Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 432

June 1992

THE LIQUIDITY PREMIUM IN AVERAGE INTEREST RATES

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ABSTRACT

This paper studies recent models of the liquidity effect of money on interest rates to determine if a systematic relationship between liquidity shocks and the economy could affect the average real interest rate.

JEL Classifications: C60, D50, E40, E50.

The Liquidity Premium in Average Interest Rates

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1. Introduction

To study the relationship between money and interest rates, the empirical regularities documented by Cagan (1966), Cagan and Gandolfi (1969), and others, and recently by Christiano and Eichenbaum (1991) and Strongin (1991),[2] point to the necessity of using a model that can generate a negative contemporaneous correlation between these two variables. Here we build on the rudiments of such a model that have been developed by Lucas (1990) and Fuerst (1992),[3] in which a liquidity effect of money on interest rates stems from imposing separate cash-in-advance constraints in the goods and asset markets and delaying the flow of money between these markets. This flow of money thus cannot immediately respond to a variety of shocks in each market, and as a result monetary shocks in the asset market lead to a negative contemporaneous correlation between open-market purchases of securities and one-period interest rates. In this setting the decision to allocate money between the goods and asset markets is made before interest rates are known; the risk that this creates in allocating money to purchase a bond thus opens up the possibility of a systematic premium in one-period interest rates that is due to the additional uncertainty created by liquidity shocks. In studying

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[2]See also Leeper and Gordon (1992).

[3]These papers build on the earlier work of Grossman and Weiss (1983) and Rotemberg (1984).

his model Lucas focused on a simple stochastic structure in which output was held constant, and consequently he did not find such a premium. By allowing monetary shocks that induce a negative correlation between output and interest rates, we find that short-term interest rates are systematically lower than forward interest rates. This model thus may help explain why short-term interest rates are low relative to the return on other assets, which has been difficult to explain with models that have Arrow-Debreu markets.

This paper is organized as follows. First, we describe the model for a rather general monetary policy conducted through open-market operations which substitute bonds of various maturities for money. Then, we prove a Modigliani-Miller-Ricardian Equivalence Theorem to justify focusing the remainder of this paper on a monetary policy which relies only on one-period bonds and lump-sum monetary transfers. We then discuss how we solve this model. In the final section we solve the model for various values of the parameters and report how average interest rates differ from their Fisherian fundamentals.

2. The Model

The model we study essentially consists of Lucas' (1990) economy in which the endowment varies stochastically over time. Each household in the economy is identical and has preferences over stochastic consumption streams $\{c_t\}$ that are given by

$$E\{\sum_{t=0}^{\infty} \beta^t u(c_t)\}.$$

The single-period utility function u is continuously differentiable, strictly increasing and strictly concave. A representative household begins period t with a portfolio of money H_t and pure-discount government bonds of various maturities, $A_{t\tau}$, $\tau \geq 1$. In this economy there are two markets, a goods market and an asset market, and each household consists of three members who have particular roles in these markets. One member of each household, the seller, proceeds to the goods market and receives an endowment ξ_t of perishable goods which he sells in exchange for money at the price P_t. We assume that no household can consume any part of its own endowment. Another member, the buyer, takes a portion $H_t - Z_t$ of the household's money balances to the goods market and purchases consumption goods from other households. The third member, the portfolio manager, takes the remaining money Z_t and bonds $A_{t\tau}$ to the asset market and rebalances the portfolio by using some of these assets to purchase bonds $W_{t\tau}$ at prices $q_{t\tau}$. Purchases of τ-period bonds at time t are redeemed for money at the beginning of the $(t + \tau)^{th}$ period. The members of the households join at the end of the period to consume the purchased goods, consolidate the assets, and share information. At the beginning of the next period the representative household receives a lump-sum monetary transfer of X_{t+1}. We assume that the price of goods and the size of the endowment are known at the beginning of the period, but changes in the supply of government securities and the prices of bonds are known only after the household divides its money between the goods and asset markets. We also impose that there is no transfer of information or assets between the goods and asset markets within a period.

