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Pressure Fluctuations as a Diagnostic Tool for Fluidized Beds

Technical Progress Report for the Period
April 1, 1996 - June 30, 1996

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Abstract

The pressure fluctuations in the transition regime between bubbling fluidization and fast fluidization were investigated using system identification techniques. The results show that the voidage waves and surface eruption effects seen in bubbling fluidized beds are also observed in the transition regime fluctuations. In addition, a third frequency phenomenon is observed in the spectrum which is hypothesized to be a surface wave phenomenon analogous to surface waves in water.

The validity of previously derived similitude parameters for bubbling fluidized beds was investigated using spectral analysis of pressure fluctuations. When BFB similitude parameters are matched in two different beds, the pressure dynamics are similar under most conditions, suggesting that the correct similitude parameters have been defined for BFBs.

Pressure Fluctuations as a Diagnostic Tool for Fluidized Beds

Robert C. Brown and Ethan Brue

Objective

The purpose of this project is to investigate the origin of pressure fluctuations in fluidized bed systems. The study will asses^s the potential for using pressure fluctuations as an indicator of fluidized bed hydrodynamics in both laboratory scale cold-models and industrial scale boilers.

Progress

Experimental Set-up

Bubbling Fluidized Bed Models

Two geometrically similar bubbling bed models were also used in this study. Both beds were constructed of Plexiglas tubing with an inside diameter of 10.2 cm and 5.08 cm for the prototype and model respectively. The column height is 64 cm and 32 cm for the large and small beds respectively. A 3 m extension was used on the prototype BFB for turbulent bed studies. Both 36 hole and 72 hole distributor plates were used on the bubbling beds with a 75 μm screen fastened to the plate. This screen not only kept particles from entering the plenum, but also increased the pressure drop across the distributor plate such that even distribution was insured. Pressure taps in the small bed were located at 2.54 cm intervals along the height of the column, from 3.8 to 8.8 cm. Pressure taps in the large bed were located at 2.54 cm intervals up the column, between the heights of 3.8 and 21.3 cm. The small bed was designed to run at pressures up to 200 kPa gage.

Background

Surface Waves in Fluidized Bed Systems

In addition to the voidage waves reported and discussed previously, another second order phenomenon that may be responsible for pressure fluctuations in fluidized beds is surface waves analogous to surface waves observed in water. As proposed by Sun et. al [1], since the hydrodynamics of fluidized bed systems exhibit many of the characteristics of liquid, surface waves are expected in a fluidized bed. Water waves are classified according to the ratio of water depth (H) to wave length (λ) [2]. For $H/\lambda < 1/20$, the waves are termed shallow waves and the frequency is dependent on both the water depth and wave length. For shallow waves, the governing wave equation (presented by Sun [1]) reduces to a simplified relation that can be used to estimate the wave frequency:

$$\omega = \frac{\sqrt{gH}}{\lambda}$$

For intermediate depth waves $1/20 > H/\lambda < 1/2$, the wave equation cannot be reduced to a simple expression for wave frequency, and must be estimated as:

$$\omega = \frac{1}{2\cdot\pi}\sqrt{\left[\frac{g}{\lambda\cdot 2\cdot\pi}\cdot\tanh\left(\frac{2\cdot\pi}{\lambda}\cdot H\right)\right]}$$

For deep waves ($H/\lambda > 1/2$), the wave equation can be again be simplified and the frequency is only dependent on the wavelength and can be estimated as:

$$\omega = \sqrt{\frac{g}{2\pi\lambda}}$$

For surface waves in a cylindrical container the wavelength is determined by the container diameter:

$$D = \frac{n}{2}\lambda$$

where n is an integer greater than zero. The fundamental frequency is represented by n = 1, with overtones represented by higher integer values. Assuming that a half-wave is established in the bed ($\lambda/2 = D$) the deep wave frequency in a fluidized bed could be estimated as:

$$\omega = \sqrt{\frac{g}{4\pi D}} \qquad (6.33)$$

This surface wave phenomenon provides additional insight into the pressure dynamics of both turbulent (transition regime) and circulating beds.

The Use of Pressure Fluctuations to Validate Similitude Parameters

The most extensive research on the subject of similitude in fluidized bed systems has been done by Glicksman [3, 4]. Using both the Buckingham Pi theorem and derivations based on fundamental equations of motion, Glicksman proposes a set of similitude parameters that govern fluidization. Glicksman assumes that if the PSDs or PDFs of pressure fluctuations match between model and prototype, then the fluidized beds are in hydrodynamic similitude. However, he does not distinguish the important characteristics of the PSD that must match in order for two beds to be governed by similar dynamics. Particularly in CFBs, Glicksman's data does not show the important spectral characteristics in the PSD due to inadequate data sampling. Furthermore, Glicksman never questioned whether pressure fluctuations were correlated to the hydrodynamic state of a fluidized bed. In addition to relating Bode plot characteristics to physical phenomena in fluidized beds, a secondary goal of this study is to reassess whether pressure fluctuations can be used to validate proposed BFB and CFB similitude parameters.

