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ABSTRACT

It is pointed out that if the Pauli principle is taken into account in the discussion of pion scattering by complex nuclei (as it ought, of course, to be) some rather implausible consequences of some earlier treatments of this problem can be avoided.

The scattering of pions by complex nuclei is commonly treated by the Kisslinger potential.[1] In momentum space, this has the form

$$\langle \vec{k}'|V|\vec{k}\rangle = a_0(\omega) + a_1(\omega)\ \vec{k}\cdot\vec{k}' \ , \tag{1}$$

where k, k' are the initial and final momenta of the pion <u>inside</u> the nucleus, ω is the energy of the pion, and $a_0 a_1$ are the s- and p-state π-nucleon scattering amplitudes. The form (1) is very natural; in particular, the second term reflects the p-wave character. As is well known, in the energy range from about 50 to 400 MeV the p-wave scattering dominates. The momentum space amplitude is then translated into a potential in coordinate space, and the Schrödinger equation is solved in this potential.

Auerbach, Fleming, and Sternheim,[2] among others, have used the Kisslinger potential to calculate the scattering of π^+ and π^- by various nuclei, and found good agreement with experiment. They introduce the new amplitudes

$$b_0 = -2(2\pi)^3 E_\pi a_0/k_0^2 \ ,$$

$$b_1 = -2(2\pi)^3 E_\pi a_1 \ , \tag{2}$$

where k_0 is the pion momentum in vacuum and $E_\pi = \omega$, its energy. They discuss how b_0 and b_1 can be calculated from pion-nucleon phase shifts. In this calculation, they take into account the difference between laboratory and center-of-mass momentum for πN scattering. Fortunately, the real parts of b_0 and b_1 do not depend strongly on ω, at least as long as E_π is well below the πN resonance (180 MeV). For b_1 this is very plausible because the main momentum dependence of the p-wave amplitude is contained in the factor $\vec{k}\cdot\vec{k}'$ in (1); for b_0 it follows from the fact that a_0 is essentially zero for $p_0 = 0$.

The theoretical values of b_0, b_1, i.e., those derived from πN scattering, are at 80 MeV

$$b_0 = -0.75 + 0.34\, i \qquad F^3 \ ,$$

$$b_1 = 6.5 + 1.8\, i \qquad F^3 \ . \tag{3}$$

The positive sign in the real part of b_1 signifies attraction, the negative b_0 repulsion. The dimension is fermi^3 for both amplitudes.

If we apply the "Schrödinger" equation of Auerbach et al. to a region of constant nuclear density[3] ρ, we obtain

$$-(1 - b_1\rho)\nabla^2\psi = k_0^2(1 + b_0\rho)\psi \ , \tag{4}$$

where ψ is the pion wave function, and p_0 its momentum in free space. Introducing the momentum k in the nuclear medium, we have

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$$k^2 = k_0^2 \frac{1 + b_0\rho}{1 - b_1\rho} . \qquad (5)$$

This has the most peculiar property, observed by many previous authors, that the real part of the denominator can change sign. That is, if we take for b_1 the theoretical value (3), then

$$\mathrm{Re}(1 - b_1\rho) < 0 \text{ if } \rho > 0.16 \text{ F}^{-3} . \qquad (6)$$

Now the density of nuclear matter is generally taken to be 0.17 F^{-3}, so the denominator is indeed negative. The numerator is close to 1 because b_0 is small, Eq. (3).

A complete catastrophe is avoided because b_1 has also an imaginary part. Using the values given in (3), and $\rho = 1/6.5$, we find

$$k^2 = p_0^2(3.2\ i - 0.19) . \qquad (7)$$

Thus the inside momentum is (a) very large, and (b) strongly complex; i.e., for 80-MeV kinetic energy, $k \approx 1.1(1 + i)F^{-1}$. When going into the nucleus, the density of pions, $|\psi|^2$, would be attenuated by a factor e in a distance of less than half a fermi, a most unlikely result. A physical consequence of this rapid attenuation is a very large probability of one-nucleon capture of the pion,

$$N + \pi \to N' + \gamma , \qquad (8)$$

where the charge of N' differs from that of the original nucleon N by that of the π. This large capture is contrary to observation. Some nuclei have densities $\rho > 0.16$ near the center; e.g., Auerbach et al.[2] use $\rho = 0.24\ F^{-3}$ at the center of C^{12}. Then the peculiar behavior (7) occurs some distance from the center, and near the center k is purely imaginary.

This note is to point out that this peculiar behavior can be avoided if the Pauli principle is taken into account in the pion scattering. In fact, this should of course be done anyway, regardless of the peculiar behavior discussed. As far as I know, the Pauli principle has not been previously considered in this problem, except in some minor way.

The fundamental interaction of a pseudoscalar pion with a nucleon is

$$H_i = (f/\mu)\vec{\sigma} \cdot \vec{k} , \qquad (9)$$

where $\vec{\sigma}$ is the nucleon spin, f the coupling constant, and μ the pion mass. Scattering occurs because H_i acts twice, yielding the form of the second term in (1). In particular, if we consider the p-wave scattering of π^+ by a proton, the pion going from momentum $\vec{k}$ to $\vec{k}'$, it proceeds in two stages,

$$P \to N + \pi^+(\vec{k}') ,$$

$$N + \pi^+(\vec{k}) \to P . \qquad (10)$$

If the initial proton has momentum $\vec{p}_i$, then the intermediate-state neutron has

$$\vec{p}_N = \vec{p}_i - \vec{k}' , \qquad (10a)$$

and the final proton momentum is

$$\vec{p}_f = \vec{p}_i + \vec{k} - \vec{k}' \equiv \vec{p}_i - \vec{q} . \qquad (10b)$$

Now in nuclear matter, the intermediate neutron state $\vec{p}_N$ may already be occupied, and then the scattering cannot take place. Likewise, if the final proton state $\vec{p}_f$ is occupied, the scattering is forbidden. These effects of the Pauli principle should be taken into account.

