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Nominal Wage Rigidities and the Propagation of Monetary Disturbances

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Abstract

Recent research has challenged the ability of sticky price general equilibrium models to generate a contract multiplier, i.e., an effect of a monetary innovation on output that extends beyond the contract interval. We show that a simple dynamic general equilibrium model that includes "Taylor-style" (1980) wage and price contracts can account for a substantial contract multiplier under various assumptions about the structure of the capital market. Most interestingly, our results do not rely on a high intertemporal labor supply elasticity or elastic supply of capital: our preference specification is standard (logarithmic), and we can account for a strong contract multiplier even when the aggregate capital stock is fixed. Finally, our analysis highlights the importance of the income elasticity of money demand in accounting for output persistence.

Keywords: Contract Multiplier, Sticky Price Model

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I. Introduction

A wide class of dynamic general equilbrium (DGE) models can account for persistent monetary nonneutrality through various mechanisms that induce price or wage stickiness. These mechanisms include Fischer wage or price contracts (Cho and Cooley, 1995), Calvo staggered and overlapping price or wage contracts (Yun, 1994, King and Watson, 1996, Woodford, 1996, Kollmann, 1996, Koenig, 1997, Rotemberg and Woodford, 1997), and menu costs (Rotemberg, 1996, Kim, 1996).

However, these models can be criticized for building in a large exogenous component of price/wage stickiness to account for monetary nonneutrality.[1] In an important paper, Chari, Kehoe, and McGrattan (1996; henceforward, CKM) construct a simple DGE model that includes staggered price-setting. In the spirit of Taylor (1980), these authors make limited and transparent assumptions about the exogenous source of price stickiness (contracts last only four periods), and ask if their model can account for endogenous output/price persistence. Their surprising conclusion is that models which incorporate standard assumptions about preferences and factor-market clearing cannot deliver a "contract multiplier," i.e., output persistence that exceeds the assumed contract length. These authors attribute the inability of their model to account for persistent output effects to the high procyclicality of marginal cost (especially labor cost) implied by standard assumptions about preferences and factor market-clearing.

In a similar vein, several recent papers investigating sticky price models (Ball and Romer, 1990, Romer, 1993, and Christiano, Eichenbaum, and Evans, 1997) conclude by suggesting that labor market frictions may play a key role in allowing sticky price models to account for a contract multiplier without having to incorporate a large exogenous component of price stickiness. In this paper, we propose a dynamic process for wage-setting that can account for acyclicality in the real wage, a feature consistent

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[1] For example, Kiley observes that even when Calvo contracts are renegotiated on average every four quarters, a large fraction of contracts (over 30 percent) remain in effect for more than four quarters.

with the results of a large empirical literature that does not suggest systematic cyclical variation in real wages (for an extensive survey, see Abraham and Haltiwanger, 1995). We then investigate whether replacing the assumption of spot labor market-clearing (as in CKM) with this wage-setting process can deliver a contract multiplier under different assumptions about the structure of the capital market.

Our wage-setting process is derived from a household optimization problem in a framework that is basically a dynamic version of Blanchard and Kiyotaki (1987). The labor inputs of different households are imperfect substitutes in production. Households behave as monopsonists in the labor market, taking their labor demand curve and the prevailing average wage as given. Households fix their nominal wage for four quarters, and agree to satisfy demand for their labor input at this wage. Wage-setting is asynchronous, as only one-quarter of households adjust their nominal wage in any given quarter. This structure attempts to capture several key empirical observations that were emphasized by Taylor in his original formulation of the staggered contract model (1980; also 1983, 1997), including the discrete nature of wage adjustment, that it tends to be asynchronous in most industrial economies, and that the prevailing average wage influences current wage-setting.

The household's first order condition for choosing a contract wage results in an equation that resembles the ad hoc wage-setting rule proposed by Taylor (1980). We show that our wage-setting rule can account for an arbitrarily high degree of nominal wage persistence as labor becomes increasingly substitutible across households. The asynchronous nature of wage adjustment plays a key role in accounting for wage inertia. Because a large fraction of wages are fixed at any given time, households that raise their wage experience a reduction in the demand for their labor. Accordingly, households that raise their wage in response to a stimulative shock are willing to accept a smaller wage increase than if their marginal cost of working rose in line with that of the average household.

We embed this wage-setting process into a general equilibrium model that largely retains the same goods market structure as CKM. Because prices are set by monopolists in a staggered fashion, our framework provides a symmetric treatment of wage and price-setting. The model's ability to accomodate sluggish nominal wage behavior enables us to calibrate it to imply an acyclical real wage.

In our baseline model (Sections II-III), the aggregate capital stock is assumed fixed. We show that a variant of the model in which capital is freely mobile across firms is able to account for only a modest contract multiplier. Even though labor costs are flat, a rise in the capital rental price increases the marginal costs of all producers by enough to dampen output persistence.

By contrast, we show that the model can account for a strong contract multiplier when we consider a version with "factor specificity" in which capital is fixed at the firm-level, so that the marginal cost curve of each firm is upward-sloping. Factor specificity works to increase price/output persistence through a mechanism that is analogous to that which accounts for nominal wage inertia in the labor market. Model simulations indicate that about half of the impact effect of a monetary shock on output remains after all producers have had a chance to reset their price (for the case of a permanent innovation to money). Interestingly, the model implies a response of the average price that is initially convex, so that the (price) inflation rate actually rises for four quarters before gradually declining. The highly persistent inflation response suggests that our model with price and wage contracts may have considerably more ability to fit the autocorrelation structure of inflation (and its cross-correlation with output) than Fuhrer-Moore (1995) found in their empirical investigation of a standard wage-contracting model.

Our results highlight that it is not necessary for the intertemporal elasticity of labor supply to be very large, or for capital to be highly elastic, in order to account for output/price persistence. Our model delivers a strong contract multiplier with very standard preference assumptions (logarithmic in consumption, leisure, and real balances), even if the aggregate capital stock is fixed. Our model accounts for price and wage rigidity (with the latter playing an important role in accounting for the former) by smoothing the marginal costs of agents resetting prices and wages relative to the marginal costs of the average firm or household (that may rise much more sharply). The smoothing is attributable to substitution effects that reduce the relative demand of agents readjusting their price or wage.

Of course, an alternative mechanism for deriving output persistence is to simply assume highly elastic aggregate factor supplies. This increases output persistence by flattening the marginal costs of all firms equally (in part by smoothing the marginal disutility of working of all households). However, a serious problem with this approach is that it appears difficult to rationalize such elastic factor responses empirically.[2] As emphasized in CKM, estimates of the intertemporal elasticity of labor supply tend to be quite low (e.g., Ghez and Becker, 1975, Mulligan, 1995, Rupert, Rogerson, and Wright, 1997). Moreover, a sizeable fraction of the capital stock consists of slowly-depreciating structures and other forms of capital for which the elasticity of the stock is probably very low over a business cycle.

In Section IV, we show that modifying our basic model to allow for aggregate capital accumulation (subject to firm-level adjustment costs) can strengthen the contract multiplier somewhat further. However, the effects are quite modest. Our analysis (in Section V) also highlights how output/price dynamics can be dramatically affected by the form of the money demand function in economies with aggregate capital accumulation. This reflects that a relatively high income elasticity of real money demand is crucial to deriving a contract multiplier, even if real wages are acyclical. To the extent that models with consumption-based money demand functions have a structure that allows consumption to be relatively insensitive to income, such models imply a low effective income elasticity of money demand -- and correspondingly generate very weak output persistence (and even oscillatory output dynamics). Accordingly, we show that with a consumption-based money demand specification, increasing adjustment costs on capital actually leads to much more persistent output dynamics by increasing the effective income elasticity of money demand.

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[2] For example, Kiley (1997) and Dotsey, King, and Wolman (1997) smooth real wages within a standard labor market-clearing framework by assuming a labor supply elasticity of infinity. CKM note "preferences with zero income effects and high labor elasticities offer a promising route to persistence," (pg. 15) but stress that such assumptions are inconsistent with micro evidence on labor supply.

Section II. Baseline Model with a Fixed Aggregate Capital Stock

Firms

Our setup of the goods market follows a standard monopolistic competition framework, as in e.g., CKM (1996) and Kim (1996). There is assumed to be a single final output good that is produced with a continuum of intermediate goods indexed by j that are distributed on the unit interval, so $j \in [0,1]$. The technology for producing final goods is given by:

(1)
$$y(s^t) = \left[\int_0^1 y(j, s^t)^{\mu} \, dj \right]^{\frac{1}{\mu}}$$

where $y(j, s^t)$ denotes the production of intermediate good j (by producer j) at date t, given the state of the world s^t, and $y(s^t)$ denotes production of the final good.

The output market for final goods is perfectly competitive. Final goods producers purchase inputs from intermediate goods producers to maximize their profits Π_F:

(2)
$$Max \;\; \Pi_F = P_A(s^t) \, y(s^t) - \int_0^1 P(j, s^t) \, y(j, s^t) \, dj$$

where $P(j, s^t)$ is the price of intermediate good j, and $P_A(s^t)$ is the price index of the final output good. Final goods producers take the input prices they face to be exogenous. Thus, the final goods market is competitive both in the product and factor markets, and producers maximize a static profit function. The solution to the profit maximization problem in [2] implies demand functions for the intermediate goods of the form:

(3)
$$y(j, s^t) = \left[\frac{P(j, s^t)}{P_A(s^t)} \right]^{1/(\mu - 1)} y(s^t)$$

Thus, each intermediate producer's share of total output depends on the ratio of its own price to the price index denoting the cost of one unit of the final good, given by:

(4) $$P_A(s^t) = \left[\int_0^1 P(j, s^t)^{\frac{\mu}{\mu-1}} dj \right]^{\frac{\mu-1}{\mu}}$$

This aggregate price index is derived by imposing the condition that final goods producers make zero profits.

The technology for the production of intermediate goods of each producer j is given by the Cobb-Douglass function:

(5) $$y(j) = k(j)^{\alpha} h(j)^{1-\alpha}$$

where k(j) is firm j's capital stock, and h(j) is the quantity of labor it employs.

Firms producing intermediate goods are assumed to behave as monopolistic competitors in their respective output market. Following CKM, producers are divided into N distinct cohorts. Each of the cohorts of producers sets its price in a staggered and overlapping fashion, holding its price fixed for N periods before resetting it. Thus, for a producer that resets its price during period t, $P(j, s^{t+K}) = P(j, s^t)$, for K = 0, ... N-1. Individual producers are indexed so that producers indexed $j \in [0, 1/N]$ set their prices during periods 0, N, 2N, producers indexed $j \in [1/N, 2/N]$ set their prices during periods 1, N+1, 2N+1, etc. A producer j that resets its price at date t is assumed to choose its nominal price to maximize its discounted profits over the life of the contract (subject to its demand curve [3]), which are given by:

(6) $$Max \;\; \Pi_I (j, s^t) = \sum_{\tau=t}^{\tau=t+N-1} \int_{s^\tau} Q(s^\tau/s^t) \left[P(j, s^t)\, y(j, s^\tau) - x(y(j, s^\tau), j, s^\tau)\right] d(s^\tau/ s^t)$$

where $Q(s^\tau/ s^t)$ is the stochastic discount factor which gives the current dollar value of a contingent claim to $1 to be paid at date τ iff state s^τ occurs, and $x(y,j,s^\tau)$ is firm j's nominal cost of producing y units of output (in state s^τ). Producers are assumed to set their price after they observe the shock at date t, so that price contracts must last at least two periods for the average price level to be sticky.

The particular form of the cost function depends on our assumption about the rental market for capital. In the first variant of our model, we assume that there is a perfect rental market for capital (as in the baseline model outlined by CKM), with intermediate goods producers taking the capital rental price (as well as the wage) as exogenously given. For this case, the cost function is derived as the solution to the following cost-minimization problem, with capital (k) and hours (h) as choice variables:

(7)
$$x(y, s^t) = Min\ W_A(s^t)\ h + P_k(s^t)\ k + q(s^t)\left[y - k^{\alpha}h^{1-\alpha}\right]$$

where $W_A(s^t)$ is the average wage, $P_k(s^t)$ the rental price of capital, and $q(s^t)$ is the Lagrange multiplier on the output constraint. We refer to this variant of the model as the "mobile capital" version below. Since all firms have the same cost function in this case, the j subscript is dropped. Alternatively, we assume that capital is fixed at the firm level. In this case, the cost function is simply given as:

(8)
$$x(y, j, s^t) = W^A(s^t)\ h(y, j)$$

where the relation between hours and output is determined by the production function [5]. We refer to this case below as the "fixed capital" or "factor specificity" version of the model, reflecting that one of the two productive factors (capital) is specific to individual firms.

For either variant of the model, the solution to producer j's maximization problem given in [6] for choosing its fixed nominal price $P(j, s^t)$ from t to t+N-1 can be represented as:

(9)
$$P(j, s^t)\sum_{\tau=t}^{\tau=t+N-1}\int_{s^\tau} Q(s^\tau/s^t)\ \frac{y(j, s^\tau)}{y(j, s^t)}\ d(s^\tau/s^t)$$
$$= \frac{1}{\mu}\sum_{\tau=t}^{\tau=t+N-1}\int_{s^\tau} Q(s^\tau/s^t)\ \sigma(j, s^\tau)\ \frac{y(j, s^\tau)}{y(j, s^t)}\ d(s^\tau/s^t)$$

Equation [9] says that producer j sets its nominal output price as a weighted average of its future expected nominal marginal costs, given by $\sigma(j, s^\tau)$, scaled up by a constant markup factor. An upward revision to the firm's expectations about nominal marginal costs (due either to a rise in real marginal costs, or to a projected rise in the aggregate price level) causes the firm to raise its nominal output price.