BFB Similitude

The Buckingham Pi theorem will be used to develop the important non-dimensional fluidized bed parameters. Using the frequency of pressure fluctuations as the dependent parameter, all independent variables important for bubbling fluidization can be defined:

$$\omega = f(U, g, D, H, d_p, \rho_s, \rho_g, \mu, \phi)$$

The dimensions are as follows:

$[\omega] = 1/T$	$[U] = L/T$	$[g] = L/T^2$	$[D] = L$
$[H] = L$	$[d_p] = L$	$[\rho_s] = M/L^3$	$[\rho_g] = M/L^3$
$[\mu] = M/LT$	$[\phi] = 1$		

If we choose U, d_p, and ρ_g as the dimensionally independent parameters the remaining variables can be non-dimensionalized based on these variables.

$$g \rightarrow \frac{g \cdot d_p}{U^2} \qquad H \rightarrow \frac{H}{d_p} \qquad D \rightarrow \frac{D}{d_p}$$

$$\rho_s \rightarrow \frac{\rho_s}{\rho_g} \qquad \mu \rightarrow \frac{\mu}{\rho_g \cdot U \cdot d_p} \qquad \omega \rightarrow \omega \cdot \frac{d_p}{U}$$

Recognizing the dimensionless g and μ as the inverse of the Froude number and Reynolds number respectively the full set of dimensionless parameters as Glicksman defines them is:

$$Fr = \frac{U^2}{g \cdot d_p} \qquad \frac{H}{d_p} \qquad \frac{D}{d_p} \qquad \frac{\rho_g}{\rho_s} \qquad \mathrm{Re}_p = \frac{\rho_g \cdot U \cdot d_p}{\mu}$$

Also, it is more convenient to modify the dependent frequency spectrum parameter by multiplying by other dimensionless groupings as shown below.

$$\omega \cdot \frac{d_p}{U} \Rightarrow \omega \cdot \frac{d_p}{U} \times \sqrt{\frac{U^2}{g \cdot d_p}} \times \sqrt{\frac{H}{d_p}} \Rightarrow \omega \cdot \sqrt{\frac{H}{g}}$$

By matching the dimensionless parameters in a 10.2 cm BFB and a 5.1 cm pressurized BFB, the corresponding non-dimensionalized Bode plots can be compared.

Another important dependent variable that should be compared in fluidized bed systems is the pressure drop per unit length. Non-dimensionalizing this dependent variable via the same Buckingham Pi approach used above yields:

$$\frac{\Delta P}{L} = \rho_s \cdot (1-\varepsilon) \cdot g \Rightarrow \rho_s \cdot (1-\varepsilon) \cdot g \cdot \frac{D}{U^2 \cdot \rho_f} = \frac{\rho_s}{\rho_f}(1-\varepsilon) \cdot Fr \Rightarrow (1-\varepsilon)$$

In addition to the Bode plot profiles of pressure fluctuations being similar, the local voidage measured in the fluidized bed should be equal.

Results and Discussion

Transition Regime Fluctuations

Pressure fluctuations in the transition regime provide an important link between the nature of fluctuations in bubbling and circulating beds. Depending on the diameter of the bed, this regime can be described as a slugging or turbulent bed. The Bode plots throughout this regime continue to represent the output of multiple second order systems (i.e. a -40 dB/decade asymptotic slope). As previously shown, the frequency of voidage waves in BFB pressure fluctuations stays relatively constant as the superficial velocity increases. This holds true in the transition regime even as the bed approaches the fast fluidization regime (U/Umf > 20.0 for the prototype BFB). This is shown in Figure 1 which plots the observed frequencies versus U/U_{mf} for the transition regime. The surface eruption frequency phenomena observed in bubbling fluidized beds is also observed in the transition regime. This surface eruption frequency approaches the voidage wave frequency as the superficial velocity increases. At high velocities near fast fluidization, these two frequencies become nearly impossible to differentiate.