The government in this economy begins each period with a stock of money M_t and bonds $B_{t\tau}$, $\tau \geq 1$, in the hands of households. In the asset market the government issues an additional supply of bonds $D_{t\tau}$ at prices $q_{t\tau}$ in exchange for money and bonds that households brought into the asset market. At the beginning of the next period the government issues a lump-sum monetary transfer of X_{t+1}. Denote the new issue of bonds relative to the supply of money as $d_{t\tau} = D_{t\tau}/M_t$, and denote the lump-sum monetary transfer relative to the supply of money as $x_{t+1} = X_{t+1}/M_t$. We assume that (ξ_t, d_{t-1}, x_t) follows a first-order finite-state Markov process with transition matrix π in which ξ is bounded away from 0.

With the assumption of a Markov process and time-separable utility it makes sense to seek a stationary equilibrium in which all nominal variables are divided by the stock of money. We follow the convention of denoting nominal variables divided by the stock of money with a corresponding lower case letter, and then denoting variables that are known at the beginning of a period without a prime and denoting variables that are first known in the asset market or in the next period with a prime. With this convention, information at the beginning of a period is summarized by $s = (\xi, d, x, b)$, where ξ is this period's endowment of goods, $d = (d_\tau)$ is the additional supply of bonds relative to the stock of money that was issued in last period's asset market, x is the beginning of period's monetary transfer relative to last period's stock of money, and $b = (b_\tau)$ is the outstanding stock of government debt relative to the current stock of money. As this also represents information available in the goods market, we seek an equilibrium normalized price of goods that can be represented as $p(s)$. Information in the asset market includes d', so we seek an equilibrium price of bonds $q = (q_\tau)$ that can be represented as $q(s,d')$. The evolution of the exogenous state variables (ξ, d, x) is given by π. If we denote the next period's stock of money relative to this period's by g', the evolutions of g' and the endogenous state variable b'_τ are then according to

(2.1) $$g' = 1 - \sum_\tau q_\tau(s,d')d'_\tau + b_1 + d'_1 + x',$$

(2.2) $$b'_\tau = (b_{\tau+1} + d'_{\tau+1})/g', \quad \tau \geq 1.$$

Denote the representative household's initial stock of money and bonds relative to the aggregate stock of money by h and a respectively. At the beginning of the period the household chooses to bring $h - z$ of the initial money to the goods market and z to the asset market. Since households can carry cash into next period through the asset market, and while in the asset market they have the option of purchasing interest-bearing bonds, clearly households will only carry enough cash into the goods market to finance their purchases: $p(s)c = h - z$. Purchases of bonds w_τ in the asset market are subject to the finance constraint $\Sigma q_\tau w_\tau \leq z$, where households only hold excess cash in the asset market if one-period interest rates are zero. Given price functions p and q, the household's problem can be solved via the dynamic programming problem

$$v(h,a,s) = \max_{z,c}\{u(c) + \beta E_s[\max_{w}\{E_{sd'}[v(h',a',s')]\}]\}$$

subject to

$$p(s)c = h - z,$$

$$\Sigma q_\tau(s,d')w_\tau \leq z,$$

$$h' = (z - \Sigma q_\tau(s,d')w_\tau + a_1 + w_1 + p(s)\xi + x')/g',$$

$$a'_\tau = (a_{\tau+1} + w_{\tau+1})/g', \quad \tau \geq 1.$$

In the above expression, subscripts of the expectation operator E denote conditioning information: E_s is the expectation conditional on information available at the beginning of the period (that is, knowing the state s) while $E_{sd'}$ is the expectation given the information (s,d') available in the asset market. For given functions p and q, the proof of the existence of a solution to this problem is standard (see, e.g., Stokey and Lucas with Prescott, 1989).

An equilibrium consists of goods prices $p(s) > 0$, bond prices $0 < q(s,d') \leq 1$, a cash allocation function $\hat{z}(h,a,s)$, and bond purchase function $w_\tau(h,a,s,d')$ such that: (i) $\hat{z}$ and w are the optimal policy functions for the household's dynamic programming problem, and (ii) at the equilibrium values $h = 1$ and $a = b$ we satisfy the feasibility condition $0 < \hat{z}(1,b,s) < 1$ and the market clearing conditions $w(1,b,s,d') = d'$ and $(h - \hat{z}(1,b,s))/p(s) = \xi$.

