



Densification of the ITRF

The NAREF Experience in North America

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Outline

- NAREF objectives
- Regional solutions & standards
- Combination of regional solutions
- Integration into IGS global network

NAREF Objectives

Densify the ITRF reference frame in NA

- Densify IGS global network
- Combine various regional and local networks

Generate coordinate solutions

- Weekly combinations of regional solutions
- Cumulative solutions with velocity estimates

Make available to public

- Scientific applications (crustal motion studies)
- Reference stations for integrating surveys into ITRF

Standards for Regional Solutions

State-of-the-art GPS software

- Advanced modelling techniques
- **Availability of full covariance matrix**
- E.g., Bernese, GAMIT, GIPSY-OASIS, MicroCosm

Fixed IGS orbits & ERPs

Ties to IGS global network

- At least 3 IGS global stations
- Preferably all in vicinity of network

Problems

Regional solutions from independent organizations

- Limited resources
- Objectives different from NAREF
- **Difficult to impose standards**
- Take what we can get

Uneven coverage & redundancy

- Some stations in all solutions
- Many stations in only one solution
- Causes uneven weighting of stations

Standards for Regional Stations

Station selection criteria

- Dual frequency data, 24 hr/day, min. 5 days/week
- Elevation mask angle 10°
- **Stable geodetic-quality monumentation (or classify?)**
- Complete & up to date station logs

Overlapping networks/solutions desired

- Stations in multiple solutions
- More reliable outlier detection
- Quantify software “noise” (average out?)

Contributors

Currently 4 solutions

- ✓ Geodetic Survey Division Bernese Regional Network
- ✓ Geodetic Survey Division GIPSY Regional Network
- ✓ Pacific Geoscience Centre WCDA
- ✓ SIO Plate Boundary Observatory

Need more for US & Mexico

- CORS network – over 200 pts (NGS ??)
- Mexican permanent GPS network – about 10 pts

GSD Bernese Regional Network (GSB)

65± Points

Bernese 4.2

Double differenced obs.

3 min. data rate

10° elevation cut off

Fixed IGS orbits

TZD every 2 hr

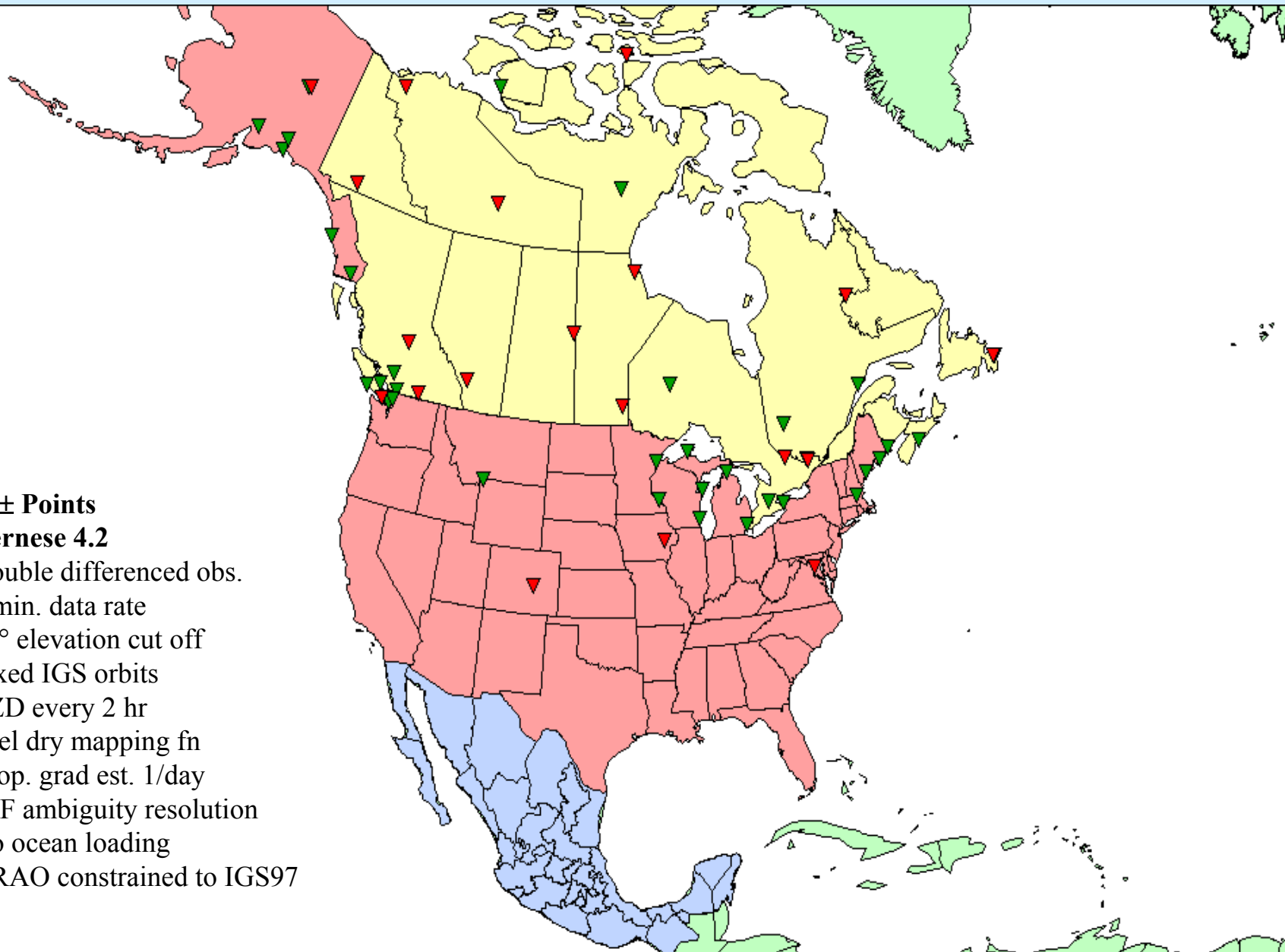
Niel dry mapping fn

Trop. grad est. 1/day

QIF ambiguity resolution

No ocean loading

DRAO constrained to IGS97



GSD GIPSY Regional Network (GSG)

28± Points

GIPSY-OASIS II

Undifferenced obs.

7.5 min. data rate

15° elev. cut off

Fixed IGS orbits

TZD random walk

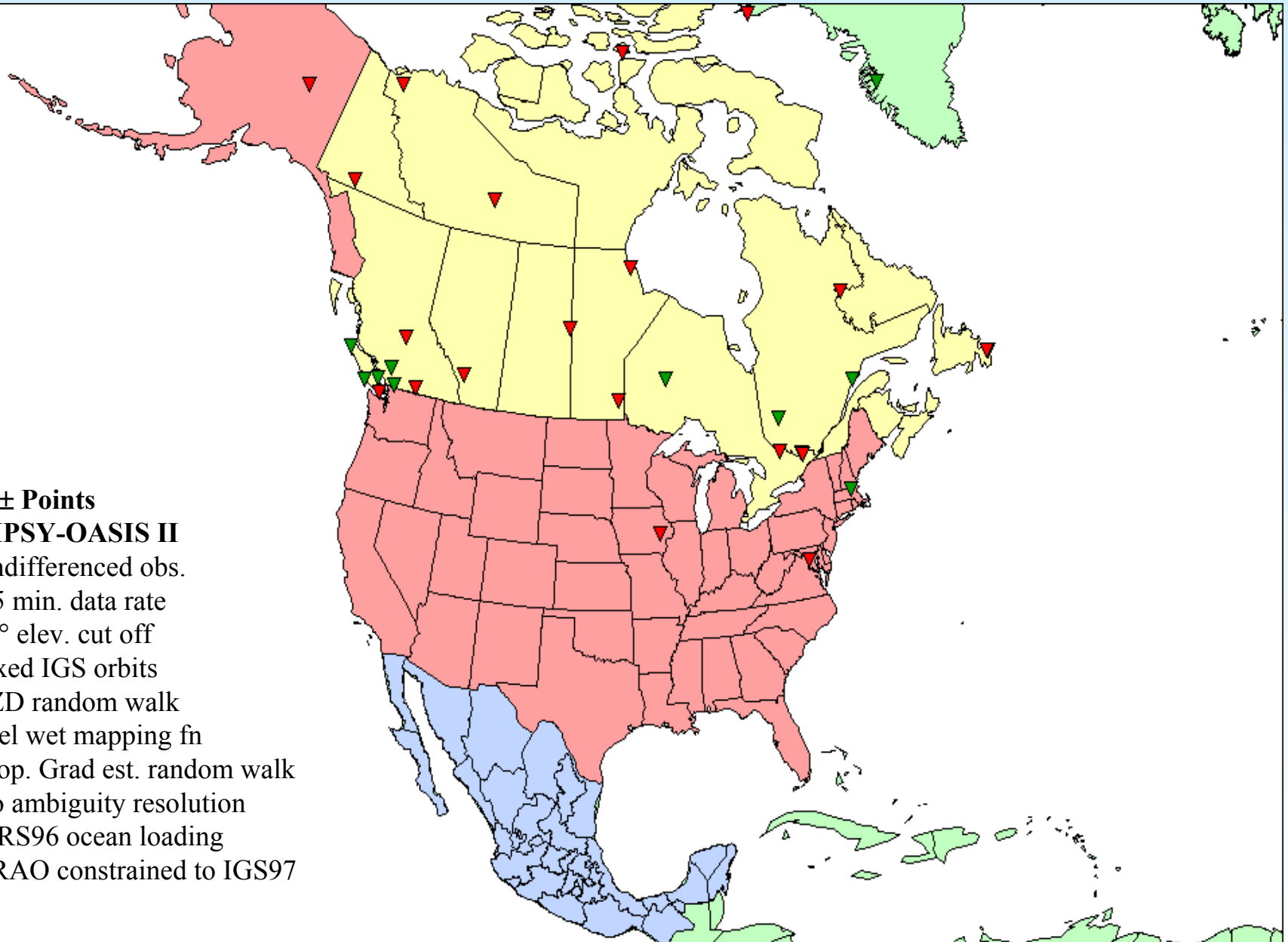
Niel wet mapping fn

Trop. Grad est. random walk

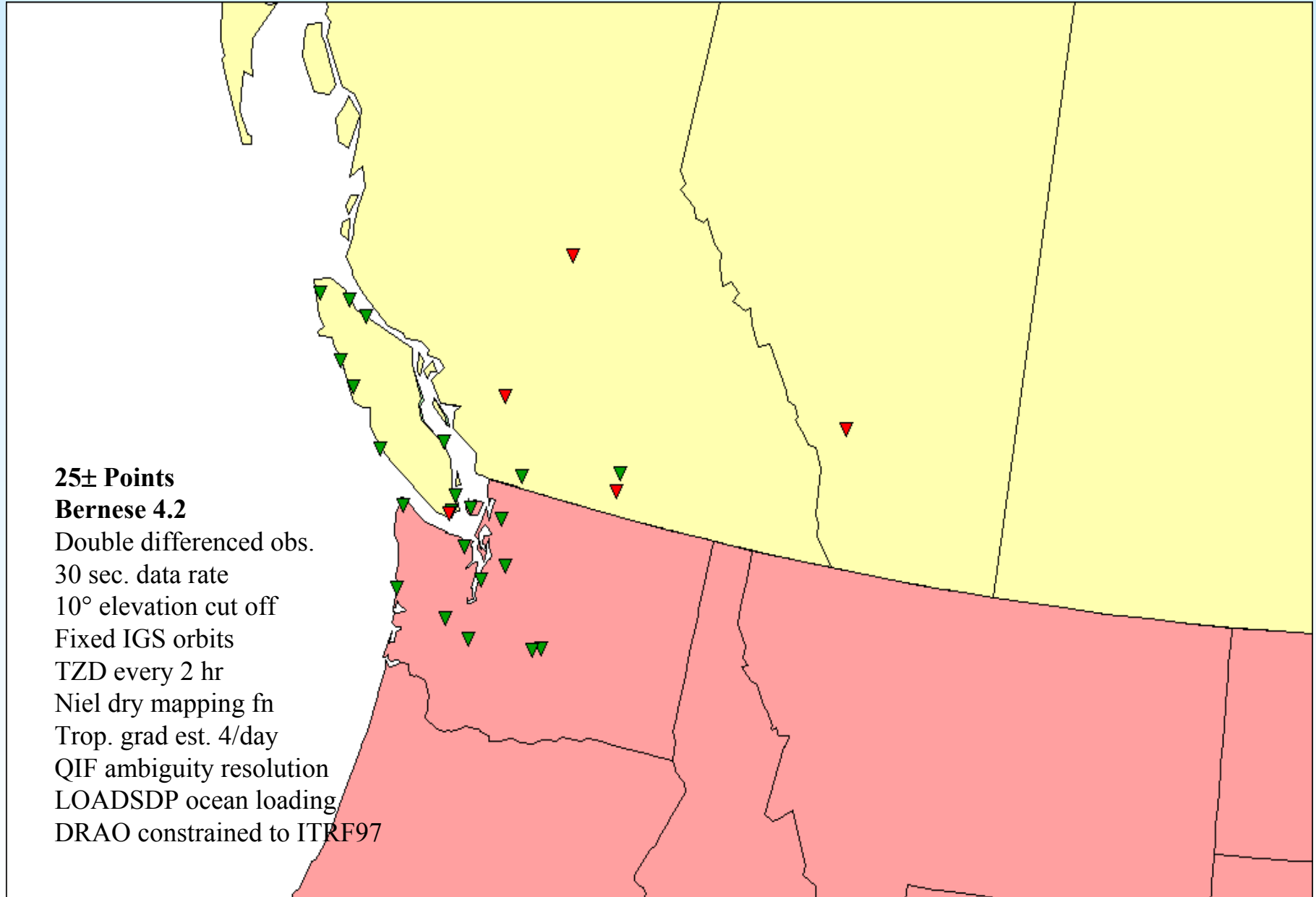
No ambiguity resolution

IERS96 ocean loading

DRAO constrained to IGS97



PGC Western Canada Deformation Array (PGC)



SIO Plate Boundary Observatory (PBO)

56± Points

GAMIT 9.72

Double differenced obs.

