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REAL EXCHANGE RATES: MEASUREMENT AND IMPLICATIONS FOR PREDICTING U.S. EXTERNAL IMBALANCES

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ABSTRACT

That international trade flows respond to changes in real exchange rates is beyond question. What is less clear is whether the measurement of real exchange rates matters for characterizing and predicting such responses. To identify the implications of choosing a given measure of the real exchange rate, I examine how the parameter estimates and the forecast performance of a *given* model vary in response to alternative measures of real exchange rates. I reject a given measure if its use (a) implies estimates inconsistent with economic theory; (b) contradicts the assumptions needed for statistical inference; (c) leads to systematic forecast errors; or (d) entails a loss of information relative to an alternative measure. Although the analysis rejects several measures of real exchange rates, it cannot identify a unique measure suitable for explaining U.S. trade: relative unit labor costs and relative consumer prices are equally suited for modeling and predicting U.S. trade.

Real Exchange Rates: Measurement and Implications for Predicting U.S. External Imbalances

Jaime Marquez[1]

1. Introduction

This paper examines the empirical implications of using alternative measures of real exchange rates for explaining U.S. international trade. This issue is of interest because no consensus exists on how to measure *the* real exchange rate. Should the nominal exchange rate be adjusted by the ratio between domestic and foreign consumer prices to emphasize factor reallocations across productive sectors, or by the ratio between domestic and foreign labor costs to focus on competitiveness? These ambiguities in measurement need not matter for trade or macro theory because definitions can be tailored to the problem at hand. But to the extent that policymakers use one measure of the real exchange rate to forecast trade imbalances, the question is which measure should it be.

Existing analyses cannot address this question because they combine different assumptions about economic behavior with alternative measures of real exchange rates. For example, the dispersion of model forecasts for the U.S. trade deficit is $60 billion for Bryant, Holtham, and Hooper (1988) and $80 billion for Marquez and Ericsson (1990). How much of their dispersions stems from differences in model structure and how much is due to differences in real exchange rates? I address this issue by examining how the parameter estimates and the forecast performance of a *given* model vary in response to alternative measures of real exchange rates. I reject a given measure if its use (a) implies estimates inconsistent with economic theory; (b) violates the assumptions needed for statistical inference; (c) leads to systematic forecast errors; or (d) entails a loss of information relative to an alternative measure.

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[1] The author is a staff economist in the Division of International Finance. I have benefited from comments by Neil Ericsson, Jon Faust, Joe Gagnon, William Helkie, David Howard, Eric Leeper, Ellen Meade, Edwin Truman, and Joseph Whitt. I am also grateful to Russell Tarver who replicated the results of this paper. All of the regression tests are performed with the software developed by David Hendry for PCGIVE and installed into TROLL by Ralph Tryon; the BIN data file for PCGIVE is available on request. The Monte Carlo simulations are generated with TROLL. The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting those of the Board of Governors of the Federal Reserve System or other members of its staff.

Taking a model as given avoids using the data for both parameter estimation and model specification but makes the conclusions model dependent. To minimize the influence of modeling idiosyncrasies, I rely on the most commonly used formulation which section 2 describes. Using data over 1976:1-1988:1 for parameter estimation, section 3 finds that the estimates are sensitive to the measure of the real exchange rate, but that they are not inconsistent with economic theory. Thus in the absence of very specific priors about their values, parameter estimates alone cannot discriminate among the alternative measures of real exchange rates. As a result, section 4 uses forecast performance over 1988:2-1990:1 as an additional selection criterion. Recognizing that predictions from econometric models are random variables, I use stochastic simulations for generating the associated forecast distributions. The analysis rejects several measures of real exchange rates as inducing biased forecasts of U.S. trade but cannot identify a single measure for forecasting purposes. Thus section 5 applies the technique of forecast encompassing to compare the informational content of the remaining measures. On the basis of this technique, relative labor costs and relative consumer prices are equally suited for modeling and predicting U.S. trade.

2. Empirical Formulation

2.1 Model Design

To study and predict the behavior of U.S. trade imbalances, I rely on the following model:

$$\ln X_{jt} = a + b\ln Y^{*}_{t-1} + c\Delta\ln Y^{*}_{t} + g\ln RPX_{i,t-1} + e\Delta\ln RPX_{it} + f\ln X_{j,t-1} + \varepsilon_{jt}, \quad (1)$$

$$\ln M_{jt} = m + n\ln Y_{t-1} + o\Delta\ln Y_{t} + p\ln RPM_{i,t-1} + q\Delta\ln RPM_{it} + r\ln M_{j,t-1} + \xi_{jt}, \quad (2)$$

$$X_t = PX_{jt} X_{jt}, \quad (3)$$

$$M_t = PM_{jt} M_{jt}, \quad (4)$$

$$NX_t = X_t - M_t, \quad (5)$$

where

X_j	=	jth measure of export volume.
Y^*	=	Foreign real GNP.
RPX_i	=	ith measure of real exchange rate for exports.

