EVALUATION OF AIR HEATER PERFORMANCE AND THE ACCURACY OF THE RESULT

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With the increased emphasis on the efficiency of fossil-fuel-fired, steam generation facilities, the performance of ancillary equipment is becoming increasingly important. The air heater is a source of lost thermal efficiency in two ways -- air leakage into flue gas side and poor heat recovery. Moreover, air inleak makes it difficult to determine the exiting flue gas temperature and the performance of the air heater. This paper addresses the issue of properly evaluating the air heater performance and the accuracy of the final result. The appendix discusses the procedures used to determine the individual measurements and the uncertainty of these measurements.

Background

As part of the U. S. Department of Energy's (DOE) Clean Coal Technology IV Demonstration Program, New York State Electric & Gas Corporation (NYSEG) selected the Milliken Station for installation of innovative SO_2 and NO_x control technologies and efficiency improvements. These improvements will allow utilities to comply with the Clean Air Act Amendments of 1990. The air heaters on Unit 2 were replaced to improve unit efficiency as part of the demonstration program. The original air heater was a regenerative Ljungstrom unit; the replacement air heater was a low pressure drop, high efficiency heat pipe. CONSOL R&D evaluated the performance of the air heater and estimated the uncertainty in the evaluation.

The American Society of Mechanical Engineers (ASME) provides a standard method of computing the performance of air heaters. This is performance test code (PTC) ASME PTC 4.3.[1] This method was specified as the standard of acceptable performance by warrantees of the new air heaters. While the ASME code is often specified in equipment warrantees, it appears to be rarely applied. Instead, performance indicators such as the measured effectiveness of the air- and gas-sides and the X-ratio are compared directly against design values. Such comparisons are poor substitutes for the ASME PTC which corrects for part of the differences between test and design conditions independently of the vendor's design algorithms. The algorithms, normally provided by the vendor as performance curves and/or correlations that predict the outlet temperature based on inlet conditions, cannot be applied directly in the ASME code. The ASME code predicts the temperature corrected to the design value while the performance curves predict the expected temperature at operating conditions. This paper provides a method of applying the vendor's performance curves to evaluate the performance corrected to design as per ASTM PTC 4.3.

An air heater is shown schematically in Figure 1. Note that the ASME PTC 4.3 nomenclature is used in Figure 1 and throughout this paper. In the air heater, energy in the flue gas is recovered by the incoming combustion air. While normally several air streams are present (primary and secondary), in this paper we examine only one section of the air heater.

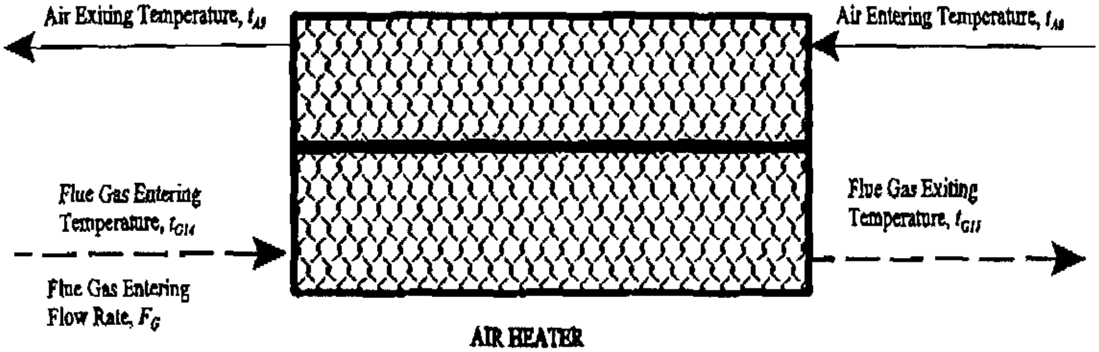

Figure 1 Air Heater Schematic

ASME PTC 4.3 calculates a "totally corrected flue gas outlet temperature" (TCFGOT), $t_{G15\,\delta\,Total}$, shown below (ASME Supplement[1] as Equation 7.12):

$$t_{G15\,\delta Total} = t_{G15\,\delta A} + t_{G15\,\delta G} + t_{G15\,\delta X} + t_{G15\,\delta\varepsilon} - 3 \cdot t_{G15} \quad (1)$$

where

$t_{G15\,\delta A}$ = Flue gas temperature leaving the air heater corrected for deviation from design entering air temperature, °F,

$t_{G15\,\delta G}$ = Flue gas temperature leaving the air heater corrected for deviation from design entering flue gas temperature, °F,

$t_{G15\,\delta XR}$ = Flue gas temperature leaving the air heater corrected for deviation from design X-ratio, °F,

$t_{G15\,\delta\varepsilon}$ = Flue gas temperature leaving the air heater corrected for deviation from design entering gas flow, °F, and

t_{G15} = Measured flue gas temperature leaving the air heater, °F.

