Data Uncertainty and the Role of Money as an Information Variable for Monetary Policy*,†

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Abstract

This paper shows that money can play an important role as an information variable when initial output data are measured with error and subject to revision. Using an estimated model of the euro area we find that current output estimates may be substantially improved by including money growth in the information set. The gain in precision, however, depends on the magnitude of the output measurement error relative to the money demand shock. We find noticable but small improvements in output estimates, if the uncertainty due to money demand shocks corresponds to the estimated variance obtained from the money demand equation. Money plays a quantitatively more important role with regard to output estimation if we allow for a contribution of monetary analysis in reducing uncertainty due to money demand shocks. In this case, money also helps to reduce uncertainty about output forecasts.

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Non-technical summary

In this study, we perform a quantitative assessment of the role of money as an indicator variable for monetary policy when aggregate data are initially measured with error and subject to revision. We start by analyzing the sequence of revisions to euro area-wide data and find that measures of real output have been subject to substantial revisions over a period of up to nine months, whereas measures of prices and money have generally been subject to relatively minor revisions that occur within a short period of the initial data release. Given this pattern of euro area data revisions, monetary aggregates have a potentially significant role in providing information about the current level of output.

To analyze the macroeconomic implications of uncertainty with regard to current output, we then utilize the euro area model developed by Coenen and Wieland (2000), augmented by the estimated money demand equation of Coenen and Vega (1999), together with a calibrated specification for the output revision process. In particular, the model incorporates rational expectations and exhibits nominal inertia due to overlapping wage contracts. Furthermore, the short-term nominal interest rate is assumed to be the instrument of monetary policy. The quantity of money is determined recursively by money demand, as a function of the chosen nominal interest rate and the output and price level consistent with this interest rate. Since the model does not assign a causal role to the money stock in influencing output or inflation (other than through nominal interest rates), this approach may be viewed as a means of providing a reasonable *lower bound* on the information content of money.

Money has a potentially useful role as an indicator variable in our model, because money demand depends on the *true* level of output whereas the monetary policymaker and private agents only receive a noisy measure of output. We use the Kalman filter to determine the optimal weight on money (as well as the other relevant information variables in the model) in estimating the true level of output. We then proceed to compute the reduction in output uncertainty that is achieved by including money in the information set.

We find that money may play an important role as an information variable and may result in major improvements in current output estimates. However, this depends on the magnitude of the output measurement errors relative to the unobserved component of the money demand shocks. If the policymaker observes the money stock but has no contemporaneous information about money demand shocks, then monetary aggregates provide relatively little information about aggregate demand. In contrast, if the policymaker conducts monetary analysis that provides contemporaneous information about money demand shocks, then the money stock provides substantial information about current output and also improves the accuracy of short-term output forecasts.

1 Introduction

Many macroeconomic time series are subject to substantial revisions, and hence such data only provide imperfect information about the true state of the economy at a given point in time. In light of these data limitations monetary policymakers and researchers alike have long been interested in identifying indicator variables that provide precise and timely information. At least since the early 1970s research on the information content of alternative indicators has highlighted the potential usefulness of monetary aggregates. Some examples of this line of research are Kareken et al. (1973), Friedman (1975, 1990), Tinsley et al. (1980) and Angeloni et al. (1994). These evaluations have typically been conducted in reduced-form models and models with adaptive expectations.

More recently, research on Taylor-style interest rate rules has re-emphasized the importance of "real-time" data uncertainty for the design of monetary policy albeit without considering money's potential role as an information variable. In particular, a number of studies with U.S. data have found that uncertainty arising from revisions of output gap and inflation measurements may lead to a significant deterioration in the performance of such interest rate rules.[1] This problem may be even more important in the euro area, for which aggregate time series have only been developed fairly recently and have been subject to ongoing refinement.

In this study, we perform a quantitative assessment of the role of money as an indicator variable for monetary policy in the euro area. Thus, we investigate the same idea as the earlier literature on the information content of money in a forward-looking model of the economy. However, as the more recent literature on interest rate rules and real-time data uncertainty we aim to obtain the best possible estimates of those variables entering the policy rule and we model the process of real-time measurements empirically to match data revisions. In analyzing the sequence of revisions to euro area-wide data, we find that

[1] See for example the evaluations of interest rate rules under data uncertainty by Orphanides (1998), Orphanides et al. (2000) and Rudebusch (2000). For a large-scale analysis of the differences between alternative vintages of U.S. macroeconomic data the reader is referred to Croushore and Stark (1999).

measures of real output have been subject to substantial revisions over a period of up to nine months, whereas measures of prices and money have generally been subject to relatively minor revisions that occur within a short period of the initial data release. Given this pattern of euro area data revisions, monetary aggregates have a potentially significant role in providing information about the current level of aggregate demand.

To analyze the macroeconomic implications of data uncertainty, we utilize the euro area model developed by Coenen and Wieland (2000), augmented by the estimated money demand equation of Coenen and Vega (1999), together with a calibrated specification for the output revision process. In particular, the model incorporates rational expectations and exhibits nominal inertia due to overlapping wage contracts. Furthermore, the short-term nominal interest rate is assumed to be the instrument of monetary policy. The quantity of money is determined recursively by money demand, as a function of the chosen nominal interest rate and the output and price level consistent with this interest rate. Thus, implicitly we assume that any effect of a change in the nominal money stock on real output and inflation comes through the associated change in the nominal interest rate. In other words, direct effects of money on output and inflation are absent from the model. This assumption is typical of the current generation of macroeconomic models[2] and is consistent with optimizing behavior if certain restrictions[3] are satisfied. Since the model does not assign a causal role to the money stock in influencing output or inflation (other than through nominal interest rates), this approach may be viewed as a means of providing a reasonable *lower bound* on the information content of money.

Money has a potentially useful role as an indicator variable in our model, because

[2]This includes most of the smaller-scale models currently used for research on monetary policy (see for example Rotemberg and Woodford (1997), Fuhrer (1997) or Orphanides and Wieland (1998)) as well as large-scale policy models such as the Federal Reserve Board's FRB/US model (see Brayton and Tinsley (1996)), the ECB's Area-Wide Model (see Fagan, Henry and Mestre (2001)), or the multi-country model of Taylor (1993a). An alternative approach, which allows for direct effects of money on inflation, would be the P^* model of Hallman et al. (1991) estimated more recently for Germany by Tödter and Reimers (1994) and for the euro area by Gerlach and Svensson (2000).

[3]These restrictions are discussed in more detail in Ireland (2001), McCallum (2000) and Leahy (2000). They include, for example, the separability of the utility function in consumption, money and leisure or the absence of transaction costs of purchases.

money demand depends on the *true* level of output whereas the central bank and private agents only receive a noisy measure of output. We use the Kalman filter to determine the optimal weight on money (as well as the other relevant information variables in the model) in estimating the true state of the economy (cf. Pearlman et al. (1986), Svensson and Woodford (2000)).[4] We then proceed to compute the reduction in output uncertainty that is achieved by including money in the information set.

We find that money may play an important role as an information variable and may result in major improvements in current output estimates. However, this depends on the magnitude of the output measurement errors relative to the unobserved component of the money demand shocks. If the policymaker observes the money stock but has no contemporaneous information about money demand shocks, then monetary aggregates provide relatively little information about aggregate demand. In contrast, if the central bank conducts monetary analysis that provides contemporaneous information about money demand shocks, then the money stock provides substantial information about current output and also improves the accuracy of short-term output forecasts.[5]

Finally, it should be emphasized that our analysis focuses solely on uncertainty regarding actual output, and does not address the problem of estimating potential output. While uncertainty about potential output has important consequences for the determination of monetary policy, we neglect this issue here because the money stock is related to actual output and thus cannot serve as a direct source of information regarding potential output.[6]

The remainder of this paper is organized as follows. Section 2 characterizes the timing and magnitude of revisions to euro area data on aggregate output, prices, and money.

[4] Our paper is related to recent work by Dotsey and Hornstein (2000) on the usefuleness of money for discretionary policy. However, they focus on impulse responses in a calibrated model of the U.S. economy with simple measurement error, while we derive measures of the information content of money in an empirically estimated model of the euro area with a more general empirical specification of measurement error. Further, there are important differences in the information structure, which we discuss later on.

[5] Details about the ECB's approach to monetary analysis may be found in the May 2001 issue of the ECB Monthly Bulletin, and in Masuch et al. (2001). For further discussion related to the United States, see Orphanides and Porter (2001).

[6] For the implications of uncertainty about output gaps and potential output (or unemployment gaps and the NAIRU) for monetary policy we refer the reader to Ehrmann and Smets (2000), Orphanides (2000) and Wieland (1998) among others.

Section 3 outlines the behavioral equations of the model, and indicates alternative representations of the output revision process. Section 4 describes our methodology for determining the optimal filtering weights and for evaluating the information content of indicator variables. Section 5 illustrates the information role of money for the case of a highly stylized money demand equation, while Section 6 uses the complete model described above to evaluate the quantitative significance of money as an indicator variable. Section 7 summarizes our conclusions and suggests several directions for future research. Finally, the Appendix reports further details of our methodology as well as additional sensitivity analysis regarding our results.

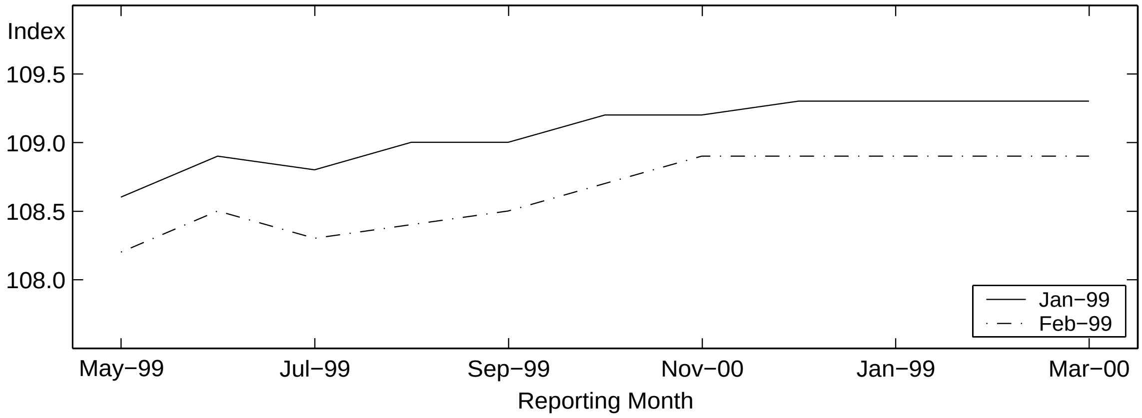
2 Data Uncertainty in the Euro Area

Some macroeconomic data series, such as nominal interest rates, exchange rates, and raw materials prices, are readily available and not subject to revision. In contrast, indicators of aggregate quantities and prices are more difficult to construct, and are frequently subject to substantial revisions as additional information becomes available to the statistical agency.[7] For the euro area, aggregate data has only become available fairly recently (with the harmonization of statistical procedures across the individual member countries), and hence the record of initial releases and revisions is necessarily limited. Nevertheless, it is useful to characterize the properties of these revisions in order to shed some light on the degree of data uncertainty in the euro area.

Thus, we proceed to analyze the timing and magnitude of revisions to euro area output, price, and money data, beginning with the advent of European Monetary Union in 1999. As measures of real output, we consider monthly data on industrial production (excluding construction) as well as quarterly data on real GDP. To measure aggregate prices, we consider monthly data on the Harmonized Index of Consumer Prices (HICP) and quarterly data on the GDP price deflator. Finally, we consider monthly data for M3; we focus on this

[7]Further revisions occur on a less frequent basis as the result of definitional changes, such as switching to a different benchmark year for the national income accounts. Such revisions often shift the entire level of a data series, but may have relatively minor implications for the determination of monetary policy.

Figure 1: Revisions to Industrial Production



Source: ECB Monthly Bulletin.

measure of money because Coenen and Vega (1999) found that the demand function for M3 has been remarkably stable. In each case, we utilize real-time data series over the period October 1998 through December 2000, as published in consecutive issues of the European Central Bank's Monthly Bulletin over the period January 1999 through February 2001.[8]

The nature of the revision process is best understood with an example. **Figure 1** shows monthly revisions of industrial production at the start of monetary union. Estimates of euro area industrial production in January and February 1999, for instance, were first published in the May 1999 issue of the ECB Monthly Bulletin. The estimates of the index reported in May were 108.6 and 108.2 for January and February industrial output, respectively. Over the following months the statistical authorities revised these estimates upwards. Revisions only ceased by the end of the year. The magnitude of the revisions over this period was 0.7 in both cases. Clearly, these revisions suggest a significant degree of data uncertainty, which persisted for some time.

[8]The ECB's monthly bulletin is a convenient source for obtaining consistent real-time data. Furthermore, each bulletin represents a reasonably accurate summary of the data available to the ECB Governing Council at its first meeting each month: the cut-off date for inclusion in the bulletin predates each meeting, and the bulletin itself is published a week later. However, in future work it would be interesting to analyze the timing of revisions as published by the statistical agency that actually compiles each data series.

