

**Finance and Economics Discussion Series
Divisions of Research & Statistics and Monetary Affairs
Federal Reserve Board, Washington, D.C.**

**Tips from TIPS: the informational content of Treasury
Inflation-Protected Security prices**

Stefania D'Amico, Don H. Kim, and Min Wei

2008-30

NOTE: Staff working papers in the Finance and Economics Discussion Series (FEDS) are preliminary materials circulated to stimulate discussion and critical comment. The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors. References in publications to the Finance and Economics Discussion Series (other than acknowledgement) should be cleared with the author(s) to protect the tentative character of these papers.

Tips from TIPS: the informational content of Treasury Inflation-Protected Security prices*

Stefania D'Amico[†], Don H. Kim[‡], and Min Wei[§]

This version: February 27, 2008

JEL Classification: G12, E31, E43

Keywords: Term structure model; Inflation expectation; Inflation risk premium; SPF; Treasury Inflation-Protected Securities (TIPS)

*Part of the work on this paper was done while one of us (Kim) was at the Bank for International Settlements. We thank Andrew Ang, Claudio Borio, Mike Chernov, Jim Clouse, Greg Duffee, Peter Hördahl, Athanasios Orphanides, Frank Packer, George Pennacchi, Jennifer Roush, Brian Sack, Jonathan Wright, and seminar participants at the 2008 AFA Annual Meetings, Dallas Fed Conference on Price Measurement for Monetary Policy, ECB Workshop on Inflation Risk Premium, the 11th International Conference on Computing in Economics and Finance, and the San Francisco Fed for helpful comments or discussions. We also thank Michelle Steinberg for providing information from the NY Fed's survey of TIPS dealers. We alone are responsible for any errors. The opinions expressed in this paper do not necessarily reflect those of the Federal Reserve Board or the Federal Reserve System or the Bank for International Settlements.

[†]Division of Monetary Affairs, Federal Reserve Board, stefania.d'amico@frb.gov, +1-202-452-2567.

[‡]Division of Monetary Affairs, Federal Reserve Board, don.h.kim@frb.gov, +1-202-452-5223.

[§]Division of Monetary Affairs, Federal Reserve Board, Min.Wei@frb.gov, +1-202-736-5619.

Abstract

We examine the informational content of TIPS yields from the viewpoint of a general 3-factor no-arbitrage term structure model of inflation and interest rates. Our empirical results indicate that TIPS yields contained a “liquidity premium” that was until recently quite large ($\sim 1\%$). Key features of this premium are difficult to account for in a rational pricing framework, suggesting that TIPS may not have been priced efficiently in its early years. Besides the liquidity premium, a time-varying inflation risk premium complicates the interpretation of the TIPS breakeven inflation rate (the difference between the nominal and TIPS yields). Nonetheless, high-frequency variation in the TIPS breakeven rates is similar to the variation in inflation expectations implied by the model, lending support to the view that TIPS breakeven inflation rates are a useful proxy for inflation expectations.

1 Introduction

Since its inception in 1997, the market for Treasury Inflation-Protected Securities (TIPS) has grown substantially and now comprises about 10% of the outstanding Treasury debt market. Almost a decade's TIPS data thus accumulated is a rich source of information to academic researchers and market participants alike. Because TIPS are securities whose coupon and principal payments are indexed to the price level, information about yields on these "real bonds" has direct implications for asset pricing models, many of which are written in terms of real consumption. Meanwhile, real-time TIPS data have attracted much attention from policy makers and market participants as a source of information about the state of economy. In particular, the differential between yields on nominal Treasury securities and on TIPS of comparable maturities, often called the "breakeven inflation (BEI) rate" or "inflation compensation", has been often used in policy circles and the financial press as a proxy for the market's inflation expectations.

However, certain complications arise in interpreting the information from TIPS yields. First, TIPS might not have been "efficiently" priced, due to their lower liquidity, the relative newness of the TIPS market, and other factors. Second, besides such institutional idiosyncrasies, the interpretation of the TIPS breakeven rate as an inflation expectation depends on the validity of the "Fisher hypothesis", which states that the nominal yield is the real yield plus expected inflation. This relation is only approximate, however, as it ignores the potential correlation between inflation and the real economy. More formally, we can define the nominal yield as the sum of the real yield, expected inflation, *and* the inflation risk premium. The last component may be substantial; indeed, the presence of an inflation risk premium in nominal bonds, translating to an additional financing cost for Treasury to issue nominal bonds, was one of the argument for issuing TIPS in the first place.

These issues naturally lead to the following questions: (1) Can we take TIPS yields at face value? Are they suitable for use as an input for other studies that involve *ex ante* real interest rates? (2) Can we take the TIPS breakeven rate as a reasonable measure of inflation expectations? Do movements in the breakeven rates reflect "fundamentals" or extraneous factors?

The purpose of our paper is to provide quantitative evidence that bears on these questions. Specifically, we model the dynamics of nominal yields, inflation, and TIPS yields in a general

no-arbitrage term structure model setting, and examine the extent to which these data are consistent with each other. Furthermore, we seek to establish some basic facts about the real term structure and the inflation risk premia implicit in nominal bond yields and to obtain an estimate of the “liquidity premium” in TIPS yields.

Although there have been other studies that use a no-arbitrage framework to model the U.S. real term structure, they have not explicitly explored the efficiency of TIPS pricing or estimated a liquidity premium in TIPS yields. Also, these studies do not employ TIPS data in estimation, with the exception of Chen, Liu and Cheng (2005).¹ Studies such as Ang, Bekaert and Wei (2007a), Chernov and Mueller (2007) and Buraschi and Jiltsov (2005) produce “shadow real yields” which could be compared with the TIPS yields. However, the estimates of real yields (as well as other quantities, like the inflation risk premium) from these studies differ a lot from each other, and in many cases are too much at odds with the priors of practitioners, possibly indicating problems in the specification or estimation of the model.

In view of the fact the the pricing mechanisms behind nominal and real bonds are not well understood, in this paper we take a *statistical* perspective, and use flexibly specified affine-Gaussian latent-factor models, which may face less misspecification concerns than some of the models in the existing literature. Also, in the estimation stage, we strive to address the small sample and overfitting problems that can lead to poor results, by utilizing additional information in survey forecasts and experimenting with different auxiliary conditions to search for robust conclusions.

Our main results can be summarized as follows. In all the cases that we have examined, estimating the model taking TIPS yields at their face value fails to produce plausible estimates of inflation expectations or inflation risk premia. The difference between the observed TIPS yields and the model-implied real yields estimated without TIPS data indicates that the “liquidity premium” was quite large in the early years of TIPS’s existence, but has become smaller recently. This liquidity premium turns out to be difficult to account for within a simple rational pricing framework, suggesting that TIPS may not have been priced efficiently in their early years. Nonetheless, time variation in TIPS-based and model-implied breakeven rates are quite similar, suggesting that changes in the TIPS breakeven rates largely reflects changes in infla-

¹ Most of the existing studies (including Risa (2001) and Evans (2003)) with inflation-indexed bond data have focused on the UK term structures. A recent paper by Hördahl and Tristani (2007) explores the euro-area term structure using the French indexed bond data.

tion expectations or in the investors' attitude toward inflation risks, rather than being random movements.

The rest of this paper is organized as follows. In Section 2, we discuss the informational content of TIPS yields in simple terms as well as the related literature. Section 3 specifies the no-arbitrage approach to modeling inflation, nominal yields, and real yields jointly. Section 4 describes the empirical strategies for estimating the model and presents the empirical results. Section 5 provides additional discussion of the model estimates and the interpretation of the TIPS information, and Section 6 concludes. Throughout the main text, we strive to keep the discussion relatively non-technical, relegating most of the technical details to the Appendices.

2 TIPS: Preliminary considerations

2.1 TIPS breakevens as measures of expected inflation

Despite potential complications associated with the inflation risk premium and the liquidity premium, TIPS breakeven rates have been frequently used as a proxy for inflation expectations by policy makers and market practitioners. The minutes of FOMC meetings often take note of changes in TIPS yields since the previous meeting,² and it is not uncommon to see explicit references to TIPS breakeven rates in Fed officials' speeches.³ Similarly, TIPS breakeven rates are frequently cited in the financial press when discussing inflation expectations.

Such usages indicate that many practitioners find TIPS breakeven rates to be a plausible measure of market inflation expectations. Indeed, empirical evidence indicates that TIPS breakeven rates respond to news arrivals or important economic events in the "right" direc-

²For example, the minutes of the June 2006 FOMC meeting includes the following sentence: "Yields on inflation-indexed Treasury securities increased by more than those on nominal securities, and the resulting decline in inflation compensation retraced a substantial share of the rise that had occurred over the preceding intermeeting period."

³Fed Vice Chairman Kohn (2006)'s speech on April, 2006, for example, includes the following remark: "[L]onger-term inflation expectations remain well contained. For example, the median expected inflation rate during the next five to ten years, as reported in the University of Michigan's survey of consumers, has barely edged up in recent years... Meanwhile, inflation compensation for investors implied by the spreads between the rates on nominal and CPI-indexed Treasury notes at both five- and ten-year maturities also has not shown any tendency to move higher on balance."

tion.⁴ It is, however, difficult to tell from such event studies whether the *magnitude* of the reaction is right, which is also of considerable interest. This question about the magnitude of TIPS breakeven rates' reaction to data announcements is a part of the larger discussion about whether the *variability* and the *level* of TIPS breakeven rates are reasonable, to which we now turn.

One way to examine the reasonableness of the level and the variability of TIPS breakeven rates is to compare them with another measure based on survey forecasts of inflation. There are largely two kinds of surveys available. One is the Michigan survey, which polls households (consumers), and the other, such as the Blue Chip survey or the SPF survey, polls "professionals" (business forecasters). Figure 1(a) shows the the Michigan survey of long-term inflation forecast and the 10-year SPF inflation forecast, together with the 10-year TIPS breakeven inflation rate.⁵ It can be seen that until recently the TIPS breakeven rate has been lower than these survey forecasts. In the case of the Michigan survey, the TIPS breakeven rate has remained substantially lower throughout the 1999-2007 period.

[insert Figure 1 about here]

Note that comparing the *level* of TIPS breakeven rates with the Michigan survey may not be fair. An inflation measure can depend significantly on the definition of the consumption basket, how quality changes are taken into account, and other issues.⁶ While the TIPS breakeven rate relates to CPI inflation, the Michigan survey asks the participants for their views about the change in the "prices of things you buy" rather than inflation based on a specific price index. However, even the 10-year SPF inflation forecast, which is specifically about CPI inflation, tended to be higher than the 10-year TIPS breakeven rate until about 2004. This is surprising: one would have expected the TIPS breakevens to be *higher*, since it includes an inflation risk premium, which is normally believed to be *positive*.

One might argue that the fault lies with the survey forecast rather than with TIPS. Survey

⁴ For example, in the working paper version of Gürkaynak, Sack, and Swanson (2005), they find that a higher-than-expected core CPI data release typically leads to a rise in the breakeven rates, suggesting an upward revision in inflation expectations.

⁵ For professional's survey forecast of inflation, we shall use the SPF forecasts only, as the Blue Chip surveys (*Blue Chip Economic Indicators* and *Blue Chip Financial Forecasts*) are quite similar.

⁶ Note, for instance, that (annual) CPI inflation has been persistently higher than PCE inflation by about 40 basis points.

forecasts cannot be expected to be a completely reliable measure of inflation expectations. That said, a quick comparison of the 1-year ahead CPI inflation forecasting performance of the 5-year TIPS breakeven rates versus the SPF survey forecast⁷ for the 1999:Q1-2007:Q3 period produces the following root-mean-square errors (RMSE):

$$RMSE(\text{TIPS}^{5Y}) = 1.07, \quad RMSE(\text{SPF}^{1Y}) = 0.92.$$

Note that the survey measure has done better than the TIPS breakeven rate, making it difficult to dismiss offhand. A recent study by Ang, Bekaert, and Wei (2007b) also provides extensive evidence that surveys perform better in forecasting inflation than various model-based measures that they have examined.

TIPS breakeven rates differ from survey forecasts not only in levels but also in variability. Figure 1(a) shows that TIPS breakeven rates exhibit greater time variation than the survey forecasts. The Michigan survey may contain a substantial amount of sampling error and other noise.⁸ However, once we look beyond the monthly noise and focus on the systematic movement, the long-horizon Michigan survey forecast seems to be less variable than the TIPS breakeven rate. An even greater contrast with TIPS is offered by the 10-year SPF inflation forecast, which has been pretty much immobile in the 1999-2006 period. This may appear suspicious: even if long-term inflation expectations are “well anchored”, it is difficult to imagine that they have become virtually immovable as suggested by the SPF survey. Nonetheless, the qualitatively similar messages from the two surveys raise the possibility that the greater variability of TIPS breakevens may be a robust fact.

One possible explanation is that part of the TIPS breakeven rate variation is due to variations in the inflation risk premium rather than changes in inflation expectations. The difficulty with this argument is that the *level* consideration seems to leave little room for inflation risk premium, at least one that is positive.

⁷ A reliable TIPS breakeven rate for near-term maturities are not available (especially in the early years of TIPS), so we use the 5-year TIPS breakeven rate as a TIPS-based forecast of the 1-year inflation.

⁸ The group of Michigan survey participants changes from month to month. Also, the distribution of forecasts is extremely wide, containing many responses that are very high or very low (negative). For example, in the Jan 2007 Michigan survey, 15% of respondents predicted 0% or negative inflation in the next 12 months, while about 10% of respondents predicted inflation of 10% or higher.

2.2 Evidence on inflation risk premium

The above discussions highlight the potential importance of the inflation risk premium. Let us now review here what is known, or has been said, about the inflation risk premium in the existing literature:

One argument for the presence of the inflation risk premium in the early literature is based on the need to make sense of the high nominal interest rates in the 1980s. Many studies from that period took note of the “extremely high level” of real interest rates in the 1980s.⁹ As can be seen from Figure 1(b), inflation in the early 1980s fell below 4%. Although inflation expectations based on survey forecasts remained fairly high, nominal interest rates were much higher, e.g., about 13% for the 10-year yield in 1984. The Fisher hypothesis then implies a very high ex ante real rate, e.g., 6% or even higher. Unlike inflation expectations which are known to be quite persistent, it is difficult to think of mechanisms that would make the ex ante real rates similarly persistent or generate such high values of real interest rates,¹⁰ thus leaving room for a positive inflation risk premium. The possibility of a positive and substantial inflation risk premium in the 1980s seems to be also in line with observations (e.g., Goodfriend (1993)) that during these years the Federal Reserve had less credibility than in later periods, bond markets thus counting in an extra premium in nominal yields beyond those attributable to inflation expectations.

Theoretically, the inflation risk premium arises from a potential correlation between inflation and the real aggregate marginal utility. One simple way to assess the magnitude of the inflation risk premium is, therefore, modeling the inflation covariance risks in a (C)CAPM framework, either by comparing the expected excess returns on the nominal and real bonds,¹¹ or by estimating the risk premium due to covariances with the expected or the unexpected inflation shocks.¹² The estimated inflation risk premium ranges from 10 to 100 basis points. The interpretation of such results, however, hinges on the validity of the proxy used for the marginal utility of wealth. The general failure of the current generation of (C)CAPM models in accounting for the time-series and cross-sectional properties of equity returns casts some doubts on this framework. This methodology also ignores information from the entire yield

⁹ See, among others, Clarida and Friedman (1984), Blanchard and Summers (1984), and Poole (1988).

¹⁰ However, the findings in the later parts of our paper (especially Sec. 5) suggest that real term premia may help explain part of the puzzle.

¹¹ See, for example, Campbell and Shiller (1996)

¹² See, for example, Chen, Roll and Ross (1986), Ferson and Harvey (1991) and Chan (1994).

curve, and, given its traditional regression-based implementation, cannot be used to accurately pin down the inflation risk premium at longer horizons or generate a consistent estimate of the term structure of the inflation risk premia.

Lastly, the inflation risk premium can be estimated using a no-arbitrage term structure model, which avoids the aforementioned criticisms of (C)CAPM-type models by adopting a no-arbitrage approach and by modeling the entire yield curve simultaneously. These can be grouped into two categories: those that use indexed-bond data and those that do not.¹³ Due to the relatively short history of the indexed debt market in the U.S., there has not been a lot of “time-consistent” no-arbitrage model-based studies with TIPS data, Chen, Liu and Cheng (2005) being the only study we are aware of.¹⁴ Many of these studies find an inflation risk premium in nominal yields that is quite substantial, has a significant cross-sectional (maturity) dependence, and exhibits substantial variation over time.¹⁵ However, if we look at specific qualitative and quantitative aspects of the inflation risk premia, there is much disagreement between various studies. This disagreement is also reflected in real yields: for example, Ang, Bekaert and Wei (2007a), Buraschi and Jiltsov (2005), and Chernov and Mueller (2007) obtain model-implied real yields that are quite different from each other, underscoring how little is established about real yields and inflation risk premia in the extant literature.

Some of these studies may face misspecification concerns. For example, the model considered by Chen, Liu and Cheng (2005), which is similar to that of Richard (1978), assumes that the nominal short rate r_t^N is the sum of two factors that both follow square-root (CIR) processes: the instantaneous inflation expectation, π_t , and the real short rate, r_t^R . Although the shocks to π_t and r_t^R in Chen, Liu, and Cheng (2005)’s model are correlated (unlike Richard (1978)’s), this type of model does not permit a flexible feedback structure between the factors. In their model π_t and r_t^R are thus restricted to be univariate; hence the term structure of inflation expectations and the real term structure are each described by only one factor. Another potential problem with this type of model is that the market price of risk specification is too

¹³ See, e.g., Chen, Liu, and Cheng (2005), Risa (2001), and Evans (2003), Hördahl and Tristani (2007) for the former, and Ang, Bekaert and Wei (2007a,b), Buraschi and Jiltsov (2005), Campbell and Viceira (2001) for the latter.

¹⁴ Jarrow and Yildirim (2003) also model nominal yields, TIPS yields, and inflation jointly in an HJM framework, taking nominal and TIPS term structures as inputs. Their focus is on hedging and inflation derivative pricing rather than on real yield and inflation risk premium modeling.

¹⁵ See, e.g., Hördahl and Tristani (2007) and Risa (2001) for the non-US case, and Chen, Liu and Cheng (2005), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2007a), Chernov and Mueller (2007) for the US case.

restrictive (see, e.g., Duffee (2002)). A recent study by Ang, Bekaert, and Wei (2007b) finds that the 3-factor no-arbitrage models that they consider perform worse than not only survey forecasts but also many simpler (regression) models, suggesting there might be problems with some of the richer no-arbitrage models in the literature as well.

Existing studies may also face technical difficulties with estimation. In particular, the small-sample problems can have serious consequences on the behavior of the estimated inflation risk premia. The conventional estimation of stationary models has a tendency to understate the true persistence of the time series, making the process appear to be converging to their long-run means sooner than they actually do. This can lead to an artificially stable long-horizon inflation expectations; as a result, the inflation risk premium might pick up part of the variability of inflation expectations that is lost in the estimation.

In the remainder of the paper, we explore a joint nominal and real no-arbitrage term structure model and try to address these specification and estimation issues. Further, differently from Chen, Liu, and Cheng (2005) who take TIPS yields at the face value, we are mindful of the potential problems with this assumption.

