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Loose Commitment

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Abstract

Due to time-inconsistency or policymakers' turnover, economic promises are not always fulfilled and plans are revised periodically. This fact is not accounted for in the commitment or the discretion approach. We consider two settings where the planner occasionally defaults on past promises. In the first setting, a default may occur in any period with a given probability. In the second, we make the likelihood of default a function of endogenous variables. We formulate these problems recursively, and provide techniques that can be applied to a general class of models. Our method can be used to analyze the plausibility and the importance of commitment and characterize optimal policy in a more realistic environment. We illustrate the method and results in a fiscal policy application.

JEL *classification*: C61, C63, E61, E62

Keywords: Commitment, Discretion, Fiscal Policy

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1 Introduction

1.1 Motivation and Contribution

In a general class of macroeconomic models, households' behavior depends on expectations of future variables. Characterizing optimal policy in such circumstances is intricate. A planner influences households' expectations through its actions, and households' expectations influence the actions of the planner. Following the seminal papers by Kydland and Prescott (1977) and Barro and Gordon (1983a), the literature has taken two different approaches to deal with this problem - commitment and discretion.

Both the commitment and discretion approaches are to some extent unrealistic. The commitment approach does not match the observation that governments have defaulted on past promises. The discretion approach rules out the possibility that the government achieves the benefits of making and keeping a promise, even if there is an *a posteriori* incentive to default. It seems more reasonable to assume that institutions and planners sometimes fulfill their promises and sometimes do not.

This paper characterizes optimal policy in two frameworks where some promises are kept while others are not. We first consider a setting where current promises will be fulfilled with a given probability. This setting can easily be extended to one where promises are only kept during a finite tenure. Lastly, we make the likelihood of default a function of endogenous variables. There may be several interpretations for the *loose commitment* settings just described. A political economy interpretation is that governments fulfill their own promises but it is possible that a new government is elected and the previous government's promises are not considered. Another interpretation is that a government commits to future plans, but defaulting becomes inevitable if particular events arise, such as wars or political instability. As it is common in the discretion literature, we consider that a default on past promises occurs whenever a reoptimization takes place. For the purposes of this paper it is inconsequential whether the reoptimization is undertaken by the same planner or by a newly appointed one. Another interpretation of these settings is that policymakers are often required to reevaluate policy. Hence, the optimal current policy recommendation should already take into account that policy will be revised in the future.

The contribution of this paper is in part methodological. We considerably generalize and extend the work of Roberds (1987) and Schaumburg and Tambalotti (2007). The methods previously available can be applied to linear quadratic models but can not usually be applied to non-linear models. The large bulk of models used in macro have non-linear features, such as non-linear utility and production functions. One could argue that non-linear models can be approximated by linear-quadratic methods. However, taking a linear quadratic approximation to a non-linear model often requires the timeless perspective assumption, which sharply contradicts the *loose commitment* assumption. This issue significantly reduces the set of models that one can solve in a *loose commitment* setting with linear quadratic techniques. We provide a methodology that can be applied to a large class of models, and we prove that the solution of these problems is recursive. In addition, we extend the *loose commitment* approach to models with endogenous state variables. This opens the possibility for each planner to set these state variables strategically and influence future planners. This issue raises some interesting questions and poses methodological challenges.

As an illustration of what can be learnt, we provide an application to fiscal policy. When the probability of keeping promises is decreased from 1 to 0.75, most variables move substantially towards discretion. Hence, in our application a full commitment framework seems unrealistic. We then discuss the effects of a default on economic variables. The main effect of reneging on past promises is an increase in the capital tax. Policy instruments are also found to change relatively more than private allocations during a default. We then discuss our default and commitment cycles in the spirit of the political business cycle literature. We characterize how the welfare gains change as a function of the probability to commit or the implied average time period before a default. In the endogenous probability model, we find that since the probability of commitment/reelection depends on endogenous state variables, the planner actively manipulates these state variables in order to enhance commitment.

1.2 Methodology

In an important contribution, Roberds (1987) considers that promises may not always be kept and proposes the probabilistic model also analyzed here. The author's model and assumptions are very specific, and his method is not generalizable to other applications. In another important contribution, Schaumburg and Tambalotti (2007) extended Roberds work to linear quadratic models and apply their methods to a Barro-Gordon type of monetary model without endogenous state variables. While their methods are useful for models that are exactly linear quadratic, most non-linear models can not be properly solved with Schaumburg and Tambalotti (2007). This is still the case if a linear quadratic approximation is performed to the non-linear model. As shown by Curdia et al. (2006), Debortoli and Nunes (2006) and Benigno and Woodfoord (2006), a correct linear-quadratic approximation can in general be derived if one imposes the timeless perspective assumption. The timeless perspective assumes that the problem is initialized at the full commitment steady state and that default never occurs. The *loose commitment* framework clearly requires a departure from the timeless perspective. In such cases, using the linear-quadratic approach with *loose commitment* is inappropriate because one considers different assumptions and the specification of the original model is violated. As discussed in the above references, this problem would occur whenever the planner can not achieve the first best allocation, for instance due to the presence of distortionary taxes. Unlike Schaumburg and Tambalotti (2007), our methodology can be applied to models that are not linear quadratic.

The tools for the analysis of time-inconsistent and time-consistent policy are recent. The key reference for solving time-inconsistent models is Marcet and Marimon (1998). Klein and Rios-Rull (2003) show how to solve for the time-consistent policy with linear quadratic techniques. Klein et al. (2007) recognize that the techniques proposed in Klein and Rios-Rull (2003) do not deliver controlled accuracy and propose a technique based on generalized Euler equations and a steady state local analysis. Judd (2004) proposes global approximation methods instead of steady state local analysis. In the solution procedure, we use a global method and generalized Euler equations taking the recent contributions of Judd (2004) and Klein et al. (2007). Besides the points presented in Judd (2004), employing a global method is specially important here. For the solution to be accurate, one needs to perform a good approximation both in commitment and default states. These two states, and the corresponding policy functions are not necessarily similar for one to be *a priori* certain that a local approximation would suffice.

Finally, we prove the recursiveness of the solution using the tools of Marcet and Marimon (1998). We show how to solve for linear and non-linear models, without and with endogenous state variables, relying only on one fixed point. As a by-product, our methodology can be used as a homotopy method to obtain the time-consistent solution.

1.3 Literature Review

Unlike the reputational equilibria literature, as in Backus and Driffill (1985), we are not aiming at building setups where a planner of a certain type resembles another type. We aim at characterizing the solution of planners that can make credible promises, but the commitment technology may become inoperative when it is time to fulfill them.

Another related topic is the trigger strategies, as in Barro and Gordon (1983b). Our paper is not aimed at building equilibria where private agents try to enforce a given equilibrium. Such strategies are quite intricate and raise enforcement and coordination issues.[1] Hence, one can think that the planner may not always be forced to fulfill its promises, as in the *loose commitment* setting.

Flood and Isard (1989) consider a central bank commitment to a rule with escape clauses. The rule does not incorporate some important shocks affecting the economy. When such shocks hit the economy, it may be better to abandon the rule. One can interpret that our probability of default is their probability of anomalous shocks. Another interpretation is that we consider policymakers who are more rational, and do not leave important shocks outside the commitment rule. In such interpretation, the rule is always better and the planner only defaults if the commitment technology becomes inoperative.

Persson et al. (2006), elaborating on an earlier proposal of Lucas and Stokey (1983), suggest a mechanism that makes the commitment solution to be time-consistent. Each government should leave its successor with a carefully chosen maturity of nominal and indexed debt for each contingent state of nature and at all maturities. Even though such strategies do eliminate the time-consistency problem, this structure of debt is not observed in reality. The view of this paper is that at certain points in time the commitment solution may be enforced, but in some contingencies discretion is unavoidable. This paper will consider a model with endogenous public good. Rogers (1989) showed that in such case debt restructuring can not enforce the commitment solution.

More importantly, most of the macroeconomics literature has either followed a commitment or discretion approach. This paper presents a general method that can potentially fill this gap. This paper can characterize policy under the more realistic description that some promises are fulfilled while others are not. These methods can be directly applied to the dynamic political economy literature, where different governments alternate in office, as in Alesina and Tabellini (1990). Due to technical reasons that this paper overcomes, such literature had always assumed a discretion approach or avoided time-inconsistency issues.

The paper is organized as follows: section 2 describes the model, section 3 analyzes the probabilistic setting, section 4 considers an extension with endogenous

[1]There are several issues on the enforcement and coordination of trigger strategies. Firstly, agents may not be able to learn such strategies because the punishment never occurs in equilibrium. Secondly, many atomistic private agents would need to develop and coordinate on highly sophisticated expectations mechanisms. Thirdly, if the punishment occurred, it is not clear that the economy would not renegotiate and enforce a better equilibrium.

probabilities and section 5 concludes.

2 The model

The methods and frameworks described in this paper can be applied to a wide set of dynamic optimization problems. Instead of discussing the methods in an abstract way, we will show an application to a fiscal policy problem. The model we are going to use has been described for instance in Martin (2007).

A representative household derives utility from private consumption (c), public consumption (g) and leisure ($1-l$). The household has 1 unit of time each period that he can allocate between leisure and labor (l). The household rents capital (k) and supplies labor (l) to a firm. Labor and capital earnings are taxed at a rate (τ^l) and (τ^k) respectively.

