - z_A^* z_0)}\, e^{-\frac{1}{4m}|z_0|^2}\, S_{z_B}^{\dagger} S_{z_A} |W\rangle\, d^2 z_A\, d^2 z_B$$

$$= \frac{\sqrt{2\pi m}}{L}\, e^{-\frac{1}{8m}|z_0|^2} \int e^{-\frac{1}{2m}|z_A|^2}$$

$$\times\, e^{\frac{1}{4m}(z_A^* z_0 - z_A z_0^*)}\, S_{z_A - z_0/2}^{\dagger}\, S_{z_A + z_0/2}\, |W\rangle\, d^2 z_A \quad . \tag{33}$$

This state is a phonon because $S^{\dagger}_{z_A - z_0/2}\, S_{z_A + z_0/2}$ effectively displaces the lattice point nearest z_A while leaving the others fixed. Its action on a single orbital is

$$e^{-\frac{1}{2}|z|^2} \left(2\frac{\partial}{\partial z} z_A^* - z_0^*/2\right)(z - z_A - z_0/2) \left[e^{\frac{1}{2}|z|^2} \phi_j(z)\right]$$

$$= \left[(x_j^* - z_A^* + z_0^*/2)(z - z_A - z_0/2) + 2\right] \phi_j(z) \quad . \tag{34}$$

When x_j is far from z_A, this operation multiplies $\phi_j(z)$ by the number $|z_A - x_j|^2$. When x_j equals z_A, on the other hand, it multiples $\phi_j(z)$ by the function

$$\left[2 - \frac{1}{4}|z_0|^2 + \frac{1}{2} z_0^*(z - x_j)\right] \cong 2\, e^{\frac{1}{4} z_0^*(z - x_j)}$$

thus displacing x_j by $z_0/2$. Calling D_{z_A} the operator which displaces the electon nearest z_A by $z_0/2$, I write

$$S^{\dagger}_{z_A - z_0/2} S_{z_A + z_0/2} |W\rangle \cong \prod_{j} |z_A - x_j|^2\, D_{z_A} |W\rangle$$

$$= e^{\frac{1}{2}|z_A|^2}\, D_{z_A} |W\rangle \quad . \tag{35}$$

We have finally

$$|z_0\rangle = \int e^{\frac{1}{4m}(z_A^* z_0 - z_A z_0^*)}\, D_{z_A} |W\rangle \quad . \tag{36}$$

which is a phonon. One may say qualitatively that the exciton and phonon are the same except at long wavelengths, where the exciton has a gap while the phonon does not. In light of this analogy it is disturbing that the exciton and phonon dispersion curves do not match near the Brillouin zone edge. (They are comparable). One would expect the liquid and solid to be indistinguishable on short length scales. Further work will be required to clarify this point.

I remark finally that spontaneous generation of excitons can lead to a distinctive type of nonlinear conductivity in samples that do not first break down by a "Cerenhov catastrophe."[9] This conductivity mechanism is qualitatively similar to tunneling from one Landau level to the next in strong electric fields, but quantitatively different in that the gap, and thus the threshold field, is smaller. In the presence of an electric field $\vec{\varepsilon}$, the exciton dispersion curve is modified by an electric dipole contribution to be

$$u = \frac{E_0(|z_0|)}{F_0(|z_0|)} - \frac{e}{m}\, \vec{\varepsilon} \cdot \vec{r}_0 \quad . \tag{37}$$

where $\vec{r}_0$ is z_0 expressed as a vector. Thus excitons with certain momenta $\vec{q}$ require energy to create and can be emitted spontaneously in the presence of a weak impurity potential. The rate of momentum loss is

$$\frac{d\vec{q}}{dt} = \frac{2\pi}{\hbar} \frac{\rho_{imp}}{2\pi a_0^2} \left(\frac{1}{2\pi}\right)^2 \int S(q)\, |\hat{V}_q|^2\, |\vec{\nabla}_q u|^{-1}\, \vec{q}\, dS_q \quad . \tag{38}$$

where ρ_{imp} is the impurity density, $S(q)$ is the static form factor given by Eq. (25), V_q is the fourier transform of the single impurity scattering potential

$$\hat{V}_q = \int V_{imp}(\vec{r})\, e^{-i\vec{q}\cdot\vec{r}}\, d\vec{r} \qquad (39)$$

$\nabla_q u$ is the momentum gradient of the dispersion relation Eq. (37), and dS_q is a volume element appropriate for integrating over the momentum surface defined by u=0. As the electrons lose momentum, they drift in the direction of $\vec{\varepsilon}$ and dissipate energy. One may obtain σ_{xx} by identifying this power dissipation with ohmic loss, in the manner

$$e\vec{\varepsilon}\cdot\left(\frac{d\vec{p}}{dt}\right) a_0^2 = \sigma_{xx}\, \varepsilon^2 L^2 \qquad (40)$$

For ease of interpretation, I shall assume that V_q is constant and express σ_{xx} in terms of the classical conductivity

$$\sigma_{xx}^{(0)} = \left[\left\{\frac{\rho_{imp}}{2\pi m a_0^2}\right\} \frac{|\hat{V}_q|^2}{(\hbar\omega_c)^2} \right] \frac{e^2}{\hbar} \qquad (41)$$

in the manner

$$R = \left(\frac{2\Delta}{\hbar\omega_c}\right)^2 \frac{\sigma_{xx}}{\sigma_{xx}^{(0)}} \qquad (42)$$

R, calculated for simplicity using the sum rule $S_{total}(q)$ given by Eq. (26), is plotted with and without exciton binding in

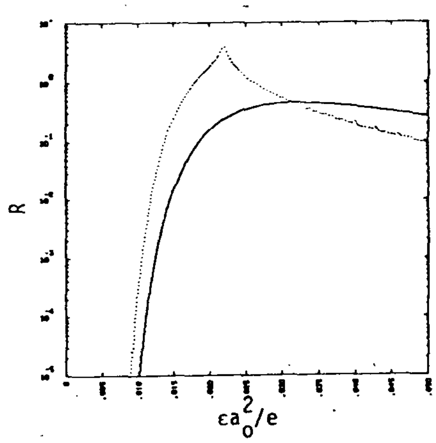

Fig. 3. R, as defined in Eq. (42) versus dimensionless electric field $a_0^2\varepsilon/e$. The dotted and solid curves are calculated with and without the excitonic binding, respectively.

Fig. 3. One sees in both cases a low-field conductivity proportional to $\exp[-\frac{1}{2}(\frac{2\Delta}{e a_0 |\vec{\varepsilon}|})^2]$ and a peak value of order 1. The singularity at 0.022 is a real effect caused by the competition between the exciton binding forces and dipole force. It occurs when $\sqrt{2m}\, q\, a_0 = 6.4$, which can be seen from Fig. 1 to be well outside the core repulsion region.

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