The PTC provides equations for the first two of the temperature corrections, $t_{G15\,\delta A}$ and $t_{G15\,\delta G}$, but not for the other two, $t_{G15\,\delta XR}$ and $t_{G15\,\delta\varepsilon}$. These latter temperature corrections are unique to a specific air heater. If these temperature corrections are not provided by the equipment manufacturer as algorithms (or plots), they can be estimated by the procedure presented in this paper. The procedure uses design performance curves and/or algorithms normally provided by the vendor to evaluate the temperature corrections required by Equation 1. The TCFGOT is then compared to the design flue gas temperature. The computed value of the TCFGOT should be less than or equal to the design flue gas temperature, if the air heater is performing properly.

The TCFGOT

The two temperature correction factors provided by the PTC are $t_{G15\,\delta A}$ and $t_{G15\,\delta G}$. These are defined in terms of design values and of measured results of a standard test of an air heater. For the deviation from the design entering air temperature, $t_{G15\,\delta A}$, this is:

$$t_{G15\,\delta A} = \frac{t_{A8D} \cdot (t_{G14} - t_{G15}) + t_{G14} \cdot (t_{G15} - t_{A8})}{(t_{G14} - t_{A8})} \quad (2)$$

where

t_{A8D} = Design air temperature entering the air heater, °F,

t_{G14} = Measured flue gas temperature entering the air heater, °F, and

t_{A8} = Measured air temperature entering the air heater, °F.

Similarly, for the deviation from the design entering flue gas temperature, the temperature correction is:

$$t_{G15\delta G} = \frac{t_{G14D} \cdot (t_{G15} - t_{A8}) + t_{A8} \cdot (t_{G14} - t_{G15})}{(t_{G14} - t_{A8})} \tag{3}$$

where

t_{G14D} = Design flue gas temperature entering the air heater, °F.

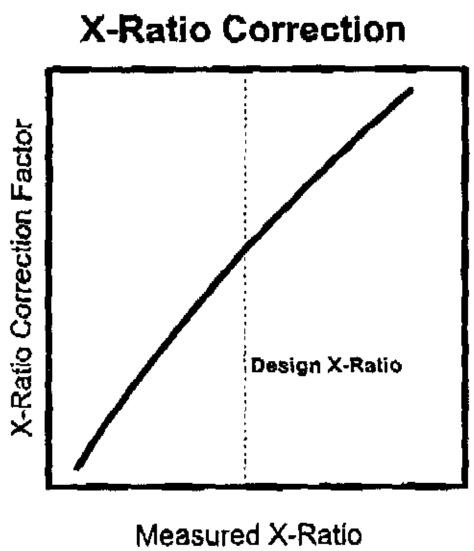

Figure 2 X-Ratio Correction

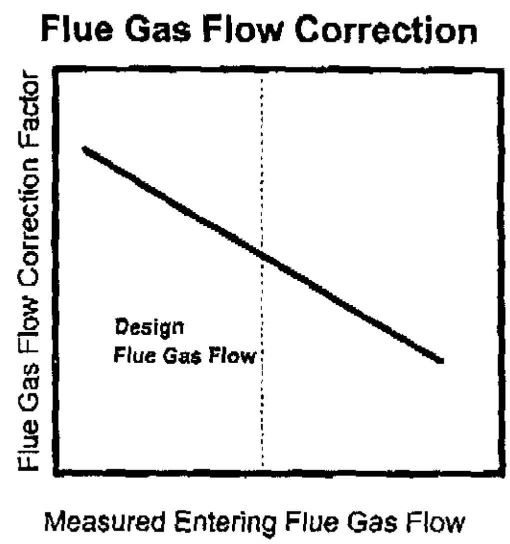

Figure 3 Flue Gas Flow Correction

The other two temperature corrections must be derived from vendor design performance curves or provided by the vendor in analytical form. In the case of the NYSEG heat pipe air heater, the vendor supplied a set of performance algorithms to be applied with performance figures similar to Figures 2 and 3, shown above. These predicted the performance temperature; that is, the expected exit flue gas temperatures for the actual operating conditions. The algorithm was of the form:

$$t_{G15} = t_{G14} \cdot \left[1 - \vartheta \cdot f_g \cdot f_X\right] + t_{A8} \cdot \vartheta \cdot f_g \cdot f_X \tag{4}$$

where

ϑ = Correlation coefficient, and

f_g, f_X = Correction factors for deviations from design flue gas flow and from design X-ratio, respectively.

For ease of analysis and of estimating the uncertainty, these plots were converted into mathematical expressions of the form:

$$f_g \approx \alpha_1 + \beta_1 \cdot F_G \tag{5}$$

for the flue gas flow, and for the X-ratio:

$$f_x \approx \alpha_2 + \beta_2 \cdot X + \delta_2 \cdot X^2 \tag{6}$$

where

$\alpha_1, \beta_1, \alpha_2, \beta_2, \delta_2$ = Correlation coefficients,

F_G = Flue gas flow rate, and

X = X-ratio for the air side.

The forms of these equations agree with the shapes of the curves in Figures 2 and 3. A least squares correlation or some other curve fitting technique can be used to evaluate the correlation constants. In the case of the Milliken study, the correlation equations agreed with results from the plots within the ability to read the plots.