An interesting result observed in Figure 1, is that an additional frequency peak, that is nearly non-existent in BFBs, begins to appear in the spectrum of transition regime beds at a frequency of 0.9 Hz in the prototype. This frequency (although significantly damped) is seen first in the pressure fluctuations recorded immediately above the bed surface as the bed moves from bubbling to fast fluidization. At U/Umf > 18 this frequency is observed in the bed fluctuation measurements as well. This suggests that this phenomenon is not solely a characteristic of fast fluidization. As the superficial velocity increases in the transition regime, a well defined bed surface is no longer observed. While some bubbles are still observed propagating through the system, the predominant motion of the bed is the sloshing motion at the surface. This sloshing motion increases in magnitude until, near the fast fluidization regime, some particles are projected 1-3 m above the original surface of the bed. Visually it is easy to relate such a motion to the wave behavior of a liquid.

According to surface wave theory, deep beds should exhibit a wave frequency inversely proportional to the $\sqrt{D}$. For the prototype BFB, the predicted frequency for surface waves is 0.45 Hz for the fundamental, and 0.9 Hz for the first harmonic. For the model BFB, the predicted frequency is 0.65 Hz for the fundamental, and 1.3 Hz for the first harmonic. These values correspond closely to the frequency measured in fluidized systems approaching the fast fluidization regime for both the model and the prototype.

In summary, the voidage fluctuation phenomena and the surface eruption frequency (seen previously in BFB Bode plots) are observed throughout the transition from turbulent to fast fluidization. A surface wave phenomena with its corresponding harmonics can be observed near the onset of fast fluidization.

Validation of BFB similitude parameters

Table 1 summarizes the results from a similitude study on the prototype and model bubbling fluidized beds over a broad range of operating conditions. The table indicates which experiments resulted in similar Bode plot profiles in the prototype and model. For hydrodynamics to be considered similar, the voidage must be equal in the two beds. Also, the dimensionless frequency and damping of the observed peaks in the fluctuation spectrum must match. The damping coefficients and system frequencies were quantitatively estimated by fitting multiple second order systems (acting in parallel) to the BFB Bode plots, as was done in previous work [5]. Table 1 rates the degree of similarity between the important dependent parameters in the prototype and model BFB under similitude. The rating for each observed frequency includes both a comparison of the damping and a comparison of the dimensionless frequency. The table includes the complete set of independent dimensionless parameters used in each run. The percent height at which the pressure measurement is taken is also given (see Appendix A for a detailed summary of these experiments). In general, matching dimensionless parameters in two BFBs results in similar pressure dynamics. The average voidage matches well in both beds under all conditions.

The only exception is that under conditions of relatively high superficial velocity, when pressure fluctuations are measured in the upper regions of the bed, the peaks that result from surface phenomena do not always show similar damping or dimensionless frequency. Evidently, the nature of bubble coalescence in the model and prototype differ as the surface eruptions begins to dominate the spectrum. Visual observation of the bubbling bed surfaces confirm the differences. The surface of the small bed is noticeably lifted by large single bubble eruptions, while the prototype surface exhibits multiple bubble eruptions across a more stationary surface.

Future Work

Future work will focus on using pressure fluctuations for system identification and verification of a revised set of similitude parameters in circulating fluidized bed models.

References

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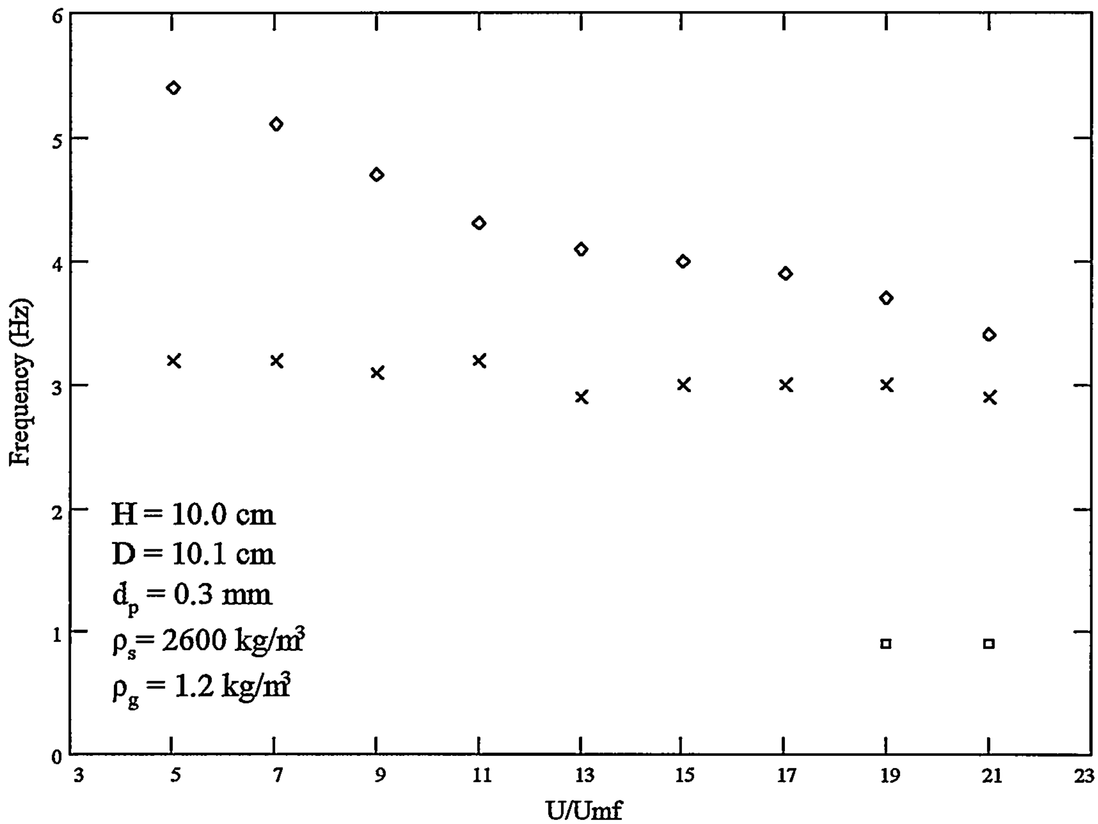