Following Greenwood et al. (1988), the household can also decide on the capital utilization rate v. Therefore, the amount of capital rented to firms will be vk. We are also going to assume that the depreciation rate of capital is a function of its utilization rate, ($\delta(v_t)$). In this model, we are going to abstract from debt. Dealing with debt and commitment issues is a topic that requires extensive treatment on itself and is beyond the scope of this paper.[2]

For given capital taxes (τ_t^k), labor taxes (τ_t^l), wages (w), and interest rate (r), the household problem is:

$$\max_{\{k_{t+1},c_t,l_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, l_t) \tag{1}$$
$$s.t: c_t + k_{t+1} = (1-\tau_t^k) r_t v_t k_t + (1-\tau_t^l) w_t l_t + (1-\delta(v_t)) k_t$$

where β denotes the discount factor. There is uncertainty in this economy because it is not know in advance whether the planner will default or not. The households' first order conditions (FOC) are:

$$u_{c,t} - \beta E_t u_{c,t+1}((1-\tau_{t+1}^k) r_{t+1} v_{t+1} + 1 - \delta(v_{t+1})) = 0 \tag{2}$$
$$u_{c,t}(1-\tau_t^l) w_t + u_{l,t} = 0 \tag{3}$$
$$(1-\tau_t^k) r_t - \delta_{v,t} = 0 \tag{4}$$

The time-inconsistency of this problem appears in Eq. (2). Today's household decisions depend on the expectation of future variables. In particular, the contemporaneous capital accumulation decision depends on future returns on capital. It is

[2]The reader is referred to Krusell et al. (2006) and Debortoli and Nunes (2007).

important to note that Eq. (4) is stating that the current capital tax is distortionary. If the planner would raise capital taxes, households could choose a lower capital utilization rate.[3] This feature is important for our model because if the planner does not keep his past promises and the capital utilization is fixed, then the capital tax could be set at an extremely high and implausible value. Martin (2007) showed that with fixed capital utilization, and for plausible calibrations, an equilibrium under discretion does not exist. There may be other reasons that inhibit the planner to choose an extremely high capital rate when it defaults, we chose this specification that guarantees the model to have a well defined solution for the commitment and discretion case.

Total output y_t is produced according to the function $F(k_t, v_t, l_t) = (k_t v_t)^\theta l_t^{1-\theta}$. Firms operate in perfectly competitive markets. Hence, wages and interest rates are given by:

$$r_t = F_{kv,t} \tag{5}$$

$$w_t = F_{l,t} \tag{6}$$

The planner provides the public good g, sets taxes τ^k and τ^l, satisfying the balanced budget constraint:

$$g_t = \tau_t^k r_t v_t k_t + \tau_t^l w_t l_t \tag{7}$$

Combining the households and governments' budget constraint one obtains the feasibility constraint:

$$y_t = c_t + g_t + k_{t+1} - (1 - \delta(v_t))k_t \tag{8}$$

To make our problem simpler we can proceed with a number of simplifications. Eq. (4) can be used to express the capital utilization as a function of other variables:

$$v_t = v(k_t, l_t, \tau_t^k) \tag{9}$$

Similarly, using the household and government budget constraint, private and public consumption can be expressed as functions:

$$c_t = c(k_{t+1}, k_t, l_t, \tau_t^k, \tau_t^l) \tag{10}$$

$$g_t = g(k_t, l_t, \tau_t^k, \tau_t^l) \tag{11}$$

[3]As discussed in Martin (2007), it is important that at least some depreciation is not tax deductible, as assumed for instance in Greenwood et al. (1998). If this is not the case, the current capital tax is not viewed by the current government as distortionary and an equilibrium in such economy may not exist. In some developed economies, there is a tax allowance for accounting depreciation, which differs from the actual depreciation. If there is excess depreciation due to a high capacity utilization such depreciation would still not be tax deductible.

Hence, the FOCs in Eqs. (2, 3) can be written in a more compact form:

$$b_1(x_t(\omega^t), k_t(\omega^t)) + \beta E_t b_2(x_{t+1}(\omega^{t+1}), k_{t+1}(\omega^{t+1})) = 0 \tag{12}$$

The vector of functions b_1, b_2 depends on several variables, where $x_t \equiv (k_{t+1}, l_t, \tau_t^k, \tau_t^l)$ is the vector of contemporaneous control variables, k_t is the state variable and ω^t is the history of events up to t.[4] Note that v_t, c_t and g_t have already been substituted in Eq. (12).

3 The probabilistic model

We will consider a model where a planner is not sure whether his promises will be kept or not. As explained above, this uncertainty can be due to several factors. For simplicity, we assume that these events are exogenous and that in any period the economy will experience default or commitment with given exogenous probabilities. In Section 4, we relax this assumption. Since it is indifferent whether it is the same or a new planner who defaults and reoptimizes, we use the terms "reelection", "new planner" and "default" interchangeably.

To make matters simple, we abstract from any shock other than the random variable s_t describing default (D) or commitment (ND) in period t. It is a straightforward generalization to include other sources of uncertainty, but the notation would be harder to follow. More formally, suppose the occurrence of Default or No Default is driven by a Markov stochastic process $\{s_t\}_{t=1}^{\infty}$ with possible realizations $\bar{s}_t \in \Phi \equiv \{D, ND\}$, and let Ω^t be the set of possible histories up to time t:

$$\Omega^t \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j \in \Phi, \forall j = 1, ..., t\} \tag{13}$$

We only consider the histories $\omega^t = \{D, \bar{s}_1, \bar{s}_2, ..., \bar{s}_t\}$, i.e. histories that start with a default on past promises. This is because in the initial period there are no promises to be fulfilled or equivalently the current government has just been settled. Before turning to the planner's problem, we describe the problem of individual agents.

[4] The class of models that our methodology is able to handle is fairly general and has the same requirements of Marcet and Marimon (1998). The separability in Eq. (12) is not necessary and the terms $k_t, k_{t+1}, x_t, x_{t+1}$ can all interact in a multiplicative way. Our methodology is also able to handle participation constraints and other infinite horizon constraints, as also described in Marcet and Marimon (1998).

3.1 Individual agents and constraints

In Eq. (12) we wrote the households' FOCs. These equations depend on future variables and hence households need to form rational expectations using available information. Given our institutional setting, households believe the promises of the current planner, but consider that if a different planner comes into play, then different policies will be implemented and past promises will not be kept. As it is common in the time-consistency literature, economic agents will take future controls that can not be committed upon as functions of the state variable, i.e. $x_{t+1}(\{\omega^t, D\}) = \Psi(k_{t+1}(\{\omega^t, D\}))$ where we use the short notation $\{\omega^t, D\}$ to denote $\{\omega^t, \bar{s}_{t+1} = D\}$. The function $\Psi(.)$ denotes the vector of policy functions that rational agents anticipate to be implemented in future periods.[5] Therefore, the constraint in Eq. (12) becomes:

$$b_1(x_t(\omega^t), k_t(\omega^t)) + \beta Prob(\{\omega^t, ND\}|\omega^t) b_2(x_{t+1}(\{\omega^t, ND\}), k_{t+1}(\{\omega^t, ND\})) \\ + \beta Prob(\{\omega^t, D\}|\omega^t) b_2(\Psi(k_{t+1}(\{\omega^t, D\})), k_{t+1}(\{\omega^t, D\})) = 0 \quad (14)$$

where we use the short notation $Prob(\{\omega^t, ND\}|\omega^t)$ to denote $Prob(\{s_j\}_{j=0}^{t+1} = \{\omega^t, ND\}|\{s_j\}_{j=0}^{t} = \omega^t)$. Note that k is a state variable and hence it is understood that $k_{t+1}(\{\omega^t, ND\}) = k_{t+1}(\{\omega^t, D\}), \forall \omega^t$.

3.2 The planner

When default occurs, a new planner is appointed and it will be taking decisions from that point onwards. Therefore, it is convenient to separate all histories ω^t with respect to the first time when default occurs. This is because we want to know which histories correspond to which planner. We now define the subset of Ω^t of histories where only commitment has occurred up to time t as:

$$\Omega^t_{ND} \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : \bar{s}_j = ND, \forall j = 1, ..., t\} \quad (15)$$

and the subsets of histories where the first default occurs in period i,

$$\Omega^t_{D,i} \equiv \{\omega^t = \{D, \{\bar{s}_j\}_{j=1}^t\} : (\bar{s}_i = D) \wedge (\bar{s}_j = ND), \forall j = 1, ..., i-1\}, \text{ if } i \leq t \quad (16)$$
$$\Omega^t_{D,i} \equiv \oslash, \text{ if } i > t$$

By construction note that $\{\Omega^t_{ND}, \Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ is a partition of the set Ω^t. Moreover, it can be seen that the sets Ω^t_{ND} and $\Omega^i_{D,i}$ are singletons.[6] Therefore, in order

[5] For further discussions on this issue see Klein et al. (2007).

[6] Ω^t_{ND} only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_t = ND\}$ and similarly the set $\Omega^i_{D,i}$ only contains the history $\{D, \bar{s}_1 = ND, \bar{s}_2 = ND, ..., \bar{s}_{i-1} = ND, \bar{s}_i = D\}$.

to avoid confusion between histories and sets of histories, we will refer to these singleton sets as ω_{ND}^t and $\omega_{D,i}^i$ respectively.

In figure 1 we show a more intuitive representation of the particular partition of histories specified above, where we use the name of the unique history ending in a given node to denote the node itself. White nodes indicate when a new planner is settled (default has occurred), while black nodes indicate the cases where the first planner is still in power (no default has occurred). We can see that in any period t there is only one history ω_{ND}^t such that commitment has always occurred in the past. Moreover, there is also only one history $\omega_{D,i}^i = \left\{\omega_{ND}^{i-1}, D\right\}$, meaning that the first default occurred in period i. In our institutional setting, a new planner is then settled from the node $\omega_{D,i}^i$ onward and it will make its choices over all the possible histories passing through the node $\omega_{D,i}^i$, that is the sets $\Omega_{D,i}^t, \forall t \geq i$.