3 Joint model of inflation and interest rates

3.1 Fisher hypothesis and beyond

It is useful to start with the Fisher hypothesis,

$$y_{t,\tau}^N = y_{t,\tau}^R + I_{t,\tau}, \quad (1)$$

where $y_{t,\tau}^N$ is the nominal yield at time t with a time-to-maturity τ , $y_{t,\tau}^R$ is the real yield (“ex ante real rate”), and $I_{t,\tau}$ is inflation expectation between time 0 and τ , i.e.,

$$I_{t,\tau} = E_t \left[\frac{1}{\tau} \log \left(\frac{Q_{t+\tau}}{Q_t} \right) \right], \quad (2)$$

where Q_t is the price level. Given the cross-sectional data on real and nominal yields at time t , one can introduce the interpolation schemes

$$y_{t,\tau}^N = f_t(\tau), \quad y_{t,\tau}^R = g_t(\tau) \quad (3)$$

and approximate inflation expectations as $I_{t,\tau} = f_t(\tau) - g_t(\tau)$. Thus, the Fisher relation (1), together with data on real and nominal bond yields, defines a simple way to compute inflation expectations for any maturity τ ;¹⁶ note that we do not need to model the dynamics of the yields in this setting.

As mentioned earlier, however, the Fisher hypothesis ignores potential correlation effects. To go beyond Fisher hypothesis, one needs to model the dynamics of real rates and inflation together. For this purpose, it is convenient to utilize a no-arbitrage relation between the so-called “pricing kernels” or “stochastic discount factors”. A real (nominal) pricing kernel, M_t^R (M_t^N), has the property that it gives today’s market value of a future payoff in real (nominal) terms.¹⁷ In particular, real and nominal bond prices, P^R and P^N , are given by

$$P_{t,\tau}^R = E_t(M_{t+\tau}^R)/M_t^R, \quad P_{t,\tau}^N = E_t(M_{t+\tau}^N)/M_t^N. \quad (4)$$

Note that in nominal terms, a real bond is an asset whose payoff is proportional to the price level. This means that the real and the nominal pricing kernels are linked by the “no-arbitrage relation”¹⁸

$$M_t^N = M_t^R Q_t^{-1}. \quad (5)$$

Using this relation, we have

$$P_{t,\tau}^N = E_t(M_{t+\tau}^R Q_{t+\tau}^{-1})/(M_t^R Q_t^{-1}). \quad (6)$$

Thus, by specifying the joint dynamics of M^R and Q , one can obtain the no-arbitrage-consistent nominal yields; equivalently, one could also specify M^N and Q to obtain the no-arbitrage-consistent real yields.¹⁹ From eq (6), it is straightforward to show that

$$y_{t,\tau}^N = y_{t,\tau}^R + I_{t,\tau} + \wp_{t,\tau}^I, \quad (7)$$

¹⁶ To have a manageable scope, in this paper we focus on inflation compensation based on *yields*. But we note that it is straightforward to compute the inflation compensation based on *forward rates* (called “forward inflation compensation”).

¹⁷ A pricing kernel (stochastic discount factor) M_t in the continuous-time formulation has the property that it prices an asset that pays X after τ period as $P_t = E_t(X_{t+\tau} M_{t+\tau})/M_t$. In discrete time, a pricing kernel for the next period $M_{t,t+1}$ gives the price of an asset that pays X at time $t + 1$ as $P_t = E_t(X_{t+1} M_{t,t+1})$.

¹⁸ See, Campbell, Lo, MacKinlay (1996, p443) for the derivation of the discrete-time version of this result.

¹⁹ “No-arbitrage” in this paper is in two senses: the consistency between real and nominal bonds as embodied by the relation (5), and the consistency between bond yields of various maturities.

where the inflation risk premium term $\varphi_{t,\tau}^I$ captures departures from the Fisher hypothesis.²⁰

It may be useful to discuss briefly the advantages and disadvantages of the no-arbitrage setup. The no-arbitrage principle is more general than the Fisher hypothesis, but in order to operationalize the no-arbitrage idea, one has to make assumptions about the dynamics of pricing kernels and the price level, and can expect to incur some amount of specification error in the process. For example, some of the studies in the literature adopt a structural specification of the real pricing kernel, M^R .²¹ While this approach has the potential benefit of making a definite connection between macroeconomic fundamentals and bond yields, it is not clear how well the current generation of structural models can capture the mechanisms that underlie asset price variation and inflation.²² Reduced-form models are not exempt from misspecification concerns, either. One example is Richard (1978)'s model, whose potential specification problems were discussed earlier in Sec. 2.2. Note also that if a model has too small a number of factors (e.g., a 2-factor model), it may have difficulty capturing the cross-section of the nominal yield curve well, resulting in sizable fitting errors and making it difficult to discuss actual market developments based on the model.

Although the Fisher relation in eq. (1) ignores the inflation risk premium, it leaves smaller room for cross-sectional fitting errors. Often it is unclear whether the specification errors in no-arbitrage models are any smaller than the error of ignoring the inflation risk premium. Indeed, we are not aware of any no-arbitrage model-based procedures for computing inflation expectations from bond yields that is as widely used by practitioners as the method based on the cross-sections of nominal and indexed bond yields (Fisher hypothesis). Even so, there are issues which the Fisher hypothesis simply cannot address. First, one cannot learn the dynamics of inflation expectations or the real and nominal term structures just from the cross-sectional

²⁰ The φ^I term is given by $\varphi_{t,\tau}^I = \tilde{J}_{t,\tau} + \tilde{c}_{t,\tau}$, where the covariance effect term is $\tilde{c}_{t,\tau} \equiv -(1/\tau) \log[1 + \text{cov}_t(M_{t+\tau}^R/M_t^R, Q_t/Q_{t+\tau}) / (E_t(M_{t+\tau}^R/M_t^R)E_t(Q_t/Q_{t+\tau}))]$ and the Jensen's inequality effect term is $\tilde{J}_{t,\tau} \equiv -(1/\tau)[\log(E_t(Q_t/Q_{t+\tau})) - E_t(\log(Q_t/Q_{t+\tau}))]$. Therefore, the Fisher relation obtains if we ignore the covariance effect and the Jensen's inequality effect.

²¹ For example, Buraschi and Jiltsov (2005) and Boudoukh (1993) use simple consumption-based models for M_t^R , of the form $M_t^R = e^{-\rho t} u'(C_t)$, where $u(C_t)$ is often a log utility or CRRA utility function. Buraschi and Jiltsov (2005) also have a specific mechanism for inflation dynamics, while studies like Boudoukh (1993) introduce an exogenous process for inflation dynamics and an exogenous correlation between inflation and consumption/output growth.

²² The still-ongoing debate about the equity premium puzzle is just one reminder of potential misspecification risk in structural models.

analysis of TIPS yields and nominal yields. Second, there are periods in which the indexed bond market did not exist, such as pre-1997 in the U.S. Third, even if the indexed bond market exists, one might wish to examine whether indexed bonds are priced efficiently. To address these issues, one needs a dynamic model like the no-arbitrage model.

3.2 No-arbitrage model

In this section we provide a “big picture” description of our model, relegating technical details of the model to Appendix A.

Basically, we view our modeling problem as a statistical one: find affine functions

$$y_{t,\tau}^N = a_\tau^N + b_\tau^{N'} x_t \quad (8)$$

$$y_{t,\tau}^R = a_\tau^R + b_\tau^{R'} x_t \quad (9)$$

$$I_{t,\tau} = a_\tau^I + b_\tau^{I'} x_t \quad (10)$$

of an n -dimensional *Gaussian* state vector $x_t = [x_{1t}, \dots, x_{nt}]'$ such that they are as general as possible, and at the same time consistent with no-arbitrage.²³ This can be done by specifying a general form for nominal pricing kernel M_t^N and the price level Q_t , and imposing suitable normalization conditions so that the model is econometrically identified (“maximally flexible”). We use the affine-Gaussian model for the nominal yields, and specify inflation as the sum of the instantaneous expectation, which is an affine function of the state vector, and the unexpected inflation. The real pricing kernel (hence the real term structure) is then determined by the no-arbitrage relationship in eq. (5). Here, the state variables x_{it} are statistical variables whose meaning is only implicitly defined by the data on nominal yields, inflation, and TIPS if available.

The affine-Gaussian model of the type used here is attractive as a model for capturing the variation in term premia (departures from the expectations hypothesis); it is reasonable to expect that some of the variation in the term premium in nominal bonds reflect variations in the inflation risk premia. Furthermore, the affine-Gaussian models allow for a flexible factor correlation structure, which is important since the departure from Fisher hypothesis involves a correlation effect.

²³ Note that the inflation risk premium, implicitly defined by eq. (7), is thus also affine in the state variables.

All risk factors (state variables) in our model are latent factors. This may sound unappealing as latent factors are not easy to interpret economically. While it would be desirable to be able to interpret the yield curve movements macroeconomically, in view of the fact that so little is empirically established about quantities like real yields and the inflation risk premia embedded in nominal yields (as discussed in Section 2.2), in this paper we set a more modest goal of “measuring” them. The latent factor approach seems attractive for this purpose, as it is more general than no-arbitrage models that use observed macro variables as risk factors. Kim (2007a) has argued that a substantial part of short-run inflation is unrelated to bond yields; hence the use of realized inflation as a risk factor (e.g., Ang, Bekaert and Wei (2007a) and Chernov and Mueller (2007)) involves strong assumptions.

In this paper, we focus on the three-factor ($n = 3$) case. Note that numerous variables could influence nominal yields, including the instantaneous inflation expectation, π_t , the real short rate, r_t^R , variables that affect their future movements (such as a time-varying perceived inflation target), variables that underlie time variation in the real term premium, the inflation risk premium, and inflation and interest rate uncertainties, among others. However, all these variables are not independent, and we can envision a relatively low-dimensional vector of latent factors summarizing the information in these variables. We do not mean to suggest that three factors are sufficient. Nonetheless, in view of the fact that many of the nominal-yields-only models in the literature have been estimated with 3 factors (e.g., Dai and Singleton (2000), Duffee (2002), Kim and Orphanides (2005)) and that fitting errors for nominal yields are fairly small for $n = 3$, the three-factor case can be viewed as an important benchmark to be explored. With fewer factors, the cross-section of yields would be fit less well, so that it might become harder to describe actual market movements. With large number of factors, on the other hand, empirical difficulties (e.g., overfitting concerns) may increase.

Traditionally, the real term structure is often modeled as of a lower dimension than the nominal term structure.²⁴ In this paper we let the real term structure have as many factors as the nominal term structure; if the real term structure is truly lower-dimensional than the nominal term structure, we let the data decide on that. A related point is that in a reduced-form setup like ours, one cannot make a distinction between the *real* and the *nominal* factors, as the correlation effect in a general model make such a distinction meaningless.

²⁴ For example, in the models of Chen, Liu, and Cheng (2005) and Campbell and Viceira (2001), the nominal term structure is described by a 2-factor model, while the real term structure is described by a 1-factor model.

While the affine-Gaussian model offers a rich and flexible framework for describing inflation and the interest rates, it does have a shortcoming, namely that it implies time-invariant interest rate volatilities and inflation uncertainties. Intuitively we would expect that the term premia in nominal and real bonds, as well as the inflation risk premia, would depend not only on the *price* of risk but also on the *quantity* of risk, but affine-Gaussian models assume the latter is constant. Nonetheless, affine-Gaussian models may still provide a reasonable estimate of various quantities of economic interest as documented by Duffee (2002) and others, despite the counterfactual assumption of constant yield volatility.

It would certainly be desirable to model time-varying uncertainty explicitly, but there may be a greater risk of misspecification as well as implementation difficulties with stochastic volatility models. Furthermore, in the affine model setting, there is a trade-off between a flexible factor correlation structure and flexible stochastic volatility modeling, as discussed by Dai and Singleton (2000). Modeling of stochastic volatility is also complicated by the debates as to whether and how to incorporate unspanned stochastic volatility (USV) effects.²⁵ Furthermore, as discussed in Kim (2007a), an explicit modeling of time-varying *inflation uncertainty* could be especially challenging, as the short-term and long-term inflation uncertainties may behave in a qualitative different manner. In view of these open issues with stochastic volatility models, we focus on the affine-Gaussian models in this paper, with the presumption that they would be useful benchmark results, before these “more advanced” issues are tackled.

4 Empirical results

4.1 Overall empirical strategy

Our model (sketched in Sec. 3.2 and spelled out in Appendix A) can be estimated *with* or *without* the TIPS data. If the model is estimated with the nominal yields and inflation but without the indexed-bond data, the resulting model-implied real yields can be viewed as the *shadow real yield*. This “hypothetical” yield can be useful in many contexts. For example, this is the relevant yield when thinking about the *ex ante* real interest rates implied by asset pricing models, the majority of which are written in real terms. In the case of pre-1997 U.S.,

²⁵ Collin-Dufresne and Goldstein (2002), Li and Zhao (2006), among others, argue for the need to incorporate the USV effects in the model, but Joslin (2007), Kim (2007b), and others, argue otherwise.

indexed bonds did not exist; therefore the “hypothetical” real yields are all one can talk about.

If indexed bonds are available, as in the U.S. post-1997, one can include them in the estimation, equating the model-implied real yields with the traded indexed-bond yield up to an error term. This can potentially improve the efficiency of the estimation. On the other hand, should the model-implied real yield and the traded indexed bond yield differ for some reason, or if there is a failure of relative pricing between the indexed bonds and the nominal bonds, the inclusion of TIPS in the estimation can lead to poorer results.

The conditions that the model be (1) Gaussian, (2) linear in some basis, and (3) maximally flexible (econometrically identified), lead to an almost unique specification that we can analyze.²⁶ However, the implementation of the model faces many challenging issues, and the estimates can depend materially on the implementation. In particular, the conventional (Kalman-filter-based) ML estimation of the model, using just nominal yields and inflation (with or without TIPS) data, leads to poor estimates which most practitioners would dismiss immediately. For example, as mentioned briefly in Sec. 2.2, one often obtains long-horizon expectations of inflation and interest rates that are too stable and fixed near the sample means of these variables. Kim and Orphanides (2005), for example, provide Monte Carlo evidence that conventional estimations tend to understate the variability of long-horizon short rate expectations and overstate the variability of term premia.

The problem is that due to the persistence of interest rates and inflation, a typical sample used in the literature (e.g., 15 years’ data) is not long enough, no matter how frequently it is sampled. Basically, we have a “small-sample problem” which manifests itself in two ways: (1) a *biased* estimate that leads to artificially stable long-horizon expectations, and (2) a very *imprecise* estimate, arising from the fact that parts of the model that are important for the description of physical (real-world) dynamics of interest rates and inflation are difficult to estimate reliably. Furthermore, for a 3-factor model, the number of parameters to be estimated is already quite large, raising concerns about overfitting.²⁷

Another difficulty is with the *evaluation* of the estimated model. Though in-sample and

²⁶ We add the term “almost”, because in addition to the popular normalization that has the mean reversion matrix \mathcal{K} with real eigenvalues, one could have other (inequivalent) normalizations in which one has the mean reversion matrix with complex eigenvalues, as discussed in Appendix A.

²⁷ On the other hand, restricting the model in ad hoc ways or using simpler models risks strong assumptions materially affecting the model output, as discussed in Sec. 3.2.

out-of-sample forecast root-mean-square errors (RMSE) can help detect problematic models, *selecting* a model (estimate) based on the “smallest RMSE” criterion would be inadvisable: in-sample RMSEs may have been artificially pushed down due to the use of the future information and the especially flexible nature of latent factor models, and out-of-sample RMSEs may have low statistical power especially in view of the considerable volatility of short-run inflation in the recent several years. Testing for a bias in the forecast is also ambiguous: an unbiased forecast has been traditionally regarded as desirable in the academic literature, but a mild bias may be a more realistic description of the market expectations in the sample period considered here.²⁸

In light of these considerations, rather than relying on a single implementation, we shall explore several different implementations of the model (different options for the data and the auxiliary conditions for addressing the small-sample problems) and seek to establish relatively robust empirical conclusions, focusing on the basic question of whether disparate pieces of input data can be made consistent with reasonable priors.

4.2 Data and estimations

We use 3- and 6-month, 1-, 2-, 4-, 7-, and 10-year nominal yields and CPI-U data from January 1990 to March 2007. When TIPS yields are used, they cover a shorter period from either January 1999 or January 2005 to March 2007, and are treated as missing observations in the rest of the sample. Both the nominal and the TIPS yields are based on zero-coupon yield curves fitted at the Federal Reserve Board²⁹ and are sampled at the weekly frequency, while CPI-U inflation is available monthly and assumed to be observed on the last Wednesday of the current month.³⁰ Due to the complications associated with the shorter-maturity TIPS yields which are discussed in detail in Appendix B, only the 10-year (zero-coupon) TIPS yield is used in estimations with TIPS data. This focus on the 10-year TIPS yield also reflects our special interest in long-term inflation expectations; as discussed in Kim (2007a), a key information embedded in bond yields is about the “trend component” of inflation, which can be better proxied by long-term, rather than short-term, inflation expectations. Because the model we

²⁸ Model evaluation difficulties are further discussed in Kim (2007a).

²⁹ See Gürkaynak, Sack, and Wright (2007a, 2007b) for details.

³⁰ Here we abstract from the real-time data issue by assuming that investors correctly infer the current inflation rate in a timely fashion.

estimate does not accommodate seasonality, we use the seasonally-adjusted CPI inflation in the estimate. TIPS are indexed to non-seasonally-adjusted CPI, but our use of seasonally adjusted CPI is not expected to matter much for a relatively long maturity like 10 years.

The sample period 1990-2007 was chosen as a compromise between having more data so as to improve the efficiency of estimation, and having a more homogeneous sample so as to avoid possible structural breaks in the relation between term structure variables and inflation (e.g., the 1979-83 episode of Fed's experiment with reserve targeting). This sample period roughly spans Greenspan's tenure and a little bit of Bernanke's as well.

When TIPS data are used in the estimation, the zero-coupon TIPS yield, denoted $y_{t,\tau}^T$, is taken to be

$$y_{t,\tau}^T = y_{t,\tau}^R + \Delta_\tau^R + \epsilon_{t,\tau}^R, \quad \epsilon_{t,\tau}^R \sim N(0, \delta_{R,\tau}^2). \quad (11)$$

where $y_{t,\tau}^R$ is the model implied real yield, Δ_τ^R is an allowance for a constant liquidity premium, and $\epsilon_{t,\tau}^R$ represents the measurement errors or the model fitting errors. The measurement error standard deviation for TIPS, $\delta_{R,\tau}$, and the constant liquidity premium term, Δ_τ^R , are determined inside the estimation, as are the measurement error standard deviation $\delta_{N,\tau}$ for nominal bond yields.³¹

To address the aforementioned small-sample and overfitting problems, we supplement the nominal yields, TIPS yields, and CPI data with survey data on the forecasts of future short-term (3-month) nominal rates in all estimations reported here.³² We experiment with additionally including the survey forecasts of inflation in the estimation.³³ These survey-based forecasts are quite straightforward to incorporate within the Kalman-filter-based maximum-likelihood framework: they are taken as the model's forecast plus a measurement error, i.e., we assume

$$E_t^{svy}(y_{t+u,3m}^N) = E_t(y_{t+u,3m}^N) + \epsilon_{t,u}^F, \quad \epsilon_{t,u}^F \sim N(0, \delta_{F,u}^2) \quad (12)$$

$$I_{t,\tau}^{svy} = I_{t,\tau} + \epsilon_{t,\tau}^I, \quad \epsilon_{t,\tau}^I \sim N(0, \delta_{I,\tau}^2), \quad (13)$$

where the superscript *svy* denotes survey forecasts.

