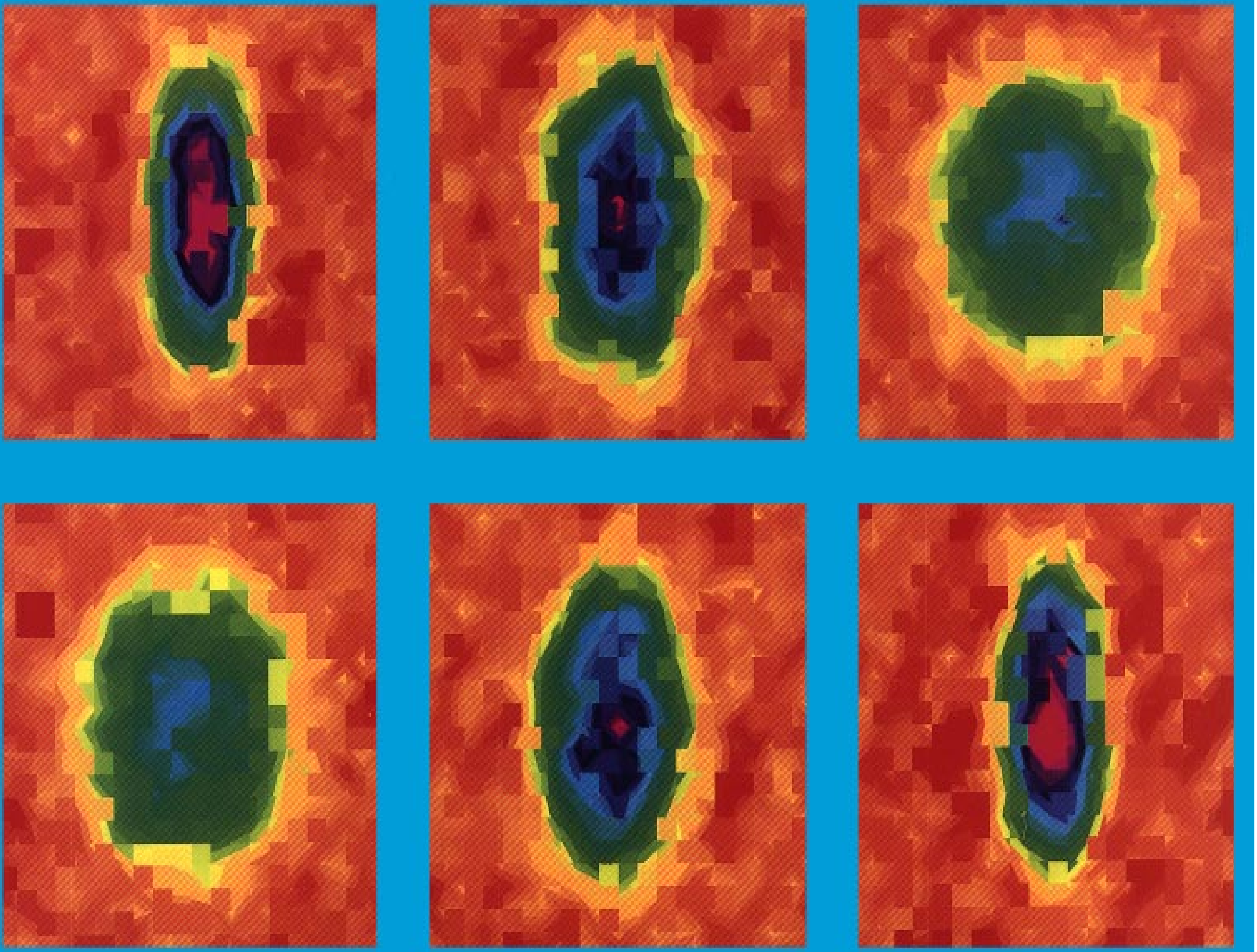
July-August 1996
Volume 101, Number 4

ISSN 1044-677X

Journal of Research of the National Institute of Standards and Technology

---

Special Issue: Bose-Einstein Condensation



United States Department of Commerce
Technology Administration
National Institute of Standards and Technology

The National Institute of Standards and Technology was established in 1988 by Congress to “assist industry in the development of technology . . . needed to improve product quality, to modernize manufacturing processes, to ensure product reliability . . . and to facilitate rapid commercialization . . . of products based on new scientific discoveries.”