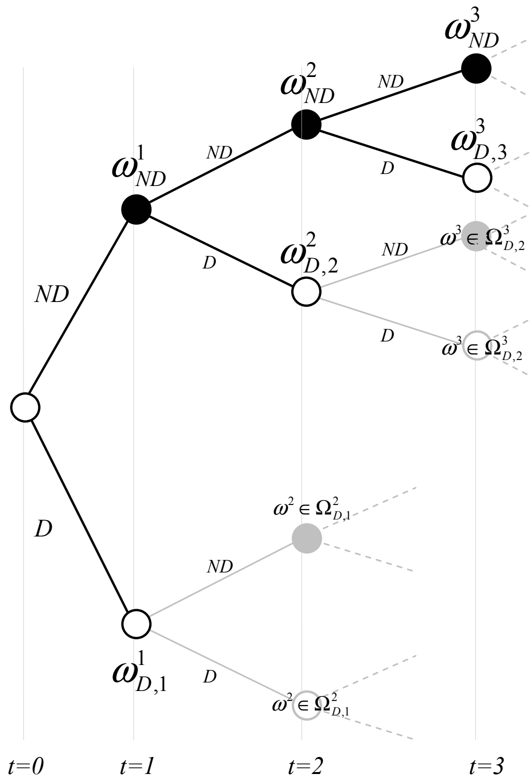

Figure 1: Diagram of the possible histories

We will now write the problem of the current planner where to simplify notation, and without loss of generality, we abstract from the presence of constraints in the maximization problem:

$$
\begin{aligned}
W(k_0) = & \max_{\substack{\{x_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} [\sum_{t=0}^{\infty} \sum_{\omega^t_{ND}} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \\
& + \max_{\substack{\{x_t(\omega^t)\}_{t=1}^{\infty} \\ \omega^t \in \Omega^t_{D,1}}} \left\{ \sum_{t=1}^{\infty} \sum_{\omega^t \in \Omega^t_{D,1}} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \right\} \\
& + \max_{\substack{\{x_t(\omega^t)\}_{t=2}^{\infty} \\ \omega^t \in \Omega^t_{D,2}}} \left\{ \sum_{t=2}^{\infty} \sum_{\omega^t \in \Omega^t_{D,2}} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \right\} \\
& + ...]
\end{aligned}
\tag{17}
$$

where we are using the short notation $Prob(\omega^t) = Prob(\{s_j\}_{j=0}^t = \omega^t)$. Equation (17) makes it explicit that inside the maximization problem of the current government there are other planners maximizing welfare during their tenures. Given that $\{\Omega^t_{ND}, \Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ is a partition of the set Ω^t, all the histories are contemplated in our formulation. Since $\forall\ t > i$, $\Omega^t_{D,i} = \{\omega^i_{D,i}, \{\bar{s}_j\}_{j=i}^t\}$, we can rewrite the probabilities for $\omega^t \in \Omega^t_{D,i}$ in the following way:

$$
Prob(\omega^t) = Prob(\omega^i_{D,i} \wedge \omega^t) = Prob(\omega^t | \omega^i_{D,i}) Prob(\omega^i_{D,i}), \forall\ \omega^t \in \Omega^t_{D,i},\ t \geq i. \tag{18}
$$

Substituting for these expressions into Eq. (17) and collecting the common term in the summation, we obtain:

$$
\begin{aligned}
W(k_0) = & \max_{\substack{\{x_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t \in \Omega^t}} \left\{ \sum_{t=0}^{\infty} \sum_{\omega^t_{ND}} \beta^t \{Prob(\omega^t) u(x_t(\omega^t), k_t(\omega^t))\} \right. \\
& \left. + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \left[\left[\max_{\substack{\{x_t(\omega^t)\}_{t=i}^{\infty} \\ \omega^t \in \Omega^t_{D,i}}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega^t_{D,i}} \beta^{t-i} \{Prob(\omega^t | \omega^i_{D,i}) u(x_t(\omega^t), k_t(\omega^t))\} \right] \right] \right\}
\end{aligned}
\tag{19}
$$

Since we are assuming that any future planner is also maximizing we can define the value functions:

$$
\xi_i(k_i(\omega^i_{D,i})) \equiv \max_{\substack{\{x_t(\omega^t)\}_{t=i}^{\infty} \\ \omega^t \in \Omega^t_{D,i}}} \sum_{t=i}^{\infty} \sum_{\omega^t \in \Omega^t_{D,i}} \beta^{t-i} \{Prob(\omega^t | \omega^i_{D,i}) u(x_t(\omega^t), k_t(\omega^t))\} \tag{20}
$$

where it was made explicit that each planner assigns probability one to its initial node. The value functions $\xi_i(k_i)$ summarize the happenings after the node $\omega^i_{D,i}$. Since $\Omega^t_{D,i} \cap \Omega^t_{D,j} = \oslash$ for $i \neq j$, the choices of future planners are independent between themselves. This formulation is very general since one can assume several institutional settings that the future planners will face. For example, one can assume that some future planners have full commitment while others do not. For simplicity we will assume that all future planners face the same institutional settings which at this stage we do not specify, thus we assume that $\xi_i(k_i) = \xi(k_i)\ \forall i$.[7] Since all the histories $\{\Omega^t_{D,1}, ..., \Omega^t_{D,t}\}$ are already being maximized by other planners, it is equivalent to consider that the initial planner maximizes over the single history $\{\omega^t : \omega^t \in \Omega^t_{ND}\} \equiv \omega^t_{ND}$ instead of $\omega^t \in \Omega^t$. We can therefore rewrite the problem at period $t = 0$ as:

$$W(k_0) = \max_{\{x_t(\omega^t_{ND})\}^{\infty}_{t=0}} \sum_{t=0}^{\infty} \Big\{ \beta^t \{Prob(\omega^t_{ND}) u(x_t(\omega^t_{ND}), k_t(\omega^t_{ND}))\} + \sum_{i=1}^{\infty} \beta^i Prob(\omega^i_{D,i}) \xi(k_i(\omega^i_{D,i})) \Big\} \tag{21}$$

We will now assume that the random variable s_t is i.i.d. to further simplify the problem. It is straightforward to generalize our formulation to Markov processes. Also to simplify notation denote $Prob(\{\omega^t, ND\}|\omega^t) = \pi$ and $Prob(\{\omega^t, D\}|\omega^t) = 1 - \pi$, which implies that:

$$Prob(\omega^t_{ND}) = \pi^t \tag{22}$$
$$Prob(\omega^t_{D,t}) = \pi^{t-1}(1 - \pi). \tag{23}$$

With this formulation at hand we are ready to show that our problem can be written as a saddle point functional equation (SPFE), and that the optimal policy functions of the planner are time-invariant and depend on a finite set of states.

[7] Debortoli and Nunes (2007), relax this assumption and focus on political disagreement issues.

3.2.1 The recursive formulation

Collecting results from the previous section, the problem of the current planner is:

$$\max_{\{x_t(\omega^t_{ND})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty}(\beta\pi)^t\{u(x_t(\omega^t_{ND}), k_t(\omega^t_{ND})) + \beta(1-\pi)\xi(k_{t+1}(\omega^{t+1}_{D,t+1}))\} \tag{24}$$
$$\begin{aligned} s.t: b_1(x_t(\omega^t_{ND}), k_t(\omega^t_{ND})) + \beta(1-\pi)b_2(\Psi(k_{t+1}(\{\omega^t_{ND}, D\})), k_{t+1}(\{\omega^t_{ND}, D\})) \\ + \beta\pi b_2(x_{t+1}(\omega^{t+1}_{ND}), k_{t+1}(\omega^{t+1}_{ND})) = 0 \end{aligned}$$

Due to the fact that we do have future controls in the constraints through the term $\beta\pi b_2(x_{t+1}(\omega^{t+1}_{ND}), k_{t+1}(\omega^{t+1}_{ND}))$, the usual Bellman equation is not satisfied.[8] Building on the results of Marcet and Marimon (1998), we show that problems of this type can be rewritten as a SPFE that generalizes the usual Bellman equation. This result is summarized in proposition 1.

Proposition 1 *Problem (24) can be written as saddle point functional equation as:*

$$W(k,\gamma) = \min_{\lambda\geq 0}\max_{x}\{H(x,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \tag{25}$$
$$s.t: \gamma' = \lambda, \ \gamma_0 = 0$$

where

$$H(x,k,\lambda,\gamma) = u(x,k) + \lambda g_1(x,k) + \gamma g_2(x,k) \tag{26}$$
$$g_1(x,k) = b_1(x,k) + \beta(1-\pi)b_2(\Psi(k'), k') \tag{27}$$
$$g_2(x,k) = b_2(x,k) \tag{28}$$

Proof. See the appendix. ■

Proposition 1 makes it clear that the current planner maximizes utility of the representative agent subject to the constraints $g_1(x,k) + \beta\pi g_2(x',k') = 0$, where the latter is incorporated in H. If there is no commitment, the continuation of the problem is $\xi(k')$. If the current promises will be fulfilled, then the continuation of the problem is $W(k',\gamma')$, and promises are summarized in the co-state variable γ'. The co-state variable is not a physical variable and the policymaker always faces the temptation to set it to zero. Also note that in our problem only the first constraint

[8] For details see Stokey et al. (1989).

contained in Eq. (12) contains future control variables. Therefore, only the first element of the vector λ needs to be included as a co-state variable. The optimal policy functions of such problem are time invariant and depend on a finite number of states, as proposition 2 describes.[9]

Proposition 2 *The solution of problem (24) is a time invariant function with state variables (k_t, γ_t), that is to say:*

$$\psi(k,\gamma) \in \arg\min_{\lambda\geq 0}\max_{x}\{H(x,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \tag{29}$$
$$s.t: \gamma' = \lambda,\ \gamma_0 = 0$$

Proof. See the appendix. ■

3.3 Equilibrium

In the institutional setting built in Eq. (24), we only assume that all planners from period 1 onward will face the same problems. From now on, we also assume that all future planners face the same institutional setting as we specify in period 0. In other words, we specify their problems in the same way as the problem of the planner in period 0. Thus we can use the following definition of equilibrium.