2 min. data rate

10° elevation cut off

Fixed SIO orbits

TZD random walk

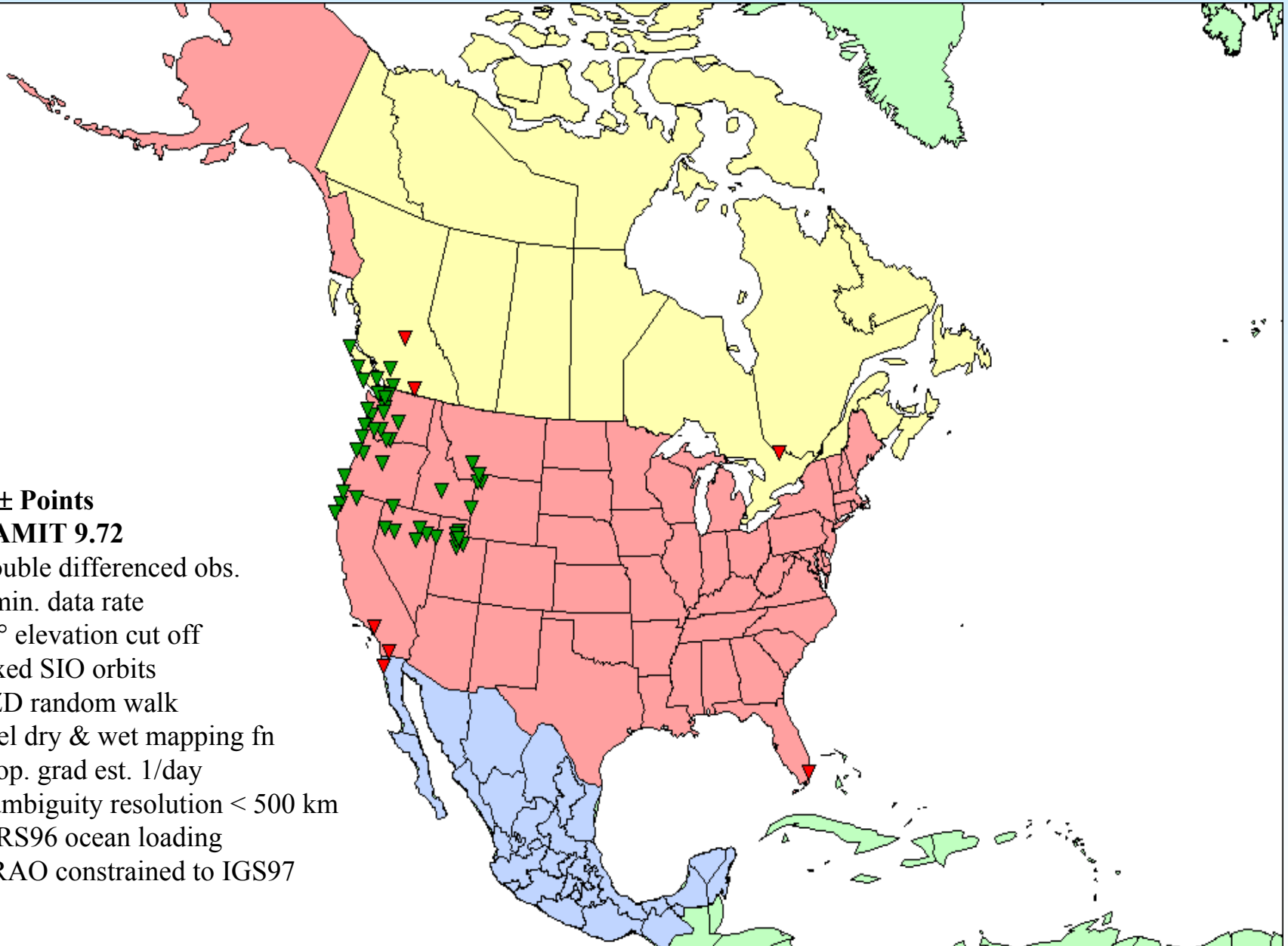
Niel dry & wet mapping fn

Trop. grad est. 1/day

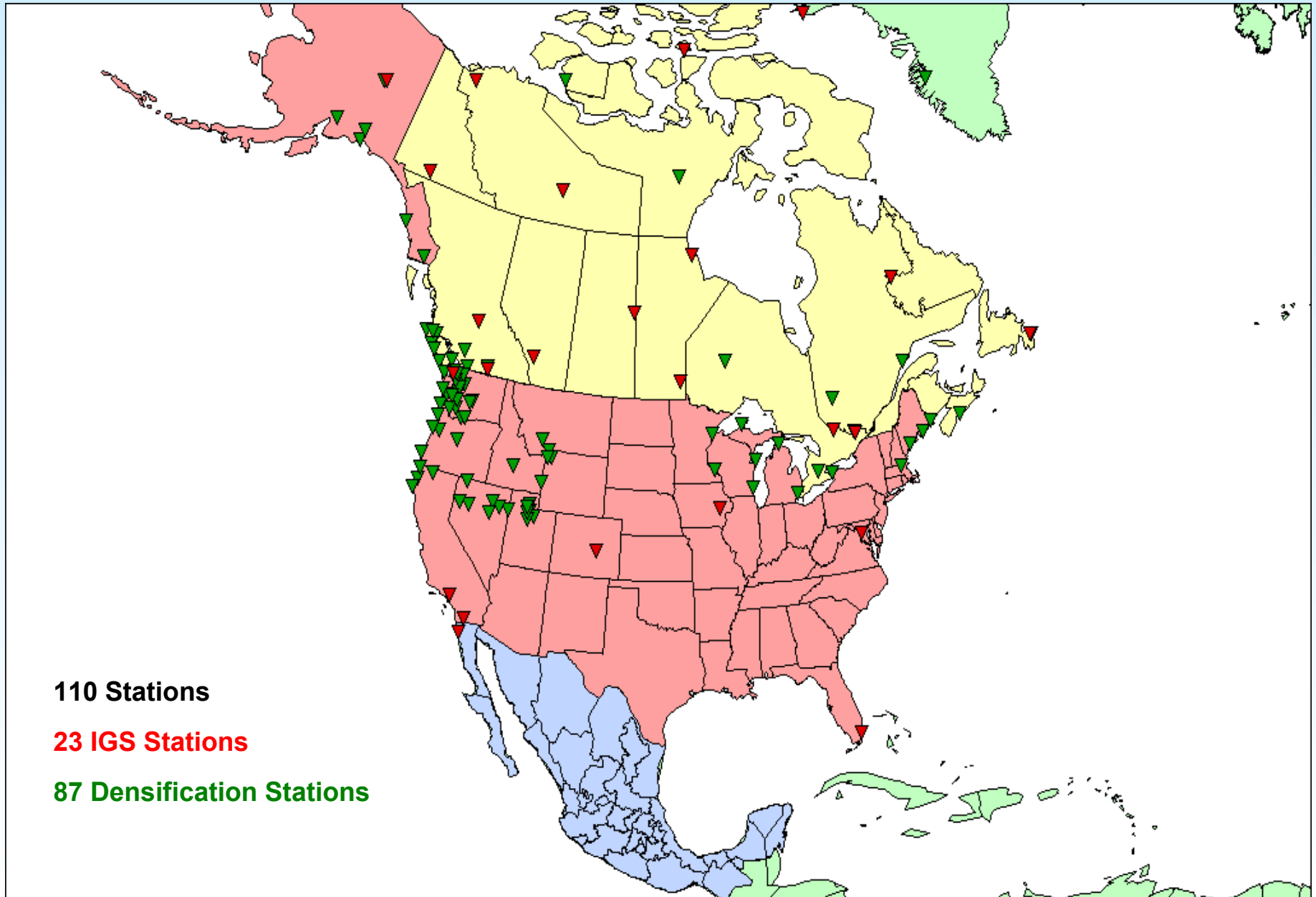
Aambiguity resolution < 500 km

IERS96 ocean loading

DRAO constrained to IGS97



NAREF Densification Network



Combination of Regional Solutions

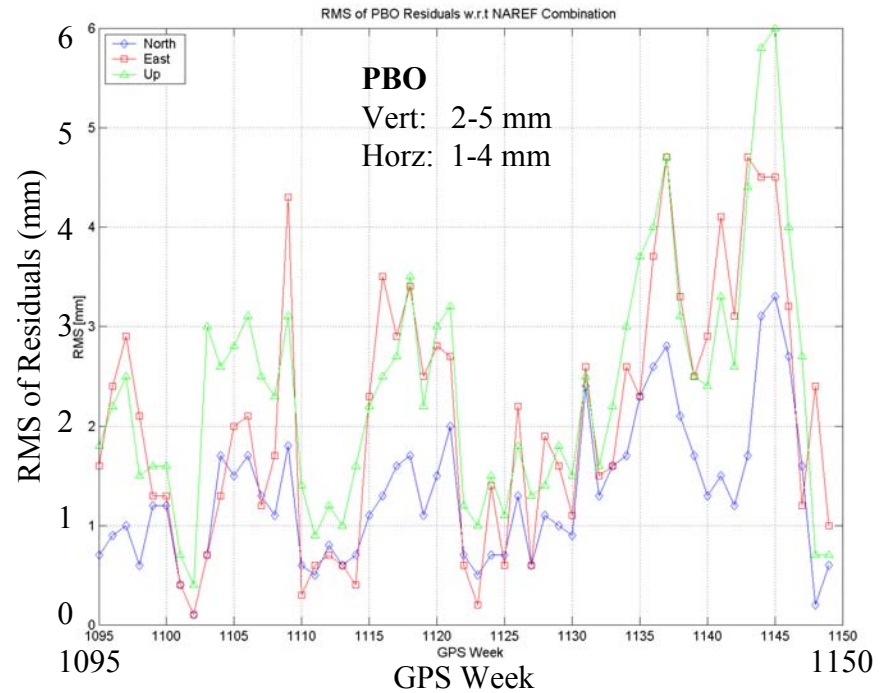
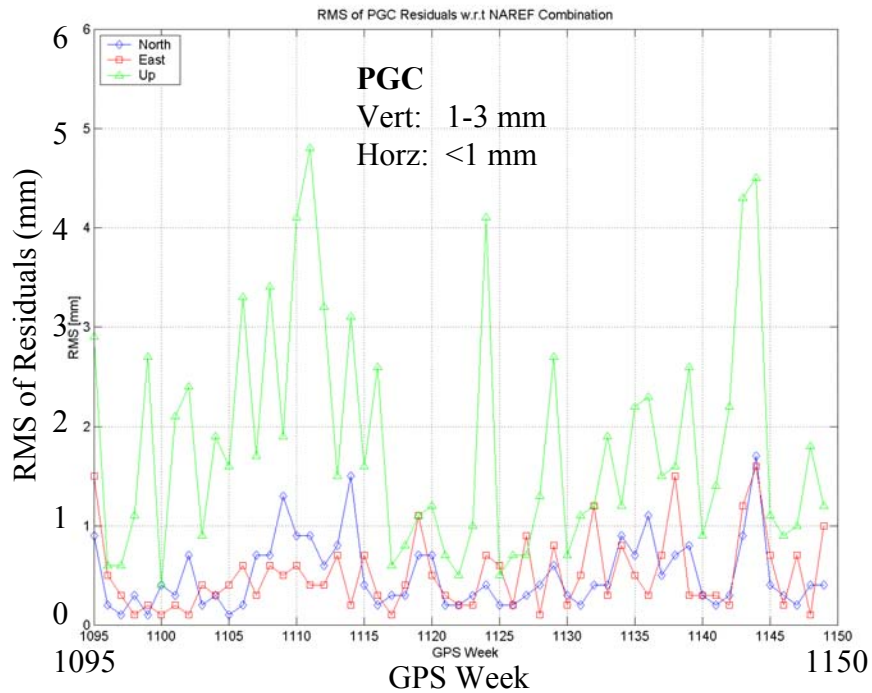
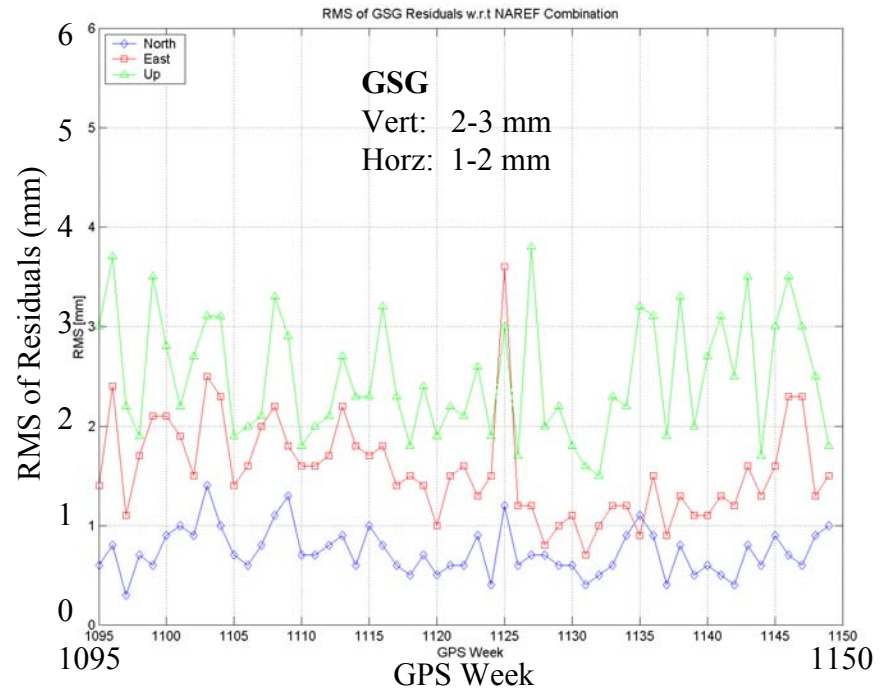
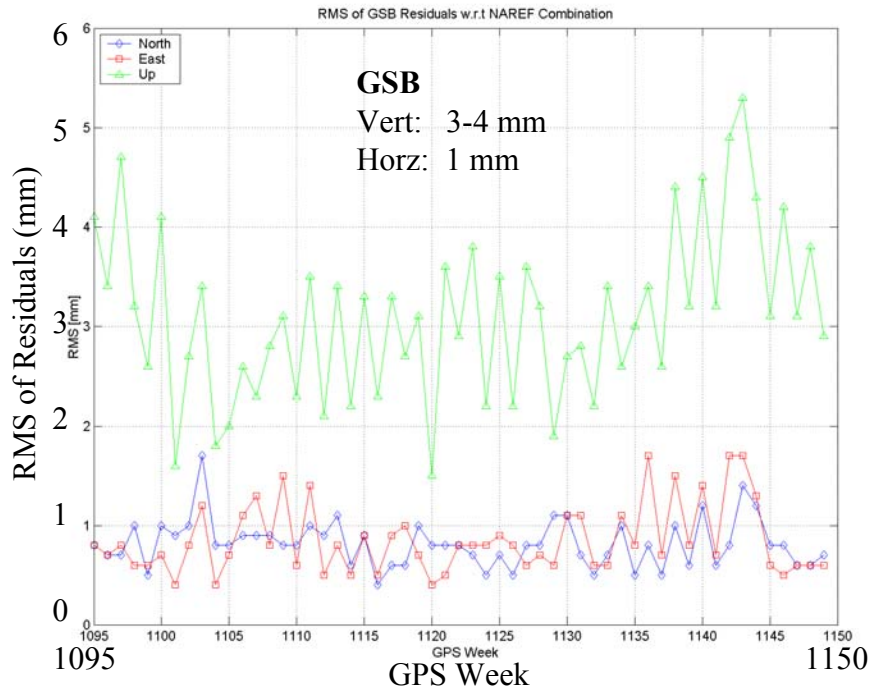
Alignment of Each Regional Solutions

1. A priori datum constraints removed
2. Aligned to IGS weekly solution (3 translations, 3 rotations, scale change)
3. Covariance matrix scaled by WRMS of residuals
4. Residuals tested for outliers (outliers removed → iterate #2-4)

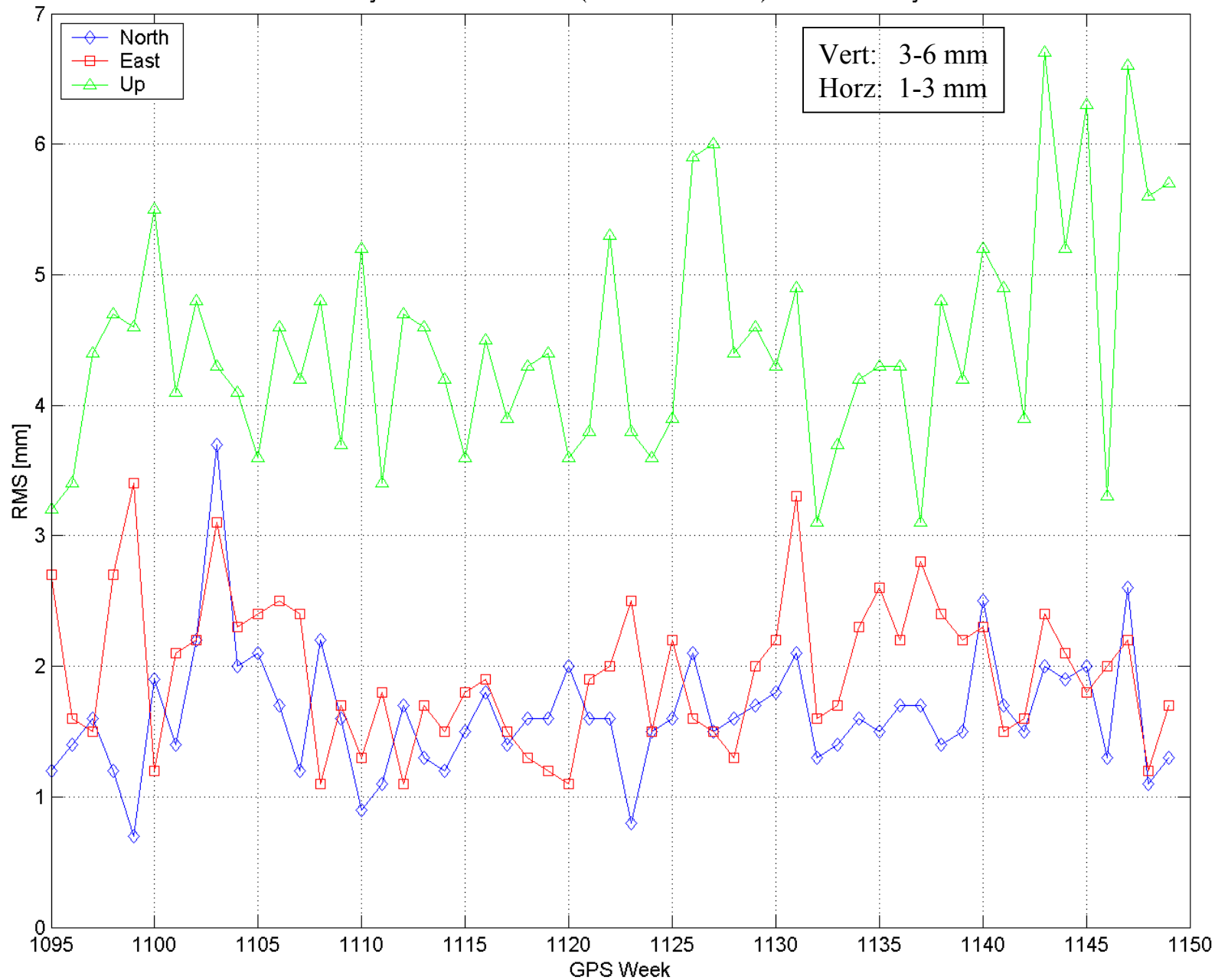
Combination of Regional Solutions

5. Summation of normals of (scaled) regional solutions
6. Aligned to IGS weekly solution (3 translations., 3 rotations, scale change)
7. Covariance matrix scaled by WRMS of residuals
8. Residuals tested for outliers (outliers removed → iterate #2-8)
9. Minimum constraint: One IGS reference frame station (DRAO) constrained to IGS97

Software: SINEX Software v1.0 by Remi Ferland (used for IGS)



RMS of Weekly NAREF Combination (Minimum constraint) w.r.t IGS Weekly Solution



Integration into Global Network (1)

Fixed constraints (or tightly weighted)

- IGS station coordinates don't change
- But distorts regional covariance matrix in both absolute and relative sense

Weighted constraints

- Equivalent to rigorous sequential adjustment of global & regional
- Summation of normals
- IGS station coordinates change

Integration into Global Network (2)

Transformation

- Align regional solution with global
- Usually done with an inner constraint solution
- IGS station coordinates change

Blaha approach

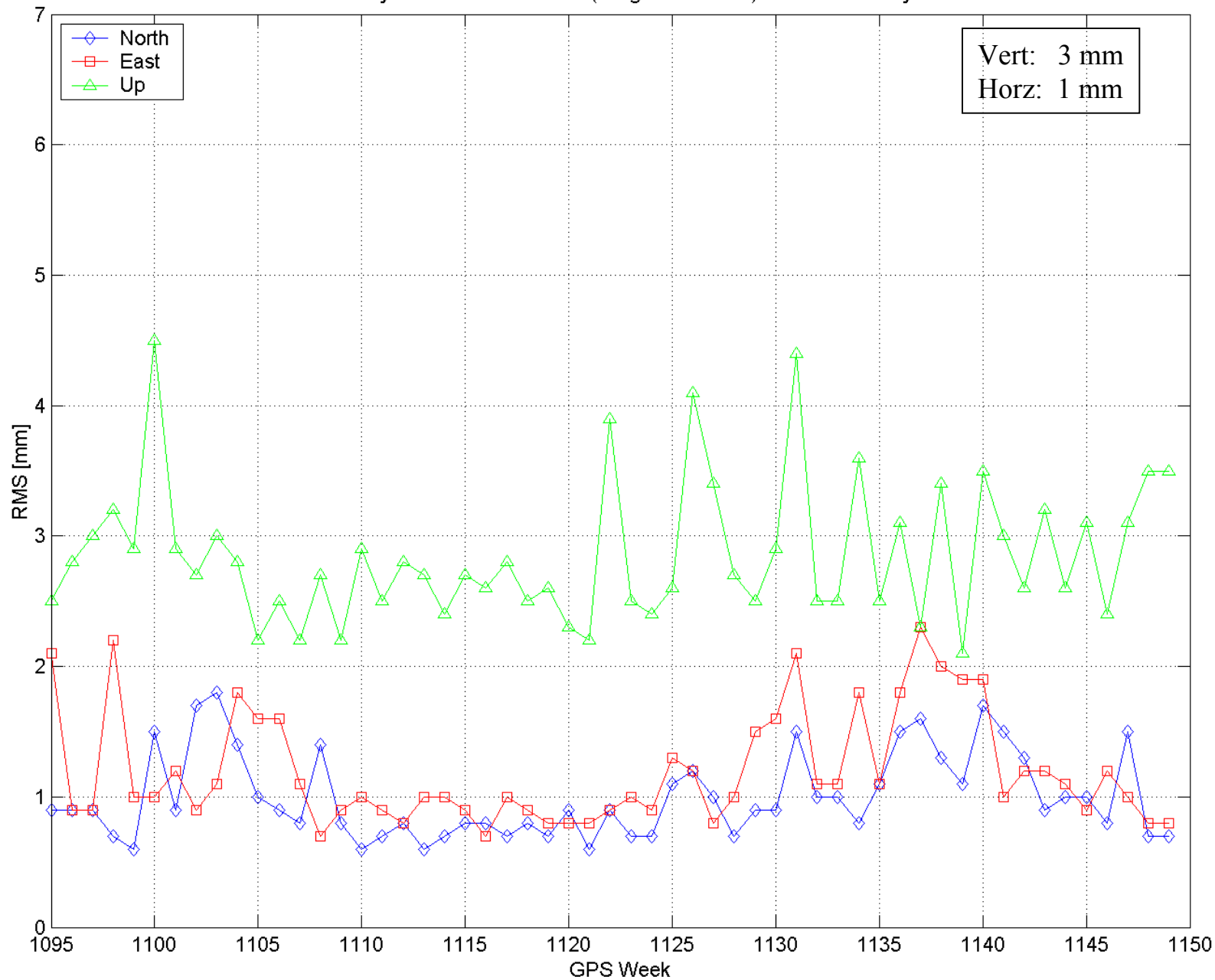
- Similar to weighted station method
- Additional condition that IGS coordinates don't change
- Difficult (impossible?) to specify constraint in SINEX APRIORI blocks – can't remove?

Integration into IGS Global Network

NAREF Approach

- Transformation + weighted constraints
- Transformation/alignment to IGS global network (already done priori to min constraint)
- Remove minimum constraint
- Apply weighted constraints (IGS weekly solution + covariance matrix of common stations)
- Constraints specified in “APRIORI” blocks of SINEX file – can be removed

RMS of Weekly NAREF Combination (Integrated to IGS) w.r.t IGS Weekly Solution



Coordinate Time Series

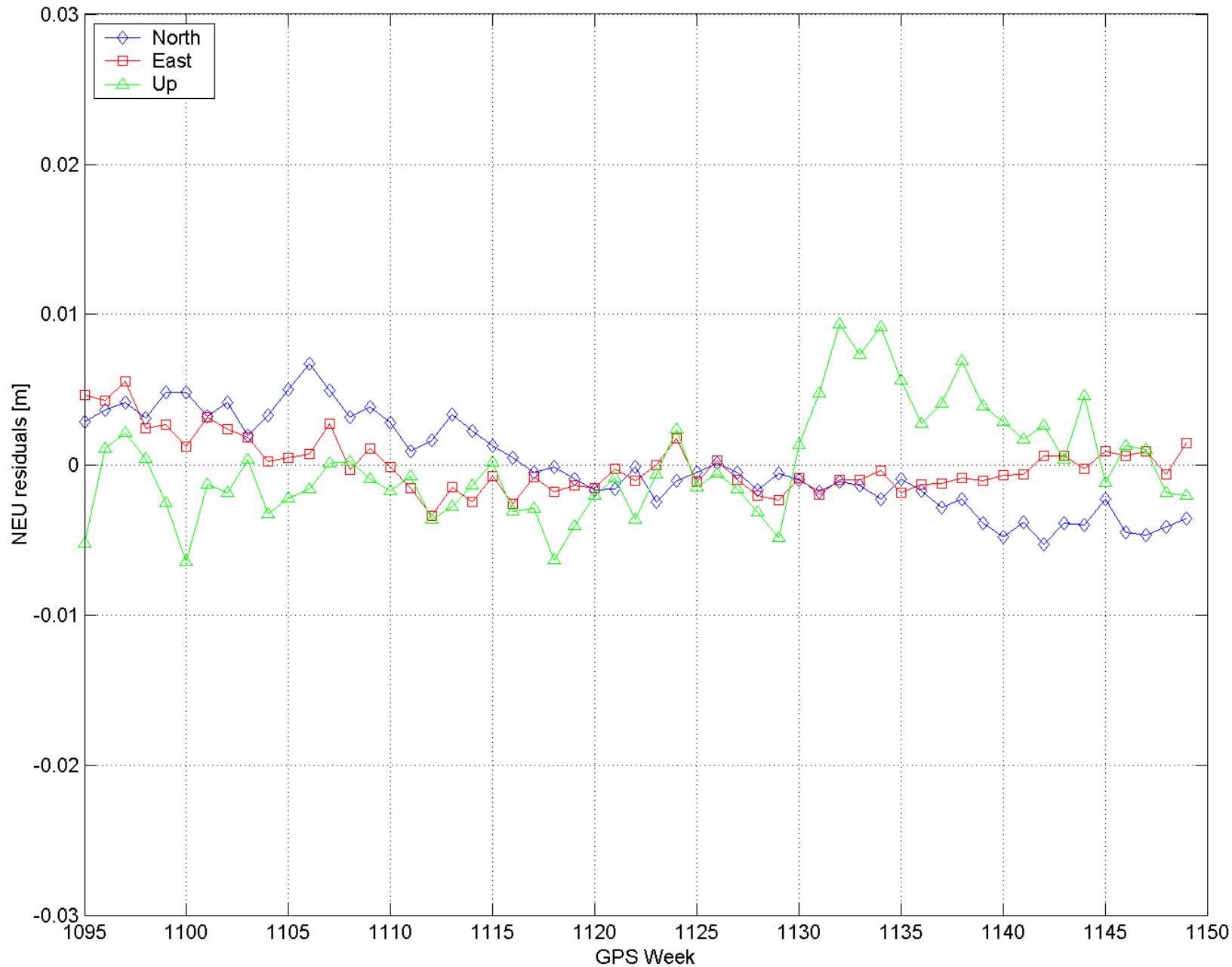
Coordinate Time Series

- From weekly NAREF combinations
- Integrated into IGS global network
- For GPS weeks 1095 to 1150 (56 weeks)

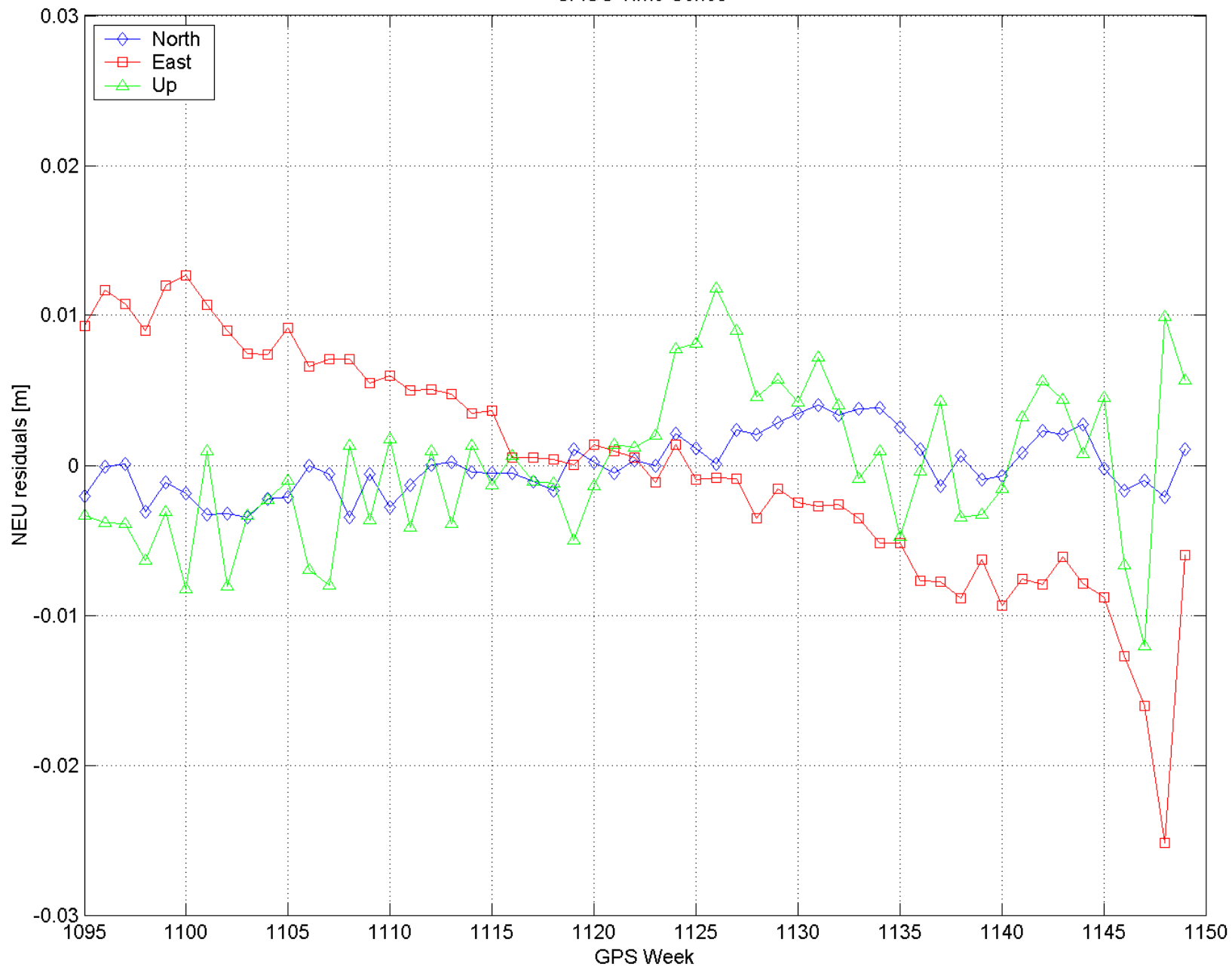
Time Series Plots

- For selected densification points
- Show typical variations in weekly solutions
- In terms of NEU (north, east, up) components
- Coordinate “residuals” are with respect to mean