□ - surface wave phenomena (α_0)
× - voidage wave phenomena (α_1)
◊ - surface eruption phenomena (α_2)

Figure 7.16: Dominant frequencies observed in turbulent bed pressure fluctuations (measured at 50% bed height)

Table 1. Summary of BFB similitude study

Exp. #	U/U_{mf}	Re_p	Fr	ρ_s/ρ_g	H/D	D/d_p	% H	ε	α_1	α_2	α_0
1	1.1	4.1	5.9	2.2	1.06	254	100	N/A	**	**	-
2	1.1	4.0	5.9	2.2	1.06	254	100	N/A	**	**	-
3	1.4	5.4	10	2.2	1.06	254	100	N/A	**	**	-
4a	1.1	4.2	5.9	2.2	1.48	254	68	**	**	-	-
4b	1.1	4.2	5.9	2.2	1.48	254	100	**	*	**	-
5a	1.4	5.3	10	2.2	1.48	254	68	*	**	*	-
5b	1.4	5.3	10	2.2	1.48	254	100	N/A	*	*	-
6a	1.8	6.9	16	2.2	1.48	254	68	**	**	-	-
6b	1.8	6.9	16	2.2	1.48	254	100	N/A	*	**	-
7a	1.1	4.2	5.9	2.2	1.97	254	25	**	**	**	-
7b	1.1	4.2	5.9	2.2	1.97	254	50	**	**	*	-
8a	1.4	5.3	10	2.2	1.97	254	25	**	**	**	-
8b	1.4	5.3	10	2.2	1.97	254	50	**	**	-	-
9a	1.8	6.9	16	2.2	1.97	254	25	**	**	-	-
9b	1.8	6.9	16	2.2	1.97	254	50	**	**	-	-
10	1.1	2.0	3.3	2.2	1.06	339	100	N/A	*	**	-
11	1.4	2.6	5.5	2.2	1.06	339	100	N/A	**	**	-
12	1.8	3.3	9	2.2	1.06	339	100	N/A	*	*	-
13	2.2	4.0	13	2.2	1.06	339	100	N/A	*	no	-
14a	1.1	2.0	3.3	2.2	1.48	339	68	**	**	*	-
14b	1.1	2.0	3.3	2.2	1.48	339	100	N/A	*	*	-
15a	1.4	2.6	5.6	2.2	1.48	339	68	**	**	*	-
15b	1.4	2.6	5.6	2.2	1.48	339	100	N/A	*	*	-
16a	1.8	3.3	9	2.2	1.48	339	68	**	**	*	-
16b	1.8	3.3	9	2.2	1.48	339	100	N/A	no	*	no

Rating system:

** Dependent parameter identical in prototype and model

* Dependent parameter is approximately the same in prototype and model

no Dependent parameter does not match in prototype and model

α_0 - surface wave phenomenon

α_1 - voidage wave phenomenon

α_2 - surface eruption frequency

(Table 7.1 continued)