The nominal marginal cost expression in the case in which capital is freely mobile is given by:

(10) $$\sigma_\tau = \alpha^{-\alpha} (1-\alpha)^{\alpha-1} W_{A\tau}^{1-\alpha} P_{K\tau}^{\alpha} \qquad [\textit{Mobile Capital}]$$

The marginal cost does not depend on an individual firm's level of output, but only on economywide factor prices, reflecting that all producers have CRTS technologies. Alternatively, when capital is fixed at the firm level, real and hence nominal marginal costs depend on the level of output of the individual firm. The marginal cost expression for this case is given by:

(11) $$\sigma_{j\tau} = (1-\alpha)^{-1} W_{A\tau} \left(\frac{y_{j\tau}}{K(j)}\right)^{\frac{\alpha}{1-\alpha}} \qquad [\textit{Fixed Capital}]$$

<u>Households</u>

There are a continuum of households distributed on the unit interval, j $\in$ [0,1]. Each household seeks to maximize a utility functional of the form:

(12) $$\sum_{\tau=t}^{\tau=\infty} \int_{s^\tau} \beta^{\tau-t} Pr(s^\tau / s^t) \; U(c(j,s^\tau),\, 1-l(j,s^\tau),\, M(j,s^\tau)/P_A(s^\tau)) \; d(s^\tau / s^t)$$

c(j,.) = consumption of the final good of household j
l(j,.) = hours worked by household j
M(j,.) = nominal cash holdings of household j

where the period utility function U of each household is assumed to be log separable in its inputs:

(13) $$U(c,\, 1-l,\, M/P_A) = \psi_1 \ln (c) + \psi_2 \ln (1-l) + \psi_3 \ln (M/P_A)$$

The household budget constraint evolves according to:

(14) $$\int_{s_{t+1}} Q\,(s^{t+1}/s^t)\; B(j,\, s^{t+1})\; d(s^{t+1}/s^t) = \int_0^1 \Pi_I(i,\, s^t) di + P_k(s^t)\; k(j)$$
$$+ W(j,\, s^t)\; l(j,\, s^t) - P_A(s^t)\; c(j,\, s^t) - M(j,\, s^t) + M(j,\, s^{t-1}) + B(j,\, s^t)$$

where:

$\Pi_l(i,.)$ = nominal profits that household j receives from intermediate goods producer i
k(j) = fixed capital stock of household j (free capital mobility case)
B(j,.) = nominal bond holdings (beginning of period)
W(j,.) = nominal wage paid to household j

Each household's revenue consists of the sum of its labor income and an aliquot share of the profits of each of the intermediate goods producers. For the version of the model with free capital mobility, households also receive capital income from renting their stock of capital to firms. In this section, we assume that each household's capital stock, and hence the aggregate capital stock, is simply fixed.[3] Each household uses its revenue to finance consumption of the nondurable final consumption good, and to increment its stock of cash balances and bonds.

A key feature of our setup is that household's act as price-setters in the labor market. Our structure of the labor market closely parallels that of Blanchard and Kiyotaki (1987) and Kim (1996). In particular, the labor market is assumed to be composed of a continuum of households that supply differentiated labor inputs. Labor inputs differ because firms that rent labor as an input into production regard each household's labor services as an imperfect substitute for the labor services of other households. The labor input that enters the production function of firms is a composite of the different labor inputs of each of the j households. The technology relating the composite labor input $l(s^t)$ to the labor input of individual households $l(j, s^t)$ is given by:

(15)
$$l\,(s^t) = \left[\int_0^1 l(j,\, s^t)^{\phi}\, dj \right]^{\frac{1}{\phi}}$$

It is useful for expositional purposes to assume a market structure in which the different labor inputs entering into [15] are "bundled" by competitive producers of the composite labor input. While each household behaves as a monopsonist supplier of its own labor input, each producer of the composite labor

[3] In the case in which capital is fixed at the firm level, capital income is effectively included in the profits of intermediate goods producers.

input regards itself as too small for its demand for labor to affect the wages of households (its suppliers). This makes the structure of the labor market isomorphic to the structure of the goods market, with the producers of the composite labor input taking the place of final goods producers, and households of the suppliers of intermediate inputs. Accordingly, competitive producers of the composite labor input purchase labor inputs $l(j,s^t)$ from each household j at a wage $W(j,s^t)$ to maximize their profits (Π_l):

(16) $$Max \quad \Pi_l = W_A(s^t)\, l(s^t) - \int_0^1 W(j, s^t)\, l(j, s^t)\, dj$$

Producers of the composite labor input in turn sell their output to firms at a nominal wage of $W_A(s^t)$ per unit of the composite input, where the nominal wage index $W_A(s^t)$ is given by:

(17) $$W_A(s^t) = \left[\int_0^1 W(j, s^t)^{\frac{\phi}{\phi-1}}\, dj \right]^{\frac{\phi-1}{\phi}}$$

The solution to the optimization problem [16] yields a demand function for the labor services of household j of the form:

(18) $$l(j, s^t) = \left[\frac{W(j, s^t)}{W_A(s^t)} \right]^{1/(\phi-1)} l(s^t)$$

Thus, the demand for household j's labor input depends on the ratio of its wage relative to the average wage in the economy, with the elasticity of demand for its labor input rising in the degree of substitutibility across the different types of labor inputs, i.e., rising in ϕ.

Instead of assuming that households can reset their wage each period subject to menu costs, as in Kim (1996), we assume that wages -- like prices -- are set in a staggered and overlapping fashion. Thus, households set wage contracts that last M periods, and agree to work whatever number of hours firms (producers of the composite labor input) demand at that fixed wage over the life of the contract. Households are indexed so that households in the interval [0, 1/M] set wage contracts at time 0, M, 2M,

..., households in the interval [1/M, 2/M] set wages at 1, M+1, 2M+1, etc.

The optimization problem faced by household j that is resetting its wage in the current period t is to choose its nominal wage rate $W(j,s^t)$ that will hold over the next M periods, and rules for consumption, nominal money balances, and bond holdings to maximize its utility functional [12] subject to its budget constraint [14] and the demand function for its labor [18]:

$$\begin{aligned}(19)\quad & \sum_{\tau=t}^{\tau=\infty} \int_{s^\tau} \beta^{\tau-t} Pr(s^\tau/ s^t) \; [U(c(j, s^\tau), 1-l(j, s^\tau), M(j, s^\tau)/P_A(s^\tau)) \\ & +\lambda(j, s^\tau) \; (\int_0^1 \Pi_I(i, s^\tau)di + P_k(s^\tau) \; k(j) + W(j, s^\tau) \; l(j, s^\tau) \\ & - P_A(s^\tau) \; c(j, s^\tau) + M(j, s^{\tau-1}) - M(j, s^\tau) + \\ & B(j, s^\tau) - \int_{s^{\tau+1}} Q(s^{\tau+1}/s^\tau) \; B(j, s^{\tau+1})] \; d(s^\tau/ s^t)\end{aligned}$$

Note that as in the case of intermediate goods producers, households setting wages at date t do so after observing the realization of the shocks.

Household j's first order conditions for choosing consumption, bonds, and money holding are given by:

$$(20) \qquad U_1(j, s^t) = \lambda(j, s^t) \; P_A(s^t)$$

$$(21) \qquad Q(s^\tau/ s^t) = \beta^{\tau-t} \; Pr(s^\tau/s^t) \left[\frac{U_1(c(j, s^\tau) \; P_A(s^t)}{U_1 c(j, s^t) \; P_A(s^\tau)}\right]$$

$$(22) \qquad \frac{U_3 \; (M(j, s^t)/P_A(s^t))}{U_1(c(j, s^t))} = 1 - \beta\int_{s^{t+1}} \left[\frac{U_1(c(j, s^{t+1})) \; P_A(s^t)}{U_1(c(j, s^t)) \; P_A(s^{t+1})}\right] d(s^{t+1}/s^t)$$

The first order condition for bond holdings [21] together with our assumption that utility is separable in consumption implies that the consumption of all households is perfectly correlated. Assuming households have the same initial level of wealth at "date 0,", their level of consumption will also be equal at all dates and in all states, regardless of the shock realization, so that $c(j,s^t) = c(s^t)$. Equations [21] and [22] link the consumption velocity of money to the nominal interest rate.

The household's first order condition determining its nominal wage is given by:

$$\frac{W(j,\, s^t)}{P_A(s^t)} \sum_{\tau=t}^{\tau=t+M-1} \int_{s^\tau} Q(s^\tau/s^t)\; \frac{l(j,\, s^\tau)}{l(j,\, s^t)}\; d(s^\tau/s^t) \tag{23}$$

$$= \frac{1}{\phi} \sum_{\tau=t}^{\tau=t+M-1} \int_{s^\tau} \beta^{\tau-t}\, \frac{U_1(c(j,\, s^\tau))}{U_1(c(j,\, s^t))} \frac{U_2(l(j,\, s^\tau))}{U_1(c(j,\, s^\tau))} \frac{l(j,\, s^\tau)}{l(j,\, s^t)}\; d(s^\tau/s^t)$$

Equation [23] says that the contract nominal wage of household j $W(j,s^t)$ is set such the real present discounted value of the household's marginal return to working (the left hand side) over the life of the contract is a constant markup ($1/\phi$) over its real discounted marginal costs (the right hand side). Note that while the nominal interest rate is the appropriate factor to discount the household's fixed nominal wage payment over the contract life, the real interest rate is the relevant factor for discounting the subjective cost of working (i.e., household j's marginal rate of substitution of leisure for consumption).

We provide a detailed discussion of wage contracts of this form in the next section. However, it is important to note two points. First, while it is assumed that labor contracts stipulate that the household will meet all demand for its labor input at its preset nominal wage over the life of the contract, the wedge between the wage and the marginal rate of substitution between leisure and consumption means that household typically benefits from working the additional hours (despite a fall in its markup of its wage over its marginal rate of substitution). Only in the case of shocks to the demand for its labor that were large enough to raise its marginal rate of substitution above its current (real) wage would household behavior be constrained by the terms of the contract. Second, in order to obtain an interior solution in which households work a positive fraction of their time endowment, households must face an elastic demand curve, so $\phi > 0$.

Solution

We assume that both price and wage contracts last four quarters,[4] with one quarter of firms and households resetting their contract price or wage during each quarter. Given that all firms (households) that reset their price (wage) during the same period face identical decision problems and hence choose the same price, we can drop j subscripts and simply denote the contract price set at t by P_t , and the wage by W_t. Thus, from [4], the average price (P_{At}) can be written in terms of contract prices as:

(24)
$$P_{At} = \left(.25\ P_t^{\frac{\mu}{(\mu-1)}} + .25\ P_{t-1}^{\frac{\mu}{(\mu-1)}} + .25\ P_{t-2}^{\frac{\mu}{(\mu-1)}} + .25\ P_{t-3}^{\frac{\mu}{(\mu-1)}} \right)^{\frac{\mu-1}{\mu}}$$

Similarly, from [17] the aggregate wage (W_{At}) may be expressed as:

(25)
$$W_{At} = \left(.25\ W_t^{\frac{\phi}{(\phi-1)}} + .25\ W_{t-1}^{\frac{\phi}{(\phi-1)}} + .25\ W_{t-2}^{\frac{\phi}{(\phi-1)}} + .25\ W_{t-3}^{\frac{\phi}{(\phi-1)}} \right)^{\frac{\phi-1}{\phi}}$$

The dynamics of our model are determined by the firm's optimality condition in setting its contract price [9], the household's first order condition for setting its contract wage [23], the household's real money demand equation [22], and an exogenous law of motion for the growth rate of money (g_t) . The first three of these equations can be represented in terms of the three endogenous state variables P_t, W_t, and y_t (and of g_t), by solving out for the output (hours worked) choices of the individual cohorts of producers (households) in terms of these state variables. Details are provided in Appendix I. The (logarithmic) growth rate of money (g_t) is assumed to follow a first order autoregression:

(26)
$$g_t = g_0 + \rho\ g_{t-1} + \omega\ e_t \qquad e_t\ \ iid\ N(0,1)$$

After scaling nominal variables by the level of the money stock (M_t), the state space representation of the log-linearized version of the model may be written:

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[4] For evidence supporting the reasonableness of these assumptions about contract length for both wage and price contracts, see a recent survey by Taylor (1997).

$$\text{(27)} \quad \Sigma_{t+1} = A_1 \Sigma_t + A_2 \mathbf{E}_{t+1}$$

where $\Sigma_t' = \{ P_{t+2}, W_{t+2}, y_t, P_{t+1}, W_{t+1}, P_t, W_t, P_{t-1}, W_{t-1}, P_{t-2}, W_{t-2}, P_{t-3}, W_{t-3}, g_t, g_{t-1}, g_{t-2}, g_{t-3}\}$. The matrix A_2 is a row vector consisting entirely of zeros except for the element premultiplying the money growth innovation (in the fourth last row). We solved the model using two different methods, and found that they yielded nearly identical results. One procedure involved solving the log-linearized system using the method of Blanchard and Kahn [1980]. A second method solved the nonlinear equations directly for the perfect foresight paths of the variables using a stacked Newton-Raphson algorithm.

Calibration

The parameters of the model include the preference parameters $\{\beta, \psi_1, \psi_2, \psi_3\}$, the technology parameter α, the markup parameters $\{\mu, \phi\}$, and the parameters governing the evolution of the growth rate of money $\{g_0, \rho, \omega\}$.

We assume that a period equals one quarter. The subjective discount factor β is set equal to .99, consistent with a steady state annualized real interest rate of about four percent. The remaining preference parameters are set to match an average fraction of the time endowment spent working of 0.3,[5] and a quarterly consumption velocity of money of 1.2. The estimate of velocity of 1.2 is roughly equal to the average quarterly consumption velocity of M1 over the past two decades in the United States.[6] We assume that money growth shocks are iid with a standard deviation of unity, so that $\rho = 0$, $\omega = 1$, and $g_0 = 1.0$. We discuss our calibration of the parameters of μ and ϕ below.

[5] The 0.3 estimate is in line with typical estimates in the real business cycle literature (Cho and Rogerson, 1988, Backus, Kehoe, Kydland, 1992) that are based on average weekly per capita hours worked relative to an assumed time endowment of 112 hours.

[6] Using other monetary aggregates that imply significantly different average values of monetary velocity has no measurable impact on our results.

Section III. Simulations of the Baseline Model

In this section, we examine whether our model can account for a contract multiplier. Since a major innovation in our setup is the incorporation of sticky wages (via equation [23]), it is useful to compare our results with an otherwise identical model that makes the standard assumption of spot labor-market clearing. Under spot labor-market clearing, our wage adjustment equation [23] is replaced by the familiar condition that the household equate its marginal rate of substitution between leisure and consumption to the real wage. In log-linearized form, this condition is:

$$\tilde{W}_{At} = \tilde{P}_{At} + \tilde{c}_t + \frac{l}{1-l}\,\tilde{l}_t \qquad \textbf{(28)}$$

where a ~ denotes the percentage deviation of a variable from its initial steady state, or baseline, level. An important feature of this condition is that it applies both to individual households and as an aggregate relation, reflecting that the hours worked of each household covary one-for-one with the aggregate.

Figure 1 presents impulse response functions (IRFs) of a benchmark model that modifies the mobile capital variant of the model outlined above by assuming spot labor market-clearing (it is referred to as the "LC" model below).   The figure shows IRFs of output, the real wage, the aggregate price level, and the contract price to a permanent one percent innovation in the level of the money stock.[7]

The figure indicates that output jumps initially by about 0.7 percent, but it is clear that the model cannot account for a contract multiplier.   Output falls well below baseline by period four, the first period in which all producers have had a chance to reset their price. Real wages increase by considerably more in percentage terms than the increase in output immediately following the shock.   The sharp rise in real factor prices induces producers to increase their own contract prices relative to the current aggregate price level. At an aggregate level, these relative price increases put upward pressure on the price level, and

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[7] The choice of μ has no first-order effects on model dynamics in the mobile capital variant of the model.

mean that nominal changes in demand translate very quickly into aggregate price increases.