Table 1: Monthly Euro Area Data Revisions (in Percent)

Month after initial publication	1	2	3	4	5	6	7	8	9
Industrial Production[a]									
largest upward revision	0.93	0.81	0.54	0.54	0.46	0.27	0.27	0.27	0.18
largest downward revision	-0.60	-0.46	-0.55	-0.27	-0.26	-0.27	-0.27	-0.27	-0.36
mean absolute revision	0.34	0.28	0.24	0.16	0.16	0.11	0.12	0.13	0.13
Consumer Prices[b]									
largest upward revision	0	0	0	0	0	0	0	0	0
largest downward revision	-0.10	-0.10	0	0	0	0	0	0	0
mean absolute revision	0.03	0.02	0	0	0	0	0	0	0
M3[c]									
largest upward revision	0.37	0.20	0.14	0.13	0.14	0.06	0.02	0.10	0.03
largest downward revision	-0.18	-0.13	-0.12	-0.03	-0.03	-0.10	-0.11	-0.04	-0.02
mean absolute revision	0.16	0.08	0.06	0.03	0.05	0.04	0.03	0.03	0.02

Source: ECB Monthly Bulletin, January 1999 to February 2001.

Note: [a] Index of Industrial Production (excluding construction), seasonally adjusted. [b] Harmonized Index of Consumer Prices, Dec. 1998 = 100, not seasonally adjusted. [c] M3 Index, Dec. 1998 = 100, seasonally adjusted; calculated from monthly differences in levels adjusted for reclassifications, other revaluations, exchange rate variations etc.

Table 1 provides summary statistics regarding the revision process for monthly euro area data. The first column is associated with the first revision (one month after the initial publication), the next column reflects the second revision (i.e., the difference between the values published one month and two months following the first publication), and so on until the revision in the tenth month following initial publication. For each series, the first row indicates the largest upward revision at each interval (as a percent of the value published in the previous month), while the second row indicates the largest downward revision, and the third row indicates the mean absolute revision.

Evidently, the industrial production data are subject to substantial and frequent revisions over the first year after the initial publication. For example, the first monthly revision of this series has a mean absolute value of 0.34 percent, with a maximum upward revision of 0.93 percent and a maximum downward revision of 0.6 percent. While the magnitude of revisions gradually declines as time passes, revisions exceeding 0.1 percent are not unusual during each of the next few months after the initial publication.

In contrast, the consumer price data are typically not revised at all; the only exceptions are apparently due to corrections of reporting errors. Clearly, the lack of revisions does not imply that these data provide an exact measure of aggregate inflation. However, measurement biases in the consumer price index have mainly been identified with longer-term factors (such as improving product quality, introduction of new goods and services, and changes in expenditure shares), and hence these biases may not be crucial in evaluating higher-frequency fluctuations in the inflation rate. In any case, as we will see below, the GDP price deflator (which is less susceptible to measurement bias than the HICP) also exhibits relatively small revisions.

Finally, the magnitude of initial data revisions is substantially smaller for M3 than for industrial output: the maximum upward and downward revisions in the first month are less than half as large (in percentage terms). Furthermore, subsequent revisions in M3 are relatively small and infrequent, so that the mean absolute revision never exceeds 0.1 percent from the second month onwards.

Table 2 reports summary statistics regarding the revision process for real GDP and the GDP price deflator, which are available on a quarterly basis. These statistics indicate that real GDP is subject to fairly large revisions.[9] For example, in the first revision (one quarter after the initial publication), the maximum upward revision exceeds a full percentage point, and the mean absolute revision is about 0.8 percent of the previously published value. Even three quarters after the initial publication, the mean absolute revision of real GDP is about 0.5 percent. In contrast, revisions of the GDP deflator are much smaller: the mean

[9] Some of these revisions have occurred as individual member countries have moved to the ESA95 harmonization of national income accounts and are likely to become smaller as the implementation process is completed in most countries.

Table 2: Quarterly Euro Area Data Revisions (in Percent)

Quarter after initial publication	1	2	3	4	5
Real GDP$^{(a)}$					
largest upward revision	1.49	1.21	1.14	0.20	0.19
largest downward revision	-0.91	-0.95	0	-0.02	-0.08
mean absolute revision	0.80	0.69	0.47	0.11	0.14
GDP Price Deflator$^{(b)}$					
largest upward revision	0.28	0.38	0.09	0.09	0.09
largest downward revision	-0.10	-0.09	0	-0.09	0
mean absolute revision	0.11	0.14	0.03	0.06	0.05

Source: ECB Monthly Bulletin, January 1999 to February 2001.
Note: $^{(a)}$ Seasonally adjusted. $^{(b)}$ Seasonally adjusted.

absolute revision is only about 0.1 percent in each of the first two quarters after the initial publication, and subsequent revisions are negligible in magnitude. Evidently, revisions to nominal GDP for the euro area are primarily due to revisions regarding real output rather than prices.

Thus, the monthly and quarterly data yield remarkably similar conclusions regarding real-time data uncertainty in the euro area. Industrial production and real GDP are each subject to relatively large revisions during the first several quarters after the initial publication, indicating that data uncertainty regarding the current level of real output is a non-trivial issue for the euro area.[10] By comparison, both measures of aggregate prices (the HICP and the GDP price deflator) and the broad money stock (M3) are subject to rela-

[10] Revisions of the growth rate of real output tend to be smaller. As discussed in the ECB's monthly bulletin of August 2001 (see pages 26-28) the average size of revisions of quarter-on-quarter growth since the first quarter of 1999 was 0.2 percentage points. Throughout this paper, however, we continue to focus on the level rather than the growth rate, because in our view, the level of output relative to the economy's potential is more relevant for determining the appropriate stance of monetary policy and its effect on inflation than the difference between actual and potential growth rates.

tively small revisions during the first quarter after the initial publication, and to negligible revisions in subsequent quarters.

These results raise the possibility that money can serve as a useful indicator in providing real-time information about fluctuations in real output. An additional advantage of money as an indicator variable is that money data typically becomes available earlier than output data.[11] In the following analysis we will primarily focus on the information gain from money in the presence of measurement error, but we will return to the gains arising from the earlier availability of money in the sensitivity analysis at the end of the paper.

3 A Rational-Expectations Model with Data Uncertainty

To quantify the information content of money, we utilize the euro area macroeconomic model of Coenen and Wieland (2000), augmented by the money demand equation estimated by Coenen and Vega (1999). Since these equations are specified at a quarterly frequency, it seems reasonable to assume (in light of the results of the previous section) that observations on aggregate output are subject to measurement error, while aggregate prices, money, and nominal interest rates are observed without measurement error.[12] It also seems reasonable to assume that the money demand of each individual household or firm depends on its own income and expenditures (which are known to that household or firm), while neither private agents nor the central bank observe the true level of aggregate output.[13] Under these assumptions, aggregate money demand will be related to the true level of aggregate income, and hence observations on the money stock can provide useful information about movements in aggregate output.

[11] This holds even for monthly data. For example, in June one learns about money growth and inflation in May but about industrial production in April.

[12] In the model considered here, measurement errors of the money stock would have the same effect as money demand shocks in reducing the information content of money as an indicator of aggregate output. Thus, one could always capture the effect of money measurement error by considering a slightly higher variance of the money demand shocks.

[13] Thus, the information structure differs from Dotsey and Hornstein (2000) who assume instead that private agents know the true level of aggregate output. To us it seems more reasonable to assume that private agents face similar uncertainty regarding aggregate data as the central bank.

3.1 The Behavioral Equations

The behavioral equations of the model are indicated in **Table 3**.[14] As shown in equation (1), the aggregate price level p_t is determined as a weighted average of overlapping nominal wage contracts signed over the past year. The estimated weighting scheme implies that a weight of 0.32 is placed on the current wage contract w_t, while smaller weights are placed on earlier contracts (e.g., w_{t-3} receives a weight of about 0.18).

The determination of nominal wage contracts is given in equation (2). As in Taylor (1980, 1993a), each wage contract is determined by expectations about aggregate prices and perceived output gaps over the duration of the contract.[15] The expectations operator $\mathrm{E}_t[\,.\,]$ indicates the optimal projection of each variable, conditional on all information available at period t. As noted above, this information set includes the true values of aggregate wages, prices, and interest rates, and noisy observations regarding aggregate output. Since our analysis is focused on the implications of data uncertainty regarding actual output q_t, we assume for simplicity that potential output q_t^* is exogenously determined and known by all private agents and by the central bank.[16] Finally, under these assumptions, it should be noted that the aggregate supply disturbance u_t^w is known to all agents (including the central bank); this implication follows from our assumptions that all agents know the current contract wage and utilize identical information in forming expectations about the variables on the right-hand side of equation (2).

As shown in equation (3), the current output gap depends on the true output gap in each of the previous two quarters and on the *ex ante* long-term real interest rate, r_t^l (which

[14]Wages, prices, output, and money are expressed in logarithmic form, and interest rates are expressed at annualized rates.

[15]Coenen and Wieland (2000) also considered relative real wage contract specifications of the type originally proposed by Buiter and Jewitt (1981) and later studied by Fuhrer and Moore (1995). We have performed sensitivity analysis and found that the results reported here concerning the information role of money are not substantially affected by using a relative real wage contract specification instead of equation (2).

[16]In fact, equations (2) and (3) were estimated by Coenen and Wieland (2000) using "final" data on euro area real GDP (that is, data available at the end of 1999), and constructing the output gap by removing a log-linear time trend. In future work, it would be interesting to reestimate these equations using real-time output gap data. However, as the authors note, constructing a real-time output gap data series for the period preceding EMU would be a "courageous undertaking".

Table 3: A Rational-Expectations Model with Data Uncertainty

Price Level	$p_t = f_0\, w_t + f_1\, w_{t-1} + f_2\, w_{t-2} + f_3\, w_{t-3},$ where $f_i = 0.25 + (1.5 - i)\, 0.0456$	(1)
Contract Wage	$w_t = \mathrm{E}_t \left[\sum_{i=0}^{3} f_i\, p_{t+i} + 0.0115 \sum_{i=0}^{3} f_i\, y_{t+i} \right] + u_t^w,$ where $y_t = q_t - q_t^*$ and $u_t^w \sim$ i.i.d.$(0, 0.0038^2)$	(2)
Aggregate Demand	$y_t = 1.2029\, y_{t-1} - 0.2225\, y_{t-2} - 0.0942\, r_t^l + u_t^d,$ where $u_t^d \sim$ i.i.d.$(0, 0.0058^2)$	(3)
Money Demand	$\Delta(m-p)_t = 0.075\, \Delta^2 q_t + 0.097\, (\Delta i_t^s + \Delta i_{t-1}^s)$ $- 0.359\, \Delta i_{t-1}^l - 1.052\, (\Delta\pi_t + \Delta\pi_{t-1})$ $- 0.136\, [(m-p) - 1.140\, q$ $+ 0.820\, (i^l - i^s) + 5.848\, \pi]_{t-2}$ $+ u_t^m,$ where $\pi_t = p_t - p_{t-1}$ and $u_t^m \sim$ i.i.d.$(0, 0.0023^2)$	(4)
Monetary Policy Rule	$i_t^s = \tilde{\pi}_t + 0.5\, (\tilde{\pi}_t - \pi^*) + 0.5\, \mathrm{E}_t\, [y_t],$ where $\tilde{\pi}_t = p_t - p_{t-4}$	(5)
Term Structure	$i_t^l = \mathrm{E}_t \left[\frac{1}{8} \sum_{j=0}^{7} i_{t+j}^s \right]$	(6)
Real Interest Rate	$r_t^l = i_t^l - \mathrm{E}_t \left[\frac{1}{2} (p_{t+8} - p_t) \right]$	(7)

Note: p: aggregate price level; w: nominal contract wage; u^w: contract wage shock; y: output gap; q: output; q^*: potential output; r^l: long-term real interest rate; u^d: aggregate demand shock; i^s short-term nominal interest rate; π^*: inflation target; i^l: long-term nominal interest rate; m nominal money balances; u^m: money demand shock.

is defined by equation (7)). This specification seems reasonable under the assumption that each household or firm makes spending decisions based on its own directly observed income path, and hence aggregate spending depends on the true path of aggregate income.[17] Nevertheless, since the true level of aggregate output is not directly observed, the aggregate demand shock u_t^d is not in the information set of private agents or the central bank.

As indicated by equation (4), real money balances $(m-p)_t$ are determined by an error correction process involving aggregate output q_t, the short-term nominal interest rate i_t^s, the long-term nominal interest rate i_t^l (defined by equation (6)), and the one-quarter inflation rate $\pi_t = p_t - p_{t-1}$.[18] As noted above, this money demand specification indicates that the money stock responds to the true level of aggregate output q_t, and hence money has a potentially useful role as an indicator variable. Of course, this role depends on the stability of the money demand. While estimates of money demand with U.S. data have tended to be unstable, results with German and euro area data have typically been more encouraging.[19]

Two additional characteristics of equation (4) are crucial in determining the specific information content of money. First, the short-run link between money and output is relatively weak, with an instantaneous income elasticity of only 0.075; evidently, the money stock would provide somewhat more information about current output if the contemporaneous relationship between these two variables were significantly larger. Second, the money demand shock u_t^m has an estimated standard error of 0.23 percent, reflecting the extent to which money balances move in response to factors other than aggregate output, inflation, and nominal interest rates. In the absence of any additional information, private agents and the central bank will have substantial difficulty in determining whether a given movement in the money stock reflects a shift in aggregate output or a shift in money demand. Thus,

[17] To the extent that individual spending decisions actually reflect agents' perceptions about the aggregate economy, then the output gap equation would need to be augmented by terms such as $\mathrm{E}_t[y_{t-1}]$ and $\mathrm{E}_t[y_{t-2}]$. We have performed some preliminary analysis of such specifications, but leave further investigation to future research. In this context, a model with more explicit microeconomic foundations that distinguishes between macro- and micro-level uncertainty will be helpful.