The first-order condition corresponding to the optimal choice of w_τ is

$$(2.3) \qquad \beta E_{sd'}[v^\tau(h',a',s')/g'] = q_\tau(s,d')(\hat{\lambda}(h,a,s,d') + \beta E_{sd'}[v^1(h',a',s')/g']), \; \tau \geq 1.$$

The function $\hat{\lambda}$ is the Kuhn-Tucker multiplier corresponding to the asset market finance constraint. The function v^{τ} is the partial derivative of v with respect to the τ^{th} component of its argument: v^1 is the partial derivative with respect to money holding and v^{τ}, $\tau > 1$, is the partial derivative with respect to holdings of $(\tau-1)$-period bonds. The multiplier $\hat{\lambda}$ determines whether or not the asset market finance constraint is binding:

(2.4) $$\sum_{\tau} q_{\tau}(s,d')d'_{\tau} \leq \hat{z}(h,a,s) \text{ with equality if } \hat{\lambda}(h,a,s,d') > 0.$$

The first-order condition for the optimal cash-allocation function $\hat{z}$ is

(2.5) $$u'((h-\hat{z}(h,a,s))/p(s))/p(s) = E_s[\hat{\lambda}(h,a,s,d') + \beta v^1(h',a',s')/g'].$$

The envelope conditions with respect to holdings of money and bonds are

(2.6) $$v^1(h,a,s) = u'((h-\hat{z}(h,a,s))/p(s))/p(s),$$

$$v^{\tau}(h,a,s) = \beta E_s[v^{\tau-1}(h',a',s')/g'], \ \tau > 1.$$

Define $z(s) = \hat{z}(1,b,s)$, $\lambda(s,d') = \hat{\lambda}(1,b,s,d')$, and $\phi_{\tau}(s) = v^{\tau}(1,b,s)$. With these definitions, we have

(2.7) $$\phi_1(s) = \xi u'(\xi)/(1-z(s)); \ \phi_{\tau}(s) = \beta E_s[\phi_{\tau-1}(s')/g'], \ \tau > 1.$$

The first-order condition (2.3) in the asset market can be written as

(2.8) $$\beta E_{sd'}[\phi_{\tau}(s')/g'] = q_{\tau}(s,d')\{\lambda(s,d') + \beta E_{sd'}[\phi_1(s')/g']\}, \ \tau \geq 1.$$

Multiply eq. (2.8) by d_τ and sum over τ, and then combine this equation with eq. (2.4) to obtain

$$(2.9)\quad \lambda(s,d') + \beta E_{sd'}[\phi_1(s')/g'] = \max\{\beta E_{sd'}[\phi_1(s')/g'],\ \beta\sum_\kappa d'_\kappa E_{sd'}[\phi_\kappa(s')/g']/z(s)\}.$$

Substituting back into eq. (2.8) yields

$$(2.10)\quad \beta E_{sd'}[\phi_\tau(s')/g'] = q_\tau(s,d')\max\{\beta E_{sd'}[\phi_1(s')/g'],\ \beta\sum_\kappa d'_\kappa E_{sd'}[\phi_\kappa(s')/g']/z(s)\},\ \tau \geq 1.$$

Using (2.9), the first-order condition (2.5) can be written as

$$(2.11)\qquad \xi u'(\xi)/(1-z(s)) = E_s[\max\{\beta E_{sd'}[\phi_1(s')/g'],\ \beta\sum_\kappa d'_\kappa E_{sd'}[\phi_\kappa(s')/g']/z(s)\}].$$

Equations (2.7), (2.10), and (2.11), along with eqs. (2.1)-(2.2), determine the equilibrium functions ϕ, q, and z, along with the equilibrium evolution of g and b.

3. A Modigliani-Miller-Ricardian Equivalence Theorem

As Lucas (1990) has noted, aside from the separation of the goods and asset markets this is a Modigliani-Miller-Ricardian world, and thus we should expect many monetary policies (as reflected in the behavior of open-market operations and transfers) to result in the same equilibrium. In particular, in the asset market the only constraint households face is that the total value of their bond purchases not exceed their portfolio holdings in that market, so we should expect the composition by maturity of the bonds the government issues to be largely irrelevant. In this section we prove a version of the Modigliani-Miller-Ricardian Equivalence Theorem that holds for this economy, and we use this theorem to justify focusing the remainder of this paper on the case where the government only issues one-period bonds.