NIST, originally founded as the National Bureau of Standards in 1901, works to strengthen U.S. industry’s competitiveness; advance science and engineering; and improve public health, safety, and the environment. One of the agency’s basic functions is to develop, maintain, and retain custody of the national standards of measurement, and provide the means and methods for comparing standards used in science, engineering, manufacturing, commerce, industry, and education with the standards adopted or recognized by the Federal Government.

As an agency of the U.S. Commerce Department’s Technology Administration, NIST conducts basic and applied research in the physical sciences and engineering, and develops measurement techniques, test methods, standards, and related services. The Institute does generic and precompetitive work on new and advanced technologies. NIST’s research facilities are located at Gaithersburg, MD 20899, and at Boulder, CO 80303. For more information contact the Public Inquiries Desk, 301-975-3058.

Cover: The figure on the cover depicts a time sequence of images showing one cycle of the ringing of a Bose-Einstein condensate (BEC) in the JILA TOP (time-averaged orbiting potential) trap after being driven by strong oscillations of trap potential. The top left image shows the earliest time, with time increasing from left to right. To create each image, a BEC was formed and then strongly driven and finally allowed to expand ballistically, after which a picture was taken. BEC excitations have recently been observed in both the JILA [D. S. Jin et al., Phys. Rev. Lett. **77**, 420 (1996)] and MIT [M.-O. Mewes et al., Phys. Rev. Lett. **77**, 988 (1996)] traps. The ringing frequencies of a BEC for weak driving agree well with zero-temperature mean-field theory. Cover photographs courtesy Eric Cornell, JILA, Boulder, CO 80309.

Journal of Research of the National Institute of Standards and Technology

Volume 101 Number 4 July–August 1996

Board of Editors

Barry N. Taylor
Chief Editor

Nancy M. Trahey, Technology Services
Richard J. Van Brunt, Electronics and Electrical Engineering Laboratory
Theodore V. Vorburger, Manufacturing Engineering Laboratory
John R. Moody, Chemical Science and Technology Laboratory
Ronald Collé, Physics Laboratory
Kenneth L. Jewett, Materials Science and Engineering Laboratory
Piotr A. Domanski, Building and Fire Research Laboratory
Alan H. Goldfine, Computer Systems Laboratory
Daniel W. Lozier, Computing and Applied Mathematics Laboratory
Matt Young, Boulder Laboratories
Chris E. Kuyatt, Washington Editorial Review Board

Donald R. Harris
Managing Editor

Julian M. Ives
Technical Production Editor

U.S. Department of Commerce—**Michael Kantor,** Secretary
Technology Administration—**Mary L. Good,** Under Secretary for Technology
National Institute of Standards and Technology—**Arati Prabhakar,** Director

Volume 101, Number 4, July–August 1996
Journal of Research of the National Institute of Standards and Technology

Bose-Einstein Condensation

Preface

Among the most remarkable effects that quantum mechanics adds to the catalog of the thermal properties of matter is the “condensation” of an ideal gas of identical particles into a single quantum state, the principle of which was discovered in the theory of statistical mechanics by Bose and Einstein in the 1920s.

Bose-Einstein condensation (BEC) is thus a mechanism for producing a macroscopic quantum system, and is exemplary of the macroscopic quantum phenomena of superconductivity and superfluidity. However, it has proven most difficult to attain BEC in the laboratory vs in the theory of the ideal gas. As discussed in the first article of this Special Issue, the basic criterion for BEC is that the typical distance between particles be comparable to or less than their deBroglie wavelengths; and in virtually every real material system, the effects of interparticle interactions become very strong when this criterion is attained, and the model of a ideal quantum gas is inapplicable.

The quest to produce BEC in dilute atomic gases began in the early 1980s with work on spin-polarized hydrogen. During the past decade, experimental techniques for optical trapping and cooling of atomic gases were developed to a high degree, and the combination of these techniques with evaporative cooling made it possible to attain BEC in dilute alkali vapors. The first successful such experiment, using rubidium, was performed at JILA, a joint institute between the National Institute of Standards and Technology (NIST) and the University of Colorado; its dramatically illustrated results were published in the scientific literature in July 1995, and immediately stimulated intense activity in the physics community and considerable interest among the public at large.

The first article in this Special Issue of the NIST Journal of Research contains the story of that experiment as told by the leader of the NIST effort, Eric Cornell. This article is an edited transcript of a colloquium presented by Dr. Cornell to a nonspecialist audience, and it conveys the sense of excitement that currently pervades the field in terms that are accessible to the general reader. The scientific rationale for BEC studies and the prospects for their practical application are aptly summarized in the concluding section of this article.

