A Size Dominance Index for Market Structure Analysis

Pascual García Alba Iduñate[1]

Abstract

Herfindahl's concentration index (H) is used by most competition authorities of the world to analyze market structure. This index is appropriate in cases in which there are no big differences in size among the firms in the relevant market. In other circumstances, it may give misleading results. In particular, H always increases with the merging of firms, although theory establishes circumstances in which mergers can be procompetitive. This paper proposes a size dominance index (P), which coincides with H when all firms are of the same size, but that under some conditions can decrease with mergers. These conditions are related to those that some theorists and antitrust practitioners suggest.

Introduction

The value of H, or for that matter, of any usual concentration index increases with any merger; notwithstanding that it is likely that some mergers, far from reducing competition, increase it. That could be the case of the merger of two moderately sized firms in a market dominated by a firm that concentrates most of the output, because of their increased capability to defend themselves from anticompetitive actions carried out by the largest firm.

In the literature about concentration and market performance, there are many examples of mergers that imply a considerable increase of the concentration measured by H, and simultaneously result in increased economic efficiency, measured as the sum of

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[1] The author is professor at El Colegio de México and a member of the Federal Competition Commission of México. I hereby thank the suggestions and support of the Commission personnel, but whatever is expressed here is solely the author´s responsibility.

profits and consumer's surplus. In this paper an alternative index is constructed that does not penalize any merger. Instead, its behavior depends on the relative size of the merging firms and on the premerger structure of the market. This index does not increase with the merging of relatively small firms, but it does with mergers of relatively large ones.

The first section starts with the specification of a family of indexes that keeps all main theoretical characteristics of the H index, except the attribute that any merger increases the value of the respective index. Next, a subset of the index family is defined, for which there are market structures where some mergers decrease "concentration". Afterwards, some mathematical properties of the proposed index are derived. The following section points out the accordance of these properties with some general results from industrial organization studies. Later, some numerical examples are considered. Finally, a procedure for dealing with real situations is suggested.

I. Specification of an Index Family

H is an element of the following index family:

$$(1.1) \qquad F(Q;p) = \sum_i \frac{Q_i^{2p}}{\left(\sum_j Q_j^{p}\right)^2} \quad , \quad p>0 \ ,$$

where Q_i is the supply by firm *i*. It is clear that H=F(Q;1). All elements of F share with H the following characteristics:

i) F is homogeneous of degree zero in Q = (Q_1 ,...,Q_n), with *n* equal to the number of firms in the market.

ii) The highest value of F is 1.0 and the minimum 1/n. The maximum is reached when one single firm k concentrates the whole market ($Q_k/\sum_i Q_i = 1$), and the minimum when all firms are of the same size ($Q_j=\sum_i Q_i/n$, for every *j)*.

iii) An increase of a "large" firm's output increases the value of F while the one of a "small" firm reduces it, where the dividing line between large and small firms depend on the distribution of Q. This can be verified from the derivative of (1):

(2)
$$\frac{\partial F}{\partial Q_r} = \frac{2pQ_r^{p-1}}{\sum_i Q_i^p}\left|\frac{Q_r^p}{\sum_i Q_i^p} - F\right|$$

II. The Effect of Mergers on *F*

The output transference from firm s to firm r, increases or decreases the value of *F* depending on $\partial F/\partial Q_r - \partial F/\partial Q_s$ being larger or smaller than zero. In the case of *H* we get from (2) with p=1, that

(3)
$$\frac{\partial H}{\partial Q_r} - \frac{\partial H}{\partial Q_s} = \frac{2(Q_r - Q_s)}{\left(\sum_i Q_i\right)^2}$$

which means that any transfer of output from a firm to another larger one, increases *H*. Therefore any merger increases *H*.[2] For that reason it is useful to analyze for which values of p the indexes F do not impose this behavior. Again, from (2):

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[2] For a discussion on the properties of the concentration indexes, which include H, see Encaoua and Jacquemin (1980), and García Alba (1990).

(4)

$$\left(\frac{\partial F}{\partial Q_r} - \frac{\partial F}{\partial Q_s}\right) \frac{\left(\sum_j Q_j^{\,p}\right)^2}{2p} = Q_r^{2p-1} - Q_s^{2p-1} - F\left(Q_r^{p-1} - Q_s^{p-1}\right) \sum_i$$

from which, if 1/2<p<1 any transference from a firm to a larger one increases F, and so any merger increases the value of the index.[3] For this implication not to hold, it is enough that p>1. Also for 0<p<1/2 some mergers could reduce the F indexes, but when 0<p<1 the mergers of small firms, such that $Q_r^p+Q_s^p<F\Sigma_i Q_i^p$, would always increase F. This results from the equivalence of (4) with:

