Comparison of Postbuckling Model and Finite Element Model with Compression Strength of Corrugated Boxes

Progress in Paper Physics, September 2002

Thomas J. Urbanik[1], Edmond P. Saliklis[2]

Abstract

Conventional compression strength formulas for corrugated fiberboard boxes are limited to geometry and material that produce an elastic postbuckling failure. Inelastic postbuckling can occur in squatty boxes and trays, but a mechanistic rationale for unifying observed strength data is lacking. This study employs a finite element model, instead of actual experiments, to generate experimentally designed data and a smoothed model. Results lead to a better understanding of how to unify elastic and inelastic failure data and form a more general box strength formula.

The objective of this study is to determine if the postbuckling formula advocated in a previous review of some historical data on box compression, including subsets of elastic and inelastic buckling, is supported by more pertinent experiments and can thus constitute a mechanistically rational more general box compression formula. The review revealed that a combination of elastic and inelastic postbuckling theory can be universally applied to the data, with different constants for each data source, provided that nonlinear material characterization is introduced and that an empirical correction is applied to panel stiffness.

Thorough experimental replication and inclusion of all geometry and material variables would be prohibitively expensive. Therefire, our approach is to numerically generate postbuckling data with finite element analysis (FEA) of buckling stress and then apply the previous formula. We it makes some sense to terminate the analysis with the FEA predictions, having a simpler, yet mechanistic, strength formula can provide the basis for actual experimental confirmation and practitioner use.

Various smoothed models were fit to the finite element predictions. An empirical correction for panel stiffness input to a broadened form of an elastic–inelasticpostbuckling model gave the best results. The finite element predictions corroborate previous experiments, and results are applicable to box geometry beyond the range of what has previously been investigated.

Objective and Scope

The objective of this study is to determine if the postbuckling formula advocated in previous work (Urbanik 1996) for combined elastic and inelastic failure is supported by more pertinent experiments and can thus constitute a mechanistically rational basis for a more general box compression formula. In the previous study (Urbanik 1996), the best model of box strength was obtained with each panel characterized by the following two-part formula:

$$\frac{\sigma_f}{\sigma_y} = \alpha\left(\frac{\sigma_{cr}}{\sigma_y}\right)^{\eta} \qquad U > 1$$

$$\frac{\sigma_f}{\sigma_y} = \alpha \qquad U \le 1 \tag{1}$$

together with the empirical correction

$$S_a = S\phi^{\tau} \tag{2}$$

[1]Research Engineer, USDA Forest Service, Forest Products Laboratory
[2]Assistant Professor, Department of Civil and Environmental Engineering, Lafayette College

In: Proceedings of the 2002 Progress in Paper Physics Seminar; 2002 September 8-13; Syracuse, NY. Rochester, NY: Rochester Institute of Technology. 4 p.

using an apparent stiffness S_a instead of *S* in the calculation of s_{cr}. Two postbuckling constants, a and h, appear m Equation (1). A third material postbuckling constant q_0 and a fourth constant t are embedded implicitly m the calculation of s_{cr}. Fits to data with calculations of a, based on nonlinear material theory (Johnson and Urbanik 1987) were more accurate than fits to data based on linear material theory.

Experimental Design

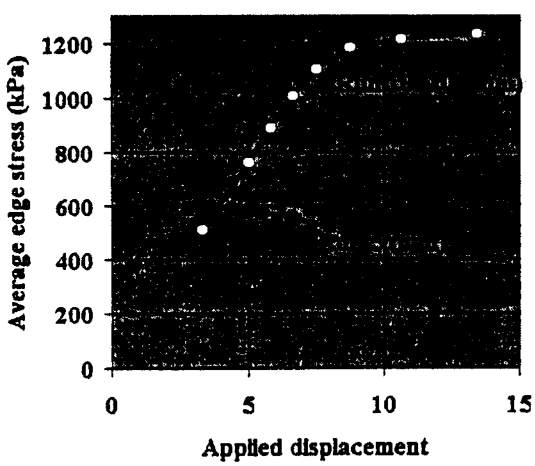
The theory of finite length plates (Urbanik 1996), representing box panels and used to determine s_{cr} has five filndamental non-dimensional inputs: q_0, *S*, f, *v*, and $\hat{c}$. In a 2^5 factorial design of these variables low and high values of each variable were selected to represent the scope of the overall characterization of box compression data utilized m our previous study (Urbanik 1996).