³¹ The observed nominal yields are modeled as $y_{t,\tau}^N + \epsilon_{t,\tau}^N$, $\epsilon_{t,\tau}^N \sim N(0, \delta_{N,\tau}^2)$.

³² Some of the output from the conventional estimation of our model (without the use of any auxiliary data) are available upon request. Kim and Orphanides (2005) present some results from a conventional estimation of the nominal-yields-only model and discuss their problematic aspects.

³³ In the term structure estimation literature, Pennacchi (1991) is the first paper to use survey forecasts of inflation.

As in Kim and Orphanides (2005), we use the 6-month- and 12-month-ahead forecasts of the 3-month T-bill yield constructed from Blue Chip Financial Forecast that are available monthly, letting the estimation decide the size of the measurement errors $\delta_{F,u=6m}$, $\delta_{F,u=12m}$. The long-horizon forecast of the 3-month T-bill yield (available twice a year) is also used, with the measurement error $\delta_{F,long-term}$ fixed at a fairly large value of 0.75% at an annual rate.

For estimations that also incorporate the survey inflation forecast information, we use the survey forecasts of business economists instead of the consumer survey forecasts, for reasons discussed earlier. Specifically, we use the 1- and 10-year inflation forecasts from the SPF. We could let the size of the measurement errors be free variables to be estimated. However, given that our interest is in uncovering information about inflation expectations contained in nominal and TIPS yields, we fix the measurement error variability at a large, but not irrelevant, value of 0.75% at an annual rate to avoid making the survey inflation forecast information too influential. This can be viewed in the Bayesian spirit as providing a quasi-informative prior. In sum, we perform two versions of estimation in this paper:

$$\text{I :} \quad \delta_{I,1y} = \infty, \quad \delta_{I,10y} = \infty \quad (14)$$

$$\text{II :} \quad \delta_{I,1y} = 0.75\%, \quad \delta_{I,10y} = 0.75\%. \quad (15)$$

The measurement error size at ∞ (or at a very large value) for version-I corresponds to not using the survey inflation forecast data.³⁴

4.3 Estimation results

In the rest of the paper we examine the results from five estimation methods with different auxiliary conditions: the version I estimation without TIPS data (denoted *NT-I*), the version II estimation without TIPS data (denoted *NT-II*), the version-I estimation with 10-year TIPS yield from 1999 (denoted *T99-I*), and the version-II estimation with 10-year TIPS yield from 1999 (denoted *T99-II*). For reasons that will become clear at the end of this section and in Sec. 5, we also examine an estimation using TIPS data starting from 2005 (denoted *T05-II*). Table 1 provides a summary.³⁵

³⁴ The choice of 75 basis points for $\delta_{I,1y}$, $\delta_{I,10y}$ in version-II, as well as $\delta_{F,long-term} = 0.75\%$, are admittedly somewhat arbitrary, but we have experimented with other choices and obtained similar results.

³⁵ In all estimations in this paper, in order to facilitate the estimation and also to make the results easily replicable by others, we perform a “pre”-estimation with only the nominal yields and the survey forecasts of 3-month

[insert Table 1 about here]

The parameter estimates and the corresponding standard errors from all five estimations are given in Appendix C, while Appendix D provides some discussions on the unconditional moments of various term structure variables as implied by the model.

4.3.1 Estimations without TIPS

Let us first examine the estimations without the TIPS data (NT-I and NT-II). As can be seen from Figures 2 and 3, the results from these two estimations are broadly similar. For example, comparing the top left panels in both figures shows that the 10-year inflation expectations in both estimations trend down from about 4% in early 1990 to about 2% by mid 2005, picked up again since then and now average around 2.5%, largely in line with the SPF long-horizon inflation forecast. This might not be surprising for NT-II, as survey inflation forecasts were used in estimation, albeit with an allowance for arguably large deviations from the true forecasts. It is encouraging to see that NT-I also generates a similar downward trend rather than a flat long-term forecast; apparently the survey *interest rate forecast* information is helping to pin down those parameters that are relevant for describing long-horizon inflation expectations. In both NT-I and NT-II, the “term spread” between the 1-year and 10-year inflation expectations is fairly narrow, a feature that is also shared with the survey-based inflation forecasts.

[insert Figure 2 about here]

The NT-I-implied inflation expectations are somewhat lower than both the model forecasts from NT-II and SPF forecasts, which in turn implies a somewhat better in-sample inflation forecasting performance by NT-I relative to that of NT-II, since realized inflation tended to be low in much of the 1990s. This does not necessarily put the NT-I results in a better light, as such a result is likely due to a look-ahead bias.³⁶

T-bill yields to obtain a preliminary estimate of the parameters underlying the nominal yield curve model. From these estimated parameters and nominal yields data, we can obtain a preliminary estimate of x_t . From the regression of monthly inflation onto x_t , we can obtain a preliminary estimate of the parameters related to the inflation dynamics. These estimates are then used as initial parameter guesses in the full (one-step) estimation of all parameters.

³⁶ When we compared the out-of-sample inflation forecast performance of NT-I and NT-II in the 1999-2007 period (estimated with samples from 1990), the NT-I estimation results were less stable over time than those of

The model-implied 10-year breakeven rates, defined as the difference between the model-implied nominal and real yields, are shown in the top right panels of Figures 2 and 3, together with the 10-year TIPS breakeven rate. Based on both NT-I and NT-II, there is a level difference between the model-implied and TIPS breakeven rates up to about 2004, with the latter being substantially lower than the former. Nonetheless, the time variation of the two are broadly similar. For example, both the model-implied and the TIPS breakeven rates peak locally at the beginning of 2000, in the middle of 2001 and 2002, and so on, and the scales of their variation are also similar. Since late 2004, the two series largely move together within 30 basis points from each other.

[insert Figure 3 about here]

The bottom left panels show the inflation risk premia at the 1- and 10-year maturities. In both NT-I and NT-II, the 10-year inflation risk premium is positive and fluctuates in a range of about 50 basis points in the 1990-2007 period. It is also encouraging that for maturities of 1-year and below, the inflation risk premium is quite small.

The model-implied real yields are plotted in the bottom right panels for the 1- and 10-year maturities. A comparison across maturities reveals that the model-implied real yield is more variable at the shorter maturity, with changes in the real yield accounting for about 75 (60) percent of the variation in the nominal yield at the 1-year (10-year) maturity, which lends some support to the usual wisdom that expected inflation affects the longer-term nominal yields to a larger extent (see, for example, Fama (1975) and Mishkin (1981)). The model-implied real yield is also highly correlated with the 10-year inflation expectation.

4.3.2 Estimations with TIPS

Next we look at how the model implications change when TIPS yields are used in the estimation at their face value, up to a constant liquidity premium. Figures 4 and 5 display the corresponding results from the estimation with 10-year TIPS yield data from 1999-2007, either with or without the survey inflation forecast data (T99-II and T99-I).³⁷ The model-implied

NT-II, and the NT-I forecasting errors were larger.

³⁷ The earlier part of the sample for which TIPS data are unavailable (i.e. 1990-98) is viewed as a case of missing data.

inflation expectations from the T99-I estimation, shown in the top left panel of Figure 4, are not very plausible. Few would take seriously the notion that the 10-year inflation expectation has not changed much in the past 15 years. The near constancy of model-implied long-term inflation expectation observed here arises from the tension between the upward trend in the TIPS breakeven rates, shown in the top right panel, and the downward trend in the long-term inflation expectation implied by nominal yields and interest rate forecasts, demonstrated by the NT-I estimation and shown in the top left panel of Figure 2. In addition, the variation in the 1-year inflation expectation is too large to be credible, and the level of the 1-year inflation expectation in early 1990s is too low. Comparison with the survey forecasts makes the last two points clear.

[insert Figure 4 about here]

The top right panel of Figure 4 points to further problems with the T99-I estimation. While the T99-I-based breakeven rate matches the “level” of the TIPS breakeven rate better than the NT-I and NT-II estimations, it misses the short-run time variations in the TIPS breakeven rates; in fact it is almost constant over time.³⁸ In addition, the model-implied inflation risk premium is *negative* and *increases* over the sample period. This is at odds with the general perception that the inflation risk premium has been historically *positive*. As discussed in Sec. 2, the inflation risk premium likely was positive and substantial in the early 1980s and probably has come down since then, whereas we observe almost the opposite behavior in the bottom right panel of Figure 4. The 95% confidence interval for the 10-year inflation risk premium in this estimation is shown in Figure A3 in Appendix C. Even allowing for sampling uncertainties, the results still appear implausible.

The results from the T99-II estimation, which uses survey inflation forecast data, are shown in Figure 5. The model-implied 1-year and 10-year inflation expectations, shown in the top left panel, are now in better accordance with survey forecasts. However, the model-implied breakeven rate, shown in the top right panel, again misses most of the short-run variability of the TIPS breakeven rate, and the inflation risk premia, shown in the bottom right

³⁸ In view of the flexible nature of latent-factor model used in this paper, there may be another local maximum of the likelihood function in which the TIPS yields are fitted better, producing a closer match between the model-implied breakeven rate and the “measured” breakeven rate. However, such a fit would have to come at the expense of other features.

panel, are even more unreasonable, implying a 10-year inflation risk premium of about -2% in 1990. Even the “short-term” (1-year) inflation risk premium departs a lot from zero, being quite negative in 1990.

[insert Figure 5 about here]

We have experimented with other auxiliary estimation conditions (e.g., using the 5-year and 7-year TIPS yields in addition to the 10-year TIPS yield, using different measurement error structure for survey forecasts, etc.), but obtained similarly implausible results for inflation expectations and/or inflation risk premia. Basically, when TIPS data from 1999 to 2007 are taken at the face value up to a constant liquidity premium, it seems impossible to obtain sensible and consistent results within a 3-factor no-arbitrage model setting.³⁹

As we can see from NT-I and NT-II estimations without TIPS data, the model-implied and TIPS breakeven rates line up reasonably well in the last several years. Thus, we also perform an estimation using the 10-year TIPS yield only from 2005 to the present (T05-II). The results from this estimation are shown in Figure 6. This shorter-sample-TIPS estimation produces more reasonable inflation risk premium estimates and agrees with the NT-I and NT-II estimation results better: The T05-II-implied 10-year breakeven rate (upper right panel of Figure 6) now shows similar variations as the TIPS breakeven rate, and contrasts sharply with those of T99-I and T99-II estimations. The 10-year inflation risk premium based on the T05-II estimation (lower left panel of Figure 6) is somewhat lower than those from NT-I and NT-II estimations, but its overall variation is similar.

[insert Figure 6 about here]

³⁹ In some sense, these findings mirror those Chen, Liu, and Cheng (2005), who take TIPS at face value and obtain results that are also quite implausible. For example, their estimate of instantaneous inflation rate drops to near-zero level in 2001 and stays there until mid-2003. (Besides potential problems with taking TIPS at face value, specification problems and other issues may have also contributed to this result.)

5 Further discussions

5.1 Real yield and inflation risk premium

One common feature among the alternative estimation results reported in the previous section is that the model-implied 10-year real yield (long-term real yield) moves around substantially, and more specifically, it mirrors much of the movement in the 10-year nominal yield. This is the case with the 10-year TIPS yield as well. Figure 7 illustrates this point by comparing the 10-year nominal yield with the model-implied 10-year real yields from the NT-II and T05-II estimations (panel (a)) and the TIPS 10-year yield (panel (b)). The strong positive covariation between the nominal yield and the model-implied real yield (or TIPS yield) works toward *reducing* the variability of the breakeven inflation (the differential between the nominal yield and the real yield); even so, the breakeven inflation rate *is* more variable than the survey long-term inflation expectation, as we have discussed in Sec. 2.1.

[insert Figure 7 about here]

Although the feature that real yields tend to track nominal yields may appear unremarkable to practitioners who largely equate the TIPS yield with the real yield, that is not the case in the extant academic literature. For example, it is harder to see a similarity between the long-term real yield and the nominal yield in Ang, Bekaert and Wei (2007a, Figures 1 and 2). Even more striking is the result of Chernov and Mueller (2007, Figure 7), in which their preferred model (Model AO) generates a 10-year real yield that stays almost constant during the entire post-1970 sample period. Note that the real yields from these studies would imply an *even more variable* breakeven inflation rate, which is even harder to reconcile with the survey evidence.

The term structure model allows us to decompose the real yield into a “real expectations” component and a “real term premium” component, i.e.,

$$y_{t,\tau}^R = y_{t,\tau}^{R,EH} + (y_{t,\tau}^R - y_{t,\tau}^{R,EH}), \quad (16)$$

where $y_{t,\tau}^{R,EH}$ is the expected average future short rates over the life of the bond,

$$y_{t,\tau}^{R,EH} = (1/\tau) \int_t^{t+\tau} E_t(r_{t+s}^R) ds, \quad (17)$$

while the terms inside the parentheses in eq. (16) represent deviations from the expectations hypotheses. The variance decomposition results, reported in the first two columns of Table 2, show that changes in the expected future short rates account for most of the variation in real yields at shorter maturities, while much of the variation in the 10-year real yield is due to movement in the real term premium.

[insert Table 2 about here]

Figure 8 plots the decomposition of the 10-year real yield, based on the NT-II and T05-II estimations. In both cases, the real expectations component displays a fairly “stationary” behavior, the variation being mainly the ups and downs associated with business cycles. The real term premium, on the other hand, displays a visible downward trend. This may be due in part to the gradual reduction in risk, manifested as declining uncertainty associated with key macroeconomic variables since the Volcker disinflation, a phenomenon often dubbed the “great moderation.”⁴⁰ The real term premium component also contains substantial short-run variation. In some sense, this is not surprising: a visual inspection of the 10-year TIPS yield and the 10-year nominal yield in the 1999-2007 period shows a lot of similar variation in them; thus, if we accept that nominal yields contains a substantial amount of term premium variation (as indicated by various expectations hypothesis tests), we can expect a similar effect for real yields as well.

[insert Figure 8 about here]

Let us now turn to the inflation risk premium. It is worth noting that, although our estimates of the inflation risk premium (NT-I, NT-II, T05-II) display interesting variation over time, our inflation risk premium is a *less prominent* driver of the 10-year nominal yield than in some other studies. For instance, Chernov and Mueller (2007)’s results imply that most of the variation in the 10-year nominal yield is due to the variations in the inflation risk premium, because of the near-constancy of their long-term real yield estimates. Our inflation risk premium estimates also differs substantially from those in Ang, Bekaert and Wei (2007a, Figure 5). In particular, their estimates sometimes show sharp jumps even though no such changes are observed in the corresponding nominal yields (e.g., early 1987, early 1990 and late 2000).

⁴⁰ See Bernanke (2004) for a discussion of this phenomenon from a policy maker’s perspective.

Intuitively, the inflation risk premium can be expected to be positively linked to inflation uncertainty. We therefore correlate our inflation risk premium estimates from the NT-II and the T05-II estimations with two measures that could be viewed as proxies for such uncertainties. Our first measure of inflation uncertainty is the dispersion of long-term inflation forecasts, constructed as the distance between the average of the top ten forecasts and that of the bottom ten forecasts of CPI inflation 5 to 10 years hence from the Blue Chip Economic Indicators survey, available twice a year since 1987. As can be seen from Figure 9(b), this measure was trending lower during much of the 1990s and has been relatively stable since 2000, suggesting diminished inflation uncertainties in recent years. Inflation risk premium estimates based on the NT-II estimation are highly correlated with this measure with a correlation coefficient of 0.55. The corresponding correlation coefficient based on the T05-II estimates, however, is negative at -0.15 over the full post-1990 sample. A closer inspection reveals that the negative correlation comes primarily from the upward trend in the T05-II inflation risk premium estimates during the early sample period. Using data after 1996, the correlation coefficient becomes highly positive at 0.45.

Our next measure is the absolute implied volatility of the 10-year swaptions with an underlying swap tenor of one year (Figure 9(b)), which has been available since 2000. This variable measures long-run uncertainty associated with future nominal interest rates, and can also be viewed as a rough measure of inflation uncertainty if we are willing to assume that investors are relatively certain about future real yields. Our inflation risk premium estimates are also positively correlated with this measure with correlation coefficients of 0.63 and 0.58, based on the NT-II and T05-II estimations, respectively.

[insert Figure 9 about here]

Finally, we could assess the relative contribution of each of the three components of the nominal yield in eq. (7) by decomposing the variance of the nominal yield into its covariances with the real yield, inflation expectation, and the inflation risk premium, respectively:

$$\text{var}(y_{t,10y}^N) = \text{cov}(y_{t,10y}^N, y_{t,10y}^R) + \text{cov}(y_{t,10y}^N, I_{t,10y}) + \text{cov}(y_{t,10y}^N, \phi_{t,10y}^I).$$

The last four columns of Table 2 report the results from this variance decomposition for yield maturities of 3 months, 1, 2, 5 and 10 years based on in-sample moments and the NT-II model estimates.⁴¹ Real yield movement accounts for about 3/4 of the variation at the shorter end of

⁴¹ Results based on the T05-II estimation are similar.

the yield curve. At longer maturities, real yield variation continues to play a dominant role, although changes in the inflation risk premia become more important. The contribution of expected inflation stays relatively stable at about 30 percent at all maturities.⁴² Note that the contribution of the inflation risk premium is modest (e.g., 6 percent for 10-year yield), consistent with our earlier remark that the inflation risk premium in our paper is a less prominent driver of nominal yields than in some other studies.

5.2 TIPS “liquidity premium”

The problems we have encountered with the T99 estimations in Sec. 4.3.2 indicate that it is difficult to equate TIPS yields with “hypothetical” real yields that are implicit in nominal bond prices and that *there exists a component in TIPS yields that is not captured by our model*. This conclusion is further supported by simple regression evidence.⁴³ A regression of the 10-year breakeven rate (1999-2007) on 3-month, 2-year, and 10-year nominal yields, i.e.,

$$y_{t,10y}^N - y_{t,10y}^T = c_o + c_1 y_{t,3m}^N + c_2 y_{t,2y}^N + c_3 y_{t,10y}^N + e_t \quad (18)$$

gives an R^2 of only 33%; i.e., a significant part of breakeven rate variation is unexplained by nominal yield curve factors.

From the more reasonable NT and T05 estimations we can obtain an estimate of the “TIPS-specific component”. Specifically, we take the the differential between the TIPS yield and the model-implied real yield, i.e.

$$L_{t,\tau} = y_{t,\tau}^T - y_{t,\tau}^R, \quad (19)$$

and explore the interpretation of this variable as a measure of the TIPS liquidity premium, though it could be also reflecting other unaccounted-for effects. We view an examination of this measure as a useful first step before attempting to model TIPS-specific factors explicitly within the no-arbitrage framework.

Figure 10 shows this object for maturities of five and ten years based on the NT-II and T05-II estimations. These liquidity premium estimates are qualitatively similar across the two estimations and exhibit several interesting features. Both the 5-year and 10-year liquidity premia were large until about 2002 and then came down to a level close to zero. The dashed

⁴² However, if unconditional variances are used instead in the decomposition, the inflation expectation component has a higher weight, because of the highly persistent nature of the inflation expectation dynamics.