Since the X-ratio is defined as the weight times heat capacity ratio of the air over that of the flue gas, the X-ratio can be approximated as the ratio of the temperature changes for the two fluids:

$$\begin{aligned} X &= \frac{w_{A9} \cdot c_{pA}}{w_{G14} \cdot c_{pG}} \\ &\approx \frac{\left(t_{G14} - t_{G15}^{NL}\right)}{\left(t_{A9} - t_{A8}\right)} \end{aligned} \tag{7}$$

where

c_{pA} = Heat capacity of air, Btu / lb-° F,

c_{pG} = Heat capacity of flue gas, Btu / lb-° F,

t_{G15}^{NL} = Average flue gas outlet temperature corrected to no-leak conditions, °F,

w_{A9} = Weight of air exiting the air heater, lb / h, and

w_{G14} = Weight of flue gas entering the air heater, lb / h.

The no-leak flue gas temperature, t_{G15}^{NL}, is calculated from the measured flue gas temperature by:

$$t_{G15}^{NL} = t_{G15} + \left[\frac{A_\lambda}{100}\right]\cdot\left[\frac{c_{pA}}{c_{pG}}\right]\cdot\left(t_{G15} - t_{amb}\right) \tag{8}$$

where

A_λ = Weight percent air leakage into the flue gas, and

t_{amb} = Temperature of the air leaking into the flue gas.

In most air heaters, the majority of the air in the flue gas is leakage from the higher pressure, air-side of the air heater. The ASME defines[1] the position of the air leak as occurring after the flue gas exits the air heater, but before t_{G15} is measured. Thus, there can be no correction to heat transfer within the air heater for air leakage. In these cases,

$$t_{amb} = t_{A8} \tag{9}$$

and t_{A8} can be substituted for t_{amb} in the following equations. However, this derivation will be general. Substituting Equation 8 into Equation 7 yields:

$$X = \frac{\left[t_{G14} - t_{G15} - \frac{A_\lambda}{100}\cdot\left[\frac{c_{pA}}{c_{pG}}\right]\cdot\left(t_{G15} - t_{amb}\right)\right]}{\left(t_{A9} - t_{A8}\right)} \tag{10}$$

Note the ASME definition for the X-ratio is based on zero leak. If the vendor bases his X-ratio correction curve on an X-ratio with a design leak, this plot should be corrected to zero leak before generating Equation 6.

For application of Equation 1, two additional, independent temperature corrections are required. These can be obtained from the vendor's air heater performance equation, Equation 4, and the associated plots -- Figures 2 and 3. Equations similar to Equation 4 can be used to estimate the effect of one parameter independent of the other parameters of the equation to obtain a temperature correction for that parameter alone. This is achieved by evaluating Equation 4 for the change in one parameter while holding the others constant. For the deviation from the design X-ratio, this procedure produces the following equation for the temperature correction, $t_{G15\,\delta XR}$:

$$t_{G15\,\delta XR} = t_{G15} + \left\| t_{G15D} - t_{G14D}\cdot\left[1 - \vartheta\cdot f_{gD}\cdot f_X\right] - t_{A8D}\cdot\vartheta\cdot f_{gD}\cdot f_X + \left[\frac{A_\lambda}{100}\right]\cdot\left[\frac{c_{pA}}{c_{pG}}\right]\cdot\left(t_{G15} - t_{amb}\right) \right\| \tag{11}$$

where

t_{G15D} = Design flue gas temperature leaving air heater, and

f_{gD} = Design flue gas flow correction factor.

For the deviation from design flow, the temperature correction, $t_{G15\,\delta g}$, is:

$$t_{G15\,\delta g} = t_{G15} + \left\| t_{G15D} - t_{G14D}\cdot\left[1 - \vartheta\cdot f_G\cdot f_{XD}\right] - t_{A8D}\cdot\vartheta\cdot f_g\cdot f_{XD} \right\| \tag{12}$$

where

f_{XD} = Design X - ratio correction factor.

Equations 11 and 12 apply the performance equations and/or curves provided by the vendor to evaluate the effect of the change in X-ratio and flue gas flow on the measured temperature. The changes from the design TCFGOT, the terms within the double lines (‖), are applied to the measured flue gas outlet temperature to provide the temperature corrections.

ASME PTC 4.3 specified the air heater temperature corrections at design leak. For the NYSEG unit, the design leak was zero. This is reflected in Equation 10 where the X-ratio is corrected to the design leak of zero before being applied to the calculation of the temperature correction. The leak correction term,

$$\left[\frac{A_\lambda}{100}\right]\cdot\left[\frac{c_{pA}}{c_{pG}}\right]\cdot\left(t_{G15}-t_{amb}\right) \tag{13}$$

is required since (1) the performance equation and factor plots were based on a zero leak design, and (2) ASME PTC 4.3 specifies comparing the TCFGOT at the design conditions, which in this case is zero leak. Therefore, the TCFGOT must be on the same basis as the design. The first four terms of Equation 1 "add" in three leak terms. The measured flue gas temperature leaving the air heater, t_{G15}, subtracts out three leak terms as this measured value contains leak. Thus, the inclusion of a leak correction term in Equation 11 evaluates the TCFGOT by Equation 1, $t_{G15\,Total}$, at zero leak, the design condition, as specified by the ASME PTC 4.3.

