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INTRODUCTION

Given the existence of chaotic oscillations in reacting chemical systems [see discussion of the Belousov-Zhabotinsky reaction in Schuster (1988), for example], it is reasonable to ask whether or not similar phenomena can occur in combustion. In this paper we present experimental evidence that kinetically driven chaos occurs in a highly simplified thermal pulse combustor.

EXPERIMENTAL APPARATUS

The thermal pulse combustor considered here has been described previously by Richards et al (1991). Briefly, the combustor is a well-stirred reactor with a tailpipe extending from one end as shown in Fig. 1. Fuel and air are injected into the combustion chamber through orifices in the end opposite the tailpipe. What makes the thermal pulse combustor more simple as compared to conventional pulse combustors is that the fuel and air flow are kept constant by maintaining a sufficiently high pressure drop over the inlet orifice to produce choked flow. Conventional pulse combustors use mechanical or aerodynamic valves to regulate the fuel and air flow depending on combustion chamber pressure.

The alumina center body (Fig. 1) has been observed to promote combustion pulsations. It is speculated that this is due to enhanced radiative heat transfer from the centerbody to the colder walls, preventing the formation of a "hot-spot" ignition source at the combustor center that would cause continuous (non-pulsing) combustion.

EXPERIMENTAL PROCEDURE

Experimental pressure data were obtained from the thermal pulse combustor facility located at the Department of Energy (DOE) Morgantown Energy Technology Center (METC). Sixteen different conditions of fuel and air flow were studied as summarized in Table I. Two parameters are used to characterize the flow rate and stoichiometry: 1) the flow time τ_f, which is the combustor volume divided by the combined fuel and air volume flow rate at inlet conditions; and 2) the equivalence ratio ϕ, which is the actual fuel to air flow ratio divided by the stoichiometric fuel to air flow ratio. Propane was the fuel used in all cases. Independent control of the wall temperature was not possible, but a wall thermocouple located approximately at the combustor midpoint was used to obtain a reference value.

Data sets 1-15 were taken at conditions expected to represent a relatively wide range of complexity in the pulsing behavior. Data set 16 was obtained for a "typical" operating condition for the METC pulse combustor facility during which a small amount of limestone and carrier gas were injected into the tailpipe. No limestone injection was used for runs 1-15.

Each experimental run was initiated by starting the combustor with the spark plug (see Fig. 1). It was necessary to operate the spark plug until the combustor temperature was sufficiently high for self-sustaining operation. For tests 1-15, the spark plug was then turned off and the combustor allowed to reach a "stationary" operating condition as indicated by a constant wall temperature. For test 16 the spark remained on.

At the stationary condition, the combustion chamber pressure was measured with a single piezoelectric pressure transducer (Kistler Model 206 Low Pressure Piezotron). The transducer could not be mounted to the wall due to the high temperature, and consequently the transducer was mounted at the end of a 60-cm-long, 4.6-mm-diameter plastic tube, water-jacketed over the last 25 cm of its length. Preliminary tests were conducted to insure that the tubing did not introduce any artifacts to the measured pressure signal [see Richards, et al (1991) for further details]. The signal leaving the transducer was subsequently processed with a Kistler Corporation Transducer Coupler, a Data Translation Corporation A/D converter, and a personal computer. Digital sampling rate was 3773 Hz for cases 1-15 and 10,000 Hz for case 16.

A shortcoming of the data used for this paper is the relative brevity of the contiguous data sets for each combustor condition as compared to the amount of data typically desired for chaos analysis (see later discussion concerning data sufficiency). This shortness did not result from any inadequacy in the data acquisition system but because it was not originally recognized that chaotic structure was present and would be of interest. Thus longer data records were not deemed important at the time. Unavailability of the METC thermal pulse combustor facility in the interim has prevented the acquisition of longer data sets. Because the thermal design is not typical of commercially important combustors, it is anticipated that future experiments will be conducted with more conventional variable-inlet-flow designs.

ANALYTICAL METHODS AND RESULTS

Power Spectra and Mutual Information

The time series pressure data were initially characterized in terms of their Fourier power spectra and mutual information functions. The power spectra are useful for observing linear structure in data, while the mutual information function reveals both linear and nonlinear features [see Abarbanel (1994)]. Results spanning the range of observed behavior are depicted in Figs. 2, 3, and 4. Complete results for all cases are summarized in Table II.