[18] Because the inflation rate π_t is not annualized, the corresponding coefficients in equation (4) appear unusually large.

[19] For a recent study regarding U.S. money demand and money's usefulness for U.S. monetary policy see Dotsey et al. (2000).

as we will see below, monetary analysis may enable the central bank to identify some of the special factors and shocks that affect money demand, and thereby enhance the information content of money.

Finally, as indicated by equation (5), we assume that the monetary authorities follow the simple interest rate rule proposed by Taylor (1993b), where $\tilde{\pi}_t = p_t - p_{t-4}$ indicates the annual average inflation rate and π^* the inflation target.[20] According to this equation, the *ex post* short-term real interest rate moves in response to deviations of inflation from target and to perceived movements in the output gap (i.e., $\mathrm{E}_t[y_t]$). Of course, the central bank would prefer to avoid making policy adjustments in response to persistent mismeasurements of aggregate output, and hence uses all available information in estimating the current value of output.

3.2 The Revision Process

In the state-space literature, a typical assumption is that each data point of a given time series is observed just once (possibly subject to some measurement error). In contrast, here we wish to represent a sequence of revisions to the real output data that gradually refines the quality of each individual data point.[21]

A general representation of the revision process can be expressed as follows:

$$q_t^{(t+j)} = q_t + v_t^{(t+j)}, \tag{8}$$

where $q_t^{(t)}$ is the initial observation of output at time t; $q_t^{(t+j)}$ is the jth revision of this observation (often referred to as the time $t+j$ "vintage" of the data); and $v_t^{(t+j)}$ represents the deviation from the true level of output, q_t. By abstracting from deviations between

[20]We make this assumption, because Taylor's rule roughly captures the systematic component of monetary policy in a number of European countries in recent years (see for example Clarida, Galí and Gertler (1998) and Gerlach and Schnabel (2000)). An alternative approach, would be to assume that the central bank implements an optimal monetary policy rule in our model.

[21]Since our analysis is focused on the behavior of private agents and the central bank, we do not explicitly model how the statistical agency determines these revisions, using new information on disaggregated variables, etc.. Sargent (1989) follows a different approach, and analyses a model in which the statistical agency uses optimal filtering to revise its data on aggregate economic variables, and hence private agents and the central bank can utilize the statistical agency's data without any further refinement.

the "final" revised data and true aggregate output, and ignoring occasional redefinitional changes in the entire time series, we may assume that the sequence of revisions for each datapoint eventually converges to the true level of output; that is, $v_t^{(t+j)} \to 0$ as $j \to \infty$.

In the simplest case, the entire revision process occurs within a single quarter (that is, $v_t^{(t+j)} = 0$ for all $j > 0$):

$$\begin{array}{rcl} q_t^{(t)} & = & q_t + v_t \\ q_t^{(t+1)} & = & q_t, \end{array} \tag{9}$$

where v_t is serially uncorrelated with mean zero and standard deviation σ_v, and is uncorrelated with the structural disturbances u_t^d, u_t^w, and u_t^m. Under this assumption about the revision process, agents learn the true value of output one period after the initial data release.

As we have seen in Section 2, however, the data on real output are subject to a sequence of substantial revisions for several quarters. Therefore, we also consider the following representation of the revision process:

$$\begin{array}{rcl} q_t^{(t)} & = & q_t + v_t^3 + v_t^2 + v_t^1 \\ q_t^{(t+1)} & = & q_t + v_t^3 + v_t^2 \\ q_t^{(t+2)} & = & q_t + v_t^3 \\ q_t^{(t+3)} & = & q_t, \end{array} \tag{10}$$

where $v_t = [\, v_t^1 \; v_t^2 \; v_t^3 \,]'$ is a vector of serially uncorrelated measurement errors with mean zero and positive semi-definite covariance matrix Σ_{vv}. According to this representation of the revision process, the period t data vintage includes error-prone observations on q_t, q_{t-1}, and q_{t-2}, as well as the true value of q_{t-3}.[22]

We calibrate the covariance matrix of v_t using the data on revisions from Section 2.[23] The estimated standard deviations are 0.97 percent, 0.77 percent, and 0.47 percent for v_t^1, v_t^2, and v_t^3, respectively. The sample correlation between v_t^1 and v_t^2 is negligible, while the sample correlations with v_t^3 are -0.638 for v_t^1 and -0.636 for v_t^2. Of course, given the short

[22]Further details regarding this representation of the revision process are provided in Appendix C.

[23]In constructing the sample covariance matrix, we only used data for which revisions were available for at least three consecutive quarters.

history of data revisions, the sample covariance matrix is not estimated very accurately, and hence in the subsequent analysis we will also consider the case in which the elements of v_t are mutually uncorrelated.

4 Evaluating the Role of Indicator Variables

4.1 The Optimal Filtering Problem

We obtain optimal estimates of output by applying the Kalman filter to our linear rational expectations model of the euro area. Given our assumption that private agents and the central bank have the same information concerning aggregate variables, we can follow the approach of Svensson and Woodford (2000), henceforth referred to as SW2000.[24]

In particular, the model can be expressed in the following form:

$$\begin{bmatrix} X_{t+1} \\ \tilde{E}\, x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + \begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix}, \tag{11}$$

where X_t is a vector of predetermined variables, x_t is a vector of non- predetermined variables and u_t is a vector of serially uncorrelated shocks with mean zero and positive semi-definite covariance matrix Σ_{uu}. The coefficient matrices A^1, A^2 and $\tilde{E}$ are matrices of appropriate dimension. For some or many of the variables, policymakers and market participants can only observe noisy measurements. The vector of observables Z_t is then given by

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t, \tag{12}$$

where v_t is a vector of serially uncorrelated measurement errors with mean zero and positive semi-definite covariance matrix Σ_{vv}. The measurement errors v_t are assumed to be uncorrelated with the shocks u_t at all leads and lags, i.e. $\mathrm{E}[u_t\, v_\tau'] = 0$ for all t and τ. The matrices D^1 and D^2 are selector matrices of appropriate dimension. Here we use $\chi_{\tau|t} = \mathrm{E}[\,\chi_\tau | I_t\,]$ to denote the rational expectation (that is, the optimal projection) of any variable χ in period

[24] A more detailed discussion of the Kalman filter and the weights given to indicator variables such as money is provided in Appendix A.

τ given information in period t. The information set in period t corresponds to

$$I_t = \left\{ Z_\tau, \tau \leq t; A^1, A^2, D^1, D^2, \tilde{E}, \Sigma_{uu}, \Sigma_{vv} \right\}.$$

SW2000 show that the non-predetermined variables fulfill the relationship

$$x_t = G^1 X_t + G^2 X_{t|t} \tag{13}$$

and that the system of equations (11), (12) can be cast into state-space form without non-predetermined variables,

$$X_{t+1} = H X_t + J X_{t|t} + u_{t+1} \tag{14}$$

$$Z_t = L X_t + M X_{t|t} + v_t, \tag{15}$$

where the matrices G^1, G^2, H, J, L and M are derived in SW2000. This transformation of course simplifies the remaining problem of forming the estimate $X_{t|t}$ considerably.[25]

Accounting for the contemporaneous effect of the estimate $X_{t|t}$ on Z_t, SW2000 show that the optimal estimate of X_t can be obtained by means of a Kalman filter updating equation. This updating equation is expressed in terms of the innovations in the transformed variables $\bar{Z}_t = Z_t - M X_{t|t}$:

$$X_{t|t} = X_{t|t-1} + K (\bar{Z}_t - \bar{Z}_{t|t-1}) \tag{16}$$

$$= X_{t|t-1} + K \left[L (X_t - X_{t|t-1}) + v_t \right]. \tag{17}$$

The steady-state Kalman gain matrix K is given by

$$K = PL' (LPL' + \Sigma_{vv})^{-1}, \tag{18}$$

where the matrix P is the steady-state covariance matrix of the innovations $X_t - X_{t|t-1}$ given information in period $t-1$ and satisfies the relation

$$P = H \left[P - PL' (LPL' + \Sigma_{vv})^{-1} LP \right] H' + \Sigma_{uu}. \tag{19}$$

[25] Having eliminated the non- predetermined variables x_t, the estimation of the predetermined variables X_t still requires solving a simultaneity problem. Simultaneity arises because the observable variables Z_t depend on the estimate of the predetermined variables $X_{t|t}$, which in turn depend on the observables used in the estimation.

We are particularly interested in the weights on the observed indicator vector Z_t under optimal filtering. While the Kalman filter estimate $X_{t|t}$ is obtained in terms of the weighted innovations in the transformed variables $\bar{Z}_t$, we can recover the optimal weights on the observations of Z_t by substituting $\bar{Z}_t = Z_t - M\, X_{t|t}$ and $\bar{Z}_{t|t-1} = Z_{t|t-1} - M\, X_{t|t-1} = L\, X_{t|t-1}$ into (16),

$$X_{t|t} \quad = \quad (I + KM)^{-1}(I - KL)\, X_{t|t-1} + (I + KM)^{-1} K\, Z_t.$$

Here we can see that the contemporaneous effect of the estimate $X_{t|t}$ on Z_t merely shows up in the premultiplication of the matrix $(I + KM)^{-1}$. When comparing the weights assigned to different information variables in the subsequent analysis we will refer to the elements of this modified Kalman gain matrix.

4.2 Measures of Information Content

We evaluate the information content of indicator variables according to the extent that they will reduce the uncertainty surrounding the estimation of structural shocks and/or prediction of key endogenous variables. One measure of *within-period estimation uncertainty* is the covariance matrix of the projection errors of the vector X_t, given the information set I_t available at period t (that is, information obtained from current and lagged values of the observed vector Z_t). As shown in Appendix B.1, this covariance matrix can be expressed as follows:

$$\mathrm{Cov}[X_t - X_{t|t}|I_t] \quad = \quad P - PL' \, (LPL' + \Sigma_{vv})^{-1} LP. \tag{20}$$

For example, one element of X_t is the unobserved aggregate demand shock, u_t^d, and the root mean-squared error (RMSE) of estimating this shock is given by the square root of the corresponding diagonal element of $\mathrm{Cov}[X_t - X_{t|t}|I_t]$. In the subsequent analysis the RMSE serves as our baseline measure of the estimation uncertainty surrounding the optimal estimate $X_{t|t}$ produced by the application of the Kalman filter. To evaluate *prediction* *uncertainty*, we will also present results regarding RMSE of multi-period-ahead predictions, for which the derivations are given in Appendix B.2.

In addition to the RMSE, it is useful to consider measuring estimation uncertainty using the concept of entropy (or "expected uncertainty") taken from the information theory literature. In doing so we follow Tinsley et al. (1980) who employ entropy as a formal measure of the information content of indicator variables. To explain the basic concept and its relationship with the coefficient of determination in linear regression models, R^2, we restate the relevant general results from Tinsley et al..[26]

Consider two vectors χ and ξ with joint density $f(\chi, \xi)$. The joint entropy of χ and ξ is given by

$$H(\chi, \xi) \quad = \quad -\,\mathrm{E}[\,\ln(f(\chi, \xi))\,].$$

The entropy or 'expected uncertainty' of χ corresponds to

$$H(\chi) \quad = \quad -\,\mathrm{E}[\,\ln(f(\chi))\,],$$

where $f(\chi)$ is the marginal density of χ, and the entropy of χ given ξ corresponds to

$$H(\chi|\xi) \quad = \quad -\,\mathrm{E}[\,\ln(f(\chi|\xi))\,]$$

with $f(\chi|\xi) = f(\chi, \xi)/f(\xi)$ denoting the conditional density of χ given ξ. Since $H(\chi)$ corresponds to the prior uncertainty associated with χ and the observation ξ may provide additional information with $f(\chi|\xi)$ describing what is known about χ after having observed ξ, $H(\chi|\xi)$ reflects the posterior uncertainty about χ given ξ. The expected information of the observation ξ with respect to χ is then defined as the difference between the prior uncertainty about χ, $H(\chi)$, and the posterior uncertainty of χ given ξ, $H(\chi|\xi)$,

$$I(\chi|\xi) \quad = \quad H(\chi) - H(\chi|\xi).$$

Using this measure of information content one can derive the expected relative information gain associated with adding a particular indicator variable ζ to the information vector

[26] For early uses of the concept of entropy in the economics literature see also Theil (1967).