Substituting

$$A_\lambda^\circ = \left[\frac{A_\lambda}{100}\right]\cdot\left[\frac{c_{pA}}{c_{pG}}\right] \tag{14}$$

into Equation 11 and then expanding Equation 1 by substituting Equations 2, 3, 11, and 12, along with the air heater performance correlations (Equations 5 and 6), results in the following revised equation:

$$
\begin{aligned}
t_{G15\,\delta Total} = {} & \frac{t_{A8D}\cdot(t_{G14}-t_{G15})+t_{G14}\cdot(t_{G15}-t_{A8})}{(t_{G14}-t_{A8})}+\frac{t_{G14D}\cdot(t_{G15}-t_{A8})+t_{A8}\cdot(t_{G14}-t_{G15})}{(t_{G14}-t_{A8})}+ \\
& \Bigg\{t_{G15}+\Bigg\|t_{G15D}-t_{G14D}\cdot\Bigg[1-\vartheta\cdot f_{gD}\cdot\Bigg\langle\alpha_2+\beta_2\cdot\left[\frac{(t_{G14}-t_{G15})-\mathring{A}_\lambda\cdot(t_{G15}-t_{amb})}{(t_{A9}-t_{A8})}\right]+ \\
& \qquad \delta_2\cdot\left[\frac{(t_{G14}-t_{G15})-\mathring{A}_\lambda\cdot(t_{G15}-t_{amb})}{(t_{A9}-t_{A8})}\right]^2\Bigg\rangle\Bigg]- \\
& \qquad t_{A8D}\cdot\Bigg[\vartheta\cdot f_{gD}\cdot\Bigg\langle\alpha_2+\beta_2\cdot\left[\frac{(t_{G14}-t_{G15})-\mathring{A}_\lambda\cdot(t_{G15}-t_{amb})}{(t_{A9}-t_{A8})}\right]+ \\
& \qquad \delta_2\cdot\left[\frac{(t_{G14}-t_{G15})-\mathring{A}_\lambda\cdot(t_{G15}-t_{amb})}{(t_{A9}-t_{A8})}\right]^2\Bigg\rangle\Bigg]+\mathring{A}_\lambda\cdot(t_{G15}-t_{amb})\Bigg\|\Bigg\}+ \\
& \Big\{t_{G15}+\big\|t_{G15D}-t_{G14D}\cdot\left[1-\vartheta\cdot\langle\alpha_1+\beta_1\cdot F_{FG}\rangle\cdot f_{XD}\right]-t_{A8D}\cdot\vartheta\cdot\langle\alpha_1+\beta_1\cdot F_{FG}\rangle\cdot f_{XD}\big\|\Big\}-3\cdot t_{G15}
\end{aligned}
\tag{15}
$$

Inspection of this equation reveals that calculation of the TCFGOT requires only four measured and 2 determined values: inlet and outlet air temperatures, inlet and outlet flue gas temperatures, entering flue gas flow and the air leak. All of the other parameters are constants. The calculated value of the TCFGOT from an air heater performance test must be equal to or less than the design value for optimal air heater performance.

Uncertainty Analysis

The uncertainty in the calculation of the TCFGOT by Equation 15 was estimated in support of a study of air heater performance conducted at the Milliken Station of New York State Electric & Gas Corporation (NYSEG) in 1995 and 1996. Details of the air heater performance and of the uncertainty analysis can be found in the referenced reports.[2,3] The uncertainty in the result of a calculation can normally be estimated directly by partial differentiation of Equation 15 with respect to each parameter. To accurately evaluate the uncertainty with an explicit equation, the equation must not be significantly nonlinear. In the case of Equation 15, the air leak introduces a significant non-linearity which invalidates this approach. Thus, a mathematical approximation was required to evaluate the uncertainty in the TCFGOT.

Errors in measurements are of two types: bias errors and random errors. Biases are associated with the measuring equipment or procedure and cannot be minimized by repeat measurements. However, the TCFGOT temperature corrections consist of differences and ratios. This tends to compensate for bias errors. Random errors are reduced by repeat measurements. The following derivation assumes only one test and thus represents the maximum estimated error.

The bias and random errors are propagated separately using Taylor series expansions for highly nonlinear equations:

$$S_{error} = \left[\sum_{\substack{i=1 \\ j=1}}^{n} \frac{\Delta f}{\Delta x_i} \cdot \frac{\Delta f}{\Delta x_j} \cdot \sigma_{x_i} \cdot \sigma_{x_j} \right]^{\frac{1}{2}} \tag{16}$$

where

$\frac{\Delta f}{\Delta x_i}$ = Incremental change in the function f with respect to x_i,

$\frac{\Delta f}{\Delta x_j}$ = Incremental change in the function f with respect to x_j,

σ_{x_i} = Error in parameter i,

σ_{x_j} = Error in parameter j, and

f = Function shown above as Equation 15.

This numerical approach of estimating the uncertainty in the TCFGOT is similar to the one that Carl James[4] uses for estimating the uncertainty in the design of a cross-flow heat exchanger. Of interest is the fact that the uncertainty in the design of a heat exchanger estimated by James is much larger than the uncertainty in the estimate of the performance. For a numerical approach, Equation 16 must approximate the surface of the function as a linear segment parallel to the true functional relationship. With independent parameters, only the $i=j$ terms of Equation 16 are non-zero, simplifying the Taylor series expansion. However, if the terms are correlatable, that is, not independent, then the sum of the cross products in Equation 16 is not zero and these terms must be included in the estimate. This is discussed further in the appendix.