Exp. #	U/U_{mf}	Re_p	Fr	ρ_s/ρ_g	H/D	D/d_p	% H	ε	α_1	α_2	α_0
17a	2.2	4.0	13	2.2	1.48	339	68	**	*	**	*
17b	2.2	4.0	13	2.2	1.48	339	100	N/A	no	*	no
18a	1.1	2.0	3.3	2.2	1.97	339	25	*	**	**	-
18b	1.1	2.0	3.3	2.2	1.97	339	50	**	*	**	-
19a	1.4	2.6	5.5	2.2	1.97	339	25	**	**	**	-
19b	1.4	2.6	5.5	2.2	1.97	339	50	**	*	**	-
20a	1.8	3.3	9	2.2	1.97	339	25	**	**	**	-
20b	1.8	3.3	9	2.2	1.97	339	50	**	*	**	-
21a	2.2	4.0	13	2.2	1.97	339	25	**	**	no	no
21b	2.2	4.0	13	2.2	1.97	339	50	**	*	*	no
22a	1.1	0.6	1.0	2.2	1.48	508	68	**	*	no	-
22b	1.1	0.6	1.0	2.2	1.48	508	100	N/A	no	*	-
23a	1.4	0.7	1.6	2.2	1.48	508	68	**	*	*	-
23b	1.4	0.7	1.6	2.2	1.48	508	100	N/A	*	no	-
24a	1.8	1.0	2.7	2.2	1.48	508	68	**	*	no	-
24b	1.8	1.0	2.7	2.2	1.48	508	100	N/A	*	no	-
25a	1.1	0.6	1.0	2.2	1.97	508	25	**	**	-	-
25b	1.1	0.6	1.0	2.2	1.97	508	50	**	**	-	-
26a	1.4	0.7	1.6	2.2	1.97	508	25	**	**	*	-
26b	1.4	0.7	1.6	2.2	1.97	508	50	**	**	*	-
27a	1.8	0.6	2.7	2.2	1.97	508	25	*	**	*	-
27b	1.8	0.6	2.7	2.2	1.97	508	50	**	**	no	-

APPENDIX A
BFB Similitude study results

Table A.1. Similitude tests for dp =0.4 mm and H = 10 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	ρ_s/ρ_g x10^{-3}	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
1	1.1	4.0 ± 0.3	5.8 ± 0.7	2.2 ± 0.1	1.06 ± 0.02	254 ± 6	100	N/A	2.5	3.7	0.2	0.3
	1.1	4.2 ± 0.3	6.0 ± 0.6	2.2 ± 0.1	1.06 ± 0.04	254 ± 13	100	N/A	2.5	3.7	0.3	0.3
2	1.1	4.0 ± 0.3	5.8 ± 0.7	2.2 ± 0.1	1.06 ± 0.02	254 ± 6	100	N/A	2.5	3.6	0.3	0.4
	1.1	4.2 ± 0.3	6.0 ± 0.6	2.2 ± 0.1	1.06 ± 0.04	254 ± 13	100	N/A	2.4	3.7	0.3	0.3
3	1.4	5.3 ± 0.4	10 ± 1	2.2 ± 0.1	1.06 ± 0.02	254 ± 6	100	N/A	2.7	3.8	0.3	0.4
	1.4	5.4 ± 0.4	10 ± 1	2.2 ± 0.1	1.06 ± 0.04	254 ± 13	100	N/A	2.6	3.7	0.3	0.4

white - parameters for prototype BFB
grey - parameters for model BFB

ω - frequency of peak in BFB Bode plot
ζ - damping coefficient of peak in BFB Bode plot

Table A.2. Similitude tests for dp =0.4 mm and H = 15 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
4a	1.1	4.1 ± 0.3	5.8 ± 0.7	2.2 ± 0.1	1.48 ± 0.02	254 ± 6	68	0.46 ± 0.03	2.4	-	0.3	-
	1.1	4.3 ± 0.3	6.1 ± 0.6	2.2 ± 0.1	1.48 ± 0.04	254 ± 13	68	0.47 ± 0.04	2.3	-	0.4	-
4b	1.1	4.1 ± 0.3	5.8 ± 0.7	2.2 ± 0.1	1.48 ± 0.02	254 ± 6	100	N/A	2.3	4.7	0.3	0.7
	1.1	4.3 ± 0.3	6.1 ± 0.6	2.2 ± 0.1	1.48 ± 0.04	254 ± 13	100	N/A	2.4	4.7	0.6	0.7
5a	1.4	5.3 ± 0.4	10 ± 1	2.2 ± 0.1	1.48 ± 0.02	254 ± 6	68	0.47 ± 0.03	2.5	3.5	0.2	0.3
	1.4	5.4 ± 0.4	10 ± 1	2.2 ± 0.1	1.48 ± 0.04	254 ± 13	68	0.50 ± 0.04	2.3	3.6	0.2	0.5
5b	1.4	5.3 ± 0.4	10 ± 1	2.2 ± 0.1	1.48 ± 0.02	254 ± 6	100	N/A	2.4	3.8	0.4	0.5
	1.4	5.4 ± 0.4	10 ± 1	2.2 ± 0.1	1.48 ± 0.04	254 ± 13	100	N/A	2.0	4.1	0.6	0.6
6a	1.8	6.7 ± 0.4	16 ± 1	2.2 ± 0.1	1.48 ± 0.02	254 ± 6	68	0.49 ± 0.03	2.5	-	0.2	-
	1.8	7.0 ± 0.4	15 ± 2	2.2 ± 0.1	1.48 ± 0.04	254 ± 13	68	0.50 ± 0.04	2.5	-	0.3	-
6b	1.8	6.7 ± 0.4	16 ± 1	2.2 ± 0.1	1.48 ± 0.02	254 ± 6	100	N/A	2.5	3.8	0.4	0.5
	1.8	7.0 ± 0.4	15 ± 2	2.2 ± 0.1	1.48 ± 0.04	254 ± 13	100	N/A	2.1	3.8	0.5	0.6