(5)

$$\left(\frac{\partial F}{\partial Q_r} - \frac{\partial F}{\partial Q_s}\right)\left(\sum_i Q_i^{\,p}\right)^2 \frac{(Q_r Q_s)^{1-p}}{2p} = Q_r - Q_s + \left(Q_r^{1-p} - Q_s^{1-p}\right)\left(F \sum_i Q_i^p - Q_r^p - Q_s^p\right)$$

Since it is seeked that F decreases with mergers of "small" firms and increases with mergers of "large" ones, it is convenient to define the subset S(Q;p) of F(Q;p), that includes the indexes that, for some Q structures, increase with mergers of relatively large firms and decrease with mergers of relatively small firms:

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[3] Mergers are analyzed keeping market shares constant. This is customary in the use of indexes, including H. Once the merger is completed, it is expected that firms modify their share. But in order to foresee its probable results, it is necessary to work with the shares before the merger.

(6) $$S(Q;p) = \sum_i \frac{Q_i^{2p}}{\left(\sum_j Q_j^2\right)^2} \quad , \qquad p > 1$$

The lowest degree element of S(Q;p), with p an integer, is P = S(Q;2), that is

(7) $$P = \sum_i \frac{Q_i^4}{\left(\sum_i Q_j^2\right)^2}$$

This index, being an element of S(Q;p) with p>1, does not necessarily increase with mergers, unlike H and all concentration indexes which always increase when a firm concentrates the market of another one by means of a merger. Except for this, all other above mentioned characteristics of H are fulfilled also by P. Since it is reasonable that a concentration through a merger increases the output concentration index, it is clear that in a strict sense, the proposed index is not a market concentration index.

III. An Interpretation of P

Before giving an interpretation of P it is convenient to offer an alternative interpretation of H, for which this index is expressed as:

(8) $$H = \sum_i w_i(q_i)$$

where $q_i = w_i = Q_i/\sum_k Q_k$. In this expression it is clear that H is a weighted average of the output shares. That is why H is an output concentration index. As for index P, it can be expressed as:

(9)

$$P = \sum_i (h_i^2)$$

where $h_i = q_i^2/\Sigma_k q_k^2$, is the portion of H that is contributed by firm *i*. Therefore P is an average of each firm's share in the concentration, measured by H. As a result P is an index of the concentration of the concentration (a Herfindahl of the contributions to the Herfindahl computed over market shares).

Mexico's Federal Law of Economic Competition determines that when the probability is evaluated, that a market structure propitiates monopolistic practices, not only the market concentrated by each firm should be considered, but also in its relation to the other firms' concentrations. Index P, about concentration of concentration, does that, through the relations $q_i^2 / \Sigma_k q_k^2$.

Also the studies of several markets suggest that large firms can exert a stronger monopolistic power, when the rest of the market is less concentrated.[3] Index P increases when the h_i shares concentrate, not necessarily when the q_i shares concentrate. That is, the effect of an output concentration is evaluated not in itself, but in the way it affects the

[3]This has been documented in the case of airlines. See Borenstein (1990).

relative position of each firm in relation to the concentration estimated by the shares in H, not in Q.

If a firm controls 60% of output, H decreases when the output concentration of the other firms also decreases. In fact, H would reach its lowest value when the remaining 40% of output is produced by an arbitrarily larger number of firms of insignificant size. But it is reasonable to assume that in many cases the mergers among the small firms would strengthen them to counteract uncompetitive practices of the largest firm to displace them from the market. On the other hand, in this example, P would reach its highest value when the firms, different from the dominant one, were of the smallest size possible. In this case P would be close to one, the magnitude corresponding to a monopoly. This would reflect that with firms other than the dominant of size close to zero, the capability of the dominant firm to exert dominion over the others would go almost uncontested.

In the opposite extreme, when in the above example the small firms merge into a single one, H would reach its highest value and P its lowest. There are reasons to assume that it is better to have a market in which a firm with 60% of the market competes with another that controls 40%, rather than with a huge number of insignificant firms with practically no competitive capability. Section V points out the concordance between P's general behavior and some results from industrial organization theory.

A firm's market power does not depend only on its relative size compared to the size of its competitors. Other variables are determinant, such as demand elasticity and barriers to entry. But under equal circumstances, the higher h_i, the greater its capability to control the market. That is why it was chosen to designate P as the *size dominance index*.

Just for participating in the market, each firm has an influence on it, which will be stronger the larger its size in comparison to the sizes of the other firms.

IV. Some General Properties of P

In this section some properties of the proposed index will be pointed out while in the following one, these properties will be discussed in the frame of industrial organization theory.

Property 1. $P \geq H$ with $P = H$ if and only if $q_i = 1/n$ for any i, in which case $P=H= 1/n$.

This property follows from the fact that $h_i \geq q_i$ for relatively large firms ($q_i \geq H$) , and $h_i \leq q_i$ for relatively small firms ($q_i \leq H$). For the distribution of q_i to duplicate the corresponding to h_i, it would be necessary to transfer production from small firms to large firms, which increases the Herfindahl (remember that P is a Herfindahl computed over the h_i).