ξ as follows:

$$G(\chi, \xi, \zeta) \;=\; [\, I(\chi|\xi, \zeta) - I(\chi|\xi)\,]/I(\chi|\xi).$$

Regarding χ and ξ as jointly distributed normal with covariance matrix Σ, Tinsley et al. show that $I(\chi|\xi)$ has a particularly simple form. In this case, using the properties of multivariate normal distributions,

$$I(\chi|\xi) \;=\; 0.5 \,\ln(\, |\Sigma_{\chi\chi}|/|\Sigma_{\chi\chi} - \Sigma_{\chi\xi}\,\Sigma_{\xi\xi}^{-1}\Sigma_{\xi\chi}|\,),$$

where $\Sigma_{\chi\chi}$, $\Sigma_{\xi\xi}$ and $\Sigma_{\chi\xi} = \Sigma'_{\xi\chi}$ are the submatrices of Σ with appropriate dimensions and $\Sigma_{\chi\chi} - \Sigma_{\chi\xi}\,\Sigma_{\xi\xi}^{-1}\Sigma_{\xi\chi}$ is the conditional covariance matrix of χ given ξ. Thus, under normality, the measure of information content, $I(\chi|\xi)$, corresponds to the log-distance between the determinants of the covariance matrices of the marginal and the conditional distribution of χ.

The case of univariate χ can then be used to develop an intuitive interpretation of the expected information content $I(\chi|\xi)$. In this case,

$$\begin{aligned} I(\chi|\xi) &= 0.5 \,\ln(\, \sigma_\chi^2/(\sigma_\chi^2 - \Sigma_{\chi\xi}\,\Sigma_{\xi\xi}^{-1}\Sigma_{\xi\chi})\,) \\ &= 0.5 \,\ln(\, 1/(1 - \beta'\Sigma_{\xi\xi}\beta/\sigma_\chi^2)\,) \\ &= 0.5 \,\ln(\, 1/(1 - R^2_{\chi|\xi})\,), \end{aligned}$$

where β is the vector of regression coefficients, and $R^2_{\chi|\xi}$ is the population coefficient of determination in the linear regression of χ on ξ. Evidently,

$$R^2_{\chi|\xi} \;=\; 1 - (\, \exp\{2I(\chi|\xi)\}\,)^{-1}\,.$$

To adapt these measures to our euro area model with rational expectations and data uncertainty, we need to obtain the joint distribution of the innovations in the observed indicator variables and the innovations in the predetermined variables. This is done in Appendix B.1.

5 Illustrating the Information Role of Money

We have now assembled the tools necessary to assess the contribution of money in the estimation of noisy output data and the underlying shocks. To illustrate the information role of money, we start with the case in which output revisions occur within a single period (as specified in equation (9)). Furthermore, we utilize the following highly stylized money demand function in place of the more complicated dynamic specification of Coenen and Vega (1999) that was presented in section 3:

$$m_t - p_t = q_t + u_t^m, \tag{21}$$

where the exogenous disturbance u_t^m is assumed to be serially uncorrelated with mean zero and standard deviation $\sigma(u^m)$.

As discussed in Section 3, we assume that money demand evolves in response to the true level of output, and hence can serve as a useful indicator variable. In fact, under our assumption that aggregate prices are known by all agents, money would be a perfect indicator of true output in the limiting case with no money demand shocks ($\sigma(u^m) = 0$). On the other hand, of course, money would provide no useful additional information if output were observed without measurement error ($\sigma(v) = 0$).

More generally, the role of money as an indicator variable will depend on the relative magnitude of money demand shocks compared with output measurement errors. As previously noted, contract wage shocks are known by all agents in our model,[27] so that the key information problem is to determine whether a given movement in output is due to an aggregate demand shock or to measurement error. Thus, in this section, we will evaluate the information content of money based on its contribution in estimating the current aggregate demand shock, u_t^d.

[27] This implication follows from our assumptions that all agents know the current contract wage and utilize identical information in forming expectations about the variables on the right-hand side of equation (2).

5.1 The Economy without Money

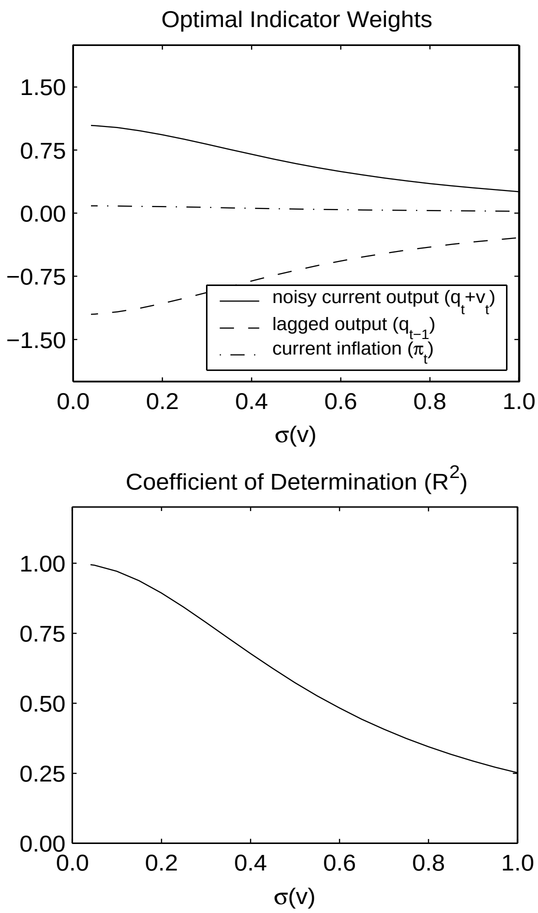
As a benchmark for comparison, we begin with the special case in which money is not in the information set, or equivalently, the variance of money demand shocks is arbitrarily large. **Figure 2** summarizes the characteristics of the information problem for a range of values of the standard deviation of the output measurement error, $\sigma(v)$. The upper-left panel shows the optimal filter weights on the noisy observation of current output ($q_t + v_t$), as well as the true values of inflation (π_t) and lagged output (q_{t-1}). The upper-right panel indicates the root mean-squared error (RMSE) of the estimate $u^d_{t|t}$ of the current aggregate demand shock, and the lower panel indicates the R^2 of a regression of u^d_t on the vector of observed variables.

Evidently, when output is measured without error ($\sigma(v) = 0$), the aggregate demand shock can be determined exactly as a function of the observed variables; that is, the RMSE equals zero and the R^2 equals unity. As the standard deviation of the measurement errors increases, the optimal filter places lower weight on the noisy observation of current output (similarly on lagged output and inflation), the RMSE rises while the R^2 falls. Finally, as noted in Section 3.2, the first-quarter revisions of euro area output have a sample standard deviation of 0.97 percent. As $\sigma(v)$ approaches this value, we see that the RMSE rises to about 0.5 percent (still a bit lower than the unconditional standard deviation of 0.58 percent for the aggregate demand shock), while the R^2 falls below 25 percent.

Figure 3 shows the behavior of this economy in response to a single aggregate demand shock of one standard deviation (which occurs at time 0). The solid line indicates the response of each variable under the baseline calibration ($\sigma(v) = 0.97$ percent), while the dot-dashed line indicates the corresponding path when output is not subject to measurement errors ($\sigma(v) = 0$). Finally, in the upper-left panel, the dotted line indicates the path of perceived output in response to the shock.

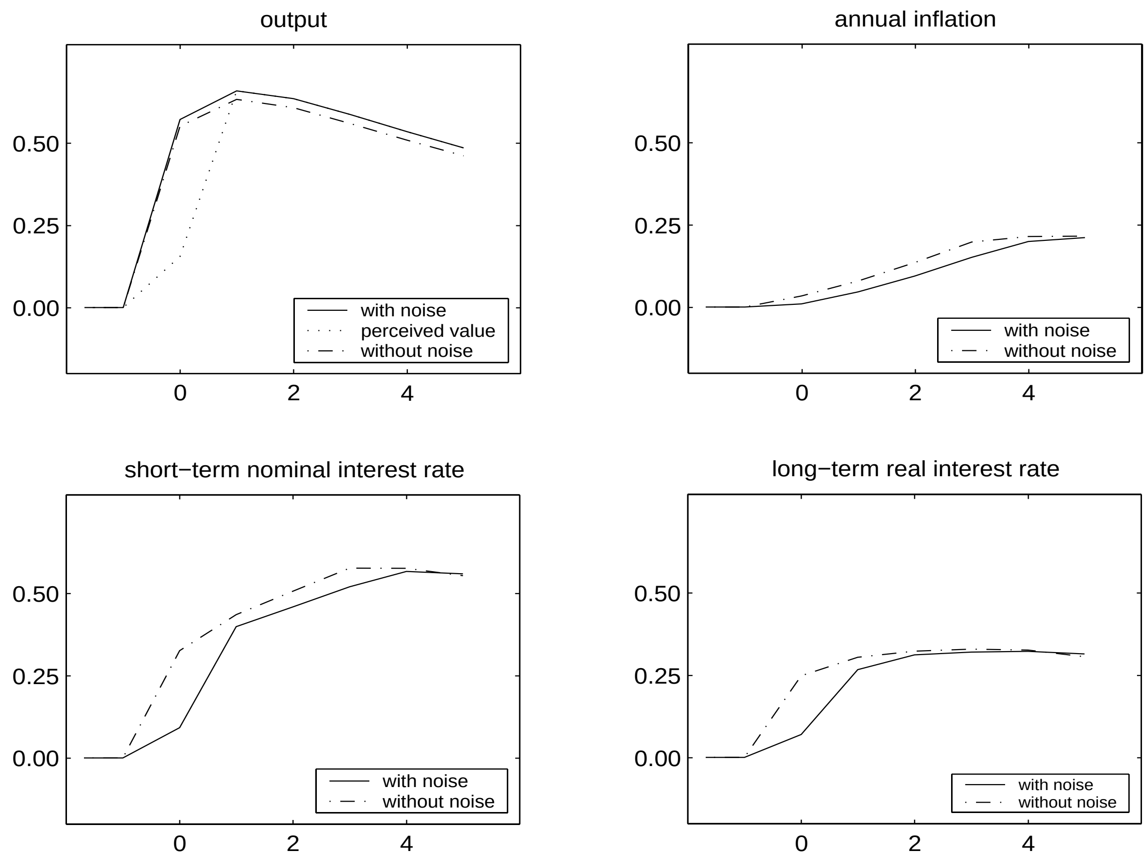
Of course, the aggregate demand shock immediately raises the level of real output. In this case, the reported level of output happens to be exactly correct, but the optimal filter

Figure 2: Optimal Filtering of the Aggregate Demand Shock without Money



causes private agents and the central bank to downweight this observation. Thus, contract wages and aggregate inflation respond slightly less than if output were not subject to measurement errors. More importantly, the central bank does not raise short-term nominal interest rates as quickly, and hence actual output rises more sharply and takes somewhat longer to return to potential. Based on these impulse responses, it is evident why indicator variables such as money can serve a useful role when output observations are noisy.

Figure 3: Impulse Responses to an Aggregate Demand Shock

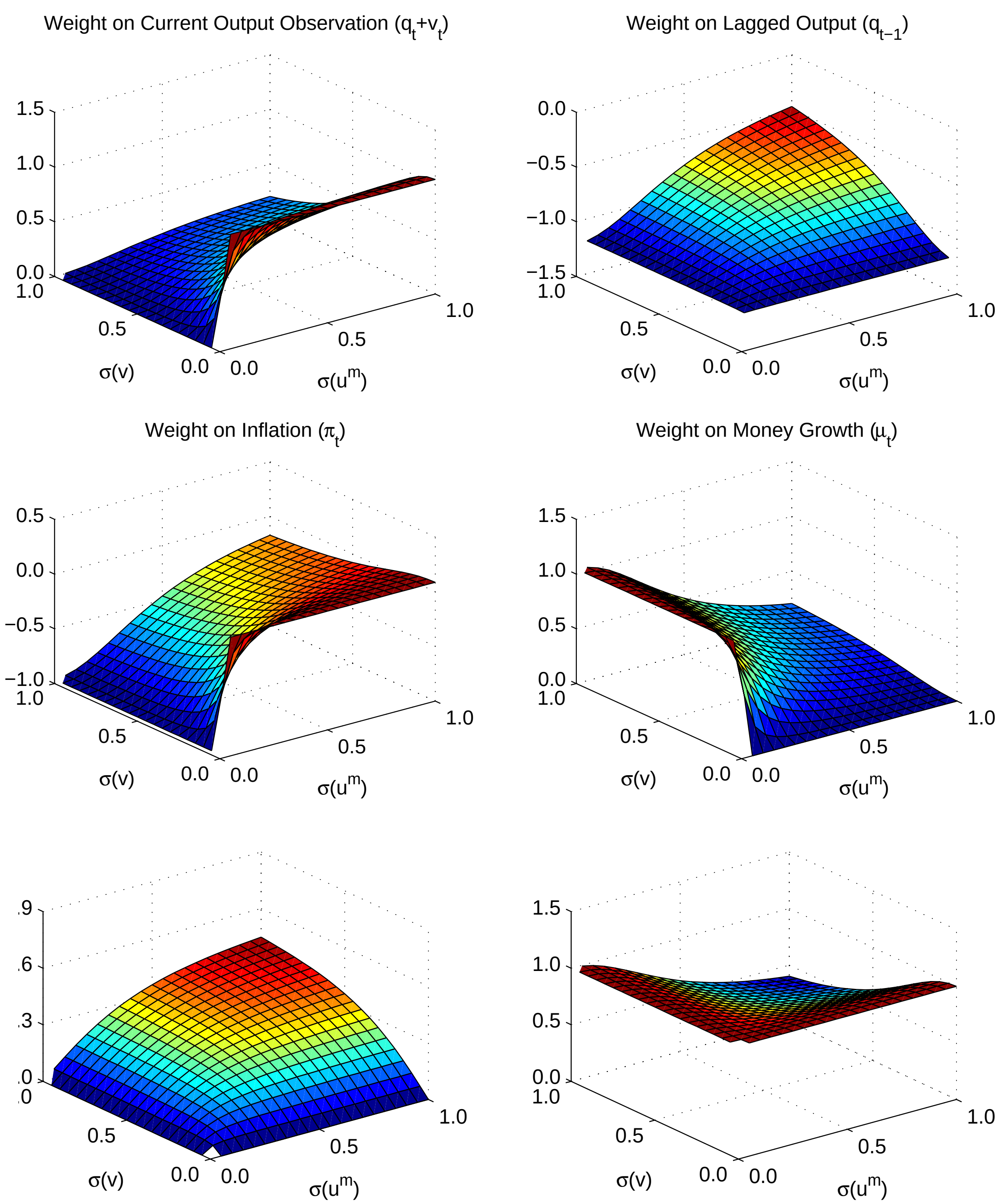


5.2 The Information Role of Money

We now consider the extent to which money can provide a more accurate estimate of the current aggregate demand shock. Since the contribution of money depends both on the standard deviations of the output measurement error and the money demand shock we report the results in a set of three-dimensional graphs. In each panel, the two axes in the horizontal plane denote the standard deviation of the measurement error on output $\sigma(v)$ and the standard deviation of the money demand shock $\sigma(u^m)$.