Before examining corresponding IRFs of (the variants of) our model outlined in the last section, it is useful to provide a more detailed comparison of how our wage-setting process in equation [23] differs from the spot labor market-clearing condition [28]. To do so, we consider a log-linearized version of [23]:

$$\textbf{(29)} \qquad \tilde{W}_t - \tilde{P}_{At} = \tilde{c}_t + \frac{1}{1+\beta}\tilde{R}_t + \frac{1}{1+\beta}\frac{l_1}{1-l_1}(\tilde{l}_{1t} + \tilde{l}_{1t+1})$$

where:

R_t = nominal interest rate
l_{1t} = hours worked by type 1 households at date t

and where:

$$\textbf{(30)} \qquad \begin{aligned} \tilde{l}_{1t} &= \frac{1}{\phi-1}(\tilde{W}_t - \tilde{W}_{At}) + \tilde{l}_t \\ \tilde{l}_{1t+1} &= \frac{1}{\phi-1}(\tilde{W}_t - \tilde{W}_{At+1}) + \tilde{l}_{t+1} \end{aligned}$$

For simplicity, contracts last only two periods. The first order conditions are relevant for a "type 1" household (half of all households), a member of the cohort that resets its wage during the current period.

Equation [29] indicates that the contract wage of a given cohort depends on the number of hours that the cohort expects to work over the life of the contract. This equation may appear to be simply the dynamic analogue of equation [28]; however, [29] has very different implications for aggregate wage behavior, due to the asynchronous nature of wage adjustment. In particular, because a large fraction of wages remain fixed during any given period, the cohort of households adjusting its wage (upward) in response to a shock expects to experience some reduction in the relative demand for its labor inputs. Accordingly, the number of hours that these households expect to work, and hence their marginal cost of working, rises less sharply than that of the average household.[8] This induces households resetting their wage to accept a smaller nominal wage increase than if they bore the marginal disutility of working of the average household.

The magnitude of this substitution effect varies directly with the parameter ϕ, which determines the degree of substitutibility between labor inputs. Our argument suggests that as ϕ rises -- implying a larger wedge between the marginal disutility of working of households resetting their wage and that of the average household -- nominal wage persistence should rise (as households become "more reluctant" to raise nominal wages), and wages should become less sensitive to aggregate demand.[9] This intuition is confirmed by substituting [30] into [29] to derive a difference equation in the contract wage of the form:

(31)
$$\tilde{W}_t = \gamma_1 \tilde{W}_{t-1} + \gamma_1 \beta E_t \tilde{W}_{t+1} + \gamma_2 \tilde{Z}_t$$

where:

Z_t = index of nominal aggregate demand

(32)
$$\gamma_1 = -\frac{1}{1+\beta}\left(\frac{b_F}{1-b_F}\right) > 0$$
$$\gamma_2 = \frac{1}{1-b_F} > 0$$
$$b_F = \frac{1}{2}\frac{l_1}{1-l_1}\left(\frac{1}{\phi-1}\right) < 0$$
$$\tilde{Z}_t = \tilde{P}_{At} + \frac{1}{1+\beta}\tilde{R}_t + \left(\frac{1}{1+\beta}\right)\left(1 + \frac{l_1}{1-l_1}\frac{1}{1-\alpha}\right)(\tilde{y}_t + E_t\tilde{y}_{t+1})$$

From [32], as ϕ rises from its minimum value of zero to its maximum of unity, the coefficient b_F

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[8] Recall that the form of the utility function implies that the marginal cost of working rises with hours worked.

[9] The reduced sensitivity reflects weaker comovement between hours worked of households resetting wages and aggregate hours as ϕ rises.

increases in absolute value. This raises the coefficient γ_1 on past and future wages (so that wages become "stickier"), while lowering the coefficient on aggregate demand γ_2. Table 1 gives values of these coefficients for different values of ϕ. Notice that as ϕ tends toward unity, the coefficient on the aggregate demand term converges to zero. Thus, the wage-setting process can accomodate arbitrarily slow nominal wage adjustment, despite standard preference assumptions and the absence of explicit costs of adjusting nominal wages.[10]

Table 1. How Parameters of Wage Equation vary with ϕ
(assuming B = 1, and l_1 = 0.3)

ϕ	b_F	wage coefficent (γ_1)	output gap coefficient[11] ($0.81\gamma_2$)
0.0	-0.21	0.09	0.66
0.5	-0.43	0.15	0.57
0.9	-2.14	0.34	0.26
0.97	-7.13	0.44	0.10
1	$-\infty$	0.5	0

We note that the linearized wage-setting equation [31] appears similar to the ad hoc wage adjustment equation proposed by Taylor (1980), with two main differences. First, in our model the aggregate demand term (Z_t) includes nominal variables in addition to the real output gap terms. A second important difference is in the coefficients of the wage and output gap terms. In Taylor's formulation, the coefficients on the wage terms (γ_1) equal 0.5, while parameterizations that deliver highly persistent output

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[10] It is worth emphasizing the radically different implications of our labor market structure under the assumption of asynchronous and multiperiod wage-setting compared with the assumption that all wage contracts are set in unison to last a single period: in the latter case, the parameter ϕ simply determines a fixed markup of the wage over the marginal cost, which is essentially irrelevant for wage dynamics (see Blanchard and Kiyotaki, 1987).

[11] Note that the coefficient on the output gap terms is equal to a product of γ_2 and the term premultiplying y_t in [32], which for our parameterization equals 0.81.

effects require a low coefficient on the output gap, roughly in the 0.05-0.1 range. Table 1 indicates that our model implies uniformly lower coefficients on the wage terms than Taylor's (0.5 is an upper bound as ϕ approaches unity), and considerably larger coefficients on the output gap terms, at least for values of ϕ under 0.97. For example, $\phi = 0.9$ implies a coefficient on wages of 0.34, and on the output gap of 0.26. This means that even though our model can accomodate complete nominal wage inertia as a limiting case, it implies considerably more rapid nominal wage adjustment than Taylor's baseline parameterization, except for relatively high values of ϕ (in excess of 0.97).

Turning to simulations, we first consider IRFs of the variant of our model with free capital mobility. These IRFs are shown in Figure 2. The parameter ϕ is set equal to 0.9 in our calibration. Although this parameterization implies fairly rapid nominal wage adjustment, it is consistent with rough acyclicality of the real wage for this variant of our sticky wage model. [12]

It is evident that incorporating sticky wages into the basic LC model has a significant effect on the dynamic responses of output and prices. Output remains 0.19 percent above its baseline value by period four, meaning that almost one-quarter of the initial output effect remains after all producers have had a chance to reset their price (compared to a -0.1 percent fall in the LC model). Nevertheless, it is

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[12] As noted in the introduction, a large literature examining the responsiveness of real wages to various cyclical measures does not suggest that real wages have displayed systematic pro- or countercyclical tendencies over long sample periods (Abraham and Haltiwanger, 1995). One problem with drawing on this literature to calibrate our model is that most of this research does not condition on the different types of shocks that have driven real wage/output behavior over various historical periods -- whereas we would like to fit the conditional response of the real wage to a monetary impulse. An exception is recent work by Fleischman (1994), which attempts to uncover the conditional dynamic pattern of real wage/output correlations that are associated with various underlying demand and supply shocks. Fleischman finds that while "technology shocks and oil price shocks result in very procyclical real wages, labor supply and aggregate demand shocks move wages countercyclically." (pg. 3). Moreover, Den Haan finds a significant negative correlation between real wages and output in the short-term, even though the correlation is positive in the longer-term.

Thus, some evidence suggests that monetary impulses may generate countercyclical real wage variation. Accordingly, our choice of calibrating ϕ to imply a roughly acyclical real wage response seems conservative from the perspective of accounting for a contract multiplier: calibrating ϕ to imply countercyclical real wages would strengthen output persistence relative to our results below.

apparent that the model can only account for a modest degree of endogenous output persistence, even though real wages are acyclical. This reflects that the increase in aggregate demand induces a sharp rise in the rental price of capital of all producers that producers rapidly pass on in the form of higher prices.

As in the case of wage-setting, what is crucial to deriving price/output persistence is to keep the marginal costs of *producers that are resetting prices* from rising too sharply. Introducing factor specificity can strengthen the contract multiplier significantly, through essentially the same mechanism as was used to account for wage inertia. To see this, it is helpful to compare the log-linearized marginal costs of a producer under the mobile and fixed capital (or "factor specificity") variants of the model:

(33)
$$\tilde{\sigma}_t = \tilde{W}_t + \gamma \left[\frac{1}{\mu-1}\frac{\alpha}{1-\alpha}\right](\tilde{P}_t - \tilde{P}_{At}) + \frac{\alpha}{1-\alpha}\tilde{y}_t$$

where:

$\gamma = 0$ mobile capital case
$\gamma = 1$ fixed capital (factor specificity) case

In the mobile capital version, all producers have the same marginal cost that depends only on economywide factor prices.[13] Thus, as all producers have CRTS, smoothing the marginal costs of producers that are resetting their prices requires smoothing the marginal costs of all producers, i.e., imposing a structure that implies a highly elastic aggregate supply of capital.[14] By contrast, factor specificity implies that producers face upward-sloping marginal cost curves at the firm-level, with the marginal cost of each producer depending inversely on its price relative to the average (the middle term of [33]). Because a large fraction of prices are fixed during any given period, producers that adjust their price upward experience a reduction in their relative demand that implies that their marginal cost rises less than proportionally with that of the average firm. Since the strength of this substitution effect

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[13] The expression for marginal cost in the mobile capital variant is obtained by solving out for the rental price of capital in terms of the aggregate wage and output.

[14] This approach is taken in Kiley (1997).

depends on the parameter μ that determines substitutibility between goods, price sluggishness becomes more pronounced (for any given sensitivity of marginal cost to output) as μ rises.[15]

Figure 3 plots IRFs corresponding to those in Figures 1-2 for the version of the sticky wage model that incorporates factor specificity. The parameter ϕ is set equal to .9 (as in Figure 2), while μ is also set equal to .9 (as in the baseline parameterization of CKM). Figure 3 shows that this version of the model can account for a much stronger contract multiplier: output remains about 0.36 percent above baseline by period four, forty percent of its initial rise. Correspondingly, price adjustment is slower. Real wages appear modestly procyclical for this parameterization.

As shown in Figure 4, this variant of the model can account for considerably greater output persistence when ϕ is increased (to .97 in the case shown), while remaining consistent with rough acyclicality of the real wage. Output remains 0.43 percent above baseline by period four, almost half the size of its initial jump.

While we have emphasized the ability of our model to generate output persistence, there are "mirror" implications for price dynamics. This reflects that with a random walk shock to money (and fixed aggregate capital stock), velocity remains constant, so that nominal income remains constant after the shock. Thus, Figure 4 shows that the gradual convergence of output is associated with a similar gradual convergence of prices (and of nominal wages, given that the real wage is close to constant).

Interestingly, because the initial response of the average price is convex, price inflation rises slowly for several periods before peaking, as shown in Figure 5. Thus, our generalized model with Taylor-style wage and price contracts seems capable of accounting for a considerable degree of persistence in the rate of price (and wage) inflation. The IRFs in Figures 4 and 5 suggest that our model may have considerably more ability to fit the autocorrelation structure of inflation than was found by

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[15] In Appendix II, we consider a simple analytic model that shows explicitly how incorporating factor specificity increases price persistence for any given degree of wage responsiveness to output.

Fuhrer and Moore (1995) in their investigation of a standard Taylor wage-contracting model. Moreover, it is intriguing that the output and inflation IRFs seem in line with sample cross-correlation functions and VARs that indicate that output shocks induce inflation to rise in the future, while inflation shocks cause output to contract (Fuhrer and Moore, 1995, Taylor, 1997). [16] [17]

Section IV. The Model With Aggregate Capital Accumulation

The previous section showed that our sticky wage model can account for a substantial contract multiplier even if the factor(s) of production other than labor is fixed in aggregate supply. Thus, an elastic supply of capital is not a prerequisite to deriving a contract multiplier. From an empirical standpoint, certain types of capital -- including the stock of structures, and capital investment with long gestation lags -- are largely fixed at business cycle frequencies. Our results suggest that even if we overstated the importance of these factors in production by assuming that all capital was of this form, monetary innovations could still induce quite persistent real effects.

It seems reasonable to expect that monetary innovations would have somewhat more persistent real effects if capital could be increased at business cycle frequencies. In this section, we investigate this

[16] The general point that staggered contract models offer a promising route to accounting for observed output/inflation dynamics -- including a highly persistent inflation response -- appears borne out in recent work by Rotemberg and Woodford (1997). These authors utilize a modified Calvo contract framework to attempt to match the empirical response of output/inflation to a monetary innovation that is identified in an unrestricted VAR. They find that their simple model provides a surprisingly good fit of the empirical IRFs.

[17] Because our focus in this paper is to develop a plausible mechanism to account for a contract multiplier, we have considered a very simple shock process, i.e., iid innovations to the money growth rate. It is worth noting that when we allow ρ, the autocorrelation coefficient of the money growth rate, to be nonzero, aggregate price dynamics change very little -- except for very high values of ρ, in the range of 0.7 or higher. Thus, assuming that the autocorrelation in money shocks is low enough that the level of the money stock nearly converges to its long-run level within a year, the price effects one year after the shock and beyond would appear very similar to those shown. However, the initial response of output would be damped relative to the responses shown in the figures, as the expansion of real output is effectively limited by the (gradual) expansion in the money stock. Thus, output persistence would appear somewhat greater than depicted in the figures.

hypothesis by allowing for aggregate capital accumulation. For simplicity, our modified model treats all capital as homogenous, with investment and final consumption goods selling at the same price (reflecting a linear aggregate transformation locus), though firms face internal costs of adjusting their capital stock.

Thus, our model with endogenous capital accumulation basically retains the structure of the model in Section II, with some modification to the problem faced by intermediate goods producers. Producers of intermediate goods purchase new investment goods from final goods producers (at the price of the numeraire consumption good). Assuming quadratic costs of adjusting capital, the problem of a monopolist intermediate goods producer j that is setting prices during the current period is to choose a contract price $P(j,s^t)$ and end of period capital stock $k(j,s^{t+1})$ to maximize:

$$\textbf{(34)} \quad \begin{aligned} Max \quad \Pi_I\,(j,\, s^t) \;=\; & \sum\nolimits_{\tau=t}^{\tau=t+N-1} \int_{s^\tau} Q(s^\tau/s^t)\; [P(j,\, s^t)\; y(j,\, s^\tau) \;-\; W_A(s^\tau)\; h(y(j,\, s^\tau),\, k_E(j,\, s^\tau)) \\ & -\; P_A(s^\tau)\; (k(j,\, s^{\tau+1}) \;-\; (1-\delta)\; k(j,s^\tau))\;]\; d(s^\tau/\, s^t) \end{aligned}$$

where the distinction between the actual capital stock k(j) held by firm j and the effective stock it uses in production $k_E(j)$ reflects internal costs of adjusting its capital stock. The effective stock of capital (of any firm j) is related to the actual stock and current period net investment by the deterministic function:

$$\textbf{(35)} \qquad k_{Et} \;=\; k_t \;-\; \xi\; \frac{(k_{t+1} \;-\; k_t)^2}{k_t}$$

where ξ is a parameter determining the real costs of net capital investment. Note that capital is quasi-fixed, in the sense that new investment is only available for production in the following period.