Table A.3. Similitude tests for dp =0.4 mm and H = 20 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
7a	1.1	4.1 ± 0.3	5.8 ± 0.7	2.2 ± 0.1	1.97 ± 0.02	254 ± 6	25	0.44 ± 0.03	2.3	4.1	0.3	1.0
	1.1	4.3 ± 0.3	6.1 ± 0.6	2.2 ± 0.1	1.97 ± 0.04	254 ± 13	25	0.46 ± 0.04	2.5	4.1	0.3	0.9
7b	1.1	4.1 ± 0.3	5.8 ± 0.7	2.2 ± 0.1	1.97 ± 0.02	254 ± 6	50	0.46 ± 0.03	2.3	3.8	0.3	0.9
	1.1	4.3 ± 0.3	6.1 ± 0.6	2.2 ± 0.1	1.97 ± 0.04	254 ± 13	50	0.46 ± 0.04	2.5	4.4	0.4	0.9
8a	1.4	5.3 ± 0.4	10 ± 1	2.2 ± 0.1	1.97 ± 0.02	254 ± 6	25	0.45 ± 0.03	2.4	4.1	0.3	1.0
	1.4	5.4 ± 0.4	10 ± 1	2.2 ± 0.1	1.97 ± 0.04	254 ± 13	25	0.47 ± 0.04	2.4	4.1	0.2	1.0
8b	1.4	5.3 ± 0.4	10 ± 1	2.2 ± 0.1	1.97 ± 0.02	254 ± 6	50	0.48 ± 0.03	2.5	-	0.3	-
	1.4	5.4 ± 0.4	10 ± 1	2.2 ± 0.1	1.97 ± 0.04	254 ± 13	50	0.47 ± 0.04	2.3	-	0.4	-
9a	1.8	6.7 ± 0.4	16 ± 1	2.2 ± 0.1	1.97 ± 0.02	254 ± 6	25	0.46 ± 0.03	2.4	-	0.3	-
	1.8	7.0 ± 0.4	16 ± 1	2.2 ± 0.1	1.97 ± 0.04	254 ± 13	25	0.48 ± 0.04	2.4	-	0.3	-
9b	1.8	6.7 ± 0.4	16 ± 1	2.2 ± 0.1	1.97 ± 0.02	254 ± 6	50	0.50 ± 0.03	2.4	-	0.3	-
	1.8	7.0 ± 0.4	16 ± 1	2.2 ± 0.1	1.97 ± 0.04	254 ± 13	50	0.49 ± 0.04	2.4	-	0.4	-

Table A.4. Similitude tests for dp =0.3 mm and H = 10 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
10	1.1	2.0 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.06 ± 0.02	339 ± 11	100	N/A	2.6	3.8	0.2	0.3
	1.1	2.1 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.06 ± 0.04	339 ± 22	100	N/A	2.8	4.1	0.5	0.3
11	1.4	2.6 ± 0.2	5.6 ± 0.7	2.2 ± 0.1	1.06 ± 0.02	339 ± 11	100	N/A	2.7	4.1	0.3	0.4
	1.4	2.6 ± 0.2	5.4 ± 0.7	2.2 ± 0.1	1.06 ± 0.04	339 ± 22	100	N/A	2.8	4.1	0.4	0.4
12	1.8	3.3 ± 0.2	9 ± 1	2.2 ± 0.1	1.06 ± 0.02	339 ± 11	100	N/A	2.2	3.1	0.5	0.3
	1.8	3.4 ± 0.3	9 ± 1	2.2 ± 0.1	1.06 ± 0.04	339 ± 22	100	N/A	1.9	3.5	0.5	0.6
13	2.2	4.0 ± 0.3	13 ± 1	2.2 ± 0.1	1.06 ± 0.02	339 ± 11	100	N/A	1.8	3.1	0.4	0.2
	2.2	4.1 ± 0.23	13 ± 1	2.2 ± 0.1	1.06 ± 0.04	339 ± 22	100	N/A	2.0	3.3	0.5	0.5

