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CRITICAL AMOUNTS OF URANIUM COMPOUNDS

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March 19, 1943

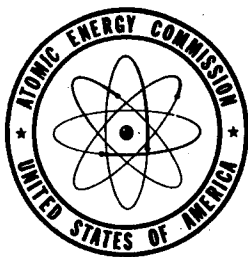Chicago. University. Metallurgical Laboratory

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CRITICAL AMOUNTS OF URANIUM COMPOUNDS

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Problem & Results:

By a critical amount of material, we mean the least amount of that
material which under the prevailing geometrical conditions is capable of
supporting a chain reaction. The critical amount depends on the geometric
shape. In the following we shall assume that the material is present in
the form of a sphere. This gives the critical amount its smallest value
since it minimizes the loss of neutrons through the surface. The critical
amount also depends on the reflection of neutrons from materials in the neigh-
borhood of the reacting substance. We shall discuss the influence of various
reflectors. Above all, the critical amount depends on the chemical and iso-
topic composition of the reacting materials. Calculations have been carried
out which are applicable to fluorides and oxides and to isotopic mixtures
ranging from 20% to 100% of 25 content.

The result of the calculation is that approximately rather more
than 100 KG of the pure 25 isotope must be present in any isotopic
mixture or chemical compound of the type discussed in order that the
substance should become chain reacting. The figure given here already
includes the effect of some neutron reflection from a container not more
than an inch in thickness and also from the walls of the room.

The above statements must be qualified in two ways. First, physi-
cal data on which the calculations are based are exceedingly rough. Thus
previous calculations which were based on measurements of Leipunsky et al.
give little more than one half the critical masses just mentioned. Lei-
punsky's data seem to be definitely much less reliable than those on
which the present calculations are based. They had been adopted in order
to obtain the smallest amount of critical materials compatible with pub-
lished measurements.

The second and even more important qualification of our statements
is that they hold only in the absence of hydrogenous materials. In dilute
solution in water and in the absence of absorbing material such as boron
less than a kilogram of 25 may become chain reacting. Calculations on
this point have been published by Christy and Wheeler (Report CP-400).
Further calculations giving similar results have been carried out and
will be mentioned below. Appropriate use of boron can prevent the danger
from hydrogenous materials.

The Constants Used*

Uranium. We shall distinguish two groups of neutrons which we shall call
the "fast" and the "slow." The "fast" neutrons are those having

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* See Report CP-334.   CF-548   1

sufficient energy to cause fission of 28. The "slow" ones have energies below the 28 fission threshold, which is at about 1 mv. The values which have been used here for the various cross sections of 25 and 28 for "fast" and "slow" neutrons follow. The cross sections are given in units of 10^{-24} cm^2:

Fission cross section of 28 for "fast" neutrons: σ_{f8} = 2/3

    "   "   "   " 25 for "fast"   " : σ_{f5} = 4/3

    "   "   "   " 25 " "slow"   " : σ_{s5} = 3

Inelastic "   "   " 28 " "fast"   " : σ_8^i = 3.3

    "   "   "   " 25 " "fast"   " : σ_5^i = 2.0

Absorption* "   "   " 28 " "slow"   " : σ_r = 0.3

Transport "   "   "   U " "fast"   " : σ_{ft} = 5

    "   "   "   "   U " "slow"   " : σ_{st} = 8

Number of "fast" neutrons emitted per fission :   w = 1.3

    "   " "slow"   "   "   "   " : w' = 0.9

The inelastic cross section is defined as the cross section of a scattering process in which the incident neutron is fast but the outgoing neutron is slow.

The inelastic scattering cross-section of 28, σ_8^i = 3.3, was chosen so as to be compatible with experiments by Snell's group in Chicago and with an absorption cross-section σ_r not greater than 0.3. The corresponding 25 cross section was made smaller primarily because in this case a larger fission cross-section competes with the inelastic scattering. When compared to the geometric cross-sections, these inelastic cross-sections may seem large and, therefore, doubtful. It is however on the safe side to assume for inelastic scattering large values since this tends to increase the proportion of "slow" neutrons, and these can take advantage of the large fission cross-section of 25 for "slow" neutrons. Of course, at the same time the fissions of 28 are lost, but that has importance only when the 25 concentration is small and the critical amounts safely large.