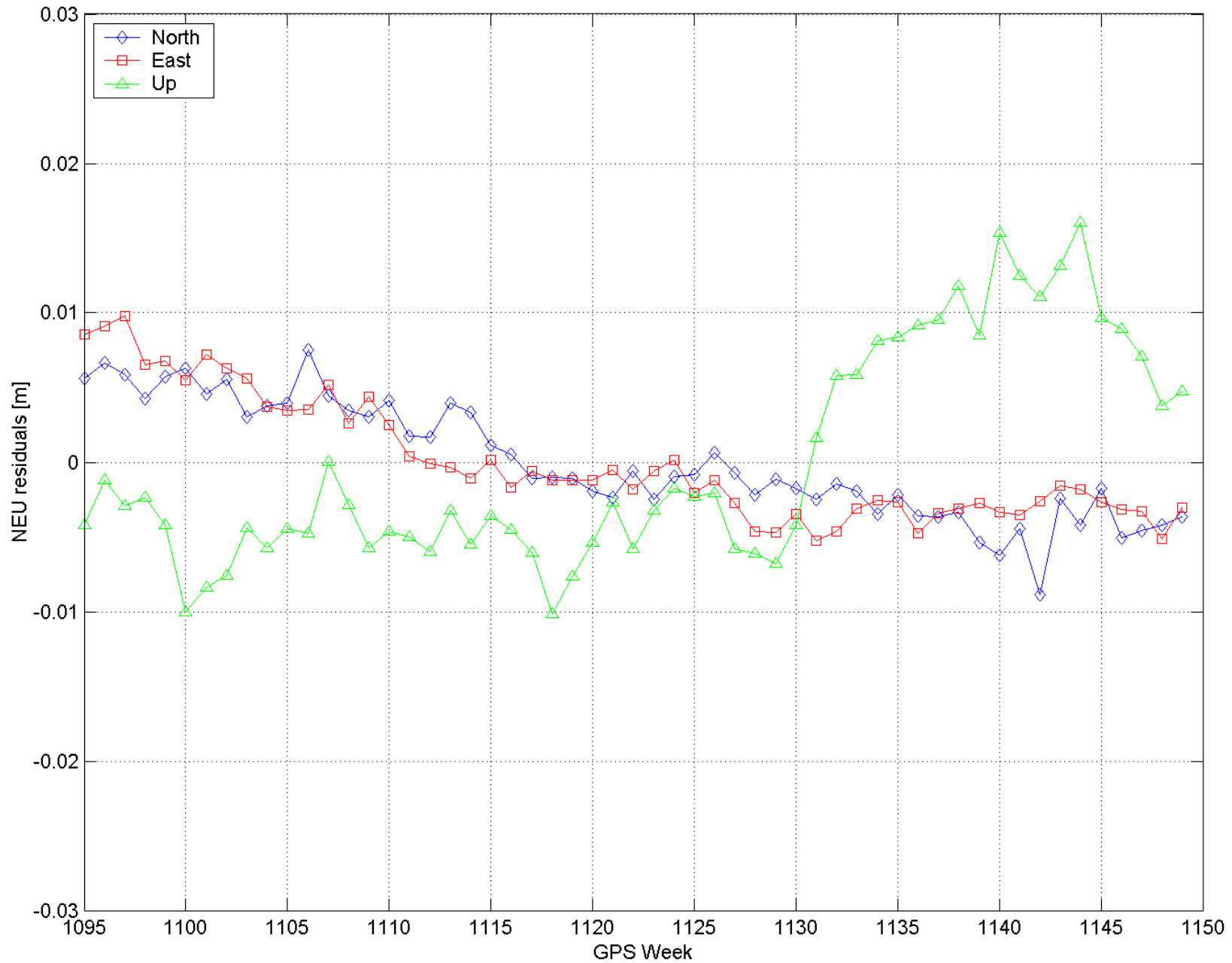
ALBH Time Series



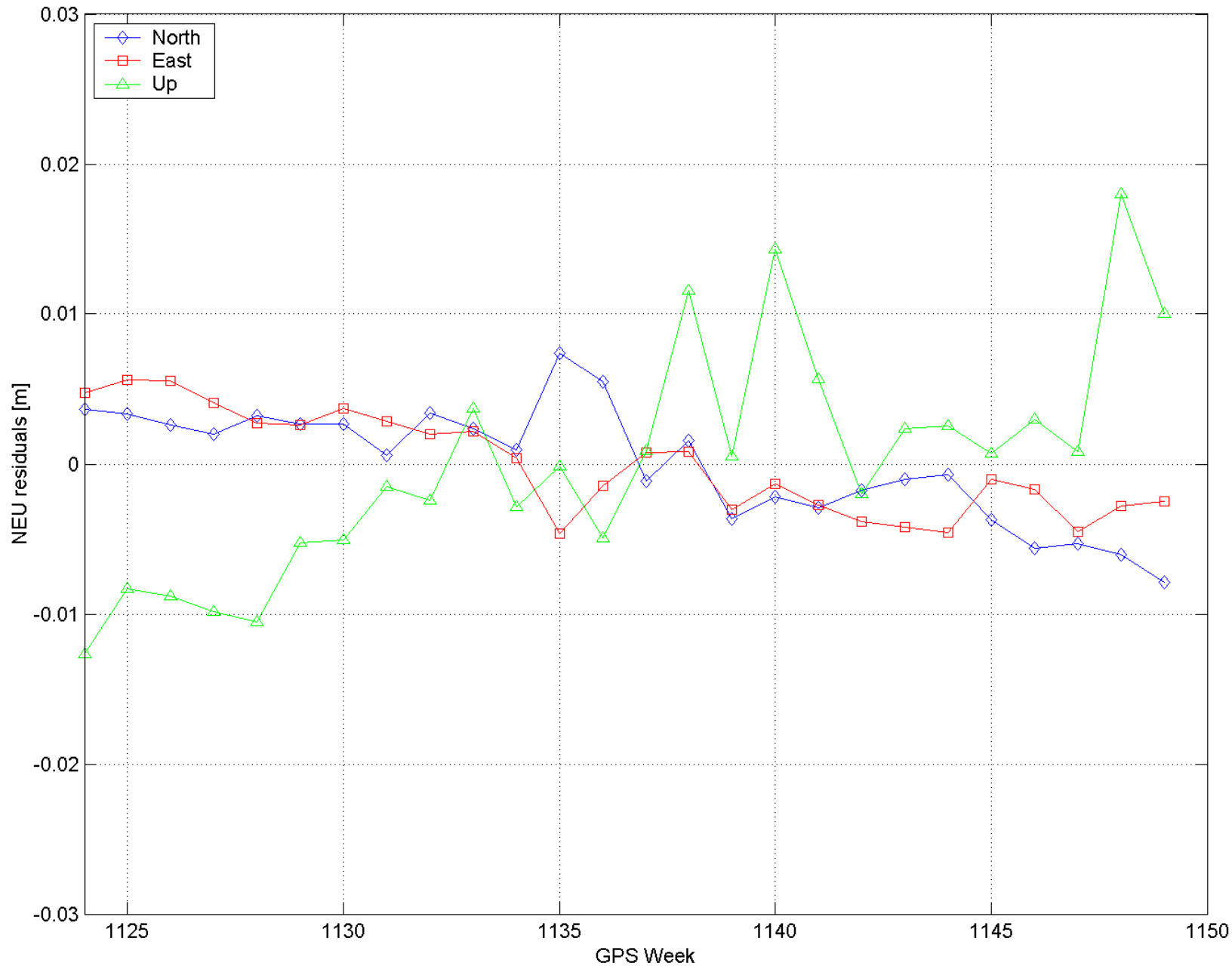
CAGS Time Series



LIND Time Series



INVK Time Series



Future Work

Incorporate other regional networks/solutions

- OSU/NGS Great Lakes CORS network (21 Stations) – in progress
- GPS at Arctic Tide Gauges (4 stations) – Summer 2002
- CORS network (would cover entire US) – ??
- Mexican GPS network (about 10 stations) – ??

Submissions to CDDIS to begin next week

- Weekly solutions since beginning of 2001

Regular cumulative solutions with velocities



EUREF in View of Regional Densification



Heinz Habrich

EUREF Analysis Coordinator

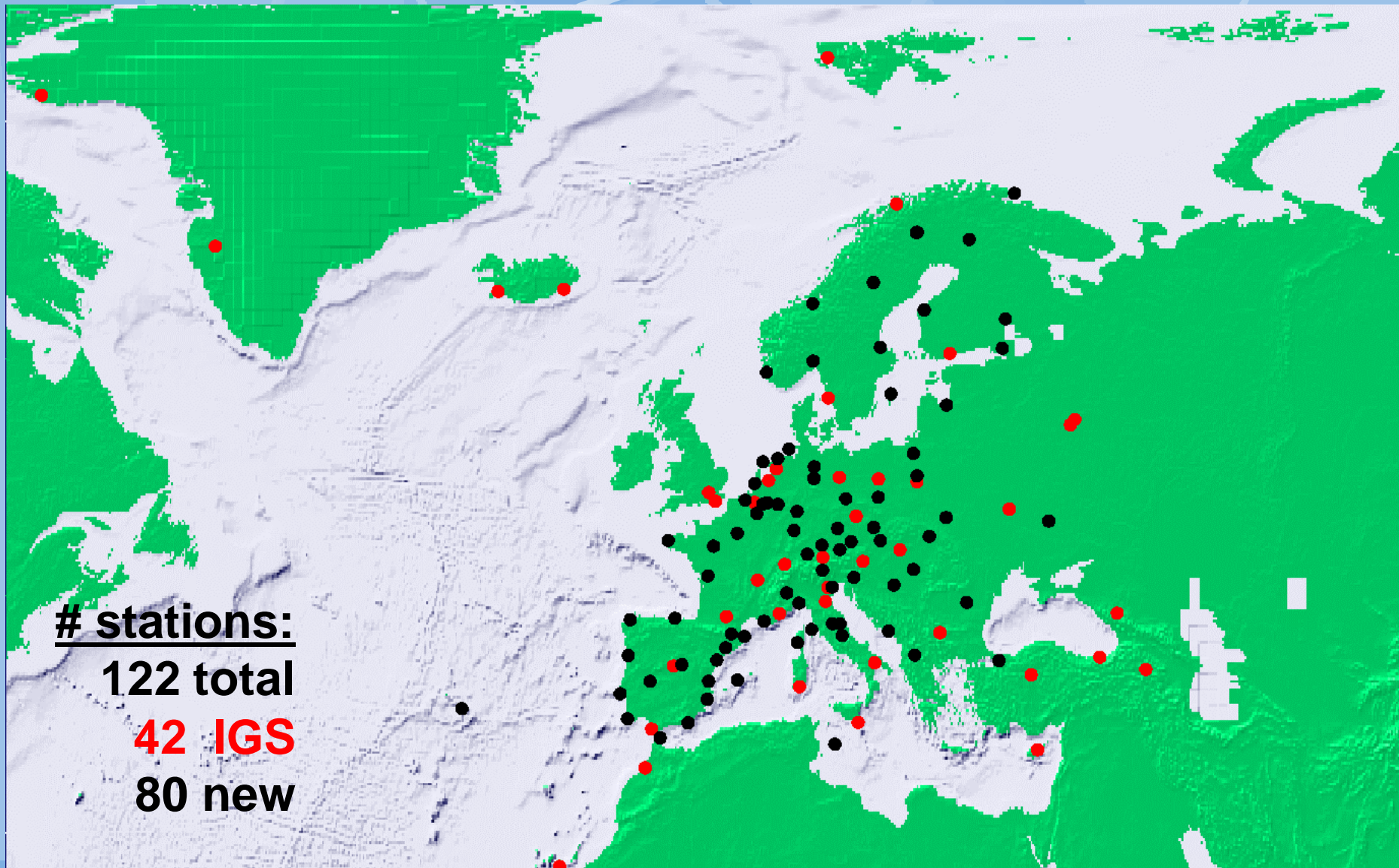
Bundesamt für Kartographie und Geodäsie
Frankfurt, Germany

2002 IGS Workshop, April 8 - 11, Ottawa

Introduction

- Network overview
- Processing scheme
- Current reference frame realization
- Additional preliminary densification products

EUREF GPS Permanent Network



EUREF GPS Permanent Network (EPN) - Organization -

EPN Coordination Group

- Network Coordinator
C. Bruyninx, ROB
- Analysis Coordinator
H. Habrich, BKG
- Dataflow Coordinator
G. Stangl, OLG
- Technical Working Group representative
W. Gurtner, AIUB
- Special projects representatives

EPN Central Bureau

Tasks:

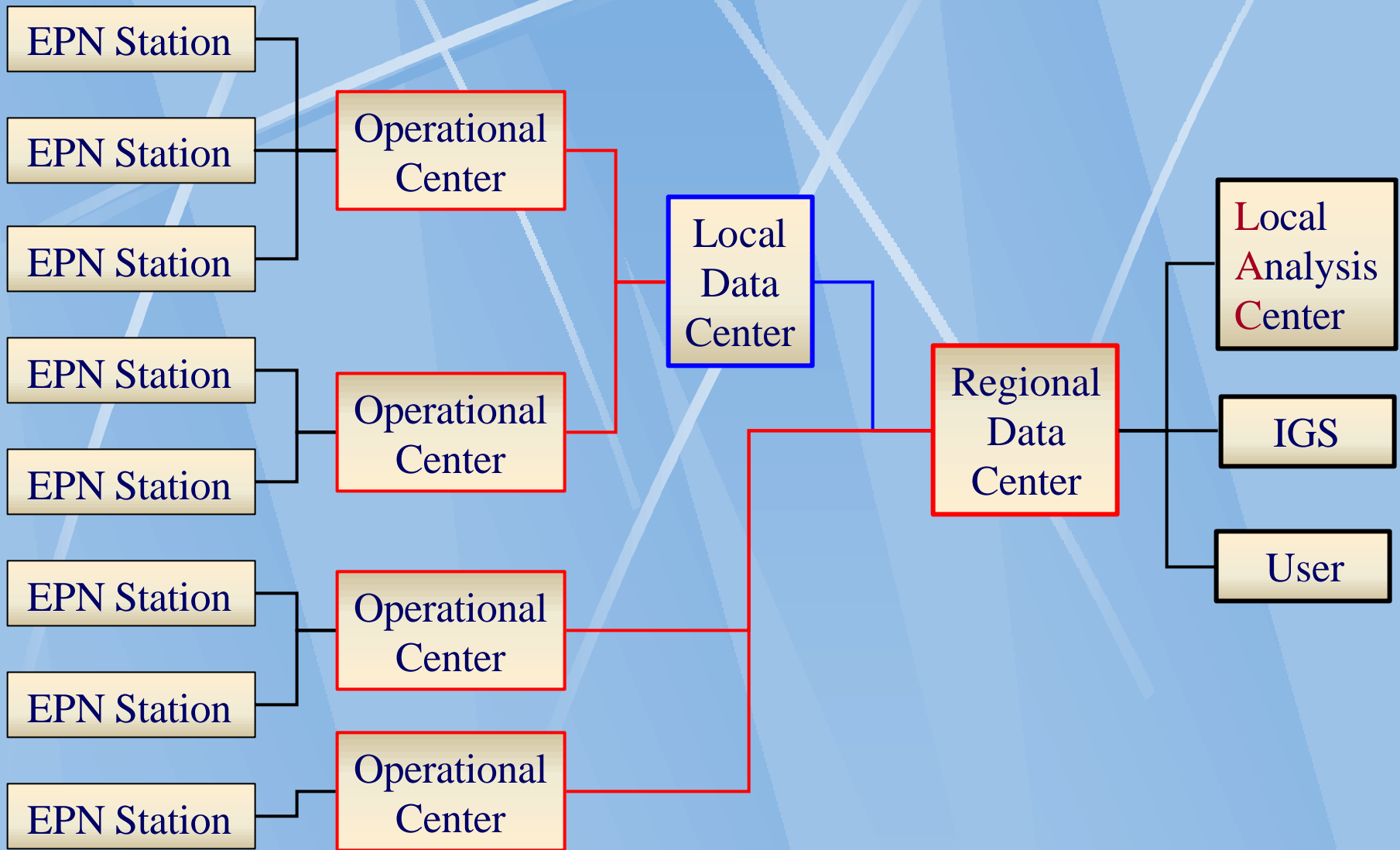
- Day-to-day general management of the EPN
- Realization of a policy as prescribed by the Technical Working Group
- Provide all EPN related information

Head: *C. Bruyninx, ROB*

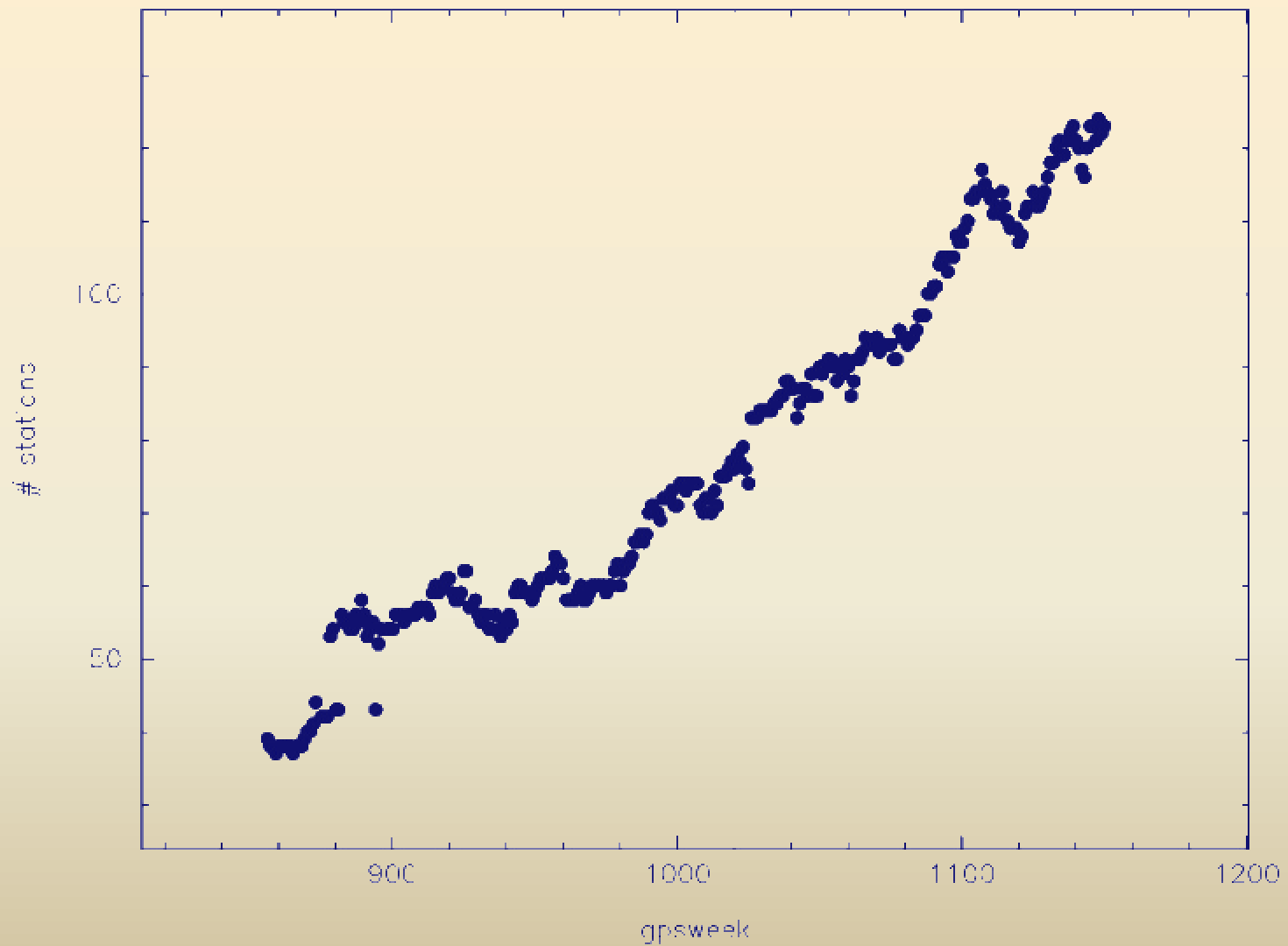
EPN Special Projects

- Time series for geokinematic
Head: *A. Kenyeres, FOMI*
- Troposphere parameter estimation
Head: *G. Weber, BKG*