From the power spectra it is clear that there are one or two dominant frequencies in each data set, but there is also a substantial amount of so-called "broadband" structure that could be indicative of deterministic chaos. This indication is supported by the mutual information results, which show the characteristic oscillations associated with the dominant frequencies and a net decay in mutual information over time that is characteristic of chaos.

Correlation Dimension and Entropy

For additional quantitative characterization of the chaotic structure in the pressure data, we evaluated the correlation dimension and entropy using the maximum liklihood algorithms recommended by Takens (1984) and Schouten et al (1994), respectively. These methods both use the so-called time delay embedding method to reconstruct the multidimensional information contained in univariate time series measurements. From the embedded pressure trajectories, the above methods provide maximum liklihood estimates of correlation dimension and entropy, which can be thought of as statistical measures of chaotic structure. Because of their statistical basis, the algorithms also provide estimates of uncertainty for the results.

Correlation dimension and entropy results are summarized in Table II. Estimated error bounds for the dimension and entropy are 0.1, and 10 bits/s, respectively. Confirmation of nonintegral (i.e., fractal) dimension and positive, finite entropy are further indications that deterministic chaos dominates the thermal pulse combustor dynamics.

Confirmation of Data Consistency and Sufficiency

Problems resulting from the data brevity are ameliorated by the fact that several data sets exist for similar conditions, allowing confirmation of near-replicate results. We also performed additional checks of each data set to confirm statistical self consistency and the sufficiency of the record length. This was done by dividing the duration of each data set in half and determining the correlation dimension and entropy for each part. If either of these statistical measures differs significantly between the two halves of a data set, one should suspect that the combustor was in a nonstationary state at the time or the data set is insufficiently long to obtain good statistical estimates of the dynamics. No significant differences were observed between the two halves of any data set, and thus we conclude that statistical consistency and record length were not major problems in our analyses.

Confirmation of Nonlinear Structure vs. Colored Noise

We further verified that the observed combustor dynamics represent deterministic chaos as opposed to colored noise (i.e., linearly filtered stochastic noise) by applying the so-called method of surrogates. Specifically, we used the method of Theiler et al (1992) for creating a "surrogate" data set from each of the original data sets by 1) transforming the original data into Fourier frequency domain; 2) randomizing the Fourier phases; and 3) inverse transforming the randomized Fourier series back to time domain. The resulting surrogate data have identical power spectra to the originals and thus contain identical linear time correlation. Any significant differences between the surrogate and original data should then be due to the loss of nonlinear (i.e., chaotic) structure resulting from the phase randomization. Lack of significant differences would indicate that the original data did not have important nonlinear structure.

As shown in Table II, the correlation dimension and entropy of the surrogate data sets are clearly different from the originals (much larger than the error associated with the dimension and entropy estimates themselves). Thus the hypothesis that the original data reflect deterministic chaos is supported.

Principal Components Analysis and Trajectory Visualization

For a more qualitative assessment of the general multivariate structure of the pulsed combustor data, we used time delay embedding and principal components analysis according to the procedure recommended by Broomhead and King (1986). This procedure involves constructing an optimal set of orthogonal axes on which to project the embedded trajectory. This method is especially convenient because it provides a way to view multidimensional dynamics in terms of standard two-dimensional projections.

Two-dimensional projections of selected pulsed combustor phase-space trajectories are shown in Figs. 5 and 6. The axes in these figures represent the indicated principal components. To better appreciate the physical significance of the axes, it is helpful to recognize that the nth principal component as determined in the Broomhead and King procedure is approximately proportional to the (n-1)th derivative of the pressure signal. Thus principal component 1 is approximately proportional to the as-measured signal, principal component 2 is approximately proportional to the first derivative of the signal, etc.

As might be expected, the trajectory projections show similarity between data sets having similar combustor operating conditions. Relatively clear data set "families" are composed of (1,2); (3,4); (5,6,7); and (12,13). It appears that data set 8 might also be grouped with 5,6, and 7 and data sets 11 and 14 with 12 and 13. Considering attractor size and shape, the data appear to exhibit definite behavioral trends.