M_j = jth measure of non-oil import volume.
Y = Real GNP.
RPM_i = ith measure of real exchange rate for imports.
X = Value of U.S. exports.
PX_j = jth measure of export deflator.
M = Value of non-oil imports.
PM_j = jth measure of non-oil import deflator.
NX = Net exports.

This model offers several useful features. First, eqs. (1) and (2) use the imperfect substitutes model (Goldstein and Khan, 1985) where trade volumes respond directly to increases in real income ($b,n>0$) and inversely to increases in real exchange rates ($g, p < 0$). Thus I reject a measure of real exchange rate that contradicts these sign restrictions. Second, reliance on a logarithmic formulation is convenient and relates this work to a tradition in forecasting U.S. trade balances.[2] Third, I follow Hendry, Pagan, and Sargan (1984) and choose a specification that differentiates dynamic responses due to changes in income from those due to changes in relative prices. Finally, I assume that both ε_{jt} and ξ_{jt} are independent, white noise disturbances and reject measures of real exchange rates contradicting these assumptions.

Equations (3)-(5) give, respectively, the value of exports, non-oil imports, and the associated balance of trade.[3] Note that eqs. (3)-(4) recognize that several measures of trade volumes and prices are consistent with a given value of trade. Recognition of this multiplicity is central to this paper because, as seen below, trade deflators are commonly used for measuring real exchange rates. In this analysis, I consider price indexes and unit values as alternative deflators:

PX_1 = Export price index.
PX_2 = Export unit value.
PM_1 = Non-oil import price index.

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2 See the trade models of Helkie and Hooper (1988), Moffet (1989), and Lawrence (1990); Marquez (1992) reviews the literature on the estimation of trade elasticities and finds that, out of 110 papers written over 1941-1991, 74 assume a logarithmic formulation.

3 I exclude oil imports because they are a perfect substitute for U.S. oil production questioning the usefulness of (2).

PM_2 = Non-oil import unit value.

Finally, by treating real income (domestic and foreign) and real exchange rates as given, eqs. (1)-(5) offer a partial-equilibrium view of the behavior of U.S. external imbalances. Endogenizing real income entails developing macroeconometric models for both the United States and the rest of the world, a task beyond the scope of this paper. Similarly, endogenizing real exchange rates means expanding (1)-(5) with additional equations the structure of which depends on the measure of the real exchange rate. This dependency precludes isolating the effects of using alternative real exchange rates for a *given* model.[4]

2.2 Measures of Real Exchange Rates

To implement the analysis, I define the real exchange rate R as EP/P*, where P is the domestic price level, E is the effective, nominal exchange rate (foreign/US$), and P* is the foreign price level. The absence of theoretical arguments restricting the choice of P and P* means that numerous measures of R are available for empirical work. This multiplicity of measures of R is relevant because they need not bear a constant relation to each other as Marston (1987) shows. In this analysis, I consider four measures of real exchange rates:

RPX_1 = IMF's relative labor costs, U.S. relative to foreign.
RPX_2 = Export price index (PX_2) relative to foreign GNP deflator.
RPX_3 = Export unit value (PX_3) relative to foreign GNP deflator.
RPX_4 = Domestic consumer price relative to foreign consumer price.

RPM_1 = IMF's relative labor costs, foreign relative to U.S.
RPM_2 = Non-oil import price index (PM_2), tariff adjusted, relative to the GNP deflator.
RPM_3 = Non-oil import unit value (PM_3), tariff adjusted, relative to the GNP deflator.
RPM_4 = Foreign consumer price relative to domestic consumer price.

This list includes measures based on trade prices relative to GNP deflators because they are commonly used in modeling and forecasting U.S. trade, which is the focus of this paper (see Marquez,

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4 Unrestricted Vector Autoregressive (VAR) formulations can address these two considerations. I do not follow this methodology for two reasons. First, the signs of the associated parameter estimates are not useful in discriminating among measures of real exchange rates. Second, the corresponding impulse-response results are sensitive to the ordering of equations, a very restrictive feature.

1992). I exclude the price of tradeables relative to the price of non-tradeables because of the practical difficulties in matching the various definitions of non-tradeables to the existing data (see Pauls and Helkie, 1987; Lipschitz and McDonald, 1991).