The "transport" cross-sections signify those cross sections which govern the diffusion. Thus the diffusion mean free path λ is connected with a transport cross section σ_t by $\lambda = 1/N\sigma_t$ where N is the number of nuclei per cm^3. The values given for the transport cross sections are measured total cross sections somewhat diminished properly to eliminate the elastic forward scattering.

Fluorides (UF_n). The entire molecule is treated as a single U nucleus having inelastic and transport cross sections which consist of the sum of the corresponding cross sections of each of the atoms in the molecule. Thus the apparently most reliable measurements yield values for the total cross section of fluorine about equal to 2 for "fast" neutrons and 3 for "slow" ones. Accordingly,

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* The absorption cross section for fast neutrons is zero.

$$\sigma_{ft} = 5 + 2n$$

$$\sigma_{st} = 8 + 3n$$

for the UF_n molecule.

Nothing is known concerning the inelastic scattering by fluorine. For reasons mentioned above it is safer for our purposes to allow for large inelastic scattering and it will be generous to assume that <u>half</u> the total cross section of 2 for "fast" neutrons is due to inelastic scattering. Accordingly,

$$\sigma_5^i = 2 + 1n$$

$$\sigma_8^i = 3.3 + 1n$$

Oxides (UO_n). The Minnesota group's measurements of the total cross section of oxygen show wide fluctuations with neutron energy. Fair averages to take for the "slow" and "fast" groups seem to be about 3.6 and 2.4, respectively. Thus,

$$\sigma_{ft} = 5 + 2.4n$$

$$\sigma_{st} = 8 + 3.6n$$

for the UO_n molecule. n is understood to be 8/3 for U_3O_8.

There is a measurement (Report C-334) of the inelastic scattering of "fast" neutrons by oxygen giving a cross section of 0.65. Accordingly we may use

$$\sigma_5^i = 2 + .65n$$

$$\sigma_8^i = 3.3 + .65n$$

Neutron reflectors with which we may be concerned are Fe, Ni, Cu and Al. We give a table of total neutron cross sections for these elements. The values listed are estimates based on the data (Report C-334) and on assumptions of 100-500 kv for the "slow" neutron energies, 1-2 mv for the "fast" neutrons. For the "fast" neutrons, an attempt is made to choose values representative of back-scattering rather than the total, since for "fast" neutrons the back-scattering may be appreciably smaller than forward scattering and is the process responsible for the reflections. On the whole, the estimates are made in a way to obtain small critical masses which means using large cross sections.

	Fe	Ni	Cu	Al
"slow"	3.7	6.6	3.6	3.7
"fast"	2	2.5	2.2	1.9

The corresponding mean free path (in cm) follow:

	Fe	Ni	Cu	Al
"slow"	3.2	1.65	3.3	4.5
"fast"	5.9	4.4	5.4	8.7

Method of Calculation:

To obtain the critical amount of reacting materials, a differential diffusion theory was employed. A pair of coupled differential equations were constructed, one expressing the balance between the diffusion of "fast" neutrons from a point in the reacting material, the net number of "fast" neutrons created there by fission processes caused by both "fast" and "slow" neutrons, and the "fast" neutrons lost by inelastic scattering. The other expressed the balance between the gains and losses at a point of the "slow" neutrons. Here the losses are through fission giving "fast" neutrons and through resonance absorption by 28. The results for spherically symmetrical "fast" and "slow" neutron distribution (n_f and n_s) within the reacting material were, respectively:

$$n_s = \frac{1}{v_s r} \left[a' \sin k'r + a'' \sinh k''r \right] \tag{1a}$$

$$n_f = \frac{1}{v_f r} \left[\alpha' \, a' \sin k'r + \alpha'' \, a'' \sinh k''r \right] \tag{1b}$$

where v_s and v_f are the "slow" and "fast" neutron velocities. a' and a'' are arbitrary constants to be determined through the application of boundary conditions. k', k'', and a' and a'' are quantities determined by the physical constants of the nuclei involved. One sees that k' is a "wave number" for the "fundamental" sinusoidal component of each distribution. The hyperbolic terms are needed to adjust the distributions to the surface conditions imposed, and since k'' is always considerably larger than k', these terms suffer rapid exponential decay with distance from the surface. They thus earn the names "transients" and "surface corrections."