Property 2. Any transfer to the largest firm m ($q_m \geq q_i$ for each i), increases P.

To demonstrate this property dP is calculated for $dq_r = -dq_s > 0$, with $q_r > q_s$. From $P = \Sigma q_i^4 / (\Sigma q_i^2)^2$

(10)
$$\frac{dP}{dq_r} = \frac{4}{H^2} [q_r^3 - q_s^3 - (q_r - q_s)PH],$$

and making use of $q_r^3 - q_s^3 = (q_r - q_s)(q_r^2 + q_r q_s + q_s^2)$,

(11) $$\frac{dP}{dq_r} = \frac{4}{H^2}(q_r - q_s)(q_r^2 + q_s^2 + q_r q_s - PH)$$

Making $q_r = q_m$, and since P is an average of h_i and max $(h_i)=h_m$, $P<h_m = q_m^2 / H$, then $dP / dq_m > 0$ results.

Lemma 2.1. The acquisition of any firm by the largest one increases P.

Lemma 2.2. Any transference from the largest firm to another one reduces P.

Property 3. If the largest firm suplies most of the output ($q_m > 1/2$), P is greater than the corresponding to a symmetric duopoly ($P > 1/2$).

Let $Z=\sum_i q_i^4 - (1/2) H^2$. Note that $Z > 0$ implies $P > 1/2$. Transfers from a firm to a larger one decrease Z if the following expression is negative when $q_r > q_s$:

(12) $$\frac{\partial Z}{\partial q_r} - \frac{\partial Z}{\partial q_s} = 4(q_r - q_s)(q_r^2 + q_s^2 + q_r q_s - H/2)$$

If $q_r \neq q_m > 1/2$, then $q_r^2 + q_s^2 + q_r q_s < H / 2$, and the former expression is in fact negative. Consequently $Z(q_m, 1 - q_m) < Z(q_m, q_2, ..., q_n)$. Since $P(q_m, 1 - q_m) > 1/2$, then $Z(q_m, 1-q_m)>0$. Moreover, $Z(q_m, q_2, ..., q_n)>0$, and therefore $P(q_m, q_2, ..., q_n) > 1/2$ when $q_m>1/2$.

Property 4. If the merger of q_r and q_s results in a share $q= q_r + q_s$ larger than the one resulting form the merger of any two other firms, the former merger increases P.

If $q_r = q_m$, this property follows from lemma 2.1. Suppose that $q_m > q_r > q_s$. Since $\sum_i q_i^2 / H = 1$, $q_r^2 + q_s^2 + q_r q_s - PH$ can be substituted by $\sum_i (q_i^2 / H)(q_r^2 + q_s^2 + q_r q_s - q_i^2)$ in (11). The assumption that $q_r + q_s > q_i + q_m$ for $i \neq m, r, s$ implies that in the previous sum the elements for $i \neq m, r, s$ are always positive. From $q_m < q$, the sum of the three remaining elements ($i = m, r, s$) is larger than $q_r q_s (q^2 - q_m^2) / H > O$. Therefore, in this case the expression (11) is positive, when $q_r > q_s$.

Lemma 4.1. If $q_r + q_s > 1/2$ the merger of q_r and q_s increases P.

Property 5. If $q_m > q_r > q_s$ and $q_r + q_s = q > 1/2$, then $P < 1/2$, the value corresponding to a symmetric duopoly.

This results from continuity of P and lemma 4.1, which imply $P(1/2, 1/2 - q_s, q_s) \leq 1/2$; as well as property 2, according to which $P(q_m, q_r > 1/2 - q_s, q_s, q_4, ..., q_n) < P(1/2, 1/2 - q_s, q_s)$.

Property 6. If $q_m > 1/2$, then any merger that does not involve the largest firm, decreases P.

This follows from (11), $P > 1/2$, and $q_r^2 + q_s^2 + q_r q_s < H/2$, when $r, s \neq m$.

Property 7. If the merger of q_r and q_s increases P, then the merger of $q_r^1 \geq q_r$ and $q_s^1 \geq q_s$, also increases P.

It is assumed that the merger between q_r^1 and q_s^1 is carried out instead of the merger between q_r and q_s (not following it).

The change in P resulting from the mergers is:

(13) $$\Delta P = \frac{R + \Delta R}{(H + \Delta H)^2} - \frac{R}{H^2}$$

were $R = \Sigma q_i^4$, R and H refer to their values before the merger;

$\Delta R=(q_r + q_s)^4-(q_r^4 + q_s^4) = 4 q_r^3 q_s + 6 q_r^2 q_s^2 + 4 q_r q_s^3$ is the increase of R as a result of the merger; and $\Delta H=2 q_r q_s$. Making use of these equivalences:

(14) $$P = \frac{\Delta H}{(H + \Delta H)^2} [2q_r^2 + 2q_s^2 + q_r q_s (3 - 2P) - 2PH$$

where P refers to its value before the merger. Because for any merger $\Delta H>0$, the sign of ΔP depends only on the sign of the expression in brackets. Since $3 - 2P > 0$, it is clear that taking larger values of q_r and q_s the expression in brackets increases. If its value is positive, it will remain so when substituting the merging firms by others of equal or larger size.