The patterns in Figs. 5 and 6 suggest that principal component projections of the phase-space trajectories could be very useful as a characterization tool. We speculate that using such patterns as input to sophisticated pattern recognition systems (e.g., neural networks) would be a highly efficient means for diagnosing combustor performance.

Comparisons Between the Data and a Previously Proposed Model

A model for thermal pulsed combustion was previously proposed by Richards et al (1991) that clearly exhibits chaotic pulsations for certain conditions near flameout. The chaos-causing mechanism in the model is the highly nonlinear dependence of the ignition in any given pulsing cycle on the behavior of previous cycles (as expressed through the Arrhenius temperature term).

Analyses of the model behavior under fully chaotic conditions [see Daw et al (1992)] have demonstrated that it exhibits slightly lower correlation dimension and similar entropy (approximately 2.2 and 210 bits/s, respectively) compared to the experimental data discussed here. Considering the significant oversimplifications used in the model we consider the agreement to be quite good.

It is also clear that the model achieves chaos through a series of period doubling bifurcations as it is driven toward flameout. Such a period doubling sequence has not been observed experimentally, and the actual pulse combustor appears to operate in a chaotic state over a wider range of conditions than does the model. Nevertheless, we believe that the source of the chaos in the actual combustor is similar to that in the model.

At this point we conjecture that the differences between the model and the data reflect a slightly higher degree of complexity in the actual experiment as compared to the assumptions of the model. Such additional complexity is likely to stem from mixing and/or heat transfer processes occurring in the combustion chamber that invalidate the assumption of complete homogeneity. Future improvements to the model should consider these factors.

CONCLUSIONS

From the above experimental data analyses, it is clear that deterministic chaos is an important factor in thermal pulse combustor dynamics. While we have only observed such behavior in this particular type combustor to date, we infer from our understanding of the origins of the chaos that it is likely to exist in other pulse combustors and even nonpulsing combustion. We speculate that realization of the importance of chaos in affecting flame stability could lead to significant changes in combustor design and control.

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TABLE I. Test conditions for selected pulse combustor tests.

Test	τ_f (ms)	φ	T_w (°K)	Data Sampling Rate (Hz)	Number of Data Points
1	51.9	0.79	819	3,773	9,434
2	52.2	0.79	824	3,773	9,434
3	45.4	0.87	801	3,773	9,434
4	45.5	0.87	810	3,773	9,434
5	43.0	0.82	788	3,773	9,434
6	43.2	0.83	796	3,773	9,434
7	42.6	0.81	807	3,773	9,434
8	40.0	0.83	824	3,773	9,434
9	39.5	0.89	943	3,773	18,868
10	34.4	0.88	1003	3,773	18,868
11	31.2	0.80	992	3,773	18,868
12	30.7	0.78	967	3,773	18,868
13	30.8	0.79	978	3,773	18,868
14	29.9	0.76	943	3,773	18,868
15	29.8	0.76	797	3,773	18,868
16*	47.9	0.87	963	10,000	10,000

* Limestone was being injected into exhaust stream, spark plug was on.

TABLE II. Analysis results for pulse combustion test data sets.

					Phase Randomized Data	
Test	Estimated Correlation Dimension	Estimated Entropy (Bits/s)	Major FFT Frequency Peak(s) (Hz)	Time to First Mutual Information Minimum (ms)	Estimated Correlation Dimension	Estimated Entropy (Bits/s)
1	2.41	164	131	2.1	5.05	389
2	2.52	216	138	1.9	5.47	611
3	2.40	138	132	2.4	5.65	364
4	2.32	127	133	1.9	5.41	349
5	2.55	255	122	2.3	6.55	722
6	2.59	260	123	2.3	6.36	723
7	2.64	282	125	2.3	6.16	745
8	2.45	258	122	2.4	6.49	654
9	2.55	88	110	2.7	4.06	179
10	3.18	152	115	2.4	3.98	182
11	2.43	84	111	2.7	5.10	314
12	2.40	85	68, 108	3.2	5.27	362
13	2.59	108	84, 108	3.0	5.67	400
14	2.68	142	66	4.0	5.98	390
15	1.34	168	40	4.5	6.58	486
16	2.18	105	133	2.5	4.88	297