Consider, then, an equilibrium z, p, and q corresponding to a particular monetary policy. The experiment we wish to conduct is where we perturb the monetary policy in such a way as to maintain the same inflows and outflows of funds to the government at the original prices. This is meant to correspond to the standard application of the Modigliani-Miller-Ricardian Equivalence Theorem in which government policy is changed but expenditures, taxes, and seignorage are kept constant. In our model there is no fiscal policy and all flows between the government and the private sector are money flows. Funds flow to the government in the asset market in an amount equal to the value of the open-market operations $\Sigma q_{t\tau} d_{t\tau}$. They flow to the private sector at the beginning of the period in an amount determined by holdings of one-period bonds and transfers; keeping this outflow fixed is equivalent to fixing the rate of growth of money.

Now select any other monetary policy $(\hat{x},\hat{d})$ such that for any possible realization (x,d) under the original policy, it is true that $\hat{g}_t = g_t$ and $\Sigma_\tau q_\tau(\hat{d}_{t\tau} - d_{t\tau}) = 0$. We want to check if the corresponding realizations p_t and q_t are also equilibrium paths under the perturbed monetary policy. A word of caution is in order. With the new monetary policy taking the place of the original one, there is no guarantee that the recursive structure of the equilibrium can be preserved with respect to the new policy variables. But from the households' point of view the only reason to care about the state and the size of the new bond issues is to be able to forecast future prices. With the distribution of prices completely unchanged, households can make the same forecasts in the new economy. To prove that the original realizations for p_t and q_t remain equilibrium paths, we check that with these prices z_t still satisfies all the equations. Suppose first that the paths for z_t and λ_t remain unchanged. From eq. (2.7) the path for $\phi_{\tau t}$ then remains unchanged, and from (2.8) the path for $\beta\Sigma_\kappa d'_\kappa E_{sd'}[\phi_\kappa(s')/g']$ remains unchanged. Eq. (2.9) reveals that λ_t indeed remains unchanged and (2.11) reveals that z_t remains unchanged. This completes the proof.

Perhaps an example would help to clarify the situation. Suppose that at some time the government wishes to replace quantity d of one-period bonds, sold at a unit price of q_1 in the financial market, with two-period bonds, selling at q_2 each. For the swap to be a zero-value transaction, the extra quantity of two-period bonds must be equal to $(q_1/q_2)d$. If that is the only change in the policy, the equilibrium will be affected because the money stock will behave differently. Under the new policy, the money stock is reduced by d the following period (when the one-period bonds would have matured) and is increased by $(q_1/q_2)d$ in the subsequent period (when the two-period bonds mature). In order to prevent this change in the money stock, the government can increase transfers by d next period and reduce them by $(q_1/q_2)d$ in the subsequent period. Now the new policy is equivalent to the old one. Note that the present value of the change in transfers is zero at the time the policy is changed. Note also that the new policy induces no change in household's wealth (bonds plus transfers) at any time, no matter how asset prices evolve.

One interpretation of this theorem is that the government has only two instruments to affect the economy: the value of open market operations and the growth rate of the money supply. Any policy shift that does not alter these two quantities is irrelevant. In particular, bond swaps intended to take advantage of an upward sloping yield curve will not change this yield curve, so long as the effect of the swap on the rate of growth of the money stock is sterilized (say, through transfers). Such a swap will in general change the value of government debt, but only because this value ignores the transfers: with proper accounting, total government debt (bonds plus money) is unchanged at any time and in any state. Clearly, policy shifts may affect the value of open-market operations or the growth rate of money. Such shifts will in general change the equilibrium, even when they are designed to keep the value of government debt (not including money balances and transfers) unchanged at the original prices. Grilli and Roubini (1991) have noticed that such policy shifts disturb the equilibrium.