Inspection of the data suggests that the choice of the real exchange rate and trade deflator is not inconsequential for explaining U.S. trade (figure 1). For example, the relation among the RPXs is approximately constant until 1985 and unstable since then with the gap among them reaching 40 percent by 1990:1. Similarly, the depreciation of the dollar that began in 1985 leaves RPM_2 and RPM_3 virtually unchanged but raises RPM_1 and RPM_4 by approximately 60 percent and 50 percent, respectively. Finally, differences in trade deflators translate into substantial and growing gaps for measuring trade volumes. For example, the gap between real exports based on prices indexes (X_1) and unit values (X_2) reaches 25 percent by 1990:1.

3. Econometric Estimation

Combining four measures of real exchange rates with two trade deflators yields eight models of interest. I estimate the associated parameters with ordinary least squares using quarterly data for 1976:1-1988:1 and the results reveal several findings of interest (tables 1 and 2).[5] First, income elasticities are positive and replicate the asymmetry noted by Houthakker and Magee (1969): the income elasticity for U.S. imports exceeds the income elasticity for U.S. exports. Second, price elasticities are negative and the sum of their absolute values exceeds one, which is consistent with the Marshall-Lerner condition for stability. Third, the estimated price elasticities are sensitive to the choice of the real exchange rate. For example, using either relative consumer prices or relative labor

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[5] Appendix A describes the construction of the data and presents the associated sources. The estimation sample starts in 1976:1 for three reasons. First, the series for the IMF's measure of real exchange rates (RPX1 and RPM1) starts in 1975:1. Second, even though earlier observations are available for the other measures, I want to control for sample size when comparing results. Third, the starting date could not be 1975:1 because the statistical tests performed below require re-estimating (1) and (2) with additional lags which involves using the observations during 1975 for testing. I have performed Granger-causality tests for all eight models and the results support real exchange rates Granger-causing trade and not vice versa. Finally, note that the relatively short span of data (1976-1988) questions the power of cointegration tests for the present application (see Campbell and Perron, 1991, p. 45; Hakkio and Rush, 1991).