Definition 1 *A Markov Perfect Equilibrium where each planner faces the same institutional setting must satisfy the following conditions.*

1. Given $\Psi(k)$ and $\xi(k)$, the sequence $\{x_t\}$ solves problem (24);
2. The value function $W(k,\gamma)$ is such that $\xi(k) = W(k,0) \equiv W(k)$;
3. The policy functions $\psi(k,\gamma)$ solving problem (24) are such that $\Psi(k) = \psi(k,0)$.

The second part of the definition imposes directly that the problem of the initial and future planners must be equal. When a planner comes to office, he has not previously made any promise and therefore the co-state variable is reset to zero. While a planner is in office, he makes promises, and faces the temptation to deviate and reoptimize. In other words, the multiplier encoding the planner's promises is

[9] As it is common in the time-consistent literature and also in the optimal taxation literature we do not prove that the optimal policy function is unique. Nevertheless, we found no evidence of multiple solutions.

not a physical state variable and could always be put to zero. We are assuming that such a deviation only occurs with probability $1 - \pi$. The third part of the definition imposes a consistency requirement in the constraints. More precisely, we require the policy functions $\Psi(k)$ that agents expect to be implemented under default to be consistent with the optimal policy function. We refer to the notion of Markov Perfect Equilibrium because the function Ψ only depends on the natural state variables k. Also, in this equilibrium neither the planner nor individual agents desire to change behavior. Individual agents are maximizing and their beliefs are correct. The planner, taking as given Ψ and $\xi = W$, is also maximizing.

3.4 Solution strategy

The previous propositions showed that the problem is recursive. But at first sight, solving this problem looks daunting. The policy functions and the value function appear in the constraints and the objective function. One could try to guess $\Psi(k)$ and $\xi(k)$, solve the problem, update $\Psi(k)$ and $\xi(k)$, and iterate until convergence. Such procedure would imply solving three fixed points, one for the problem itself and two for $\Psi(k)$ and $\xi(k)$. In addition, this problem faces simultaneously all the difficulties present in the commitment and discretion literature. One has to include the lagrange multiplier as a state variable, and the derivatives of policy functions matter for the solution.

In this section, we discuss how to solve the problem in an easier way. We use the FOCs of the associated lagrangian formulation. Our generic problem is:

$$W(k_0) = \max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} (\beta\pi)^t \left[u\left(x_t, k_t\right) + \beta\left(1-\pi\right)\xi\left(k_{t+1}\right)\right] \tag{30}$$

$$s.t.\ g_1\left(x_t, k_t\right) + \beta\pi g_2\left(x_{t+1}, k_{t+1}\right) = 0 \qquad \forall t = 0, ..., \infty$$

where g_1 and g_2 are defined by Eqs. (27, 28) respectively.

Details on the FOCs can be found in the appendix. The term $\xi_{k,t+1}$ appears in the FOCs, because the current planner will try to influence future planners. The value function $\xi\left(k_{t+1}\right)$ summarizes the welfare that agents will achieve with a planner appointed at $t+1$. From the perspective of the planner appointed at $t+1$, the state variables k_{t+1} can not be changed. Nevertheless, from the perspective of the current planner, who is in charge at period t, k_{t+1} is not given and can be set strategically.[10]

[10]Note that, when default occurs, the lagrange multiplier is set to zero and cannot be used to

The FOCs expressed in Eqs. (46-48) allows us to solve for the optimal policy. As described in Definition 1, we are particularly interested in the formulation where future planners face the same problem as the current planner, i.e. where $\xi(k_t) = W(k_t)$ and hence $\xi_{k,t+1} = W_{k,t+1}$. We will show a solution method that only relies on solving one fixed point. To obtain the derivative $W_{k,t+1}$ we can use envelope results, which are summarized in result 1.

Result 1 *Using envelope results it follows that:*

$$\frac{\partial W(k_t)}{\partial k_t} = \frac{\partial u(x_t(k_t), k_t)}{\partial k_t} + \lambda_t g_{1,k_t,t} \tag{31}$$

where all variables are evaluated using the optimal policy of a planner appointed in period t, given the state k_t.

Result 1 uses the fact that all the planners are maximizing the same function, which allows the use of envelope principles.[11] It is important to note that in Eq. (31) all the variables are evaluated with the optimal policy of a newly elected government.

By Definition 1, the policy functions that the current and future planners implement are equal. If we use the envelope result to substitute $\xi_{k,t+1} = W_{k,t+1}$, the FOCs only depend on the functions $\psi(k_t, \lambda_{t-1})$ and $\Psi(k)$. Using Definition 1 and Proposition 1 we know that $\Psi(k) = \psi(k, 0), \forall k$, which also considerably simplifies the problem. We use a collocation method to solve for the optimal policy functions. Hence, using Result 1, Proposition 1 and 2, we can solve the problem relying on only one fixed point. Note that, unlike Schaumburg and Tambalotti (2007), we have endogenous state variables. Only in this case the derivative of the policy and value function appear in the FOCs, creating further difficulties.

We want to stress that in our framework the global solution methods proposed in Judd (1992) and Judd (2004) are more appropriate than local approximations. Besides the arguments presented by Judd, there are other reasons specific to our problem. The value function derivative, the levels and derivative of policy under default are present in the FOCs. The allocations under default and commitment are not likely to be similar. Demanding for a local approximation to deliver a good approximation in distant points to levels, derivatives and value functions is quite demanding.

The linear quadratic approximation proposed in Benigno and Woodfoord (2006) is only valid in a timeless perspective. The timeless perspective assumes that initial

influence incoming planners.

[11] A proof of this envelope result is available upon request.

commitments are equal to the steady-state commitment. There are several reasons that make the timeless perspective approach inappropriate in our framework. Firstly, we consider that commitments may be broken and consequently we need to focus on transition dynamics at that point. Secondly, our model does not have a deterministic steady state point around which one can take an approximation. Indeed, shutting down uncertainty completely changes the problem. Thirdly, under discretion the allocations can be very far from the commitment steady-state. Our method is more suitable and it is also simpler. Even for an exactly linear quadratic model, Schaumburg and Tambalotti (2007) only solved a model with no endogenous state variables, and suggested a procedure to handle endogenous state variables relying on three fixed points. The method presented here relies on only one fixed point in policy functions.

Besides these considerations, there is a crucial drawback of applying the linear-quadratic timeless perspective approach to study problems with *loose commitment* settings, as suggested by Schaumburg and Tambalotti (2007). This point was already discussed in the introduction and its methodological discussion.

3.5 Results

In order to proceed to the numerical solution, we specify a per-period utility function:

$$u(c_t, g_t, l_t) = (1 - \phi_g)\left[\phi_c \log(c_t) + (1 - \phi_c)\log(1 - l_t)\right] + \phi_g \log(g_t) \tag{32}$$

and a depreciation function:

$$\delta(v_t) = \frac{\chi_0}{\chi_1} v_t^{\chi_1} \tag{33}$$

We use a standard calibration for an annual model of the US economy. Table 1 summarizes the values used for the parameters. The parameters χ_0 and χ_1 imply that in steady state the capital utilization rate (v) is about 0.8, and the depreciation rate ($\delta(v)$) is about 0.08.

Table 2 presents the long run average for several variables, and across different parameterizations of π. The column with $\pi = 1$ and $\pi = 0$ correspond to full commitment and full discretion respectively. In the full commitment model, the capital tax is zero, a result common in the optimal taxation literature with full commitment. With full discretion, the capital tax is roughly 19%. In the discretion model, once capital has been accumulated the government has a temptation to tax it. Due to the possibility of changing the capital utilization rate, capital is

Table 1: Parameter values

Parameter	Value	Description
β	.96	Discount factor
ϕ_c	.285	Weight of consumption vs. leisure
ϕ_g	.119	Weight of public vs. private consumption
θ	.36	Capital share
χ_0	.171	Depreciation function parameter
χ_1	1.521	Depreciation function parameter

Table 2: Average Values

	1.00	**0.75**	**0.50**	**0.25**	**0.00**
k	1.122	0.947	0.899	0.880	0.870
λ	-0.536	-0.177	-0.080	-0.030	0.000
g	0.093	0.076	0.072	0.070	0.069
c	0.196	0.216	0.220	0.222	0.224
y	0.378	0.368	0.364	0.363	0.362
τ^k	0.000	0.131	0.163	0.178	0.187
τ^l	0.384	0.251	0.218	0.203	0.191
l	0.233	0.245	0.248	0.250	0.250
u	0.798	0.799	0.801	0.800	0.799

Note: The values refer to long run averages.

not an entirely fixed factor of production that can be heavily taxed. The average capital utilization rate does not change much with π. But as we will discuss later, if private agents are surprised with higher than expected capital taxes, then the capital utilization rate is lowered.