EPN Data Flow



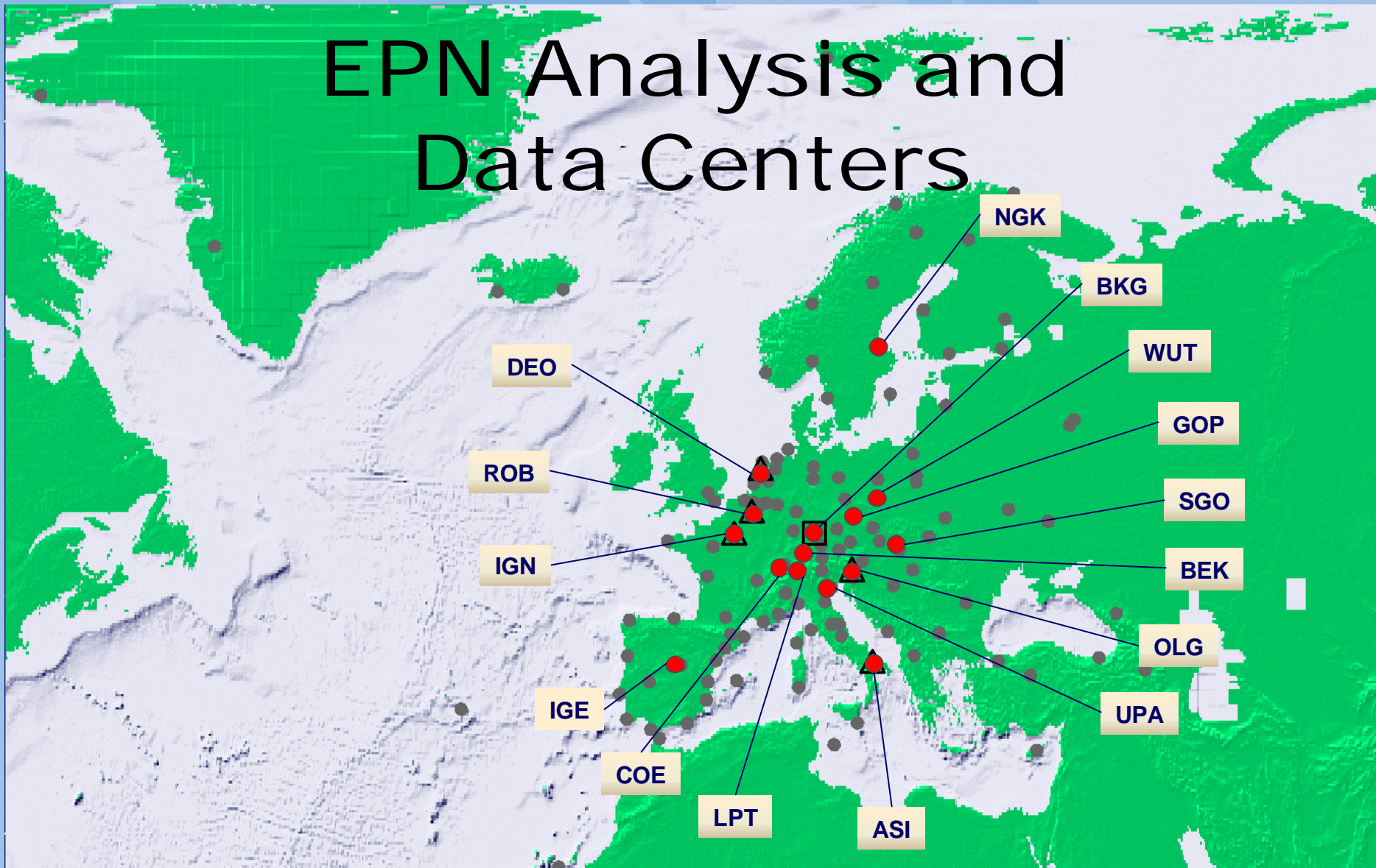
Number of Stations in EUREF Permanent Network



Distributed Processing Approach

- EPN sub-networks are analysed by **15 Local Analysis Centers (LACs)**
- Each station is processed by **at least 3 LACs**
- Combination of 15 sub-networks
- Benefit:
 - **Quality check** because of redundant results
 - **Know-How Transfer** between European Nations
 - National participation increases the **national acceptance** of the EUREF products

EPN Analysis and Data Centers



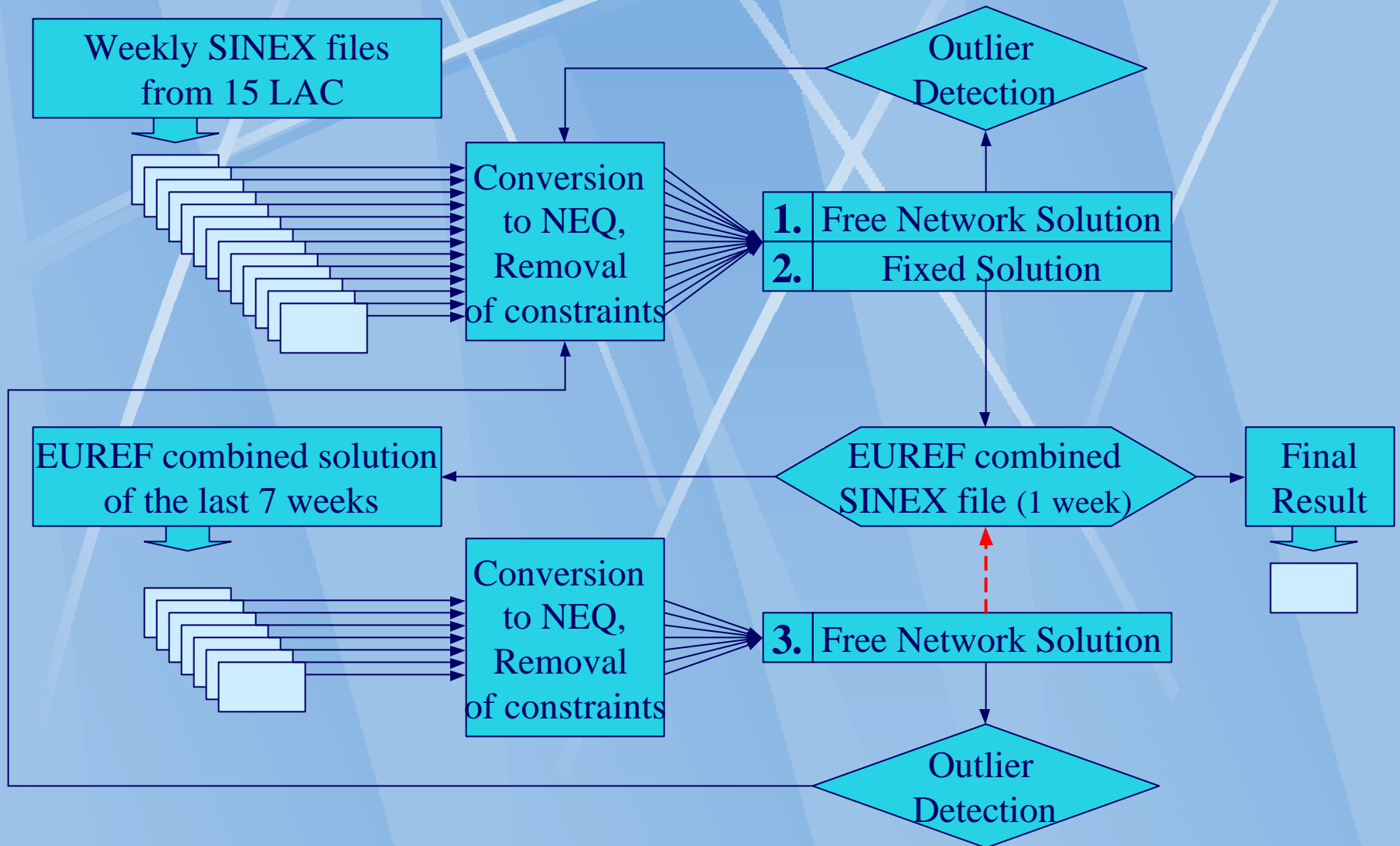
● Local Analysis Center

□ Regional Data Center

△ Local Data Center



Combination Scheme



Weighting of Solutions in Bernese Software

SINEX Files
COVA/CORR
RMS of Unit Weight (σ)
Unknowns
Observations

$$1/\sigma^2$$

NEQ
Files

Combined
Solution

External
Reference

Weight
File

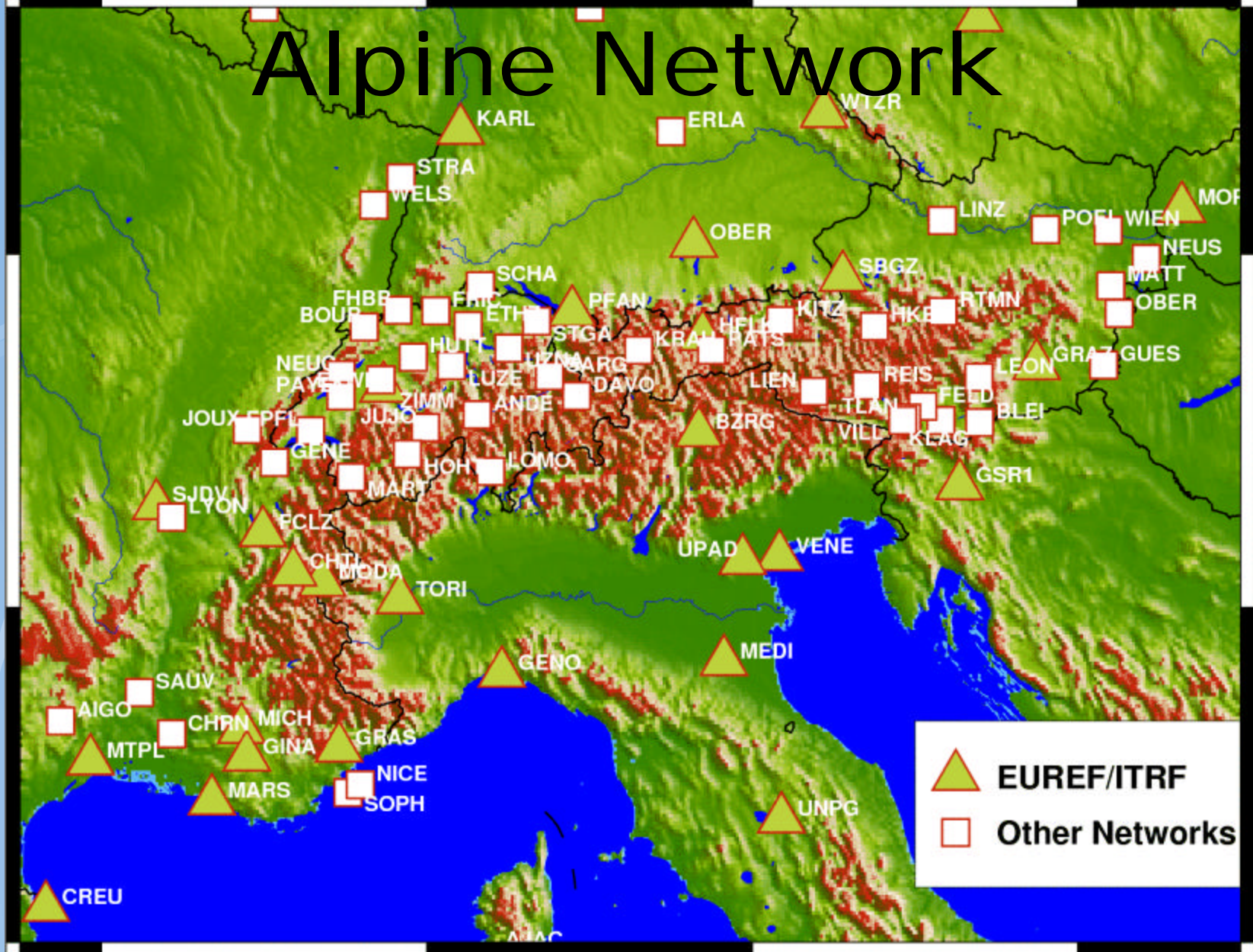
Estimation
of Factor
(Repeatability/
RMS ratio)

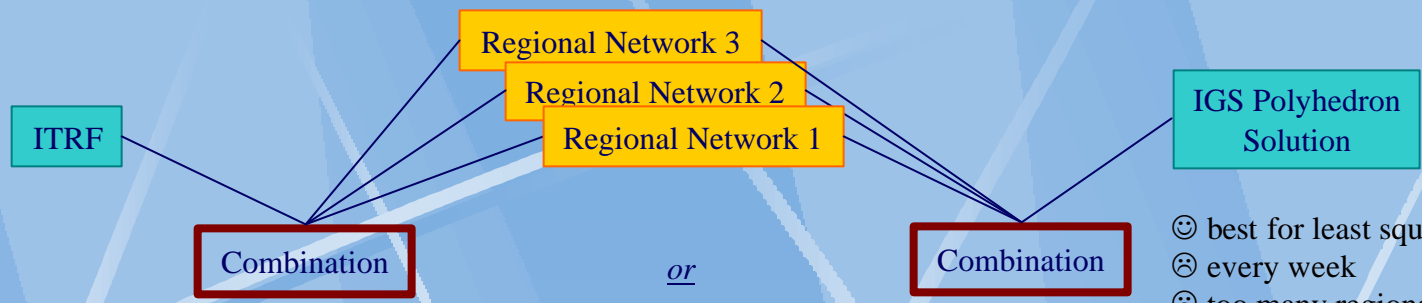
► Remark: Improvement in preparation ◀

Reference Frame Alignment of Weekly EPN Solutions

- ITRF-2000 coordinates of **12 EPN stations** are hold **fixed**
- Generation of weekly SINEX files
- A-priori constraints are included in SINEX files
- Product usage:
 - Direct extraction of coordinates and covariances
 - Removal of constraints for new reference frame definition *or* cumulative solutions
- Example for usage:
 - Contribution to ITRF-2000 (cumulative solution)
 - Alignment of the **,'Alpine Network'** to EPN

Alpine Network



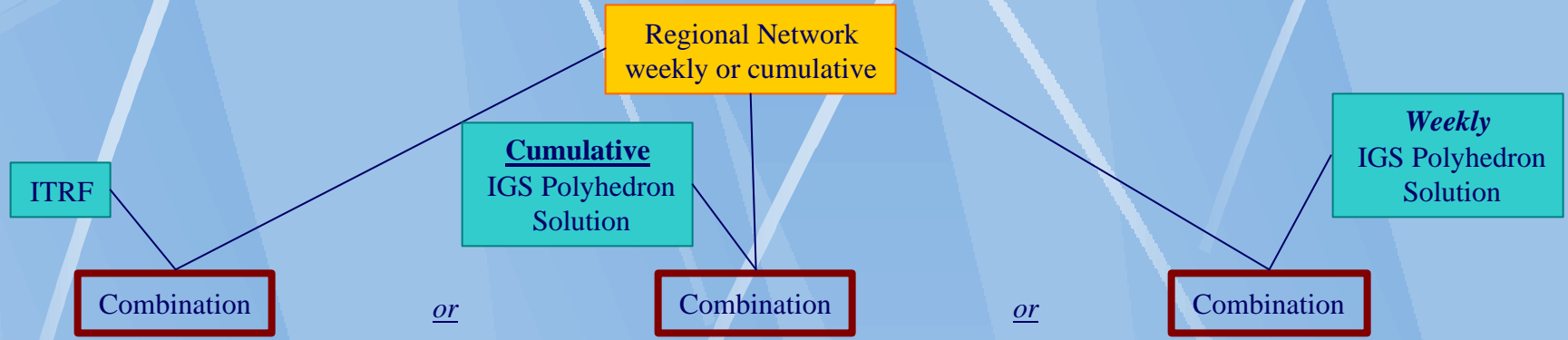


or

Centralized Densification

- ☺ best for least squares adj.
- ☺ every week
- ☹ too many regional solutions
- ☹ too many stations

De-centralized Densification



- ☹ lag of 1-2 years between observations and solution
- ☹ temporary variations not modeled

☹ lag of a couple of weeks

- ☺ Close to real time
- ☹ integration instability due to number of overlapping stations (network dependent), and alignment of IGS weekly solution to ITRF (51 RF stations)

IGS global station solution fixed in integrated solution

☺ **Candidate !**

or

Full variance-covariance matrix of IGS Polyhedron solutions of common stations introduced into integrated solution.

☹ Coordinates of common stations will change



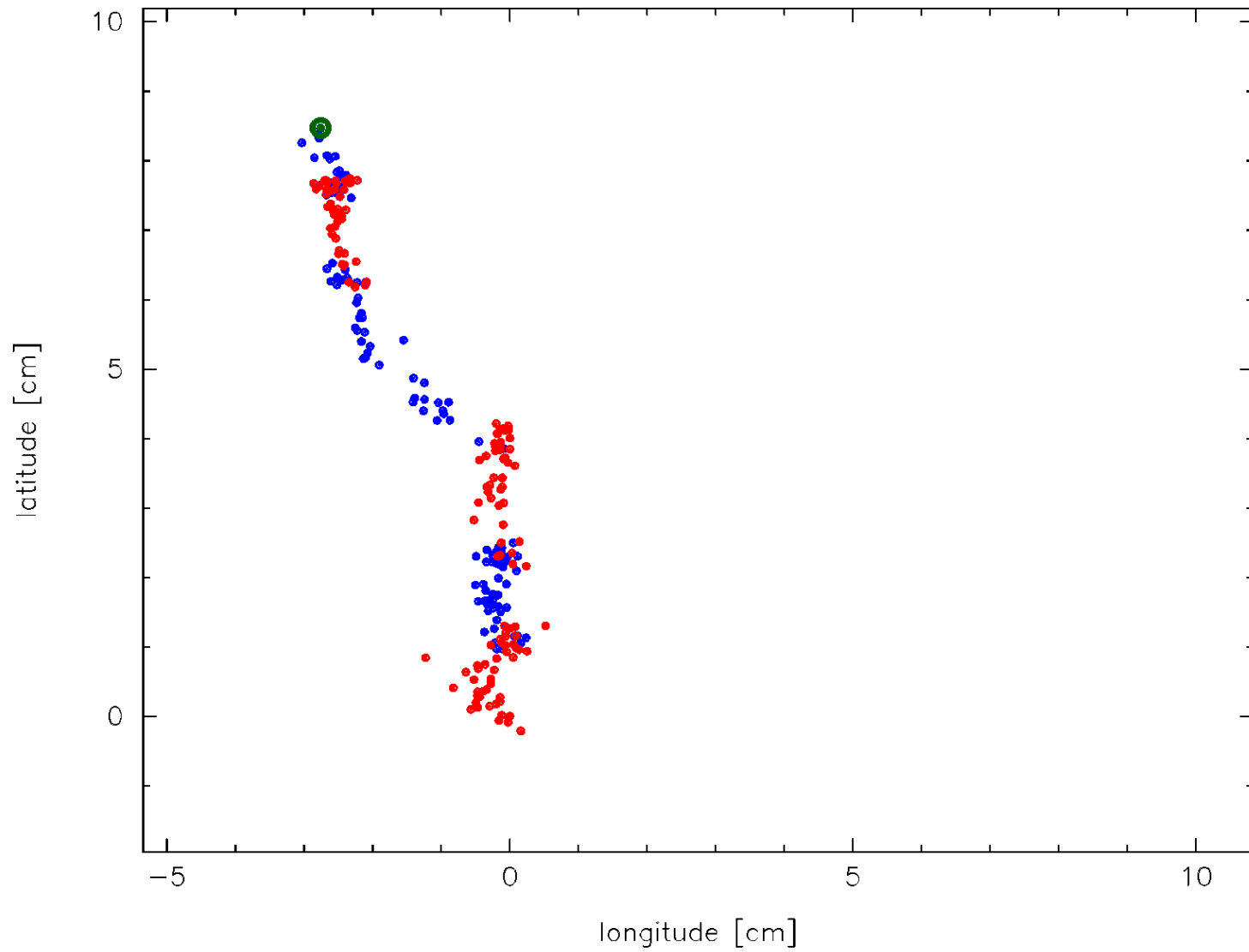
Realization of the IGS Polyhedron and Preliminary European Densification

	Weekly	Cumulative
IGS	Coordinates and velocities of 51 high quality global distributed stations aligned to ITRF-2000 \hat{E} IGS Polyhedron	
	~ 130 global stations	~ 130 global stations
EUREF	IGS Polyhedron fixed	
	~ 40 European stations	~ 40 European stations
	\hat{E} EPN Polyhedron	
	~ 120 European stations	~ 120 European stations

EPN Cumulative Solution Characteristics

- Currently 288 weekly solutions, starting gpsweek 860
- Relative velocity between substations (set up of new coordinates after station inconsistency) constraint to be zero
- Generation of 2 intermediate and 1 final solution
 - ❖ Free network constraints, residuals show velocities
 - ❖ Free network constraints, velocities removed in residuals
 - ❖ Solution fixed to the global IGS solution (= Final Solution)

Track of Horizontal Displacement for ANKR



(c) BKG euref 5-Mar-2002 15:20

Matrix of Solutions

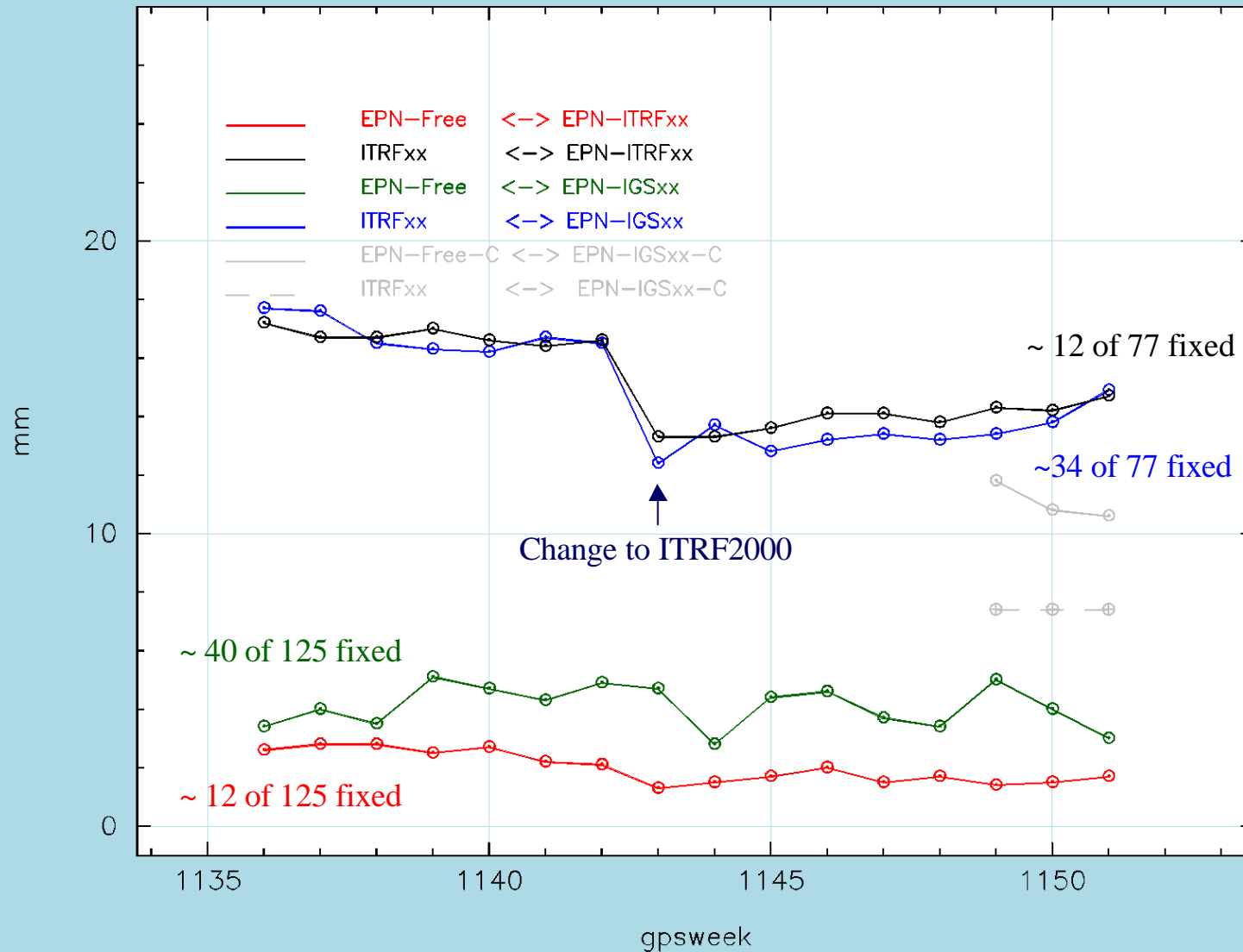
Global Reference Frame Solutions	Free Network Conditions	Fixed Reference Stations
ITRF-2000	EPN-Free	EPN-ITRFxx
IGS-Weekly	(EPN-Free)	EPN-IGSxx
IGS-Cumulative	EPN-Free-C	EPN-IGSxx-C