⁴³ We thank Greg Duffee for this point.

lines in the figure mark the “old level” and the “new level”. The 5-year liquidity premium appears more jagged, possibly due in part to the fact that the 5-year TIPS yield is more “contaminated” by the indexation lag effect. The 10-year liquidity premium also contains some relatively high-frequency variations, perhaps reflecting the limitation of our 3-factor model. Still, the large magnitude of $L_{t,\tau}$ in the early years and the clearly visible trend component appear to be fairly robust findings.⁴⁴

[insert Figure 10 about here]

To examine the validity of our interpretation of this variable as primarily a measure of TIPS liquidity premium, we regress $L_{t,10y}$ (from NT-II and T05-II) onto a set of variables that may be related to the liquidity conditions in the TIPS market: the 3-month moving average of the weekly turnover in TIPS (defined as the ratio of TIPS transaction volume over the total amount of TIPS debt outstanding), the implied volatilities derived from options on ten-year Treasury note futures and on the S&P 500 stock index, and the spread between the overnight LIBOR and the effective federal funds rate. The results from these regressions, reported in Table 3, show that $L_{t,10y}$ loads significantly on all variables with intuitive signs: the coefficient on the turnover is negative (a higher trading turnover implies a more liquid TIPS market and hence a lower liquidity premium), the coefficients on Treasury and S&P implied volatilities are positive (a higher liquidity premium during periods of heightened market uncertainties), and the coefficient on the LIBOR spread is also positive (a higher liquidity premium during periods of strains in the LIBOR market). Together the four variables account for more than 80 percent of the variations in our liquidity premium estimates.

[insert Table 3 about here]

The weekly turnover measure exhibits the highest explanatory power, with a one percent increase in this variable leading to a 25 and 44 basis point decrease in the liquidity premia. As can be seen in Figure 11, the trading turnover in TIPS remained low up to 2002 and then rose substantially in 2003, the latter coinciding roughly with the decline in the TIPS liquidity premium (Figure 10), suggesting an improvement in the liquidity in the TIPS market in recent

⁴⁴ Meyer and Sack (2005) also find a liquidity premium with similar declining trend (their Chart 5). Their liquidity premium is smaller than ours because their setup does not include an inflation risk premium.

years.⁴⁵ We caution, however, that the turnover as a liquidity proxy may be quite imperfect and likely affected by other factors as well. We do not have a measure of TIPS bid-ask spread that is available regularly enough to be used in our regression. However, a recent survey by the NY Fed finds that the TIPS bid-ask spread has narrowed modestly since its last survey in 2003, consistent with the decline in our measure of $L_{t,\tau}$.⁴⁶

[insert Figure 11 about here]

While the qualitative behavior of the TIPS liquidity premium thus seems plausible, it is questionable whether our estimates of L_t are consistent with rational pricing. One issue is the *magnitude* of the TIPS liquidity premium in early years. A comparison with corporate bonds may be useful here. Corporate bonds, including those with the highest credit rating (AAA/AA), tend to trade infrequently, e.g., once a day. TIPS have traded more often than AAA/AA corporate bonds even during the early years when liquidity was poorer; the bid-ask spread in TIPS has also been substantially smaller than those of corporate bonds. Thus the TIPS liquidity premium should be *smaller* than the liquidity premium on an AAA/AA corporate bond. A typical magnitude of the AAA/AA spread (over the swap yield or the Treasury yield) is 50 ~ 100 basis points; the liquidity premium on a AAA/AA bond would be some fraction of that.⁴⁷ Alternatively, one can also estimate the liquidity premium in the corporate bond by taking the difference between the CDS premium and the bond spread;⁴⁸ this number also tends to be 50 basis points or less for AAA/AA bonds. Therefore, a TIPS liquidity premium exceeding 1% in the early years (in Figure 10) may be difficult to reconcile with the usual concept of liquidity premium (*a la* Amihud and Mendelson (1986)).

One possibility is that the TIPS liquidity premium reflects some amount of “mispricing” which took time to get corrected. Such mispricing is not unheard of in financial markets;

⁴⁵ The decline in the liquidity premium in the 2003-2004 period may have been also helped by the increased market attention to inflation risk amid a booming economy and rising oil prices.

⁴⁶ An informal survey of seven primary dealers by NY Fed in 2007 found that the TIPS bid-ask spread is about 1/2-1 tick at the two-year maturity, 1 tick at the five-year maturity, 1-2 ticks at the 10-year maturity, 4-6 ticks at the twenty-year maturity, and 6-10 ticks at the thirty-year maturity. A similar survey conducted by NY Fed in 2003 and quoted in Sack and Elsasser (2004) found a TIPS bid-ask spread of 1/2-1 ticks for maturities of five years or less, 2 ticks for maturities of five to ten years, and 4-16 ticks for maturities beyond ten years. One tick is 1/32s of a point, where a point roughly equals 1 percent of the security’s face value.

⁴⁷ The AAA yield may also contain some amount of default premium and tax premium.

⁴⁸ See, e.g., Longstaff et al (2005).

a notable example of securities being mispriced in a relatively new market is the convexity premium mispricing in the swap/eurodollar-futures markets prior to 1995 or so.⁴⁹ A lesson there is that when a relatively complex security is first introduced, it could be mispriced to some extent and the mispricing might last for some time. The case of the TIPS market bears some similarity to the early swap-eurodollar markets in the sense that TIPS was a new security, and is a fairly complex one involving elaborate calculations of indexed coupon and principal payments with the reference CPIs. Furthermore, a popular belief that TIPS are tax-disadvantageous for taxable investors⁵⁰ may have further depressed the demand for TIPS.

The results in this section imply a considerable challenge for modeling the TIPS-specific factors along the lines of liquidity premium modeling in the reduced-form defaultable bond pricing literature (e.g., Driessen (2005) and Longstaff, Mithal and Neis (2005)). Note from Figure 10 that in early years not only is $L_{5y,t}$ large but also $L_{10y,t}$ is. In order to explain this in terms of the (physical or risk neutral) expectation of a liquidity factor l_t (e.g., $L_t = (1/\tau)E_t^Q(\int_t^{t+\tau} l_s ds)$) it has to follow a unit-root process (or something close to it). However, the unit-root-like description of the liquidity process may be unpalatable on intuitive grounds, especially if a substantial part of the downward movement seen in Figure 10 reflects a one-time adjustment associated with the inception of the TIPS market. Appendix E discusses these issues further.

The existing TIPS data is still rather short; more data in the future would certainly help shed more light on the sources of the TIPS-specific variation.

5.3 Interpreting the TIPS breakeven rates

We now revisit the question of whether TIPS breakeven rates are too variable and whether they are informative about inflation expectations of the bond market participants. Although the *level* of TIPS breakeven rates may have been too low in its early years due to nontrivial liquidity premium, the *changes* in TIPS breakeven rates at weekly or monthly frequencies may still be informative, as the adjustment of the TIPS yields to a more normal level may be a slow process.

Indeed, as discussed in Section 4, a visual inspection of the 10-year model-implied breakeven

⁴⁹ See Burghardt and Hoskins (1995) and Gupta and Subrahmanyam (2000) for details.

⁵⁰ See, for example, the discussion in Hein and Mercer (2003).

rates from the NT and the T05 estimations reveal that, apart from the different long-term trend, their time variations are quite similar to those of the 10-year TIPS breakeven rate; in this sense, TIPS breakeven rates might not be necessarily be excessively volatile. Part of this volatility in TIPS breakeven rates is due to the variation of the inflation risk premium (as seen in the bottom left panels of Figures 3 and 6), which in turn may be linked to time variation in inflation uncertainty (the changing perception of the credibility of monetary policy, the normal cyclical variation in business uncertainties, etc.) and in the price of inflation risk.

Figure 12(a) shows in blue circles a scatter plot of the weekly changes in the TIPS-based and the NT-II model-implied 10-year breakeven rates. The two breakeven rates line up closely against the 45-degree line, with a correlation coefficient of 0.76. A regression of the weekly changes in the TIPS breakeven rate onto the weekly changes in the model-implied breakeven rate produces a beta coefficient of 1.06, representing a roughly one-for-one relationship. On the other hand, the weekly changes in the breakeven rate implied by the T05-II model and those in the TIPS breakeven rate, shown in red pluses in Figure 12(a), are somewhat further away from the one-for-one relationship, with the beta coefficient in the same regression being 1.32.

It is also interesting to compare the weekly changes in the model-implied 10-year inflation expectation with the weekly changes in the 10-year TIPS breakeven rates, plotted in Figure 12(b). Both the NT-II and the T05-II estimations reveal a fairly clear positive relation between the weekly changes in the TIPS breakeven rate and those in the model-implied inflation expectation, with the correlation in both cases being about 0.7. A regression of the weekly changes in the TIPS breakeven rate onto the weekly changes in the model-implied inflation expectation gives a beta coefficient of 1.4 and an R^2 of 46% for NT-II, and a beta coefficient of 1.3 and an R^2 of 50% for T05-II. Thus, the TIPS breakeven rates are informative about the direction of the change in inflation expectations (but somewhat overstating the magnitude), though a substantial part of the weekly changes in TIPS breakeven rates remains to be accounted for.

[insert Figure 12 about here]

6 Concluding remarks

In this paper we analyze the inflation-related information in nominal and TIPS yields from a “measurement perspective”, i.e., using a flexibly-specified reduced-form model which has little structure beyond the no-arbitrage assumption. Our framework allows for nontrivial term premia and inflation risk premia, as opposed to the use of the expectations hypothesis and the Fisher hypothesis. We were particularly motivated by two questions: whether TIPS “breakeven rates” are informative about market inflation expectations, and whether TIPS yields are consistent with the economic fundamentals (or with no-arbitrage pricing).

We find that, at least from the viewpoint of a flexibly-specified 3-factor term structure model, we cannot reconcile TIPS data with reasonable priors about inflation expectations and inflation risk premia. TIPS yields seem to have been too high in the early years, possibly due to the newness of the security, the poor liquidity, and some degrees of mispricing. This implies that it may be problematic to use early years’ TIPS data as “no-arbitrage consistent” real yields in other applications. On a more encouraging note, we find that the liquidity premium component of TIPS yields has become much smaller in recent years, suggesting that the TIPS information can be taken at its face value currently and in the future (assuming no deterioration of the TIPS market liquidity conditions going forward) more than in the past.

The answer to the question of whether the TIPS breakeven rate can be taken as inflation expectation is more complicated. We find that the weekly changes in the model-implied 10-year inflation expectation tend to line up with the weekly changes in the 10-year TIPS breakeven rate. However, we also find that time variation in the inflation risk premium and the TIPS liquidity premium, the latter of which may also include other unaccounted-for effects, are often significant enough to drive a wedge between the qualitative behavior of the breakeven rates and inflation expectations.

Our findings in this paper provide support for the use of TIPS breakeven rate information as a proxy for inflation expectations, but also provide a justification for caution. Indeed, in speeches that touch on inflation, policy makers often refer to the TIPS breakeven rate, but they also recognize that the interpretation of this measure is complicated by inflation risk premia and liquidity issues and then continue to monitor a large number of variables to gauge inflation expectations and underlying inflation pressures.⁵¹ More data and more work on TIPS

⁵¹ See Bernanke (2007), for example.

modeling in the future will undoubtedly shed more light on the informational content of TIPS prices.

Appendix

A Joint no-arbitrage model of inflation and interest rates

The n -dimensional vector of state variables $x_t = [x_{1t}, \dots, x_{nt}]'$ follows a multivariate Gaussian process,

$$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t, \quad (\text{A-1})$$

where B_t is an n -dimensional vector of standard Brownian motion, μ is an n -dimensional constant vector, and \mathcal{K}, Σ are $n \times n$ constant matrix. The nominal pricing kernel and the price level processes are specified as

$$dM_t^N / M_t^N = -r^N(x_t)dt - \lambda(x_t)' dB_t, \quad (\text{A-2})$$

$$d \log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^\perp dB_t^\perp. \quad (\text{A-3})$$

Eq. (A-2) is a standard way of specifying the nominal pricing kernel, which describes nominal interest rates that follow diffusion processes. The nominal term structure in this paper is described by the “essentially affine” $A_0(3)$ specification of Duffee (2002), i.e., the nominal short rate $r^N(x_t)$ and the market price of risk $\lambda(x_t)$ are specified to be affine functions of the state variables:

$$r^N(x_t) = \rho_0^N + \rho_1^N' x_t \quad (\text{A-4})$$

$$\lambda(x_t) = \lambda_0^N + \Lambda^N x_t, \quad (\text{A-5})$$

where ρ_0^N is a constant, ρ_1^N and λ_0^N are both n -dimensional constant vectors, and Λ^N is an $n \times n$ constant matrices. The specification (A-3) is also a standard specification of the diffusion model for inflation, consisting of the (instantaneous) expected inflation, $\pi(x_t)$, and the “unexpected inflation” (or the inflation shock), $\sigma_q' dB_t + \sigma_q^\perp dB_t^\perp$. The instantaneous expected inflation $\pi(x_t)$ is also specified as an affine function of the state variables, i.e.,

$$\pi(x_t) = \rho_0^\pi + \rho_1^\pi' x_t. \quad (\text{A-6})$$

The “unexpected inflation” is allowed to depend on shocks that move the nominal interest rates (or expected inflation), dB_t , and also on an orthogonal shock, dB_t^\perp (i.e., $dB_t dB_t^\perp = 0$).⁵²

In some simple cases, for example, as in Campbell and Viceira (2001), the state vector x_t can be represented intuitively: $x_t = [\pi_t, r_t^R]'$, $\rho_1^\pi = [1, 0]'$, etc. In general, however, the state variables x_{it} 's

⁵² The dB_t^\perp part is included to accommodate short-run inflation shocks that are not spanned by yield curve movements, discussed in Kim (2007a)

are statistical variables whose meaning of determined only implicitly by the data on yields and inflation. This is similar to “nominal yields only” model of Duffee (2002) and Dai and Singleton (2000); as we shall discuss below, some normalization restrictions have to be imposed to obtain an econometrically identified model. By having a joint model of nominal yields and inflation (eqs. (A-2) and (A-3)), we are giving more meaning to the state variables (than the nominal-yields-only model), though they still remain “latent variables.”

Applying Ito’s lemma to eq. (5), we can derive the real pricing kernel process as

$$dM_t^R/M_t^R = dM_t^N/M_t^N + dQ_t/Q_t + (dM_t^N/M_t^N) \cdot (dQ_t/Q_t) \quad (\text{A-7})$$

$$= -r^R(x_t)dt - \lambda^R(x_t)'dB_t - (\cdot)dB_t^\perp \quad (\text{A-8})$$

where

$$r^R(x_t) = \rho_0^R + \rho_1^{R'} x_t \quad (\text{A-9})$$

$$\lambda^R(x_t) = \lambda_0^R + \Lambda^R x_t \quad (\text{A-10})$$

with

$$\rho_0^R = \rho_0^N - \rho_0^\pi - \frac{1}{2}(\sigma_q' \sigma_q + \sigma_q^{\perp 2}) + \lambda_0^{N'} \sigma_q \quad (\text{A-11})$$

$$\rho_1^R = \rho_1^N - \rho_1^\pi + \Lambda^{N'} \sigma_q \quad (\text{A-12})$$

$$\lambda_0^R = \lambda_0^N - \sigma_q \quad (\text{A-13})$$

$$\Lambda^R = \Lambda^N. \quad (\text{A-14})$$

It is straightforward to show that in this formulation, the nominal yields, the real yields and inflation expectations all take the affine form – eq. (8) to (10), where factor loadings $a^N, b^N, a^R, b^R, a^I, b^I$ depend on the basic parameters of the model.⁵³ Specifically, the time- t τ -period nominal and real bond yields, $y_{t,\tau}^N$ and $y_{t,\tau}^R$, are given by eqs. (8) and (9) with

$$a_\tau^i = -A_\tau^i/\tau, \quad b_\tau^i = -B_\tau^i/\tau,$$

where

$$\begin{aligned} \frac{dA_\tau^i}{d\tau} &= -\rho_0^i + B_\tau^{i'} (\mathcal{K}\mu - \Sigma\lambda_0^i) + \frac{1}{2} B_\tau^{i'} \Sigma \Sigma' B_\tau^i \\ \frac{dB_\tau^i}{d\tau} &= -\rho_1^i - (\mathcal{K} + \Sigma\Lambda^i)' B_\tau^i \end{aligned}$$

⁵³ See Dai and Singleton (2000), for example.

with the initial conditions $A_0^i = 0$ and $B_0^i = 0_{n \times 1}$, for $i = N, R$. The factor loadings a^I and b^I for inflation expectation, $I_{t,\tau}$, in eq. (10) are given by

$$\begin{aligned} a_\tau^I &= \rho_0^\pi + (1/\tau)\rho_1^\pi \int_0^\tau ds (I - e^{-\mathcal{K}s})\mu \\ b_\tau^I &= (1/\tau) \int_0^\tau ds e^{-\mathcal{K}'s} \rho_1^\pi, \end{aligned}$$

where e^M (M being a square matrix) denotes the matrix exponential. These expressions for factor loadings a^i, b^i ($i = N, R, I$) can be further worked out to yield analytical expressions.

To make the model operational, we need to impose some identification restrictions (normalizations) to rule out “equivalent” models. In this paper, we impose the normalization restriction

$$\mu = 0_{3 \times 1}, \quad \Sigma = \begin{bmatrix} 0.01 & 0 & 0 \\ \Sigma_{21} & 0.01 & 0 \\ \Sigma_{31} & \Sigma_{32} & 0.01 \end{bmatrix}, \quad \mathcal{K} = \begin{bmatrix} \mathcal{K}_{11} & 0 & 0 \\ 0 & \mathcal{K}_{22} & 0 \\ 0 & 0 & \mathcal{K}_{33} \end{bmatrix}, \quad (\text{A-15})$$

(other parameters $\rho_0^N, \rho_1^N, \lambda_0^N, \Lambda^N, \rho_0^\pi, \rho_1^\pi, \sigma^q, \sigma_q^\perp$ remain unrestricted). It can be shown that any specification of the affine Gaussian model that has a \mathcal{K} matrix with all-real eigenvalues can be transformed to the form (A-15).⁵⁴ Note that this does not exhaust the empirical possibilities for all affine-Gaussian models. For example, the K matrix could contain complex eigenvalues,

$$\mathcal{K} = \begin{bmatrix} \mathcal{K}_a & -\mathcal{K}_b & 0 \\ \mathcal{K}_b & \mathcal{K}_a & 0 \\ 0 & 0 & \mathcal{K}_c \end{bmatrix}. \quad (\text{A-16})$$

However, the normalization (A-15) does cover a large set of possibilities, and is equivalent to those used in other studies such as Duffee (2002); therefore we focus on this case in this paper.⁵⁵

B More on the TIPS data

This appendix is devoted to a more detailed discussion on the TIPS data. Figure A1 shows the smoothed TIPS par yield curves on June 9, 2005 in the top panel and on June 9, 1999 in the bottom panel, together with the actual traded TIPS yields that were used to create the smoothed TIPS par yield and zero

⁵⁴ With normalization (A-15), the specification we estimate in this paper can be shown to be equivalent to that of Sangvinatsos and Wachter (2005). The main difference from their paper is empirical: they use a much longer sample (assuming the stability of inflation-yields relationship) and do not use survey forecast information.

