Studies in Health Technology and Informatics, vol. 81
(*Proceedings of Medicine Meets Virtual Reality 2001.* J. D. Westwood, *et al., eds.*),
Amsterdam: IOS Press, pp. 594-600.

Anatomic Modeling from Unstructured Samples Using Variational Implicit Surfaces

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Abstract. We describe the use of variational implicit surfaces (level sets of an embedded generating function modeled using radial basis interpolants) in anatomic modeling. This technique allows the practitioner to employ sparsely and unevenly sampled data to represent complex biological surfaces, including data acquired as a series of non-parallel image slices. The method inherently accommodates interpolation across irregular spans. In addition, shapes with arbitrary topology are easily represented without interpolation or aliasing errors arising from discrete sampling. To demonstrate the medical use of variational implicit surfaces, we present the reconstruction of the inner surfaces of blood vessels from a series of endovascular ultrasound images.

1. Introduction

Medical data analysis and visualization often requires the representation of known segments or objects in an easily processed form. The primitives used for these representations are often either a binary voxel map or a polygonal or polyhedral representation. Each of these modeling primitives has its limitations. As a modeling representation, voxel maps are not invariant with respect to rotation or changes in scale. When arbitrarily rotating or scaling a binary voxel map, grey values will be introduced near the boundary as voxels become partially covered by the rotated or scaled bitmap, making necessary a type change from a binary-valued array to a scalar valued system. The alternative is to enforce a binary mapping of the voxel representation, accepting the subsequent sampling errors.

Surface representations modeled either with polygons (or with volume structuring primitives such as tetrahedra) are invariant with respect to rotation and scale. However, if you create a close-up view of the surface representation, the discretization of the surface into planar primitives becomes apparent, and the viewed surface no longer closely approximates the desired smoothness for the representation of the model. Thus although invariant with respect to changes in scale, polygonal surface models are susceptible to artifacts and errors in the piecewise planar approximation to smooth surfaces when scaled.

What is desired is a smooth representation that is easily captured from a binary segmentation and that is invariant with respect to rotation, translation, and scale. Moreover, it should support resampling of the surface model at any arbitrary sampling rate to support visualization at any level of zoom or scale. We have explored the use of variational implicit surfaces as a modeling primitive for binary anatomical objects. These systems provide the necessary smooth differentiable surface models with the desired properties for robust representation of complex biological structures.

2. Background and History

When viewing the 3D relationships among anatomical structures identified within medical data, researchers often apply direct volume rendering techniques [6]. The generation of surface models from volumetric information is an alternative to direct rendering and is also a common practice in the visualization of anatomy. Reconstruction of piecewise planar polygonal surface models from rectilinear volume data is frequently achieved using computationally efficient methods such as Marching Cubes [7]. However, when either of these techniques is applied to objects that are represented as binary bitmaps, sampling and discretization errors arise leading to terracing and jaggies, a prominent problem in medical imaging.

Recent work in surface fitting has addressed these issues. Gibson extracts smooth surface models treating the existing binary data as a constraining element in an energy-minimizing deformable surface system [4]. The resulting data structure can be used either to create Euclidean distance maps for direct volume rendering or employed directly as a polygonal surface model [5]. Whitaker has modified the constrained deformable surface model to a constrained level-set model, which creates smooth models while bypassing the need for a separate surface representation [14]. However, while these methods generate smooth representations, both the level set model and the surface net remain discretely sampled and retain the problem that they are not zoom invariant. A different modeling primitive is still needed.

Turk and O'Brien adapt earlier work on thin plate splines [1][2] and radial basis interpolants [3][10] to create a new technique in generating implicit surfaces [11]. Their method allows direct specification of a complex surface from sparse, irregular surface samples. The method is quite flexible and has been extended to higher dimensions to support shape interpolation [12]. However, their technique as described cannot be used to model surfaces where large numbers of surface points are included, making it unsuitable for medical applications where range data or tomographic reconstruction often lead to data described by hundreds of thousands of surface points.