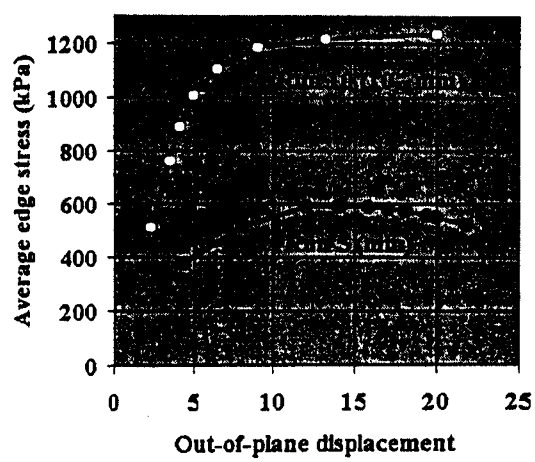
Physical properties representing a standard 205 g/mm^2, C-flute corrugated fiberboard, as was investigated in a previous study (Urbanik 2001), were adjusted for isotropic behavior such that the computed *S* and f of a plate would remain the same for both isotropic and anisotropic cases. The non-dimensional inputs combined with the standard corrugated fiberboard result m physical properties that provide the material and geometry inputs for FEA characterization.

Finite Element Procedure

Our finite element postbuckling analyses used 8-noded isoparametric shell elements. Eight elements per buckled shape (sinusoidal half-wave) were used in typical mesh sizes. This mesh size was chosen based on mesh refinement exercises. Isotropic material characterization enabled input of the exact stress-strain curve, $s = c_1 \tanh(c_2 e/c_1)$.

To more realistically simulate experimental laboratory results, the postbuckling analysis imposed a downward displacement at the top of the panel, which simulated the head movement of a testing machine. The bottom of the panel was not allowed to translate vertically. All edges were restrained from out-of-plane movement, but they were free to rotate. As the top of the panel was forced downward, the panel bulged outward into a number of half-sine waves. Stresses increased throughout the analysis, until convergence could no longer be achieved. Maximum out-of-plane displacement was recorded throughout the analysis, as was the final number of half-sine waves. The total force along the loaded edge was recorded throughout the analysis. Typical results for applied and out-of-plane displacement are shown below for run 5 (s_f 3 s_{cr}) and run 10 (s_f £ s_{cr}). Note the difference m displacement scales. For run 5, the maximum average stress was approximately 586 kPa even though the panel had not yet collapsed. Final collapse occurred shortly afterwards, at 520 kPa





Results

Various forms of Equation (1) were fit to the data.

$$\sigma_f = \alpha \sigma_y{}^{1-\eta} \sigma_{cr}{}^{\eta} = \alpha c_1 \theta_0{}^{\eta-1} \hat{\sigma}^{\eta} \qquad (3)$$

$$\sigma_f = \alpha \sigma_1 \theta_0^{\eta-1} \hat{\sigma}_a^{\eta} \qquad (4)$$

$$\sigma_f = \alpha \sigma_y{}^{1-\eta} \sigma_{cr}{}^{\eta} = \alpha c_1 \theta_0{}^{\eta-1} (CS)^{\eta} \qquad (5)$$

$$\sigma_f = \alpha c_1 \theta_0{}^{\eta-1} (CS_a)^{\eta} \qquad (6)$$

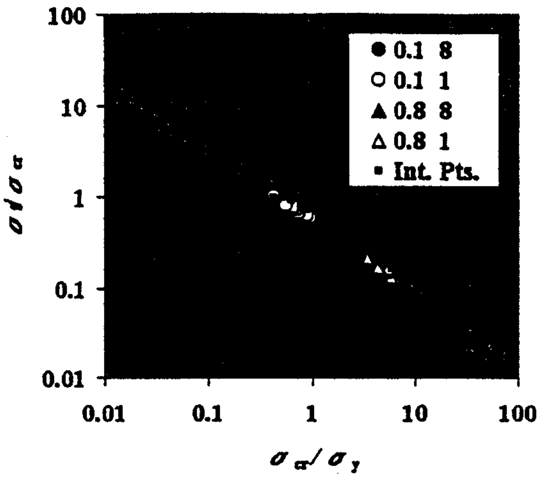
A summary of the models obtained from Equations (3) to (6) is given m Table 1. For comparison, the first model is simply the McKee formula from McKee et al. (1963) rearranged into Equation (3) with constants a and h taken from Urbanik (1997). Results of the best fitting model 8 are shown m the following graphs.