The year following the original discovery of alkali gas BEC has seen the production of BEC in at least one other gas, a growth by three orders of magnitude in the maximum attainable number of condensate atoms, and the beginning of experiments that probe BEC properties in detail. Such work continues at a rapid pace as this issue goes to press. About 200 papers on BEC have appeared on electronic preprint archives and in the scientific literature since the initial JILA report. Much of the excitement in this field is driven by its combination of elements of different subfields of physics—atomic, optical, condensed matter, statistical, even elementary particle—whose separate practitioners do not often identify a common topic as a cutting edge of research in their own fields. This Special Issue of the NIST Journal of Research is an attempt to assemble such diverse views in one package, which is a challenging enough task at this time—it will take somewhat longer to see broad agreement on issues emerge from such a group!

Dr. Cornell’s paper is a general introduction to the field of gaseous BEC. The other papers in this issue deal with four of the major themes with which this field is concerned, and they are presented in groups. The first group consists of reinvestigations of standard BEC theory, typically focussing on issues (such as inhomogeneity of the density) that were peripheral in the original theory of a uniform ideal gas, but are central to understanding the behavior of alkali BECs. The second group deals with the physics of atomic collisions at ultracold temperature, a topic that is extremely challenging both in theory and experiment, and is key to the understanding of alkali BEC at the microscopic level. Third is a group of papers that deal with quantitative modeling of BECs based on the microscopic parameters that are determined by collision physics; these provide the specific tests of current theoretical understanding against the results of experiment. Last are several papers that explore the BEC/laser analogy mentioned in the conclusion of Cornell’s article; practical exploitation of this analogy to develop a coherent source of matter waves or “atom laser” is one of the principal goals of current laboratory work.

Though by no means comprehensive, this collection of papers provides a representative introduction to current work on BEC. For secretarial and editorial assistance in its production, the editors are grateful to Mrs. Aija Roess, Mr. Robert Dodd, and the production staff of the NIST Journal of Research.

Keith Burnett
Mark Edwards
Charles W. Clark
Special Issue Editors

Contents

Special Issue: Bose-Einstein Condensation

Keith Burnett, Mark Edwards, and Charles W. Clark, Editors

Preface iii

Articles

Very Cold Indeed: The Nanokelvin Physics of Bose-Einstein Condensation	**Eric Cornell**	419
Methods for a Nonuniform Bose Gas	**Kerson Huang and Paolo Tommasini**	435
Theory of Interacting Quantum Gases	**H. T. C. Stoof, M. Bijlsma, and M. Houbiers**	443
Generalized Mean Fields for Trapped Atomic Bose-Einstein Condensates	**N. P. Proukakis and K. Burnett**	457
Effective Action Approach to Bose-Einstein Condensation of Ideal Gases	**Klaus Kirsten and David J. Toms**	471
Cold Binary Atomic Collisions in a Light Field	**Paul S. Julienne**	487
A Spectroscopic Determination of Scattering Lengths for Sodium Atom Collisions	**Eite Tiesinga, Carl J. Williams, Paul S. Julienne, Kevin M. Jones, Paul D. Lett, and William D. Phillips**	505
Estimating Bounds on Collisional Relaxation Rates of Spin-Polarized ^{87}Rb Atoms at Ultracold Temperatures	**Frederick H. Mies, Carl J. Williams, Paul S. Julienne, and Morris Krauss**	521
The Condensate Wave Function of a Trapped Atomic Gas	**F. Dalfovo, L. Pitaevskii, and S. Stringari**	537
Approximate Solutions of the Nonlinear Schrödinger Equation for Ground and Excited States of Bose-Einstein Condensates	**R. J. Dodd**	545
Zero-Temperature, Mean-Field Theory of Atomic Bose-Einstein Condensates	**Mark Edwards, R. J. Dodd, Charles W. Clark, and K. Burnett**	553
Aspect Ratio Analysis for Ground States of Bosons in Anisotropic Traps	**Kirill N. Ilinski and Alexander Moroz**	567
Near-Resonant Imaging of Trapped Cold Atomic Samples	**L. You and Maciej Lewenstein**	575
Atomic Phase Conjugation From a Bose Condensate	**Elena V. Goldstein, Katja Plättner, and Pierre Meystre**	583
Why a Condensate Can Be Thought of as Having a Definite Phase	**S. M. Barnett, K. Burnett, and J. A. Vaccarro**	593