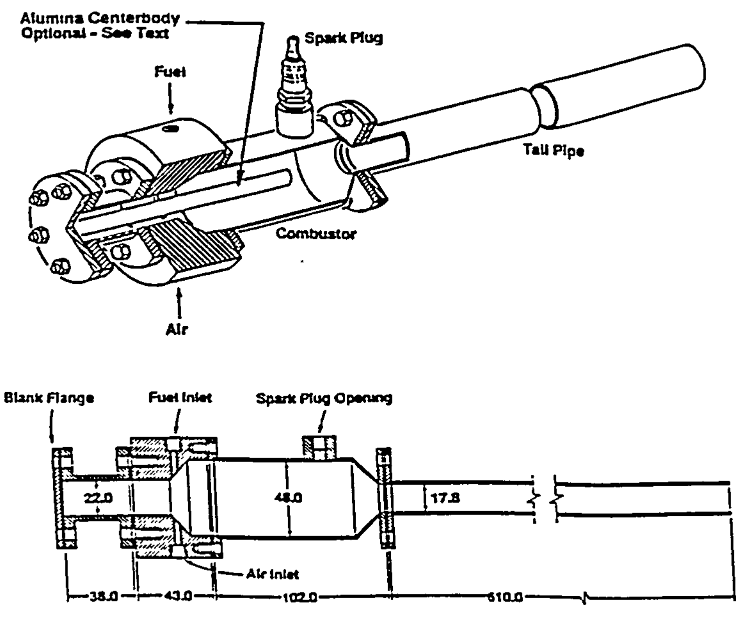

Fig. 1. Experimental combustor geometry. Dimensions are millimeters.

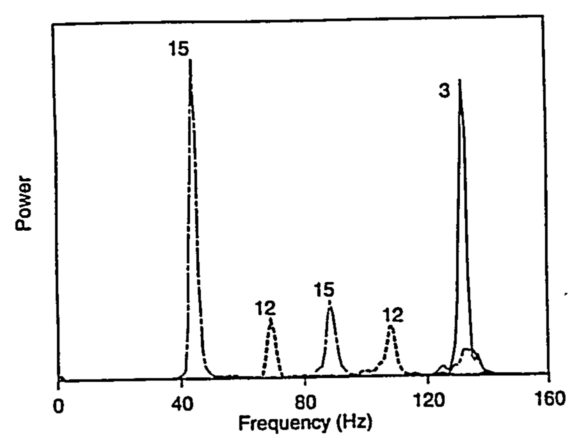

Fig. 2. The major FFT frequency peaks for data sets 3, 12 and 15 show a broad range of behavior.

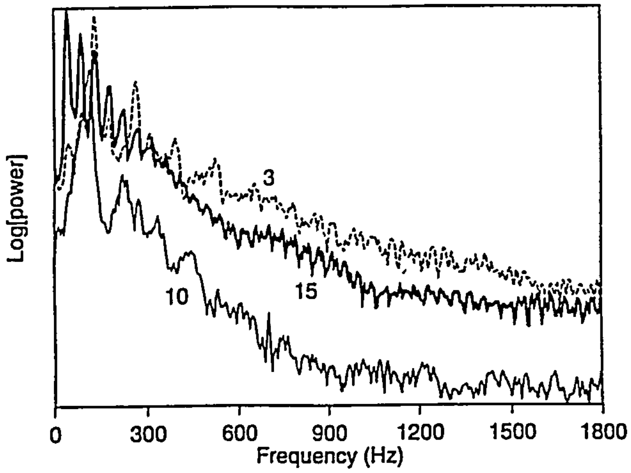

Fig. 3. The frequency spectra for data sets 3, 10 and 15 show the broad range of dynamic behavior.