4. Computing the Equilibrium

For the remainder of this paper we focus on the special case where the government only issues one-period bonds, but where eq. (2.10) permits us to compute the equilibrium prices of bonds of arbitrary maturity. In the previous section we proved that in general there is a Modigliani-Miller-Ricardian equivalence between such an economy and one in which multi-period bonds are actually issued. As in Lucas (1990), we also focus on the liquidity effects of open-market operations by abstracting from the effects on the aggregate money stock. We thus assume that lump-sum monetary transfers are such that the aggregate stock of money grows at the constant rate $\bar{g}$.

To ease the notational burden we will write d_1' as simply d', so the relevant state variables in s are (ξ,d). Equation (2.10) can then be written as

$$q_1(s,d') = \min\{1, z(s)/d'\}. \tag{4.1}$$

Substituting (2.7), along with the equation just derived, into eq. (2.11) and using the law of iterated expectations, yields

$$\xi u'(\xi)/(1-z(s)) = (\beta/\bar{g})E_s[\max\{1, d'/(z(s)\}\xi' u'(\xi')/(1-z(s'))]. \tag{4.2}$$

To compute a solution z to eq. (4.2) we found it sufficient to impose the following assumption.

Assumption 1. $(\beta/\bar{g})E_s[\max\{1, d'\}] < 1.$

The term $E_s[\max\{1, d'\}]/\bar{g}$ captures the expected real return to allocating cash to the asset market, so Assumption 1 restricts this real return to be less than the rate of time preference.

In the appendix we prove that with Assumption 1 there exists one, and only one, solution $0 < z < 1$ to eq. (4.2). Furthermore, we prove that for any function $0 < z_0 < 1$, the sequence of functions $\{z_n\}$ defined recursively by

$$\xi u'(\xi)/(1-z_{n+1}(s)) = (\beta/\bar{g})E_s[\max\{1, d'/(z_{n+1}(s)\}\xi' u'(\xi')/(1-z_n(s'))]$$

converges to the equilibrium z.

5. The Premium in One-Period Interest Rates

As can be seen from eq. (4.1), and as discussed by Lucas, movements in one-period interest rates may not have anything to do with the Fisherian fundamentals of the real interest rate or expected inflation. Extracting information on the real interest rate or expected inflation from the behavior of short-term interest rates thus requires an understanding of how interest rates deviate from their Fisherian fundamentals. Since this deviation is due to an excess supply or demand for money in the asset market, in the sense that households would like to transfer money between the goods and asset markets, Lucas referred to this deviation as a liquidity effect. Understanding the liquidity effect on interest rates may also be important when considering average values of short rates, as this liquidity effect may lead to a systematic, and even time-varying, premium to holding these bonds. In this section we study the determination of such a premium when households perceive a systematic relationship between open-market operations and subsequent output shocks.

Define $\delta(s,s') = \beta u'(\xi')/u'(\xi)$ as the marginal rate of substitution, $\mu(s,s') = p(s)/(p(s')g')$ as the rate of deflation, and $R(s,d') = 1/q(s,d')$ as the gross interest rate. In general we can write interest rates as

$$E_s[R(s,d')] = (E_s[R(s,d')]E_s[\mu(s,s')\delta(s,s')])/E_s[\mu(s,s')\delta(s,s')],$$

where $1/E_s[\mu(s,s')\delta(s,s')]$ is the Fisher nominal interest rate and the term

$$\gamma(s) = E_s[R(s,d')]E_s[\mu(s,s')\delta(s,s')]$$

is the gross premium to interest rates. For this model, the first-order conditions (2.5) and (2.8) can be seen to imply that

$$1 = E_s[R(s,d')\mu(s,s')\delta(s,s')],$$

from which it follows that

$$\gamma(s) = 1 - \mathrm{Cov}_s[R(s,d'), \mu(s,s')\delta(s,s')]. \tag{5.1}$$

Note that if d' is known at the beginning of a period, a situation which corresponds to the standard cash-in-advance model with integrated markets, then the premium $\gamma(s)$ is always one. Since its departure from one is due to the liquidity effect, we will refer to it as the liquidity premium.[4] As can be seen from (5.1), in general this premium depends on the correlation between the liquidity shocks to interest rates and shocks to the marginal value of money in the next goods market. Consider the following example to help clarify this dependence.