For reasons discussed below, we replace the money demand function that is based on household consumption [22] with one based on income that implies a unitary elasticity of real money demand:

$$\textbf{(36)} \qquad \frac{M_t}{P_{At}} \;=\; y_t$$

The solution to the modified model is somewhat more complicated than the model of Section II. This reflects that the pricing equation of each cohort of producers depends on that cohort's capital stock, so that the dynamic system includes a separate contract pricing equation of the form [9] for each of the four cohorts of producers. In addition, the equation system includes the (single) wage equation [23], and first order conditions determining the evolution of the capital stock of each of the four cohorts.[18] [19] The parameter δ is set equal to .025 in our simulations, while μ is set equal to 0.9, and ϕ to 0.97.

Simulation results (for output only) are shown in Figure 6. Our calibration in Figure 6 fits the adjustment cost parameter by allowing the initial response of investment to be about three times as large as the initial response of output ("moderate adj. costs").[20] For comparison, we also include a plot of the output IRF for the case in which adjustment costs are set at an arbitrarily high level, in which case the model converges to the fixed capital model considered in the last section ("very high adj. costs").[21] It is evident that allowing for capital accumulation can increase output persistence by a modest degree relative to the fixed capital model in Section II, though the effects are fairly small. The model that allows for aggregate capital accumulation implies that output remains at roughly 55 percent of the level to which it initially jumps by period four, or conversely, that price convergence is nearly half complete by period four. As shown in the figure, lowering adjustment costs further essentially has a negligible effect on

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[18] It is assumed that all firms can adjust their capital stocks each period, including firms in cohorts that do not adjust their output price.

[19] The model is solved using using the same stacked Newton-Raphson algorithm as in Section II.

[20] This is in the range of estimates of the relative volatility of investment and output using Hodrick-Prescott filtered data (Backus and Kehoe, 1992, provide estimates for a broad set of countries).

[21] Convergence is not quite exact between the two specifications, reflecting that there is positive gross investment in the model with capital accumulation, even when there are arbitrarily high adjustment costs. However, this difference has trivial implications for relative output dynamics.

output persistence ("very low adj. costs").[22]

It is important to note that our specification implies that capital has an important firm-specific component. This reflects that the capital stock of each firm is fixed in the impact period of the shock, and that there are costs of moving capital across firms subsequently. Factor specificity continues to play a noticeable role in the ability of the model to generate persistent output effects. This can be seen in Figure 7, which compares the output IRF for the model above in which all adjustment costs are internal to the firm with an otherwise identical "mobile capital" model in which there is only a cost of adjusting the aggregate capital stock.[23] However, it is apparent that the difference between these two specifications is less striking than when the aggregate capital stock is fixed (though it becomes more pronounced than in Figure 7 when wage adjustment is faster, i.e., for lower values of ϕ).

Taken together, our results suggest that under plausible assumptions about capital -- that some capital can be augmented over the business cycle, and that capital has considerable firm-specific attachment -- our model can deliver a strong contract multiplier.

Section V. Output Persistence and the Form of the Money Demand Function

The model's ability to account for highly persistent output IRFs (as depicted in Figure 6) is of course somewhat parameter dependent. But even fairly significant variation in the parameters that have been highlighted in our analysis -- including the parameter ϕ determining wage persistence and μ that determines the importance of factor specificity -- does not imply a dramatic difference in the strength of

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[22] Output persistence actually decreases slightly as the adjustment cost parameter declines when the latter is very small. This reflects that the partial effect of "weaker factor specificity" more than offsets the positive effect of a slightly larger aggregate investment response (which is the dominant effect at higher levels of adjustment costs). This accounts for why the IRFs of output in Figure 6 for the "low" and "moderate" adjustment cost cases are virtually indistinguishable, although the capital stock rises by somewhat more in the former case.

[23] The adjustment cost parameter for each specification is calibrated so that the initial response of investment is three times as large as output (the "moderate adj. costs" case shown in Figure 6).

the contract multiplier. For example, much lower values of ϕ can still account for a fairly strong contract multiplier. This is illustrated in Figure 8, which allows only ϕ to vary from the "moderate adj. costs" parameterization in Figure 6. The lower values of ϕ (of 0.8 and 0.2) imply significantly stronger cyclical variation in real wages: real wages (not shown) increase nearly as much as output by period four.

By contrast, the model's ability to account for output persistence can be sensitive to the form of the money demand function -- in particular, whether the demand for real balances depends on income or consumption (assuming a coefficient of unity on either variable). This sensitivity is illustrated in Figure 9. The solid line plots the IRF of output for the income-based money demand function of the last section for the case of very low adjustment costs (taken from Figure 6, the "low adjustment costs case"). As discussed above, this model specification yields a highly persistent IRF. But the dotted line -- which plots the IRF of output for the same model, replacing the income-based money demand function [36] with the consumption-based specification [22] -- appears strikingly different. Although a one percent shock to money causes output to rise by about 10 percent in the period of the shock, the output rise is not persistent. Output exhibits the same oscillatory dynamics as in the CKM paper, with output crashing below baseline by period four.

The key to understanding these seemingly odd results is to recognize that output persistence varies inversely with the income elasticity of money demand. In this section, we develop a simple analytic (labor only) model that links output dynamics to the income (or output) elasticity of money demand. We show that the model can account for the "boom-bust" behavior of output depicted in Figure 9 when the income elasticity of money demand is sufficiently low, even if the output elasticity of marginal cost is low. The model is then used to help interpret why a model with a consumption-based money demand function may yield very different implications than an otherwise identical model with an income-based money demand function when the model allows for capital accumulation.

Our model consists of the following four equations:

(37) $$(1+\beta)\ \tilde{P}_t \quad = \quad \tilde{\sigma}_t \quad + \quad \beta\ \tilde{\sigma}_{t+1}$$

(38) $$\tilde{\sigma}_t \quad = \quad \tilde{W}_t \ + \ \gamma \left[\frac{1}{\mu-1}\frac{\alpha}{1-\alpha}\right] (\tilde{P}_t - \tilde{P}_{At}) \ + \ \frac{\alpha}{1-\alpha}\ \tilde{y}_t$$

(39) $$\tilde{W}_t \quad = \quad \tilde{P}_{At} \quad + \quad \epsilon_1\ \tilde{y}_t$$

(40) $$\tilde{M}_t \quad = \quad \tilde{P}_{At} \quad + \quad \epsilon_2\ \tilde{y}_t$$

where:

$\gamma = 1$ in equation [38]: fixed capital case
$\gamma = 0$ in equation [38]: mobile capital case

All equations are represented in log percentage deviation from baseline form. Equation [37] is the log-linearized contract pricing relation [9], expressing the contract price as a weighted average of current and future marginal costs (σ_t). Contracts are assumed to last only two periods to permit the model to be solved analytically. Equation [38] is the equilibrium marginal cost expression. Equation [39] expresses the real wage as a function of output, with the parameter ε_1 determining the responsiveness of real wages to output fluctuations. Similarly, equation [40] represents real money demand as a function of income, with ε_2 the income elasticity of money demand.

Substituting equations [38], [39], [40], and the definition of the average price in terms of the contract price into equation [37] yields a second order difference equation in the contract price, with the money stock as an exogenous forcing process. This difference equation for the mobile capital model (no factor specificity, so $\gamma = 0$) can be represented as:

(41) $$\tilde{P}_t \quad = \quad \frac{1}{2}\left(\frac{b_R}{1-b_R}\right)\tilde{P}_{t+1} \quad + \quad \frac{1}{2}\left(\frac{b_R}{1-b_R}\right)\tilde{P}_{t-1} \quad + \quad \left(\frac{\epsilon_3}{1-b_R}\right)\tilde{M}_t$$

where:

(42)

$$b_R = \frac{1 - \epsilon_3}{2}$$

$$\epsilon_3 = \left(\epsilon_1 + \frac{\alpha}{1-\alpha}\right)\frac{1}{\epsilon_2}$$

In deriving [41], the log of the stock of money is assumed to follow a random walk. For expositional purposes, we set the discount factor B = 1.

It is evident from [41]-[42] that price dynamics are determined by the composite parameter ε_3. Table 2 shows how the coefficients of the price and money supply terms in [41] vary with ε_3.

ε_3	b_R	price coefficient	money coefficient
0	1/2	1/2	0
1	0	0	1
2	-1/2	-1/6	4/3
4	-3/2	-3/10	8/5

Although we defer a more rigorous treatment of model dynamics to Appendix II (including the model with factor specificity), it is evident that the contract price becomes progressively less persistent, and also more responsive to monetary innovations, as ε_3 rises. The case $\varepsilon_3 = 1$ is an important special case, as the difference equation [41] reduces to:

(43)

$$\tilde{P}_t = \tilde{M}_t$$

implying immediate adjustment of the contract price to its long-run level (with the average price, and hence output, adjusting completely in the following period). As suggested by the last column of the table, larger values of ε_3 imply that the contract price must overshoot its long-run level. Figure 10 shows the response of the contract and average price for the case $\varepsilon_3 = 4$. The overshooting of the contract price is associated with an overshooting of the average price in the period following the shock, implying that real balances, and hence output, must contract.

It is evident from the hyperbolic form of the composite parameter ε_3 that there is a "large" and inversely-related set of values of the output elasticity of the real wage (ε_1) and income elasticity of money demand (ε_2) that give rise to identical price dynamics. Accordingly, even with a weak responsiveness of wages (hence marginal cost) to output, a low enough income elasticity of money demand can rationalize the sort of price-overshooting depicted in Figure 10. But given the evolution of prices, and hence real balances, the behavior of output is determined solely by the output elasticity of money demand parameter ε_2 (by [40]). This means that parameterizations that imply a given price response will have much more volatile output implications to the extent that the income elasticity of money demand is lower. The output IRF shown in Figure 11 for the case of a low income elasticity of money demand (of 0.15) is much more volatile than its counterpart with a much higher income elasticity (of 1), even though $\varepsilon_3 = 4$ in each case. Note that the output elasticity of the real wage in the former case is only 0.17. Thus, the model can account for a boom-bust cycle in output, and highly volatile output fluctuations, even if the output elasticity of marginal cost is quite low.[24]

This simple model's basic implications carry over to models with considerably more complicated wage/price dynamics. Thus, if the average price level is somewhat sticky in the short-run, output must rise sharply if the demand for real balances is not very sensitive to output. Moreover, unless prices are very sticky, the output rise tends to be reversed very quickly. This reflects that although the (average) price rises in the period(s) subsequent to the shock (as seen, e.g., in Figure 10), prices would have to rise by even more if producers expected to maintain output at close to its initial post-shock level (i.e., to allow producers to cover their comparatively higher marginal costs that would be associated with this higher

[24] Appendix II also discusses the version of the model with factor specificity. Factor specificity increases price/output persistence for any given output elasticity of the real wage and income elasticity of money demand. Nevertheless, even this variant of the model implies oscillatory output/price dynamics for a sufficiently low income elasticity of money demand.

output level). But such an outcome would violate the condition that nominal demand remain constant at its post-shock level. The satisfaction of both the optimality condition of producers and the money demand equation requires the more modest (average) price rise shown in Figure 10, and corresponding sharp contraction of output (by [40]).

This analysis is helpful in understanding the potential sensitivity of output IRFs to whether real balances are specified as proportional to consumption or income in models with capital accumulation (as was illustrated in Figure 9). The consumption-based money demand function effectively implies a very low income elasticity of money demand in the "low adjustment costs" parameterization, as consumption varies much less than output in response to a transitory shock. Thus, given sticky prices in the short-run, consumption must rise by enough to allow the demand for real balances to adjust to the higher nominal supply. This requires a large output spike. But output crashes subsequently, because producers would demand much larger prices increases than occur in equilibrium to keep output at close to its initial peak. In fact, the slow adjustment of consumption actually forces slower price adjustment than in the labor-only model, inducing an even more severe output crash.[25]

Our analysis suggests that output persistence can be strengthened (and output volatility correspondingly damped) by imposing a structure that makes consumption more responsive to income, thereby increasing the effective income elasticity of money demand. One way to accomplish this is to impose costs of adjusting the capital stock, as in the model of Section IV. In Figure 12, we consider the effects of allowing for costs of adjusting capital in the same model as outlined in Section IV, except with a consumption-based money demand specification. The figure indicates that when adjustment costs are

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[25] This simple model highlights why CKM are unable to account for output persistence: not only is marginal cost highly procyclical in their model, but the effective income elasticity of money demand is very low due to their consumption-based money demand specification (with a unitary elasticity). Moreover, their lack of success in various attempts to generate greater output/price persistence may be attributable to the low income elasticity "swamping" the effects of these modifications.

set high enough to deliver reasonable implications for the relative magnitude of investment and output (on the order of three), the model can in fact account for highly persistent output effects.[26] Although higher adjustment costs have a negative "partial effect" on output persistence to the extent that they damp the investment response, this effect is more than offset by the positive effect associated with increasing the effective elasticity of money demand.

Section VI. Conclusion

Recent research (CKM, 1996, Kiley, 1997) highlights the importance of factor price and quantity responses for deriving a contract multiplier. It suggests that standard assumptions about factor market-clearing imply too high a sensitivity of marginal cost to output to account for *endogenous* output persistence in response to monetary innovations.

The first contribution of this paper is to derive a wage-setting process in the spirit of Taylor (1980), by making very limited and transparent assumptions about the exogenous component of wage persistence in the model. We show that a Taylor-like wage-setting process derived from a household optimization problem can account for a high degree of nominal wage persistence, even though we make very standard assumptions about household preferences.

We then investigate whether smoothing labor costs via this mechanism can allow our sticky price/ wage model to account for a substantial contract multiplier. We find that our model can account for a strong contract multiplier even if the capital stock is fixed in aggregate supply. Allowing for capital accumulation can generate even somewhat stronger persistence. Factor specificity plays a significant role in strengthening the contract multiplier, particularly when capital is essentially fixed in aggregate supply

[26] Without adjustment costs, a rise in real interest rates tends to strongly damp the response of consumption relative to income (in response to a positive money innovation). However, as discussed in Kim (1996), adjustment costs reduce the demand for capital, and hence can allow real rates to fall in response to a positive monetary innovation. This markedly increases the response of consumption to income.

over business cycle frequencies.