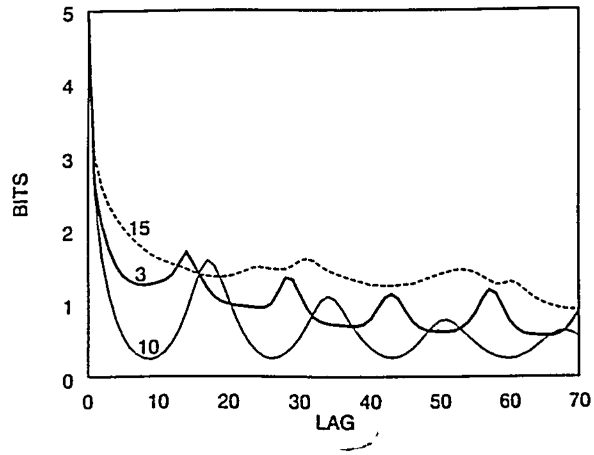

Fig. 4. The mutual information function displays major oscillations with decay indicative of chaotic behavior. The range in behavior is seen from data sets 3, 10 and 15.

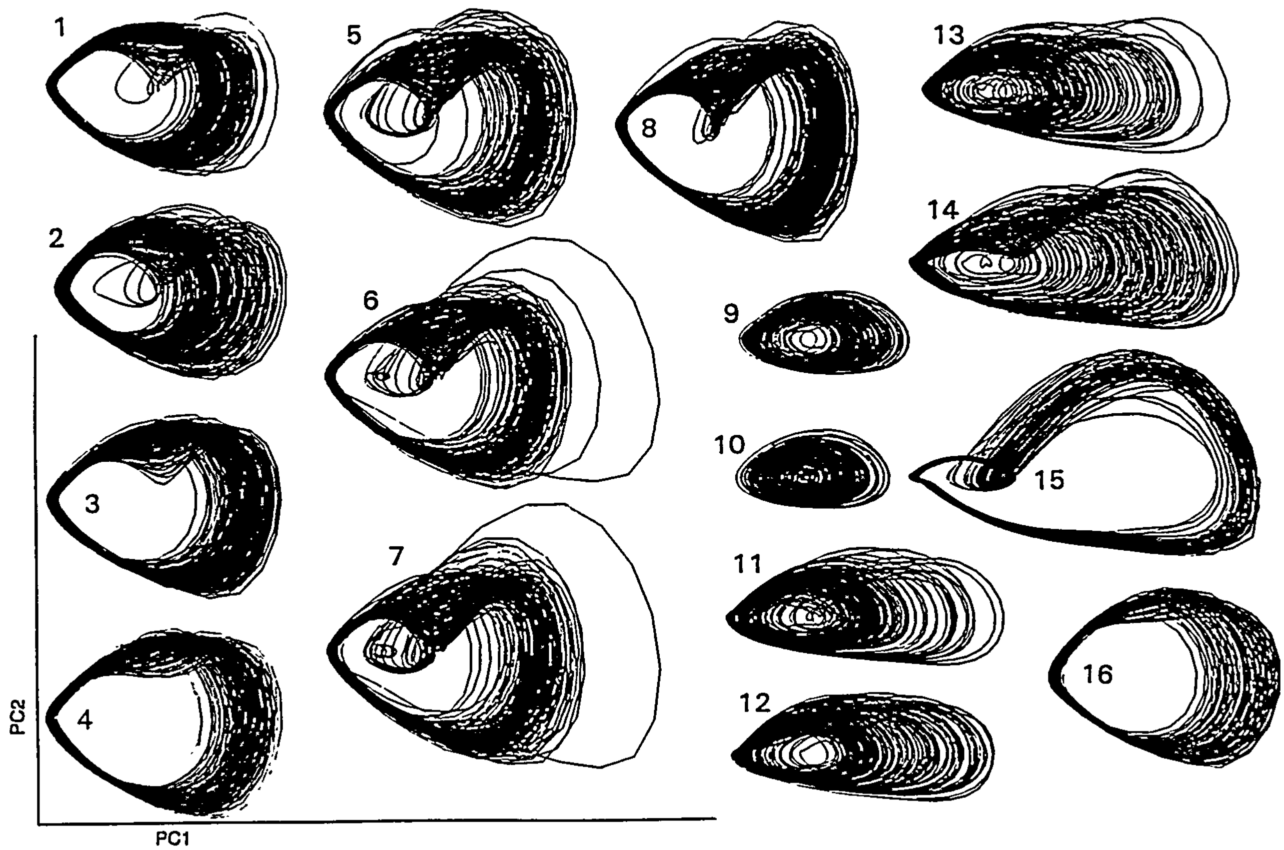

Fig. 5. The phase space trajectory projections formed by plotting principal component 1 (PC1) versus principal component 2 (PC2) are shown for each of the combustor tests (test number is shown). All trajectory projections are plotted to the same scale. Dynamical "families" and changes are clearly recognized.