This expansion is used to evaluate the bias and random error contributions separately. The bias and random errors are summed separately to form the bias error statistic and the random error statistic, and then combined to estimate the total uncertainty by:

$$U = \left[B^2 + (t \cdot S)^2 \right]^{\frac{1}{2}} \tag{17}$$

where

U = Uncertainty interval,

B = Overall bias error statistic,

S = Overall random error statistic, and

t = Appropriate *Student's t* value. (For 95 % significance, $t \approx 2.0$ for a reasonable sample size.)

The parameter values used for the estimation of the uncertainty of the TCFGOT and the bias and random errors associated with them are shown below in Table I. The bias and random errors were estimated by separate error propagation calculations for standard, multipoint sampling arrays in the inlet and outlet ducts of the air heater. These multipoint samples were used to evaluate average temperatures and compositions in the ducts. The appendix presents a brief discussion of this with a more detailed discussion available in the project reports.[5] As discussed in the Appendix, examination of the derivation of these sample errors suggests that for standard, multipoint traverses of utility-scale equipment, the bias and random errors shown in Table II for these average temperatures and compositions are typical.

Table I

Value of Parameters and Their Associated Bias and Random Errors

Parameter	Unit	Value	Bias Error	Random Error
Air Temperature @ Inlet and Ambient	°F	100	1.00	0.15
Air Temperature @ Outlet	°F	644	6.44	0.74
Flue Gas Temperature @ Inlet	°F	680	6.81	0.81
Flue Gas Temperature @ Outlet	°F	285	2.85	0.35
Flue Gas Flow	1,000 lb/h	157	9.82	0.72
Air Leak	percent	6	0.05	0.77

These parameters are propagated using Equation 16. The bias and random errors are propagated separately and summed to form the B and S components of Equation 17. Equation 17 is then used to estimate the overall uncertainty interval.

The following example shows the evaluation of one of the incremental change terms required by Equation 16. To evaluate the bias error associated with the air temperature at the inlet:

1. The TCFGOT is calculated at the base temperature, 100 °F, plus three times the bias error.
2. Then the TCFGOT is calculated at 100 °F minus three times the bias error.
3. Designating these two values of the TCFGOT as f_α and f_β, respectively, the contribution to the bias error of the TCFGOT for the inlet air temperature is evaluated by the following:

$$\begin{aligned}\Theta_i &= \frac{f_\alpha - f_\beta}{6 \cdot \sigma_i} \\ &= \frac{312.31 - 313.17}{6 \cdot 1} \\ &= \pm 0.143\end{aligned} \tag{18}$$

All other parameters are held constant at the values shown in Table I during this calculation. Equation 15 is used to calculate the TCFGOT. Since the parameters f_α and f_β were evaluated at three times the bias error, σ_i, greater and three times the bias error lower than the actual value of the temperature of the inlet air, the total delta is six times σ_i. That is, the difference between f_α and f_β is divided by six times the bias error.

To be an accurate estimate of the error, the function equation, f_i, must be relatively linear over the range of the error. That is, if f° is the value of TCFGOT at an inlet air temperature of 100 °F, then if,

$$\begin{gathered} f_\alpha - f^\circ \approx f^\circ - f_\beta \\ (312.31 - 312.74) \approx (312.74 - 313.17) \\ 0.436 \approx 0.422 \end{gathered} \tag{19}$$

then the assumption of linearity and, in turn, the validity of the estimate is confirmed.

This calculation is repeated for the other parameters listed in Table I and the products summed as shown in Equation 15 to produce the resulting bias and random errors shown in Table II. This is the estimate of the uncertainty from Equation 17 in the determination of the "totally corrected flue gas outlet temperature," or TCFGOT, for an air heater. The estimate of the error in the determination of the totally corrected flue gas temperature is ±4.75 °F for the specific conditions shown in Table I. As a percentage, ~2%, this uncertainty can be applied to evaluation of other air heaters.

Table II

Estimate of the Uncertainty of the Totally Corrected Flue Gas Temperature

Parameter	Unit	Bias Error	Random Error	Uncertainty
TCFGOT	°F	4.57	0.66	±4.75

Conclusions

The ASME PTC 4.3 provides a standardized method for evaluating the performance of utility air heaters. It provides a mathematically correct means of evaluating the performance which aids in minimizing disputes between suppliers and purchasers when guarantee performance evaluations are conducted. In operating plants, it is generally impossible to establish design conditions to verify performance. To overcome this, the PTC specifies that the measured flue gas outlet temperature must be corrected for differences from design inlet air temperature, design inlet flue gas temperature, design X-ratio, and design flue gas rate. Once these corrections are determined, the "totally corrected flue gas outlet temperature" can be calculated and compared with the design outlet temperature. Calculation of the first two temperature corrections is explicitly defined by the ASME code. The determination of the temperature corrections for differences from design X-ratio and design flue gas flow are left to the supplier or purchaser to determine. Normally the manufacturer will supply the purchaser with design performance curves or equations, but not with those to calculate the temperature corrections specified by the PTC. This paper provides a method for evaluating the remaining two temperature corrections using the performance curves. Should the manufacturer also provide procedures for calculating the specified PTC temperature corrections, the results can be checked using the proposed procedure. This was done for the Milliken air heater performance testing with good agreement found between the two methods.