2 Regular EPN Products

3 Preliminary Products



RMS of 7 Parameter Helmert Transformation

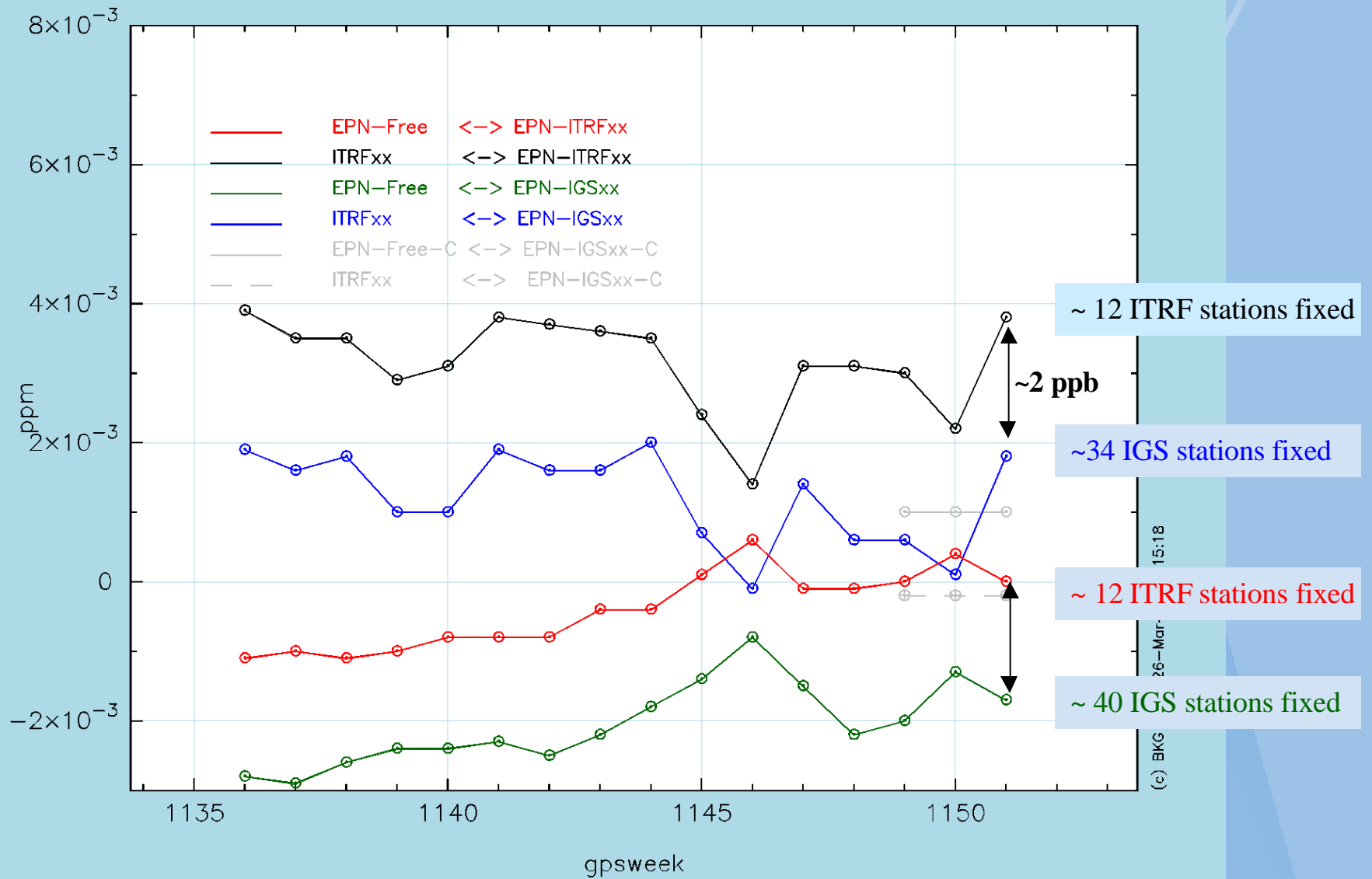


(c) BKG euref 26-Mar-2002 13:53

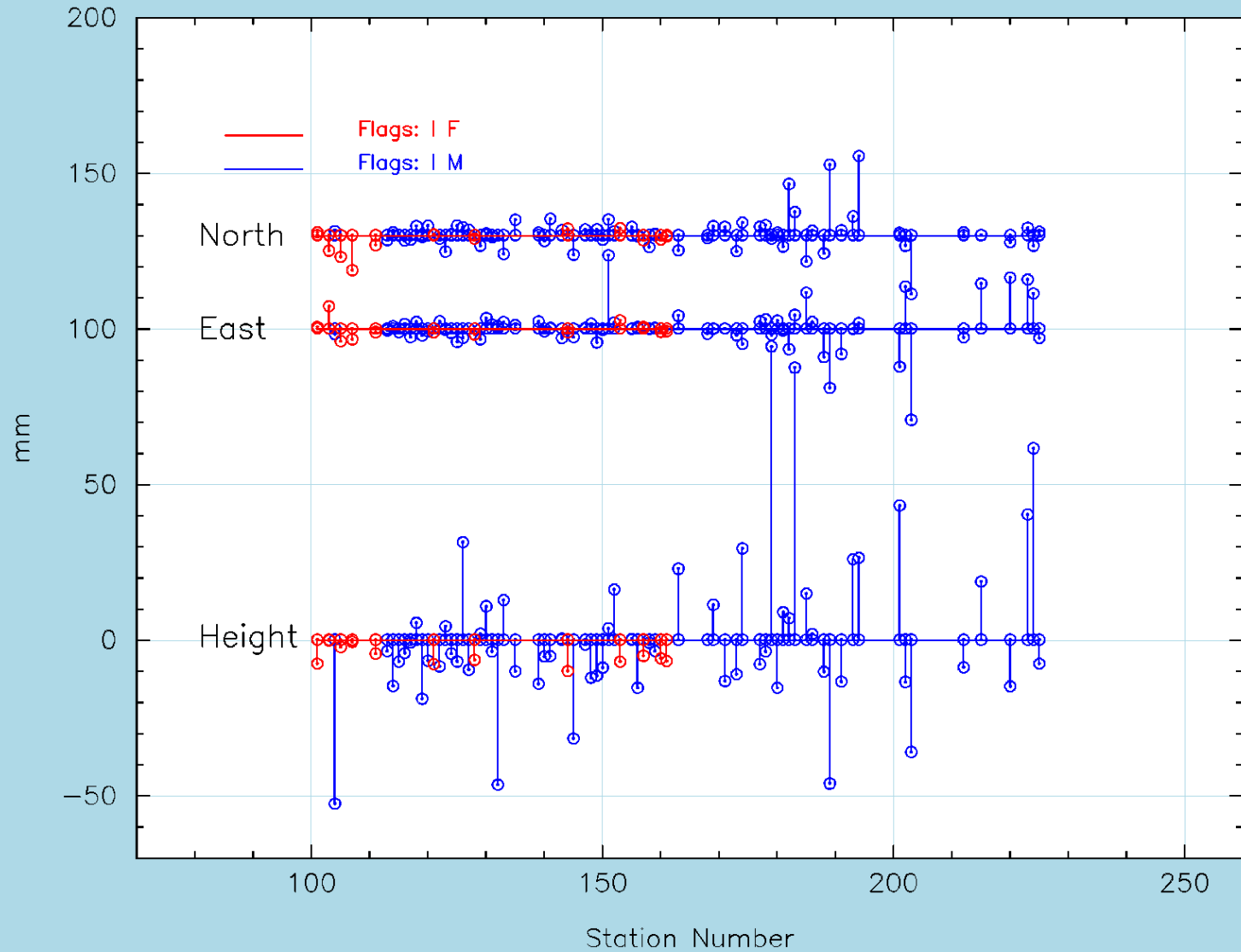
Meaning: ~## reference stations of ### total transformed stations fixed in EPN



Scale Factor in 7 Parameter Helmert Transformation



Residuals of 7 Parameter Helmert Transformation file EUS11511.RES



Conclusions

- Continue to generate a regional densification in the IGS-2000 to study the quantity of differences
- Method for densification not finally defined
- Additional products are different from current products and require guidelines describing how to use the products
- The Helmert transformation between different solutions shows some outstanding stations. Those comparisons should be repeated for solutions using the variance/covariance information for densification processing



ITRF2000 Summary and Some Regional Solutions Evaluation

- Summary of ITRF2000 datum definition
- How to express a weekly regional solution in ITRF ?

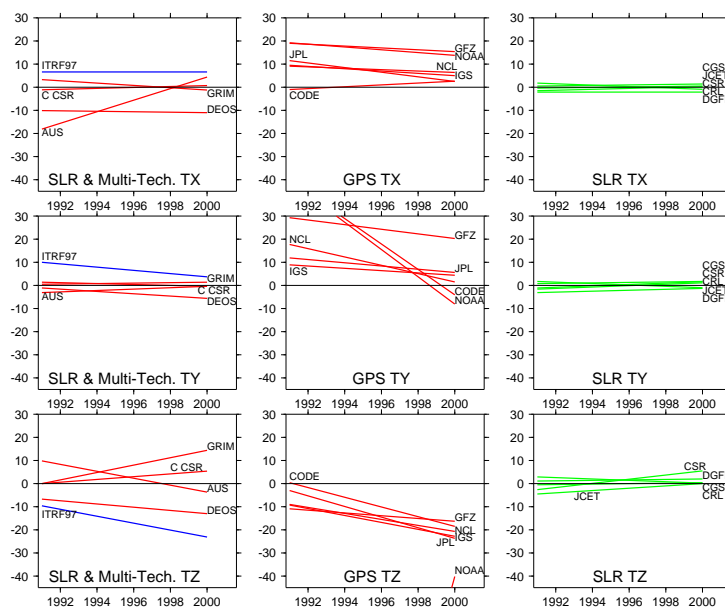
Key Words: TRF Datum Definition; Network Effect

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Institut Géographique National, FRANCE
<http://lareg.ensg.ign.fr/ITRF/ITRF2000>

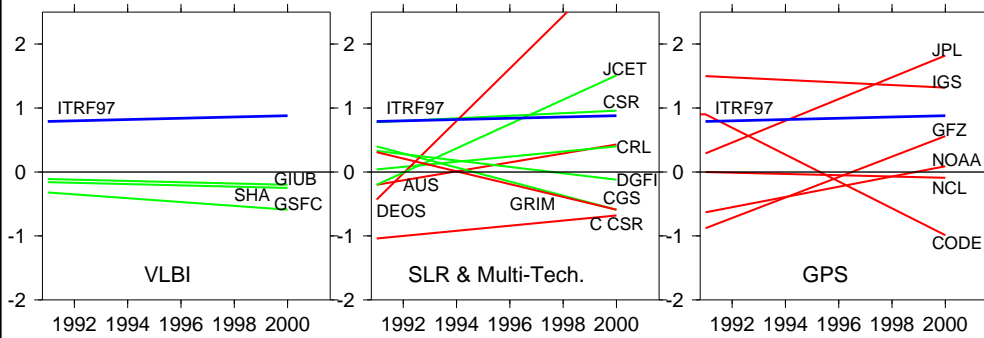


ITRF2000: Translation Variations (mm)





ITRF2000: Scale Variations (ppb = 10⁻⁹)



(1 ppb ≈ 6 mm shift in station heights)

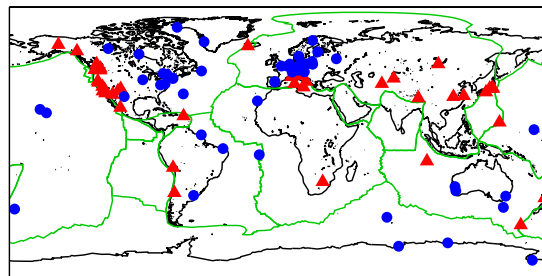


ITRF2000 orientation & rate definition

- Orientation: ITRF97 at 1997.0 (1 mm)
 - Rate: NNR-NUVEL-1A (1 mm/y)
- Using a Datum Eq.: $B(X - X_0) = 0$

$$B = (A^T A)^{-1} A^T$$

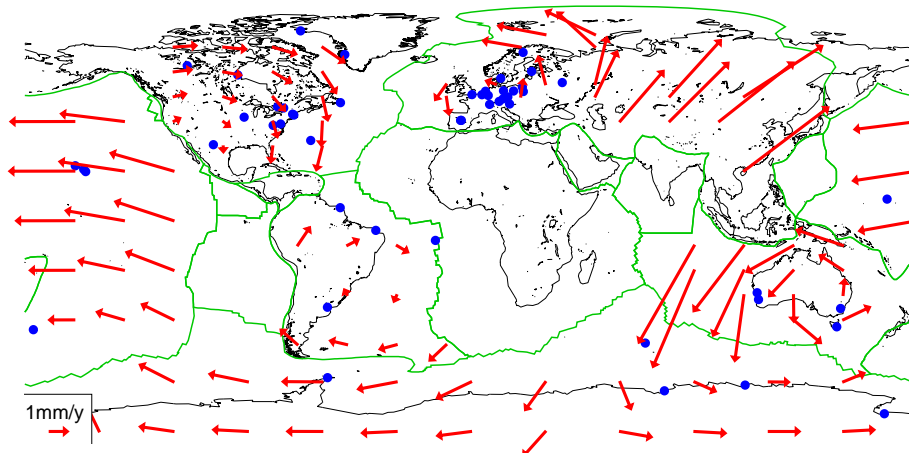
$$A = \begin{pmatrix} \cdot & \cdot & \cdot \\ 0 & z_a^i & -y_a^i \\ -z_a^i & 0 & x_a^i \\ y_a^i & -x_a^i & 0 \\ \cdot & \cdot & \cdot \end{pmatrix}$$



- X_0 : Positions (ITRF97) or Velocities (NNR-NUVEL-1A)
- X : Estimated Positions/Velocities



Differences ITRF2000 - NNR-NUVEL-1A



How to express a weekly regional solution in ITRF ? (1)

Selection of ITRF stations of high quality

- Optimal distribution over the regional network
- Long observing history (at least 3 years)
- ITRF residuals less than 5 (10!) mm for positions and 3 mm/y for velocities for at least 3 different solutions contributed to the ITRF solution.

The selected stations should be under constant surveillance to detect/identify possible discrepancy between ITRF and the weekly solutions.



How to express a weekly regional solution in ITRF ? (2)

Using the subset of ITRF stations:

1. Constraining coordinates of the selected stations ($\sigma \approx 10^{-5}$ m) to ITRF values (EUREF adopted Scheme)
2. Alignment using 7 transformation parameters
 - Advantage of (1): the weekly solution is well expressed in ITRF.
 - Disadvantage of (1): the coordinates of the selected stations are entirely determined by ITRF.
 - (2) should be used with care: network effect may bias or/and yields imprecise 7-parameter estimation
 - (2) if used with minimum constraints approach, preserves the original characteristics of the weekly solution



Alignment using Minimum Constraints (1)

The relation between a Weekly solution (X_W) and ITRF (X_I), over selected stations, could be written as:

$$X_I = X_W + A\theta \quad (1)$$

L.S. yields:
$$\theta = \overbrace{(A^T A)^{-1} A^T}^B (X_I - X_W)$$

To have X_W be expressed in ITRF at Σ_θ level, a "datum definition" equation could be written as:

$$B(X_I - X_W) = 0 \quad (\Sigma_\theta) \quad (2)$$

and in terms of normal equation: $B^T \Sigma_\theta^{-1} B(X_I - X_W) = 0$



$$A = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 1 & 0 & 0 & x_a^i & 0 & z_a^i & -y_a^i & \\ 0 & 1 & 0 & y_a^i & -z_a^i & 0 & x_a^i & \\ 0 & 0 & 1 & z_a^i & y_a^i & -x_a^i & 0 & \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \quad \theta = \begin{pmatrix} T_x \\ T_y \\ T_z \\ D \\ R_x \\ R_y \\ R_z \end{pmatrix}$$



Alignment using Minimum Constraints (2)

The initial NEQ system of the W-solution could be written as:

$$N_{unc}(DX) = K \quad (3)$$

where $DX = X - X_{apr}$

Selecting ITRF sub-set of stations (X_I), MC equation is:

$$B^T \Sigma_\theta^{-1} B(DX) = B^T \Sigma_\theta^{-1} B(X_I - X_{apr}) \quad (4)$$

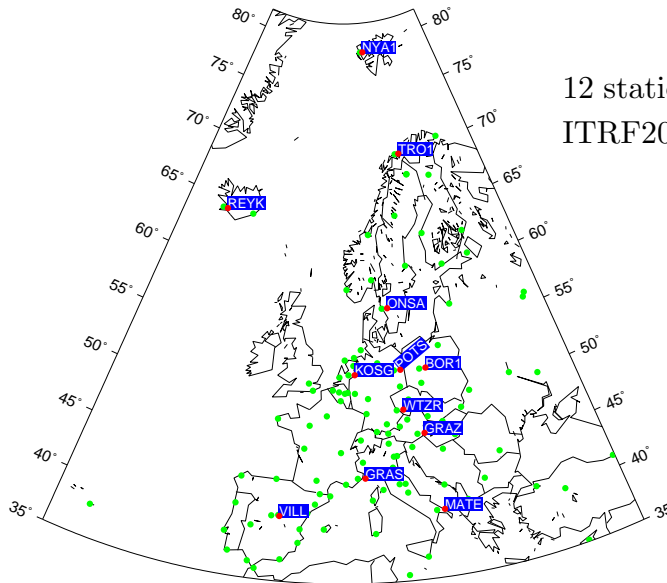
Cumulating (3) and (4) yields:

$$(N_{unc} + B^T \Sigma_\theta^{-1} B)(DX) = K + B^T \Sigma_\theta^{-1} B(X_I - X_{apr})$$



Numerical Application

EUREF Combined Solution for GPS Week 1149



12 stations are constrained to ITRF2000 values ($\sigma = 10^{-5}$)



Using classical 7-parameter estimation

From ITRF2000 to Unconst'd EUREF-W-Solution (12 Sta)

$T1$	$T2$	$T3$	D	$R1$	$R2$	$R3$
cm	cm	cm	10^{-8}	0.001''	0.001''	0.001''

Unweighted L.S. adjustment

-16.60	-4.77	-23.00	.948	.117	-.149	-.027
$\pm .44$	0.83	.40	.061	.244	.148	.179

Weighted L.S. adjustment

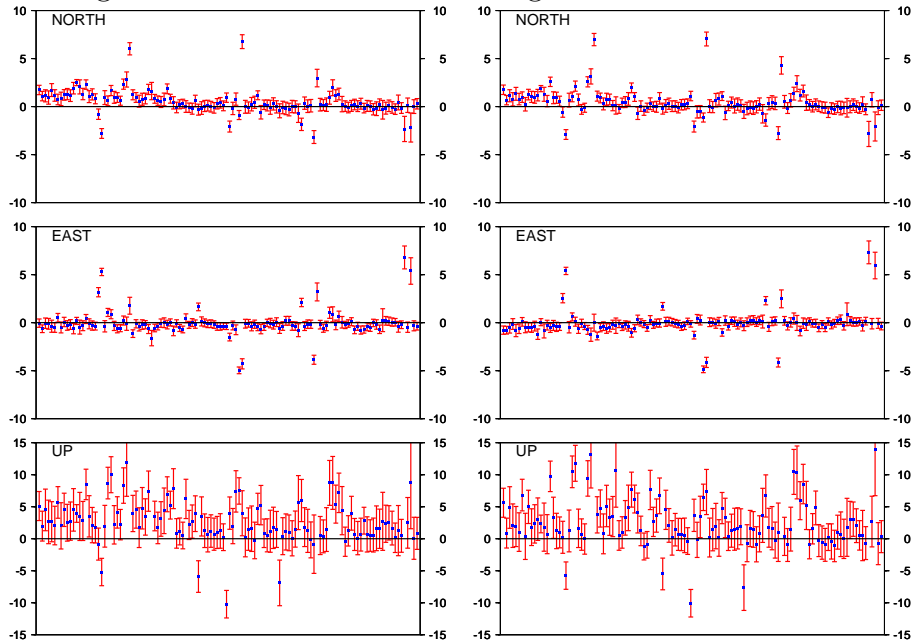
-16.36	-3.75	-23.11	.922	.412	-.225	-.186
$\pm .66$	1.08	.42	.045	.306	.236	.201