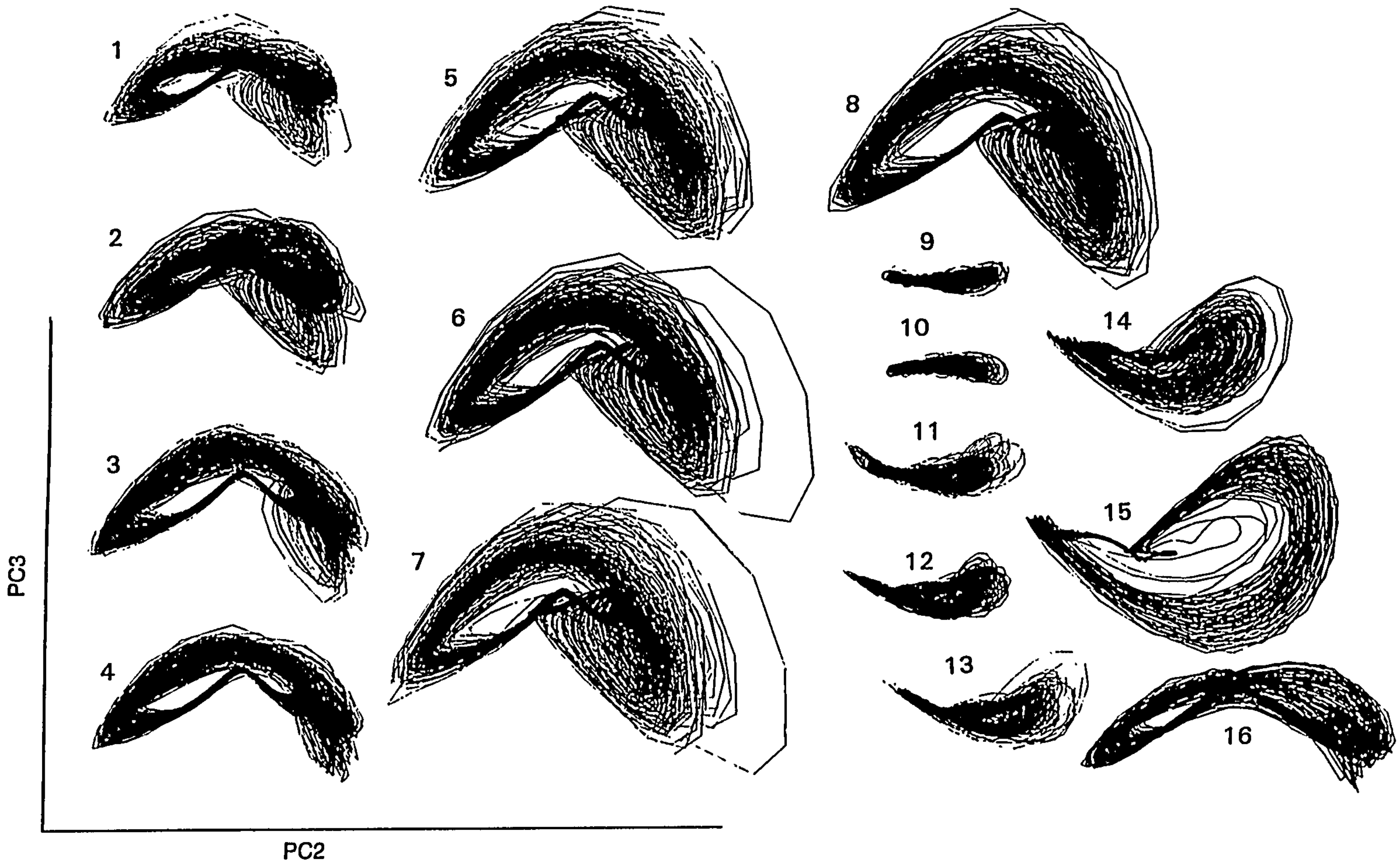

Fig. 6. Phase space trajectory projections using principal component 2 and 3 (PC2 vs PC3) also clearly depict dynamical similarities and changes.