[4]Note that the liquidity premium is not only a risk premium, as it may be different from one when utility is linear or output is constant.

Example 1. Suppose output ξ' and open-market operations d' are iid over time, although not necessarily independent of each other. Suppose also that lump-sum transfers keep the stock of money constant and that households exhibit constant relative risk aversion utility., $u'(c) = c^{-\sigma}$, $\sigma > 1$. The solution for the equilibrium cash allocation function z satisfies

$$\xi^{1-\sigma}/(1-z(\xi)) = \beta E[\max\{1, d'/z(\xi)\}\xi'^{1-\sigma}/(1-z(\xi'))].$$

In the appendix we prove that $\xi^{1-\sigma}/(1-z(\xi))$ is a decreasing function of ξ if $\sigma > 1$, it is constant if $\sigma = 1$, and it is increasing if $\sigma < 1$. If we assume that a high d', which leads to high interest rates, also is associated with low output next period, then for $\sigma > 1$ it follows that d' and $\xi'^{1-\sigma}/(1-z(\xi'))$ are positively correlated. Since the liquidity premium can be written as

$$\gamma(s) = 1 - \mathrm{Cov}_s[\max\{1, d'/z(\xi)\}, \beta((1-z(\xi))/(1-z(\xi')))(\xi'/\xi)^{1-\sigma}/\bar{g}],$$

it follows that the liquidity premium $\gamma(s)$ is less than one for $\sigma > 1$. Short term interest rates are then systematically lower than their Fisherian counterparts. Note that for log utility ($\sigma = 1$) there is no liquidity premium in short term interest rates, and for less risk averse households ($\sigma < 1$) the liquidity premium is positive. Fundamentally, the liquidity premium is an insurance premium that is driven by the negative correlation between shocks to the nominal return on a bond and future consumption. This negative correlation induces a liquidity premium less than unity, unless prices behave in such a way as to undo this correlation when cast in terms of the real return on a bond. We expect the liquidity premium to be less than one due to this negative correlation. When $\sigma > 1$ the correlation is negative, but when $\sigma \leq 1$ the behavior of prices changes this correlation.

In the above example the correlation between prices and output due to movements in the cash allocation function z complicated the analysis. Although not strictly covered by Theorem 1 in the appendix, consider the case when open-market operations d' and the growth rate of output $\rho' = \xi'/\xi$ follow a finite-state Markov process. When households exhibit constant relative risk aversion utility, $u'(c) = c^{-\sigma}$, eq. (4.2) can be written as

$$1/(1-z(s)) = (\beta/\bar{g})E_s[\max\{1, d'/(z(s)\}\rho'^{1-\sigma}/(1-z(s'))].$$

Example 2. Suppose output growth $\rho' = \xi'/\xi$ and open-market operations d' are iid over time, although not necessarily independent of each other, and that households exhibit constant relative risk aversion utility, $u'(c) = c^{-\sigma}$, $\sigma > 1$. The solution for the equilibrium cash allocation function is the constant z that satisfies

$$1 = (\beta/\bar{g})E[\max\{1, d'/z\}\rho'^{1-\sigma}].$$

If we assume that a high d', which leads to high interest rates, also is associated with low output growth ρ', then d' and $\rho'^{1-\sigma}$ are positively correlated. Since the liquidity premium can be written as

$$\gamma(s) = 1 - \mathrm{Cov}_s[\max\{1,d'/z\}, \beta\rho'^{1-\sigma}/\bar{g}],$$

it again follows that the liquidity premium $\gamma(s)$ is less than one for $\sigma > 1$.

Despite a correlation between open-market operations and subsequent output, the result of Lucas that forward rates are unaffected by the liquidity shocks still obtains. To see this for the forward rate $f_1(s,d') = q_1(s,d')/q_2(s,d')$, note that

$$f_1(s,d') = E_{sd'}[u'(\xi')\mu(s,s')]/\beta E_{sd'}[u'(\xi'')\mu(s,s')\mu(s',s'')].$$

The form of this equation is the same whether or not d' is known at the beginning of the period. This is because the forward price $1/f_1$ is the price set in the current asset market for the purchase of a one-period bond in the next asset market. Money to purchase this bond is withdrawn from the next goods market, and the bond pays off in money that is available in the following goods market. This transaction thus reflects a transfer of money from the next goods market to the following one, which by construction (the economy is fully integrated at the beginning of each period) is unaffected by liquidity shocks. The presence of a correlation between monetary shocks and output shocks thus may alter the average shape of the yield curve, as this correlation directly affects average short rates but not forward rates.