In this model, governments cannot issue debt and have to balance their budgets every period. In Chamley (1986), there is a big incentive to tax capital very highly in earlier periods to obtain large amounts of assets and eliminate distortionary taxation in later periods.[12] By imposing a balanced budget, higher capital taxation revenues have to be used immediately. Even though in our model the incentives to tax capital under discretion are mitigated, it is still the case that capital taxes under discretion

[12]The zero long run tax on capital still holds for a variety of cases (including balanced budget) as shown by Judd (1985).

are higher. For an example, where the reverse can happen the reader is referred to Benhabib and Rustichini (1997) and Klein et al. (2007).

Since capital taxes are lower under commitment, the level of capital is higher. As a direct consequence of no capital tax revenues, labor taxes need to be higher. Higher labor taxes induce households to work less. Private consumption is lower in the full commitment economy, while public consumption is higher. Obviously, the allocations in terms of leisure, private and public consumption are more efficient in the full commitment economy.

We now turn into commenting the *loose commitment* settings. The main purpose of this paper is to provide a theoretical basis for this kind of models. Providing a definitive answer on the probability π or empirically estimating these models is beyond the scope of this paper. Nevertheless, we can provide some evidence on the probability π. A probability of reelection π implies an expected tenure of $1/(1-\pi)$. A value of 0.75 corresponds to a planner being in office for 4 years on average. A calibration based on the political history of the US implies a value of 0.8, while the political history of Italy would imply a calibration around 0. The numbers above are excluding the possibility that there were no broken promises during the tenure, therefore a value of 0.75 might be considered an upper bound. Rallings (1987) tried to obtain a measure of how many manifesto pledges were actually implemented. Some of these pledges often reveal political options, such as the composition of expenditures, and may not always be related to time-inconsistency issues. The average number reported by the author is 0.63 and 0.71 for Britain and Canada respectively. Arguably, these estimates may be considered higher than the actual π. Ideological promises are easier to fulfill than promises where a temptation to default actually exists. While Rallings (1987) estimates includes both types of promises, the measure π only refers to the latter. Obviously, the measure π depends on the country being studied and the specific policy and institutional settings involved.

In our analysis, all variables seem to be much closer to the discretion solution rather than to the commitment solution. If the probability of committing is 0.75, average allocations only move about 31% of the distance from discretion to commitment. Table 3 computes this value for all the allocations. The other way to interpret the table is that small reductions in the probability π from the full commitment solution have dramatic effects. It may be expected that decreasing the probability of commitment from 1 to 0.75 would lead allocations to move 25% of the difference between commitment and discretion. But in fact, the absolute drop in capital is 69% of the difference between full commitment and discretion.

We now describe the transition dynamics. Figure 2 plots the average path during the first 25 quarters, initializing capital at its steady state value under discretion

Table 3: Relative difference from full commitment

	1.000	**0.750**	**0.500**	**0.250**	**0.000**
k	1.000	0.306	0.115	0.040	0.000
λ	1.000	0.330	0.149	0.056	0.000
g	1.000	0.292	0.125	0.042	0.000
c	1.000	0.286	0.143	0.071	0.000
y	1.000	0.375	0.125	0.063	0.000
τ^k	1.000	0.299	0.128	0.048	0.000
τ^l	1.000	0.311	0.140	0.062	0.000
l	1.000	0.294	0.118	0.000	0.000

Note: The values are computed with the formula: $(x_{\pi} - x_{\pi=0})/(x_{\pi=1} - x_{\pi=0})$.

and considering that no promises had been made. Since the economy starts with a relatively low level of capital, the utilization rate of capital is relatively high. This occurs despite the fact that capital taxes are high in early periods as commitment starts to build. In initial periods, since capital taxes are relatively high, labor taxes are lower. As a consequence, labor is higher in initial periods. As time evolves, the picture confirms the results of table 2, since for later periods the variables are relatively closer to the discretion path.

We should finally comment on the path of the lagrange multiplier. This variable may not have economic interest per se, since it is unobservable. We should nevertheless mention that as all other variables, the lagrange multiplier is also much closer to the discretion steady state of 0. This suggests that a local approximation around the full commitment, as performed in the timeless perspective approach may be a poor approximation to *loose commitment* frameworks. Also our analysis suggests that characterizing allocations in a full commitment analysis may be less realistic than in a full discretion approach.

So far the analysis has only referred to average paths. We now analyze what happens in the specific periods when governments renege on their past promises. For this purpose, in figure 3, we plot the paths followed in a given history. We consider the history where by chance a new planner is reappointed every four years. When default occurs the planner breaks the promise of a low capital tax. Capital is a relatively inelastic tax base and hence capital taxes are increased. Capital accumulation is almost unchanged but households immediately react by decreasing the capital utilization rate. Since there are more capital tax revenues, labor taxes are cut, leading to more labor. Overall, output increases mainly fueled by the increase in labor input.

Our analysis can be related to the literature on political cycles, as described for instance in Drazen (2000). The empirical analysis of political cycles mentions that output or private consumption do not move much with the political cycle. In our model, both output and private consumption are not found to move much in relative terms. It is argued that policy instruments are more affected by the political cycle. Our model has the same prediction. Public consumption, capital taxes and labor taxes are the policy instruments and these face greater variations in relative terms. This empirical literature has also found that at the end of a tenure output and private consumption are not higher than average, a feature also predicted in our model. It is also found that at the end of the tenure there are not significant tax cuts leading to less tax revenues, which also conforms with our model. There is some evidence that public expenditure is increased before the elections, being the evidence stronger for government transfers. This feature is also replicated in our model, public consumption is higher as the tenure evolves. It is not our aim to match political cycles, because electoral competition is absent from the model. But our simple model does not contradict the empirical evidence found in that literature.

3.6 Welfare Calculations

In this section, we turn to measure the welfare implications of building commitment. In our framework, this means considering the welfare gains of increasing π. Consider two regimes, an alternative regime (a) and benchmark regime (b). The life-time utility W of the representative agent in both regimes is given by:

$$W^i(k_{-1}, 0) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, l_t^i, g_t^i) \qquad i = a, b \tag{34}$$

where $\{c_t^i, l_t^i, g_t^i\}_{t=0}^{\infty}$ is the optimal allocation sequence in regime i. The expectation operator covers all the commitment and default states. Define ϖ as the increase in private consumption in the benchmark regime that makes households indifferent between the benchmark and an alternative regime. More formally ϖ is implicitly defined as:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^a, l_t^a, g_t^a) = E_0 \sum_{t=0}^{\infty} \beta^t u((1+\varpi) c_t^b, l_t^b, g_t^b) \tag{35}$$

For the calculations that follow we considered the benchmark regime to be the full discretion case and we initialized capital at the steady state prevailing when $\pi = 0.5$.[13] The welfare improvement from discretion to commitment is equivalent to an increase in private consumption of 3.65%. Figure 4 shows the welfare gains for different probabilities of commitment. When the probability of default increases from 0 to 0.25, only 13% of the benefits of commitment are achieved. We plot the relative welfare gains as a function of π in figure 4. The function is convex suggesting that increasing π from low to intermediate levels results in relative small welfare gains. Most of the gains from enhancing commitment can only be achieved when π is already high. In figure 5, we plot the relative welfare gains as a function of the expected time before a default occurs $(1/(1-\pi))$. In this metric the welfare gains function is concave. The welfare gains per unit of time of moving from 1 to 2 years are higher than the gains of moving from 2 to 3 years. This result may seem more intuitive, since as the expected commitment period increases there are less welfare gains to be achieved.

In a related work on optimal monetary policy, Schaumburg and Tambalotti (2007) found qualitatively different results. First, allocations move linearly in the probability π. For instance, when π moves from 1 to 0.75 inflation goes 25% of the distance towards discretion. Secondly, most of the welfare gains are achieved at low levels of commitment. In other words, the welfare is always concave regardless of the metric used. When π is 0.75, about 90% of the welfare gains from commitment are obtained. Comparing absolute welfare measures in our model and theirs is unclear, but the welfare gains of moving from discretion to commitment are also much higher in their model.[14]

One possible reason for the different results obtained here and in Schaumburg and Tambalotti (2007) is that fiscal and monetary policy are simply different in this respect. We have investigated other potential sources of differences between our results and theirs. First, figure 5 suggests that some differences could occur if

[13] We also considered initializing capital at other steady states or expressing ϖ in consumption units of the alternative regime. This means computing ϖ as $E_0\sum_{t=0}^{\infty}\beta^t u((1-\varpi)c_t^a, l_t^a, g_t^a) = E_0\sum_{t=0}^{\infty}\beta^t u(c_t^b, l_t^b, g_t^b)$. The results remain unchanged.

[14] In Barro-Gordon models the welfare loss penalizes quadratically deviations of inflation from zero and deviations of the output gap from a target level. The inflation and output gap under commitment are nearly zero. Under discretion the inflation is quite high and the output gap is still zero. Since standard calibrations give a much higher weight to inflation deviations in the loss function, the gains from commitment are substantial.

one would change the time period of the model. The original calibration used in Schaumburg and Tambalotti (2007) is quarterly. We also tried an annual calibration of their model, and the results do not change qualitatively. Another potential difference relates to the role of endogenous state variables. An endogenous state variable, provides the planner with an additional instrument to influence future decisions. Therefore, it may be expected that in a model with endogenous state variables, the benefit of adding commitment is smaller. While our model has one endogenous state variable their model has none. To test this hypothesis we also considered their model with an hybrid Phillips curve as in Galí and Gertler (1999). The results do move slightly in the direction that we predict but a significant difference between their model and ours remains. Our results suggest that there may be an important difference between monetary and fiscal policy in this respect.