Property 8. If P decreases with the merger of q_r and q_s, it will also decrease if instead of this merger another one is carried out in which $q_r^1 \leq q_r$ and $q_s^1 \leq q_r$.

This property also follows from (14), and as well as property 7, it doesn't have a sequential character.

V. Index P and Industrial Organization Theory

Ideally, the indexes to evaluate market structure should be deduced from theory (Schmalensee, 1989). However, practical limitations exist. Results from theory are different for different assumptions regarding firms' behavior. Even when a specific behavior is assumed, results depend on the demand and cost functions.

In spite of these problems it is desirable that indexes be compatible with, or at least not utterly contradictory to the most general results from theory. This section shows that in this respect index (P) is better than index H, and indeed better than any traditional output concentration index. These indexes increase their value with any merger, when theory points out that there can be mergers that even in the absence of synergies arising from them, result in increased welfare.[4]

When all firms are of the same size, it has been demonstrated (Tirole, 1990, p. 223) that with price competition of the Bertrand type, the number of firms has no relation with welfare, while these two variables are positively correlated in Cournot type competition[5]. This confers certain appeal to index H, which in case of equal firms is the reciprocal of the number of competitors. This appeal is shared by index P. With equal firms both indexes coincide (P=H=1/n), in accordance to P's property 1, pointed out in the previous section.

[4]See for example Salant, Switzer and Reynolds (1983); Farrell and Shapiro (1990), and Levin (1990).

[5] See also Willig (1991).

Apart from the case of equal firms, the behaviors of both indexes are in general quite dissimilar, because any merger increases H while the merger of relatively small firms could decrease P. This property of P agrees with Werden (1991), in that increases of H do not necessarily diminish welfare because:

> In the asymmetric Cournot equilibrium (a) production is inefficiently allocated among firms; with the smaller firms overproducing relative to the large ones; (b) firms below a certain relative size actually produce more than they would under competition; and (c), because of (a) and (b) it is possible for a merger to enhance welfare even though it results in an output restriction.

This last possibility could be reflected by P -but not by H- because as pointed out in section I, P decreases with mergers of relatively small firms. This also agrees with properties 6 and 8 of P which were demonstrated in section IV.

In a similar fashion, it can be argued that mergers among relatively large firms reduce welfare.[6] With Cournot competition, large firms produce less than what would be the social optimum. A merger would lead them to reduce their output even more. In fact, It has been established that a merged firm's output, in the absence of considerable synergies, is less than the sum of what the merging firms produced before the merger (Farrell and Shapiro, 1990).

It is therefore convenient that an index increases with mergers of relatively large firms, the way it was shown in section I that happens with the indexes of family S, of which P is an element. Likewise, properties 2, 4 and 7 of P, presented in section IV, limit the

[6]This in the absence of efficiencies, synergies or scale returns that the merger might make possible. But these considerations cannot reasonably fit into the restrictions to specify an index. These should be studied in a detailed analysis of each case. The structure indexes - as P or H - have to be considered always as supporting elements of the analysis, not as substitutes for it.

cases of mergers that can reduce the value of this index, by excluding from this possibility mergers that involve relatively large firms.

It is convenient to analyze more closely property 2, that points out in lemma 2.1 that any merger involving the largest firm in the market always increases P. That is because some studies specify general conditions for mergers of this kind to increase welfare (Salant *et al.*, 1983; Farrell and Shapiro, 1990; and Levin, 1990).

Those studies analyze the case in which firms face constant unit costs, demand is linear and strategy is Cournot. Under these assumptions, they conclude that any profitable merger (in which merged firm's profits are larger than the premerger profits of the merging firms) of firms with a joint market share no larger than ½, increases welfare. Clearly this can be compatible with a merger that engages the largest firm.

But if the value of an index always decreased with mergers involving less than half the market, a contradiction would result with the notion that all mergers in the case of equally sized firms decrease welfare (in the absence of synergies or increasing returns). A generalized and inappropriate application of the mentioned studies' results (consisting in imposing to the index that any merger of firms with less than half of market should reduce its value) would imply that it would always be better to concentrate the market in no more than three firms, than to have an atomistic and, therefore, perfect competition.