Diff. (Unconst'd & transformed) *minus* Constrained

Unweighted Transformation

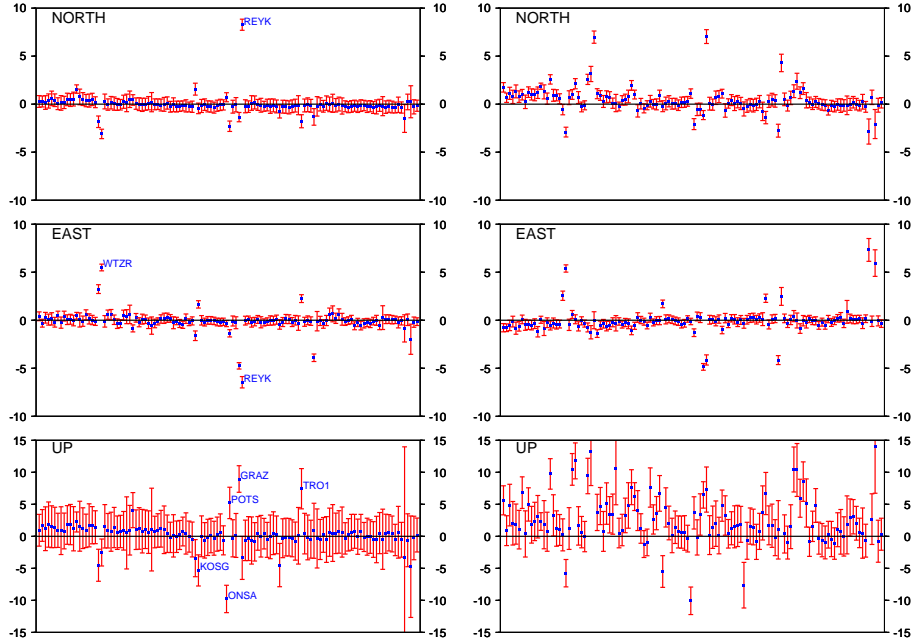
Weighted Transformation

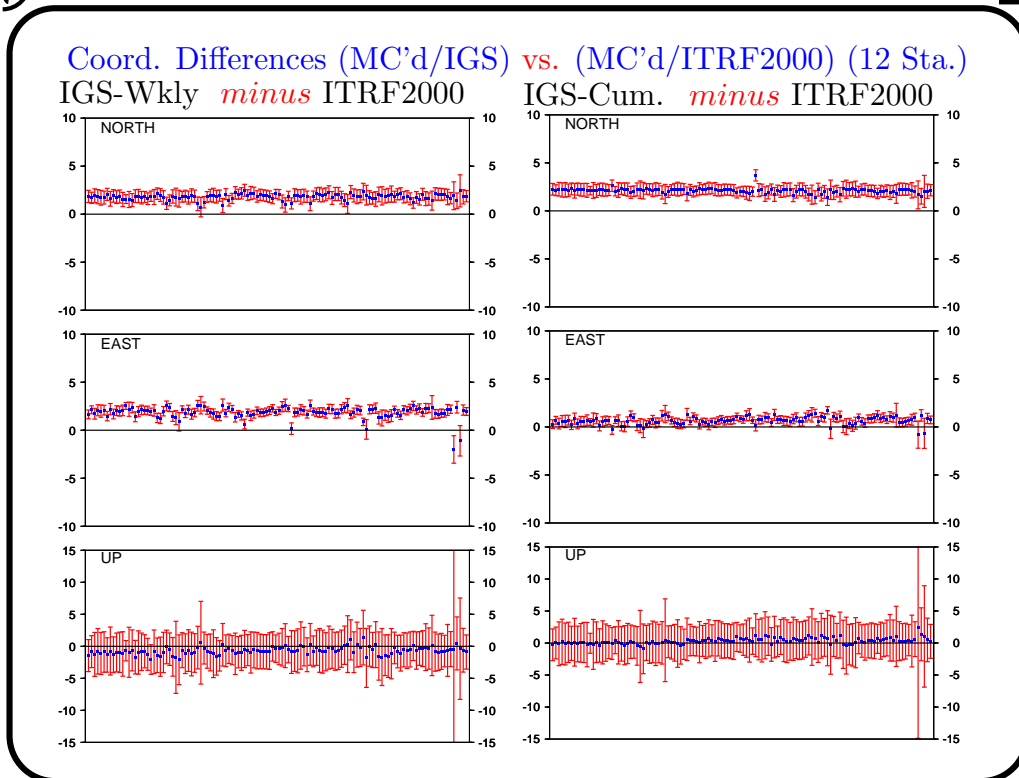
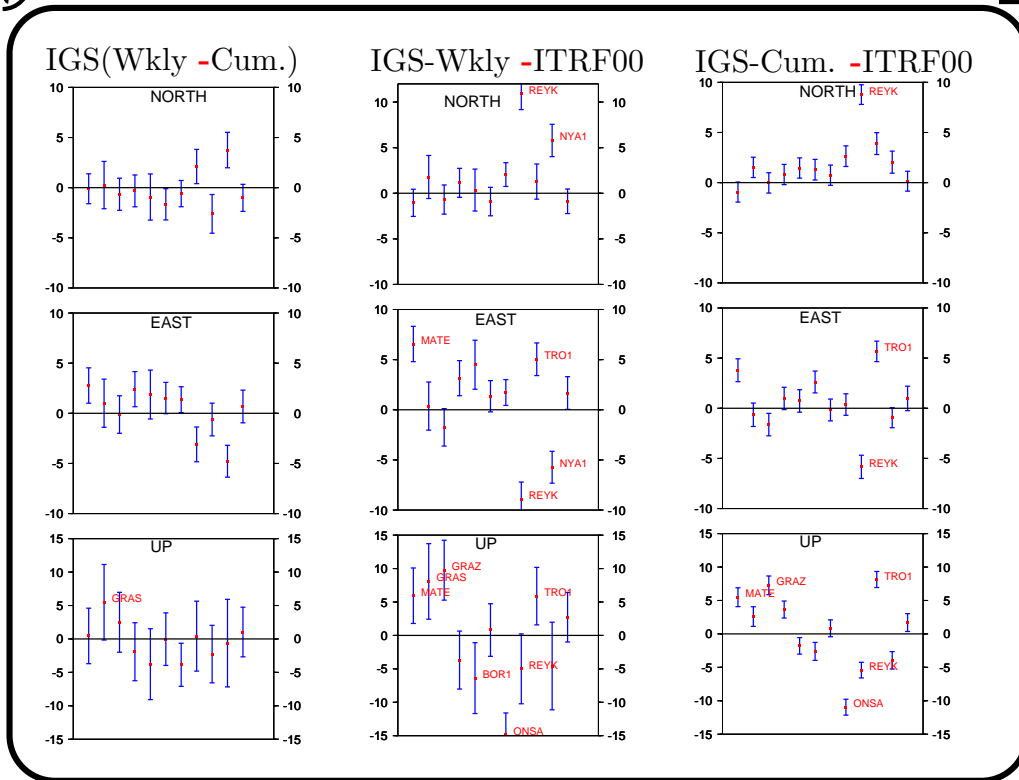


Alignment using Minimum Constraints

MC'd *minus* Constrained

Transformed *minus* Constr'd



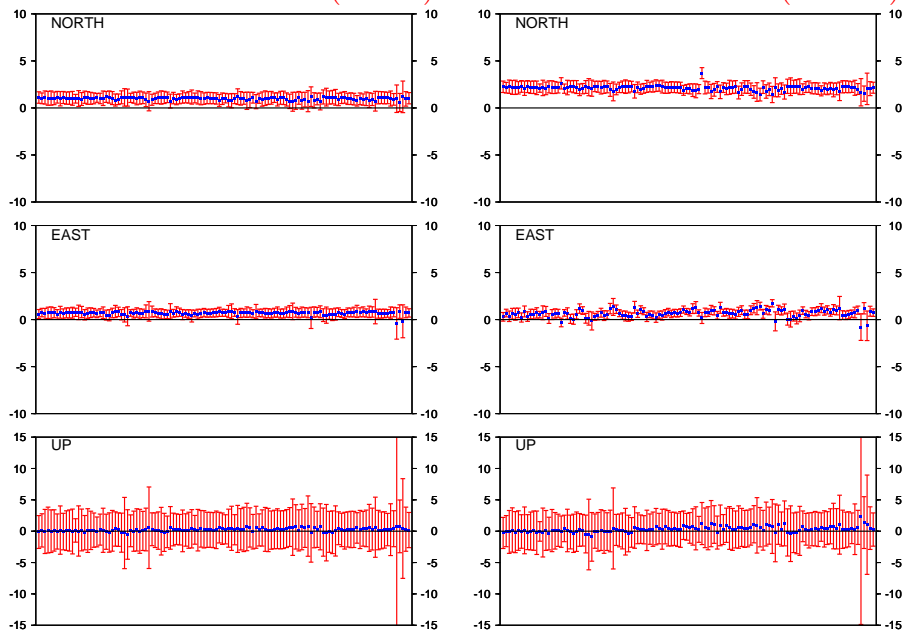




Coord. Differences (MC'd/IGS) vs. (MC'd/ITRF2000)

IGS-Cum. - ITRF2000 (5 Sta.)

IGS-Cum.- ITRF2000 (12 Sta.)



Conclusions

- Fixing coordinates (strictly speaking) should be avoided
- In terms of the TRF datum definition, Constraining or minimum constraints alignment are fully equivalent, but are dependant on the selected RF stations
- MC alignment has the advantage to preserve the original characteristics of the regional solution
- Suggest that regional agencies test/use the MC approach
- Using IGS00 or ITRF2000 subset of stations may yield a datum shift of a few mm due to the network effect
- Station position jumps deserves serious consideration and investigation

INTRODUCTION

The SINEX acronym was suggested by Blewitt et al. (1994) and the first versions, 0.04, 0.05, 1.00 evolved from the work and contributions of the SINEX Working Group of the IGS. The IGS Analysis Centres and Associated Analysis Centres use the SINEX format for their weekly solutions since mid 1995. Although the SINEX format was developed by the IGS, the ILRS and IVS decided to use it for their pilot projects as well because SINEX was designed to be modular and general enough to handle GPS as well as other techniques. To meet all the requirements for SLR and VLBI solutions some new elements and more detailed specifications were added by the ILRS Analysis Working Group and by the IVS. These extensions were merged with the previous SINEX version 1.00 to get a unique format definition for all space geodetic techniques, and after an intensive discussion the new version called SINEX 2.00 could be finalized. We have to thank the IGS Reference Frame Working Group chaired by R. Ferland, the ILRS Analysis Coordinator R. Noomen and the ILRS Analysis Working Group, the IVS Analysis Coordinator A. Nothnagel and Z. Altamimi from the ITRF section of IGN for their contributions and advice concerning a new SINEX format definition. The changes from version 1.00 to 2.00 are given in the next section of this document. The complete and detailed format definition can be seen in APPENDIX I, and the relevant least squares adjustment formulas with their relations to the SINEX format are summarized in APPENDIX II.

CHANGES FROM VERSION 1.00 TO 2.00

- 1) The version number in the header line changes to 2.00.
- 2) The list of allowed estimated parameter types has increased. As additional parameters you can include in your solution:

```
- XGC      \  
- YGC      - = coordinates of the geocenter [m]  
- ZGC      /  
- RS_RA    = right ascension of a radio source [rad]  
- RS_DE    = declination of a radio source [rad]  
- RS_RAR   = rate of right ascension of a radio source [rad/y]  
- RS_DER   = rate of declination of a radio source [rad/y]  
- RS_PL    = radio source parallax [rad]  
- NUT_LN   = nutation total in longitude [rad]  
- NUT_OB   = nutation total in obliquity [rad]  
- NUTRLN   = nutation rate in longitude [rad/d]  
- NUTROB   = nutation rate in obliquity [rad/d]  
- TGNWET   = troposphere gradient in north for the wet part [m]  
- TGNDRY   = troposphere gradient in north for the dry part [m]  
- TGNTOT   = total troposphere gradient in north (wet + dry part) [m]  
- TGEWET   = troposphere gradient in east for the wet part [m]  
- TGEDRY   = troposphere gradient in east for the dry part [m]  
- TGETOT   = total troposphere gradient in east (wet + dry part) [m]  
- AXI_OF   = antenna axis offset [m]  
- RBIAS    = range bias [m]  
- TBIAS    = time bias [ms]  
- SBIAS    = scale bias [ppb]  
- ZBIAS    = troposphere bias in zenith [m]
```

The parameters LODR and UTR (LOD and UT1-UTC reduced for the short periodic terms up to 35 days) are no official SINEX parameters and should not be used.

The coordinates of geocenter already appear in some IGS solutions, so we decided to define these parameters as officially allowed.

The coordinates of radio sources (right ascension, declination) and their rates, the parallax of radio sources, the nutation parameters and their rates, the troposphere gradients and the antenna axis offsets are new parameters requested for VLBI solutions.

The four bias parameters are taken from the ILRS implementation of SINEX.

3) Due to these new parameters the field 'Solution Contents' in the HEADER LINE and in the block INPUT/HISTORY must be changed.

All parameters belonging to the stations are summarized in only one character:

'S' = station coordinates (STAX, STAY, STAZ),
station velocities (VELX, VELY, VELZ),
all four bias parameters (RBIAS, TBIAS, SBIAS, ZBIAS),
geocenter coordinates (XGC, YGC, ZGC).

That means, the character 'X' for station coordinates and 'V' for station velocities are dropped.

A new character is defined for all parameters belonging to the celestial reference frame:

'C' = right ascension and declination of the radio sources (RS_RA, RS_DE),
rates for right ascension and declination of the radio sources
(RS_RAR, RS_DER),
parallax of radio sources (RS_PL).

The other new parameters can be attached easily to the existing characters of SINEX version 1.00:

'T' = for all troposphere parameters (including the new parameters for the troposphere gradients TGNWET, TGNDRY, TGNTOT, TGEWET, TGEDRY, TGETOT),
'E' = for all earth orientation parameters (including the new nutation parameters NUT_LN, NUT_OB, NUTRLN, NUTROB).

The orbit parameters 'O' are not changed.

4) The block SOLUTION/STATISTICS is now RECOMMENDED if the requested values are available because for a further combination of solutions it is necessary to have the complete statistical information.

The preference is given to the original values like 'NUMBER OF OBSERVATIONS' and 'NUMBER OF UNKNOWNNS' instead of 'DEGREE OF FREEDOM'.

The 'NUMBER OF OBSERVATIONS' should represent only the number of 'real' observations.

A new value became necessary if unconstrained normal equations are stored because the variance factor contains the constraints of the solution. Therefore the weighted square sum of the vector 'observed minus computed' should be given in the SOLUTION/STATISTICS block to become independent of the influence of the constraints on the variance factor: $(o-c)' P (o-c)$, where $(o-c)$ represents the vector 'observed minus computed' and P denotes the weight matrix. This new value can be stored under the name

WEIGHTED SQUARE SUM OF O-C

5) The list of allowed parameter types in the block SOLUTION/APRIORI is extended following some IGS solutions: if you apply inner constraints to your solution you can add the constrained transformation parameters to the SOLUTION/APRIORI block.

That means for the particular fields of this block:

- Parameter Type:

TX, TY, TZ for translation restrictions in x, y and z direction
RX, RY, RZ for rotation restrictions around the x, y and z axis
SC for scale restriction
TXR, TYR, TZR for restrictions on the rates of the translation in x, y and z
RXR, RYR, RZR for restrictions on the rates of the rotation around the
x, y and z axis
SCR for restriction on the rate of the scale

- The fields Site Code, Point Code, Solution ID are filled with '-'

- Time:

the reference epoch of the inner constraints

- Parameter Units:

m for translation parameters, i.e. [m]
mas for rotation parameters, i.e. [mas]

ppb for the scale, i.e. [ppb]
m/y for the rates of translation parameters, i.e. [m/y]
ma/y for the rates of rotation parameters, i.e. [mas/y]
pb/y for the rate of the scale, i.e. [ppb/y]

- Constraint Code:
0 for tight constraints

- Parameter Apriori:
the value on which you constrained the transformation parameter or its rate for the transformation of your solution according to the apriori reference frame (e.g. if the apriori reference frame represents the desired reference frame for your solution the apriori parameters are 0.0)

- Parameter Standard Deviation:
the sigma you choose for constraining the particular transformation parameter or its rate

To decide which stations were contributing to the inner constraints, the appropriate station parameters (coordinates and velocities if the rates are given as well) must be given in the block SOLUTION/APRIORI as well and should contain a '1' in the field 'CONSTRAINT CODE'.

6) With the new SINEX version the delivery of normal equations will be defined more precisely. We have now three possibilities include normal equation systems in the SINEX file:

a) In principle it was already possible in version 1.00 to store normal equation matrices in the two SOLUTION/MATRIX blocks (ESTIMATE and APRIORI) if you use matrix type INFO. And together with the two vectors in SOLUTION/ESTIMATE and SOLUTION/APRIORI you are able to reconstruct the original (reduced) normal equation system without constraints. But this procedure of removal the constraints and compute the right hand side of the normal equation system is always a little bit critical, and in addition to that problem, the procedure depends on the solution vector given in SOLUTION/ESTIMATE.

For a further combination of several solutions there would be less problems if the original normal equation system without any constraints can be stored directly in the SINEX file. In that case you have the advantage that the constraints applied in the individual solutions (SOLUTION/MATRIX_APRIORI) and the resulting solution vector (SOLUTION/ESTIMATE) have no influence on the combination. These considerations led to two other possibilities of storing normal equations in the SINEX file, but for both we had to introduce two new blocks for the original normal equation system:

- SOLUTION/NORMAL_EQUATION_VECTOR

This block contains the vector of the right hand side of the reduced normal equation system

$$b = A' P l$$

where

A' is the transposed of the Jacobi-Matrix,

P is the weight matrix of the observations and

l is the vector observed minus computed with apriori values.

- SOLUTION/NORMAL_EQUATION_MATRIX

This block contains the reduced normal equation matrix WITHOUT constraints (i.e. the 'free' / original solution):

$$N = A' P A$$

The structure of this block is similar to the other two MATRIX blocks.

The indices of both new blocks must be consistent with the indices in SOLUTION/ESTIMATE.

With these two additional blocks the second and third possibility of storing normal equations look as follows:

b) You store the complete information about your solution in the following blocks:

- original normal equation matrix $N = A' P A$: SOLUTION/NORMAL_EQUATION_MATRIX
- vector of right hand side of original normal equation $b = A' P l$: SOLUTION/NORMAL_EQUATION_VECTOR
- apriori values of the unknown parameters x_0 : SOLUTION/APRIORI
- normal equation matrix of applied constraints in your solution dN : SOLUTION/MATRIX_APRIORI
- resulting unknown parameters of the constrained solution

$x = x_0 + \text{inv}(N + dN) b$: SOLUTION/ESTIMATE

The advantage of this method is the availability of the whole information, i.e. the original normal equation can be used for a further combination without any problems of constraints removal like in method a), and other users who are interested in the parameters of the constrained solution itself can take the vector in SOLUTION/ESTIMATE.

A problem might occur when generating such a SINEX file because you need both, the original normal equation system as well as the solution estimate (as in case a)) and in most software packages the normal equation matrix might already be inverted at the time when you have the solution estimate available.

c) The third possibility is storing only the original normal equation system in the SINEX file, i.e.

- original normal equation matrix $N = A' P A$: SOLUTION/NORMAL_EQUATION_MATRIX

- vector of right hand side of original normal equation $b = A' P l$:

SOLUTION/NORMAL_EQUATION_VECTOR

- a priori values of the unknown parameters x_0 : SOLUTION/APRIORI

For a further combination with other solutions this would be enough information and there wouldn't be any problems with constraints removal.

On the other hand the documentation of the estimated parameters is missing.

To reconstruct the statistical information about the original solution for the last two possibilities (points b) and c)) it is necessary to store the weighted square sum of the vector observed minus computed, i.e. $l' P l$ in the SOLUTION/STATISTICS block because only this part of the variance factor is independent of the constrained solution and can be taken for a combination

$(v' P v = l' P l - (x - x_0)' b)$:

WEIGHTED SQUARE SUM OF O-C

(see as well point 4) of the changes from version 1.00 to 2.00)

7) For more clearness and with regards to a good documentation how the solution in SOLUTION/ESTIMATE was created, the block SOLUTION/APRIORI is now mandatory. The block SOLUTION/MATRIX_APRIORI is only mandatory if the matrix in SOLUTION/MATRIX_ESTIMATE contains some constraints.

If you deliver normal equations in your SINEX file some more blocks are mandatory, depending on the method of storing normal equations:

For method 6a)

- SOLUTION/MATRIX_APRIORI (INFO type)

- SOLUTION/MATRIX_ESTIMATE (INFO type)

- SOLUTION/STATISTICS (#observations, #unknowns, weighted square sum of o-c)

For method 6b)

- SOLUTION/MATRIX_APRIORI

- SOLUTION/NORMAL_EQUATION_MATRIX

- SOLUTION/NORMAL_EQUATION_VECTOR

- SOLUTION/STATISTICS (#observations, #unknowns, weighted square sum of o-c)

For method 6c)

- SOLUTION/NORMAL_EQUATION_MATRIX

- SOLUTION/NORMAL_EQUATION_VECTOR

- SOLUTION/STATISTICS (#observations, #unknowns, weighted square sum of o-c)

8) Besides the new blocks for normal equations we have introduced some other new blocks in the SINEX format 2.00:

- NUTATION/DATA:

to store the information about the nutation model used in the analysis;

This block contains two fields: one for the name of the nutation model and one for some comments.

- PRECESSION/DATA:

to store the information about the precession model used in the analysis;

This block contains two fields: one for the name of the precession model and one for some comments.

- SOURCE/ID:

to provide information about the radio sources observed with VLBI;

There are 3 fields for source names: the Source Code (used for SINEX internal referencing), the IERS designation and the ICRF designation.

- BIAS/EPOCHS:
important if bias parameters are included in the solution (from SLR solutions)

9) The matrix type SRIF (for Square Root Information Filter Matrix) in the two blocks SOLUTION/MATRIX_APRIORI and SOLUTION/MATRIX_ESTIMATE is no longer allowed.

10) The longitude sign definition in the SITE/ID block is redefined according to the ISO6709 definition (that is the way the information was already stored in most of the SINEX files):

- positive longitudes have to be used for east direction with respect to the Greenwich meridian
- following the ISO6709 specification, the range of longitude should be $[-180^{\circ} +180^{\circ}]$

11) The value "-----" in the field 'Antenna Serial Number' of the SITE/GPS_PHASE_CENTER block is redefined:

it signifies that the phase center offsets for L1 and L2 that are given in the following columns apply to ALL antennas of the same type that is indicated in the field 'Antenna Type'.

As a consequence, if the phase center offsets for one antenna name and the given model are the same for all antenna serial numbers, it is enough to store only one data line (with "-----" for the 'Antenna Serial Number') in the SITE/GPS_PHASE_CENTER block for each antenna type that appears in the SITE/ANTENNA block.

SINEX SYNTAX

SINEX is an ASCII file with lines of 80chars or less. It consists of a number of blocks which are mutually referenced (related) through station codes/names, epochs and/or index counters. Some blocks consist of descriptive lines (starting in Col.2) and/or fixed format fields with numerous headers and descriptive annotations.

The first line is MANDATORY and must start with "%" in col 1, and contains information about the agency, file identification, solution spans, techniques, type of solution, etc. (for more details see the Appendix I or II). The last line ends with "%ENDSNX".

The SINEX format consists of a number BLOCKS which start with "+" in the first col. followed by a standardized block labels, and each block ends with "-" and the block label. Each block data starts in the column 2 or higher. Blocks can be in any order, provided that they start with (+) and end with (-) block labels. The first header line and most blocks are related through epochs or time stamps in the following format:

YY:DOY:SECOD YY-year; DOY- day of year; SECOD -sec of day;

E.g. the epoch 95:120:86399 denotes April 30, 1995 (23:59:59UT). The epochs 00:00:00000 are allowed in all blocks (except the first header line) and default into the start or end epochs of the first header line which must always be coded. This is particularly useful for some blocks, such as the ones related to hardware, occupancy, which should be centrally archived by IGSCB with 00:00:00000 as the end (current) epochs, and which should be readily usable by ACs for SINEX and other analysis/processing as official (authoritative) IGS information.