3.7 Discussion on monetary and fiscal policy commitment

In the early 70's economics and policymakers were still not aware of the importance of commitment and its use in policy design. The commitment in both fiscal and monetary policy were low at that time. After the seminal contributions in the late 70's and early 80's regarding time-inconsistency, there has been a concern to increase institutional commitment. The reforms to increase monetary policy commitment were far more intense than the ones relative to fiscal policy. In fact, we observed a pattern of inflation that is consistent with low commitment in the 70's and high commitment today. In the 70's, the inflation level was much closer to the steady state level of discretion than the one of commitment. Nowadays, the opposite is true.

The patterns in fiscal policy are somehow different. We have not observed institutional reforms aimed at increasing commitment comparable to the ones in monetary policy. Also, as discussed in Klein and Rios-Rull (2003), the level of taxes both in the 70's and today are much closer to the full discretion prediction than to the commitment one. This evidence suggests that fiscal policy commitment was low in the 70's and is still low today, while monetary policy commitment was low in the 70's and is high today.

Combining our results with those of Schaumburg and Tambalotti (2007), may shed light on this issue. It seems that when commitment is low, the welfare gains of more commitment are higher in monetary policy than in fiscal policy. Also, it may be very hard to implement the full commitment solution, since defaults are always possible and flexibility is necessary. While most welfare improvements can be achieved at intermediate levels of commitment in monetary policy, the same is not true for fiscal policy.

It may also be argued that it may be easier to achieve high levels of commitment in monetary policy than in fiscal policy. The turnover in presidents is higher than in central bank governors. More importantly, it may be difficult to establish a fiscal authority with full commitment, because such an institution would have to rule out democratic choices as they violate the commitment plan. The intuition that achieving high commitment levels is easier in monetary policy rather than in fiscal policy together with our results may also help in explaining the data.

4 Endogenous probability Model

We are finally going to consider an extension where the probability of defaulting depends on the states of the economy. Since capital is the only natural state variable in the economy and all allocations depend on capital, we will consider that the probability of defaulting today depends on the current capital stock. The planner and households will consider that the probability of commitment in the next period is $P(k_{t+1})$ instead of π. Following the steps in earlier sections, the objective function of the planner becomes:

$$\sum_{t=0}^{\infty} \beta^t \frac{\Pi_{j=0}^{t}(P(k_j))}{P(k_0)} (u(x_t, k_t) + \beta(1 - P(k_{t+1}))W(k_{t+1})) \tag{36}$$

As before, the probability of being in charge in the first period is 1. The special term $P(k_0)$ in the objective function does not induce any time-inconsistency problem because k_0 is predetermined.[15]

Households understand that the probability of committing depends on aggregate capital. A single household can only decide his own single capital accumulation. This means that each household is atomistic and takes the aggregate capital stock as given. Therefore, as it seems reasonable, the individual household capital accumulation decision does not incorporate the effect in the commitment technology.[16] Hence, the constraints that the planner faces are:

[15]If the probability function depended on a non-predetermined variable, then the FOCs would be different when a planner starts and when it is already in power. This would introduce another source of time inconsistency.

[16]More formally, one could model a continuum of agents on a real interval between 0 and 1. All agents would be equal, and therefore their decisions would be equivalent to a representative agent, who takes aggregate capital as given.

$$b_1(x_t(\omega_{ND}^t), k_t(\omega_{ND}^t)) + \beta(1 - P(k_{t+1}))b_2(\Psi(k_{t+1}(\{\omega_{ND}^t, D\})), k_{t+1}(\{\omega_{ND}^t, D\})) + \beta P(k_{t+1})b_2(x_{t+1}(\omega_{ND}^{t+1}), k_{t+1}(\omega_{ND}^{t+1})) = 0 \tag{37}$$

We need to prove that this setting can also be written as a SPFE. This result is done in Proposition 3, and details are available in the appendix.

Proposition 3 *The problem of a planner maximizing Eq. (36) subject to Eq. (37) can be written as saddle point functional equation.*

Proof. See the appendix. ■

With proposition 3 at hand, it then follows that the solution to the problem is a time-invariant function. In the appendix, we also describe the FOCs and some simplifications that are very useful for computational work. In comparison with the exogenous probability case, the FOCs with respect to all variables except to capital remain unchanged. In the FOC with respect to capital, some new terms appear reflecting that the commitment probability can be influenced. Unlike households, the planner does not take aggregate capital as given. One extra term refers to the appearance of $P(k_{t+1})$ in the constraints of households. The other term refers to the expected change in utility, induced by the change in the commitment probability. This is captured by the term $P_{k_{t+1}}\beta(W(k_{t+1}, \lambda_t) - W(k_{t+1}, 0))$. If capital is increased, the commitment probability is increased by $P_{k_{t+1}}$. This increases the chances of the current planner to obtain tomorrow's continuation value under commitment $W(k_{t+1}, \lambda_t)$. Nevertheless, it decreases the chances of obtaining tomorrow's continuation value under discretion $W(k_{t+1}, 0)$.

This model raises an extra difficulty in terms of computational work. As explained above, the level of the value function appears in the FOCs. Hence, one needs to approximate the value function as well. The exogenous probability model could be solved as one fixed point. The endogenous probability model needs to be solved as two fixed points.

In what follows, we will consider a probability function such that when capital is higher there is a higher probability of commitment. This assumption could be justified on political economy grounds. More capital implies more output and a higher probability of reelection. We will consider the following probability function:

$$P(k_t) = 1 - \frac{1}{(\frac{k_t}{\tilde{k}})^\rho + 1} \tag{38}$$

The parameter $\tilde{k}$ is a normalization such that $P(\tilde{k}) = 0.5$. The higher is ρ, the easier it is for the planner to influence its reelection probability. In the case of $\rho = 0$, the probability is always constant. We can use a homotopy from the model in section 3 to this model by changing ρ from 0 to the desired value. We chose $\rho = 30$ and $\tilde{k}$ to be equal to the average capital allocation when $\pi = 0.5$. Our normalization of $\tilde{k}$ allows us to directly compare the results with the probabilistic model when $\pi = 0.5$.

Results are presented in table 4. In the endogenous probability model, capital is now higher. Since the probability of commitment is increasing in capital, the planner has a further motive to accumulate capital. The incentives to accumulate more capital need to be provided by the planner. Households by themselves do not strategically increase their capital in order to increase the commitment probability. In order to make households accumulate more capital, the planner mainly reduces capital taxes. Since more commitment is achieved, average allocations move towards the full-commitment equilibrium.

In the endogenous probability model the welfare gain relative to discretion is 2.6%. This value is higher than the welfare gains obtained in the benchmark case of $\pi = 0.5$. One reason is that the commitment probability is higher. The other reason is that the welfare gains function is convex in π. A varying probability around a mean may therefore induce some additional welfare gains. In a political economy interpretation, our model would suggest that governments accumulate more capital to be reelected; and this is a good policy since it reduces political turnover thus increasing the commitment probability.

Table 4: Endogenous Probability - Average Values

	$\pi = 0.5$	**End. Prob.**
k	0.899	0.932
λ	-0.080	-0.064
g	0.072	0.082
c	0.220	0.209
y	0.364	0.365
τ^k	0.163	0.153
τ^l	0.218	0.268
l	0.248	0.246
u	0.801	0.790
$\bar{\pi}$	0.500	0.738

5 Conclusions

The time-consistent and time-inconsistent solutions are to some extent unrealistic. This paper tried to characterize optimal policy in a setting where some promises are fulfilled while others are not. One interpretation of such setting is based on political turnover, where governments make promises but may be out of power when it is time to fulfill them. Alternatively, governments may make promises but when certain events arise the commitment technology breaks. This framework can also be thought as providing an optimal policy prescription knowing that at a later date policy is going to be revised.

From the methodological point of view, our contribution is to show a solution technique for problems of *loose commitment* with the following main features. First, it can be applied to a wide class of non-linear models, with or without endogenous state-variables keeping the model's micro-foundations structure intact. While there were other works on similar *loose commitment* settings, such methods could not be used in the standard non-linear macro models. For instance, the fiscal policy problem described here has to be solved with the methods developed in this paper. Second, building on the results of Marcet and Marimon (1998), we proved that the solution to our problem is recursive. Third, we implemented an algorithm which is relatively inexpensive, and makes use of global approximation techniques which are pointed out in the literature as more reliable. Finally, as a by-product, our procedure can be used as a homotopy method to find the time-consistent solution.

We show that in the optimal taxation model under *loose commitment*, average allocations seem to be closer to the time-consistent solution. We have also characterized the economic consequences of reneging on promises. Our results suggest that when promises are not fulfilled capital taxes will be raised and labor taxes will be lowered. As a consequence, the capital utilization rate drops and labor input increases. We then showed that several features of the model are in accordance with some empirical results on political cycles.

We then considered that the probability of committing would be a function of the endogenous state variables. In such a model, the government would try increase the commitment probability through the state variables. The intuition for such results is that having more commitment is welfare improving, therefore the government tries to increase commitment.

Regarding welfare, we find that for an upper bound of the probability of commitment around 0.75, most of the gains from commitment are not achieved. While the welfare gains are a concave function of the expected time before a default, they are a convex function of the probability of commitment. These results are different from those obtained in the literature on monetary policy. This may explain the observation that more effort has been devoted to building commitment in monetary rather than fiscal policy.