When there are four firms or more, there will always be at least two with a joint share smaller or equal to one half. If the mentioned standard were applied, it would always be possible to come to a situation of only three firms by means of mergers no larger than one half of the market. If besides that, a Levin (1990) result was considered (that with constant and equal costs any profitable merger of two firms in a market of three always increases welfare), as requiring that an index should always decrease when going from three to two firms, it would implicitly be accepted that there is a duopolic concentration not less desirable, and probably better than perfect competition. This would contradict one of the fundamental theorems of neoclassical economic theory, that perfect competition is optimal in the Pareto sense (Varian, 1894). To avoid such a contradiction, property 2 is justified.

The apparent unsuitability of the considered results is due to the assumptions of the models and, especially, to the condition that mergers be profitable. Scherer (1991) insisted that in real world, mergers which turn out to be unprofitable, are too frequent to be considered as exceptions. Furthermore, in the foregoing models, a Cournot strategy is assumed when firms decide on their production scale; that is, each competitor asumes that the output of the other firms will remain constant. In the Cournot model firms that do not participate in a merger increase their output after the merger takes place. In this way, gains from the merger would always be lower than expected by the merging parts, as long as they stick to the Cournot assumption. There is no reason whatsoever to expect more rationality when buying a firm than when deciding the scale of production. Both decisions impinge on the value of the firm. If non profitable mergers are discarded because they are irrational, congruence would force the whole Cournot framework to be discarded.

Nevertheless, it is convenient to show that P reflects much of the intuition behind the result that a merger involving less than half of the market - in terms of the shares prior to the merger -, can be favorable. Property 6 of P agrees with the idea that any merger of less than half of the market is desirable, but conditioned to the existence of a dominant firm that supplies most of that market. This last condition avoids contradiction with property 1, that with equal size firms, the larger their number, the smaller the value of P. This last property is rasonable because in the absence of increasing returns, the best structure is one that promotes perfect competition; that is, competition on equal terms among many. This condition also avoids contradiction with property 2, justified by the mentioned result, that the large firms, in the Cournot equilibrium, produce less than the optimal, and any merger among them would lead them to produce even less. This is especially true for the largest firm in the market.

Lemma 4.1 states that mergers of two firms that control more than half the market will always increase P. In this regard, Farrell and Shapiro (1990) concluded, for the case of constant and equal costs and linear demand, that (parentheses added):

> ...the net external effect (on consumers' surplus plus profits of non participating firms in the merger)... is surely negative if the postmerger outputs satisfy $X_I \geq X_0$ (the participants' output is larger than the output of non-participants. Note that if after the merger $X_I \geq X_0$, this condition is also fulfilled before the merger, because in the Cournot model the participants in the merger reduce their share in the market.)... This does not necessarily mean that such merger should ever be approved, but there is certainly a case to answer.

The use of concentration indexes such as H, for which P is proposed as an alternative, is supposed to support market analysis and especially that of mergers, by pointing out cases in which an increase of the index seems to indicate risks of uncompetitive behavior (see section VI). In this sense lemma 4.1 implies an increase of P in situations in which there is a risk against competition.

In the analyses of the performance of indexes it is common to impose restrictive conditions in order for the models to show precise results. This has led to asume a Cournot strategy in most studies on the properties of H. The difficulty to precisely model other structures has implied that fundamental aspects of market behavior have been left out. For Scherer (1991), a result of mergers is that merging firms can adopt more uncompetitive strategies because of their increased market power.This is left out from analyses that suppose a fixed strategy, disregarding the relative size of dominant firms.

In this respect, it can be emphasized that the interpretation of index P offered in section III, was that of an average of the relative size of each firm. This relative size is computed from $h_i=q_i^2/H$; that is, the contribution to concentration measured by H. This is congruent with the idea that the market dominance a firm can exert depends not only on its size, but also on its relation with the size of the competing firms.

Another idea worth analyzing was suggested by Bork (1978, pp. 221-222), in the sense that the probability of a merger being uncompetitive, could depend on whether, once the merger has been carried out, there is not space left for non participating firms to engage in other mergers of the same size. It would be more unlikely that the purpose of a merger was to take advantage of a monopolistic position against competitors, if these could, in their turn, cancel that effect by means of mergers among themselves.

If any number of firms was allowed in the mergers considered in the criterion suggested by Bork, this would be equivalent to the suggestion of opposing only mergers involving more than 50% of the market. However, reaching a certain share could be easier with the merger of only two big firms, than with the merger of a large number of relatively small ones, because of the coordination problems that could arise. Property 4 of P, that its value increases with the merger of two firms of such size that the merger of any two other firms can not attain a merged firm of at least the same size, agrees with Bork's proposal when limited to mergers between pairs of firms. Lemma 4.1 would be related to the case of mergers among any number of firms. According to this lemma, any merger of more than half of the market increases P.