Figure 1: Alternative Measures of Real Exchange Rates and Trade Volumes

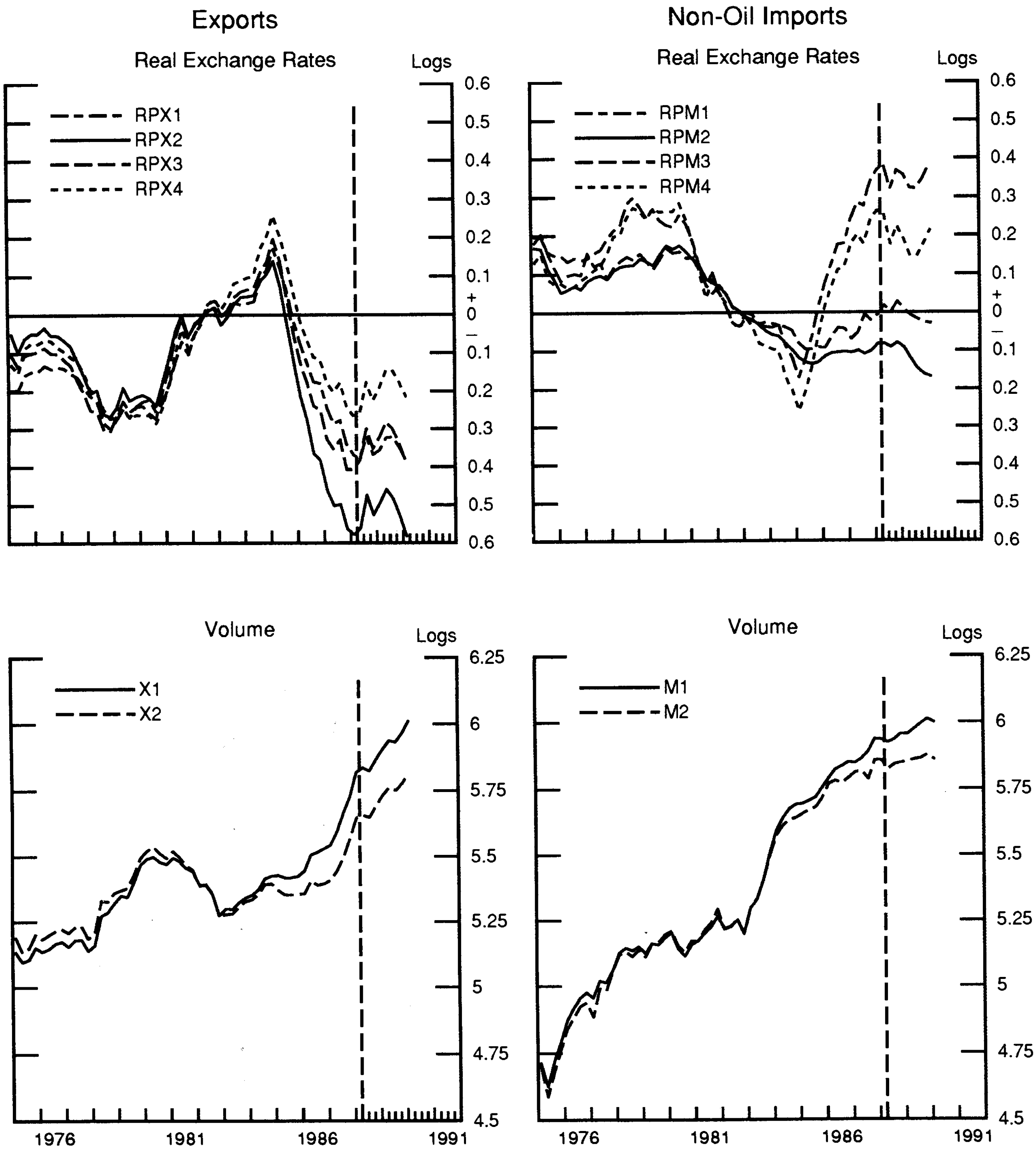

Table 1
Income and Price Elasticities for U.S. Exports[1]
Alternative Measures of Real Exchange Rates
(t-statistics in parentheses)

Explanatory Variables	Estimation Model							
	M1	M2	M3	M4	M5	M6	M7	M8
Lagged Real GNP	0.610	0.332	0.454	0.618	0.285	0.120	0.197	0.322
	(4.4)	(2.8)	(3.7)	(4.2)	(3.1)	(1.5)	(2.4)	(3.4)
ΔReal GNP	1.122	1.077	1.277	1.666	1.161	1.184	1.360	1.499
	(1.6)	(1.4)	(1.7)	(2.4)	(1.4)	(1.3)	(1.6)	(1.8)
Lagged Relative Price	-0.214	-0.138	-0.155	-0.186	-0.174	-0.115	-0.129	-0.165
	(-5.2)	(-4.4)	(-4.7)	(-4.7)	(-4.1)	(-3.7)	(-3.8)	(-4.1)
ΔRelative Price	0.076	0.111	0.079	0.011	0.062	0.116	0.037	0.024
	(0.6)	(1.0)	(0.8)	(0.1)	(0.4)	(0.9)	(0.3)	(0.2)
Lagged Dep. Variable	0.729	0.810	0.792	0.732	0.819	0.870	0.865	0.794
	(9.7)	(11.3)	(11.1)	(9.3)	(11.3)	(12.7)	(12.4)	(10.5)
$\bar{R}^2$	0.971	0.967	0.968	0.969	0.928	0.922	0.923	0.927
SER	0.026	0.028	0.027	0.027	0.030	0.031	0.031	0.030
Mean of Dep. Variable	5.387	5.387	5.387	5.387	5.371	5.371	5.371	5.371
Diagnostics[2]								
Normality	0.73	0.78	0.86	0.89	0.81	0.77	0.77	0.72
Serial Independence	0.85	0.85	0.90	0.90	0.77	0.87	0.83	0.86
Homoskedasticity	0.45	0.28	0.46	0.67	0.64	0.63	0.64	0.69
Dynamic Specification	0.69	0.83	0.84	0.88	0.53	0.69	0.69	0.69
Parameter Constancy	0.63	0.54	0.44	0.49	0.82	0.66	0.59	0.69
Long-run Elasticities								
Income	2.25	1.75	2.18	2.31	1.57	0.92	1.46	1.56
Relative prices	-0.79	-0.73	-0.75	-0.69	-0.96	-0.88	-0.96	-0.80

[1] The estimates are based on OLS with quarterly data for 1976:1-1988:1. The description of the alternative models is:

	Deflator for Volume	Relative Prices
Model M1	Price Indexes	Relative labor costs
Model M2	Price Indexes	Price indexes relative to foreign GNP deflator
Model M3	Price Indexes	Unit values relative to foreign GNP deflator
Model M4	Price Indexes	Relative consumer price levels
Model M5	Unit Values	Relative labor costs
Model M6	Unit Values	Price indexes relative to foreign GNP deflator
Model M7	Unit Values	Unit values relative to foreign GNP deflator
Model M8	Unit Values	Relative consumer price levels

[2] These entries represent one minus the significance level needed to reject the associated null hypothesis: If the entry is 0.95 then the corresponding hypothesis can be rejected with a 5 percent significance level.