⁵⁵ We have also examined the empirical contents of the specification with \mathcal{K} given by eq. (A-16), and obtained similar results.

coupon yield curves. The smoothing is done by assuming that the zero-coupon TIPS yield curve follows the 4-parameter Nelson-Siegel (1987) functional form up to the end of 2003 and the 6-parameter Svensson (1994) functional form thereafter,⁵⁶ and minimizing the fitting error for the actual traded TIPS securities. The substantial increase in the number of points in the top panel reflects the growth of the TIPS market. Note that in 1999 there is essentially one data point on the curve between the 0 and 5 years maturity (corresponding to the 5-year TIPS issued in 1997), thus the TIPS term structure in the short-maturity region of 0-5 years cannot be determined reliably. With more points across the maturity spectrum in 2005, shorter maturity TIPS yields can be determined more reliably than in 1999.

[insert Figure A1 about here]

Still, the analysis of the short-maturity TIPS are complicated by the indexation lag and the seasonality issues. Note that the TIPS payments are indexed to the CPI 2.5 months earlier, thus the TIPS yields contain some amount of realized inflation, often referred to as the “carry effect”. The yield that is more relevant to policy makers is the one that takes out this realized inflation—the so-called carry-adjusted yields—which can be heuristically represented as

$$y_{t,\tau}^{T,CA} = y_{t,\tau}^T + (\delta/\tau)\pi_{t-\delta,t}, \quad (\text{B-17})$$

where $\pi_{t-\delta,t} = \log(Q_t/Q_{t-\delta})/\delta$ denotes the inflation realized between time $t - \delta$ and t , with $\delta = 2.5$ months.⁵⁷ Because the realized inflation $\pi_{t-\delta,t}$ can be quite volatile, the carry-unadjusted yield y^T and the carry-adjusted yield $y^{T,CA}$ can differ significantly, though the difference becomes smaller with an increasing maturity, due to the δ/τ factor in eq. (B-17). Figure A2 shows the carry-adjusted and the unadjusted TIPS yields for the 5-year and 10-year maturities. It can be seen that indeed the 5-year carry-adjusted and unadjusted TIPS yields show greater discrepancies than the 10-year ones. This has been particularly the case in 2005, during which large fluctuations in oil prices caused sharp short-term fluctuations in inflation. The expression (B-17) for the carry adjustment is only a schematic one. The actual carry-adjustment is further complicated by the fact that TIPS is indexed to the seasonally-unadjusted CPI, rather than the seasonally-adjusted CPI. While one could in principle explicitly model seasonality (and carry effects) within the dynamic model of inflation and term structure, such a procedure may be more prone to specification errors than the case in which these effects are corrected at the input stage.⁵⁸

⁵⁶ In comparison, the zero-coupon nominal yield curve is assumed to follow the 6-parameter Svensson (1994) functional form during the entire sample period. In the case of TIPS, however, there were not enough securities in the early years to pin down as many parameters. See Gürkaynak, Sack, and Wright (2007a, 2007b) for details.

⁵⁷ Note that eq. (B-17) takes out the realized inflation in the previous 2.5 months but makes no adjustment for the lack of inflation protection during the last 2.5 months prior to the maturity date.

⁵⁸ See Ghysels (1993) for a discussion of the Sims-Sargent debate that bears on this issue.

[insert Figure A2 about here]

As noted in the main text, in view of the reliability problems and the indexation lag (carry adjustment) problems for shorter maturity TIPS, in this paper we focus mainly on the 10-year maturity (“long-maturity”) TIPS yield for which the effects of the indexation lag and seasonality are less important. While the analysis of the shorter-maturity TIPS yields is an important problem in itself, it deserves a fuller treatment elsewhere.⁵⁹ The 10-year (carry-unadjusted) TIPS yield used in the estimation in this paper is viewed as the carry-corrected TIPS yield plus a measurement error (as suggested by eq. (B-17)).

C More on the Estimation and the Estimates

Here we show how to rewrite the model in a state-space form, which consists of a state equation that describes the dynamic of the state variables, and an observation equation that describes the relationship between the observable variables and the unobservable state variables.

When the time interval h is small, we can discretize the continuous-time equation (A-1) as

$$x_t = \kappa\mu h + (I - \kappa h)x_{t-h} + \Sigma\eta_t = K + HX_{t-h} + \Sigma\eta_t, \quad (\text{C-18})$$

where $\eta_t \sim N(0, hI_{3 \times 3})$. Similarly, the discretized process for the price level is

$$\log Q_t = \log Q_{t-h} + (\rho_0^\pi + \rho_1^{\pi'} X_{t-h}) h + \sigma_q' \eta_t + \sigma_q^\perp \eta_t^\perp. \quad (\text{C-19})$$

Define the state vector $s_t = [\log Q_t, x_t']'$. We can write the Kalman-filter state equations in a matrix form

$$s_t = G_h + \Gamma_h s_{t-h} + \eta_t^s. \quad (\text{C-20})$$

where

$$G_h = \begin{bmatrix} \rho_0^\pi h \\ K \end{bmatrix}, \quad \Gamma_h = \begin{bmatrix} 1 & \rho_1^{\pi'} h \\ 0 & H \end{bmatrix} \quad \text{and} \quad \eta_t^s = \begin{bmatrix} \sigma_q' \eta_t + \sigma_q^\perp \eta_t^\perp \\ \Sigma \eta_t \end{bmatrix}$$

At each time point, we observe N_1 nominal yields and N_2 TIPS yields, both with measurement errors. We also observe the price level as well as survey forecasts of future nominal short rate or inflation, all at a lower frequency than that of the yields. We denote the vector of nominal yields as y_t^N , the vector of TIPS yields as y_t^T , the vector of survey forecasts of the 3-month yield (the vector of $E^{svy}(y_{t+u,3m}^N)$

⁵⁹ Taking a proper account of the seasonality and carry effects is important to TIPS traders, but here in this paper we are concerned with more basic questions.

for various u 's) as $y_t^{svy,N}$, and the vector of survey inflation forecast as I_t^{svy} , respectively. Define the observation vector as

$$z_t = [\log Q_t, y_t^{N'}, y_t^{I'}, y_t^{svy,N'}, I_t^{svy}']'. \quad (\text{C-21})$$

When all variables are available (at time t), we have an observation equation that takes the form

$$z_t = a + F s_t + \nu_t \quad (\text{C-22})$$

where

$$a = \begin{bmatrix} 0 \\ a^N \\ a^R \\ a^{svy,N} \\ a^{svy,I} \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 \\ 0 & b^N \\ 0 & b^R \\ 0 & b^{svy,N} \\ 0 & b^{svy,I} \end{bmatrix} \quad \text{and} \quad \nu_t = \begin{bmatrix} 0 \\ \epsilon_t^N \\ \epsilon_t^R \\ \epsilon_t^{svy,N} \\ \epsilon_t^{svy,I} \end{bmatrix}$$

where a^i and b^i stack the a_τ^i and b_τ^i terms for nominal and real yields, for $i = N, R$, $a^{svy,i}$ and $b_u^{svy,i}$ collect the factor loadings of survey forecasts of future nominal short rates and inflation, for $i = N, I$, and ϵ 's are measurement errors. When only a subset of z_t is available, either due to the less frequent observations of $\log Q_t$, $y_t^{svy,N}$ or I_t^{svy} , or due to the TIPS yields not available or not used in the estimation, the Kalman filter is run with the available subset of the data only.

We estimate the model using the maximum likelihood method with the Kalman filter.⁶⁰ Table A1 reports the parameter estimates and the corresponding standard errors for the NT-I, NT-II, T99-I, T99-II and T05-II estimations. The confidence intervals (standard errors) for quantities of economic interest, such as the inflation risk premium, can be computed using the delta method. As an example, in Figure A3 we plot the 95% confidence intervals for the 10-year inflation risk premium based the NT-I and the T99-I estimates.

Because the state variables in the model are statistical variables that are only defined up to an invariant transformation, individual parameters of the model are not easy to interpret. However, we note that in all estimations in Table A1, there is a diagonal element (eigenvalue) of the \mathcal{K} matrix which is quite small (e.g., $\min(\text{diag}(\mathcal{K}))=0.0419$ for NT-II); this is a necessary condition for the model to generate long-horizon expectations that have substantial variation over time. Note also that the size of the nominal yield measurement errors ($\delta_{N,\tau}$'s) is quite small (e.g., the 10-year nominal yield measurement error is about 5 basis points in all five estimations). In other words, the model fits the nominal yield curve fairly well. Lastly, note that the size of the ‘‘orthogonal shock’’ to inflation (σ_q^\perp) is quite large; this could lead to a material difference between the latent-factor model of the present paper and models in the literature that do not accommodate such a shock.

⁶⁰ The x_t part of the state vector $s_t = [\log Q_t, x_t']'$ is started from the unconditional distribution of x_t , while the $\log Q_t$ is started from a diffuse prior as it is nonstationary.

[insert Table A1 about here]

[insert Figure A3 about here]

D Term structure of real and nominal yields

Table A2 reports the model-implied unconditional moments for the nominal yield, the nominal term premium, the real yield, the real term premium, expected inflation and the inflation risk premium at maturities of 1 quarter, 1-, 2-, 5- and 10 years, based on the NT-II estimation.⁶¹ As can be seen from the third column, this model implies a slightly upward sloping nominal term structure, with the mean levels of yields gently rising from 4.31% at the 1-quarter maturity to 4.74% at the 10-year maturity. The model-implied real term structure is essentially flat, similar to what Ang, Bekaert and Wei (2007a) find, implying a real term premium that is close to zero at all maturities. The nominal term premium can therefore be attributed almost entirely to an upward sloping term structure of inflation risk premia, as shown in the last column, with the point estimates of the inflation risk premia ranging from 10 to 50 basis points for maturities up to ten years. The steady-state CPI inflation is estimated to be around 2.4%. After adjusting for a typical difference of around 50 basis points between the CPI and core CPI inflation measures, this estimate falls just within the 1% to 2% range for core CPI inflation commonly referred to as the Fed’s “comfort zone.”

[Insert Table A2 about here]

We do note, however, that these results about unconditional moments should be interpreted with much caution: there is a general consensus among practitioners and policy makers that the expectations of inflation and interest rates, even at long horizons, have moved around substantially in the past few decades, which means that the data are close to the unit-root boundary (nonstationarity) where the unconditional moments are ill-defined.

E Liquidity premium modeling

In the corporate bond pricing literature (Duffie and Singleton (1999), Longstaff et al (2005), Driessen (2005)), the liquidity premium component is often modeled via a “liquidity factor” tacked on to the

⁶¹ The results from the NT-I and T05-II estimations are qualitatively similar.

short rate in the discount function. We can analogously try to express the TIPS yields as

$$y_{t,\tau}^T = -(1/\tau) \log E_t^Q(e^{-\int_t^{t+\tau} (r_s^R + l_s) ds}), \quad (\text{E-23})$$

where l_t is the liquidity factor. If we assume as in Driessen (2005) and Longstaff et al (2005) that l_t is independent of factors driving the real short rate r_t^R , we have

$$y_{t,\tau}^{TIPS} = y_{t,\tau}^R + \wp_{t,\tau}^L, \quad (\text{E-24})$$

$$\wp_{t,\tau}^L = -(1/\tau) \log E_t^Q(e^{-\int_t^{t+\tau} l_s ds}), \quad (\text{E-25})$$

where $\wp_{t,\tau}^L$ is the liquidity premium for TIPS of τ maturity. Driessen (2005) and Longstaff et al (2005) model the liquidity factor as a univariate process under the Q measure:⁶²

$$dl_t = \alpha(l_t)dt + \beta(l_t)dW_t^Q. \quad (\text{E-26})$$

It can be seen from Figure 10 that stationary specifications of eq. (E-26) are not likely to be a promising model of the liquidity premium in the historical TIPS yields: the liquidity premia seem to contain a quite large secular (nonstationary) component.

Another possibility is to adopt a deterministic process for l_t , i.e., $l_t = F(t)$, such that the liquidity premium term structure at a certain point in time is matched exactly, analogous to the practice of introducing deterministic components in term structure modeling to fit the current yield curve exactly (see, e.g., Hull and White(1990)). For example, motivated by the idea that the liquidity discounts that were large at the inception of the TIPS market would disappear over time, one could write

$$l_t = c_1 e^{-c_2(t-t_0)}, \quad t > t_0, \quad c_1, c_2 > 0 \quad (\text{E-27})$$

or

$$l_t = c_1(1 - \tanh(c_2(t - t_0 - c_3))), \quad t > t_0, \quad c_1, c_2, c_3 > 0, \quad (\text{E-28})$$

where t_0 is denotes the start date of the TIPS market. In both cases, l_t would monotonically decay to 0. However, TIPS liquidity premium implied by these forms are not consistent with observed time variation and term structure of $\wp_{t,\tau}^L$. For example, assuming a liquidity factor of the form (E-27), the liquidity premium on a τ -year TIPS can be derived as

$$\wp_{t,\tau}^L = (1/\tau) \int_t^{t+\tau} ds c_1 e^{-c_2(s-t_0)}, \quad (\text{E-29})$$

implying little variation in $\wp_{t,\tau}^L$ for large τ , or a more significant term structure (τ -dependence) of $\wp_{t,\tau}^L$ than is seen in Figure 10.

⁶² Some examples are the CIR model $dl_t = k(\theta - l_t)dt + \sigma\sqrt{l_t}dW_t^Q$ and the Vasicek model $dl_t = k(\theta - l_t)dt + \sigma dW_t^Q$.

Another way to introduce deterministic dynamics of l_t is to fit the term structure of $\varphi_{t,\tau}$ at a point in time, say, at $t = t^*$. That can be done by setting

$$l_t = \frac{\partial}{\partial \tau}(\tau \cdot \varphi_{t^*,t+\tau}^L), \quad (\text{E-30})$$

where $\varphi_{t^*,\tau}^L$ is the term structure of liquidity premium at time t^* . However, this would be a problematic description of φ^L at another time, say t^{**} , a phenomenon known as the “time inconsistency problem.”

References

- [1] Amihud, Y. and H. Mendelson, (1986), "Asset Pricing and the Bid-Ask Spread," *Journal of Financial Economics*, 17, 223-249.
- [2] Ang, A., G. Bekaert, and M. Wei (2007a), "The term structure of real rates and inflation expectation", *Journal of Finance*, forthcoming.
- [3] Ang, A., G. Bekaert, and M. Wei (2007b), "Do Macro Variables, Asset Markets or Surveys Forecast Inflation Better?" *Journal of Monetary Economics*, 54, 1163-1212.
- [4] Bernanke, B. (2004), "The Great Moderation", speech at the Meetings of the Eastern Economic Association, Washington, DC, February 2004.
- [5] Bernanke, B. (2007), "Inflation Expectations and Inflation Forecasting," speech at the Monetary Economics Workshop of the NBER Summer Institute, Cambridge, MA, July 2007.
- [6] Blanchard, O. J. and L. Summers (1984), "Perspectives on High World Real Interest Rates", *Brookings Papers on Economic Activity*, 2, 273-334.
- [7] Boudoukh, J. (1993) "An equilibrium model of nominal bond prices with inflation-output correlation and stochastic volatility", *Journal of Money, Credit, and Banking*, 25, 636-665.
- [8] Buraschi, A. and A. Jiltsov, (2005), "Inflation Risk Premia and the Expectations Hypothesis," *Journal of Financial Economics*, 75, 429-90.
- [9] Burghardt, G., and B. Hoskins, (1995), "Question of Bias", *Risk* , 8, 63-70.
- [10] Campbell, J.Y., A. W. Lo., and A. C. MacKinlay (1996), *The Econometrics of Financial Markets*, Princeton U. Press.
- [11] Campbell, J.Y., and R.J. Shiller, (1996), "A Scoreboard for Indexed Government Debt", *NBER Macroeconomics Annual*, 155-208.
- [12] Campbell, J., and L. Viceira, (2001), "Who Should Buy Long Term Bonds?" *American Economic Review*, 91, 99-127.
- [13] Chan, L. K. C. (1994), "Consumption, Inflation Risk, and Real Interest Rates: An Empirical Analysis," *Journal of Business*, 67(1), 69-96.

- [14] Chen, N.F., R. Roll, and S.A. Ross, (1986), "Economic Forces and the Stock Market," *Journal of Business*, 59(3), 383–403.
- [15] Chen, R.-R., B. Liu and X. Cheng, (2005), "Inflation, Fisher Equation, and the Term Structure of Inflation Risk Premia: Theory and Evidence from TIPS," Working Paper, Rutgers Business School.
- [16] Chernov, M. and P. Mueller, (2007), "The Term Structure of Inflation Forecasts," Working Paper.
- [17] Clarida, R. H. and B. M. Friedman (1984), "The Behavior of U. S. Short-Term Interest Rates since October 1979," *Journal of Finance* , 39, 671-82.
- [18] Collin-Dufresne, P. and R. Goldstein (2002), "Do bonds span the fixed income markets?: Theory and evidence for unspanned stochastic volatility", *Journal of Finance* 57, p1685-1729.
- [19] Dai, Q. and K. J. Singleton (2000), "Specification analysis of affine term structure models", *Journal of Finance*, 55, 1943-78.
- [20] Driessen, J. (2005), "Is Default Event Risk Priced in Corporate Bonds?" *Review of Financial Studies* , 18, 165-95.
- [21] Duffee, G. R., (2002), "Term Premia and Interest Rate Forecasts in Affine Models," *The Journal of Finance*, Vol. 57 (1), 405-443.
- [22] Duffie, D., and K. J. Singleton (1999), "Modeling Term Structures of Defaultable Bonds," *Review of Financial Studies* , 12, 687-720.
- [23] Evans, M., (2003), "Real Risk, Inflation Risk, and the Term Structure," *The Economic Journal*, Vol. 113, 345-389.
- [24] Fama, E. F. (1975), "Short-Term Interest Rates and Predictors of Inflation," *American Economic Review*, Vol. 65, 269-82.
- [25] FERSON, W. E., AND C. R. HARVEY (1991): "The Variation of Economic Risk Premiums," *Journal of Political Economy*, 99(2), 385–415.
- [26] Ghysels, E. (1993), "Seasonality and Econometric Models: Editor's Introduction", *Journal of Econometrics*, 55, 1-8.
- [27] Goodfriend, M. (1993), "Interest Rate Policy and the Inflation Scare Problem: 1979-1992," *Economic Quarterly (Federal Reserve Bank of Richmond)*, 79(1), 1–24.

- [28] Gupta, A. and M. Subrahmanyam (2000), "An Empirical Investigation of Convexity Bias in the Pricing of Interest Rate Swaps," *Journal of Financial Economics*, Vol. 55, 239-79.
- [29] Gürkaynak, R., B. Sack, and E. Swanson (2005), "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models", *American Economic Review*, 95, 425-36.
- [30] Gürkaynak, R., B. Sack, and J. Wright (2007a), "The U.S. Treasury Yield Curve: 1961 to the Present", *Journal of Monetary Economics*, forthcoming.
- [31] Gürkaynak, R., B. Sack, and J. Wright (2007b), "The TIPS Yield Curve and Inflation Compensation", working paper.
- [32] Hein, S. E. and J. M. Mercer (2003), "Are TIPS really tax disadvantaged?", working paper.
- [33] Hördahl, P. and O. Tristani (2007), "Inflation risk premia in the term structure of interest rates", BIS working paper.
- [34] Hull, J. and A. White (1990), "Pricing Interest Rate Derivative Securities", *Review of Financial Studies*, 3, 573-92.
- [35] Jarrow, R. and Y. Yildirim, (2003), "Pricing treasury inflation protected securities and related derivatives using an HJM model," *Journal of Financial and Quantitative Analysis*, 38(2):337-358.
- [36] Joslin, S. (2007), "Pricing and hedging volatility risk in fixed income markets", working paper.
- [37] Kim, D. H. (2007a), "Challenges in Macro-Finance Modeling," BIS Working Paper, No. 240.
- [38] Kim, D. H. (2007b), "Spanned stochastic volatility in bond markets: A reexamination of the relative pricing between bonds and bond options," BIS Working Paper, No. 239.
- [39] Kim, D. H. and A. Orphanides, (2005), "Term Structure Estimation with Survey Data on Interest Rate Forecasts," *Finance and Economics Discussion Series*, 2005-48.
- [40] Longstaff, F., S. Mithal, E. Neis, (2005), "Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market", *Journal of Finance*, 60, 2213-53.
- [41] Li, H. and F. Zhao (2006), "Unspanned stochastic volatility: evidence from hedging interest rate derivatives", *Journal of Finance*, 61, 341-378.