As part of the Milliken air heater test program, the uncertainty in the ASME PTC 4.3 equation for calculating the TCFGOT was determined. Because of the non-linearity of the final equation, numerical approximations were used to determine the differentials needed for the propagation procedure. For the example presented, the estimated uncertainty is 4.75 °F for the TCFGOT at a 95% confidence level. This shows that the uncertainty in the code procedure is relatively small, about 2% of the design outlet temperature as expressed in degrees Fahrenheit.

References

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APPENDIX

Estimation of Uncertainty in the Individual Parameters Required for the Evaluation of the ASME PTC 4.3 "Totally Corrected Flue Gas Outlet Temperature"

The uncertainty analyses discussed in this paper are for the American Society of Mechanical Engineering (ASME) procedures for testing the performance of air heaters and, specifically, for the equation to predict the "totally corrected flue gas outlet temperature" (TCFGOT). The estimates of bias errors and random errors for the individual parameters were derived for the equipment and methodology used in obtaining the data required for a test program. This test program focused on evaluating the performance of an air heater recently installed in the Milliken Station of New York State Electric & Gas. The methods followed in deriving the estimates of the uncertainty of the individual parameters are published by ASME.[1] Comprehensive discussion of all of the calculations is published elsewhere.[2]

Milliken Station Unit 2 is a 150 MW_e pulverized coal-fired boiler with twin, parallel air heaters. Each air heater heats both primary and secondary air for half of the unit in separate sections with the flue gas mixed before and after the air heater. The uncertainty analysis presented below contains the results for both the primary and secondary sides of the air heater. The design of the air heater was such that all of the air leakage occurred at sootblower ports. Air leaked from outside of the air heater into the flue gas heating the primary air. Leakage into the side heating the secondary air was insignificant and was ignored in the following evaluation.

Test Procedure

The general test procedure followed in the determination of the TCFGOT was the ASME Performance Test Code (PTC) PTC 4.1[3] and PTC 4.3[4]. Individual parameters required by the PTC 4.3 were measured following generally accepted methods, normally U. S. Environmental Protection Agency (EPA) methods. For gas velocity, EPA Method 2[5] was used along with EPA Method 1[6]. The gas composition was determined generally following the procedures of EPA Method 3[7]. Since the ASME procedure bases the flue gas and air flow rates on the coal feedrate and gas properties, rather than on the measured gas and air velocities, the derivation of the errors of the individual parameters is complex. However, using the coal feedrate, from calibrated feeders, and gas compositions as a base creates a common bond between the air and flue gas flows. This creates a consistent basis for the calculations.

Background

Error propagation is calculated by Taylor Series expansion of the resultant function. In general, if $r = f(x_1, x_2, \ldots, x_i, \ldots, x_n)$, then the error statistics, S_{error}, for either the bias error or the random error is calculated by

$$S_{error} = \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial f}{\partial x_i} \cdot \frac{\partial f}{\partial x_j} \cdot \sigma_{x_i} \cdot \sigma_{x_j} \right)^{\frac{1}{2}} \qquad \textbf{(A1)}$$

where

$\frac{\partial f}{\partial x_i}, \frac{\partial f}{\partial x_j}$ = Partial derivatives of f with respect to x_i (or x_j), and

$\sigma_{x_i}, \sigma_{x_j}$ = Error with respect to x_i (or x_j).

When the parameters are independent, only the $i=j$ terms are significant. For many of the parameters examined in this work, the parameters were not independent and all of the terms in Equation A1 were evaluated. Note that using a single thermocouple to measure all of the temperatures in the traverse of a plane in a duct creates a dependency between these measurements. The bias error associated with the thermocouple is the same for all points. Thus, it is dependent. To illustrate the calculation complexity for the estimate of the errors of the individual parameters, a step-by-step calculation of the estimate of the uncertainty for a weight average temperature of a gas is shown below. The average is based on a traverse of an inlet (or outlet) duct. For the details of the estimation of the uncertainties of other parameters, refer to the final Milliken project report.[8] Only the errors and uncertainty for these other evaluations are presented here.

Temperature Traverse Uncertainty Calculation

The weight average temperature of a gas flowing in a duct is based on a flow weighted average of the temperatures obtained from a standard traverse of the duct. That is,

$$T_{avg} = \frac{\sum_{i=1}^{n} A_i v_i \rho_i T_i}{\sum_{i=1}^{n} A_i v_i \rho_i} \tag{A2}$$

where

A_i = Cross sectional area for point i, ft^2,

i = Traverse point number,

T_i = Temperature measured at point i, $^\circ\text{R}$,

v_i = Velocity in area A_i determined at point i, fps, and

ρ_i = Fluid density in area A_i, lb/ft^3.