News Briefs

GENERAL DEVELOPMENTS 601

APEC Seminar on TBT Agreement
NIST Conducts Training on Price Verification Procedure
Energy-Related Inventions Program Recommendations

NIST Role Ensures Agreement between U.S., IEC Standards for Measurement of Electric and Magnetic Fields in Support of Low-Frequency Bioeffects Research 602
MBE Metrology Chamber Established in Response to Industry Needs
Round Robin Confirms Accuracy of NIST-Developed Josephson Voltage Standards

NIST-Industry Consortium Formed to Provide U.S. Semiconductor Industry With Needed Overlay Measurement Capability 603
NIST Publishes New Edition of Directory of Law Enforcement Associations
Small Business Puts STEP in Production
High-Speed Milling

New Technology for Preparative DNA Separations 604
NIST-EPRI Ultrasonic Technology Assessment Program for Flow Measurement
Precision Na Lifetime Measurement
Atoms Form Finely Spaced Structures in Optical Standing Wave

NIST Develops a New Broadband Polarizer 605
Intrinsically Absolute Radiometry Using Correlated Photons

NIST Hosts Meeting of the IAEA Program on Quality Assurance in Radiation Processing 606
State-of-the-Art Neutron Diffraction Residual Stress Instrument
Patent for a Welding Control System Is Granted to NIST Researchers
Chemically Assisted Diamond Machining of Advanced Materials Project Enters Plant Trial Stage

Industry Workshop Addresses On-line Composite Process Monitoring 607
FASTLite in Demand
Management Guidance on Financial Systems Issued
Workshop on Quality and Uncertainty Sets Tone for Measurement Quality Conference

New Technique for Performing Static Analysis of Programs Developed 608
New Publication Describes Interoperability Experiments With CORBA
Five Additional Federal Implementation Guidelines for Electronic Data Interchange Issued
New Publication Describes Software Verification and Validation (V&V) in the Health Care Environment
Schroedinger's Cat in an Atomic Cage

Directory Lists Labs Meeting International Standards 609
NIST Semiconductor Metrology Efforts Listed
Russia, NIS Standards Executives Hosted by NIST
Future WTO Members Learn to Provide Data

First View of Molecular Assembly Captured 610
Report on ISO Environmental Standards Published
Arc-Light Sensing Makes Sense for Welding
CIRMS Plans Fifth Annual Meeting at NIST

New Consortium Eyes Integrated ‘‘Toolkit’’ 611
Quality Federal Procurement Policy Recommended
Entities Sought for EU Conformity Assessments
Upgrade to Improve Time Signal’s Coverage

New Intense Source of Cooled Ions Developed 612
Energy-Related Inventions Program Makes Recommendations
Installation of a High-Speed Machining Testbed at Private Company Production Facility Completed
Self-Lithographic Processing Used in Fabricating Multicomponent Arrays for Gas Sensing

NIST Hosts Workshop for International Atomic Energy Agency 613
NIST Hosts Workshop on Low-Level Light Standards for Luminometry

Highly Charged Ions Put to Use 614
Color Display Calibration Standard
Industrial Collaboration Results in Award

Unraveling Anisotropic Microstructures in Plasma-Sprayed Ceramic Deposits 615
Evaluation of Corrosion Fatigue Resistance of Alloys Used in the Pulp and Paper Industry
NIST Hosts Concept Planning Workshop for a Construction Technology Extension Program (CTEP)

New Edition of Life-Cycle Costing Handbook for Buildings and Building Systems 616
NIST Staff Find Effective Fire Retardant
Flickering Flames Documented by NIST Scientists

NIST Scientist Advances Knowledge in Field of Crystal Growth 617
NIST Begins Testing of Automated Software Test Generation Methods
Minimum Interoperability Specification for PKI Components
Report Introduces New Parallel Program Performance Tool

STANDARD REFERENCE MATERIALS 617
Human Serum SRM Now New and Improved

Standard Reference Material (SRM) 1249—Inconel 718 618
Standard Reference Material (SRM) 2433—Titanium Base Alloy 8Al-1Mo-1V

STANDARD REFERENCE DATA 618
Atomic-Scale Surface Structure Database Expanded

In Memoriam

Dr. Jean W. Gallagher of Technology Services served with distinction as a member of the Board of Editors of the Journal of Research for the past 5 years. We are saddened by her recent death.