Finally, we show that while smoothing marginal costs (at least to producers/household resetting prices) seems crucial to derive a contract multiplier, it is not sufficient. In particular, a high enough income elasticity of money demand also seems an important ingredient. This feature has important implications for how models with consumption-based money demand functions must be structured if they are to account for a contract multiplier.

References

Abraham, K.C., and J.C. Haltiwanger. "Real Wages and the Business Cycle." *Journal of Economic Literature* 33, 1215-1264 (1995).

Altonji, J. "Intertemporal Substitution in Labor Supply: Evidence from Micro Data." *Journal of Political Economy* 94 (1986).

Backus, David K. and Patrick J. Kehoe. "International Evidence on the Historical Properties of Business Cycles." *American Economic Review* 82, 864-888 (1992).

Backus, David K. and Patrick J. Kehoe, and Kydland, Finn. "International Real Business Cycles." *Journal of Political Economy* 4, 745-775 (1992).

Ball, Laurence. "Credible Disinflation with Staggered Price-Setting." *American Economic Review* 84, 282-289 (1994).

Ball, Laurence, and Romer, David. "Real Rigidities and the Nonneutrality of Money." *Review of Economic Studies* 57, 183-204 (1990).

Bils, Mark. "Real Wages over the Business Cycle: Evidence from Panel Data." *Journal of Political Economy* 93, 666-689 (1985).

Bils, Mark. "The cyclical behavior of marginal cost and price." *American Economic Review* 77, 838-57 (1987).

Blanchard, Olivier J., and Charles Kahn, "The Solution of Linear Difference Equations under Rational Expectations," *Econometrica* 48, 1305-1313 (1980).

Blanchard, Olivier J., and Kiyotaki, Nobuhiro. "Monopolistic Competition and the Effects of Aggregate Demand." *American Economic Review* 77, 647-666 (1987).

Calvo, Guillermo. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics* 12, 383-398 (1983).

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan. "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" Federal Reserve Bank of Minneapolis Staff Report #217 (September, 1996).

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan. "Monetary Shocks and Real Exchange Rates in Sticky Price Models of Interational Business Cycles." Federal Reserve Bank of Minneapolis Staff Report #223 (December, 1996).

Christiano, Lawrence J., Eichenbaum, Martin, and Evans, Charles L. "Sticky Price and Limited Participation Models of Money: A Comparison." *European Economic Review* 41 1201-1249 (1997).

Cho, Jang-Ok and Thomas F. Cooley. "The Business Cycle with Nominal Contracts." *Economic Theory* 6, 13-33 (1995).

Cho, Jang-Ok and Richard Rogerson. "Family Labor Supply and Aggregate Fluctuations." *Journal of Monetary Economics* 21, 233-245 (1988).

Den Haan, Wouter J. "The Comovements between Real Activity and Prices at Different Business Cycle Frequencies." Manuscript (April, 1996).

Dotsey, Michael, Robert G. King, and Alexander L. Wolman. "Menu Costs, Staggered Price-Setting, and Elastic Factor Supply." Manuscript (May, 1997).

Fischer, Stanley. "Long Term Contracts, Rational Expectations, and the Optimal Money Supply Rule." *Journal of Political Economy* 85, 191-205 (1977).

Fleischmann, Charles A. "The Causes of Business Cycles and the Cyclicality of Real Wages." Manuscript (December, 1994).

Ghez, G. and Gary Becker. *The Allocation of Time and Goods Over the Life Cycle.* New York: NBER (1975).

Kiley, Michael T. "Staggered Price Setting, Partial Adjustment, Real Rigidities, and Sunspots." Manuscript (March, 1997).

Kim, Jinill. "Monetary Policy in a Stochastic Equilibrium Model with Real and Nominal Rigidities." Manuscript (1996).

Kimball, Miles. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit, and Banking* 27, 1241-1277 (1995).

King, R.G. and M. Watson. "Money, Output, Prices, and Interest Rates." *Review of Economics and Statistics* 78, 35-53 (1996).

Kollmann, Robert. "The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation." Working Paper. International Monetary Fund (January, 1997).

Koenig, Evan F. "Aggregate Price Adjustment: The Fischerian Alternative." Manuscript (March, 1997).

Levin, Andrew. "The Macroeconomic Significance of Nominal Wage Contract Duration." Working Paper. University of California at San Diego (1990).

Mulligan, Casey. "Intertemporal Substitution: What Does the Evidence Say?" Manuscript (1995).

Romer, David. "The New Keynesian Synthesis." *Journal of Economic Perspectives* 7, 5-22 (1993).

Romer, David. *Advanced Macroeconomics.* New York: McGraw Hill (1996).

Rotemberg, J.J. "Prices, Output, and Hours: An Empirical Analysis Based on a Sticky Price Model." *Journal of Monetary Economics* 37, 505-533 (1996).

Rotemberg, J.J. and Michael Woodford. "Markups and the Business Cycle." *NBER Macroeconomics Annual*, ed. Olivier Blanchard and Stanley Fischer, MIT Press, 63-129 (1994).

Rotemberg, Julio and Michael Woodford. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy." Manuscript (April, 1997).

Rupert, Peter, Richard Rogerson, and Randall Wright. "On the Macroeconomic Implications of Models with Home Production." Manuscript (March, 1997).

Taylor, John B. "Aggregate Dynamics and Staggered Contracts." *Journal of Political Economy* 88, 1-23 (1980).

Woodford, Michael. "Control of the Public Debt: A Requirement for Price Stability?" NBER Working Paper no. 5684 (July, 1996).

Yun, Tack. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles." *Journal of Monetary Economics* 37, 345-370 (1994).

Appendix I. Model Solution

The pricing equation derived from the optimization problem of producer j (equation [9]) can be regarded as the pricing equation for all "cohort 1" producers that set prices during the current period t. This pricing equation may be rewritten as (replacing integrals over states of the world with conditional expectations operators where appropriate):

$$\textbf{(44)}\quad \mu\ P_t\ E_t\left(1 + Q_{t+1,t}\ \frac{y_{1t+1}}{y_{1t}} + Q_{t+2,t}\ \frac{y_{1t+2}}{y_{1t+1}} + Q_{t+3,t}\ \frac{y_{1t+3}}{y_{1t}}\right) = E_t\left(\sigma_{1t} + Q_{t+1,t}\ \sigma_{1t+1}\ \frac{y_{1t+1}}{y_{1t}} + Q_{t+2,t}\ \sigma_{1t+2}\ \frac{y_{1t+2}}{y_{1t+1}} + Q_{t+3,t}\ \sigma_{1t+3}\ \frac{y_{1t+3}}{y_{1t}}\right)$$

with the demand for the output of cohort 1 producers at each date over the life of the contract given by (from equation [3]):

$$\textbf{(45)}\qquad y_{1t+\tau} = \left(\frac{P_t}{P_{At+\tau}}\right)^{\frac{1}{(\mu-1)}} y_{t+\tau} \qquad \tau = 0,\ 1,\ 2,\ 3$$

where the nominal marginal cost terms are given in equations [10] or [11], for the mobile and fixed capital variants of the model, respectively, and the average price level (P_{At}) is defined in [24].[27]

Similarly, the wage equation of cohort 1 households that reset their wage during the current period is given by (after substituting the form of the utility function into the household's first order condition for wage determination in [23]):

$$\textbf{(46)}\quad \phi\ \frac{W_t}{P_{At}}\ E_t\left(1 + Q_{t+1,t}\ \frac{l_{1t+1}}{l_{1t}} + Q_{t+2,t}\ \frac{l_{1t+2}}{l_{1t+1}} + Q_{t+3,t}\ \frac{l_{1t+3}}{l_{1t}}\right) = E_t\left(\frac{y_t}{1-l_{1t}} + Q_{t+1,t}\ \frac{y_{t+1}}{1-l_{1t+1}}\ \frac{l_{1t+1}}{l_{1t}} + Q_{t+2,t}\ \frac{y_{t+2}}{1-l_{1t+2}}\ \frac{l_{1t+2}}{l_{1t+1}} + Q_{t+3,t}\ \frac{y_{t+3}}{1-l_{1t+3}}\frac{l_{1t+3}}{l_{1t}}\right)$$

where the demand function facing cohort 1 households at each date over the life of the contract is given by:

---

[27] The discount factor term is given by the first order condition [21] as:

$$Q_{t+\tau,t} = \beta^{\tau}\left(\frac{y_t\ P_{At}}{y_{t+\tau}\ P_{At+\tau}}\right)$$

(47) $$l_{1t+\tau} = \left(\frac{W_t}{W_{At+\tau}}\right)^{\frac{1}{(\phi-1)}} l_{t+\tau} \qquad \tau = 0, 1, 2, 3$$

Note that aggregate output has been substituted for the consumption of cohort 1 households in equation [46], reflecting that the consumption of each cohort equals per capita aggregate consumption, and that total consumption equals output. The average wage (W_{At}) is defined in [24]. Finally, the equation determining the evolution of the household's real money balances may be written:

(48) $$\frac{\psi_3}{\psi_1}\left(\frac{y_t\, P_{At}}{M_t}\right) = 1 - E_t\, Q_{t+1,t}$$

In order to represent the three equations [44], [46], and [48] as functions only of the endogenous state variables P_t, W_t, and y_t (and the money stock M_t) , we must solve for the controls h_t , l_t, and P_{kt} (in the mobile capital case). It is possible to solve for P_{Kt} in terms of aggregate labor hours (h_t or l_t), as all producers equate their labor to capital ratio to the ratio of the rental price of capital to the wage:

(49) $$P_{Kt} = \frac{\alpha}{1-\alpha}\, W_t\, \frac{h_t}{K}$$

The control l_t can be eliminated immediately, since the total effective hours used by firms in production (h_t) must equal the quantity of hours produced by the producer of effective labor services (l_t). Finally, the final control h_t can be solved for as a function of the state variables using the production function for final goods producers (equation [1], which links total output to the output of each intermediate goods producer), the identity that total effective hours h_t equals the sum of hours worked by each of four cohorts of firms, and the form of the firm level production functions.

Appendix II. Dynamics of the Model in Section V

The second-order difference equation in the contract price that obtains in the general case of the model (given by [37]-[40]) that allows for factor specificity can be written as:

(50) $$\nu_1\, \tilde{P}_t = \left(\frac{\nu_2}{\beta}\right) \tilde{P}_{t-1} + \nu_2\, \tilde{P}_{t+1} + \frac{\epsilon_3(1+\beta)}{\beta}\, \tilde{M}_t$$

where:

(51) $$\nu_1 = \frac{1+\beta}{\beta}\left(1 - b_R + \frac{1}{2}\epsilon_4\right) > 0$$

(52) $$\nu_2 = b_R + \frac{1}{2}\,\epsilon_4$$

(53) $$b_R = \frac{1}{2}\,(1 - \epsilon_3)$$

(54) $$\epsilon_3 = (\epsilon_1 + \frac{\alpha}{1-\alpha})\,\frac{1}{\epsilon_2}$$

(55) $$\epsilon_4 = -\gamma\left(\frac{1}{\mu-1}\right)\left(\frac{\alpha}{1-\alpha}\right) > 0 \quad \textit{if } \gamma = 1 \quad (\textit{factor specificity})$$
$$= 0 \quad \textit{if } \gamma = 0 \quad (\textit{capital mobility})$$

where the structural parameters appearing in [50]-[55] are defined:

ε_1 = output elasticity of the real wage
ε_2 = income elasticity of money demand
μ = parameter determining substitutibility between goods
α = share of fixed factor

The solution to this difference equation can be written in the form:

(56) $$\tilde{P}_t = \lambda_2\,\tilde{P}_{t-1} + \lambda_M\,\tilde{M}_t$$

In this section, we relate the (real) root λ_2 in [56] to the coefficient on the price term ν_2 in [50]. Noting that $\nu_1 > 0$, we show that:

i) the sign of λ_2 is the same as the sign of ν_2, so that the model is consistent with price inertia if $\nu_2 > 0$, and oscillatory dynamics if $\nu_2 < 0$.

ii) the magnitude of the coefficient on λ_2 increases monotonically in ν_2, so that persistence increases as ν_2 rises.

We then highlight the dependence of the key parameter ν_2 on underlying structural parameters.

We begin by rearranging [50] slightly to put it in the form:

(57) $$\tilde{P}_{t+1} - \frac{\nu_1}{\nu_2}\tilde{P}_t + \frac{1}{\beta}\tilde{P}_{t-1} = -\frac{\epsilon_3\ (1+\beta)}{\nu_2\beta}\tilde{M}_t$$

or:

(58) $$\tilde{P}_{t+1}\ (1 - s_1\ L + \frac{1}{\beta}L^2) = \tilde{Z}_t = \frac{-\epsilon_3\ (1+\beta)}{\nu_2\beta}\tilde{M}_t$$

where:

(59) $$s_1 = \frac{\nu_1}{\nu_2} = \left(\frac{1+\beta}{\beta}\right)\left(\frac{1 - b_R + \frac{1}{2}\epsilon_4}{b_R + \frac{1}{2}\epsilon_4}\right)$$

The characteristic polynomial in [58] can be factorized as:

(60) $$1 - s_1\ L + \frac{1}{\beta}L^2 = (1 - \lambda_1 L)\ (1 - \lambda_2 L) = 1 - (\lambda_1+\lambda_2)L + \lambda_1\lambda_2 L^2$$

Hence, the roots λ_1 and λ_2 are determined by the pair of equations:

(61) $$\lambda_1 + \lambda_2 = s_1$$

(62) $$\lambda_1\ \lambda_2 = \frac{1}{\beta}$$

The solution for the roots is given by the quadratic formula:

(63) $$\lambda_1,\ \lambda_2 = \frac{s_1 \pm \sqrt{s_1^2 - 4/\beta}}{2}$$

To demonstrate points i-ii) above, we consider the following cases:

I. ν_2 **> 0** (1/2 >= $b_R > -1/2\varepsilon_4$)

Since $s_1 > 0$ (noting its definition in [59], and recalling $\nu_1 > 0$), both roots are positive. This establishes that when $\nu_2 > 0$, $\lambda_2 > 0$. Noting $b_R <= 1/2$, s_1 reaches a minimum of (1+B)/B at $b_R = 1/2$. At this value of b_R, it is evident that the roots solving [61]-[62] are given by λ_1 = 1/B and λ_2 = 1 (with λ_1 chosen to be the larger root). As ν_2 declines due to a fall in either of its components, s_1 rises monotonically, with $s_1 \rightarrow \infty$ as $b_R \rightarrow -1/2\varepsilon_4$ from above. Correspondingly, [63] indicates that the smaller root λ_2 declines from its maximum of unity to zero (while the larger root λ_1 rises from its minimum of 1/B toward infinity). Thus, the stable root λ_2 declines monotonically in ν_2.

II. ν_2 **= 0** ($b_R = -1/2\varepsilon_4$)

For this case, equation [50] indicates that the difference equation reduces to a static equation in the current period contract price, so that $\lambda_2 = 0$ in [56].