COMMENT lines starts with "*" in Col. 1 and can be anywhere within or outside a block, though for the clarity sake, beginning and ends of blocks are preferable. For increased portability, the floating number exponent of "E" should be used rather than "D" or "d" which is not recognized by some compiler/installations. Fields not coded should be filled with "-" characters to allow efficient row and column format readings.

The most important blocks are the SOLUTION blocks. They are in fixed format (For more information on the format, see APPENDIX I).

The mandatory SOLUTION blocks depend on the contents of the SINEX file.

If you deliver variance-covariance matrices or correlation matrices in your SINEX files the blocks

A P P E N D I X I

S I N E X

V E R S I O N 2 . 0 0

D E T A I L F O R M A T D E S C R I P T I O N

1. INTRODUCTION
2. DATA STRUCTURE
3. HEADER LINE
4. FILE/REFERENCE BLOCK
5. FILE/COMMENT BLOCK
6. INPUT/HISTORY BLOCK
7. INPUT/FILES BLOCK
8. INPUT/ACKNOWLEDGEMENTS BLOCK
9. NUTATION/DATA BLOCK
10. PRECESSION/DATA BLOCK
11. SOURCE/ID BLOCK
12. SITE/ID BLOCK
13. SITE/DATA BLOCK
14. SITE/RECEIVER BLOCK
15. SITE/ANTENNA BLOCK
16. SITE/GPS_PHASE_CENTER BLOCK
17. SITE/ECCENTRICITY BLOCK
18. SOLUTION/EPOCH BLOCK
19. BIAS/EPOCHS BLOCK
20. SOLUTION/STATISTICS BLOCK
21. SOLUTION/ESTIMATE BLOCK
22. SOLUTION/APRIORI BLOCK
23. SOLUTION/MATRIX_ESTIMATE BLOCK
24. SOLUTION/MATRIX_APRIORI BLOCK
25. SOLUTION/NORMAL_EQUATION_VECTOR BLOCK
26. SOLUTION/NORMAL_EQUATION_MATRIX BLOCK
27. FOOTER LINE

1. Introduction

This document describes the Software Independent Exchange (SINEX) format. It started in early 1995 with an effort by a number of IGS participants and it was designed to be easily extended. For the new IERS structure, operational since January 1, 2001, and due to the use of SINEX by the ILRS (pilot project 'positioning and earth orientation') and the IVS as well, some extensions were made with the purpose to have a unique format description for all techniques.

2. Data Structure

Each SINEX line has at most 80 ASCII characters. The SINEX file is subdivided in groups of data called blocks. Each block is enclosed by a header and trailer line. Each block has a fixed format. The blocks contain information on the file, its input, the sites and the solution. All elements within a line are defined. A character field without information will have "-"s within its field and a missing numerical element will have a value of 0 within its field. Therefore the SINEX file is accessible "column-wise" as well as "line-wise". Character fields should be left hand justified whenever applicable.

The first character of each line identify the type of information that the line contains. Five characters are reserved. They have the following meaning when they are at the beginning of a line, they identify:

Character Definition

"%"	Header and trailer line,
"*"	Comment line within the header and trailer line,
"+"	Title at the start of a block
"-"	Title at the end of a block
" "	Data line within a block

No other character is allowed at the beginning of a line!

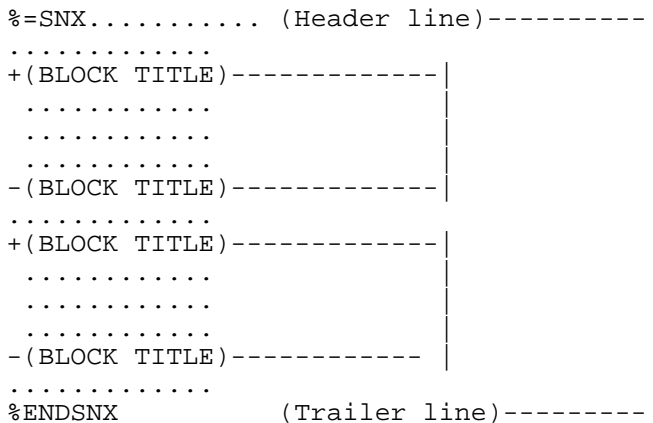
A SINEX file must start with a header line and ends with a footer line.

The following blocks are defined:

- FILE/REFERENCE
- FILE/COMMENT
- INPUT/HISTORY
- INPUT/FILES
- INPUT/ACKNOWLEDGEMENTS
- NUTATION/DATA
- PRECESSION/DATA
- SOURCE/ID
- SITE/ID
- SITE/DATA
- SITE/RECEIVER
- SITE/ANTENNA
- SITE/GPS_PHASE_CENTER
- SITE/ECCENTRICITY
- BIAS/EPOCHS
- SOLUTION/EPOCH
- SOLUTION/STATISTICS
- SOLUTION/ESTIMATE
- SOLUTION/APRIORI
- SOLUTION/MATRIX_ESTIMATE {p} {type}
- SOLUTION/MATRIX_APRIORI {p} {type}
- SOLUTION/NORMAL_EQUATION_VECTOR
- SOLUTION/NORMAL_EQUATION_MATRIX {p}

Where: {p} L or U
 {type} CORR or COVA or INFO

These block titles are immediately preceded by a "+" or a "-" as they mark the beginning or the end of a block. The block titles must be in capital letters. After a block has started(+) it must be ended(-) before another block can begin. The general structure is as follow:



Most fields within a SINEX line are separated by a single space. In the following sections, each SINEX line is defined by its field name, a general description and the (FORTRAN) format.

A comment line (not to be confused with the FILE/COMMENT Block) can be written anywhere within the header and the footer line. All comment lines must start with a "*" in the first column. With the use of this character information can be hidden from the software reading the file without deleting it from the file. A comment line is defined as follow:

C O M M E N T D A T A L I N E		
Field	Description	Format
Comment	Any general comment relevant to the SINEX file.	1H*,A79
		80

Some fields are found in several blocks. To keep the description short, they are described in detail here, and will be referred to in the sections with additional information added when necessary. The fields defined below will be referenced to by putting them within square brackets [] when encountered in the following sections.

Field	Description	Format
Time	YY:DDD:SSSSS. "UTC" YY = last 2 digits of the year, if YY <= 50 implies 21-st century, if YY > 50 implies 20-th century, DDD = 3-digit day in year, SSSSS = 5-digit seconds in day.	I2.2, 1H:,I3.3, 1H:,I5.5
Constraint Code	Single digit indicating the constraints: 0-fixed/tight constraints, 1-significant constraints, 2-unconstrained.	A1
Parameter Type	Type of parameter. List of allowed parameters and the units provided: STAX - station X coordinate, m STAY - station Y coordinate, m STAZ - station Z coordinate, m VELX - station X velocity, m/y VELY - station Y velocity, m/y VELZ - station Z velocity, m/y XGC - geocenter X coordinate, m YGC - geocenter Y coordinate, m ZGC - geocenter Z coordinate, m RS_RA - radio source right ascension, rad RS_DE - radio source declin., rad RS_RAR - radio source right ascension rate, rad/y RS_DER - radio source declination rate, rad/y RS_PL - radio source parallax, rad LOD - length of day, ms UT - delta time UT1-UTC, ms XPO - X polar motion, mas YPO - Y polar motion, mas XPOR - X polar motion rate, mas/d YPOR - Y polar motion rate, mas/d NUT_LN - total nutation in longitude, rad NUT_OB - total nutation in obliquity, rad NUTRLN - nutation rate in longitude, rad/d	A6

	<p>NUTROB - nutation rate in obliquity rad/d</p> <p>SAT__X - Satellite X coord., m</p> <p>SAT__Y - Satellite Y coord., m</p> <p>SAT__Z - Satellite Z coord., m</p> <p>SAT_VX - Satellite X velocity, m/s</p> <p>SAT_VY - Satellite Y velocity, m/s</p> <p>SAT_VZ - Satellite Z velocity, m/s</p> <p>SAT_RP - Radiation pressure,</p> <p>SAT_GX - GX scale,</p> <p>SAT_GZ - GZ scale,</p> <p>SATYBI - GY bias, m/s²</p> <p>TROTOT - wet + dry Tropo delay, m</p> <p>TRODRY - dry Tropo delay, m</p> <p>TROWET - wet Tropo delay, m</p> <p>TGNTOT - troposphere gradient in north (wet + dry), m</p> <p>TGNWET - troposphere gradient in north (only wet), m</p> <p>TGNDRY - troposphere gradient in north (only dry), m</p> <p>TGETOT - troposphere gradient in east (wet + dry), m</p> <p>TGEWET - troposphere gradient in east (only wet), m</p> <p>TGEDRY - troposphere gradient in east (only dry), m</p> <p>RBIAS - range bias, m</p> <p>TBIAS - time bias, ms</p> <p>SBIAS - scale bias, ppb</p> <p>ZBIAS - troposphere bias at zenith, m</p> <p>AXI_OF - antenna axis offset, m</p>	
Site Code	<p>- For stations: Call sign for a site.(It should be consistent with ITRF, see below).</p> <p>- For satellites: Use "PRXX" where XX is the PRN number.</p>	A4
Point Code	<p>- For stations: A two character code identifying physical monument within a site. Typically has a code A, but could vary if the site has more than one monument.</p> <p>- For bias parameters: satellite ID L1, L2 for LAGEOS-1 or -2; LC for combined LAGEOS; E1, E2 for Etalon 1 or 2; EC for combined Etalon;</p>	A2
Solution ID	<p>Character identifying the solution given for a point at a site. "-----" applies to all.</p>	A4
Observation Code.	<p>A single character indicating the technique(s) used to arrive at the solutions obtained in this SINEX file. It should be consistent with the IERS convention. This character code may be: C-Combined techniques used. D-DORIS, L-SLR, M-LLR,</p>	A1

P-GPS, R-VLBI.

Comment:

For the official IERS sites the values of the 'Site Code' are listed in ftp://lareg.ensg.ign.fr/pub/itrf/iers_dir.sta

'Per year' means 365.25 days in this context.

3. Header Line (Mandatory)

Description

The header line must be the first line in a SINEX file.

Contents:

H E A D E R L I N E		
Field	Description	Format
First Character	Single character '%' in column #1. No other character than '%' is allowed.	A1
Second Character	Single character '=' in column #2. Indicates 'resultant' solution. No other character than '=' is allowed.	A1
Document Type	Three characters 'SNX' in columns 3 to 5. Indicates that this is a SINEX document.	A3
Format Version	Four digits indicating the version of SINEX format used. '2.00' for this version.	1X,F4.2
File Agency Code	Identify the agency creating the file.	1X,A3
[Time]	Creation time of this SINEX file.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Agency Code]	Identify the agency providing the data in the SINEX file	1X,A3
[Time]	Start time of the data used in the SINEX solution Value 00:000:00000 should be avoided.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	End time of the data used in the SINEX solution Value 00:000:00000 should be avoided.	1X,I2.2, 1H:,I3.3, 1H:,I5.5

[Observation Code]	Technique(s) used to generate the SINEX solution	1X,A1
Number of Estimates	Number of parameters estimated in this SINEX file. Mandatory field.	1X,I5.5
[Constraint Code]	Single character indicating the constraint in the SINEX solution. Mandatory field.	1X,A1
Solution Contents	Solution types contained in this SINEX file. Each character in this field may be one of the following: S - all station parameters, i.e. station coordinates, station velocities, biases, geocenter, O - Orbits, E - Earth Orientation Parameter T - Troposphere, C - Celestial Reference Frame BLANK	5(1X,A1)
		77

Relationship with other blocks:

This line is duplicated as the resultant line of the INPUT/HISTORY block with the exception of its first character.

4. FILE/REFERENCE Block (Mandatory)

Description:

This block provides information on the Organization, point of contact, the software and hardware involved in the creation of the file.

Contents:

F_I_L_E R_E_F_E_R_E_N_C_E D_A_T_A L_I_N_E		
Field	Description	Format
Information Type	Describes the type of information present in the next field. May take on the following values: 'DESCRIPTION' - Organization(s) gathering/altering the file contents. 'OUTPUT' - Description of the file contents. 'CONTACT' - Address of the relevant contact. e-mail 'SOFTWARE' - Software used to generate the file. 'HARDWARE' - Computer hardware on which above software was run. 'INPUT' - Brief description of the input used to generate this solution.	1X,A18

	Any of the above fields may be and in any order.	
Information	Relevant information for the type indicated by the previous field.	1X,A60
		80

5. FILE/COMMENT Block (Optional)

Description:

This block can be used to provide general comments about the SINEX data file.

Contents:

F_I_L_E_C_O_M_M_E_N_T_D_A_T_A_L_I_N_E		
Field	Description	Format
Comment	Any general comment providing relevant information about the SINEX file.	1X,A79
		80

6. INPUT/HISTORY Block (Recommended)

Description:

This block provides information about the source of the information used to create the current SINEX file.

Contents:

I_N_P_U_T_H_I_S_T_O_R_Y_D_A_T_A_L_I_N_E		
Field	Description	Format
File Code	Only one of the following characters is permitted: '+' - This character indicates that the information that follows identify an input solution contributing to this SINEX file. '=' - This character indicates that the information that follows identify the output solution file.	1X,A1
Document Type	Three characters 'SNX' in columns 3 to 5. Indicates that this is a SINEX document.	A3
Format Version	Four digits indicating the version of SINEX format used. '2.00' for this version.	1X,F4.2
[Agency Code]	Identify the agency creating the file.	1X,A3

[Time]	Creation time of this SINEX file.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Agency Code]	Identify the agency providing the data in the SINEX file.	1X,A3
[Time]	Start time of the data used in the SINEX solution.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	End time of the data used in the SINEX solution.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Observation Technique]	Technique(s) used to generate the SINEX solution.	1X,A1
Number of Estimates	Number of parameters estimated in this SINEX file.	1X,I5.5
[Constraint Code]	Single digit indicating the constraint in the SINEX solution.	1X,A1
Solution Contents	Solution types contained in this SINEX file. Each character in this field may be one of the following: S - all station parameters, i.e. station coordinates, station velocities, biases, geocenter, O - Orbits, E - Earth Orientation Parameter T - Troposphere, C - Celestial Reference Frame BLANK	5(1X,A1)
		77

Comment:

The final data line "=" describes the current SINEX file and match the header line with the exception of the first character.

7. INPUT/FILES Block (Optional)

Description:

This block identify the input files (and the current SINEX file) and allow for a short comment to be added to describe those files.

Contents:

I N P U T F I L E S D A T A L I N E		
Field	Description	Format

[Agency Code]	Agency creating the solution described in this data line.	1X,A3
[Time]	Time of creation of the input SINEX solution	1X,I2.2, 1H:,I3.3, 1H:,I5.5,
File Name	Name of the file containing the solution described in the current data line.	1X,A29
File Description	General description of the file referred to on this data line.	1X,A32
		80

Comments:

There must be exactly one INPUT/FILES data line for every INPUT/HISTORY data line. The final data line must describe this current SINEX file.

8. INPUT/ACKNOWLEDGEMENTS Block (Optional)

Description:

This block defines the agency codes contributing to the SINEX file.

Contents:

I N P U T A C K N O W L E D G M E N T S D A T A L I N E		
Field	Description	Format
[Agency Code]	Agency(ies) contributing to this SINEX file.	1X,A3
Agency Description	Description of agency code.	1X,A75
		80

9. NUTATION/DATA Block (Mandatory for VLBI)

Description:

This block contains the nutation model used in the analysis procedure.

Contents:

N U T A T I O N D A T A L I N E		
Field	Description	Format
Nutation Code	Code for nutation reference: IAU1980 IERS1996 IAU2000a IAU2000b	1X,A8
Comments	General description of the	1X,A70

	nutaton model used	
		80

COMMENT:

It must be a generally accepted model which is accessible to all users.

10. PRECESSION/DATA Block (Mandatory for VLBI)

Description:

This block contains the precession model used in the analysis procedure.

Contents:

P_R_E_C_E_S_S_I_O_N D_A_T_A L_I_N_E		
Field	Description	Format
Precess. Code	Code for precession reference: IAU1976 IERS1996	1X,A8
Comments	General description of the nutaton model used	1X,A70
		80

COMMENT:

It must be a generally accepted model which is accessible to all users.

11. SOURCE/ID (Mandatory for VLBI)

Description:

This block contains information about the radio sources estimated in the analysis, especially the names used in ICRF and for IERS.

Contents:

R_A_D_I_O S_O_U_R_C_E D_A_T_A L_I_N_E		
Field	Description	Format
Source Code	Call sign for a source	1X,A4
IERS des.	IERS designation of the radio source	1X,A8
ICRF des.	ICRF designation of the radio source	1X,A16
Comments	Comments or other names of the radio source	1X,A68
		80

12. SITE/ID Block (Mandatory)

Description:

This block provides general information for each site containing estimated parameters.

Contents:

S I T E I D D A T A L I N E		
Field	Description	Format
[Site Code]	Call sign for a site.	1X,A4
[Point Code]	Physical monument used at a site	1X,A2
Unique Monument Identification	Unique alpha-numeric monument identification. For ITRF purposes, it is a nine character DOMES/DOMEX number (five/six digits, followed by the single letter 'M' or 'S', followed by four/three digits)	1X,A9
[Observation Code]	Observation technique(s) used.	1X,A1
Station Description	Free-format description of the site, typically the town and/or country.	1X,A22
Approximate Longitude	Approximate longitude of the site in degrees(E/+), minutes and seconds.	1X,I3, 1X,I2, 1X,F4.1
Approximate Latitude	Approximate latitude of the site in degrees(NS/+-), minutes and seconds.	1X,I3, 1X,I2, 1X,F4.1
Approximate Height	Approximate height of the site in metres.	1X,F7.1
		75

Comments:

For DOMES numbers and station description as well as for Site Codes please refer to ftp://lareg.ensg.ign.fr/pub/itrf/iers_dir.sta

If a DOMES number is not available (e.g. for a new station), please ask Zuheir Altamimi for a DOMES number (altamimi@ensg.ign.fr).

Use the minus sign for negative approximate longitude or latitude only in the "degrees" component and don't repeat it in the "minutes" and "seconds" component.

Following the ISO6709 specification, the range of longitude should be [-180° +180°].

13. SITE/DATA Block (Optional)

Description:

This block gives the relationship between the estimated station parameters in the SINEX file and in the input files.

Contents:

S I T E D A T A L I N E		
Field	Description	Format
[Site Code]	Site Code for solved station coordinates.	1X,A4
[Point Code]	Point Code for solved station coordinates.	1X,A2
[Solution ID]	Solution number to which the input in this data line is referred to.	1X,A4
[Site Code]	Site Code from an input SINEX file	1X,A4
[Point Code]	Point code from an input SINEX file.	1X,A2
[Solution ID]	Solution Number for a Site/Point from an input SINEX file.	1X,A4
[Observation Code]	Observation Code for a Site/Point/Solution Number from an input SINEX file.	1X,A1
[Time]	Time of start of data for the input SINEX file.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Time of end of data for the input SINEX file.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Agency Code]	Creation Agency Code for the input SINEX file.	1X,A3
[Time]	Creation time for the input SINEX file.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
		71

Comment:

Times must refer to observation epochs.

14. SITE/RECEIVER Block (Mandatory for GPS)

Description:

List the receiver used at each site during the observation period of

interest.

Contents:

S_I_T_E R_E_C_E_I_V_E_R D_A_T_A L_I_N_E		
Field	Description	Format
[Site Code]	Site code for which some parameters are estimated.	1X,A4
[Point Code]	Point Code at a site for which some parameters are estimated.	1X,A2
[Solution ID]	Solution Number at a Site/Point code for which some parameters are estimated.	1X,A4
[Observation Code]	Identification of the observation technique used.	1X,A1
[Time]	Time since the receiver has been operating at the Site/Point. Value 00:000:00000 indicates that the receiver has been operating at least since the "File Epoch Start Time".	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Time until the receiver is operated at a Site/Point. Value 00:000:00000 indicates that the receiver has been operating at least until the "File Epoch End Time".	1X,I2.2, 1H:,I3.3, 1H:,I5.5
Receiver Type	Receiver Name & model.	1X,A20
Receiver Serial Number	Serial number of the receiver. Takes on value '-----' if unknown.	1X,A5
Receiver Firmware	Firmware used by this receiver during the epoch specified above. Takes on value '-----' if unknown.	1X,A11
		80

Comments:

- For IGS standard receiver names please refer to ftp://igsb.jpl.nasa.gov/igsb/station/general/rcvr_ant.tab

15. SITE/ANTENNA Block (Mandatory for GPS)

Description:

List of antennas used at each site used in the SINEX file.

Contents:

S_I_T_E A_N_T_E_N_N_A D_A_T_A L_I_N_E		
---------------------------------------	--	--

Field	Description	Format
[Site Code]	Site code for which some parameters are estimated.	1X,A4
[Point Code]	Point Code at a site for which some parameters are estimated.	1X,A2
[Solution ID]	Solution Number at a Site/Point code for which some parameters are estimated.	1X,A4
[Observation Code]	Identification of the observation technique used.	1X,A1
[Time]	Time since the antenna has been installed at the Site/Point. Value 00:00:00000 indicates that the antenna has been installed at least since the "File Epoch Start Time".	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Time until the antenna is installed at a Site/Point. Value 00:00:00000 indicates that the antenna has been installed at least until the "File Epoch End Time".	1X,I2.2, 1H:,I3.3, 1H:,I5.5
Antenna Type	Antenna name & model.	1X,A20
Antenna Serial Number	Serial number of the antenna. Takes on value '-----' if unknown.	1X,A5
		68

Comments:

- For IGS standard antenna names please refer to ftp://igscb.jpl.nasa.gov/igscb/station/general/rcvr_ant.tab

16. SITE/GPS_PHASE_CENTER Block (Mandatory for GPS)

Description:

List of GPS phase center offsets for the antennas described in the Site Antenna block. The offset is given from the Antenna Reference Point (ARP) to the L1 and L2 phase centers respectively.