Table 2
Income and Price Elasticities for U.S. Non-oil Imports[1]
Alternative Measures of Real Exchange Rates
(t-statistics in parentheses)

Explanatory Variables	Estimation Model							
	M1	M2	M3	M4	M5	M6	M7	M8
Lagged Real GNP	0.429	1.313	0.760	0.630	1.240	1.428	1.372	1.447
	(1.4)	(3.9)	(2.6)	(2.0)	(3.2)	(3.9)	(3.7)	(3.7)
ΔReal GNP	1.712	1.575	1.697	1.774	1.940	1.402	1.720	1.93
	(3.4)	(3.8)	(4.0)	(3.6)	(3.6)	(2.8)	(4.1)	(3.8)
Lagged Relative Price	-0.101	-0.611	-0.475	-0.123	-0.183	-0.516	-0.602	-0.198
	(-2.3)	(-4.8)	(-4.1)	(-2.9)	(-3.5)	(-4.2)	(-4.3)	(-4.0)
ΔRelative Price	0.086	-0.336	-0.335	0.064	0.202	-0.579	-0.750	0.139
	(0.6)	(-1.3)	(-1.8)	(0.5)	(1.4)	(-1.8)	(-3.9)	(1.1)
Lagged Dep. Variable	0.859	0.461	0.675	0.794	0.594	0.425	0.448	0.524
	(9.7)	(3.7)	(6.6)	(8.4)	(5.0)	(3.2)	(3.3)	(4.3)
$\bar{R}^2$	0.991	0.993	0.993	0.991	0.988	0.989	0.992	0.988
SER	0.032	0.027	0.028	0.031	0.035	0.032	0.028	0.033
Mean of Dep. Variable	5.370	5.370	5.370	5.370	5.348	5.348	5.348	5.348
Diagnostics[2]								
Normality	0.60	0.38	0.85	0.63	0.76	0.39	0.20	0.76
Serial Independence	0.85	0.96	0.84	0.86	0.79	0.79	0.91	0.86
Homoskedasticity	0.71	0.16	0.25	0.73	0.37	0.07	0.49	0.50
Dynamic Specification	0.68	0.55	0.98	0.56	0.53	0.11	0.95	0.35
Parameter Constancy	0.45	0.03	0.35	0.70	0.44	0.43	0.08	0.46
Long-run Elasticities								
Income	3.04	2.44	2.34	3.06	3.05	2.48	2.49	3.04
Relative prices	-0.72	-1.13	-1.46	-0.60	-0.45	-0.90	-1.09	-0.42

[1] The estimates are based on OLS with quarterly data for 1976:1-1988:1. The description of the alternative models is:

	Deflator for Volume	Relative Prices
Model M1	Price Indexes	Relative labor costs
Model M2	Price Indexes	Price indexes relative to foreign GNP deflator
Model M3	Price Indexes	Unit values relative to foreign GNP deflator
Model M4	Price Indexes	Relative consumer price levels
Model M5	Unit Values	Relative labor costs
Model M6	Unit Values	Price indexes relative to foreign GNP deflator
Model M7	Unit Values	Unit values relative to foreign GNP deflator
Model M8	Unit Values	Relative consumer price levels

[2] These entries represent one minus the significance level needed to reject the associated null hypothesis: If the entry is 0.95 then the corresponding hypothesis can be rejected with a 5 percent significance level.

costs gives the lowest price elasticities (in absolute terms) for imports. Fourth, the choice of trade deflator influences the estimated elasticities. Specifically, the import-price elasticities of models M5-M8 (unit values) are lower, in absolute terms, than those of models M1-M4 (price indexes); price elasticities for exports exhibit the opposite pattern.

Fifth, fit alone cannot discriminate among these models because they all have high $\bar{R}^2$s. Note, however, that the standard error of the regression is lower for models deflating trade with price indexes (models M1-M4) than for models deflating trade with unit values (models M5-M8). Sixth, the data cannot reject the assumptions embodied in estimation for six of the eight models. That is, the disturbances satisfy normality, serial independence, and homoskedasticity; the parameter estimates are constant; and the choice of dynamic specification is not rejected by the data. The two exceptions are the import equation of model M2, which has serially correlated residuals, and the import equation for model M3, which rejects the choice of dynamic specification.[6] Apart from these two exceptions, tables 1 and 2 do not offer a compelling case against any of the remaining measures of real exchange rates, and thus I turn to forecast performance as an additional criterion. For this purpose, however, I consider all eight models to highlight the consequences of violating the assumptions maintained in estimation, especially serial independence of disturbances.