III. ν_2 **< 0** ($b_R < -1/2\varepsilon_4$)

Since $s_1 < 0$ (recalling $\nu_1 > 0$), both roots are negative. This establishes that when $\nu_2 < 0$, $\lambda_2 < 0$. This case is essentially the "mirror image" of the $\nu_2 > 0$ case. As $b_R \rightarrow -\infty$ (as does ν_2), s_1 reaches a minimum in absolute value of -(1+B)/B. For this case, the roots solving [61]-[62] are given by λ_1 = -1/B and λ_2 = -1 (with λ_1 chosen to be the larger root in absolute value) -- implying highly oscillatory dynamics. As ν_2 rises toward zero (i.e., declines in absolute value) as $b_R \rightarrow -1/2\varepsilon_4$ from below, $s_1 \rightarrow -\infty$ monotonically. This implies (by [63]) that the smaller root in absolute value λ_2 also rises monotonically from its minimum of -1 toward zero (while the unstable root $\lambda_1 \rightarrow -\infty$). Thus, oscillations become progressively damped as ν_2 rises.

Finally, there are two interesting points to make about the form of the composite parameter ν_2 given in [52]. First, it is evident that factor specificity increases price (and hence output) persistence for any given values of the output elasticity of the real wage and income elasticity of money demand (given that $\varepsilon_4 > 0$, factor specificity increases ν_2 for any particular value of b_R). Factor specificity plays a more important role in enhancing persistence, to the extent that goods are close substitutes (since a high value of μ raises ε_4 by [55]). Second, given the additive form in which the factor specificity term ε_4 enters into ν_2, it is evident that it remains possible to set the income elasticity of money demand low enough to cause ν_2 to be negative (so that the effects of a low income elasticity of money demand may "swamp" the effects of factor specificity, as noted in our interpretation of the CKM results in the footnote on page 30).

Figure 1. Labor-Market Clearing Model w/o Factor Specificity

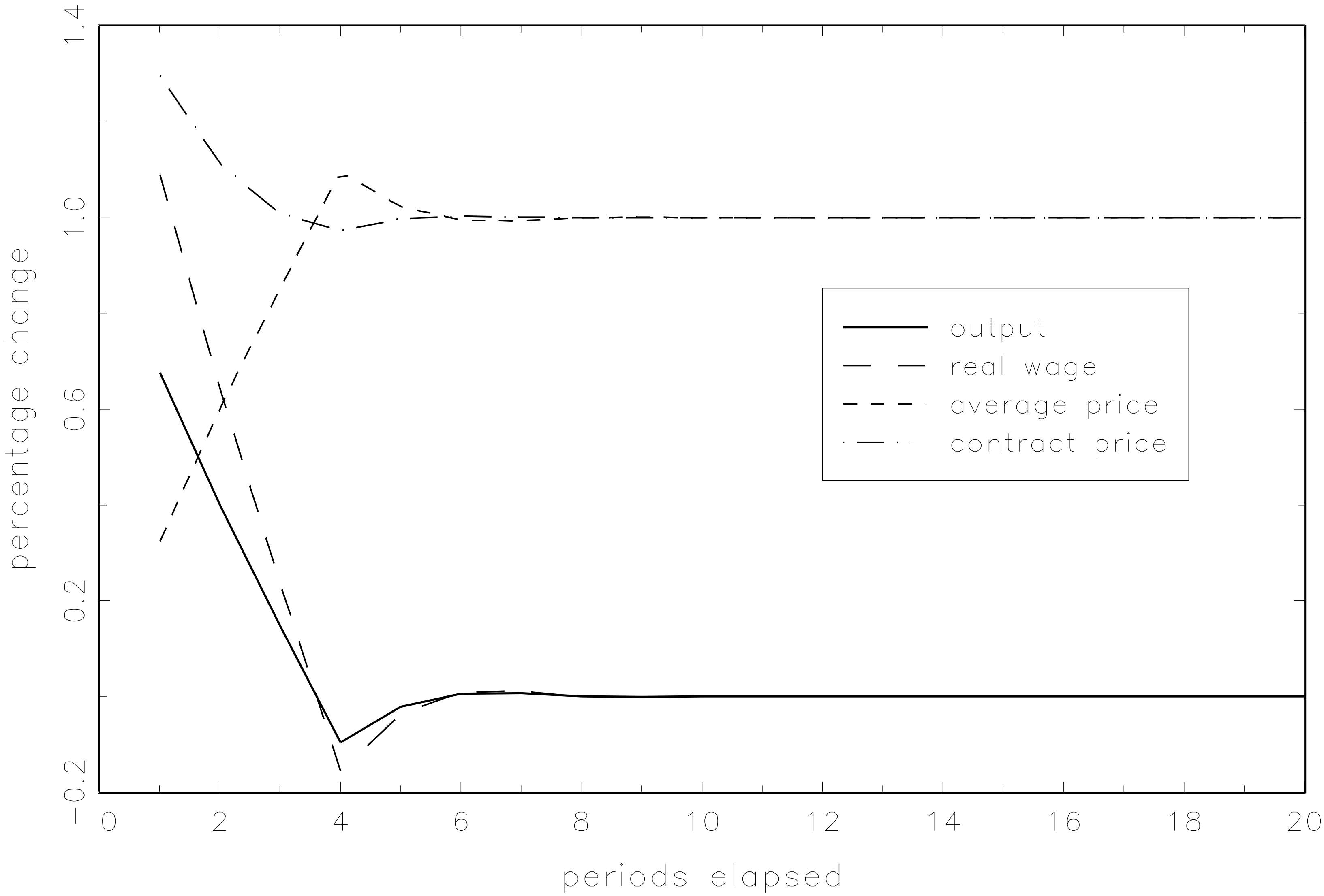

Figure 2. Sticky Wage Model w/o Factor Specificity (phi = .9)

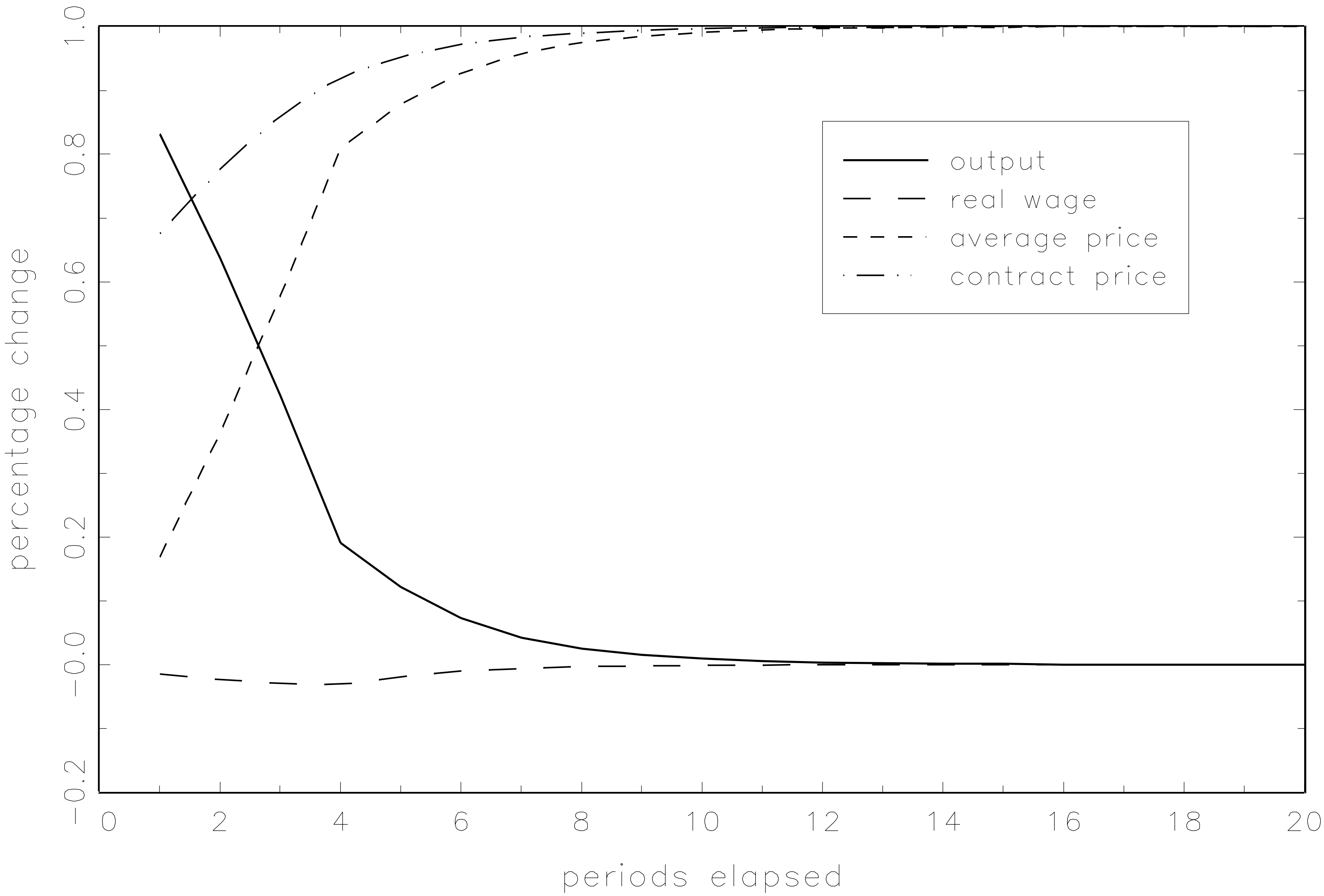

Figure 3. Sticky Wage Model with Factor Specificity (phi = .9)

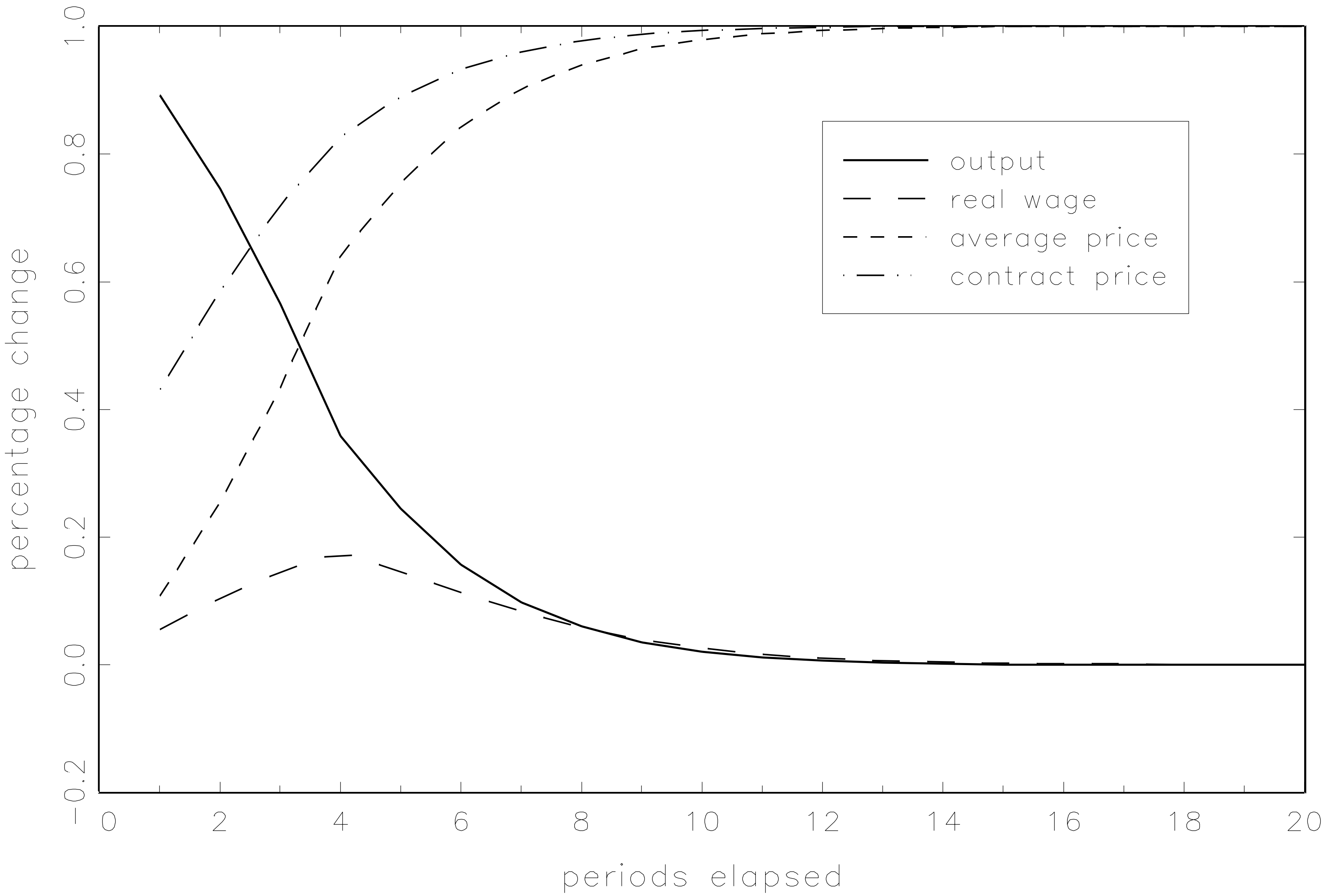

Figure 4. Sticky Wage Model with Factor Specificity (phi = .97)