6. Numerical Illustrations

To get a sense of the quantitative importance of the liquidity premium in this model, we study explicit solutions that are computed along the lines suggested by Theorem 1. In selecting the model's parameters, we choose the period length short enough to reflect lags in the adjustment of portfolios in response to open-market operations, and long enough to capture some perceived relationship between money and output. We choose $\beta = .9999$, which with a quarterly frequency corresponds to an annual rate of time preference of .04 percent, and we choose constant relative risk aversion utility, $u'(c) = c^{-\sigma}$. Throughout this exercise we model d as following a 2-state Markov process with states $(d_1,d_2) = (.050,.052)$, and transition matrix

$$\pi_d = \begin{bmatrix} \theta & 1-\theta \\ 1-\theta & \theta \end{bmatrix}.$$

We choose d_1 and d_2 so that $d_2/d_1 = 1.04$, which corresponds to a 4 percentage point interest rate differential between the two states if interest rates are always positive. We also choose lump-sum monetary transfers so that money grows at the constant gross rate 1.005.

We solve three versions of the model. In all versions open-market sales of securities and subsequent output shocks are perfectly negatively correlated. In the first version we assume the level of output takes on two values, $(\xi_1,\xi_2) = (1.0,1.05)$, and that when d_1 is realized in the asset market then ξ_2 is realized in the next goods market, and d_2 is similarly associated with ξ_1. In the second version we assume the growth rate of output takes on two values, $(\rho_1,\rho_2) = (.98,1.03)$, where d_1 leads to ρ_2 and d_2 leads to ρ_1. In the third version, which we use as a benchmark, we use the same stochastic process as in the second version except that we allow households to know the value of d' at the beginning of the period. Interest rates are then determined exclusively by Fisher effects. Each of these models is solved for three values of the parameters (θ,σ), and the results are reported in Table 1. The numbers in Table 1 reflect expectations with respect to the ergodic distribution of the state variables, and are expressed at annual rates assuming the model's period consists of a quarter. The variable r_1^* refers to the one-period Fisher nominal interest rate (in percent, computed as $r_1^*(s) = 1/E_s[\mu(s,s')\delta(s,s')]$). The term r_1 refers to the model's nominal interest rate, and f_1 refers to the forward rate (again in percent).

In general we find an insignificant liquidity premium in Model 1, where the level of output and open-market operations evolve according to the Markov matrix determined by θ. In Model 2, where the growth rate of output and open-market operations evolve according to this Markov matrix, however, we find that the liquidity premium begins to be quantitatively significant. In the iid case ($\theta = .5$), when $\sigma = 4$ the nominal one-period interest rate is on average 48 basis points below the corresponding Fisher nominal interest rate (6.27 versus 6.75), and 104 basis points below the nominal interest rate when d' is known (6.27 versus 7.31). For log utility the liquidity premium vanishes, and lowering risk aversion even further ($\sigma = .25$) leads to an insignificant premium (1 basis point higher). Keeping $\sigma = 4$, for serially correlated shocks ($\theta = .75$), the liquidity premium remains significant (41 basis points), and the interest rate is also on average 41 basis points below the one-period interest rate in the model where d' is known. In model 3, of course, all interest rates are determined by Fisherian factors and there is no liquidity premium ($\gamma(s) = 1$). It shows, however, that the presence of a liquidity effect can lower the whole yield curve and not only the short rate relative to the long rate.