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[6] The paper uses the Jarque-Bera statistic (Jarque and Bera, 1980) to test normality and the ARCH statistic (Engle, 1982) to test homoskedasticity. For serial independence, I test the hypothesis that the coefficients of an AR(4) for the residuals are jointly equal to zero. Testing for parameter constancy involves re-estimating (1) and (2) with a sample excluding data for 1986:2-1988:1 and testing the null hypothesis that the mean of the associated one-step ahead forecast errors is zero. I also apply recursive least squares to (1) and (2) and generate recursive Chow tests with both decreasing and increasing forecast horizons (see Hendry, 1989, p. 44 for a description). The results cannot reject the hypothesis of parameter constancy. To test for dynamic misspecification, I include additional lags of all predetermined variables as regressors and test for the joint significance of the additional coefficients.

4. Forecast Performance

4.1 Monte Carlo Methodology

Predictions based on (1)-(5) are subject to uncertainty because they depend on realizations of ε_t and ξ_t (see eqs. 1 and 2) which are random variables. Thus evaluating forecast performance entails testing whether the actual values in the forecast period come from the same distribution as that assumed to generate the in-sample values. This testing needs estimates of the mean and variance of the forecasts which I calculate using Monte Carlo simulations because of the nonlinearities embodied in (1) and (2). Specifically, I follow Fair (1984) and simulate 1000 times eqs. (1)-(5) over 1988:2-1990:1 by drawing random values of ε_t and ξ_t from their distribution NI(0, $\hat{\Omega}$) where $\hat{\Omega}$ is the estimate of Ω; appendix B describes this procedure in detail. These simulations use the model to generate the values of the lagged endogenous variables and take as given both the coefficient estimates and the paths for the exogenous variables.[7]

Taken as a whole, these simulations constitute a random sample of forecast paths that I use to estimate, for each variable, the mean forecast at time T+s, $\tilde{\mu}_{T+s}$; the mean forecast error, $\tilde{v}_{T+s}$, computed as the actual value minus the estimated mean prediction; the variance at time T+s, $\tilde{\sigma}^2_{T+s}$; and the variance for the *whole* forecast path, $\tilde{\Phi}$; T is the last observation in the estimation sample (1988:1); s=1,..., S, where S is the number of observations in the forecast sample (12). Given these estimates, I test whether the forecast errors are substantially larger than anticipated. For a forecast at T+s, the associated test statistic is the mean forecast error $\tilde{v}_{T+s}$ divided by its estimated standard error $\tilde{\sigma}_{T+s}$. The corresponding test statistic for the *whole* forecast path is (Hendry, 1979) $\tau=\tilde{v}'[\tilde{\Phi}]^{-1}\tilde{v}$, where $\tilde{v}=(\tilde{v}_{T+1}, ..., \tilde{v}_{T+S})'$. Finding a large value of τ means that the path of forecast errors, $\tilde{v}$, differs significantly from the model's own measure of uncertainty as embodied in $\tilde{\Phi}$; in this case, the model is said to exhibit a

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7 These assumptions are consistent with current practice for generating model forecasts. A full investigation of the forecasting performance, however, would have to address the question of the accuracy with which information on exogenous variables can be obtained or how well can they be projected.

predictive failure. Under the null hypothesis that the model is correctly specified over both the estimation and forecast samples, τ is approximately distributed as χ^2 (S).

By itself, however, τ cannot uniquely identify the source of predictive failures. To detect whether they stem from systematic forecast errors, I follow Marquez and Ericsson (1990) and compute $\tau^* = (\sqrt{S})(\tilde{v}'[\tilde{\Phi}]^{-1/2}\iota/S)$, where ι is an Sx1 vector of ones. This statistic (a) transforms the forecast errors into errors which (under the null hypothesis of correct model specification) are standardized and uncorrelated, (b) averages those transformed errors, and (c) rescales the average by $\sqrt{S}$. Under the null hypothesis of zero mean in the forecast error, τ^* has a t-distribution with S-1 degrees of freedom and is designed to have power against the alternative of a non-zero mean, constant across forecasts; large values of τ^* indicate that the model's forecasts are biased.

4.2 Empirical Results

The results reveal that the choice of real exchange rate is responsible for a substantial dispersion in export forecasts (table 3). For example, reliance on relative labor costs (models M1 and M5) overpredicts exports whereas reliance on relative consumer prices (models M4 and M8) produces the opposite result. The associated dispersion in forecasts is $31 billion for 1991:1, slightly less than 10 percent of the actual value of exports. The uncertainty embodied in these predictions, however, is large enough to prevent a sharp distinction between these two measures of real exchange rates. Specifically, exports' forecast errors are smaller than their own uncertainty, a feature reflected in low values for the τ and τ^* statistics. Thus reliance on these two statistics cannot discriminate among these formulations.