The effect of the Pauli principle in an intermediate state may be unfamiliar. However, a simple example is the scattering of light from an atom with several shells occupied. In the hydrogen atom, there is an absorption line from the 1s to the 2p state, and in the neighborhood of this line, theory predicts enhanced light scattering (anomalous dispersion). In an atom in which both 1s and 2p shells are fully occupied, there is no absorption line 1s → 2p, and therefore also no enhanced light scattering in the neighborhood of the line. Thus in light scattering, which is a two-stage process just like pion scattering, the process is forbidden if the intermediate state 2p is occupied.[4]

The effect of the Pauli principle on foward scattering is easy to calculate. Let us assume that neutrons and protons have the same Fermi momentum p_F. Then, since $\vec{k}' = \vec{k}$, we must have

$$p_N = |\vec{p}_i - \vec{k}| > p_F > p_i . \qquad (11)$$

Thus p_i must be inside a sphere around the origin of radius p_F, but outside a sphere of the same radius centered at $+\vec{k}$. The volume contained between those two spheres can be calculated geometrically; its ratio to $(4\pi/3)p_F^3$ gives the fraction of the protons which can contribute to forward scattering. This ratio is

$$F = \frac{3}{4}\,\frac{k\left(p_F^2 - \frac{1}{12}k^2\right)}{p_F^3}\,, \qquad k < 2p_F\ . \tag{12}$$

For $k > 2p_F$, we have $F = 1$, i.e. no Pauli effect. For $k \ll p_F$, however,

$$F \to 3k/4p_F \qquad k \ll p_F\ . \tag{12a}$$

In nuclear matter, $p_F = 1.35\ F^{-1}$, whereas an 80-MeV pion in free space has the momentum $k_0 = 0.85\ F^{-1}$; in this case,

$$F = 0.456\ . \tag{12b}$$

So the effect is large. Even for $k = p_F$,

$$F = 0.688\ . \tag{12c}$$

This k, for a free-space meson, corresponds to a kinetic energy of 165 MeV.

The amplitude b_1 in (4) must now be multiplied by F. This will remove the possibility of a sign change of the denominator in (5), at least if ρ is the nuclear-matter density and the meson momentum k is not too large. The internal meson momentum k is therefore now much closer to the external momentum k_0. For larger k_0, the meson energy will come close to the resonance; then the imaginary part of b_1 becomes large and $|k|$ is still not very far from k_0. In this way, the peculiar phenomena discussed after (7) are eliminated.

Inelastic scattering involves the permanent removal of a nucleon from the normally occupied shells (if we disregard excitation of collective motions). With this latter restriction, inelastic scattering through small angles is strongly impeded because $\vec{q}$ in (10b) is small. The use of $\vec{q}$ is most appropriate for quasi-elastic scattering, which looks like scattering of the pion from a free nucleon. This effect of the Pauli principle on the inelastic scattering is familiar. Of course, even at the large angles where (10b) does not restrict inelastic scattering much, its amplitude is still diminished by the Pauli effect in the intermediate state, (10a). This could be calculated similarly to (12) but is somewhat more complicated. Thus we expect the cross section for inelastic scattering at moderate pion energy (<200 MeV, say) to be considerably reduced compared with the Kisslinger theory, and the Im b_1 to be reduced likewise.

Because inelastic scattering is reduced, the resonance will be shifted. For a quantitative theory it would be best to repeat the Chew-Low calculation in nuclear matter of various densities, taking the Pauli principle into account.

In a finite nucleus, the amplitude of the forward $p\pi^+$ scattering should be reduced by an amount proportional to[5]

$$\sum_n \sum_p \left|\langle n|e^{i\vec{k}\cdot\vec{r}}|p\rangle\right|^2\ , \tag{13}$$

where n and p label the neutron and proton shell model states, and the sums are over all occupied states; $\vec{k}$ is again the meson momentum. Equation (13) may be rewritten

$$\sum_n \sum_p \int \psi_n^*(\vec{r})e^{i\vec{k}\cdot\vec{r}}\psi_p(\vec{r})d^3r \int \psi_p^*(\vec{r}\,')e^{-i\vec{k}\cdot\vec{r}\,'}\psi_n(\vec{r}\,')d^3r'$$

$$= \int \rho_n(\vec{r},\vec{r}\,')\rho_p(\vec{r}\,',\vec{r})e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')}d^3r\,d^3r'\ , \tag{14}$$

where ρ_n, ρ_p are the mixed densities of neutrons and protons, respectively. Negele and Vautherin[6] have recently given convenient expressions for these mixed densities in terms of $|\vec{r} - \vec{r}\,'|$, the ordinary density at the midpoint $\frac{1}{2}(\vec{r} + \vec{r}\,')$, and other quantities describing the local state of nuclear matter. It should therefore be possible to express the Pauli effect in terms of local parameters in the nucleus.

A disturbing feature of our Pauli effect is that it will change the parameters in the Kisslinger theory quite significantly. But Auerbach et al.[2] obtained good agreement with experiment, using the