The top four panels of **Figure 4** indicate the weights (measured on the vertical axis) on the noisy current output observation $q_t^{(t)}$, as well as the true values of lagged output q_{t-1}, inflation π_t, and money growth $\mu_t = m_t - m_{t-1}$.

Figure 4: Optimal Filtering of the Aggregate Demand Shock with Money



With regard to the weights on current and lagged output and inflation as a function of the output measurement error, we confirm the findings for the case without money. As the measurement error regarding output increases the weights assigned to noisy current output, lagged output and inflation decrease (in absolute terms). This is the case for any level of the standard deviation of the money demand shock as can be seen by moving from right to left along the dimension which corresponds to the output measurement error.

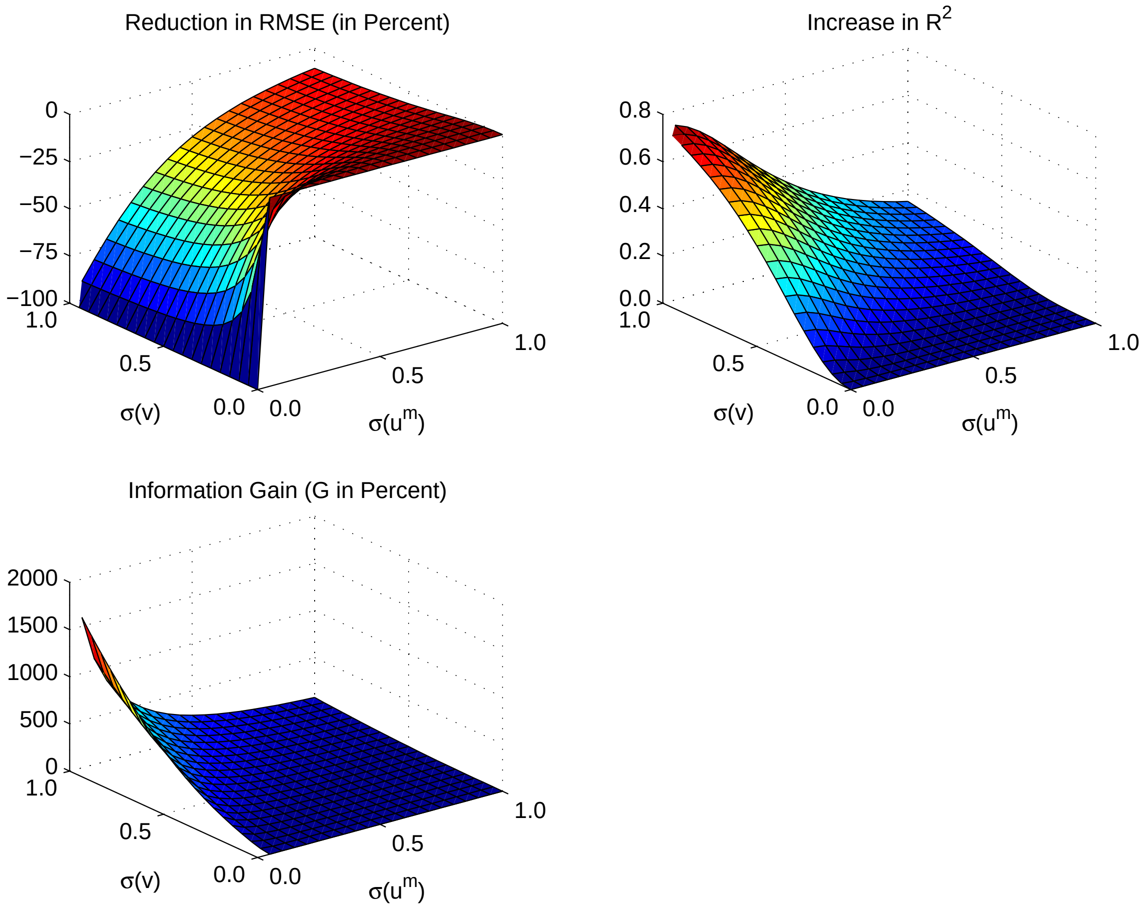
We find that money can play an important role in estimating the current aggregate demand shock if the relative magnitude of the money demand shock is not too large. Not surprisingly, the weight assigned to money is largest in the absence of money demand shocks (that is, $\sigma(u^m) = 0$). In this case, the decision maker can infer the true value of output (and consequently, the aggregate demand shock) directly from the money growth rate, since lagged output and inflation are observed exactly.

As $\sigma(u^m)$ increases, however, the weight on money growth in the optimal estimate of output declines. With the possibility of money demand shocks, the decision maker cannot be sure whether a money growth observation that seems inconsistent with observed output is an indication of a mismeasurement of actual output or of a money demand shock. While the weight on money declines, one can see that it declines more slowly the greater the standard deviation of the output measurement error. Finally, it is of interest to note that the weight on inflation and lagged output also decreases in absolute value with the weight on money, as the standard deviation of the money demand shock increases.

The bottom two panels of **Figure 4** show the degree of uncertainty associated with the contemporaneous estimate of the aggregate demand shock, as indicated by the RMSE and the R^2. Of course, along either axis in the horizontal plane, the RMSE is zero and the R^2 is equal to one, because either $\sigma(v)$ or $\sigma(u^m)$ equals zero. When both $\sigma(v)$ and $\sigma(u^m)$ are strictly positive, the RMSE is positive and the R^2 is less than unity.

Figure 5 indicates three measures of the information content of money in estimating the aggregate demand shock: the reduction in RMSE (in percentage points), the improvement in R^2, and the expected relative gain in information G. When output measurement errors

Figure 5: Information Content of Money in Estimating the Aggregate Demand Shock



are relatively large compared with money demand shocks (that is, $\sigma(v) > 0.5$ percent and $\sigma(u^m) < 0.5$ percent), the improvement in the quality of the estimate can be substantial: in such cases, money reduces the RMSE by 20 percent or more and raises the R^2 by at least 0.18. The information gain from utilizing money as an information variable is also very high under these conditions.

6 The Quantitative Significance of Money as an Indicator Variable

Having illustrated the role of money in a somewhat simplified model, we now proceed to quantify the information content of money using the full model given in Section 3. In particular, we utilize the empirical money demand equation given in equation (4) with the estimated standard deviation of the money demand shock, and we consider two variants of the 3-quarter output revision process given in equation (10), as well as the simpler 1-quarter revision process (given by equation (9)) that was used in the previous section.

6.1 Results for the Baseline Estimated Model

6.1.1 The Optimal Indicator Weights

Table 4 indicates the optimal weight on each indicator variable used in estimating the current aggregate demand shock, u_t^d. The upper panel shows these weights when money is not included in the information set, while the lower panel indicates the weights when current money growth is utilized in constructing the optimal estimate. In each case, we consider three alternative assumptions about the output revision process.

When the revision process is completed in a single period, the previous period's output is known with certainty (that is, $q_{t-1}^{(t)} = q_{t-1}$). In this case, longer lags of output do not contain any additional information regarding the period t aggregate demand shock. Thus, as shown by the first row of the upper panel, the optimal filter places non-zero weight on the noisy current output observation $q_t^{(t)}$, the previous period's output level q_{t-1}, and the current inflation rate π_t. (As noted previously, we are assuming that current inflation is known by all agents and hence can always serve as a perfect indicator variable.)

In contrast, when the revision process takes three periods, the current and previous two output observations contain measurement error, while the true value of q_{t-3} is revealed in the latest data vintage. Hence, in this case, the optimal filter places non-zero weight on all four output observations (that is, $q_t^{(t)}$, $q_{t-1}^{(t)}$, $q_{t-2}^{(t)}$, and $q_{t-3}^{(t)}$) as well as the current inflation rate. The second row of the upper panel shows the optimal weights when the revisions are

Table 4: Optimal Indicator Weights for Estimating the Aggregate Demand Shock

Revision Process	$q_t^{(t)}$	$q_{t-1}^{(t)}$	$q_{t-2}^{(t)}$	$q_{t-3}^{(t)}$	π_t	μ_t
Filtering without Money						
One-Period	0.267	-0.309	0	0	0.022	—
Three-Period Uncorrelated	0.136	-0.059	-0.054	-0.021	0.011	—
Three-Period Correlated	0.271	-0.127	-0.054	0.016	0.022	—
Filtering with Money						
One-Period	0.265	-0.268	0	0	0.044	0.354
Three-Period Uncorrelated	0.138	-0.053	-0.081	-0.052	0.035	0.387
Three-Period Correlated	0.275	-0.115	-0.089	0.012	0.046	0.374

uncorrelated but their variances are set to the estimated values described in Section 3.2., while the third row indicates the weights when we use the complete estimated covariance matrix.

When current money growth is included in the information set, we see that this indicator variable receives substantial weight in estimating the current aggregate demand shock. The exact weight varies somewhat depending on the specific representation of the output revision process, but the notable point is that the magnitude of this weight is roughly similar to that placed on the noisy current output observation. Of course, interpreting the specific pattern of filtering weights is rather difficult, and hence we now proceed to consider the measures of information content described in Section 4.2.

6.1.2 Measures of Information Content

Table 5 characterizes the information role of money in estimating the current aggregate demand shock under each of the three alternative assumptions about the revision process. As a benchmark for comparison, the first two columns indicate the RMSE of the demand

Table 5: The Information Role of Money in the Estimated Model

			Information Content		
Revision Process	RMSE	R^2	%ΔRMSE	ΔR^2	Info. Gain
One-Period	0.50	0.26	-1.29	0.02	8.51
Three-Period Uncorrelated	0.54	0.13	-1.46	0.03	20.42
Three-Period Correlated	0.50	0.27	-1.59	0.02	10.29

shock estimate (that is, the square root of $\mathrm{E}_t[u^d_{t|t} - u^d_t]^2$) and the associated R^2 when money growth is not included in the information set. Evidently, the precision of the demand shock estimates is not very high regardless of how the output revision process is specified.

The remainder of **Table 5** provides three measures of the extent to which current money growth increases the precision of the estimated demand shock. By including money as an indicator variable, the RMSE is reduced by about 1.5 percent, and the R^2 rises by about 0.02. Measured in terms of lower entropy, the information gain is somewhat more impressive: about 10 to 20 percent, depending on the specification of the output revision process.

Based on these results, one would reasonably conclude that money has noticeable but not remarkably high information content in estimating the current aggregate demand shock. However, our analysis thus far has assumed that the central bank is unable to identify any of the underlying factors that generate the contemporaneous money demand shock. As we will see below, monetary analysis can dramatically raise the usefulness of money as an indicator variable.

6.2 The Role of Monetary Analysis

Central banks tend to expend significant resources to gain a better understanding of ongoing monetary developments on a very detailed level, thereby identifying factors that would not be well-explained by a standard money demand model. This function of monetary analysis is highlighted in the May 2001 issue of the ECB Monthly Bulletin:

> *"The decomposition of monetary growth into its macroeconomic determinants also indicates the extent to which monetary growth is not explained by the [money demand] model. Hence it may reveal additional information contained in monetary aggregates which is not captured by the other macroeconomic variables. ... Ideally, a detailed institutional analysis can provide some additional insight by providing information concerning special events, thus reducing the unexplained part of monetary growth."* [ECB Monthly Bulletin, May 2001, pp. 46/47.]