3. Methods & Tools

An *implicit surface* is defined by $\{\vec{x}: f(\vec{x}) = k\}$, $k \in \mathbb{R}$, for some characteristic embedding function $f: \mathbb{R}^3 \to \mathbb{R}$. Given a set of surface points $C = \{\vec{c}_1, \vec{c}_2, \vec{c}_3, \ldots \vec{c}_n\}$, the variational implicit technique interpolates the smoothest possible scalar function $f(\vec{x})$ whose zero level set, $\{\vec{x}: f(\vec{x}) = 0\}$, passes through all points in C. That is, find the smoothest function f such that $f(\vec{x}_i) = 0$ for each known surface point $\vec{x}_i$, and $f(\vec{y}_i) = 1$ for one or more points $\vec{y}_i$ known to be inside the shape. Following Turk and O'Brien's generalization of the problem, ., given a set of positions $\vec{c}_i$ and corresponding values h_i, solve for an embedding function f such that $f(\vec{c}_i) = h_i$. by employing radial basis interpolating functions $\phi(r)$ in a critically constrained linear system of n equations and n unknowns. Specifically, given a radial basis interpolating function $\phi(r)$ and a series of known points where the desired function f is constrained to be $f(\vec{c}_i) = h_i$, solve the following equation for the unknown weights d_j.

$$f(\overline{c}_i) = \sum_{j=1}^{n} d_j \phi(\|\overline{c}_i - \overline{c}_j\|) + P(\overline{x}) = h_i \tag{1}$$

Expanding $\vec{c}_i = (c_i^x, c_i^y, c_i^z)$, the entire linear system can be expressed as an $(n+4) \times (n+4)$ matrix M where:

$$\begin{bmatrix} \phi_{11} & \phi_{12} & \dots & \phi_{1n} & c_1^x & c_1^y & c_1^z & 1 \\ \phi_{21} & \phi_{22} & \dots & \phi_{2n} & c_2^x & c_2^y & c_2^z & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_{n1} & \phi_{n2} & \dots & \phi_{nn} & c_n^x & c_n^y & c_n^z & 1 \\ c_1^x & c_2^x & \dots & c_n^x & 0 & 0 & 0 & 0 \\ c_1^y & c_2^y & \dots & c_n^y & 0 & 0 & 0 & 0 \\ c_1^z & c_2^z & \dots & c_n^z & 0 & 0 & 0 & 0 \\ 1 & 1 & \dots & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \\ p^x \\ p^y \\ p^z \\ 1 \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_n \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

The resulting matrix is known to be positive semi-definite, and can be solved with a linear decomposition system. Once the weights d_i are found, the embedding function can be written as:

$$f(\overline{x}) = \sum_{i=1}^{n} d_i \phi(\|\overline{x} - \overline{c}_i\|) + P(\overline{x}) \tag{3}$$

Depending on the differentiability of the radial basis function, $\phi(r)$, $f(\vec{x})$ is an implicit surface generating function that can be resampled with arbitrary precision and remains invariant under rotation, translation, and zoom. (For most of this work, we have used the same radial basis function, $\phi(r) = r^3$ for $r \geq 0$, used by Turk and O'Brien from related research on thin-plate splines.) If the constraints $\vec{c}_i$ for $f(\vec{x})$ are abstracted from either a grey-level or a binary bit mask volume sampled volume, the resulting variational implicit surface is a more compact analytical representation of the same data.

4. Results

We have applied this method to the modeling of a bovine aorta from data acquired using an endovascular ultrasound transducer. This particular modality acquires noisy 2D image slices, sampled at uneven intervals with non-parallel orientations. The transducer is drawn slowly through the vessel, acquiring cross-sectional sonographic images (see Figure 1).

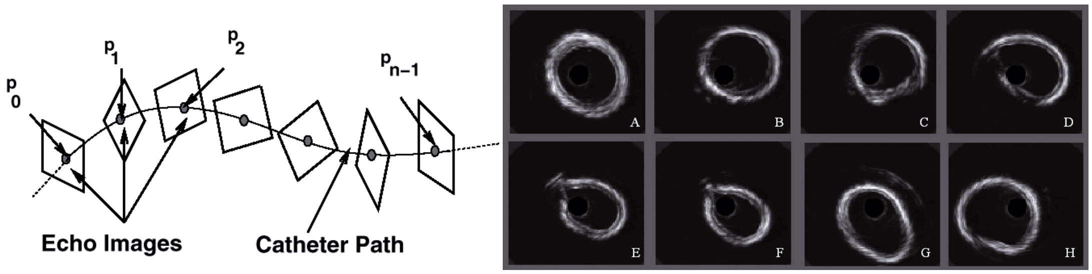


Figure 1. Cross-sectional ultrasound images of an ex-vivo bovine aorta. The slices are acquired at angles perpendicular to the endovascular catheter path. The imaging path is determined by the geometry of the blood vessel.