The methods provided in this paper are general and can be applied to a variety of macroeconomic problems. The setups formulated here can be easily brought into the dynamic political economy literature. This literature has commonly abstracted from the presence of time-inconsistency or assumed a discretion approach. Our paper allows consideration of the more realistic setting in which governments can fulfill promises when they are reelected. This paper also sets up the base for addressing problems where different governments do not face the same institutional settings or disagree on policy objectives. Finally, the applications of our methodology are not restricted to optimal policy problems. Indeed, it can be used in many dynamic problems where commitment plays an important role, like the relationship between firms and their customers and shareholders, or in other principal-agent problems.

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A Proofs

Proof. of Proposition 1

Drop history dependence and define:

$$
\begin{aligned}
r(x_t, k_t) &\equiv u(x_t, k_t) + \beta(1-\pi)\xi(k_{t+1})) \\
g_1(x_t, k_t) &\equiv b_1(x_t, k_t) + \beta(1-\pi)b_2(\Psi(k_{t+1}), k_{t+1}) \\
g_2(x_{t+1}, k_{t+1}) &\equiv b_2(x_{t+1}, k_{t+1})
\end{aligned}
$$

Our problem is thus:

$$
\begin{aligned}
&\max_{\substack{\{x_t(\omega^t)\}_{t=0}^{\infty} \\ \omega^t=\omega^t_{ND}}} \sum_{t=0}^{\infty} (\beta\pi)^t \{r(x_t, k_t)\} \qquad (39) \\
&s.t: \; g_1(x_t, k_t) + \beta\pi g_2(x_{t+1}, k_{t+1}) = 0
\end{aligned}
$$

which fits the definition of Program 1 in Marcet and Marimon (1998). To see this more clearly note that our discount factor is $\beta\pi$ and we have no uncertainty. Since ω^t_{ND} is a singleton, we have previously transformed our stochastic problem into a non-stochastic problem. Therefore, we can write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying

$$
\begin{aligned}
W(k,\gamma) &= \min_{\lambda \geq 0} \max_{x} \{h(x,k,\gamma,\lambda) + \beta\pi W(k',\gamma')\} \qquad (40) \\
&s.t: \; \gamma' = \lambda, \; \gamma_0 = 0
\end{aligned}
$$

where

$$
h(x,k,\lambda,\gamma) = r(x,k) + \lambda g_1(x,k) + \gamma g_2(x,k) \qquad (41)
$$

or in a more intuitive formulation define:

$$
H(x,k,\lambda,\gamma) = u(x,k) + \lambda g_1(x,k) + \gamma g_2(x,k) \qquad (42)
$$

and the saddle point functional equation is:

$$
\begin{aligned}
W(k,\gamma) &= \min_{\lambda \geq 0} \max_{x} \{H(x,k,\lambda,\gamma) + \beta(1-\pi)\xi(k') + \beta\pi W(k',\gamma')\} \qquad (43) \\
&s.t: \gamma' = \lambda, \; \gamma_0 = 0
\end{aligned}
$$

■

Proof. of Proposition 2: Using Proposition 1, this proof follows trivially from the results of Marcet and Marimon (1998). ■

Proof. of Proposition 3

Define an additional variable η, The law of motion for η is:

$$\eta_{t+1} = \eta_t P(k_{t+1}) \tag{44}$$

with $\eta_0 = 1$. The problem of the planner can then be rewritten as:

$$\begin{aligned}\min_{\{\lambda_t,\varphi_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathscr{L} = \sum_{t=0}^{\infty} & \beta^t \eta_t (u(x_t, k_t) + \beta (1 - P(k_{t+1})) \xi (k_{t+1})) \\ & + \lambda_t (g_1 (x_t, k_t) + \beta P(k_{t+1}) g_2 (x_{t+1}, k_{t+1})) \\ & + \varphi_t (\eta_{t+1} - \eta_t P(k_{t+1}))\end{aligned} \tag{45}$$

Using similar redefinitions as in the proof of proposition 1 the result follows. The condition $\eta_0 = 1$ signals that a new planner is in charge. Eq. (44) is still valid because it only refers to the evolution of η through commitment states. Finally, note that in terms of notation $\Psi(k) \equiv \Psi(k, \lambda = 0, \eta = 1)$.[17] ■

B First Order Conditions

B.1 Probabilistic Model

To solve the problem set up the Lagrangian:

$$\begin{aligned}\min_{\{\lambda_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathscr{L} = \sum_{t=0}^{\infty} & (\beta\pi)^t (u (x_t, k_t) + \beta (1 - \pi) W (k_{t+1})) \\ & + \lambda_t (g_1 (x_t, k_t) + \beta \pi g_2 (x_{t+1}, k_{t+1}))\end{aligned}$$

[17] For the purpose of this proof one has to include η_t as a state variable. This is only convenient for this proof and as discussed later is not necessary for the numerical work.

The FOCs are[18]:

$$\frac{\partial \mathscr{L}}{\partial z_t} : u_{z_t,t} + \lambda_t g_{1,z_t,t} + \lambda_{t-1} g_{2,z_t,t} = 0 \tag{46}$$

$$\frac{\partial \mathscr{L}}{\partial k_{t+1}} : u_{k_{t+1},t} + \beta\left(1-\pi\right) W_{k_{t+1},t+1} + \lambda_t (g_{1,k_{t+1},t} + \beta\pi g_{2,k_{t+1},t+1}) \tag{47}$$

$$+ \beta\pi\left(u_{k_{t+1},t+1} + \lambda_{t+1} g_{1,k_{t+1},t+1}\right) - \lambda_{t-1} g_{2,k_{t+1},t} = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_t} : g_1\left(x_t, k_t\right) + \beta\pi g_2\left(x_{t+1}, k_{t+1}\right) = 0 \tag{48}$$

$$\forall t = 0, ..., \infty \qquad \lambda_{-1} = 0$$

where $z_t \equiv (l_t, \tau_t^k, \tau_t^l)$, and using Eqs. (27, 28) it follows that $g_{1,x_t,t} = b_{1,x_t,t}$, $g_{2,x_t,t} = b_{2,x,t}$, $g_{1,k_t,t} = b_{1,k_t,t}$, $g_{2,k_t,t} = b_{2,k_t,t}$, $g_{2,k_{t+1},t} = b_{2,k_{t+1},t}$ and

$$g_{1,k_{t+1},t} = b_{1,k_{t+1},t} + \beta\left(1-\pi\right)\left[\left(b_{2,x_{t+1},t+1}\Psi_{k_{t+1}} + b_{2,k_{t+1},t+1}\right)\right]$$

B.2 Endogenous Probability Model

To simplify the problem, it is useful to multiply the second constraint by η_t, which does not change the solution. Set up the Lagrangian:

$$\min_{\{\lambda_t,\varphi_t\}_{t=0}^{\infty}} \max_{\{x_t\}_{t=0}^{\infty}} \mathscr{L} = \sum_{t=0}^{\infty} \beta^t \{\eta_t (u\left(x_t, k_t\right) + \beta\left(1 - P(k_{t+1})\right) W\left(k_{t+1}\right)) \tag{49}$$

$$+ \lambda_t \eta_t \left(g_1\left(x_t, k_t\right) + \beta P(k_{t+1}) g_2\left(x_{t+1}, k_{t+1}\right)\right)$$

$$+ \varphi_t (\eta_{t+1} - \eta_t P(k_{t+1}))\}$$

The FOCs are:

[18]The symbol $f_{x_t,i}$ denotes the partial derivative of the function $f(m_i)$ with respect to x_t. We suppressed the arguments of the functions for readability purposes.

$$\frac{\partial \mathscr{L}}{\partial z_t} : u_{z_t,t} + \lambda_t g_{1,z_t,t} + \lambda_{t-1} g_{2,z_t,t} = 0 \tag{50}$$

$$\frac{\partial \mathscr{L}}{\partial k_{t+1}} : u_{k_{t+1},t} + \beta \left(1 - P(k_{t+1})\right) W_{k_{t+1},t+1} + \lambda_t (g_{1,k_{t+1},t} + \beta P(k_{t+1}) g_{2,k_{t+1},t+1}) \tag{51}$$

$$+ \beta P(k_{t+1}) \left(u_{k_{t+1},t+1} + \lambda_{t+1} g_{1,k_{t+1},t+1}\right) - \lambda_{t-1} g_{2,k_{t+1},t}$$
$$- \beta P_{k_{t+1}}(k_{t+1}) W(k_{t+1}) - \varphi_t P_{k_{t+1}}(k_{t+1})$$
$$+ \lambda_t P_{k_{t+1}}(k_{t+1})(b_2(x_{t+1}, k_{t+1}) - b_2(\Psi(k_{t+1}), k_{t+1})) = 0$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_t} : g_1\left(x_t, k_t\right) + \beta P(k_{t+1}) g_2\left(x_{t+1}, k_{t+1}\right) = 0 \tag{52}$$

$$\frac{\partial \mathscr{L}}{\partial \varphi_t} : \eta_{t+1} - \eta_t P(k_{t+1}) = 0 \tag{53}$$

$$\frac{\partial \mathscr{L}}{\partial \eta_{t+1}} : \beta \left(u_{t+1} + \beta \left(1 - P(k_{t+2})\right) W\left(k_{t+2}\right)\right) + \varphi_t - \beta \varphi_{t+1} P(k_{t+2}) = 0 \tag{54}$$

$$\forall t = 0, ..., \infty \qquad \lambda_{-1} = 0$$

To obtain Eqs. (50, 51) one has to divide the original FOC by $\beta^t \eta_t$, and use Eq. (53).[19] One can solve Eq. (54) forward and obtain:

$$-\varphi_t = \beta \sum_{i=t+1}^{\infty} \beta^{i-1} \frac{\Pi_{j=0}^{i}(P(k_j))}{P(k_{t+1})} (u(x_i, k_i) + \beta(1 - P(k_{i+1}))W(k_{i+1})) \tag{55}$$