Specifically,

$$k'^2 = 1/2 \, (A+D) + \sqrt{1/4 \, (A-D)^2 + BC} \tag{2a}$$

$$k''^2 = -\left[1/2 \, (A+D) + \sqrt{1/4 \, (A-D)^2 + BC} \right] \tag{2b}$$

$$\alpha' = \frac{k'^2 - D}{C} \tag{3a}$$

$$\alpha'' = \frac{-k''^2 - D}{C} \tag{3b}$$

where

$$A = -3\sigma_{ft} \left[\{ \sigma_8^i - \sigma_{f8}(w-1) \} \, (1-x) + \{ \sigma_5^i - \sigma_{f5}(w-1) \} \, x \right] N^2$$

$$B = 3\sigma_{ft} \left[\sigma_{s5} \, wx \right] N^2$$

$$C = 3\sigma_{st} \left[\left(\sigma_{f8} \, w' + \sigma_8^i \right)(1-x) + \left(\sigma_{f5} \, w' + \sigma_5^i \right) x \right] N^2$$

$$D = -3\sigma_{st} \left[\sigma_{s5} \, (1-w') \, x + \sigma_r (1-x) \right] N^2$$

$$\left. \right\} \tag{4}$$

Here x is the fraction of the U nuclei of the isotope 25. It is immediately apparent that A and D are negative and so k'' is greater than k'. The quantity

α', giving the ratio of "fast" to "slow" neutrons in the "fundamental"
is usually small, becoming as great as 35 percent or so only for material
free of 28. It turns out that α'' is always negative, showing that the
"fast" and "slow" distributions must be "corrected" near the surface in
opposite directions.

The most widely applicable boundary conditions are best obtained
by considering the neutron reflection coefficients of the surroundings of
the reacting material. We represent the resultant reflection coefficient,
more conveniently called the "albedo," by γ for the "slow" neutrons, $\epsilon\gamma$
for the "fast." ϵ will be less than unity since "fast" neutrons penetrate
farther out into the surroundings and have less chance of returning than
the "slow" neutrons.

The boundary conditions may be expressed in terms of the albedo as
follows. We represent the density of "slow" neutrons directed outward by
n_o, those directed inward by n_i so that

$$n_s = n_o + n_i \tag{5}$$

The net outgoing current will then be

$$\frac{1}{\sqrt{3}} \ (n_o\, v_s \ = \ n_i\, v_s)$$

in which the $1/\sqrt{3}$ comes from averaging the directions of the neutrons.
According to the diffusion theory the net outgoing current is expressed
in terms of the distribution n_s by

$$-\frac{\lambda_s\, v_s}{3}\ \frac{dn_s}{dr} \ = \ \frac{1}{\sqrt{3}}\ (n_o v_s - n_i v_s) \tag{6}$$

We now make use of the definition of albedo, γ or $\epsilon\gamma$, as the ratio
of incoming to outgoing neutron currents. We shall assume that all "fast"
neutrons become "slow" upon reflection. In many cases this assumption is
justified and in general it is a safe assumption for our purposes since
it tends to make the critical mass smaller. Then

$$\frac{1}{\sqrt{3}}\ n_i v_s \ = \ \gamma\, \frac{1}{\sqrt{3}}\ n_o v_s + \epsilon\gamma \left(- \frac{\lambda_f v_f}{3}\ \frac{dn_f}{dr}\right) \tag{7}$$

since the entire "fast" neutron current

$$^-(\lambda_f v_f/3)\ (dn_f\,(dr))$$

is outgoing. The two equations (5), (6) relating n_o, n_i, and n_s can now
be used to eliminate n_i and n_o:

$$n_s + \frac{\lambda_s}{\sqrt{3}}\ \frac{dn_s}{dr} = \gamma \left(n_s - \frac{\lambda_s}{\sqrt{3}}\ \frac{dn_s}{dr}\right) - 2\epsilon\gamma \frac{\lambda_f}{\sqrt{3}}\ \frac{v_f}{v_s}\ \frac{dn_f}{dr} \tag{8a}$$

This equation is to hold at the surface of the active material ($r=R$). One
sees that for no reflection ($\gamma=0$), n_s does not vanish at the boundary but

$$n_s + \frac{\lambda_s}{\sqrt{3}}\ \frac{dn_s}{dr} = 0 \text{ for } \gamma = 0$$

which is equivalent to saying that n_s linearly extrapolated to a distance $\lambda/\sqrt{3}$ outside the boundary vanishes. The "fast" neutrons have no reflected part in any case so that the boundary condition to be generally used in addition to (8a) is:

$$n_f = 0 \quad \text{at } R + \lambda_f/\sqrt{3} = 0 \tag{8b}$$

i.e., at a distance $\lambda/\sqrt{3}$ outside the boundary (at radius R). The two conditions (8a) and (8b) are sufficient to determine the ratio

$$a'/a''$$

in the distribution (1) and in addition the <u>critical radius R</u>. This will be the radius for which all conditions are <u>fulfilled</u> so that the balance of lost and gained neutrons described in the first paragraph of this section is maintained. If the critical radius is exceeded, not enough neutrons will find their way out through the surface and a chain reaction will begin.