The results reported in **Table 6** indicate that monetary analysis can play an important role in enhancing the information role of money when contemporaneous output estimates are afflicted by substantial measurement error. In the first column, the standard deviation of the money demand shock $\sigma(u^m)$ is equal to the estimated value from Coenen and Vega (1999). In the remaining three columns, we assume that monetary analysis is able to identify factors accounting for a substantial fraction of the variation in the money demand shock. In such cases, the information content of current money growth increases dramatically.

When monetary analysis is able to identify factors accounting for 75 percent of the variance of the money demand shock, then utilizing money as an indicator variable reduces the RMSE of the current aggregate demand shock estimate by about 10 to 15 percent, while the associated R^2 rises by about 0.2, and the entropy measure of information increases by more than 100 percent. When the central bank succeeds in identifying the factors that account for 7/8 of the money demand shock, then the RMSE is reduced by 30 to 40 percent; the other measures of information content also rise dramatically in this case.

When monetary analysis is relatively effective, we also find that current money growth provides substantial benefits in predicting future levels of output.[28] To illustrate this benefit, we consider the case in which the central bank can identify factors that account for three-fourths of the variation in the money demand shock. The underlying output revision process

[28] Table D1 in Appendix D indicates the RMSE of output and inflation forecasts for the case in which output is measured without error.

Table 6: Monetary Analysis and the Information Content of Money

Information Measure	Revision Process	Reduction in the Std. Dev. of the Money Demand Shock 0%	50%	75%	82.5%
%ΔRMSE	One-Period	-1.29	-4.88	-16.13	-38.99
	Three-Period Uncorrelated	-1.46	-4.69	-11.91	-26.01
	Three-Period Correlated	-1.59	-5.41	-14.41	-28.77
ΔR^2	One-Period	0.02	0.07	0.22	0.46
	Three-Period Uncorrelated	0.03	0.08	0.19	0.39
	Three-Period Correlated	0.02	0.08	0.20	0.36
Information Gain	One-Period	8.51	32.79	115.12	323.36
	Three-Period Uncorrelated	20.42	66.81	176.11	418.47
	Three-Period Correlated	10.29	36.05	100.58	219.55

is assumed to take three periods with the revisions being correlated. The first column of **Table 7** indicates the RMSE of the output prediction for a range of forecast horizons when money growth is not included in the information set, while the fourth column indicates the reduction in RMSE that results from utilizing money as an indicator variable. Evidently, money growth enhances the precision of the contemporaneous output estimate by about 5 percent, and improves the accuracy of the two-quarter-ahead output forecast by about 2 percent. Of course, as the forecast horizon increases, output exhibits a higher degree of unpredictable variation, and hence no indicator variable would be expected to have very much predictive power.

As discussed in Appendix B.2, the MSE can be decomposed into a component which relates to the within-period estimation error of the predetermined variables (component I), and the propagation of unpredictable future disturbances which will affect the evolution of the predetermined variables as well as their within-period estimates in the future (component II). Component II increases with the prediction horizon and converges to the

Table 7: The Role of Money in Predicting Output

Forecast Horizon	RMSE			%ΔRMSE		
	Overall	I	II	Overall	I	II
0	0.56	0.56	0	-5.42	-5.42	0
1	0.84	0.62	0.57	-3.44	-5.97	-0.57
2	1.05	0.59	0.87	-2.25	-6.14	-0.52
4	1.33	0.50	1.24	-1.32	-6.15	-0.55
8	1.60	0.34	1.56	-0.83	-6.15	-0.59
16	1.75	0.15	1.74	-0.65	-6.15	-0.61

unconditional standard deviation of output. The contribution to the RMSE coming from component I dominates initially but has a declining contribution to the overall RMSE as the forecast horizon increases.

Turning to the final two columns of **Table 7**, we see that using money as an indicator variable causes a 6 percent reduction in the RMSE associated with component I, regardless of the forecast horizon. It is interesting to note that having money in the information set also changes the dynamic behavior of actual output, and hence has a small effect in reducing the RMSE associated with component II.[29]

6.3 Further Sensitivity Analysis

Now we briefly summarize some additional sensitivity analysis regarding the results presented above.

First, it is worthwhile to consider the implications of alternative money demand specifications with a stronger contemporaneous relationship between money and output. For the preceding analysis, we have used the demand function for M3 because this money aggregate has exhibited reasonable stability in the euro area over the past two decades (cf. Coenen

[29] Results regarding inflation prediction are given in Table D2 of Appendix D. With regard to inflation, component I is not very important. Even though component I decreases by including money growth in the information set, component II actually increases slightly. With regard to the overall MSE money growth does not help as a result. Of course, one needs to keep in mind that the information role of money for inflation here is rather limited.

and Vega (1999)). However, the estimated coefficient that determines the instantaneous income elasticity of M3 is rather small. Narrower money aggregates (such as M1) typically have a much tighter relationship with current output. As shown in Table D3 of Appendix D, we find that a higher instantaneous income elasticity substantially raises the information content of money. Thus, to the extent that monetary analysis can identify structural changes and special factors that generate shifts in the demand function for a narrow aggregate (such as M1), the central bank would be able to utilize such an aggregate in reducing the data uncertainty associated with current output.

Second, we have assumed for simplicity that private agents and the central bank are able to utilize a noisy estimate of contemporaneous output at each point in time. Given the actual time delays in releasing GDP data, however, it may be more realistic to assume that no output estimate is available until the subsequent period (especially since our model is specified at a quarterly frequency). As shown in Appendix Tables D4 and D5, the importance of using money as an indicator variable increases in this case.

Finally, the results reported above have been derived using a structural macroeconomic model. This approach provides a clear description of the transmission mechanism of monetary policy, and explicitly considers the evolution of market participants' expectations. Nevertheless, we recognize that structural assumptions are always somewhat controversial. Therefore, we have also measured the information content of money using the non-structural time series model of the euro area estimated by Coenen and Vega (1999). As shown in Appendix Tables D6 through D9, the implications of the time series model are remarkably similar to those of the structural macroeconometric model.

7 Conclusion

To explore the information role of money in the presence of data uncertainty we have extended the euro area macroeconomic model of Coenen and Wieland (2000) by incorporating the euro area-wide money demand model of Coenen and Vega (1999) and an empirically calibrated model of the revision process of aggregate euro area output. Using this framework we have found that money can play an important role as an information variable and may result in major improvements in current output estimates. However, the specific nature of this role depends on the magnitude of the output measurement error relative to the money demand shock.

In particular, we have found noticable but small improvements in output estimates due to the inclusion of money growth in the information set when the standard deviation of money demand shocks equals the estimated value from Coenen and Vega (1999). Money plays a quantitatively more important role with regard to output estimation if we allow for a contribution of monetary analysis in reducing uncertainty due to money demand shocks. In this case, money also helps to reduce uncertainty about output forecasts. Of course, as the construction of euro area aggregate output data is improved over time, the magnitude of the revisions discussed in Section 2 is likely to decline over time. Nevertheless, evidence concerning U.S. data vintages collected by Croushore and Stark (1999) indicates that data uncertainty will remain an important issue even once the data collection technology has matured.

Throughout the paper we consider a relatively limited role of money by focusing exclusively on the information content of money with respect to output measurement and by excluding the possibility of a direct role of money in output and inflation determination. In this sense, our quantitative results only indicate a lower bound on the usefulness of money. An alternative model that allows for significant direct effects of money on inflation and could be used in future research is the so-called P^* model.

Also, as noted earlier, we have focused attention on a framework with symmetric information regarding aggregate output data as far as private market participants and the central bank are concerned. We have also conducted some exploratory analysis under the assumption of asymmetric information regarding aggregate data that is used by Dotsey and Hornstein (2000) and Svensson and Woodford (2001). However, in our view this assumption is undesirable if it implies that a representative agent by knowing his individual income can also infer aggregate income and demand while the policymaker only observes a noisy estimate of aggregate demand. We plan to study the asymmetric case in more detail in the future in a model that would allow us to differentiate more carefully between individual and aggregate uncertainty.

Finally, another interesting avenue for future research would be to compare optimal filtering to simple filtering rules in keeping with the recent debate on optimal versus simple monetary policy rules. For example, one could investigate the performance of simple rules that respond only to observed output growth, inflation and money growth instead of optimal estimates of the output gap. A recent study that considers an example of a simple filtering rule in the context of NAIRU uncertainty is Meyer, Swanson and Wieland (2001).

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Appendix A Kalman filtering à la Svensson and Woodford

This appendix restates and specializes the setup of Svensson and Woodford (2000) – henceforth SW – so that the Kalman filter can be applied to the variants of the wage contracting models analysed in the main text. In particular, we now work with a generic linear rational expectations model and a model of measurement which permit us to formulate the wage contracting models in state-space form to which the Kalman filter is applicable.

A.1 The state-space representation

The generic linear rational expectations model takes the form

$$\begin{bmatrix} X_{t+1} \\ \tilde{E}\, x_{t+1|t} \end{bmatrix} = A^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + A^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + \begin{bmatrix} u_{t+1} \\ 0 \end{bmatrix}, \tag{A.1}$$

where X_t is a vector of predetermined variables, x_t is a vector of non-predetermined variables and u_t is a vector of serially uncorrelated shocks with mean zero and positive semi-definite covariance matrix Σ_{uu}. The coefficient matrices A^1, A^2 and $\tilde{E}$ are matrices of appropriate dimension.

Regarding the measurement of the predetermined and non-predetermined variables, let Z_t denote a vector of observables given by

$$Z_t = D^1 \begin{bmatrix} X_t \\ x_t \end{bmatrix} + D^2 \begin{bmatrix} X_{t|t} \\ x_{t|t} \end{bmatrix} + v_t, \tag{A.2}$$

where v_t is a vector of serially uncorrelated measurement errors with mean zero and positive semi-definite covariance matrix Σ_{vv}. The measurement errors v_t are assumed to be uncorrelated with the shocks u_t at all leads and lags, i.e. $\mathrm{E}[u_t\, v_\tau'] = 0$ for all t and τ. The matrices D^1 and D^2 are selector matrices of appropriate dimension.

Information in period t is supposed to be given by

$$I_t = \left\{ Z_\tau, \tau \le t; A^1, A^2, D^1, D^2, \tilde{E}, \Sigma_{uu}, \Sigma_{vv} \right\}$$

and we let $\chi_{\tau|t} = \mathrm{E}[\,\chi_\tau | I_t\,]$ denote the rational expectation of any variable χ in period τ given information in period t.

SW show that the non-predetermined variables fulfill the relationship

$$x_t = G^1\, X_t + G^2\, X_{t|t} \tag{A.3}$$

and that the system of equations (A.1), (A.2) can be cast into state-space form without non-predetermined variables,

$$X_{t+1} = H\, X_t + J\, X_{t|t} + u_{t+1} \tag{A.4}$$

$$Z_t = L\, X_t + M\, X_{t|t} + v_t, \tag{A.5}$$

where the matrices G^1, G^2, H, J, L and M are provided in SW.

This transformation turns out to simplify the remaining problem of forming the estimate $X_{t|t}$ considerably.

A.2 The Kalman filter

After having eliminated the non-predetermined variables x_t there is a simultaneity problem to be solved when estimating the predetermined variables X_t because the observable variables Z_t depend on the estimate of the predetermined variables $X_{t|t}$, while the latter depends on the observables used in the estimation.

Accounting for the contemporaneous effect of the estimate $X_{t|t}$ on Z_t, SW show that the optimal estimate of X_t can be obtained by a Kalman filter updating equation in terms of the innovations in the transformed variables $\bar{Z}_t = Z_t - M\,X_{t|t}$,

$$\begin{aligned} X_{t|t} &= X_{t|t-1} + K\,(\bar{Z}_t - \bar{Z}_{t|t-1}) && \text{(A.6)} \\ &= X_{t|t-1} + K\,\Big[L\,(X_t - X_{t|t-1}) + v_t\Big], && \text{(A.7)} \end{aligned}$$

where the second line uses that $\bar{Z}_t - \bar{Z}_{t|t-1} = L\,(X_t - X_{t|t-1}) + v_t$.[30]

The steady-state Kalman gain matrix K is given by

$$K = PL'\,(LPL' + \Sigma_{vv})^{-1}, \tag{A.8}$$

where the matrix P is the steady-state covariance matrix of the innovations $X_t - X_{t|t-1}$ given information in period $t-1$ and fulfills[31]

$$P = H\,\Big[P - PL'\,(LPL' + \Sigma_{vv})^{-1}LP\Big]\,H' + \Sigma_{uu}. \tag{A.9}$$

In section B.1 below it is verified that the term in square brackets is the covariance matrix of the updating errors $X_t - X_{t|t}$ given information in period t, which may serve as a measure of the estimation uncertainty surrounding the optimal estimate $X_{t|t}$ produced by the application of the Kalman filter.

The evolution over time of the predetermined variables X_t and their estimates $X_{t|t}$ is simultaneously determined by the transition equation (A.4) and the Kalman filter updating equation (A.7) in combination with the prediction formula $X_{t|t-1} = (H+J)\,X_{t-1|t-1}$ being derived from the former.