This equation states that $-\varphi_t$ is equal to β times the value function starting at period t+1. Note that it represents the value function when past promises are kept, because all the terms considered refer to commitment states. Simplifying notation:

$$-\varphi_t = \beta W(k_{t+1}, \lambda_t) \tag{56}$$

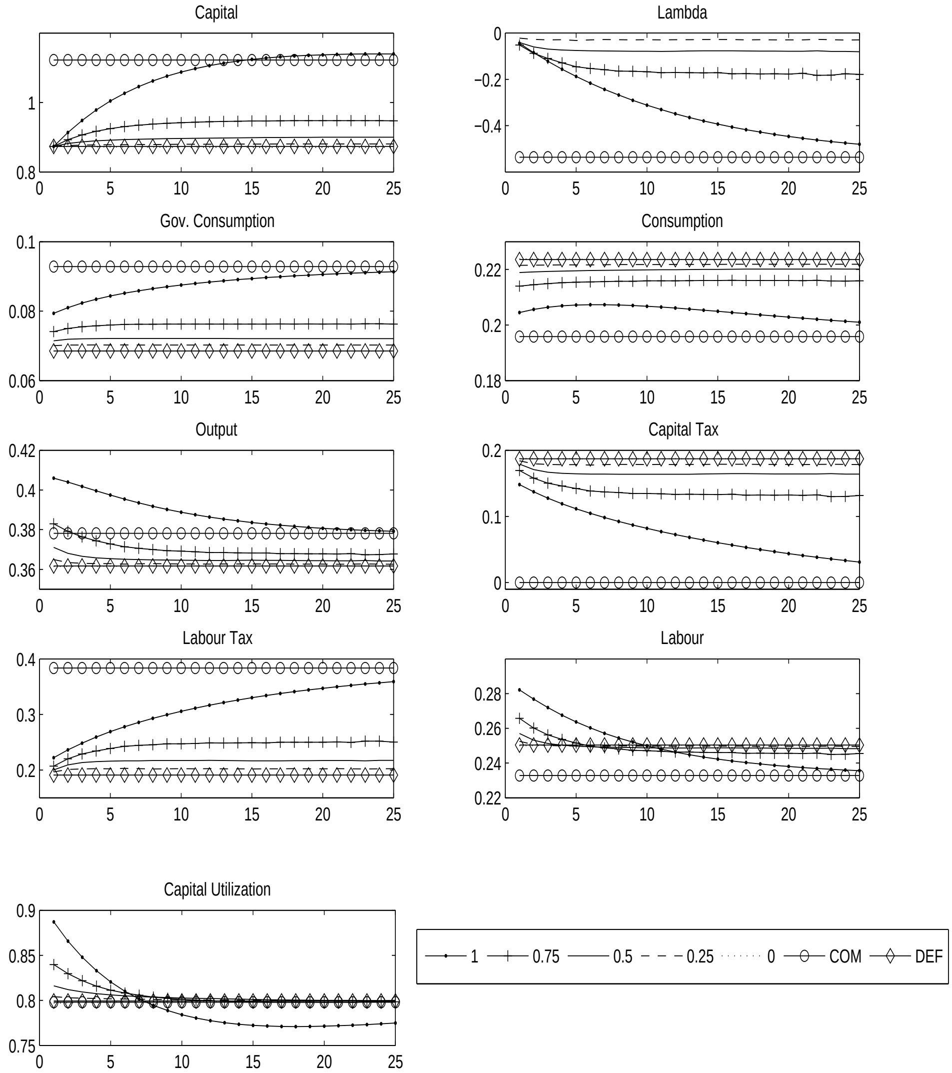
One can use the equation above to eliminate φ_t in Eq. (51).

For the numerical work all the simplifications above are convenient to reduce the number of equations in the system. Since η_t is eliminated from the problem, this variable is not a state variable necessary for the numerical approximation. Intuitively, at each point in time the planner that is in charge only needs to know the current promises summarized by λ_{t-1}, and the capital stock k_t. The probability of

[19]Note that in performing such operation the term η_{t-1} multiplies λ_{t-1}. If there is a default that term disappears. If there is no default it is still the case that $\eta_t = \eta_{t-1} P(k_t)$. Therefore, the original FOC and the changed FOC in Eq. (50, 51) are equivalent.

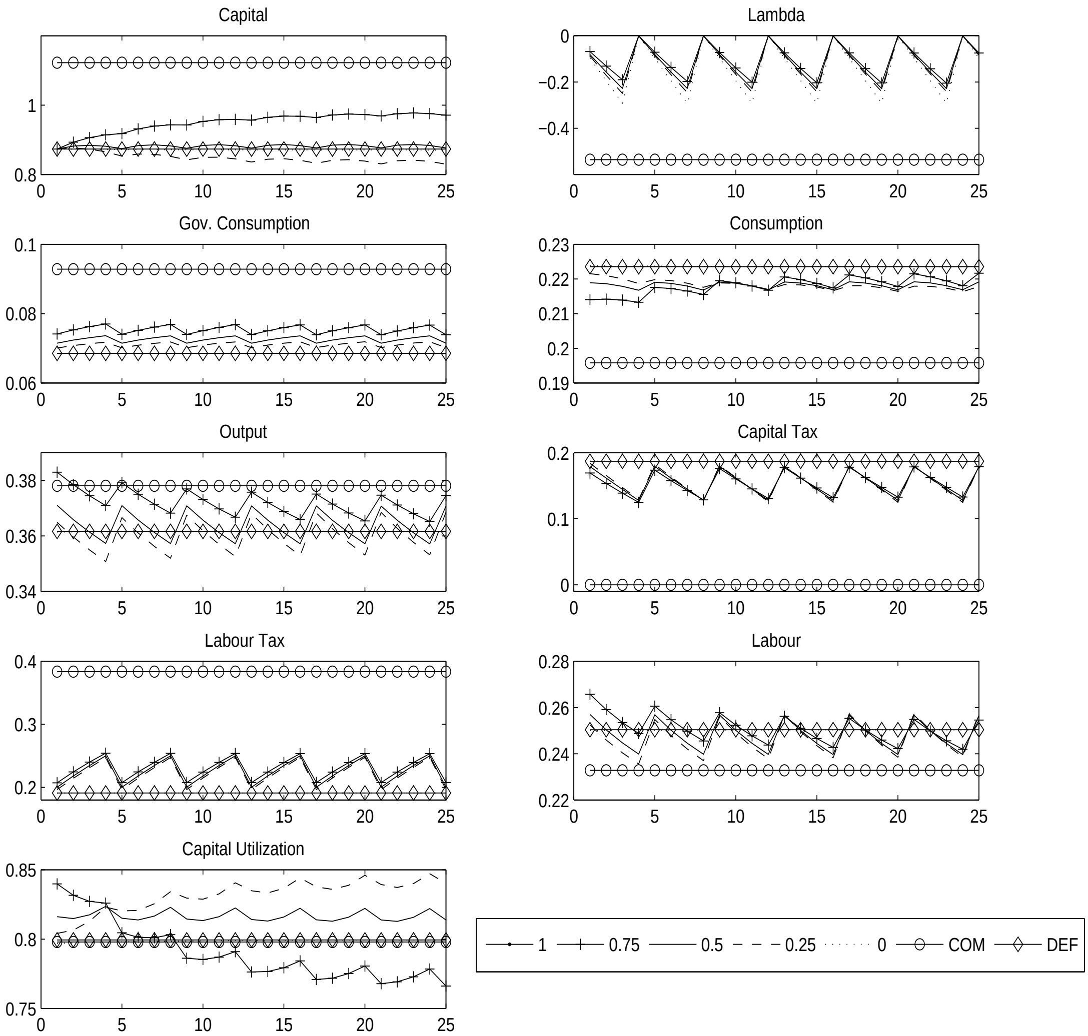
committing between $t-1$ and t is a bygone. The probability of committing between t and $t+1$, $P(k_{t+1})$, is not predetermined.

Figure 2: Average Allocations



Note: The figure plots for several values of π the average path across realizations. Capital is initialized at the discretion steady state. The lagrange multiplier is initialized at zero, considering that there were no previous promises in the first period.

Figure 3: Particular History: Default every 4 periods



Note: The figure plots for several values of π a particular history realization. In this history a default occurs every four periods. Capital is initialized at the discretion steady state. The lagrange multiplier is initialized at zero, considering that there were no previous promises in the first period.

Figure 4: Welfare Gains on π axis

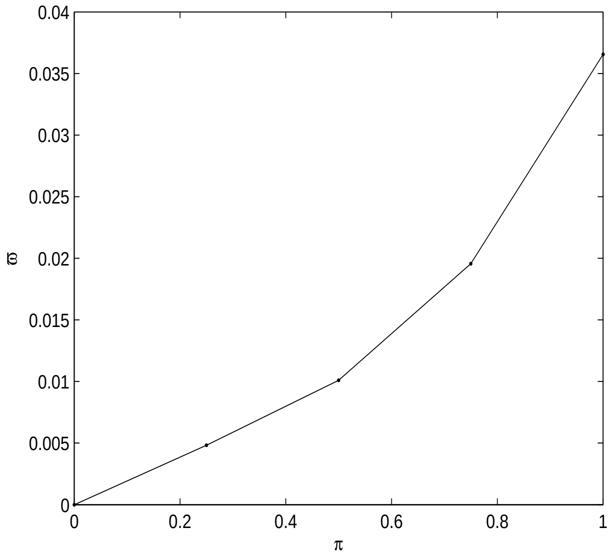


Figure 5: Welfare Gains on expected time axis