Other criteria have been mentioned in a report of Mexico's Federal Competition Commission (Comisión Federal de Competencia, 1994, pp. 19-20). In the context of cellular telephony the Commission pointed out that a situation could be more uncompetitive, when a single firm dominates half or more of the market and the rest was fragmented among too many firms, than when the latter were concentrated. The reason is that a larger firm would have more power to counteract the possible uncompetitive practices of the firm that controls the main part of the market, than a set of relatively small firms. Therefore the Commission concluded that it is probable that a duopoly be more advantageous than a fragmented competition, when the dominating firm has a share of more than 50% of the market. This is precisely what is reflected by property 3 of P.

The Commission, also in the context of the cellular phone system in Mexico, points out that as a consequence of the above, mergers among firms other than the largest, when this one controls most of the market, can be procompetitive. This way the dominant firm will be forced to face a more vigorous competition from other firms, in a situation where the first one has the strongest power to put into practice uncompetitive displacement policies, compared to the rest of the firms. In the case of P this is compatible with its property 6.

To conclude this section, it could be emphasized that in the case of symmetric firms (as far as their size), H is a suitable index for the analysis of market structure. In this case P is equal to H. In other cases index P seems more attuned with several results from theory, as well as with some considerations by antitrust experts.

VI. Mergers in a Market With Three Firms

P, as well as H, increases when the largest firm of the market merge with any other. For other mergers it is probable that P decreases, while H always increases. Chart 1 shows results of mergers of the two smallest firms, when *n*=3.

CHART 1

Results of several mergers for n = 3

Before the merger					After the merger				Changes	
q_1	q_2	q_3	H	P	q_1	q_2	H	P	ΔH	ΔP
0.90	0.05	0.05	0.815	0.9878	0.90	0.10	0.820	0.9759	0.0050	-0.0119
0.80	0.10	0.10	0.660	0.9408	0.80	0.20	0.680	0.8893	0.0200	-0.0515
0.70	0.15	0.15	0.535	0.8424	0.70	0.30	0.580	0.7378	0.0450	-0.1046
0.60	0.20	0.20	0.440	0.6860	0.60	0.40	0.520	0.5740	0.0800	-0.1120
0.50	0.25	0.25	0.375	0.5000	0.50	0.50	0.500	0.5000	0.1250	0.0000
0.40	0.30	0.30	0.340	0.3616	0.40	0.60	0.520	0.5740	0.1800	0.2124

Chart 1 shows that mergers of the two small firms always increase H, but P decreases when the resulting firm of the merger is smaller than the largest firm ($q_2 + q_3 < q_1$). This result follows from property 6, that when q_m>1/2, any merger that does not involve the largest firm reduces P. On the other hand, when the firm resulting from the merger becomes the largest of the market, P increases, because when this happens starting from three firms, the merged firm's output share is greater than ½ (lemma 4.1). As long as the firm resulting from the merger is smaller than the largest, such merger can be viewed as purely defensive against the dominant firm. In this case, by merging, both firms can defend themselves better against unilateral decisions of the dominant firm, without becoming themselves a dominant firm.

In practice it is accepted, in some way, that H is inflexible to help evaluate concentrations, and that this comes precisely form the inability of this index to reflect that some mergers can increase competition in the market. To correct this, value ranges for H are specified, within which mergers that increase H in less than a fixed amount for each range, are not considered to impose a risk for competition. In this context, the *Merger Guidelines* of the Department of Justice and the United States Federal Trade Commission establish:

i) If mergers keep concentration in a value corresponding to a non concentrated market (H less than 0.1), it is unlikely that they have uncompetitive effects.

ii) When mergers result in a moderately concentrated market (postmerger H between 0.1 and 0.18), it is improbable that an increase of H of less than .01, provides a reason to worry.

iii) For a merger resulting in a highly concentrated market (H greater than 0.18), the corresponding increase of H should be less than 0.005, in order for those mergers not to be considered, in general, worrisome.

This approach to overcome the inflexibility of H so as to aid merger evaluations, has some difficulties, apart from its evident arbitrariness. One of them is the discontinuity combined with invariability of criteria within each range of values of H. Thus, a merger that increased H in a little more than 0.005 could be considered worrisome, if postmerger H was only somewhat higher than 0.18, while another merger, with a much larger increase of H (close to 0.01), and postmerger H a little smaller than 0.18, would not set off the alarm.

Another problem with this approach is that a certain increment of H could be objected to, if it was carried out in a single step, but not if the modification of the distribution was made or studied in two or more steps. As a matter of fact there are multiple combinations of mergers through which H could increase gradually to cause worry, without the H *Guidelines* prompting the authorities' intervention. This could happen even if the mergers involved different set of firms.

In spite of these problems, analyzing the results of the above chart in terms of the *Guidelines* is illustrative. In all mergers considered there, the *Guidelines* would signal anticompetitive risks. This contrasts with the fact that having n = 3, P decreases when the aggregated share of the merging firms is less than 0.5.[7]

VII. Mergers in a Market With Four Firms

In the previous section examples were given that with three firms, the merger of the two smallest ones would decrease P, when it was purely defensive, in the sense that the merged firm did not become itself the dominant one. Nevertheless, if a fourth firm exists, it can happen that the merger is defensive in relation to the dominant firm, but not in relation to the other firm; especially if this one is smaller than any of the two that merge.