- [42] Litterman, R. and J. Scheinkman (1991), "Common Factors Affecting Bond Returns", *Journal of Fixed Income*, 1, 62-74.
- [43] Meyer, L. and Sack, B. (2005), "What drives breakeven inflation rates?", *Macroeconomic Advisors Fixed Income Focus*.
- [44] Mishkin, F. S. (1981), "The Real Rate of Interest: An Empirical Investigation," *Carnegie-Rochester Conference Series on Public Policy*, 15, 151-200.
- [45] Nelson, C. R. and A. F. Siegel (1987), "Parsimonious Modeling of Yield Curves," *Journal of Business*, 60, 473-489.
- [46] Pennacchi, G. (1991), "Identifying the dynamics of real rates and inflation: Evidence using survey data", *Review of Financial Studies*, 4, 53-86.
- [47] Poole, W. (1988), "Monetary policy lessons of recent inflation and disinflation", *Journal of Economic Perspectives*, 2, 73-100.
- [48] Richard, S. F. (2001), "An arbitrage model of the term structure of interest rates," *Journal of Financial Economics*, 6, 33-57.
- [49] Risa, S. (2001), "Nominal and Inflation Indexed Yields: Separating Expected Inflation and Inflation Risk Premia," working Paper, Columbia University.
- [50] Sack, B. and R. Elsasser, (2004), "Treasury Inflation-Indexed Debt: A Review of the U.S. Experience," *Economic Policy Review*, Vol. 10 (1), 47-63, Federal Reserve Bank of New York (May).
- [51] Sangvinatsos, A. and J. Wachter (2005), "Does the Failure of the Expectations Hypothesis Matter for Long-Term Investors?", *Journal of Finance*, 60, 179-230
- [52] Svensson, Lars E. O. (1994): "Estimating and Interpreting Forward Rates: Sweden 1992-1994," National Bureau of Economic Research Working Paper 4871.

Table 1: Summary of Estimations

Estimation	Using TIPS yields?	Using survey forecasts of inflation?
NT-I	no	no
NT-II	no	yes
T99-I	since 1999	no
T99-II	since 1999	yes
T05-II	since 2005	yes

Table 2: In-Sample Variance Decomposition of Nominal and Real Yields (NT-II)

maturity (qtr)	real yield		nominal yield		
	real EH component	real term premium	real yield	expected inflation	inflation risk premium
1	1.00 (0.01)	0.00 (0.01)	0.73 (0.00)	0.26 (0.02)	0.01 (0.02)
4	0.94 (0.03)	0.06 (0.03)	0.72 (0.00)	0.26 (0.03)	0.02 (0.03)
8	0.82 (0.05)	0.18 (0.05)	0.69 (0.00)	0.27 (0.04)	0.03 (0.04)
20	0.49 (0.08)	0.51 (0.08)	0.62 (0.00)	0.33 (0.06)	0.04 (0.06)
40	0.25 (0.11)	0.75 (0.11)	0.58 (0.00)	0.36 (0.11)	0.06 (0.11)

Note: This table reports in-sample variance decompositions of real yields into the real expectations component and the real term premium component, and in-sample variance decompositions of nominal yields into the real yield, expected inflation and the inflation risk premium, all based on NT-II model estimates. Variance decompositions of the real yields are calculated according to

$$1 = \frac{\text{cov}(y_{t,\tau}^R, y_{t,\tau}^{R,EH})}{\text{var}(y_{t,\tau}^R)} + \frac{\text{cov}(y_{t,\tau}^R, y_{t,\tau}^R - y_{t,\tau}^{R,EH})}{\text{var}(y_{t,\tau}^R)},$$

while variance decompositions of the nominal yields are calculated according to

$$1 = \frac{\text{cov}(y_{t,\tau}^N, y_{t,\tau}^R)}{\text{var}(y_{t,\tau}^N)} + \frac{\text{cov}(y_{t,\tau}^N, I_{t,\tau})}{\text{var}(y_{t,\tau}^N)} + \frac{\text{cov}(y_{t,\tau}^N, \wp_{t,\tau}^I)}{\text{var}(y_{t,\tau}^N)}.$$

Standard errors calculated using the delta method are reported in parentheses.

Table 3: Model-Implied TIPS Liquidity Premium

$L_{t,10y}$	NT-II	T05-II
Constant	0.74 [13.10]	-0.16 [-2.81]
Daily Turnover	-0.45 [-28.96]	-0.26 [-16.38]
Ten-year implied volatility	0.07 [9.03]	0.09 [11.75]
S&P500 implied volatility	0.02 [11.77]	0.03 [12.10]
Libor-FFR	0.05 [1.42]	0.07 [1.72]
R^2	0.87	0.81

Note: This table regresses model-implied TIPS liquidity premia, $L_{t,10y}$, from NT-II and T05-II estimations on measures of liquidity conditions in the TIPS market. OLS t-statistics are reported in brackets.

Table A1: Parameter Estimates

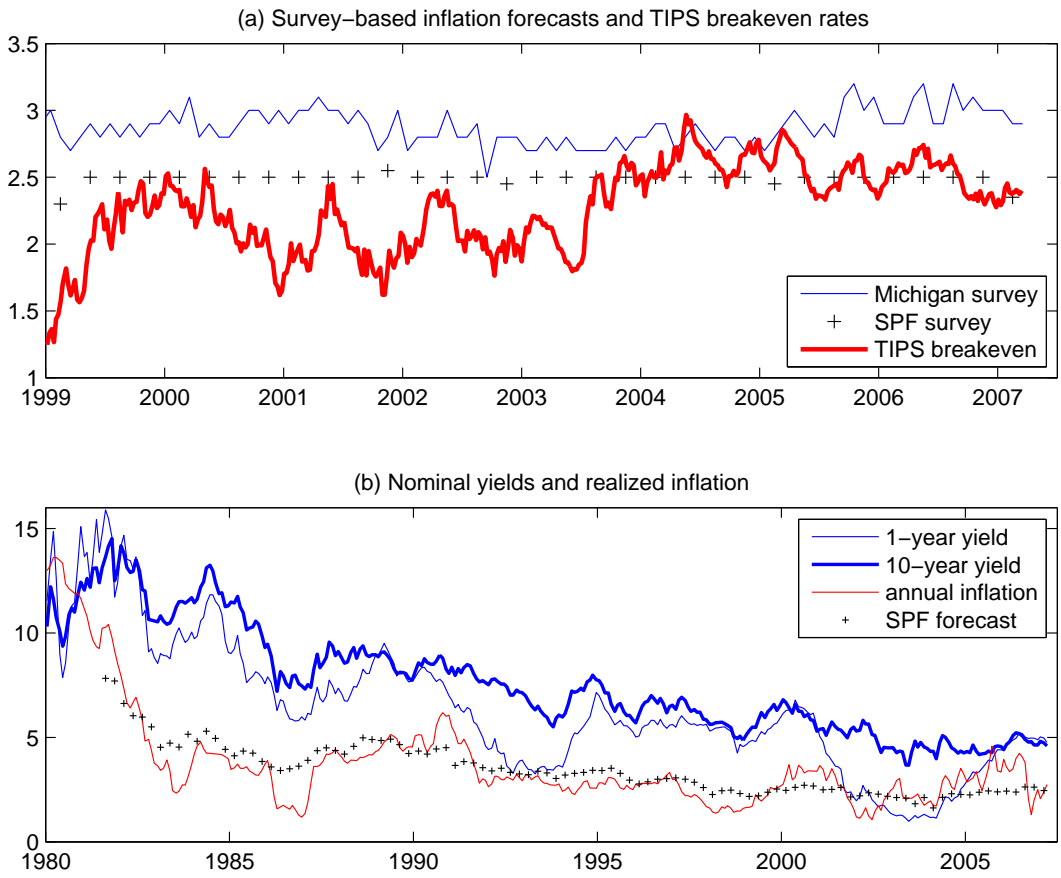
	NT-I	NT-II	T99-I	T99-II	T05-II
State Variables					
\mathcal{K}_{11}	0.8597(0.5198)	0.8169(0.4666)	0.8630(0.1368)	0.8433(0.1555)	0.8720(0.4394)
\mathcal{K}_{22}	0.0481(0.0677)	0.0419(0.0657)	0.0521(0.0810)	0.0488(0.0693)	0.0595(0.0620)
\mathcal{K}_{33}	1.5080(0.8180)	1.4558(0.7827)	1.5175(0.1711)	1.5542(0.3420)	1.6065(0.7212)
$100 \cdot \Sigma_{21}$	-0.3216(0.3141)	-0.4376(0.3257)	-0.3385(0.2728)	-0.5437(0.2805)	-0.2812(0.2602)
$100 \cdot \Sigma_{31}$	-4.6373(9.5310)	-4.7168(9.1773)	-4.6357(1.0617)	-4.6394(2.7987)	-4.2647(6.6484)
$100 \cdot \Sigma_{32}$	-0.5415(0.2378)	-0.5558(0.2465)	-0.5649(0.2574)	-0.6195(0.2409)	-0.5114(0.2162)
Nominal Pricing Kernel					
ρ_0^N	0.0414(0.0137)	0.0429(0.0132)	0.0418(0.0138)	0.0438(0.0115)	0.0496(0.0057)
$\rho_{1,1}^N$	2.9036(5.5599)	3.0242(5.4197)	2.8956(0.5941)	2.9736(1.6467)	2.6107(3.8015)
$\rho_{1,2}^N$	0.4756(0.1130)	0.4747(0.1160)	0.4881(0.1246)	0.5206(0.1144)	0.4771(0.1024)
$\rho_{1,3}^N$	0.6183(0.0396)	0.6216(0.0338)	0.6135(0.0288)	0.6095(0.0258)	0.6090(0.0353)
$\lambda_{0,1}^N$	0.4459(0.2510)	0.3845(0.2091)	0.4283(0.1844)	0.3593(0.1862)	0.3330(0.3409)
$\lambda_{0,2}^N$	-0.1673(0.8515)	-0.2632(0.7712)	-0.1785(0.8719)	-0.2939(0.8008)	-0.7861(0.4900)
$\lambda_{0,3}^N$	0.0656(3.4014)	-0.3082(3.2911)	-0.0031(3.4588)	-0.4506(2.9862)	-1.9781(1.3698)
$[\Sigma^N]_{11}$	-0.5324(1.7803)	-0.4950(1.6063)	-0.5637(0.3480)	-0.7460(0.6339)	-0.5476(1.4149)
$[\Sigma^N]_{21}$	1.7443(5.0050)	1.5625(4.3385)	1.7923(1.0253)	2.0343(1.4944)	1.8403(4.0156)
$[\Sigma^N]_{31}$	3.9083(16.7852)	3.8860(15.8683)	3.9766(2.1698)	4.6583(5.8307)	3.3807(1.5160)
$[\Sigma^N]_{12}$	-0.0368(0.2438)	-0.0396(0.2299)	-0.0193(0.1014)	0.0772(0.1054)	-0.0247(0.2011)
$[\Sigma^N]_{22}$	-0.2734(0.1321)	-0.2499(0.1296)	-0.2886(0.1053)	-0.3761(0.0782)	-0.3128(0.1013)
$[\Sigma^N]_{32}$	-0.6572(0.8385)	-0.6737(0.8239)	-0.7407(0.4363)	-1.2661(0.6819)	-0.6546(0.7059)
$[\Sigma^N]_{13}$	-0.0684(0.3218)	-0.0662(0.2910)	-0.0777(0.0935)	-0.1527(0.0950)	-0.0858(0.2643)
$[\Sigma^N]_{23}$	0.6062(0.2421)	0.5632(0.2340)	0.6181(0.2161)	0.7250(0.1882)	0.7139(0.2149)
$[\Sigma^N]_{33}$	0.6822(2.1513)	0.7232(2.0002)	0.6877(0.3571)	0.9391(0.8475)	0.6115(1.7395)
Inflation					
ρ_0^π	0.0232(0.0096)	0.0239(0.0108)	0.0268(0.0048)	0.0250(0.0078)	0.0281(0.0059)
$\rho_{1,1}^\pi$	0.0764(0.6658)	0.0993(0.5811)	-1.0700(0.5303)	-1.2517(1.1124)	-1.3801(2.7343)
$\rho_{1,2}^\pi$	0.2786(0.1283)	0.3557(0.0923)	0.1051(0.0925)	0.2942(0.0709)	0.4084(0.0657)
$\rho_{1,3}^\pi$	-0.0174(0.2035)	-0.0081(0.1702)	-0.5109(0.1582)	-0.5595(0.1642)	-0.5348(0.2232)
$\sigma_{q,1}$	-0.1136(0.0755)	-0.1043(0.0761)	-0.1774(0.0675)	-0.1515(0.0722)	-0.1821(0.0710)
$\sigma_{q,2}$	0.0814(0.0767)	0.0896(0.0761)	0.0524(0.0812)	0.0451(0.0810)	0.1410(0.0764)
$\sigma_{q,3}$	0.0295(0.0619)	0.0320(0.0589)	0.0038(0.0549)	-0.1021(0.0503)	0.0820(0.0591)
σ_q^\perp	0.7144(0.0241)	0.7168(0.0237)	0.7545(0.0271)	0.7661(0.0285)	0.7368(0.0264)
Measurement Errors and Liquidity Premium					
$100 \cdot \delta_{N,3m}$	0.1011(0.0026)	0.1011(0.0026)	0.1009(0.0026)	0.1009(0.0026)	0.1010(0.0026)
$100 \cdot \delta_{N,6m}$	0.0220(0.0017)	0.0220(0.0017)	0.0224(0.0017)	0.0223(0.0017)	0.0223(0.0017)
$100 \cdot \delta_{N,1y}$	0.0530(0.0016)	0.0530(0.0016)	0.0531(0.0017)	0.0532(0.0017)	0.0530(0.0017)
$100 \cdot \delta_{N,2y}$	0	0	0	0	0
$100 \cdot \delta_{N,4y}$	0.0293(0.0012)	0.0293(0.0012)	0.0292(0.0013)	0.0292(0.0013)	0.0293(0.0012)
$100 \cdot \delta_{N,7y}$	0	0	0	0	0
$100 \cdot \delta_{N,10y}$	0.0491(0.0019)	0.0491(0.0019)	0.0491(0.0019)	0.0491(0.0019)	0.0491(0.0019)
$100 \cdot \delta_{F,6m}$	0.1757(0.0134)	0.1756(0.0133)	0.1757(0.0134)	0.1761(0.0135)	0.1763(0.0136)
$100 \cdot \delta_{F,12m}$	0.2259(0.0195)	0.2260(0.0195)	0.2259(0.0196)	0.2255(0.0195)	0.2266(0.0197)
$100 \cdot \delta_{R,10y}$			0.3028(0.0132)	0.3036(0.0132)	0.0868(0.0088)
$100 \cdot \Delta_{10y}^R$			0.0519(0.2251)	-0.1385(0.1846)	-0.1292(0.1810)

Note: Standard errors based on the BHHH formula are given in parentheses.

Table A2: Unconditional Moments of Real and Nominal Term Structure (NT-II)

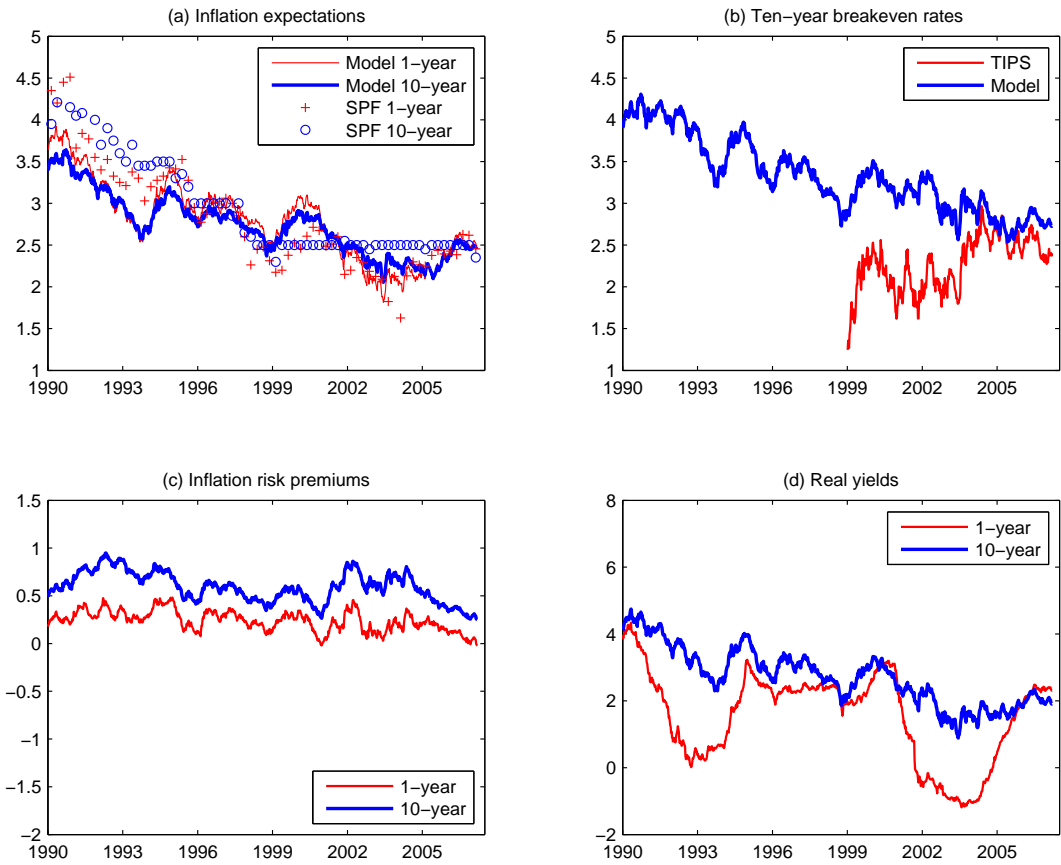
maturity (qtr)	moment	nominal yield	nominal term premium	real yield	real term premium	expected inflation	inflation risk premium
1	mean	4.31 (1.48)	0.03 (0.23)	1.84 (0.29)	0.02 (0.22)	2.39 (1.07)	0.08 (0.17)
	std. dev.	2.03 (1.23)	0.29 (0.20)	0.89 (0.14)	0.27 (0.20)	1.32 (1.00)	0.24 (0.20)
4	mean	4.33 (1.92)	0.04 (0.66)	1.83 (0.69)	0.01 (0.64)	2.39 (1.07)	0.11 (0.18)
	std. dev.	2.52 (1.67)	0.84 (0.60)	1.11 (0.48)	0.78 (0.60)	1.30 (1.02)	0.28 (0.20)
8	mean	4.32 (2.22)	0.03 (0.97)	1.78 (0.98)	-0.04 (0.94)	2.39 (1.07)	0.15 (0.18)
	std. dev.	2.86 (1.98)	1.23 (0.88)	1.36 (0.80)	1.12 (0.90)	1.28 (1.03)	0.31 (0.19)
20	mean	4.38 (2.53)	0.09 (1.28)	1.72 (1.32)	-0.10 (1.29)	2.39 (1.07)	0.27 (0.15)
	std. dev.	3.20 (2.32)	1.65 (1.17)	1.69 (1.18)	1.51 (1.27)	1.21 (1.08)	0.34 (0.14)
40	mean	4.74 (2.51)	0.45 (1.27)	1.88 (1.40)	0.06 (1.37)	2.39 (1.07)	0.46 (0.06)
	std. dev.	3.16 (2.32)	1.74 (1.10)	1.76 (1.28)	1.59 (1.35)	1.09 (1.12)	0.32 (0.14)

Note: This table reports the unconditional mean and standard deviations of nominal yields, nominal term premium, real yields, real term premia, expected inflation and inflation risk premia based on NT-II model estimates. Standard errors calculated using the delta method are reported in parentheses.