Contents:

G_P_S_P_H_A_S_E_C_E_N_T_E_R_D_A_T_A_L_I_N_E		
Field	Description	Format
Antenna Type	Antenna name & model.	1X,A20
Antenna Serial Number	Serial number of the antenna. Takes on value '-----' if the phase center offsets apply to all	1X,A5

	antennas of the same type.	
L1 Phase Center Up Offset	Up(+) offset from the ARP to the L1 phase center in meters.	1X,F6.4
L1 Phase Center North Offset	North(+) offset from the ARP to the L1 phase center in meters.	1X,F6.4
L1 Phase Center East Offset	East(+) offset from the ARP to the L1 phase center in meters.	1X,F6.4
L2 Phase Center Up Offset	Up(+) offset from the ARP to the L2 phase center in meters.	1X,F6.4
L2 Phase Center North Offset	North(+) offset from the ARP to the L2 phase center in meters.	1X,F6.4
L2 Phase Center East Offset	East(+) offset from the ARP to the L2 phase center in meters.	1X,F6.4
Antenna Cali- bration model	Name of the antenna model used in the correction of the observations for phase center variations.	1X,A10
		80

Comments:

For IGS purposes see the IGS Central Bureau Information System for ARPs and antenna phase center offsets:

directory: igs/b/station/general/
files: antenna.gra and rcv_ant.tab

If the phase center offsets for one antenna type (antenna name and the given model) are the same for all antenna serial numbers, it is enough to store only one data line (with '-----' for the 'Antenna Serial Number') in this block for each antenna type that appears in the SITE/ANTENNA block.

17. SITE/ECCENTRICITY Block (Mandatory)

Description:

List of antenna eccentricities from the Marker to the Antenna Reference Point (ARP) or to the intersection of axis.

Contents:

S_I_T_E _ E_C_C_E_N_T_R_I_C_I_T_Y _ D_A_T_A _ L_I_N_E		
Field	Description	Format
[Site Code]	Site code for which some parameters are estimated.	1X,A4
[Point Code]	Point Code at a site for which some parameters are estimated.	1X,A2
[Solution ID]	Solution ID at a Site/Point code for which some parameters	1X,A4

	are estimated.	
[Observation Code]	Identification of the observation technique used.	1X,A1
[Time]	Time since the antenna has been installed at the Site/Point. Value 00:000:00000 indicates that the antenna has been installed at least since the "File Epoch Start Time".	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Time until the antenna is installed at a Site/Point. Value 00:000:00000 indicates that the antenna has been installed at least until the "File Epoch End Time".	1X,I2.2, 1H:,I3.3, 1H:,I5.5
Eccentricity Reference System	Reference system used to describe vector distance from monument benchmark to the antenna reference point or intersection of axis: 'UNE' - Local reference system Up, North, East. 'XYZ' - Cartesian Reference System X, Y, Z. All units are in meters.	1X,A3
Up / X Eccentricity	Up / X offset from the marker to the Antenna reference point (ARP).	1X,F8.4
North / Y Eccentricity	North/Y offset from the marker to the Antenna reference point (ARP).	1X,F8.4
East / Z Eccentricity	East/Z offset from the marker to the Antenna reference point (ARP).	1X,F8.4
		72

Comments:

- At the moment the local ties used in the ITRF2000 primary combination are only available throughout a login/password ftp access. To get access to these values please contact Zuheir Altamimi (altamimi@ensg.ign.fr).
- An older (March 23, 1999) listing of the official values for IERS sites is available at
<ftp://lareg.ensg.ign.fr/pub/itrf/iers.ecc>
- For GPS (antenna heights) you can refer to the daily generated IGS SINEX template:
<ftp://igs.cb.jpl.nasa.gov/pub/station/general/igs.snz>
- For VLBI solutions you can use
http://gemini.gsfc.nasa.gov/solve_save/ECCDAT.ecc
- The official ILRS eccentricity file is
<ftp://cddisa.gsfc.nasa.gov/pub/slrocc/slrecc.txt>

18. SOLUTION/EPOCHS Block (Mandatory)

Description:

List of solution epoch for each Site Code/Point Code/Solution Number/Observation Code (SPNO) combination.

Contents:

S O L U T I O N E P O C H S D A T A L I N E		
Field	Description	Format
[Site Code]	Site code for which some parameters are estimated.	1X,A4
[Point Code]	Point Code at a site for which some parameters are estimated.	1X,A2
[Solution ID]	Solution Number at a Site/Point code for which some parameters are estimated.	1X,A4
[Observation Code]	Identification of the observation technique used.	1X,A1
[Time]	Start time for which the solution identified (SPNO) has observations	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	End time for which the solution identified (SPNO) has observations	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Mean time of the observations for which the solution (SPNO) is derived.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
		54

19. BIAS/EPOCHS Block (Mandatory if bias parameters are included)

Description:

List of epochs of bias parameters for each Site Code/Point Code/Solution Number/Bias Type (SPNB) combination for which a bias parameter is solved.

Contents:

B I A S E P O C H S D A T A L I N E		
Field	Description	Format
[Site Code]	Site code for which some biases are estimated (station ID)	1X,A4
[Point Code]	satellite ID for which some biases are estimated: e.g. L1, L2 for LAGEOS-1 and -2 respectively	1X,A2
[Solution ID]	sequential number of the bias for this particular station	1X,A4

	(if just one bias is solved for a particular station, this parameter remains "1").	
Bias Type	Specification of the type of bias: R - range bias T - time bias S - scale bias Z - troposphere bias at zenith	1X,A1
[Time]	Epoch of 1st observation of the solution identified (SPNB)	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Epoch of last observation of the solution identified (SPNB)	1X,I2.2, 1H:,I3.3, 1H:,I5.5
[Time]	Weighted mean time of the observations for which the solution (SPNB) is derived.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
		54

20. SOLUTION/STATISTICS Block (Recommended if available)

Description:

Statistical information about the solution contained in the SINEX file.

Contents:

S O L U T I O N S T A T I S T I C S L I N E		
Field	Description	Format
Information Type	Describes the type of information present in the next field. May take on the following values: 'NUMBER OF OBSERVATIONS' # of observations used in the adjustment. 'NUMBER OF UNKNOWNNS' # of unknowns solved in the adjustment. 'SAMPLING INTERVAL (SECONDS)' Interval in seconds between successive observations. 'SQUARE SUM OF RESIDUALS (VTPV)' Sum of squares of residuals. (V'PV); V - resid. vector; P - weight matrix 'PHASE MEASUREMENTS SIGMA' Sigma used for the phase measurements. 'CODE MEASUREMENTS SIGMA' Sigma used for the code (pseudo-range) measurements. 'NUMBER OF DEGREES OF FREEDOM' # of observations minus the # of unknowns (df) 'VARIANCE FACTOR'	1X,A30

	Sum of squares of residuals divided by the degrees of freedom (V'PV/df). Equivalent to Chi-squared/df. 'WEIGHTED SQUARE SUM OF O-C' Sum of squares of the vector 'observed minus computed': (o-c)'P(o-c) with P - weight matrix	
Information	Relevant information for the type indicated by the previous field.	1X,F22.15
		54

Comments:

The mentioned 'Information Types' may be in any order but the name of the fields should be identical to the names listed above.

Note that the NUMBER OF UNKNOWNNS contains all parameters of the adjustment, i.e., not only the parameters stored in the SINEX file but as well all the pre-eliminated parameters.

The NUMBER OF OBSERVATIONS should contain only the real observations and not the pseudo-observations used for constraining.

In principle the estimated variance-covariance matrix should be normalized / multiplied by the VARIANCE FACTOR of this statistic block.

21. SOLUTION/ESTIMATE Block (Mandatory)

Description:

Estimated parameters.

Contents:

SOLUTION ESTIMATE DATA LINE		
Field	Description	Format
Estimated Parameters Index	Index of estimated parameters. values from 1 to the number of parameters.	1X,I5
[Parameter Type]	Identification of the type of parameter.	1X,A6
[Site Code]	Site code for which some parameters are estimated.	1X,A4
[Point Code]	Point Code at a site or the satellite ID for which some parameters are estimated.	1X,A2
[Solution ID]	Solution ID at a Site/Point code for which some parameters are estimated.	1X,A4
[Time]	Epoch at which the estimated parameter is valid. For bias parameters the beginning	1X,I2.2, 1H:,I3.3, 1H:,I5.5

	of the pass (identical to the BIAS/EPOCHS block).	
Parameter Units	Units used for the estimates and sigmas. The notations are: m (metres), m/y (metres per year), m/s2 (metres per second**2), ppb (parts per billion), ms (milliseconds), msd2 (milliseconds per day**2), mas (milli-arc-seconds), ma/d (milli-arc-seconds / day), rad (radians), rd/y (radians per year), rd/d (radians per day).	1X,A4
[Constraint Code]	Constraint applied to the parameter.	1X,A1
Parameter Estimate	Estimated value of the parameter.	1X,E21.15
Parameter Standard Deviation	Estimated standard deviation for the parameter.	1X,E11.6
		80

Comments:

For the demanded units of the estimated parameters and their standard deviation please refer to the list of possible parameter types (2. Data Structure).
'Per year' means 365.25 days in this context.

22. SOLUTION/APRIORI Block (Mandatory)

Description:

Apriori information for estimated parameters, either the used apriori values for the adjustment or the parameters of a Helmert Transformation for applied inner constraints with the constraint given in the field 'Standard Deviation'.

Contents:

S O L U T I O N A P R I O R I D A T A L I N E		
Field	Description	Format
Parameter Index	Index of apriori parameters.	1X,I5
[Parameter Type]	Identification of the type of parameter. For apriori values of estimated parameters see parameter list above; For inner constraints: TX - Translation in X-direction m TY - Translation in Y-direction m TZ - Translation in Z-direction m RX - Rotation around X-axis mas RY - Rotation around Y-axis mas RZ - Rotation around Z-axis mas	1X,A6

	SC - Scale ppb TXR - Rate for translation in X-direction m/y TYR - Rate for translation in Y-direction m/y TZR - Rate for translation in Z-direction m/y RXR - Rate for rotation around X-axis mas/y RYR - Rate for rotation around Y-axis mas/y RZR - Rate for rotation around Z-axis mas/y SCR - Rate for scale ppb/y	
[Site Code]	Site code with apriori parameter estimate. ---- for inner constraints	1X,A4
[Point Code]	Point Code with apriori parameter estimate. -- for inner constraints	1X,A2
[Solution ID]	Solution ID at a Site/Point code with apriori parameter estimate. ---- for inner constraints	1X,A4
[Time]	Epoch at which the apriori parameter or the inner constraint is valid.	1X,I2.2, 1H:,I3.3, 1H:,I5.5
Parameter Units	Units used for the aprioris and sigmas. For the demanded units look at SOLUTION/ESTIMATE block. For the inner constraints the units should be: m for translations, mas for rotations, ppb for scale, m/y for translation rates, ma/y for rotation rates, pb/y for scale rates	1X,A4
[Constraint Code]	Constraint applied to the parameter. If inner constraints are applied: 1 for contributing stations	1X,A1
Parameter Apriori	Apriori value of the parameter or transformation parameter for the inner constraint.	1X,E21.15
Parameter Standard Deviation	Apriori standard deviation for the parameter or applied inner constraint.	1X,E11.6
		80

Comments:

If inner constraints are applied to the solution (i.e. parameters like TX, TY, TZ, RX,... are included) the stations contributing to these inner constraints should be listed with a '1' in the field 'Constraint Code' in this SOLUTION/APRIORI block.

23. SOLUTION/MATRIX_ESTIMATE Block (Mandatory)

Description:

The Estimate Matrix can be stored in an Upper or Lower triangular form. Only the Upper or Lower portion needs to be stored because the matrix is always symmetrical.

The matrix contents can be:

CORR - Correlation Matrix

COVA - Covariance Matrix

INFO - Information Matrix (of Normals), i.e. COVA⁽⁻¹⁾

The distinction between the form and its contents is given by the title block which must take one of the following forms:

SOLUTION/MATRIX_ESTIMATE L CORR

SOLUTION/MATRIX_ESTIMATE U CORR

SOLUTION/MATRIX_ESTIMATE L COVA

SOLUTION/MATRIX_ESTIMATE U COVA

SOLUTION/MATRIX_ESTIMATE L INFO

SOLUTION/MATRIX_ESTIMATE U INFO

Contents:

SOLUTION/MATRIX_ESTIMATE DATA LINE		
Field	Description	Format
Matrix Estimate Row Number	Row index for the Matrix Estimate. It must match the parameter index in the SOLUTION/ESTIMATE block for the same parameter.	1X,I5
Matrix Estimate Column Number	Column index for the Matrix Estimate. It must match the parameter index in the SOLUTION/ESTIMATE block for the same parameter.	1X,I5
First Matrix Estimate Element	Matrix element at the location (Row Number , Column Number).	1X,E21.14
Second Matrix Estimate Element	Matrix element at the location (Row Number , Column Number + 1).	1X,E21.14
Third Matrix Estimate Element	Matrix element at the location (Row Number , Column Number + 2).	1X,E21.14
		78

Comment:

The Matrix Estimate Row/Column Number correspond to the Estimated Parameters Index in the SOLUTION/ESTIMATE block.

If the CORR matrix is used, standard deviations must be stored in the diagonal elements instead of 1.000.

Missing elements in the matrix are assumed to be zero (0); consequently, zero elements may be omitted to reduce the size of this block.

NOTE: The same scale (variance) factor MUST be used for both MATRIX_ESTIMATE and MATRIX_APRIORI, as well as for the standard deviations in the ESTIMATE and APRIORI Blocks. This scale factor should be stored as 'Variance Factor' in the SOLUTION/STATISTICS block.

If you use the INFO type this block should contain the constrained normal equation matrix of your least square adjustment.

24. SOLUTION/MATRIX_APRIORI Block (Recommended/Mandatory)

Description:

The Apriori Matrix can be stored in an Upper or Lower triangular form. Only the Upper or Lower portion needs to be stored because the matrix is always symmetrical. Mandatory if any significant constraint have been applied to the SOLUTION/ESTIMATE.

The matrix contents can be:

CORR - Correlation Matrix

COVA - Covariance Matrix

INFO - Information Matrix (of Normals), i.e. COVA⁽⁻¹⁾

The distinction between the form and its contents is given by the title block which must take one of the following forms:

SOLUTION/MATRIX_APRIORI L CORR

SOLUTION/MATRIX_APRIORI U CORR

SOLUTION/MATRIX_APRIORI L COVA

SOLUTION/MATRIX_APRIORI U COVA

SOLUTION/MATRIX_APRIORI L INFO

SOLUTION/MATRIX_APRIORI U INFO

Contents:

SOLUTION/MATRIX_APRIORI DATA LINE		
Field	Description	Format
Matrix Apriori Row Number	Row index for the Matrix Apriori. It must match the parameter index in the SOLUTION/APRIORI block for the same parameter.	1X,I5
Matrix Apriori Column Number	Column index for the Matrix Apriori. It must match the parameter index in the SOLUTION/APRIORI block for the same parameter.	1X,I5
First Matrix Estimate Element	Matrix element at the location (Row Number , Column Number).	1X,E21.16
Second Matrix Estimate Element	Matrix element at the location (Row Number , Column Number + 1).	1X,E21.16
Third Matrix Estimate Element	Matrix element at the location (Row Number , Column Number + 2).	1X,E21.16
		78

Comment:

The Matrix Apriori Row/Column Number correspond to the Apriori Parameters Index in the SOLUTION/APRIORI block. If the apriori constraint matrix is diagonal and no loss of significant digits occurs by using the Parameter Standard Deviation in the SOLUTION/APRIORI block, then, this block becomes redundant.

If the CORR matrix is used, Standard deviations must be stored in the diagonal elements instead of 1.000.

Missing elements in the matrix are assumed to be zero (0); consequently, zero elements may be omitted to reduce the size of this block.

NOTE: The same scale (variance) factor MUST be used for both MATRIX_ESTIMATE and MATRIX_APRIORI, as well as for the standard deviations in the ESTIMATE and APRIORI Blocks. This scale factor should be stored as 'Variance Factor' in the SOLUTION/STATISTICS block.

If you use the INFO type this block should contain the normal equation matrix of the constraints applied to your solution in SOLUTION/ESTIMATE.

25. SOLUTION/NORMAL_EQUATION_VECTOR Block (Mandatory for normal equations)

Description:

If the SINEX file shall provide the normal equation directly this block is mandatory and contains the vector of the right hand side of the unconstrained (reduced) normal equation.

Contents:

<u>SOLUTION_N O R M A L _ E Q U A T I O N _ V E C T O R _ DATA_LINE</u>		
<u>Field</u>	<u>Description</u>	<u>Format</u>
Estimated Parameters Index	Index of estimated parameters. Values from 1 to the number of parameters. It must match the parameter index in the block SOLUTION/ESTIMATE for the same parameter.	1X,I5
[Parameter Type]	Identification of the type of parameter.	1X,A6
[Site Code]	Site code for which some parameters are estimated.	1X,A4
[Point Code]	Point Code at a site or the satellite ID for which some parameters are estimated.	1X,A2
[Solution ID]	Solution ID at a Site/Point code for which some parameters are estimated.	1X,A4
[Time]	Epoch at which the estimated parameter is valid. For bias parameters the beginning of the pass (identical to the BIAS/EPOCHS block).	1X,I2.2, 1H:,I3.3, 1H:,I5.5
Parameter Units	see SOLUTION/ESTIMATE	1X,A4
[Constraint Code]	Constraint applied to the parameter.	1X,A1
Right hand side of normal equation	Value of the right hand side of the normal equation for the corresponding parameter.	1X,E21.15
		68

Comment:

The indices correspond to the indices of the SOLUTION/ESTIMATE block.

26. SOLUTION/NORMAL_EQUATION_MATRIX Block (Mandatory for normal equations)

Description:

This block is mandatory if the normal equation is to be provided directly in the SINEX file.

The block should contain the original (reduced) normal equation matrix (i.e., without constraints).

The normal equation matrix can be stored in an Upper or Lower triangular form. Only the Upper or Lower portion needs to be stored because the matrix is always symmetrical. The distinction between the forms is given by the title block which must take one of the following forms:

SOLUTION/NORMAL_EQUATION_MATRIX L

SOLUTION/NORMAL_EQUATION_MATRIX U

Contents:

<u>SOLUTION_N O R M A L _ E Q U A T I O N _ M A T R I X _ DATA_LINE</u>		
<u>Field</u>	<u>Description</u>	<u>Format</u>
NEQ-Matrix Row Number	Row index for the normal equation matrix. It must match the parameter index in the SOLUTION/ESTIMATE block for the same parameter.	1X,I5
NEQ-Matrix Column Number	Column index for the normal equation matrix. It must match the parameter index in the SOLUTION/ESTIMATE block for the same parameter.	1X,I5
First Matrix Element	Matrix element at the location (Row Number , Column Number).	1X,E21.14
Second Matrix Element	Matrix element at the location (Row Number , Column Number + 1).	1X,E21.14
Third Matrix Element	Matrix element at the location (Row Number , Column Number + 2).	1X,E21.14
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Comment:

The NEQ-Matrix Row/Column Number correspond to the Estimated Parameters Index in the SOLUTION/ESTIMATE block.

Missing elements in the matrix are assumed to be zero (0); consequently, zero elements may be omitted to reduce the size of this block.

27. Footer Line (Mandatory)

Description:

Marks the end of the SINEX file.

Contents:

$$(5) \quad b = A' P l .$$

The resulting unknown parameters can be determined with

$$(6) \quad x = x_0 + \text{inv}(A' P A) A' P l = x_0 + \text{inv}(N) b$$

where inv stands for the inverse matrix and x_0 are the a priori values.

The residuals can be computed with equation (1) and the a posteriori variance factor is then

$$(7) \quad s_0 = (v' P v) / (n_{\text{obs}} - n_{\text{unk}}) .$$

The weighted square sum of the vector l (= observed minus computed) can be obtained with

$$(8) \quad \begin{aligned} l' P l &= v' P v + dx' b \\ &= v' P v + dx' A' P l . \end{aligned}$$

The variance-covariance matrix of the unknowns results in

$$(9) \quad K = s_0 \text{inv}(N) .$$

If you introduce constraints as pseudo-observations with n_{constr} linearized observation equations

$$(10) \quad v_c = H dx - h$$

with

n_{constr} number of constraints as pseudo-observations
 v_c residuals over the constraints
 H Jacobian matrix for pseudo-observation equations
 h vector 'observed' minus 'computed' for the constraints.

P_c denotes the weight matrix for your pseudo-observations.

The least square methods lead to the normal equation for the pseudo-observations

$$(11) \quad H' P_c H dx = H' P_c h$$

with normal equation matrix of constraints

$$(12) \quad N_{\text{constr}} = H' P_c H$$

and vector of the right hand side of normal equation for constraints

$$(13) \quad b_{\text{constr}} = H' P_c h .$$

The complete normal equation system for the constrained solution can easily be computed:

$$(14) \quad (A' P A + H' P_c H) dx = A' P l + H' P_c h$$

with the constrained normal equation matrix

$$(15) \quad N_{\text{total}} = A' P A + H' P_c H = N + N_{\text{constr}}$$

and the vector of the right hand side of the constrained normal equation system

$$(16) \quad b_{\text{total}} = A' P l + H' P_c h = b + b_{\text{constr}} .$$

The unknown parameters of the constrained solution can be computed with

$$(17) \quad x_c = x_0 + \text{inv}(N_{\text{total}}) b_{\text{total}} .$$

After computing the residuals over the constraints with equation (10) the weighted square sum of residuals of the constrained normal equation system can be obtained with

$$(18) \quad v' P v + v_c' P_c v_c$$

and the number of degrees of freedom of the constrained normal equation system is

$$(19) \text{ dof} = n_{\text{obs}} + n_{\text{constr}} - n_{\text{unk}} .$$

The a posteriori variance-factor for the constrained normal equation system is then

$$(20) s0_c = (v' P v + v_c' P_c v_c) / \text{dof} .$$

The variance-covariance matrix for the unknowns of this constrained normal equation system can be computed with

$$(21) K_xx = s0_c \text{ inv}(N_total)$$

And the variance-covariance matrix for the constraints is

$$(22) K_constr = s0_c \text{ inv}(N_constr) .$$

IMPLEMENTATION IN SINEX

The different elements belonging to the normal equations can be stored in SINEX files in the following way:

SOLUTION/STATISTICS block:

n_unk	=	NUMBER OF UNKNOWNNS
n_obs	=	NUMBER OF OBSERVATIONS
(20) s0_c	=	VARIANCE FACTOR
(18) v' P v + v_c' P_c v_c	=	SQUARE SUM OF RESIDUALS (VTPV)
(19) dof	=	NUMBER OF DEGREES OF FREEDOM

SOLUTION/ESTIMATE block:

(17) x_c in field "Parameter Estimate"

SOLUTION/APRIORI block:

x0 in field "Parameter Apriori"

SOLUTION/MATRIX_ESTIMATE block:

(21) Type COVA: K_xx
Type CORR: correlation matrix of K_xx
(15) Type INFO: N_total = N + N_constr

SOLUTION/MATRIX_APRIORI block:

(22) Type COVA: K_constr
Type CORR: correlation matrix of K_constr
(12) Type INFO: N_constr

SOLUTION/NORMAL_EQUATION_VECTOR block:

(5) b = A' P l

SOLUTION/NORMAL_EQUATION_MATRIX block:

(4) N = A' P A