Each individual slice is then segmented, and the aggregate contours from the tilted slices are processed to form a variational implicit surface. The resulting analytic description can be sampled and rendered using volume rendering approaches, or it can be interrogated and tesselated into a polygonal or parametric surface representation. Figure 2 shows the two views of the interior surface of the bovine aorta from Figure 1 reconstructed as a variational implicit surface. The zero-set of the implicit model has been extracted using a surface tiler, rendering them as polygons at the resolution desired for the magnification shown. If close-up views are desired, the model can easily be re-interrogated and tiled in the surface rendering case or the model simply re-rendered in a direct volume rendering system with arbitrary precision, eliminating aliasing artifacts in the representation of the model.

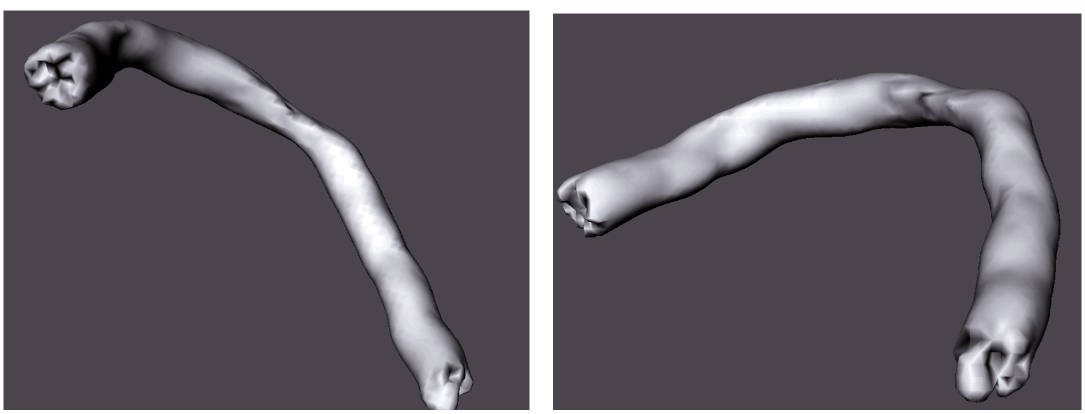
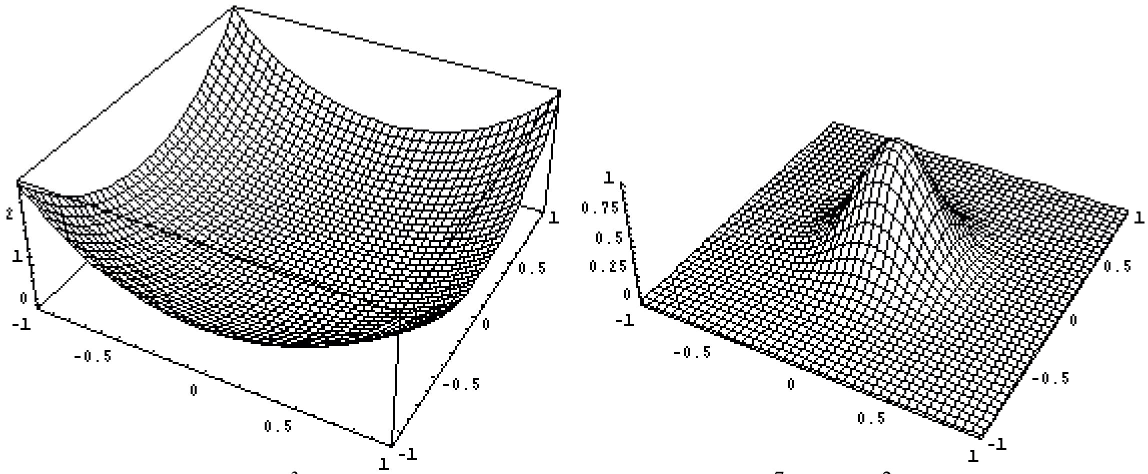


Figure 2. Surface renderings of the interior surfaces of the ex-vivo bovine aorta reconstructed as a variationalimplicit surface from the ultrasound slices in Figure 1. The models are not subject to aliasing artifacts generated by comparable surface or volume. In addition, these models are generated from sparse, non-uniformly sampled non-parallel ultrasound slices.