Table 1

Money, Output, and the Liquidity Premium

(all numbers are annualized)

		Model 1 s=(d,ξ)	Model 2 s=(d,ρ)	Model 3 s=(d,ρ), d$'$ known
$\theta = .5$, $\sigma = 4$				
	z	.1114	.0505	.0997
	γ	.9999	.9955	1.0000
	r_1^*	2.06	6.75	7.31
	r_1	2.05	6.27	7.31
	f_1	2.06	6.75	6.75
$\theta = .5$, $\sigma = .25$				
	z	.0633	.0519	.0679
	γ	1.0001	1.0001	1.0000
	r_1^*	2.06	0.56	0.57
	r_1	2.07	0.57	0.57
	f_1	2.06	0.56	0.56
$\theta = .75$, $\sigma = 4$				
	z	.1075	.0919	.1514
	γ	0.9999	.9962	1.0000
	r_1^*	2.05	5.62	5.62
	r_1	2.04	5.21	5.62
	f_1	2.05	5.62	5.04

7. Concluding Remarks

The essential point of this exercise is to show that, in an economy constructed to display a liquidity effect, monetary shocks that are correlated with output shocks induce a premium, positive or negative, in short-term interest rates relative to long-term interest rates. The source of this correlation, or the direction of causation, is unimportant in determining this premium; what solely matters is the size of this correlation. In particular, monetary shocks could either lead to output shocks, or reflect an immediate response by monetary authorities to these shocks. That there exists a positive money-output correlation, due to one or both of these factors, is widely accepted (see, e.g., Sims (1972), among many others); and that short-term interest rates are on average lower than long-term interest rates is also true (from 1953 to 1991 the yield on 3-month Treasury bills averaged 5.66 percent per year while the yield on 10-year Treasury bonds averaged 6.87 percent). This paper explored the possibility of a link between the positive money-output correlation and relatively low short-term interest rates.

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Appendix

Theorem 1. With Assumption 1 there exists one, and only one, solution $0 < z < 1$ to eq. (4.1).

Proof. Choose η such that $\max\{\xi u'(\xi)\} < \eta < \infty$ and

$$1 \geq (\beta/\bar{g})E_s[\max\{1, d'/(1-\xi u'(\xi)/\eta)\}].$$

By Assumption 1 this can be done. Define $\bar{w}(s) = \xi u'(\xi)/\eta$, and for any vector w that satisfies $\bar{w}(s) \leq w(s) \leq 1$ for every s, define a new vector Aw by

$$\xi u'(\xi)/(Aw)(s)) = (\beta/\bar{g})E_s[\max\{1, d'/(1-(Aw)(s)\}\xi' u'(\xi')/w(s')]. \tag{4.3}$$

Clearly a fixed point $w = Aw$ corresponds to an equilibrium $1 - z$. Note that A is well-defined, continuous, monotone in the partial ordering $w_1 \leq w_2$ if $w_1(s) \leq w_2(s)$ for every s, $A(\bar{w}) > \bar{w}$, and $A(1) \leq 1$. This establishes the existence of an equilibrium $0 < z < 1$ as corresponding to the limit, e.g., of $A^n(1)$.

To prove that there cannot exist two equilibria, suppose there exists two fixed points w_1 and w_2. Relabeling the vectors if necessary, choose $0 < t < 1$ such that $w_1 \geq tw_2$ and $w_1(s) = tw_2(s)$ for some s (hence $w_1 > tw_2$ does not hold). Note that $A(tw) > tA(w)$ for any $0 < t < 1$. It then follows that $w_1 = A(w_1) \geq A(tw_2) > tA(w_2) = tw_2$, which is a contradiction. QED.

Theorem 2. The equilibrium function z that satisfies (Example 1)

$$\xi^{1-\sigma}/(1-z(s)) = \beta E[\max\{1, d'/z(\xi)\}\xi'^{1-\sigma}/(1-z(\xi'))]$$

is such that $\xi^{1-\sigma}/(1-z(\xi))$ is a decreasing function of ξ if $\sigma > 1$, it is constant if $\sigma = 1$, and it is increasing if $\sigma < 1$.

Proof. Consider first the case $\sigma > 1$. Note that if $\xi^{1-\sigma}/(1-z(\xi))$ is a decreasing function of ξ then so is $\xi^{1-\sigma}/(A(1-z))(\xi)$, and thus the equilibrium z satisfies this condition. The remaining properties are proven in the same way. Q.E.D.

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