For later reference, it is convenient to express this system of dynamic equations more compactly as

$$\begin{bmatrix} X_t \\ X_{t|t} \end{bmatrix} = \begin{bmatrix} H & J \\ KLH & (I-KL)H+J \end{bmatrix} \begin{bmatrix} X_{t-1} \\ X_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} I & 0 \\ KL & K \end{bmatrix} \begin{bmatrix} u_t \\ v_t \end{bmatrix}. \tag{A.10}$$

[30] Note that $\bar{Z}_t = Z_t - M\,X_{t|t} = L\,X_t + v_t$ and $\bar{Z}_{t|t-1} = Z_{t|t-1} - M\,X_{t|t-1} = L\,X_{t|t-1}$, and thus $\bar{Z}_t - \bar{Z}_{t|t-1} = L\,(X_t - X_{t|t-1}) + v_t$.

[31] Note that $(LPL' + \Sigma_{vv})^{-1}$ may be replaced by a generalized inverse if $(LPL' + \Sigma_{vv})$ is singular.

Appendix B Measuring the information content of indicator variables

To evaluate the information content of indicator variables, we assess to which extent indicator measurements will reduce the uncertainty surrounding the estimation and/or prediction of variables of interest such as output and inflation. To do so, we build on a measure from information theory, the entropy, to evaluate the within-period estimation uncertainty and compute the Mean Square Error (MSE) of multi-period ahead predictions to evaluate the prediction uncertainty. In section B.1 and section B.2 below, we show how to properly adapt the necessary computations to the generic linear rational expectations model with imperfectly observed indicator variables.

B.1 Within-period estimation

Drawing on results from information theory, Tinsley et al. (1980) employ the entropy, or 'expected uncertainty', as a formal measure of the information content of indicator variables. We have restated the relevant results in Tinsley et al. in the main text and adapt them here to the generic linear rational expectations model with imperfect indicator measurements.

In order to do so we need to obtain the joint distribution of the innovations in the observed indicator variables, $Z_t - Z_{t|t-1}$, and the innovations in the predetermined variables, $X_t - X_{t|t-1}$. As an intermediate step, we first consider the case of the transformed indicator variables $\bar{Z}_t \equiv Z_t - M\,X_{t|t}$, where the contemporaneous effect of the estimate $X_{t|t}$ is subtracted from the observed variables Z_t.

Assuming that the shocks u_t and the measurement errors v_t are normal and that the steady-state covariance matrix P from the application of the Kalman filter is given, it is straightforward to show that the innovations in the transformed indicators, $\bar{Z}_t - \bar{Z}_{t|t-1}$, and the innovations in the predetermined variables, $X_t - X_{t|t-1}$, are jointly normally distributed with

$$\left[\begin{array}{c} \bar{Z}_t - \bar{Z}_{t|t-1} \\ X_t - X_{t|t-1} \end{array}\right] \Big| I_{t-1} \;\sim\; \mathrm{N}\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} LPL' + \Sigma_{vv} & LP \\ PL' & P \end{array}\right]\right), \tag{B.1}$$

where we recall that $\bar{Z}_t - \bar{Z}_{t|t-1} = L\,(X_t - X_{t|t-1}) + v_t$ and $P = \mathrm{Cov}[X_t - X_{t|t-1}|I_{t-1}]$.

Then, observing that the innovations in the observed indicators are a linear transformation of the innovations in the transformed indicators, $Z_t - Z_{t|t-1} = (I + MK)\,(\bar{Z}_t - \bar{Z}_{t|t-1})$,[32]

[32]Notice that

$$\begin{aligned} Z_t - Z_{t|t-1} &= Z_t - M\,X_{t|t} + M\,X_{t|t} - Z_{t|t-1} - M\,X_{t|t-1} + M\,X_{t|t-1} \\ &= \bar{Z}_t - \bar{Z}_{t|t-1} + M\,(X_{t|t} - X_{t|t-1}) \\ &= \bar{Z}_t - \bar{Z}_{t|t-1} + MK\,\left[\,L\,(X_t - X_{t|t-1}) + v_t\,\right] \\ &= \bar{Z}_t - \bar{Z}_{t|t-1} + MK(\bar{Z}_t - \bar{Z}_{t|t-1}) \end{aligned}$$

it follows immediately from (B.1) that the innovations in the observed indicators and the innovations in the predetermined variables are also jointly normally distributed with

$$\left[\begin{array}{c} Z_t - Z_{t|t-1} \\ X_t - X_{t|t-1} \end{array}\right] \Big| I_{t-1} \quad \sim \quad \mathrm{N}\left(\left[\begin{array}{c} 0 \\ 0 \end{array}\right], \left[\begin{array}{cc} N(LPL' + \Sigma_{vv})N' & NLP \\ PL'N' & P \end{array}\right] \right),$$

where $N = I + MK$.

Hence, using the properties of multivariate normal distributions, the conditional covariance matrix of $X_t - X_{t|t-1}$ given information in period t is

$$\begin{aligned} \mathrm{Cov}[X_t - X_{t|t-1}|I_t] &= \mathrm{Cov}[X_t - X_{t|t-1}|I_{t-1}, Z_t - Z_{t|t-1}] \\ &= P - PL'N' \left(N(LPL' + \Sigma_{vv})^{-1} N' \right)' NLP \\ &= P - PL'(LPL' + \Sigma_{vv})^{-1} LP. \end{aligned} \qquad \text{(B.2)}$$

Apparently, while the weights of the observed indicator variables Z_t in computing the conditional mean $X_{t|t} = \mathrm{E}[X_t|I_t]$ are affected by the contemporaneous effect of $X_{t|t}$ on Z_t (as shown below), the computation of the conditional covariance $\mathrm{Cov}[X_t - X_{t|t-1}|I_t]$ is not. Indeed, the conditional covariance matrix $\mathrm{Cov}[X_t - X_{t|t-1}|I_t]$ is equal to the conditional covariance matrix $\mathrm{Cov}[X_t - X_{t|t-1}|\bar{Z}_t - \bar{Z}_{t|t-1}, I_{t-1}]$ which can be obtained from (B.1).

In principle, one could also aim at measuring the information content of indicator variables starting from the covariance matrices of the innovations in the non-predetermined variables,

$$\begin{aligned} x_t - x_{t|t-1} &= G^1 (X_t - X_{t|t-1}) + G^2 (X_{t|t} - X_{t|t-1}) \\ &= G^1 (X_t - X_{t|t-1}) + G^2 K \left[L (X_t - X_{t|t-1}) + v_t \right] \\ &= (G^1 + G^2 KL) (X_t - X_{t|t-1}) + G^2 K\, v_t, \end{aligned}$$

where we have used equations (A.3) and (A.7).

However, since the covariance matrix of $x_t - x_{t|t-1}$ given information in period $t-1$ will depend on the choice of the vector of indicator variables via the Kalman gain matrix K and the covariance matrix of the measurement errors Σ_{vv}, it will not be feasible to measure the information content of the indicator variables by measuring the distance between the covariance matrices of $x_t - x_{t|t-1}$ given information in t and $t-1$, respectively, since a simultaneity problem, entering via K and Σ_{vv}, exists.

Finally, since $\mathrm{Cov}[X_t - X_{t|t}|I_t] = \mathrm{Cov}[X_t - X_{t|t-1}|I_t]$,[33] equation (B.2) also provides a measure of the estimation uncertainty surrounding the optimal estimate $X_{t|t}$ produced

$$= \quad (I + MK)(\bar{Z}_t - \bar{Z}_{t|t-1})$$

where the step from the second line to the third makes use of the Kalman filter updating equation (A.7) and the step from the third to the fourth uses the relationship shown in footnote 30.

[33] Observing that

$$\mathrm{Cov}[X_t - X_{t|t}|I_t] \quad = \quad \mathrm{Cov}[(X_t - X_{t|t}) - (X_{t|t} - X_{t|t-1})|I_t]$$

by the application of the Kalman filter, i.e. the covariance matrix of the updating error $X_t - X_{t|t}$.

B.2 Multi-period predictions

When evaluating the information content of individual indicator variables it is also of interest to asses to which extent the indicator variables may reduce the uncertainty surrounding the multi-period predictions of variables of interest such as output and inflation. To this end this section shows how to compute the Mean Square Error (MSE) of the h-period ahead predictions of the predetermined and non-predetermined variables in linear rational expectations models when the indicator variables are subject to measurement error. In particular, it is shown that the MSE can be decomposed in a component which relates to the within-period estimation error of the predetermined variables and the propagation of unpredictable future disturbances which will affect the evolution of the predetermined variables as well as their within-period estimates in the future. In general, the magnitude of both components depends on the system matrices describing the joint dynamics of the predetermined variables and their within-period estimates.

To simplify the calculations we restate the dynamic system (A.10) describing the joint evolution of the vector of predetermined variables X_t and its within-period estimate $X_{t|t}$ more compactly as

$$Y_{t+1} \quad = \quad A\,Y_t + B\,w_{t+1} \tag{B.3}$$

with $Y_t = [\,X_t',\, X_{t|t}'\,]'$, $w_t = [\,u_t',\, v_t'\,]'$ and appropriately defined matrices A and B.

Then, iterating the dynamic system (B.3) forward, we can express the realisation in period $t+h$, Y_{t+h}, in terms of the current realisation Y_t and the future disturbances $w_{t+1}, \ldots, w_{t+h}$,

$$Y_{t+h} \quad = \quad A^h\,Y_t + \sum_{i=0}^{h-1} A^i\,B\,w_{t+h-i}.$$

The h-period ahead prediction $Y_{t+h|t}$, given the available information in period t, is

$$Y_{t+h|t} \quad = \quad A^h\,Y_{t|t}$$

$$\begin{aligned} &= \mathrm{Cov}[(X_t - X_{t|t}) - KL\,[\,(X_{t|t} - X_{t|t-1}) + v_t\,]|I_t] \\ &= \mathrm{Cov}[(I - KL)(X_t - X_{t|t-1}) - K\,v_t|I_t] \\ &= (I - KL)\,P\,(I - KL)' + K\,\Sigma_{vv}\,K' \end{aligned}$$

with $P = \mathrm{Cov}[X_t - X_{t|t-1}|I_{t-1}]$ and $K = PL'\,(LPL' + \Sigma_{vv})^{-1}$, we obtain after some algebra

$$\begin{aligned} \mathrm{Cov}[X_t - X_{t|t}|I_t] &= P - PL'(LPL' + \Sigma_{vv})^{-1}LP \\ &= \mathrm{Cov}[X_t - X_{t|t-1}|I_t]. \end{aligned}$$

and, thus, the h-period ahead prediction error amounts to

$$Y_{t+h} - Y_{t+h|t} \;=\; A^h \, (Y_t - Y_{t|t}) + \sum_{i=0}^{h-1} A^i \, B \, w_{t+h-i}. \tag{B.4}$$

Observing that

$$Y_t - Y_{t|t} \;=\; \begin{bmatrix} X_t - X_{t|t} \\ X_{t|t} - X_{t|t} \end{bmatrix} = \begin{bmatrix} X_t - X_{t|t} \\ 0 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} (X_t - X_{t|t}) \tag{B.5}$$

it follows that the h-period ahead prediction error (B.4) has two sources: first, the within-period prediction error of the predetermined variables, $X_t - X_{t|t}$; and, second, the unpredictable future disturbances $w_{t+1}, \ldots, w_{t+h}$. Obviously, the impact of the within-period estimation error $X_t - X_{t|t}$ dies out with increasing prediction horizon h since A^h converges to zero with increasing h given that all eigenvalues of A have modulus less than one.

Since

$$\begin{aligned} X_t - X_{t|t} \;&=\; H \, X_{t-1} + J \, X_{t-1|t-1} + u_t \\ &\qquad - \Big[KLH \, X_{t-1} + ((I - KL)H + J) \, X_{t-1|t-1} + KL \, u_t + K \, v_t \Big] \\ &=\; (I - KL)H \, (X_{t-1} - X_{t-1|t-1}) + [\, (I - KL) \;\; -K \,] \, w_t \end{aligned}$$

we can express $X_t - X_{t|t}$ as the weighted sum of current and past disturbances by solving the above equation backwards,

$$X_t - X_{t|t} \;=\; \sum_{i=0}^{\infty} [\, (I - KL)H \,]^i \, [\, (I - KL) \;\; -K \,] \, w_{t-i}. \tag{B.6}$$

Then, combining (B.5) and (B.6) with (B.4), we can express the h-period ahead prediction error $Y_{t+h} - Y_{t+h|t}$ as a weighted sum of past, current and future disturbances,

$$Y_{t+h} - Y_{t+h|t} \;=\; A^h \begin{bmatrix} I \\ 0 \end{bmatrix} \sum_{i=0}^{\infty} [\, (I - KL)H \,]^i \, [\, (I - KL) \;\; -K \,] \, w_{t-i} + \sum_{i=0}^{h-1} A^i \, B \, w_{t+h-i}.$$

Since $Y_t - Y_{t|t}$ is uncorrelated with w_{t+h-i}, for $h - i > 0$, the MSE matrix of the h-period ahead prediction $Y_{t+h|t}$ is

$$\begin{aligned} \mathrm{MSE}[\, Y_{t+h|t} \,] \;&=\; \mathrm{E} \Big[\, (Y_{t+h} - Y_{t+h|t}) \, (Y_{t+h} - Y_{t+h|t})' \, \Big] \\ &=\; A^h \begin{bmatrix} I \\ 0 \end{bmatrix} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} [\, (I - KL)H \,]^i \, [\, (I - KL) \;\; -K \,] \, \mathrm{E} \Big[\, w_{t-i} \, w_{t-j}' \, \Big] \\ &\qquad \times [\, (I - KL) \;\; -K \,]' \, \Big([\, (I - KL)H \,]^j \Big)' \begin{bmatrix} I \\ 0 \end{bmatrix}' \Big(A^h \Big)' \end{aligned}$$

$$
\begin{aligned}
& + \sum_{i=0}^{h-1} \sum_{j=0}^{h-1} A^i \, B \, \mathrm{E}\left[w_{t+h-i} \, w'_{t+h-j} \right] B' \left(A^j \right)' \\
= \; & A^h \begin{bmatrix} I \\ 0 \end{bmatrix} \sum_{i=0}^{\infty} [\, (I-KL)H \,]^i \; [\, (I-KL) \;\; -K \,] \begin{bmatrix} \Sigma_{uu} & 0 \\ 0 & \Sigma_{vv} \end{bmatrix} \\
& \qquad \times [\, (I-KL) \;\; -K \,]' \left([\, (I-KL) \, H \,]^i \right)' \begin{bmatrix} I \\ 0 \end{bmatrix}' \left(A^h \right)' \\
& + \sum_{i=0}^{h-1} A^i \, B \begin{bmatrix} \Sigma_{uu} & 0 \\ 0 & \Sigma_{vv} \end{bmatrix} B' \left(A^i \right)',
\end{aligned}
$$

where it has been used that

$$
\mathrm{E}\,[\, w_t \, w'_\tau \,] \;=\; \begin{bmatrix} \Sigma_{uu} & 0 \\ 0 & \Sigma_{vv} \end{bmatrix}
$$

for $t = \tau$ and 0 else.