5. Discussion and Future Work

The radial basis function, $\phi(r) = r^3$ for $r \geq 0$, advocated by Turk and O'Brien is infinite in extent. It has advantages when interpolating across unpredictable spans, making it ideal for morphing and shape interpolation. However, the infinite extent leads to ill-conditioned matrices and increased computational complexity. A naïve approach is easily order $O(n^3)$.



$\phi(r) = r^3,\ r \geq 0$ $\qquad$ $\phi(r) = (1 - r/s)^7(16(r/s)^2 + 7(r/s) + 1),\ r < s$

Figure 3. Two radial basis functions. The left function $\phi(r) = r^3,\ r \geq 0$ has infinite extent. The right function $\phi(r) = (1 - r/s)^7(16(r/s)^2 + 7(r/s) + 1)$ is clamped to zero outside a specified radius of support s, and is proven to be C^4 continuous. The compactly supported function on the right leads to more stable numerics and faster solutions of the variational implicit surface model.

In related work, we address the computational complexity of variational implicit surface modeling. Instead of $\phi(r) = r^3$ as the underlying interpolant, we address select from among a family of radial basis functions presented by Wendland [13] to find a function with the necessary continuity but also with finite, compact local support. Figure 3 shows a comparison of the 3D thin-plate-spline radial basis function and the suggested compactly supported radial basis functions of our current research. The shift from a radial basis function of infinite extent to one that has compact local support has created dramatic gains in memory utilization and computational complexity. Previous work described solutions for systems of equations of order $O(n^3)$ complexity with iterative solutions capable of achieving order $O(n^2)$. The shift to finite interpolants and sparse matrices has shifted the bulk of the computation toward order $O(n)$, depending on the complexity of the model and the uniformity of the density of the surface constraints. We have measured the complexity of the matrix solution for some test cases as $O(n^{1.5})$. For more details, see Morse [8].

The use of compactly supported radial basis functions imposes a restriction on the maximum distance allowed between systems of surface constraints. This trade-off between speed and the granularity of the surface samples is the topic of some of our future research in this area. The infinite radial basis function permits interpolation across wide spans and with arbitrary orientations to the subsets of constraints that comprise the initial surface description. However, the costs of using this system rise with the number of points required to faithfully represent the surface. This trade-off has ramifications in the choice and precision of the segmentation algorithm to be used and the complexity of the objects to be represented.

Future work on this topic includes the development of hybrid representations that incorporate both infinite and compact radial basis functions. In addition, we will explore advanced slice-based segmentation techniques with confidence in our ability to interpolated smoothly between 2D segments. It should be noted that variational implicit surfaces can be generated as the output of segmentation systems; however, as a means of smoothly interpolating between sample slices, they can be used to initialize a deformable contour for segmenting an unknown intermediate slice. The use of variational implicit surfaces to help generate priors for segmentation systems is a current focus for some of our group. Finally, we are investigating algorithms for the automatic generation of variational implicit surface models for the anatomic structures currently identified as part of the Visible Human Project (VHP) [9]. Four hundred thirty-five (435) hand-segmented structures comprise the current segmented thorax database, with each segment modeled using an uncompressed binary bitmask. For example, the bitmask for the heart is over 400 kilobytes, alone. We are seeking a means of automatically generating compact analytical descriptions of these models using the techniques described here.

6. Conclusions

Given a set of surface points $C = \{\vec{c}_1, \vec{c}_2, \vec{c}_3, \ldots \vec{c}_n\}$, the variational implicit technique interpolates the smoothest possible scalar function $f(\vec{x})$ whose zero level set, $\{\vec{x}: f(\vec{x}) = 0\}$, passes through all points in C. These surface constraints or points can be generated from a variety of sources including segmentation systems. We show that variational implicit surfaces can be effectively used to model anatomic structures. Earlier limitations of computational and memory efficiency can been solved through a judicious selection of interpolant and improved numerics. Examples including noisy data from modalities generating curvilinear gridded data, such as endovascular ultrasound, demonstrate the utility of this technique.

7. Acknowledgements

This work was performed in large part at the National Library of Medicine under a visiting faculty program supporting both Dr. Morse and Dr. Subramanian. Dr. Rheingans was supported in part by NSF CAREER Grant #9996043. We would like to thank Greg Turk for his useful conversations and for making his code available to us, upon which our implementation is based.

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