Table A.5. Similitude tests for dp =0.3 mm and H = 15 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
14a	1.1	2.0 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	68	0.45 ± 0.03	2.3	3.1	0.3	0.9
	1.1	2.1 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	68	0.46 ± 0.04	2.3	3.8	0.4	0.8
14b	1.1	2.0 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	100	N/A	2.4	4.1	0.3	0.8
	1.1	2.1 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	100	N/A	2.6	4.1	0.4	0.4
15a	1.4	2.6 ± 0.2	5.6 ± 0.5	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	68	0.46 ± 0.03	2.5	3.5	0.3	0.7
	1.4	2.6 ± 0.2	5.4 ± 0.5	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	68	0.47 ± 0.04	2.5	3.8	0.4	0.6
15b	1.4	2.6 ± 0.2	5.6 ± 0.5	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	100	N/A	2.5	4.1	0.4	0.7
	1.4	2.6 ± 0.2	5.4 ± 0.5	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	100	N/A	2.7	4.1	0.5	0.3
16a	1.8	3.3 ± 0.2	9 ± 1	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	68	0.47 ± 0.03	2.8	4.1	0.3	0.7
	1.8	3.4 ± 0.2	9 ± 1	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	68	0.47 ± 0.04	2.8	3.8	0.5	0.6
16b	1.8	3.3 ± 0.2	9 ± 1	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	100	N/A	2.5	4.0	0.6	0.6
	1.8	3.4 ± 0.2	9 ± 1	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	100	N/A *(third peak)*	- *0.2*	4.1	- *1.2*	0.8
17a	2.2	4.0 ± 0.3	13 ± 1	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	68	0.48 ± 0.03	2.8	4.1	0.2	0.7
	2.2	4.1 ± 0.3	13 ± 1	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	68	0.49 ± 0.04 *(third peak)*	2.8 *1.3*	4.1	0.5 *0.8*	0.7
17b	2.2	4.0 ± 0.3	13 ± 1	2.2 ± 0.1	1.48 ± 0.02	339 ± 11	100	N/A *(third peak)*	2.8 *1.8*	4.1	0.4 *0.7*	0.7
	2.2	4.1 ± 0.3	13 ± 1	2.2 ± 0.1	1.48 ± 0.04	339 ± 22	100	N/A *(third peak)*	2.2 *1.1*	3.8	0.7 *0.9*	0.7

Table A.6. Similitude tests for dp =0.3 mm and H = 20 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
18a	1.1	2.0 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	25	0.44 ± 0.03	2.4	4.1	0.3	1.0
	1.1	2.1 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	25	0.47 ± 0.04	2.4	4.1	0.3	1.0
18b	1.1	2.0 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	50	0.45 ± 0.03	2.4	3.8	0.3	1.0
	1.1	2.1 ± 0.2	3.3 ± 0.5	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	50	0.44 ± 0.04	2.4	4.1	0.7	1.1
19a	1.4	2.6 ± 0.2	5.6 ± 0.7	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	25	0.45 ± 0.03	2.4	3.5	0.3	1.0
	1.4	2.6 ± 0.2	5.4 ± 0.7	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	25	0.46 ± 0.04	2.5	3.5	0.3	1.0
19b	1.4	2.6 ± 0.2	5.6 ± 0.7	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	50	0.47 ± 0.03	2.4	3.5	0.3	0.5
	1.4	2.6 ± 0.2	5.4 ± 0.7	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	50	0.46 ± 0.04	2.4	3.5	0.5	0.6
20a	1.8	3.3 ± 0.2	9 ± 1	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	25	0.46 ± 0.03	2.4	3.1	0.3	0.8
	1.8	3.4 ± 0.2	9 ± 1	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	25	0.46 ± 0.04	2.5	3.1	0.3	0.8
20b	1.8	3.3 ± 0.2	9 ± 1	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	50	0.49 ± 0.03	2.4	3.3	0.3	0.5
	1.8	3.4 ± 0.2	9 ± 1	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	50	0.47 ± 0.04	2.4	3.5	0.5	0.6
21a	2.2	4.0 ± 0.3	13 ± 1	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	25	0.46 ± 0.03	2.5	3.5	0.2	0.7
	2.2	4.1 ± 0.3	13 ± 1	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	25	0.47 ± 0.04 *(third peak)*	2.5 *1.8*	3.1	0.2 *0.7*	0.3
21b	2.2	4.0 ± 0.3	13 ± 1	2.2 ± 0.1	1.97 ± 0.02	339 ± 11	50	0.51 ± 0.03	2.5	3.5	0.2	0.7
	2.2	4.1 ± 0.3	13 ± 1	2.2 ± 0.1	1.97 ± 0.04	339 ± 22	50	0.49 ± 0.04 *(third peak)*	2.5 *1.6*	3.1	0.5 *0.7*	0.6