The explicit connection between radius and albedo is obtained by using the distributions (1) in the expressions (8a) and (8b). A somewhat simpler formula is obtained by introducing the extrapolated values of n_s and dn_s/dr at a point $\lambda/\sqrt{3}$ beyond the physical surface of the reacting material.

If R_0 is the radius at this new point, the conditions

$$n_f = 0 \tag{9a}$$

and

$$n_s = \gamma\left(n_s - 2\,\frac{\lambda_s}{\sqrt{3}}\,\frac{dn_s}{dr}\right) - 2\epsilon\gamma\,\frac{\lambda_f}{\sqrt{3}}\,\frac{v_f}{v_s}\,\frac{dn_f}{dr} \tag{9b}$$

are obtained at $r = R_0$. After calculating R_0 one gets the physical radius R from

$$R = R_0 - \lambda_s/\sqrt{3} \tag{10}$$

We use λ_s rather than λ_f or an average λ because the "slow" neutrons always far outnumber the "fast" ones.

It will be simpler to write an equation expressing the albedo γ in terms of R_0 instead of expressing R_0 as a function of γ.

$$\frac{1-\alpha'\alpha''}{(1-\alpha'/\alpha'')(1+2\lambda_s/\sqrt{3}\,R_0)-(\lambda_s/\sqrt{3}+\epsilon\alpha'\lambda_f/\sqrt{3})2k'\cot k'R_0+2k''(\lambda_s\alpha'/\sqrt{3}\,\alpha''+\epsilon\alpha'\lambda_f/\sqrt{3}} \tag{11}$$

In deriving this coth $k''R_0$ was replaced by unity, which is justified by the large value of k''. One should not expect this formula to be valid for critical radii R_0 less than $\lambda/\sqrt{3}$. However, radii so small are of little interest since they could be reached only with an albedo much closer to

unity ($\gamma \approx 0.97$) than any albedo known.

Results and Examples

     The evaluation of the relation (11) between critical radius and albedo has been carried out numerically for UF_6 containing concentrations of 20%, 50%, and 100% of 25 among the U nuclei. For small albedo ($\gamma \leq 0.5$) simple empirical formulas are obtainable.

$$x = 0.2 \quad R = 34.8 - 4.25\,\gamma - 10.0\gamma^2 \quad \text{cm.}$$
$$x = 0.5 \quad R = 28.0 - 4.8\,\gamma - 8.8\gamma^2 \quad \text{cm.} \qquad (UF_6\ \gamma \leq .5)$$
$$x = 1.0 \quad R = 22.0 - 2.7\,\gamma - 12.7\gamma^2 \quad \text{cm.}$$

We have used a "fast" neutron reflection half as great ($\epsilon = 1/2$) as for the slow neutrons. The radius does not depend sensitively on ϵ because "fast" neutrons are always in a minority. The density of UF_6 was taken to be 4.68 gm/cm^3.

     More interesting than the critical sizes are perhaps the critical masses,

$$M = \frac{4\pi}{3}\,R^3\rho$$

Now $R \sim 1/k' \sim 1/N$ (see (2) and (4)), so that $R \sim \mu/\rho$ (μ = molecular weight, ρ = density), and $M \sim (\mu/\rho)^3\rho$, so that the quantity $\rho^2 M/\mu^3$ is independent of the density and the molecular weight of the substance. This quantity is plotted as a function of γ for UF_6 in the accompanying graph, for the 25 concentrations $x = 0.2$, $x = 0.5$, and $x = 1.0$. The ordinates given are for M in grams and ρ in grams per cm^3.

     For concreteness, we list the critical masses of UF_6 for $\gamma = 0$ and $\gamma = 0.3$:

	$\gamma = 0$	$\gamma = 0.3$
$x = 0.2$	835 kg.	700 kg.
$x = 0.5$	435 kg.	340 kg.
$x = 1.0$	215 kg.	160 kg.