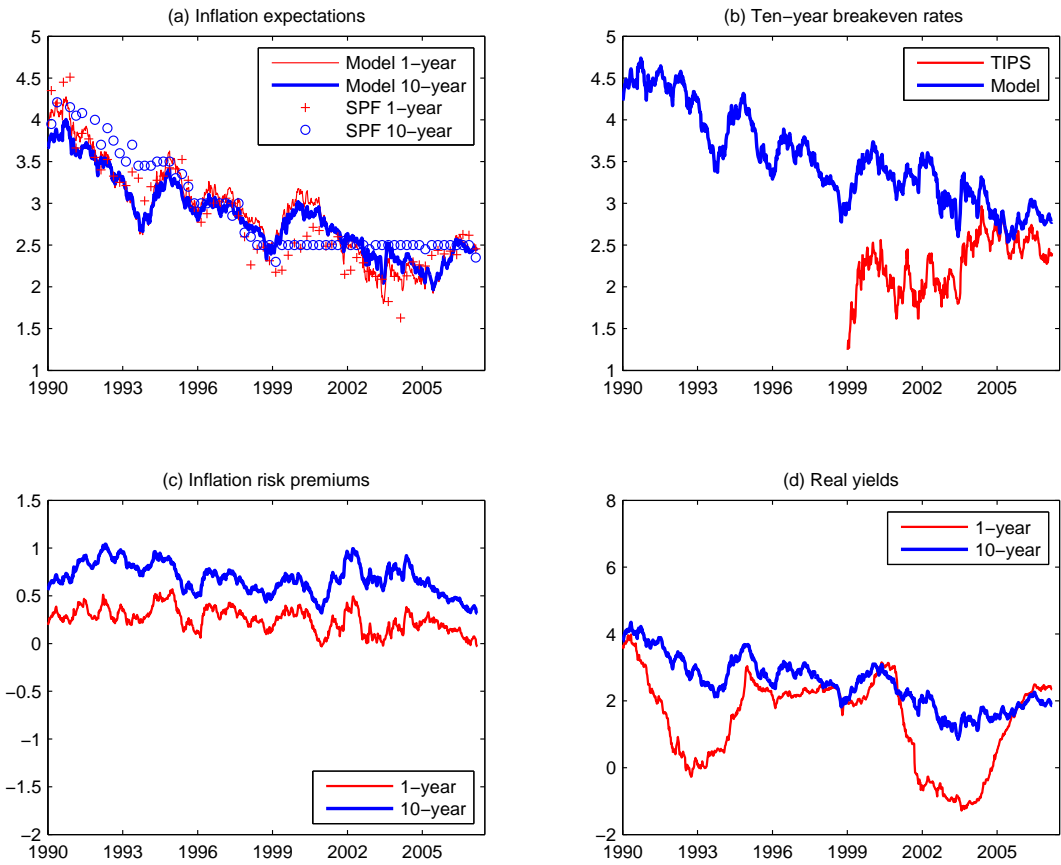
Note: The top panel plots the 10-year TIPS breakeven rate (red line), long-horizon Michigan inflation forecast (blue line), and 10-year SPF inflation forecast (black pluses). The bottom panel plots the 1-year (thin blue line) and 10-year (thick blue line) nominal yields, together with the realized annual inflation (red line) and the corresponding SPF forecast (black pluses).

Figure 1: Nominal and TIPS Yields and Inflation



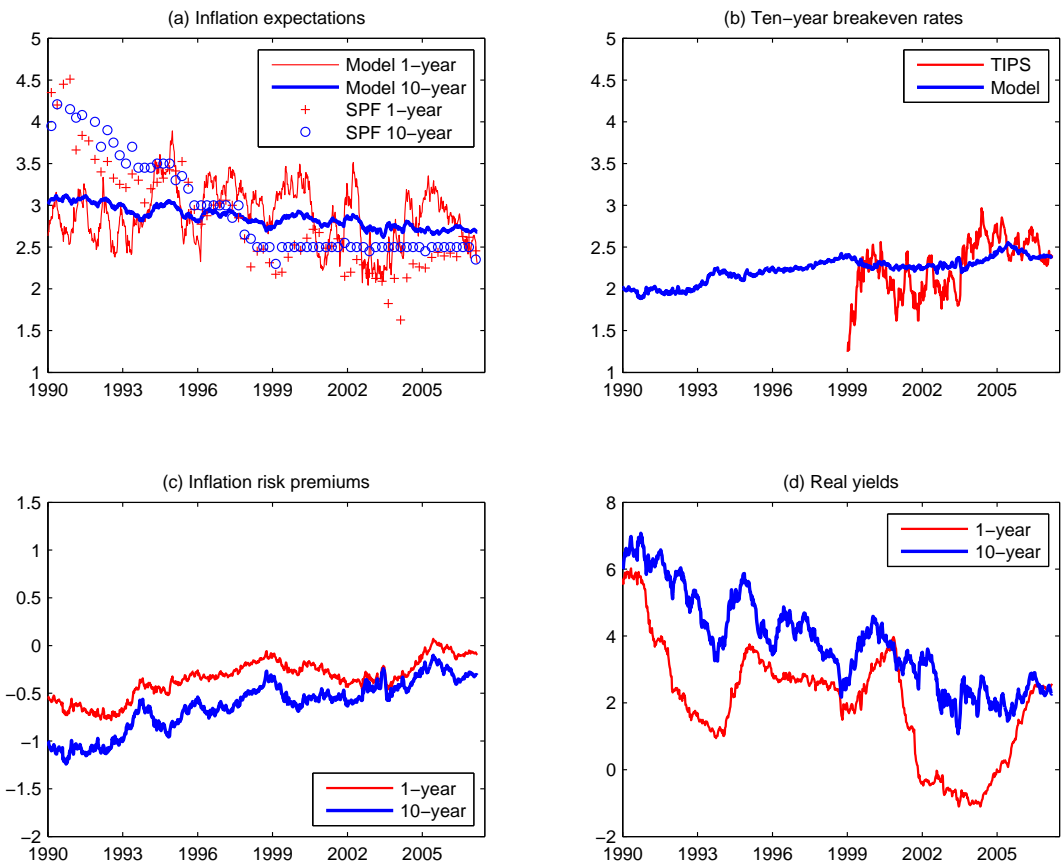
Note: Results from the NT-I estimation: (a) Model-implied 10-year (thick blue line) and 1-year (thin red line) inflation expectation. The 10-year and 1-year SPF survey inflation forecasts are shown in blue circles and red plus signs, respectively. (b) Model-implied 10-year breakeven rate (thin red line) and TIPS breakeven rate (thick blue line). (c) Model-implied 1-year (thin red line) and 10-year (thick blue line) inflation risk premia. (d) Model-implied 1-year (thin red line) and 10-year (thick blue line) real yields.

Figure 2: Results from the NT-I Estimation



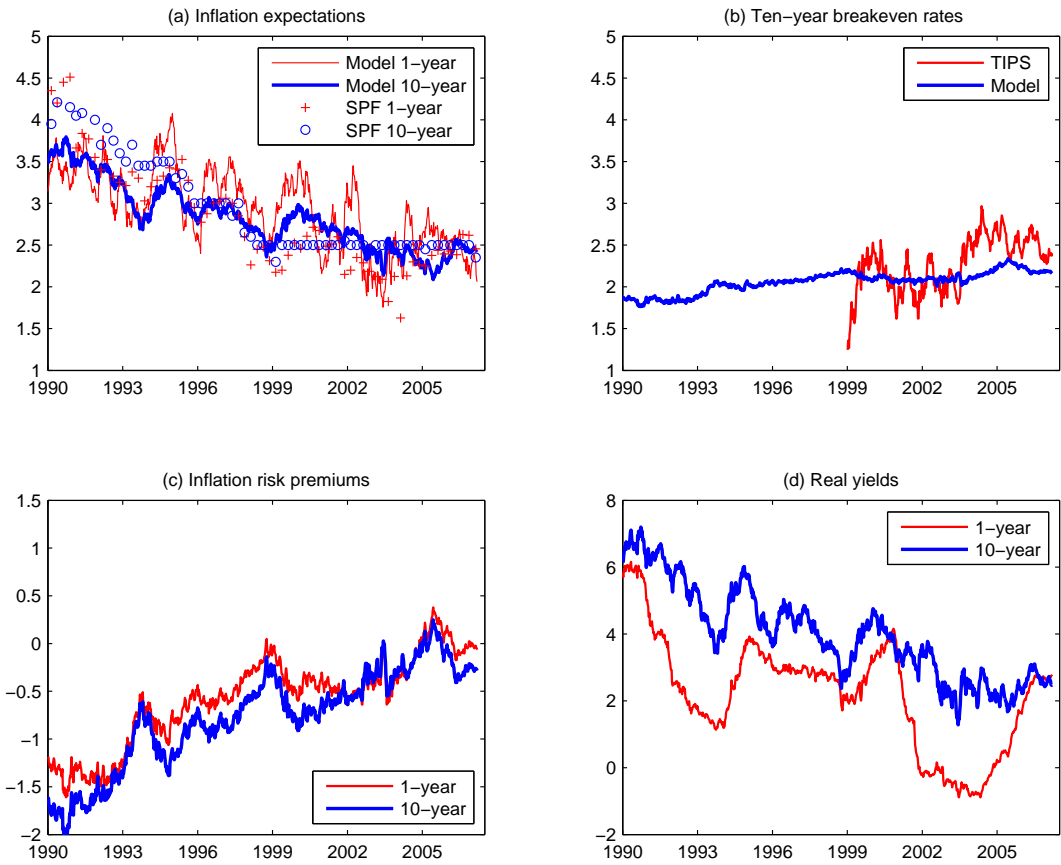
Note: Results from the NT-II estimation: (a) Model-implied 10-year (thick blue line) and 1-year (thin red line) inflation expectation. The 10-year and 1-year SPF survey inflation forecasts are shown in blue circles and red plus signs, respectively. (b) Model-implied 10-year breakeven rate (thin red line) and TIPS breakeven rate (thick blue line). (c) Model-implied 1-year (thin red line) and 10-year (thick blue line) inflation risk premia. (d) Model-implied 1-year (thin red line) and 10-year (thick blue line) real yields.

Figure 3: Results from the NT-II Estimation



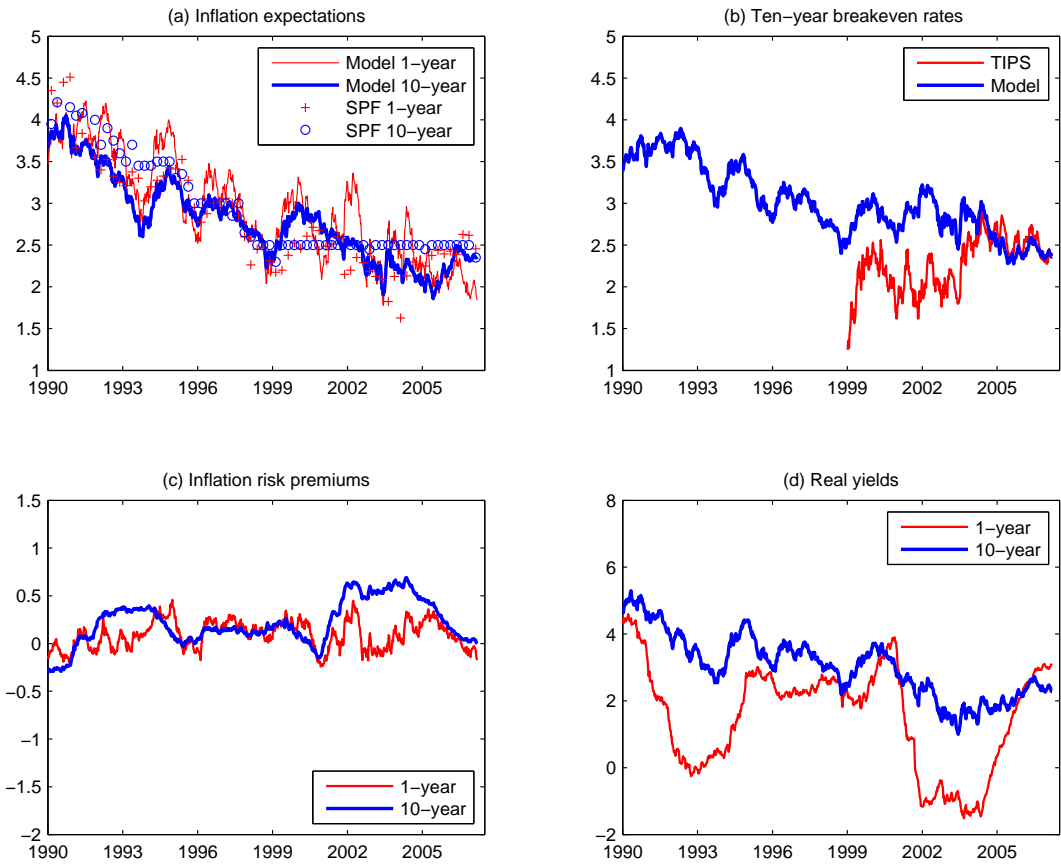
Note: Results from the T99-I estimation: (a) Model-implied 10-year (thick blue line) and 1-year (thin red line) inflation expectation. The 10-year and 1-year SPF survey inflation forecasts are shown in blue circles and red plus signs, respectively. (b) Model-implied 10-year breakeven rate (thin red line) and TIPS breakeven rate (thick blue line). (c) Model-implied 1-year (thin red line) and 10-year (thick blue line) inflation risk premia. (d) Model-implied 1-year (thin red line) and 10-year (thick blue line) real yields.

Figure 4: Results from the T99-I Estimation



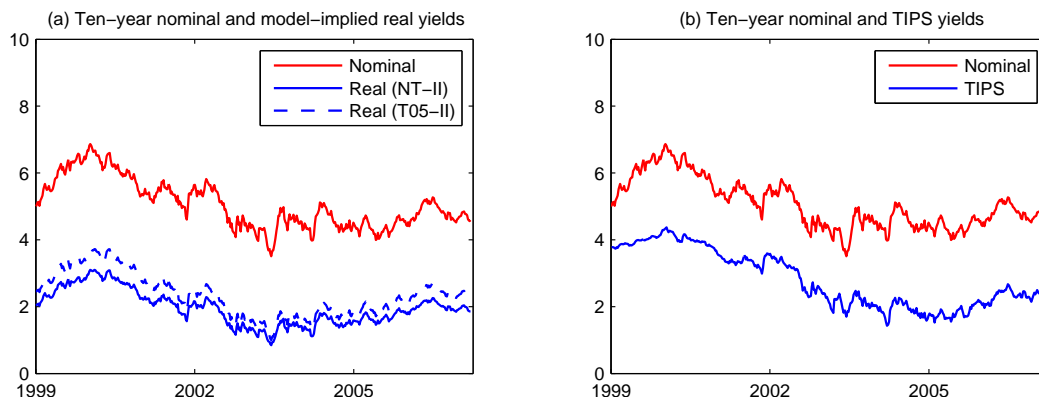
Note: Results from the T99-II estimation: (a) Model-implied 10-year (thick blue line) and 1-year (thin red line) inflation expectation. The 10-year and 1-year SPF survey inflation forecasts are shown in blue circles and red plus signs, respectively. (b) Model-implied 10-year breakeven rate (thin red line) and TIPS breakeven rate (thick blue line). (c) Model-implied 1-year (thin red line) and 10-year (thick blue line) inflation risk premia. (d) Model-implied 1-year (thin red line) and 10-year (thick blue line) real yields.

Figure 5: Results from the T99-II Estimation



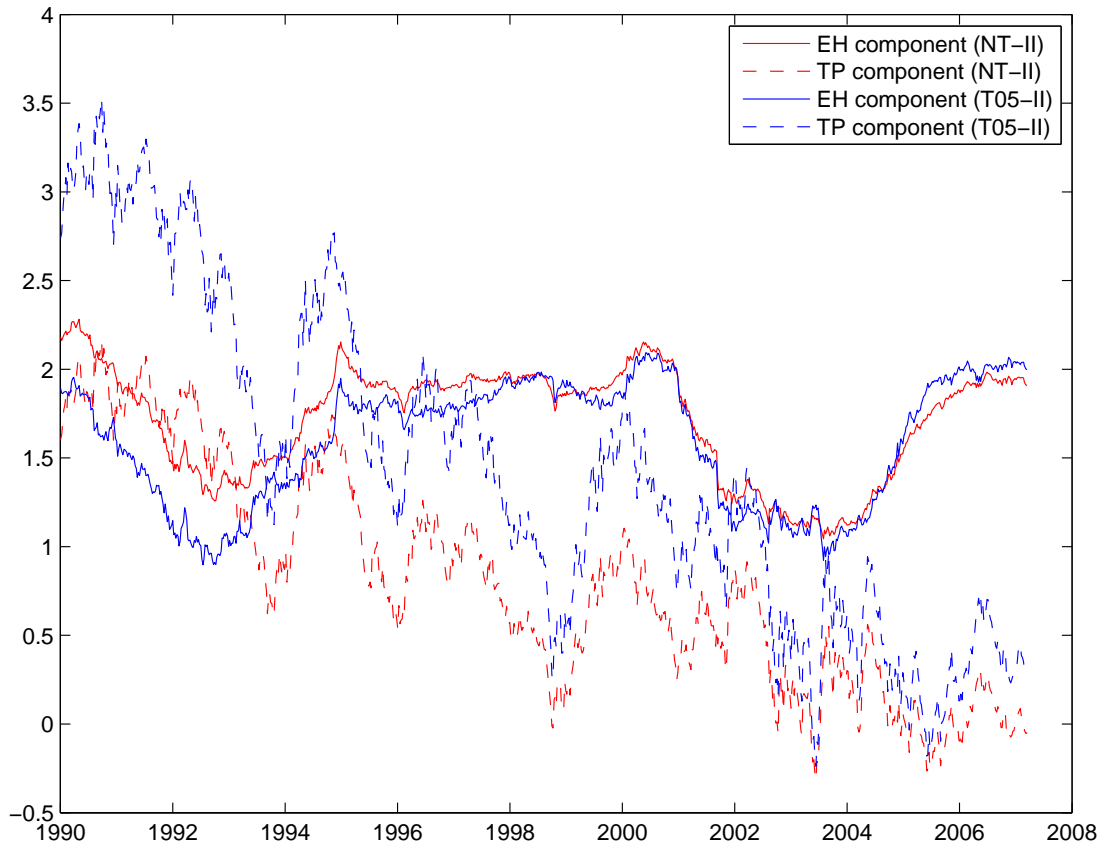
Note: Results from the T05-II estimation: (a) Model-implied 10-year (thick blue line) and 1-year (thin red line) inflation expectation. The 10-year and 1-year SPF survey inflation forecasts are shown in blue circles and red plus signs, respectively. (b) Model-implied 10-year breakeven rate (thin red line) and TIPS breakeven rate (thick blue line). (c) Model-implied 1-year (thin red line) and 10-year (thick blue line) inflation risk premia. (d) Model-implied 1-year (thin red line) and 10-year (thick blue line) real yields.