Table A.7. Similitude tests for dp =0.2 mm and H = 15 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
22a	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.48 ± 0.02	508 ± 25	68	0.46 ± 0.03	2.1	-	0.4	-
	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.48 ± 0.04	508 ± 50	68	0.44 ± 0.04	2.0	3.5	0.7	0.7
22b	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.48 ± 0.02	508 ± 25	100	N/A	2.2	3.1	0.4	0.8
	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.48 ± 0.04	508 ± 50	100	N/A	2.2	3.5	1.0	0.5
23a	1.4	0.7 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.48 ± 0.02	508 ± 25	68	0.47 ± 0.03	2.4	-	0.3	-
	1.4	0.8 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.48 ± 0.04	508 ± 50	68	0.45 ± 0.04	2.4	3.5	0.5	0.7
23b	1.4	0.7 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.48 ± 0.02	508 ± 25	100	N/A	2.4	3.5	0.3	0.5
	1.4	0.8 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.48 ± 0.04	508 ± 50	100	N/A	2.7	4.3	0.5	0.3
24a	1.8	1.0 ± 0.1	2.7 ± 0.5	2.2 ± 0.1	1.48 ± 0.02	508 ± 25	68	0.47 ± 0.03	2.5	-	0.3	-
	1.8	1.0 ± 0.1	2.6 ± 0.5	2.2 ± 0.1	1.48 ± 0.04	508 ± 50	68	0.47 ± 0.03	2.4	4.4	0.7	0.8
24b	1.8	1.0 ± 0.1	2.7 ± 0.5	2.2 ± 0.1	1.48 ± 0.02	508 ± 25	100	N/A	2.4	5.0	0.4	0.8
	1.8	1.0 ± 0.1	2.6 ± 0.5	2.2 ± 0.1	1.48 ± 0.04	508 ± 50	100	N/A	2.6	4.4	0.6	0.4

Table A.8. Similitude tests for dp =0.2 mm and H = 20 cm prototype conditions

Exp. #	U/U_{mf}	Re_p	Fr	$\rho_s/\rho_g x10^{-3}$	H/D	D/d_p	% H	ε (avg.)	ω_{n1}	ω_{n2}	ζ_1	ζ_2
25a	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.97 ± 0.02	508 ± 25	25	0.45± 0.03	2.2	-	0.6	-
	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.97 ± 0.04	508 ± 50	25	0.45 ± 0.04	2.2	-	0.6	-
25b	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.97 ± 0.02	508 ± 25	50	0.44± 0.03	2.2	-	0.8	-
	1.1	0.6 ± 0.1	1.0 ± 0.3	2.2 ± 0.1	1.97 ± 0.04	508 ± 50	50	0.43± 0.04	2.2	-	1.2	-
26a	1.4	0.7 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.97 ± 0.02	508 ± 25	25	0.45± 0.03	2.3	-	0.4	-
	1.4	0.8 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.97 ± 0.04	508 ± 50	25	0.47 ± 0.03	2.3	3.8	0.4	0.9
26b	1.4	0.7 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.97 ± 0.02	508 ± 25	50	0.46± 0.03	2.3	-	0.4	-
	1.4	0.8 ± 0.1	1.6 ± 0.4	2.2 ± 0.1	1.97 ± 0.04	508 ± 50	50	0.45± 0.04	2.3	3.8	0.6	0.8
27a	1.8	0.6 ± 0.1	2.7 ± 0.5	2.2 ± 0.1	1.97 ± 0.02	508 ± 25	25	0.45± 0.03	2.4	-	0.5	-
	1.8	0.6 ± 0.1	2.6 ± 0.5	2.2 ± 0.1	1.97 ± 0.04	508 ± 50	25	0.48 ± 0.03	2.3	3.8	0.4	0.9
27b	1.8	0.6 ± 0.1	2.7 ± 0.5	2.2 ± 0.1	1.97 ± 0.02	508 ± 25	50	0.47± 0.03	2.5	-	0.5	-
	1.8	0.6 ± 0.1	2.6 ± 0.5	2.2 ± 0.1	1.97 ± 0.04	508 ± 50	50	0.45± 0.04	2.3	4.1	0.5	0.7