We see that a reflection of 30% of the neutrons decreases the critical mass by about 25%.

     The cross sections of UF_4 are about 25% smaller than those of UF_6. This means that $k' \sim \sqrt{\sigma t}$ will be some 12% smaller and $R \sim 1/k'$ about that much larger. A deviation of 12% is not large compared to the uncertainties in our choices of the various nuclear constants. It is sufficient, therefore, to use the UF_6 curves in the figure also for UF_4, and to keep in mind that the masses obtained from the curves are most probably some 40% smaller than the actual critical masses of UF_4. The explicit dependence of M on ρ obtainable from the curves, is useful in this instance since the bulk density of UF_4 (or other solids that may be formed) varies according to the way the solid is obtained. For U_3O_8, the cross sections are smaller than for UF_6 by some 30%. The curves will thus give critical masses for this substance about 50% too small. The finding for UO_3 will be about the same.

We now discuss the values one must attribute to the albedos. Most simple to discuss will be not too thick layers of reflecting material surrounding the active mass. If the thickness Δ of the layer is about equal to the mean free path of a neutron in it, then on the average the neutron will suffer one collision and have about a 50% chance of returning to the interior. Thus,

$$\gamma \approx \frac{1}{2} \frac{\Delta}{\lambda} \quad \text{for } \Delta < \lambda$$

Mean free paths for Fe, Ni, Cu and Al are listed in the last table of the second section. One sees that the albedo of Fe will probably be less than a half up to thicknesses of 3 cm or so. One sees that here the "fast" albedo is nearly one half of the "slow" one.

It should be emphasized that as the layer thickness Δ is increased behond the value λ, then the albedo may increase beyond 1/2 but much less than proportionally to Δ.

It may also be important to know what part the walls of a room may plan in neutron reflections. If the wall area S_w is large compared to the surface S_c of the container of the active mass, it is easy to show that the albedo will be given by:

$$\gamma \approx \gamma_c + \frac{(1 - \gamma_c)^2}{(1 - \gamma_w)} \frac{S_c}{S_w}$$

in which γ_c and γ_w are albedos of the container and the wall, respectively. For a thick mass like a wall Fermi's well-known albedo formula might be applied:

$$\gamma = 1 - 2/\sqrt{N}$$

where N is the average number of collisions a neutron makes in the wall material before it is captured. A value ~ 100 for N will be of the right order of magnitude for the usual wall materials, giving $\gamma_w = 0.8$. If we now also use the thin layer albedo for γ_c the resultant albedo becomes:

$$\gamma \approx 1/2 \quad \Delta/\lambda \left[+ 5\left(1 - \frac{\Delta}{\lambda}\right) \frac{S_c}{S_w} \right]$$

Influence of hydrogenous substances.

Neutrons thrown back by a hydrogenous reflector have greatly diminished velocities. An appreciable fraction of the reflected neutrons are thermal. Since the fission cross section in 25 increases with decreasing velocity, neutrons reflected in such a way will give fission with practical certainty and such fission will take place on the surface of the reacting material. The greatly increased fission cross section of the reflected neutrons might lead to the result that in the presence of hydrogenous reflectors the critical amount is greatly reduced.

If the reacting material is surrounded by an infinite sphere of hydrogenous material, say water, the critical mass cannot be calculated with the help of diffusion theory because the critical radii become smaller than the mean free path. The critical radius R can be estimated as follows: We consider a fission taking place on the surface of a sphere. It creates w fast neutrons and w' slow ones. If these neutrons penetrate into the water without causing further fission, the number of neutrons returning will be $w\gamma_f + w'\gamma_s$. Here γ_f and γ_s are the albedos for fast and slow neutrons. All the returning neutrons cause fission. If, therefore,

$$w\gamma_f + w'\gamma_s \geq 1 \qquad (12)$$

then the chain reaction will be self-supporting. For γ_f and γ_s we find the approximate expressions

$$\gamma_f = \frac{0.5}{1+4/R}, \quad \gamma_s = \frac{0.7}{1+1/R}$$

where R is the radius of sphere measured in centimeters. Eq. (12) has to be corrected for fissions and slowing down processes taking place in the interior of the sphere. This leads to the corrected expression