Figure 6: Results from the T05-II Estimation



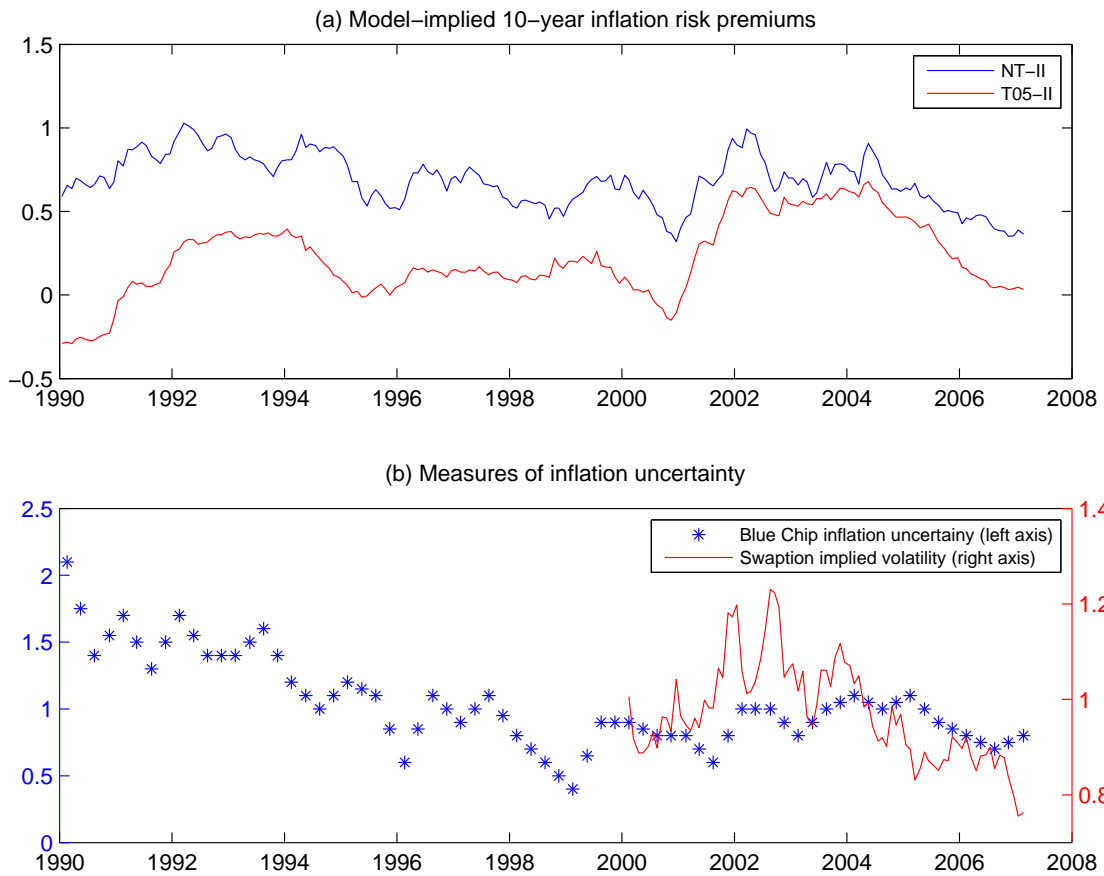
Note: (a) The red line plots the actual 10-year nominal yield, and the blue solid (dashed) line plots the implied 10-year real yield based on Model NT-II (T05-II). (b) The red and the blue lines plot the 10-year nominal and TIPS yields, respectively.

Figure 7: Ten-Year Nominal and Real Yields



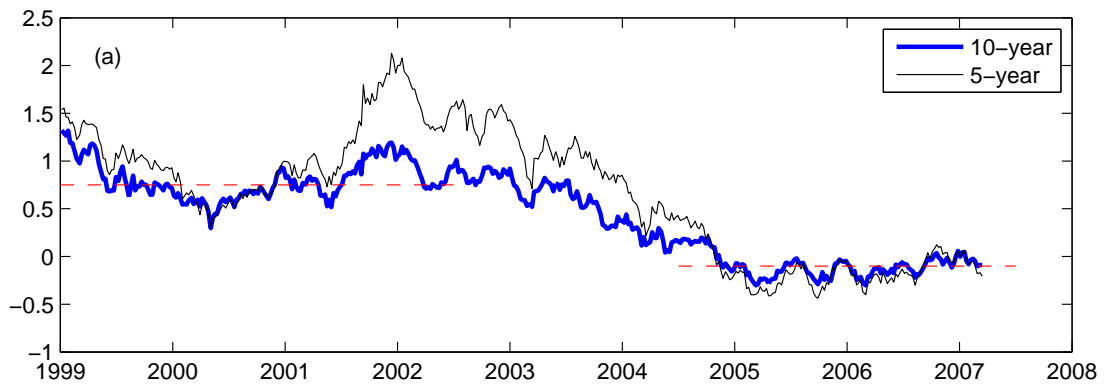
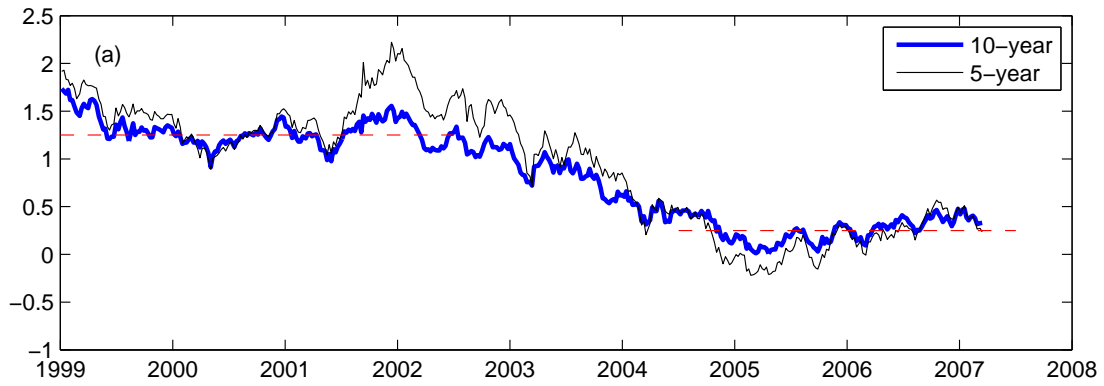
Note: Decomposition of the 10-year real yield into the real expectations component (solid lines) and the real term premium component (dashed lines). The red lines are based on the NT-II estimation, and the blue lines are based on the T05-II estimation.

Figure 8: Decomposition of Ten-year Real Yield



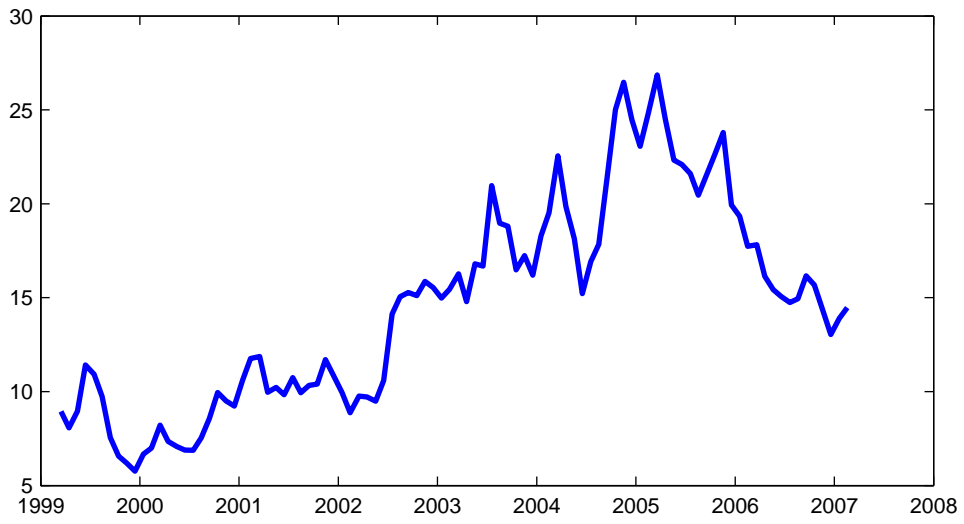
Note: (a) Model-implied 10-year inflation risk premia based on NT-II (blue line) and T05-II (red line) estimations. (b) The blue asterisks plot the distance between the average of the top ten forecasts and that of the bottom ten forecasts of CPI inflation 5 to 10 years ahead from Blue Chip Economic Indicator survey. The red line plots the basis-point implied volatility (absolute implied volatility) from 10-year swaptions with an underlying swap length of 1 year.

Figure 9: Inflation Risk Premium and Measures of Inflation Uncertainty



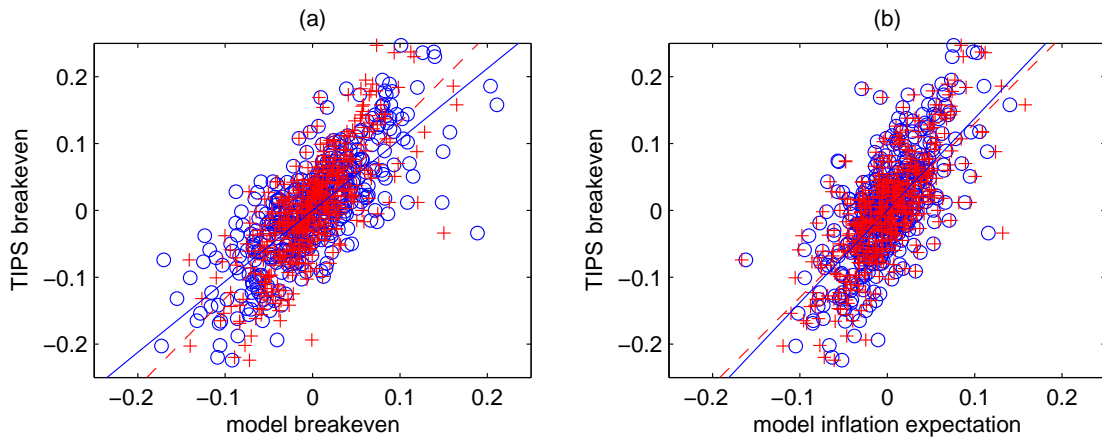
Note: (a) NT-II, (b) T05-II. The black (blue) line plots the model-implied 5-year (10-year) liquidity premium.

Figure 10: Model-Implied Liquidity Premium



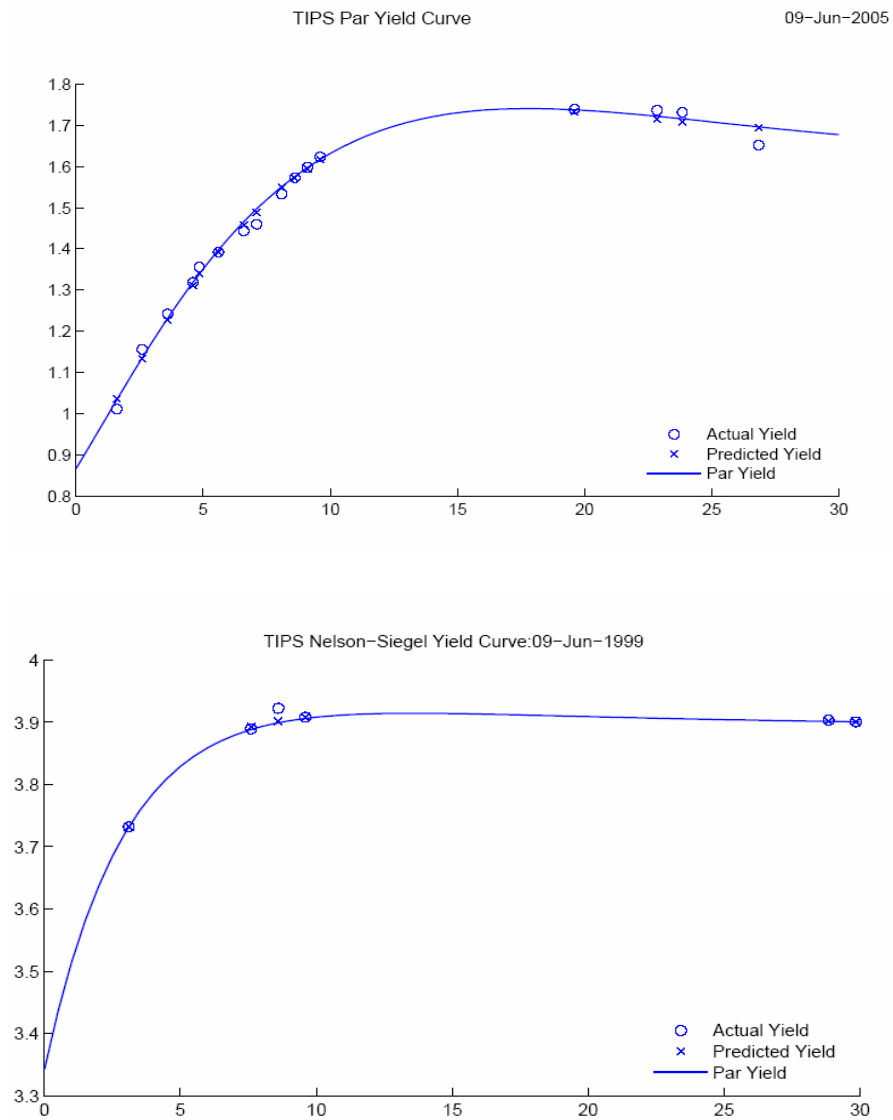
Note: in millions of dollars divided by TIPS outstanding in the corresponding month.

Figure 11: Weekly turnover of TIPS



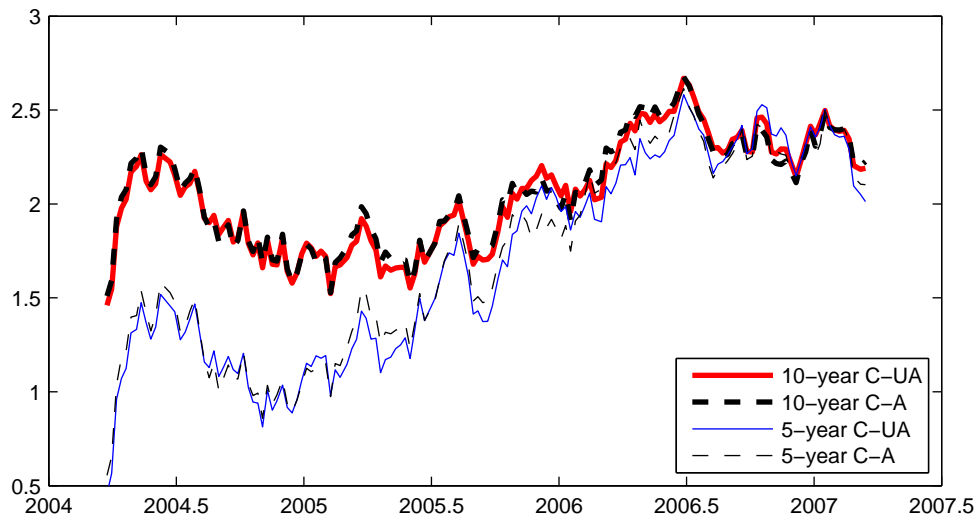
Note: (a) Scatter plots of TIPS 10-year breakeven rate against the NT-II (circles) and the T-05-II (pluses) model-implied 10-year breakeven rates. (b) Scatter plots of TIPS 10-year breakeven rate against the NT-II (circles) and the T05-II (pluses) model-implied 10-year inflation expectations.

Figure 12: TIPS and Model-Implied Breakeven Rates



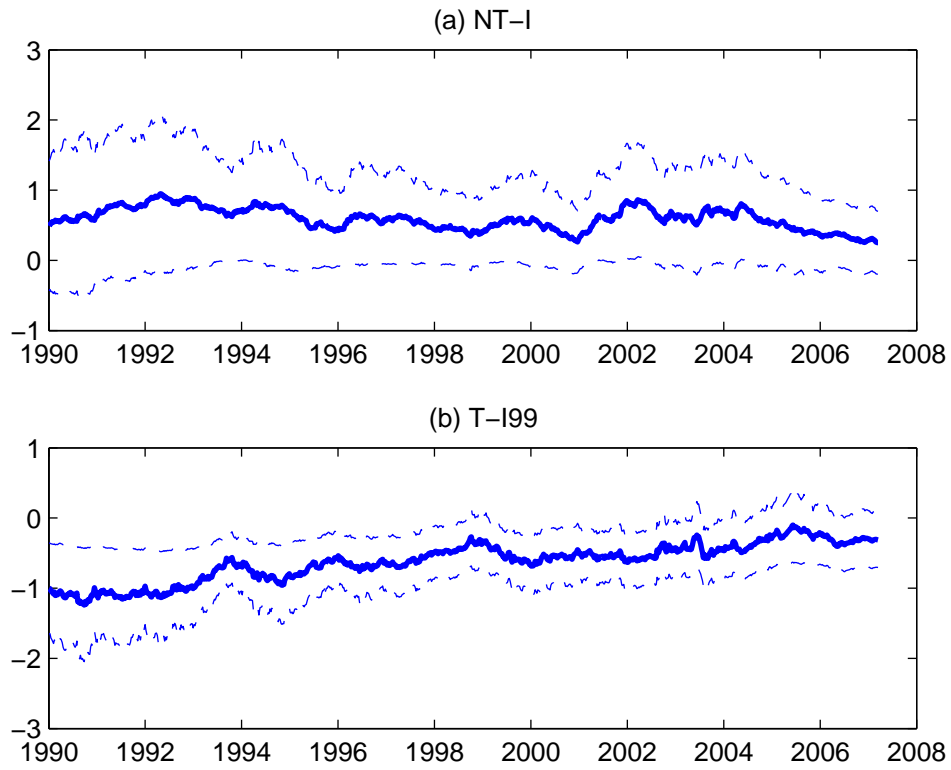
Note: The top (bottom) panel plots the fitted TIPS par yield curve together with individual TIPS yields on June 9, 2005 (June 9, 1999).

Figure A1: TIPS Yield Curves



Note: This figure plots 10-year carry-unadjusted (carry-adjusted) TIPS yields in red solid (black dashed) line and 5-year carry-unadjusted (carry-adjusted) TIPS yields in blue solid (gray dashed) line.

Figure A2: TIPS Yields with and without Carry Adjustment



Note: The top (bottom) panel plots the 10-year inflation risk premium together with the 95% confidence bands as implied by the NT-I (T99-I) estimation.

Figure A3: Ten-Year Inflation Risk Premia