The fluid velocity is determined by a Pitot tube measurement. The gas is assumed to behave ideally and the velocity is constant over the entire cross-sectional area A_i. The velocity is calculated by:

$$v_i = 85.49 \cdot CP_i \cdot \left[\frac{\Delta P_i \cdot T_i}{P_{si} \cdot M_i}\right]^{\frac{1}{2}} \tag{A3}$$

where

CP_i = Pitot tube flow coefficient, dimensionless,

ΔP_i = Velocity head in area i, inches W. C.,

P_{si} = Static pressure in area i, inches Hg, absolute, and

M_i = Gas mole weight in area i, lb / lb - mol.

Similarly, the gas density is calculated:

$$\rho_i = \frac{0.04578 \cdot M_i \cdot P_{si}}{T_i} \tag{A4}$$

Substituting the formulas for v_i (Equation A3) and ρ_i (Equation A4) into Equation A2 and simplifying yields:

$$T_{avg} = \frac{\sum_{i=1}^{n} CP_i \cdot A_i \cdot \left(\Delta P_i \cdot M_i \cdot P_{si} \cdot T_i\right)^{\frac{1}{2}}}{\sum_{i=1}^{n} CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot M_i \cdot P_{si}}{T_i}\right)^{\frac{1}{2}}} \tag{A5}$$

Equation A5 is partially differentiated with respect to A_i, CP_i, ΔP_i, M_i, P_{si}, and T_i, and the resulting partial summed as indicated in Equation A1. Equation A5 produces six sets of partial differential equations. If the denominator of Equation A5 is set equal to *Sum1* and the numerator equal to *Sum2* to simplify the resulting equations, these partial differentials are:

$$\frac{\partial T_{avg}}{\partial A_i} = \frac{CP_i \cdot \left(\Delta P_i \cdot M_i \cdot P_{si} \cdot T_i\right)^{\frac{1}{2}} \cdot Sum1 - CP_i \cdot \left(\frac{\Delta P_i \cdot M_i \cdot P_{si}}{T_i}\right)^{\frac{1}{2}} \cdot Sum2}{Sum1^2} \tag{A6}$$

$$\frac{\partial T_{avg}}{\partial CP_i} = \frac{A_i \cdot \left(\Delta P_i \cdot M_i \cdot P_{si} \cdot T_i\right)^{\frac{1}{2}} \cdot Sum1 - A_i \cdot \left(\frac{\Delta P_i \cdot M_i \cdot P_{si}}{T_i}\right)^{\frac{1}{2}} \cdot Sum2}{Sum1^2} \tag{A7}$$

$$\frac{\partial T_{avg}}{\partial \Delta P_i} = \frac{CP_i \cdot A_i \cdot \left(\frac{M_i \cdot P_{si} \cdot T_i}{\Delta P_i}\right)^{\frac{1}{2}} \cdot Sum1 - CP_i \cdot A_i \cdot \left(\frac{M_i \cdot P_{si}}{\Delta P_i \cdot T_i}\right)^{\frac{1}{2}} \cdot Sum2}{2 \cdot Sum1^2} \tag{A8}$$

$$\frac{\partial T_{avg}}{\partial M_i} = \frac{CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot P_{si} \cdot T_i}{M_i}\right)^{\frac{1}{2}} \cdot Sum1 - CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot P_{si}}{M_i \cdot T_i}\right)^{\frac{1}{2}} \cdot Sum2}{2 \cdot Sum1^2} \qquad \textbf{(A9)}$$

$$\frac{\partial T_{avg}}{\partial P_{si}} = \frac{CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot M_i \cdot T_i}{P_{si}}\right)^{\frac{1}{2}} \cdot Sum1 - CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot M_i}{P_{si} \cdot T_i}\right)^{\frac{1}{2}} \cdot Sum2}{2 \cdot Sum1^2} \qquad \textbf{(A10)}$$

$$\frac{\partial T_{avg}}{\partial T_i} = \frac{CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot M_i \cdot P_{si}}{T_i}\right)^{\frac{1}{2}} \cdot Sum1 + CP_i \cdot A_i \cdot \left(\frac{\Delta P_i \cdot M_i \cdot P_{si}}{T_i^3}\right)^{\frac{1}{2}} \cdot Sum2}{2 \cdot Sum1^2} \qquad \textbf{(A11)}$$

These individual differentials are multiplied and summed as shown by Equation A1. The bias errors and random errors, σ_i, for this calculation are listed in Table A-I. Table A-I also lists the source of the bias and random errors for each of the parameters. As previously mentioned, many of the cross product terms must be included in the bias calculations since the same equipment was used to measure a parameter. The inclusion of cross products, $i \neq j$ terms, adds significantly to the number of terms that must be evaluated. If there were no cross product terms, a duct traverse of 12 sample points in Equation A5 would require 72 terms. With the cross products, this increases to 864 terms. In the case of the bias error, the cross product terms account for essentially all of the error in determining the average temperature. Since the bias errors are not reduced by taking multiple measurements, the bias errors account for most of the uncertainty in the final average temperature as shown in Table A-II. In the case of the secondary air inlet, which has only four traverse points, the bias error is 90% of the uncertainty in the determination of the average temperature.