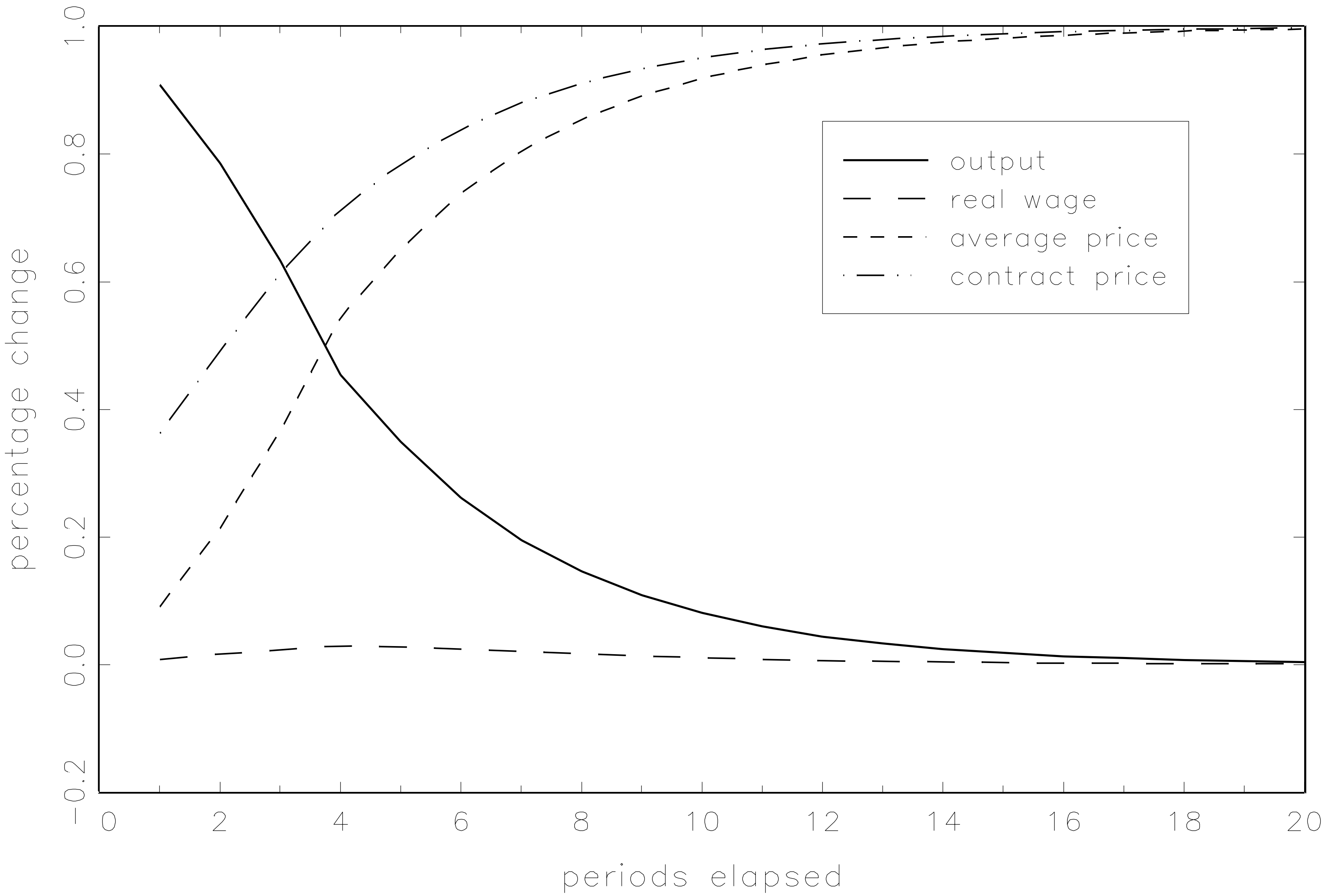

Figure 5. Sticky Wage Model with Factor Specificity (phi = .97)

IRF of Price Inflation

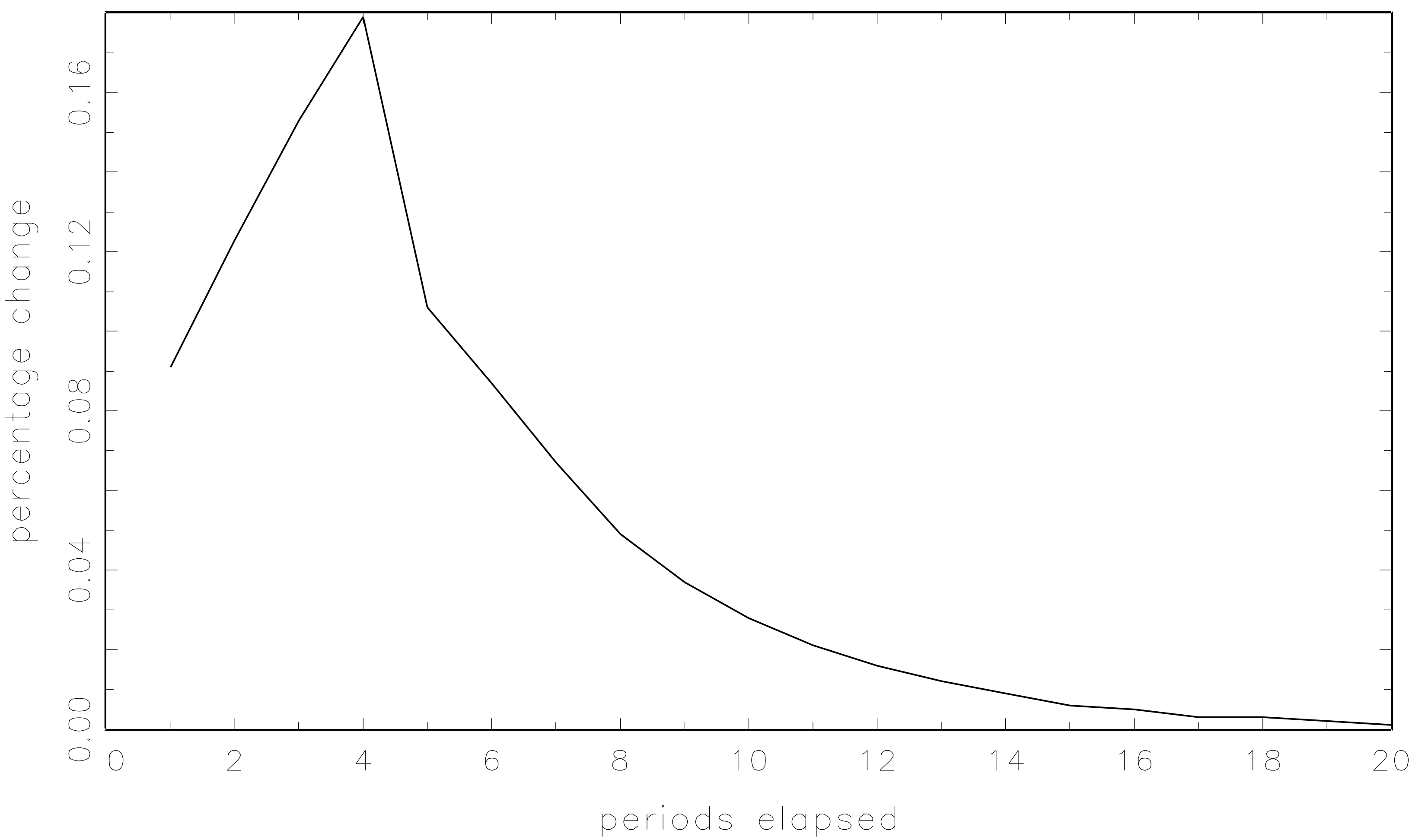

Figure 6. SW Model with Capital Accumulation: Output IRFs

(money demand function depends on income)

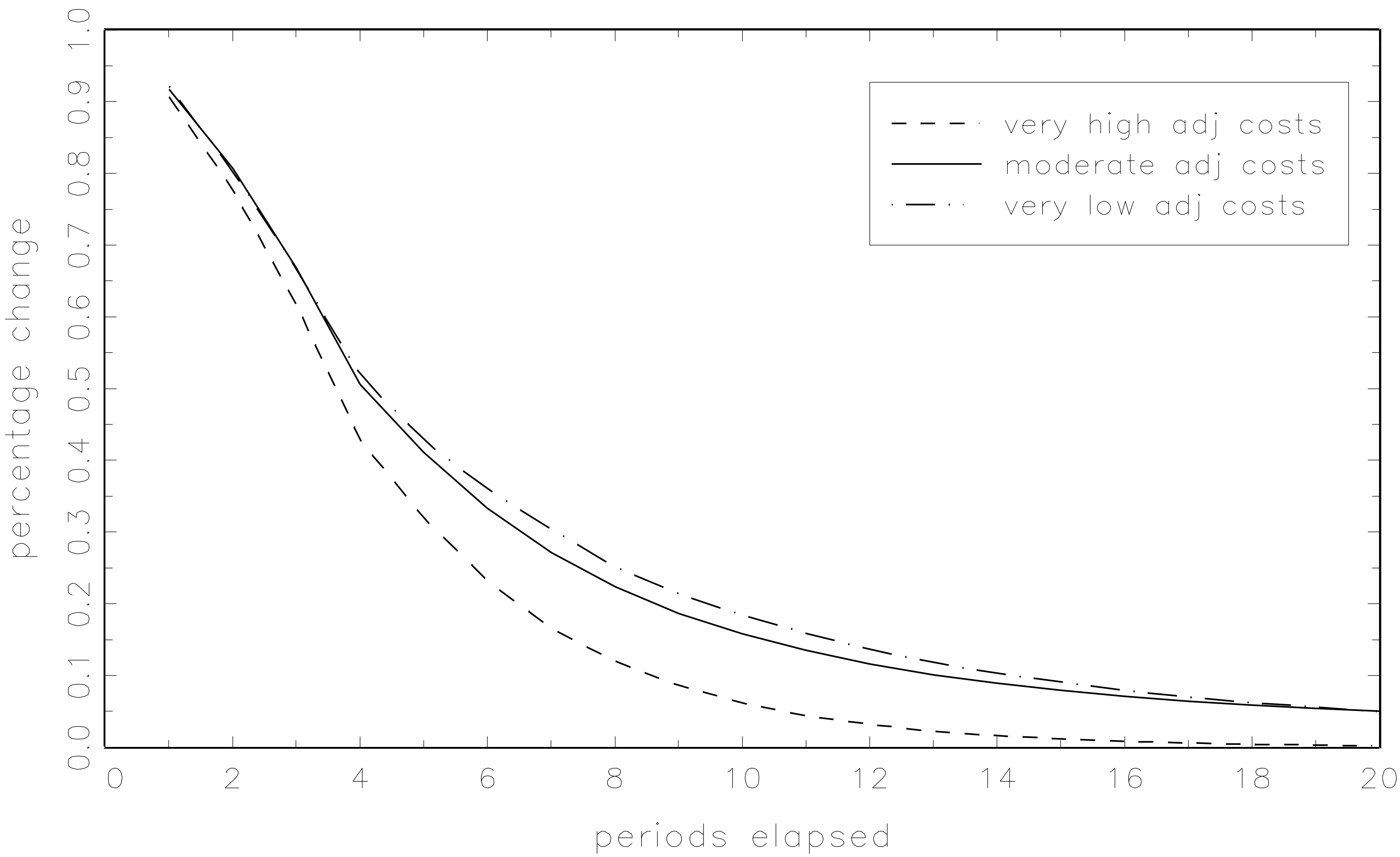

Figure 7. SW Model with Capital Accumulation: Output IRFs (phi=.97)

(money demand function depends on income, moderate adj costs)

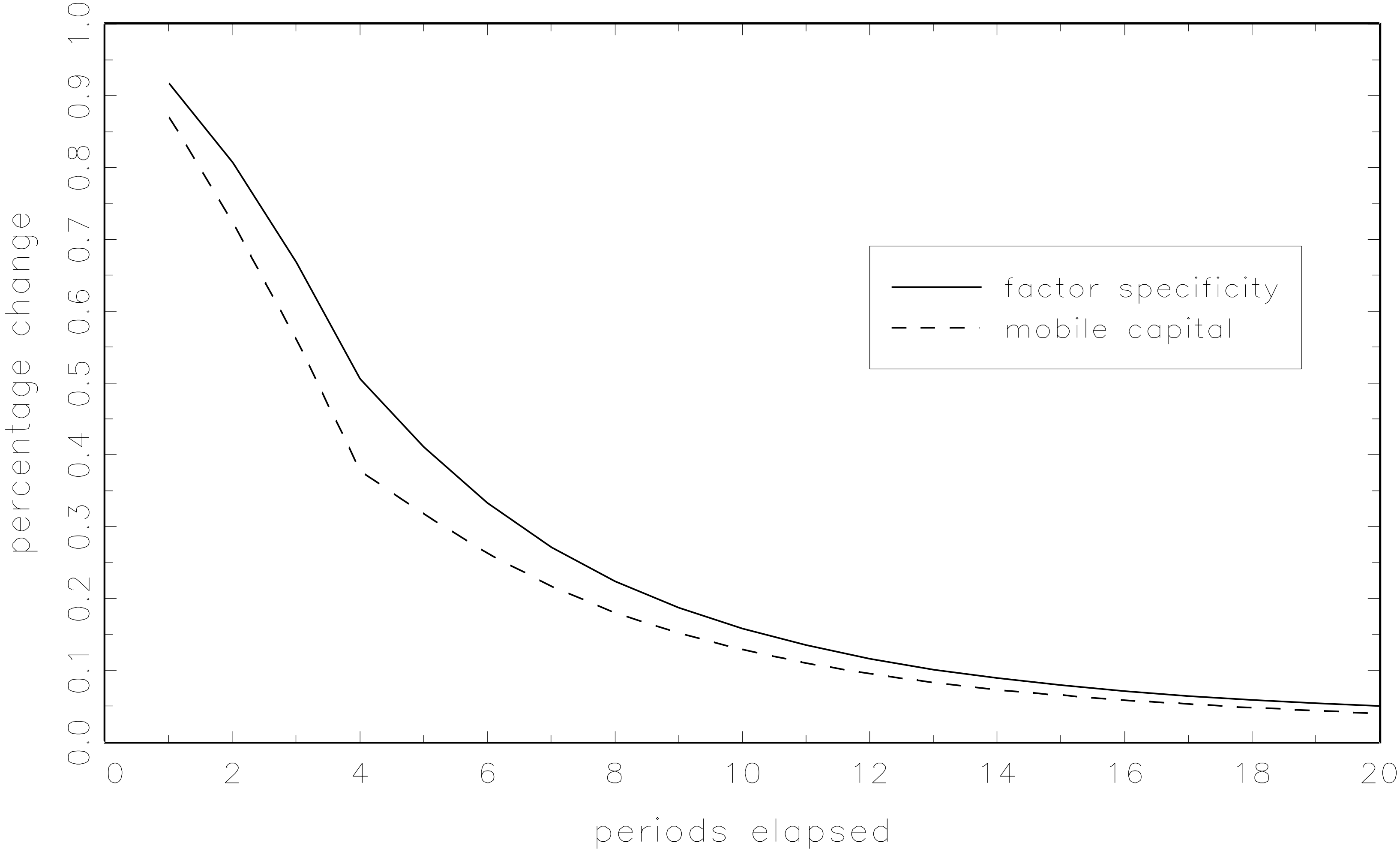

Figure 8. SW Model with Capital Accumulation: Output IRFs

(for different values of phi, moderate adj costs)