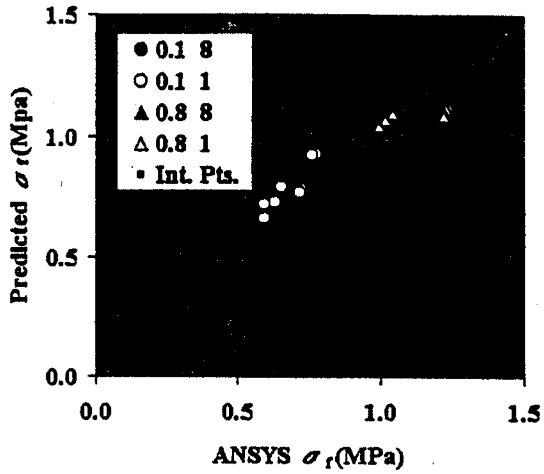
Table I—Parameter values of 15 models fit to ANSYS data

Law	Model	t	U £ U_b		U_b	U ³ U_b		Ave. |error| (%)	r^2
			a	h		a	h		
McKee	1	0	0.394	0.254	—	0.394	0.254	25.7	0.669
Linear Eq.(5)	2	0	0.606	0.126	—	0.606	0.126	18.5	0.764
	3	0	—	0	1	0.835	0.494	17.9	0.715
	4	0	0.695	0.082	1	0.695	0.353	10.8	0.860
	5	0	0.696	0.078	1.314	0.620	0.290	9.78	0.867
Hybrid Eq.(6)	6	0.555	0.607	0.147	—	0.607	0.147	16.4	0.813
	7	0.600	0.716	0.088	1	0.716	0.419	9.59	0.912
	8	0.554	0.684	0.093	1.136	0.644	0.330	7.85	0.915
Nonlinear Eq.(3)	9	0	0.705	0.126	—	0.705	0.126	23.8	0.766
	10	0	—	0	1	0.850	0.494	16.8	0.767
	11	0	0.766	0.081	1	0.766	0.288	15.3	0.821
	12	0	0.749	0.080	1.613	0.614	0.288	13.8	0.767
Nonlinear Eq.(4)	13	0.555	0.727	0.162	—	0.727	0.162	22.4	0.788
	14	0.600	0.708	0.147	1	0.708	0.415	9.77	0.878
	15	0.565	0.743	0.088	1.276	0.654	0.351	9.19	0.900

Conclusions

The postbuckling of plates with nonlinear material and subjected to axial compression was analyzed with a finite element model. Various smoothed models were fit to the finite element predictions to determine a practitioner form of a more general strength formula applicable to corrugated containers. An empirical correction for plate stiffness input to a broadened form of an elastic–inelastic postbuckling model gave the best results. The finite element predictions corroborate previous experiments, and results are applicable to box geometry beyond the range of conventional strength formulas.





References

Johnson, M. W., Jr. and T. J. Urbanik. 1987. Buckling of axially loaded, long rectangular plates. Wood Fiber Sci. 19(2):135–146.

McKee, R. C., J. W. Gander, and J. R. Wachuta. 1963. Compression strength formula for corrugated boxes. Paperboard Packaging (August).

Urbanik, T.J. 1996. Review of buckling mode and geometry effects on postbuckling strength of corrugated containers. Development, validation, and application of inelastic methods for structural analysis and design, PVP vol. 343, American Society of Mechanical Engineers, pp. 85–94.

Urbanik, T. J. 1997. Linear and nonlinear material effects on postbuckling strength of corrugated containers. AMD vol. 221/MD vol. 77, Mechanics of cellulosic, American Society of Mechanical Engineers, pp. 93–99.

Urbanik, T. J. 2001. Effect of corrugated flute shape on fiberboard edgewise crush strength and bending stiffness. J. Pulp Paper Sci. 27(10):330–335.

Nomenclature

C	Linear buckling constant	U_b	Elastic-inelastic-breakpoint	ν	Geometric mean Poisson's ratio
c_1, c_2	Stress–strain constants	α, η	Postbuckling constants	σ	Stress
$\hat{c}$	Normalized shear modulus	ε	Strain	σ_{cr}	Average critical stress at bifurcation
S	Normalized plate stiffness	θ_0	Nonlinear material postbuckling constant	σ_f	Average failure stress
S_a	Apparent S	ϕ	Effective plate aspect ratio	σ_y	Material yield stress
U	Universal plate slenderness	τ	Stiffness correction exponent	$\hat{\sigma}$	Normalized buckling stress