Table A-II summarizes the uncertainty estimates for the Milliken air heater for the average air and gas temperatures. The bias error is responsible for the majority of the uncertainty even with only a four-point traverse. Repetitive measurements tend to reduce the random error.

Table A-III shows the errors for the other parameters required to evaluate the TCFGOT. The uncertainty is shown as a percent of the final calculated value. All uncertainty estimates are at the 95% confidence limit.

Table A-I

Summary of Bias Errors and Precision Indices for Uncertainty Calculations

Parameter	Bias Error	Random Error (One Stand. Dev.)	Comments / Basis
Dimension			
Width	0.5" (0.042")	0.5" (0.042")	Assumed
Length	0.5" (0.042")	0.5" (0.042")	Assumed
Temperature	1 % of °F Reading	½ % of °F Reading	Bias - Typical for Type K Thermocouples, Random - ASME PTC 1
Pressure			
Barometric	0.04" Hg	0.04" Hg	Calibration of Aneroid Barometer Scale
Static	0.05" WC	0.05" WC	Water Manometer Scale
Vel. Head, ΔP	2 % of Avg. Reading	0.00005" WC	Shortridge Air Data Multimeter, Model ADM-870 Bias - Instrument Design Spec. Random - ½ Design Spec.
Pitot Factor, CP	0.01	0.0	Calibration Accuracy
Coal Analysis			
Moisture	3.9 % rel.	$\frac{(0.20 + 0.012*Mf)}{(2 * 1.414)}$	Bias - Assumed Same as Ash Random - ASTM Repeatability
C	3.9 % rel.	$\frac{(0.64)}{(2 * 1.414)}$	Bias - Assumed Same as Ash Random - ASTM Repeatability
H	3.9 % rel.	$\frac{(0.16)}{(2 * 1.414)}$	Bias - Assumed Same as Ash Random - ASTM Repeatability
N	3.9 % rel.	$\frac{(0.11)}{(2 * 1.414)}$	Bias - Assumed Same as Ash Random - ASTM Repeatability
S	1.9 % rel.	$\frac{(0.06 + 0.035*S)}{(2 * 1.414)}$	Bias - From Washability Data Random - ASTM Repeatability
Ash	3.9 % rel.	$\frac{(0.07 + 0.02*Ash)}{(2 * 1.414)}$	Bias - From Washability Data Random - ASTM Repeatability

Table A-I

Summary of Bias Errors and Precision Indices for Uncertainty Calculations

Parameter	Bias Error	Random Error (One Stand. Dev.)	Comments / Basis
Coal Analysis (Cont.)			
C in Ash	25 % rel.	10 % rel.	Bias - Experience with Milliken Unit 2 Loss On Ignition Data Random - Assumed
Coal Rate	5 % rel.	0.25 % rel	Bias - Assumed Random - Typical, PTC 4.1
Gas Analysis			
O_2	0.05 % abs.	0.05 % abs.	Bias - Calibration Gas Spec Random - Low O_2 Instrument Design Spec.
CO	20 ppm	10 ppm	Same
CO_2	0.1 % abs.	3 % rel.	Orsat Meter Bias - Burette Scale Division Random - Experience, PTC 4.1
Air Moisture	10 % rel.	20 % rel.	Bias - Error of 1 °F in Reading Wet Bulb Temperature (WBT) Random - Error of 2 °F in Reading WBT
Molecular Wt.			
Flue Gas	0.05	0.07	Combined Uncertainty of Analysis for Ash and Flue Gas
Air	0.025	0.05	Combined Uncertainty for Humidity

Table A-II

Uncertainty Estimates for Average Duct Temperatures Based upon Multi-Point Traverses

Location	No. of Traverse Points	Bias Error, %°F	Random Error, %°F	Uncertainty, %°F
Primary Air Inlet	12	1.00	0.15	1.05
Primary Air Outlet	20	1.00	0.11	1.03
Secondary Air Inlet	4	1.00	0.25	1.11
Secondary Air Outlet	24	1.00	0.11	1.03
Flue Gas Inlet	20	1.00	0.12	1.03
Flue Gas Outlet	24	1.00	0.14	1.04

Table A-III

Uncertainty Estimates for Other Parameters Required to Evaluate TCFGOT

Location	Bias Error, %	Random Error, %	Uncertainty, %
Primary Air Flow, Inlet	3.31	2.93	6.72
Primary Air Flow, Outlet	2.61	2.08	4.92
Secondary Air Flow, Inlet/Outlet	5.02	0.68	5.20
Flue Gas Flow, Inlet	6.28	0.75	6.46
Flue Gas Flow, Outlet	6.25	0.46	6.32
Flow Split Between Air Heaters	0.43	0.49	0.94
Air Leak @ 6.87% Leak	0.85	12.60	25.21

Conclusions

Two conclusions can be reached after examining these results. The estimates of the uncertainties shown in Table A-II and A-III are valid for all air heaters, when a valid duct traverse can be performed. The uncertainty for a duct traverse with as few as 4 points is still dominated by the bias errors. Secondly, since the dominant errors in the raw data are expressed as percentages, the results shown in Tables A-II and A-III, and in the main body of this paper, are independent of the absolute values of the parameters. Thus, they apply to any air heater.

Appendix References

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