$$w\gamma_f + w'\gamma_s = + \frac{w}{2} RN\alpha \left[\sigma_{ff}\right] \quad \left[(w-1)\sigma_f + w'\gamma_s\right]$$
$$+ \frac{w'}{2} RN\alpha \quad \sigma_{sf} \quad \left[w\gamma_f + (w'-1)\gamma_s\right] \qquad (13)$$
$$+ \frac{w}{2} RN\beta \quad \sigma^i(\gamma_s - \gamma_f) = 1$$

where N is the number of nuclei per cm^3; $\sigma\beta$ are two factors which take into account the fact that the neutrons are scattered within the sphere. We have used 1.5 for α and 1.3 for β as very approximate values. The values are very approximate but they occur only in the small correction terms. σ_{ff} and σ_{sf} are the cross sections for fast and slow fission per molecule respectively. σ^i is the cross section for inelastic scattering. The correction terms are small, hence the critical radius is to a good approximation independent of the dilution factor. This approximation is valid to the extent that an incident neutron may be considered to produce fission with certainty. If the dilution becomes too great $(x = 0.1)$ this will no longer be so. In this region, the calculation will be further complicated by the resonance absorption of ordinary uranium. It is certain that in the latter region more absorbing material will be needed than for high isotopic concentrations.

In the case of UF_6, the correction term turns out to be .094, the critical radius 6.75 cm, the critical mass 6.05 kg.

A water or paraffin layer of 10 cm thickness produces about the same effect as an infinite hydrogenous reflector. Thinner layers produce smaller effects; very thin layers produce effects which tend to zero more rapidly than the thickness of the layer.* Thus, the effect of very thin

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* This is due to the fact that protons give no back-scattering. The effect of the non-hydrogenous material in the layer gives of course, an effect proportional to the thickness of the layer.

hydrogenous layers is insignificant.

A similar picture is obtained if the hydrogenous material is intermixed with the reacting material. If the hydrogenous material is present in sufficient bulk to slow down neutrons to thermal energies before they escape then a very small concentration of 25 is sufficient to give fission with a large fraction of these slow neutrons. Thus a chain reaction may be sustained.

If the material is in the shape of a hollow sphere of about 2 or 3 mm thickness, then the principal modification of Eq. (13) is in the small correction terms. Hence the outer radius will be approximately the same as for a solid sphere. As a result much smaller masses will become chain reacting. It must be remarked that these smaller masses will not become smaller (few hundred grams) than the masses needed to sustain a chain reaction in a water solution (discussed below). If a mass of the same order of magnitude as that obtained immediately above has a planar shape of some 5 mm thickness and is surrounded by a sufficient amount of hydrogenous material, the material will sustain a chain reaction.

According to calculations of Oppenheimer, Serber, Christy, and Wheeler a few hundred grams of 25 dissolved in a few liters of water are sufficient to give a chain reaction. Actually such small amounts become chain-reacting only if the solution is surrounded by pure water which serves as a reflector. But even if such a reflector is absent about 3 kilograms of 25 dissolved in about 50 liters of water will be probably chain-reacting. Even in the vapor state, chain reaction may take place if the tank is surrounded by hydrogenous materials. If we set

ν the number of neutrons per fission
γ the average albedo of fission neutrons on
        the hydrogenous wall
λ the fission mean free path of the reflected
        neutrons in the chain reacting gas
R   the radius of the tank containing the chain
        reaction gas