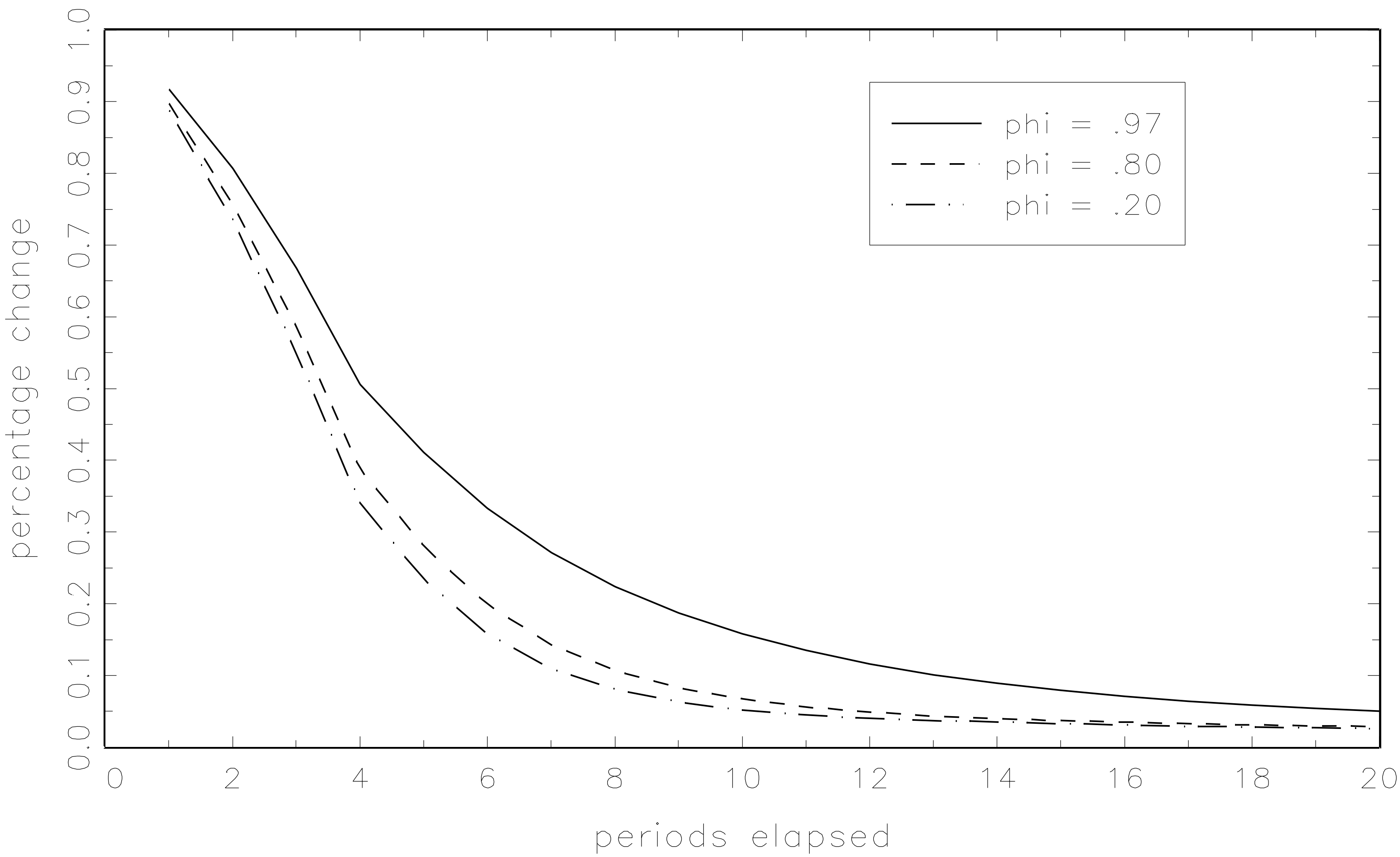

Figure 9. SW Model with Capital Accumulation: Output IRFs (phi=.97)

(comparison of different money demand functions, low adj costs)

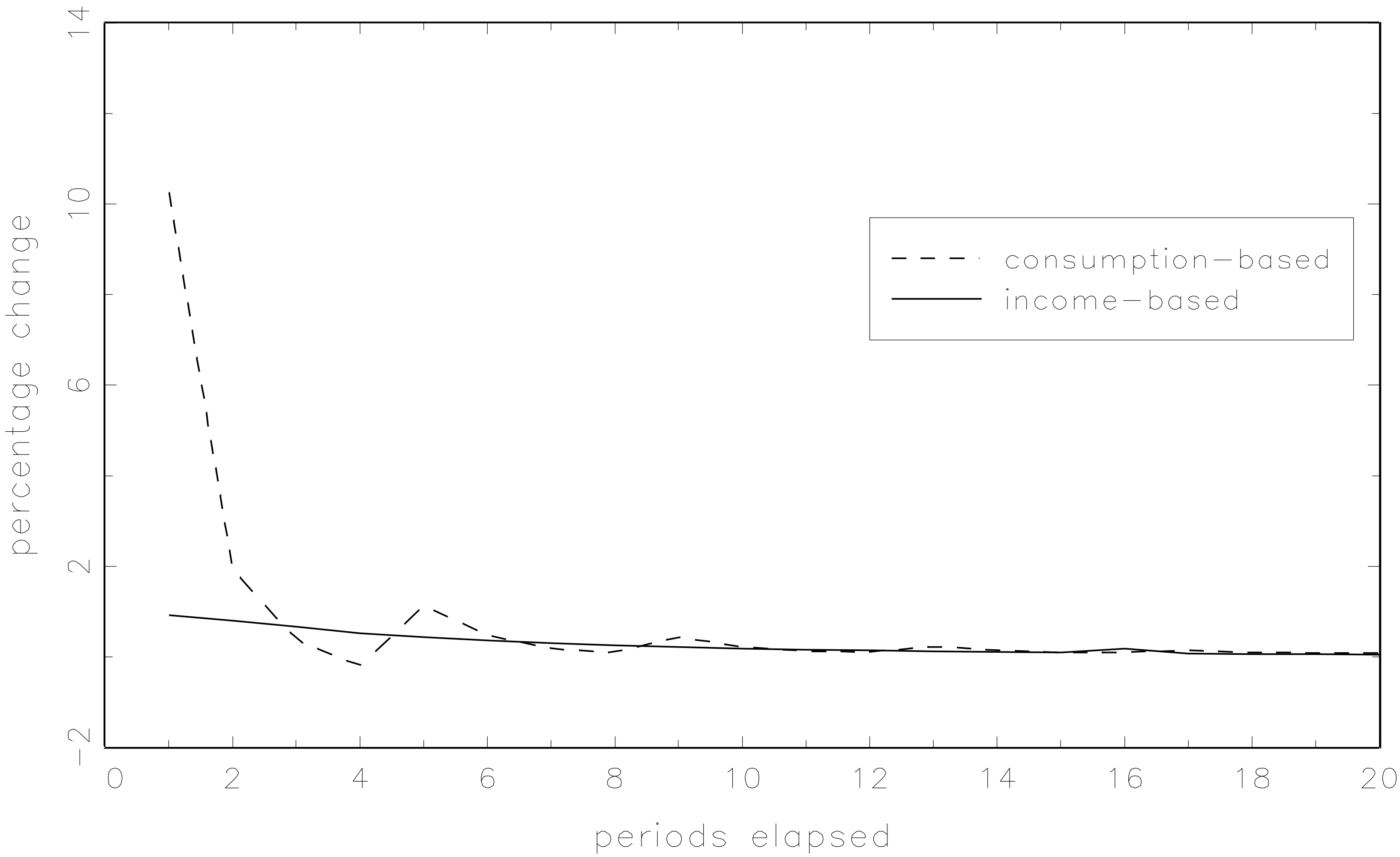

Figure 10. Analytic Model w/o Factor Specificity: Price IRFs

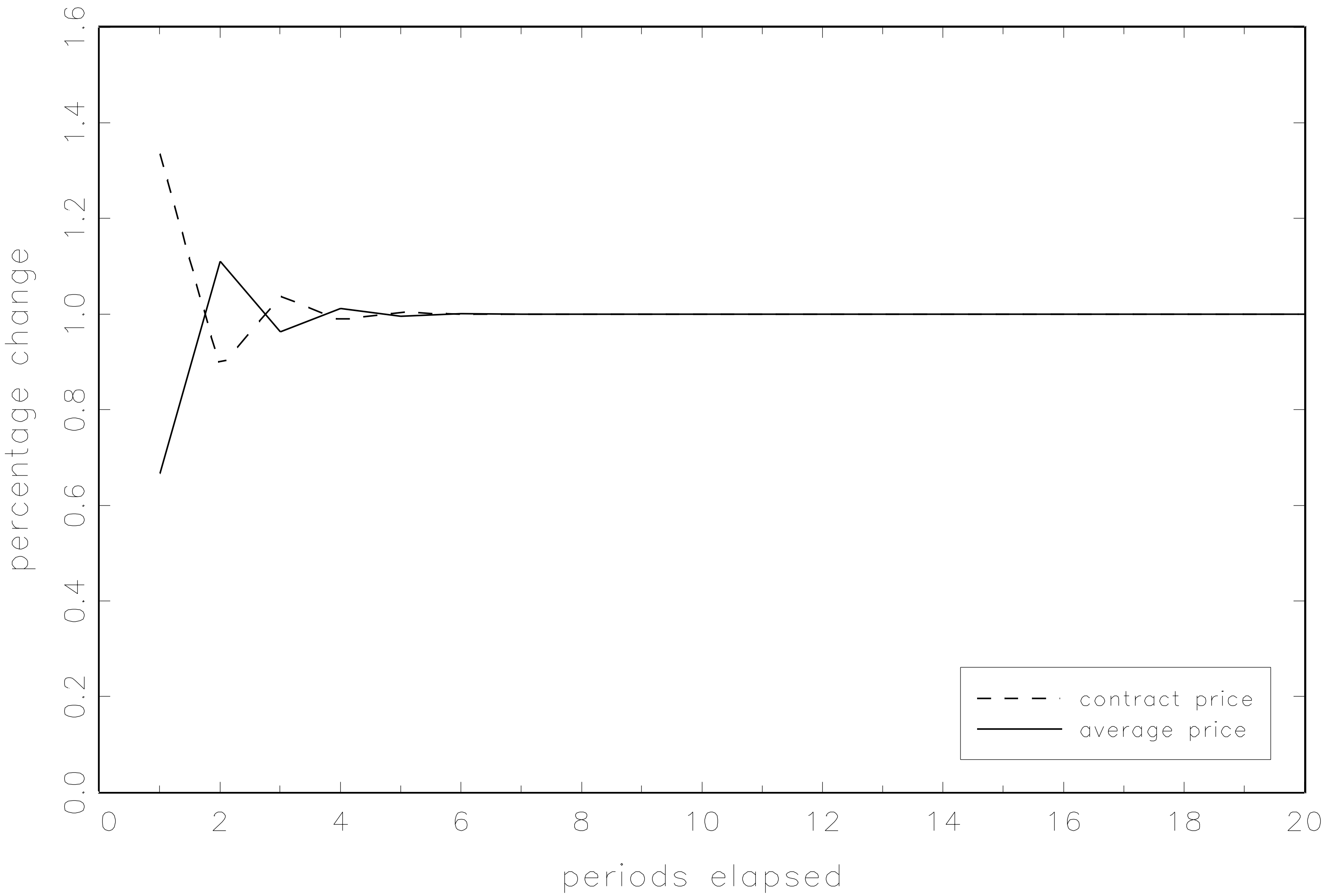

Figure 11. Analytic Model w/o Factor Specificity: Output IRFs

(comparison of different income elasticities of money demand)

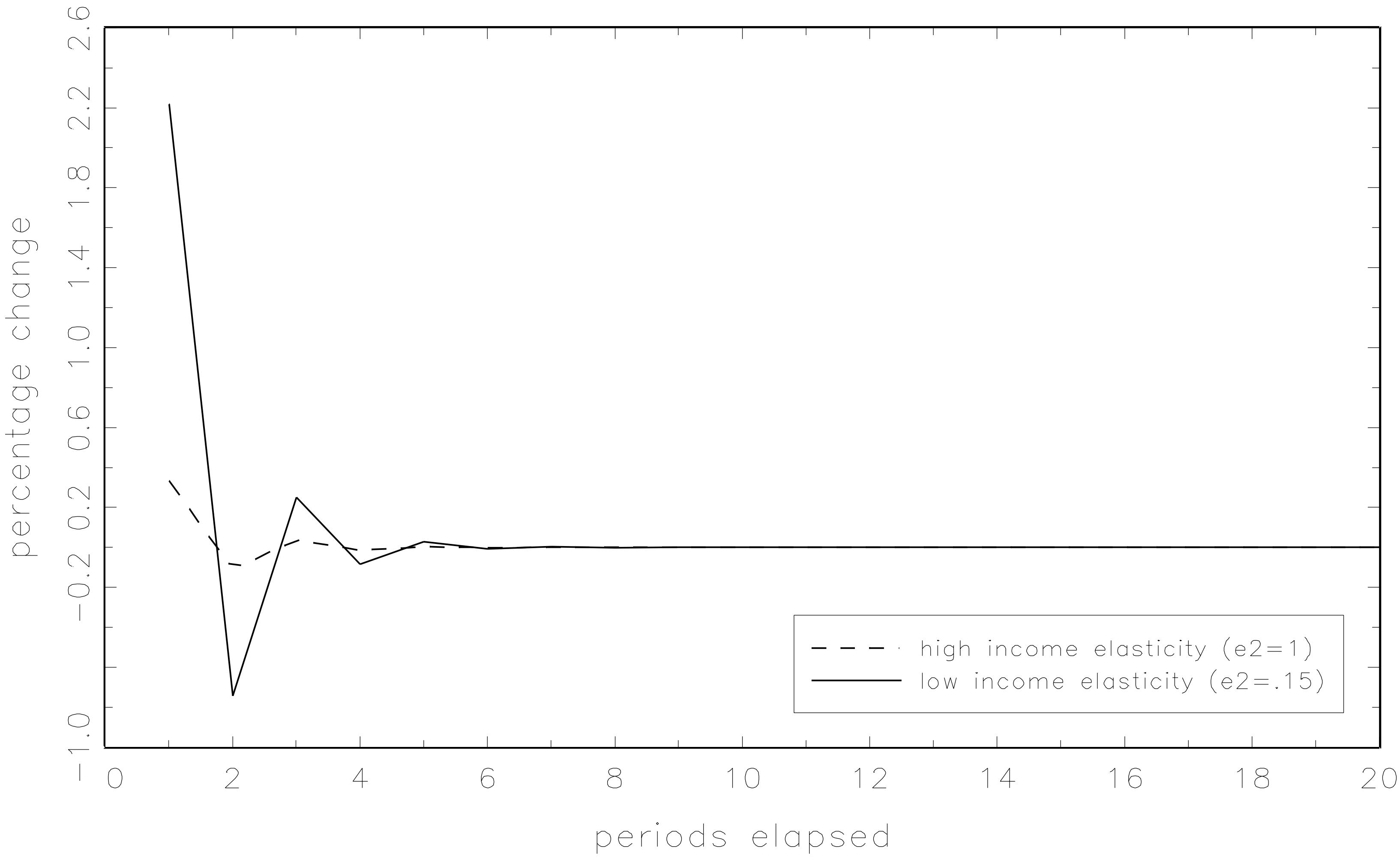

Figure 12. SW Model with Capital Accumulation: Output IRFs (phi=.97)

(comparison of different money demand functions, moderate adj costs)