The first term of the MSE matrix vanishes for $h \to \infty$ since the impact of the within-period estimation error dies out with increasing prediction horizon h.[34] By contrast, the second term is monotonically non-decreasing and for $h \to \infty$ it approaches the unconditional covariance matrix of Y_t. It is interesting to note that, even if the first term of the overall MSE matrix has vanished, the second term and, ultimately, the unconditional variance of Y_t will generally be affected by the choice of indicator variables via the Kalman gain matrix K and the covariance matrix of the measurement errors Σ_{vv}. In particular, this will hold true for the h-period ahead predictions of the predetermined variables X_t unless $J \neq 0$, i.e. unless the dynamic system in X_t and $X_{t|t}$ is decoupled.

Once we have determined the MSE matrix of the h-period ahead prediction $Y_{t+h|t}$, we can easily recover the MSE matrix of the h-period ahead prediction of the vectors of predetermined and non-predetermined variables, $X_{t+h|t}$ and $x_{t+h|t}$, from the former. Obviously, the MSE matrix of the h-period ahead forecast of the vector of predetermined variables, $\mathrm{MSE}[\, X_{t+h|t} \,]$, is the upper left block of $\mathrm{MSE}[\, Y_{t+h|t} \,]$.

To obtain the MSE matrix of the h-period ahead forecast of the vector of non-predetermined variables, $\mathrm{MSE}[\, x_{t+h|t} \,]$, we recall that $x_t \;=\; G^1 \, X_t + G^2 \, X_{t|t}$. Thus, the realisation of the vector of non-predetermined variables in period $t+h$ is given by

$$
\begin{aligned}
x_{t+h} \;&=\; G^1 \, X_{t+h} + G^2 \, X_{t+h|t+h} \\
&=\; \begin{bmatrix} G^1 & G^2 \end{bmatrix} \begin{bmatrix} X_{t+h} \\ X_{t+h|t+h} \end{bmatrix} \\
&=\; \begin{bmatrix} G^1 & G^2 \end{bmatrix} Y_{t+h}
\end{aligned}
$$

[34] The above calculations can be simplified by observing that $\mathrm{Cov}[X_t - X_{t|t} | I_t] = P - PL'(LPL' + \Sigma_{vv})^{-1} LP$ – as shown in section B.1 – though this would not give insight how the first term of the MSE depends on the current and past disturbances of the dynamic system.

and the h-period ahead prediction, given the information available in period t, is

$$\begin{aligned}
x_{t+h|t} &= (G^1 + G^2)\, X_{t+h|t} \\
&= \begin{bmatrix} G^1 & G^2 \end{bmatrix} \begin{bmatrix} I \\ I \end{bmatrix} (H + J)^h\, X_{t|t} \\
&= \begin{bmatrix} G^1 & G^2 \end{bmatrix} A^h\, Y_{t|t} \\
&= \begin{bmatrix} G^1 & G^2 \end{bmatrix} Y_{t+h|t}.
\end{aligned}$$

Consequently, the h-period ahead prediction error of the vector of non-predetermined variables equals

$$x_{t+h} - x_{t+h|t} = \begin{bmatrix} G^1 & G^2 \end{bmatrix} (\, Y_{t+h} - Y_{t+h|t}\,)$$

and the MSE matrix of the h-period ahead prediction amounts to

$$\begin{aligned}
\mathrm{MSE}[\, x_{t+h|t}\,] &= \mathrm{E}\Big[(x_{t+h} - x_{t+h|t})\, (x_{t+h} - x_{t+h|t})' \Big] \\
&= \begin{bmatrix} G^1 & G^2 \end{bmatrix} \mathrm{E}\Big[(Y_{t+h} - Y_{t+h|t})\, (Y_{t+h} - Y_{t+h|t})' \Big] \begin{bmatrix} G^1 \\ G^2 \end{bmatrix} \\
&= \begin{bmatrix} G^1 & G^2 \end{bmatrix} \mathrm{MSE}[\, Y_{t+h|t}\,] \begin{bmatrix} G^1 \\ G^2 \end{bmatrix}.
\end{aligned}$$

Apparently, even if the MSE matrix of $X_{t+h|t}$ does not, the MSE matrix of the h-period ahead prediction of the non-predetermined variables will depend on the choice of the indicator variables at all horizons, because the future values of the non-predetermined variables will depend on the path of the within-period estimates of the predetermined variables.

Appendix C The representation of the revision process

According to the model of the revision process in Section 3, information in period t will comprise error-corrupted observations on q_t, q_{t-1}, q_{t-2} and the true value q_{t-3},

$$
\begin{aligned}
q_t^{(t)} &= q_t + v_t^3 + v_t^2 + v_t^1 \\
q_{t-1}^{(t)} &= q_{t-1} + v_{t-1}^3 + v_{t-1}^2 \\
q_{t-2}^{(t)} &= q_{t-2} + v_{t-2}^3 \\
q_{t-3}^{(t)} &= q_{t-3}
\end{aligned}
$$

and the application of the Kalman filter will simultaneously determine a *preliminary* estimate of q_t (the component $q_{t|t}$), two *revised* estimates of last and next to last periods' output (the components $q_{t-1|t}$ and $q_{t-2|t}$), and a *final* estimate of output three periods ago (the component $q_{t-3|t}$) which will be equal to the true output value q_{t-3}.

Apparently, our model of measurement shows persistence since the initial measurement error drops out only gradually and it takes three periods to learn about the true value of output. To make this setup conformable with the measurement model (12) in Section 4, we augment the vector of measurement errors v_t to include lagged values of the measurement errors themselves and incorporate the augmented vector of measurement errors in the generic model (11) in the standard way by embedding the vector of measurement errors in the vector of predetermined variables X_t. As a consequence, the vector of measurement errors disappears from the obervation equation (12) and, instead, the properly redefined matrix D^1 picks off the current and past values of the measurement errors affecting the observations on current and past output.

Appendix D Results of further sensitivity analysis

Table D1: Predictions in the Absence of Measurement Error

Forecast Horizon	RMSE (in percent)	
	Output	Inflation
0	0	0
1	0.55	0.23
2	0.84	0.41
4	1.18	0.80
8	1.49	0.93
16	1.66	0.99

Note: This table shows the RMSE of output predictions ($q_{t+h|t}$) and annual average inflation predictions ($\tilde{\pi}_{t+h|t}$) at a given forecast horizon h in the absence of measurement error.

Table D2: The Role of Money in Predicting Inflation

Forecast Horizon	RMSE			%ΔRMSE		
	Overall	I	II	Overall	I	II
0	0	0	0	0	0	0
1	0.23	0.02	0.23	0.01	-8.43	0.08
2	0.41	0.06	0.41	0.02	-8.13	0.17
4	0.80	0.16	0.78	0.07	-7.54	0.32
8	0.92	0.16	0.91	0.09	-5.89	0.25
16	0.98	0.07	0.98	0.10	-6.15	0.12

Note: This table shows the implications for predicting the annual average inflation rate ($\tilde{\pi}$) at a given forecast horizon when ouput is subject to measurement error.

Table D3: Sensitivity to the Income Elasticity of Money Demand

Information Measure	Revision Process	Scaling Factor for the Income Elasticity of Money Demand			
		1	2	4	8
%ΔRMSE	One-Period	-1.29	-4.88	-16.13	-38.99
	Three-Period Uncorrelated	-1.46	-5.71	-17.58	-38.08
	Three-Period Correlated	-1.59	-5.84	-17.46	-37.51
ΔR^2	One-Period	0.02	0.07	0.22	0.46
	Three-Period Uncorrelated	0.03	0.10	0.28	0.53
	Three-Period Correlated	0.02	0.08	0.23	0.45
Information Gain	One-Period	8.51	32.79	115.12	323.36
	Three-Period Uncorrelated	20.42	81.53	268.61	665.83
	Three-Period Correlated	10.29	38.96	124.21	304.21

Table D4: Optimal Indicator Weights when Current Output is Unobserved

Revision Process	$q_t^{(t)}$	$q_{t-1}^{(t)}$	$q_{t-2}^{(t)}$	$q_{t-3}^{(t)}$	π_t	μ_t
One-Period	—	0.052	0	0	0.030	0.474
Three-Period Uncorrelated	—	0.007	-0.029	-0.039	0.028	0.442
Three-Period Correlated	—	0.022	-0.038	-0.019	0.029	0.457

Note: This table shows the optimal indicator weights when money is included in the information set.

Table D5: The Information Role of Money when Current Output is Unobserved

			Information Content		
Revision Process	RMSE	R^2	%ΔRMSE	ΔR^2	Info. Gain
One-Period	0.58	0	-1.74	0.03	n.d.
Three-Period Uncorrelated	0.58	0	-1.64	0.03	n.d.
Three-Period Correlated	0.58	0	-1.69	0.03	n.d.

Note: n.d.: The measure of information gain is not defined.

Table D6: Optimal Indicator Weights for the Time Series Model

Revision Process	$q_t^{(t)}$	$q_{t-1}^{(t)}$	$q_{t-2}^{(t)}$	$q_{t-3}^{(t)}$	π_t	i_t^s	i_t^l	μ_t
Filtering without Money								
One-Period	0.167	-0.227	0	0	-0.050	-0.113	0.334	—
Three-Period Uncorrelated	0.084	-0.032	-0.023	-0.028	-0.055	-0.101	0.323	—
Three-Period Correlated	0.178	-0.073	-0.032	-0.020	-0.051	-0.082	0.273	—
Filtering with Money								
One-Period	0.165	-0.194	0	0	-0.028	-0.111	0.291	0.286
Three-Period Uncorrelated	0.084	-0.030	-0.044	-0.051	-0.029	-0.110	0.289	0.306
Three-Period Correlated	0.181	-0.065	-0.064	-0.016	-0.026	-0.089	0.239	0.299

Table D7: The Information Role of Money in the Time Series Model

			Information Content		
Revision Process	RMSE	R^2	%ΔRMSE	ΔR^2	Info. Gain
One-Period	0.40	0.21	-1.38	0.02	11.81
Three-Period Uncorrelated	0.42	0.12	-1.71	0.03	22.93
Three-Period Correlated	0.40	0.21	-1.83	0.03	15.46

Table D8: Weights for the Time Series Model with Unobserved Current Output

Revision Process	$q_t^{(t)}$	$q_{t-1}^{(t)}$	$q_{t-2}^{(t)}$	$q_{t-3}^{(t)}$	π_t	i_t^s	i_t^l	μ_t
Filtering without Money								
One-Period	—	-0.054	0	0	-0.059	-0.134	0.404	—
Three-Period Uncorrelated	—	-0.009	0.004	0.007	-0.060	-0.124	0.385	—
Three-Period Correlated	—	-0.021	0.010	0.005	-0.059	-0.127	0.390	—
Filtering with Money								
One-Period	—	-0.027	0	0	-0.034	-0.134	0.348	0.343
Three-Period Uncorrelated	—	-0.009	-0.019	-0.023	-0.033	-0.132	0.343	0.335
Three-Period Correlated	—	-0.014	-0.019	-0.007	-0.033	-0.132	0.344	0.342

Table D9: Money's Role in the Time Series Model with Unobs. Current Output

			Information Content		
Revision Process	RMSE	R^2	%ΔRMSE	ΔR^2	Info. Gain
One-Period	0.44	0.05	-1.62	0.03	56.20
Three-Period Uncorrelated	0.44	0.05	-1.46	0.03	60.57
Three-Period Correlated	0.44	0.05	-1.48	0.03	60.64