Forecasts for non-oil imports (table 4) are also sensitive to the choice of real exchange rate. For example, using relative labor costs (model M1) underpredicts imports whereas relying on relative consumer prices (model M4) gives the opposite result; the resulting dispersion of forecasts is $28 billion by 1990:1, slightly below 7 percent of the actual value of non-oil imports. The results also indicate that the forecast errors of model M2 exceed their period-by-period standard deviation by a

Table 3
Export Forecast Performance:
Alternative Measures of Real Exchange Rates[1]

	Forecasting Horizon								Summary Measures[2]		
	1988Q2	1988Q3	1988Q4	1989Q1	1989Q2	1989Q3	1989Q4	1990Q1	MAE	τ	τ^*
Historical (US$ Billion)	317.6	322.0	335.7	353.1	364.4	357.4	367.0	384.2			
Mean Forecast Error (US$ Billion)											
Model M1	-7.1	-28.8	-21.1	-22.6	-19.5	-26.7	-20.3	-13.6	20.0	9.5	-2.1
Model M2	-8.2	-30.4	-21.3	-21.8	-16.7	-21.7	-14.8	-6.8	17.7	7.6	-1.2
Model M3	-6.0	-25.1	-14.8	-14.3	-7.9	-12.6	-5.9	1.3	11.0	6.2	-0.7
Model M4	-1.6	-17.0	-6.1	-3.9	5.9	2.4	10.3	17.3	8.1	5.9	0.5
Model M5	-10.1	-31.5	-25.6	-27.6	-26.6	-37.2	-34.3	-30.4	27.9	8.1	-2.1
Model M6	-10.2	-31.9	-23.6	-24.6	-21.6	-29.4	-25.1	-19.4	23.2	6.3	-1.3
Model M7	-8.0	-25.7	-17.5	-16.9	-11.9	-19.9	-16.4	-12.1	16.0	4.6	-1.0
Model M8	-5.5	-21.0	-10.3	-7.5	0.6	-4.5	2.3	9.2	7.6	4.4	-0.1
Forecast Standard Error (US$ Billion)											
Model M1	8.5	11.2	12.2	13.4	14.4	14.3	14.2	14.8			
Model M2	9.1	12.5	14.0	15.6	16.9	17.0	17.1	17.8			
Model M3	8.8	11.9	13.3	14.7	15.8	15.8	15.8	16.5			
Model M4	8.6	11.2	12.2	13.2	14.0	13.8	13.6	14.3			
Model M5	9.7	13.4	15.3	17.1	18.8	19.2	19.5	20.5			
Model M6	10.1	14.3	16.6	18.9	20.9	21.5	22.1	23.3			
Model M7	10.0	13.9	16.0	18.2	20.0	20.6	21.1	22.3			
Model M8	9.6	12.9	14.4	15.8	17.0	17.0	17.0	17.8			

[1] The results are based on 1000 stochastic dynamic simulations allowing only residual uncertainty.

	Deflator for Volume	Relative Prices
Model M1	Price Indexes	Relative labor costs
Model M2	Price Indexes	Price indexes relative to foreign GNP deflator
Model M3	Price Indexes	Unit values relative to foreign GNP deflator
Model M4	Price Indexes	Relative consumer price levels
Model M5	Unit Values	Relative labor costs
Model M6	Unit Values	Price indexes relative to foreign GNP deflator
Model M7	Unit Values	Unit values relative to foreign GNP deflator
Model M8	Unit Values	Relative consumer price levels

[2] MAE: Mean Absolute Error.
τ: statistic testing against predictive failure; $\tau \sim \chi^2(8)$. If $\tau < 15.5$, then reject the hypothesis of predictive failure.

τ^*: t-statistic testing against non-zero mean forecast error. If $|\tau^*| > 2.4$, then reject the hypothesis of zero mean forecast error.

Table 4
Import Forecast Performance:
Alternative Measures of Real Exchange Rates[1]