In the first two lines of chart 2, P increases with the merger of q_2 and q_3. This is due

[7]In theory there is nothing that assures that under any circumstances a duopoly is worse than a structure with a dominant firm that concentrates more than half of the output. In cases in which there are a few firms in the market, it is convenient to base the analysis on additional considerations, as it is proposed in the last section. As it was discussed in section V, the theory suggests, if at all, that when there are three firms, and the smaller two have a joint share of less than 50%, it is probable that a merger of the latter is not socially undesirable.

to the fact that in those cases the joint share of the merging firms is larger or equal to ½ (property 4, and lemma 4.1). On the other hand, P decreases in the last four lines. This is because there is one firm that controls half or more of the market and does not participate in any of the supposed mergers (property 6).

CHART 2.

***Results of several mergers when n* = 4**

Before the merger						After the merger of 3 with 2					Changes	
q_1	q_2	q_3	q_4	H	P	q_1	q_2	q_4	H	P	ΔH	ΔP
0.30	0.30	0.30	0.10	0.280	0.3112	0.30	0.60	0.10	0.460	0.6512	0.1800	0.3400
0.40	0.25	0.25	0.10	0.295	0.3851	0.40	0.50	0.10	0.420	0.5000	0.1250	0.1150
0.50	0.20	0.20	0.10	0.340	0.5692	0.50	0.40	0.10	0.420	0.5000	0.0300	- 0.0692
0.60	0.15	0.15	0.10	0.415	0.7590	0.60	0.30	0.10	0.460	0.6512	0.0450	- 0.1077
0.70	0.10	0.10	0.10	0.520	0.8891	0.70	0.20	0.10	0.540	0.8292	0.0200	- 0.0599
0.80	0.05	0.05	0.10	0.655	0.9550	0.80	0.10	0.10	0.660	0.9408	0.0050	-0.0142

While in the exercises considered in this section, P increases sometimes, and decreases in others, H always increases, as it was known already from its properties. Moreover, in none of the examples in chart 2 the criteria of the *Guidelines* would refrain from suggesting competition risks. It seems strange that authorities should worry about mergers of firms with a share of 5% that imply an additional accumulation of 5%, when there is a virtual monopoly with at least 80% of the market.

VIII. An Illustration from Real World

Mergers have recently been carried out in Mexico, in a market with a size distribution similar, but not identical, to the one in chart 3, in which starting from the original situation considered in the first column, the effects of different mergers on indexes H and P are analyzed.

CHART 3

Mergers in an approach to a real distribution

Original		Mergers of					
		1 & 3	2 & 4	4 & 5	4 & 7	5 & 6	6 & 7
q_1	0.25	0.40	0.25	0.25	0.25	0.25	0.25
q_2	0.20	0.20	0.35	0.20	0.20	0.20	0.20
q_3	0.15		0.15	0.15	0.15	0.15	0.15
q_4	0.15	0.15		0.25	0.20	0.15	0.15
q_5	0.10	0.10	0.10		0.10	0.20	0.10
q_6	0.10	0.10	0.10	0.10	0.10		0.15
q_7	0.05	0.05	0.05	0.05		0.05	
P	0.2327	0.4650	0.3710	0.2506	0.2283	0.2251	0.2199
ΔP		0.2323	0.1382	0.0179	-0.0044	-0.0076	-0.0128
H	0.1700	0.2450	0.2300	0.2000	0.1850	0.1900	0.1800
ΔH		0.0750	0.0600	0.0300	0.0150	0.0200	0.0100

This chart shows how P increases with mergers between relatively large firms, and decreases with mergers between relatively small firms. It is interesting that it could also decrease with the merger of one relatively large firm (15%) and a small one (5%), as happens when q_4 and q_7 merge. On the other hand, H always increases in such a magnitude as for the *Guidelines* to indicate uncompetitive risks for all possible mergers.

It does not seem convincing, in the case of the merger of the two smallest firms (10 and 5%), that authorities should worry about uncompetitive effects, when there are five firms larger than the ones that merge. Even after the merging, there would still be four firms equal or larger than the merged one. It can be suggested that the merger would tend to propitiate a stronger competition when a more relevant competitor comes up in exchange for the disappearance of a firm of little significance in terms of its size (only one fifth of the largest, and only half of the smallest non participating firm).

Final Considerations

By no means P could replace a detailed case by case analysis, when there are reasons to suspect that in spite of P increasing or decreasing considerably, there are other elements that impinge significantly on efficiency and competition. What is proposed is that P be used as a filter to signal cases that should be analyzed in more detail. A procedure like the following is suggested to analyze concentrations that exceed the threshold stipulated by law, in order for them to be notified to the Mexican Federal Competition Commission.