The critical condition becomes

$$\nu\gamma\left[1 + \frac{\lambda}{R}\, e^{-2R/\lambda}\,\left(1 + \frac{\lambda}{2R}\right) - \frac{\lambda^2}{2R^2}\right] = 1$$
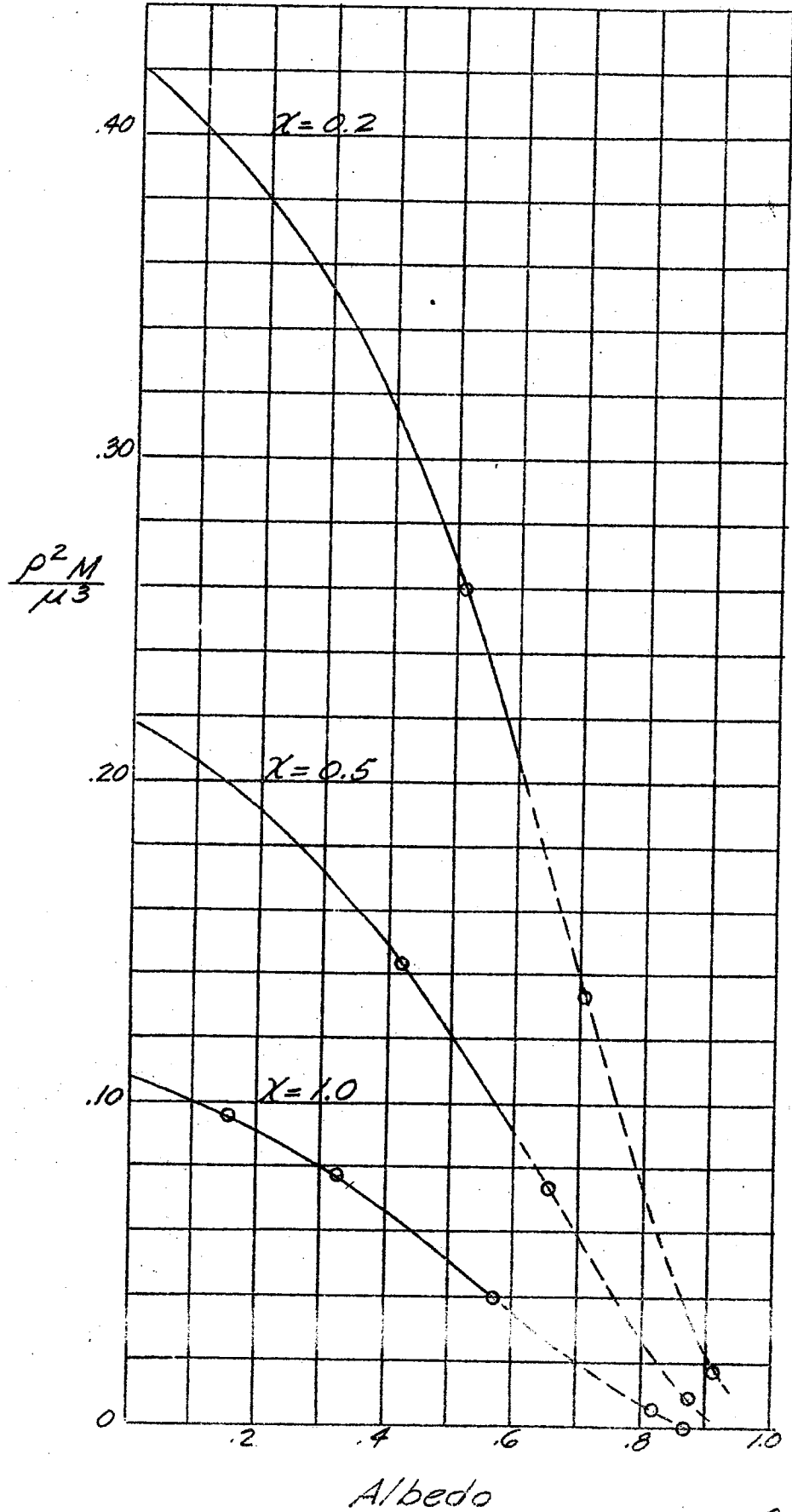
For ν we have the fairly reliable value 2.2. The albedo γ is more uncertain but may be estimated as 0.55. Therefore, the chain reaction will set in for $R \approx 1.5\,\lambda$. The mean free path λ is inversely proportional to the density of the gas, to the fission cross section and to the isotopic abundance x of 25. If we set the density equal to that of an ideal gas at NTP and use for the cross section the observed thermal fission cross section 600×10^{-24} cm^2, then λ in pure 25 is 60 cm and the critical radius becomes 90 cm. Of course, not all the neutrons reflected from the walls will have thermal velocities and thus the actual fission cross section will be smaller. In addition, the gas to be used is of a smaller density. Therefore, the actual critical radius will be several meters.

The dangers caused by hydrogenous substances can be reduced by the use of boron compounds. The tank should be coated by a boron compound. This greatly decreases danger that may arise if some hydrogenous material (for instance a person) comes near to the tank. One gram of boron per cm^2 should provide adequate protection. Even 0.1 gram/cm^2 is very helpful.

Any hydrogenous material which might become intermixed with the chain reacting material should be mixed with boron. If the mass concentration of boron is chosen as 1/10 of the maximum mass concentration with which 25 might appear in the hydrogenous material no chain reaction will occur.

Boron should be used in such a way as not to get mixed in the normal operation with the process gas. Even minute concentrations of boron in the final product are harmful.

Dependence of Critical Masses on Albedo