Step 1. Estimate the value of P before and after the concentration requested for approval.

Step 2.1. If P after the merger is larger than 0.25 (the value corresponding to four firms of the same size), subject the decision to an analysis of other circumstances in which the merger is carried out, regardless of whether P increases or decreases.[8]

[8]It has been suggested that the threshold to consider an impugnation be, in terms of H, on the order of 0.25 (see Hay and Werden, 1993). This threshold coincides with the one proposed here in the case in which P = H, that is, if all firms are the same size. In other circumstances, since $P \geq H$, the limit in terms of P is more restrictive than for H.

Step 2.2. If P increases, but its value after the merge is less or equal to 0.25, allow the merger, except in cases of flagrant reasons to conclude that the merger pursues goals opposed to competition, without important procompetitive elements.

Step 2.3. If P does not increase, and after the merger $P \leq 0.25$, allow the merger, except maybe in really exceptional situations in which the burden of the proof would be left on the opposing party, if any.

These criteria overcome the problems discussed above regarding the *Guidelines*. Firstly, they are based on an index (P) which has a more solid theoretical basis than the one used in the *Guidelines* (H). Secondly, the proposed criteria are neither discontinuous, as far as computations are concerned, nor are they vulnerable to gradualist merging processes, as could happen with the *Guidelines.* If originally a sector has a P smaller than 0.25, there is no way for a series of secuencial mergers to increase P above that value not to set off the alarm, even when those mergers are carried out by different sets of firms..

Index P can also be useful as a support to evaluate whether there is a significant power in the pertinent market when monopolistic practices are being analyzed. For example, if a presumed transgressor *i* has a market share such that $q_i^2/H > P$, the index suggests that its power is probably larger than the average power of the other market participants. Since Mexican Federal Law of Economic Competition points out that the power of the presumed transgressor should be examined, taking into account the power of the other participants in the corresponding market, comparing q_i^2 with PH could be used as a preliminary base for a more detailed analysis. Also in this case the threshold of $q_i^2/H > 0.25$ can be useful as an indicator of a probable significant market power.

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References

1. Borenstein, Severin, "Airline Mergers, Airport Dominance, and Market Power", American Economic Review, May 1990 (Papers and Proceeding), 80, pp. 400-404.

2. Bork, Robert, The Antitrust Paradox, Nueva York: Basic Books, 1978.

3. Comisión Federal de Competencia, Anual Report 93-94, México, 1994.

4. Encaoua, David and Alexis Jacquemin, "Degree of Monopoly, Indices of Concentration and Threat of Entry", International Economic Review, February 1980, 21, pp. 87-105.

5. Farrell, Joseph and Carl Shapiro, "Horizontal Mergers: An Equilibrium Analysis", American Economic Review, March 1990, 80, pp. 107-126.

6. Farrell, Joseph and Carl Shapiro, "Horizontal Mergers: Reply", American Economic Review, September 1991, 81, pp. 1007-11.

7. García Alba Iduñate, Pascual, "Un Enfoque para Medir la Concentración Industrial y su Aplicación al Caso de México", El Trimestre Económico, April-June 1990, 57 (2), pp. 317-341.

8. Hay, George and Gregory J. Werden, "Horizontal Mergers: Law, Policy, and Economics", American Economic Review, May 1993, (Papers and Proceedings), 83, pp. 173-7.

9. Levin, Dan, "Horizontal Mergers: The 50 percent benchmark", American Economic Review, December 1990, 80, pp. 1238-45.

10. Scherer, F. M., "Comment by F. M. Scherer", in Robert Willig (1991), pp. 323-7.

11. Schmalensee, Richard, "Inter-Industry Studies of Structure and Performance", in Richard Schmalensee and Robert D. Willig, eds., Handbook of Industrial Organization, Vol. II, Amsterdam: North-Holland, 1989, pp. 952-1009.

12. Salant, Stephen, Sheldon Switzer and Robert Reynolds, "Losses From Merger: the Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", Quarterly Journal of Economics, May 1983, 98, pp. 185-199.

13. Secretaría de Comercio y Fomento Industrial, "Ley Federal de Competencia Económica", Diario Oficial de la Federación, December 24th, 1992.

14. Tirole, Jean, The Theory of Industrial Organization, Cambridge, Mass.: MIT, 1990.

15. U.S. Department of Justice, "Merger Guidelines", reproduced in Trade Regulation Report, Chicago: Commerce Clearing House, 1984.

16. Varian, Hal R., Microeconomic Analysis, 2nd. edition, New York: Norton, 1984.

17. Werden, Gregory J., "Horizontal Mergers: Comment", American Economic Review, September 1991, 81, pp. 1002-6.

18. Willig, Robert, "Merger Analysis, Industrial Organization Theory, and Merger Guidelines", Brookings Papers: Macroeconomics, 1991, pp. 281-332.