	Forecasting Horizon								Summary Measures[2]		
	1988Q2	1988Q3	1988Q4	1989Q1	1989Q2	1989Q3	1989Q4	1990Q1	MAE	τ	τ^*
Historical (US$ Billion)	400.8	405.6	424.3	421.9	423.3	423.9	428.6	427.2			
Mean Forecast Error (US$ Billion)											
Model M1	-5.1	1.6	4.9	0.7	6.0	8.4	11.0	3.1	5.1	1.1	0.2
Model M2	-14.6	-20.2	-20.2	-30.6	-36.4	-45.4	-53.5	-70.7	36.4	36.1	-5.5
Model M3	-2.7	0.3	8.9	3.2	3.2	1.7	1.0	-10.8	4.0	1.7	-0.2
Model M4	-5.7	-1.9	-3.1	-10.4	-9.8	-12.6	-14.2	-25.1	10.4	2.0	-1.0
Model M5	-15.9	-5.2	-19.2	-23.6	-26.2	-37.2	-43.4	-56.6	28.4	10.9	-2.8
Model M6	-33.6	-41.4	-48.9	-60.3	-75.3	-92.4	-101.9	-118.9	71.6	75.8	-8.3
Model M7	-16.2	-20.4	-18.4	-30.4	-41.7	-52.9	-58.0	-71.5	38.7	36.6	-5.5
Model M8	-19.3	-14.9	-34.4	-40.6	-48.5	-64.1	-72.1	-85.0	47.4	29.3	-4.9
Forecast Standard Error (US$ Billion)											
Model M1	12.7	17.3	20.7	22.8	23.3	23.8	24.2	24.3			
Model M2	10.9	12.6	13.6	14.0	13.6	13.9	14.1	14.2			
Model M3	11.0	13.7	15.2	15.9	15.6	15.6	15.6	15.6			
Model M4	12.4	16.4	19.3	21.0	21.2	21.5	21.7	21.7			
Model M5	14.2	16.8	19.2	19.8	19.3	19.6	19.8	19.8			
Model M6	13.9	15.9	17.4	17.8	17.6	18.3	18.6	18.7			
Model M7	11.6	13.2	14.3	14.7	14.4	14.8	14.9	14.9			
Model M8	13.9	16.1	18.4	18.8	18.4	18.9	19.2	19.2			

[1] The results are based on 1000 stochastic dynamic simulations allowing only residual uncertainty.

	Deflator for Volume	Relative Prices
Model M1	Price Indexes	Relative labor costs
Model M2	Price Indexes	Price indexes relative to foreign GNP deflator
Model M3	Price Indexes	Unit values relative to foreign GNP deflator
Model M4	Price Indexes	Relative consumer price levels
Model M5	Unit Values	Relative labor costs
Model M6	Unit Values	Price indexes relative to foreign GNP deflator
Model M7	Unit Values	Unit values relative to foreign GNP deflator
Model M8	Unit Values	Relative consumer price levels

[2] MAE: Mean Absolute Error.

τ: statistic testing against predictive failure; $\tau \sim \chi^2(8)$. If $\tau < 15.5$, then reject the hypothesis of predictive failure.

τ^*: t-statistic testing against non-zero mean forecast error. If $|\tau^*| > 2.4$, then reject the hypothesis of zero mean forecast error.

substantial margin and that, viewed as a path, these errors exceed the model's own measure of uncertainty as reflected in large values of τ and τ^*. This failure could have been predicted from the regression results because the estimated residuals of this equation are serially correlated (see table 2). Finally, models deflating trade with unit values (models M5 - M8) overpredict imports by a substantial margin, a feature reflected in statistically significant values for τ and τ^*. This poor forecast performance might arise from changes in either the trade elasticities or the product composition of U.S. non-oil imports not adequately reflected in unit values. These failures, however, could not have been predicted from the regression diagnostics because they support the assumptions maintained for estimation.

Finally, the results for the balance of trade (table 5) indicate that, for 1990:1, differences in the measure of real exchange rates are responsible for a $81 billion dispersion in forecasts for the trade balance: the predicted trade deficit ranges from $107 billion for model M2 to $26 billion for model M1. The results also highlight that what is true for the parts need not be true for the whole. Specifically, model M5 offers the most accurate trade-balance forecast and does not exhibit a predictive failure even though the import forecasts from this model have systematic forecast errors (high τ^* statistic). This accuracy stems from a canceling of the forecast errors generated by the trade equations of model M5. Such canceling is not, however, present in the remaining models using unit values (models M6-M8), which have the largest forecast errors.

Overall, the evidence suggests two conclusions. First, differences in measures of real exchange rates contribute substantially to the dispersion of forecasts. Second, not all of these measures are equally suited for forecasting: using predictive failures in either imports or exports as a rejection criterion, I exclude models using import unit values as a trade deflator (models M5-M8). In addition, the import equations of models M2 and M3 violate the assumptions under which they are estimated (serial independence and correct dynamic specification, respectively). Thus only models M1 and M4 are left for comparison. Further discrimination between these two models requires the application of forecast-encompassing tests, to which I now turn.

Table 5
Partial Trade-balance Forecast Performance:
Alternative Measures of Real Exchange Rates[1]

	Forecasting Horizon								Summary Measures[2]		
	1988Q2	1988Q3	1988Q4	1989Q1	1989Q2	1989Q3	1989Q4	1990Q1	MAE	τ	τ^*
Historical (US$ Billion)	-83.3	-83.6	-88.5	-68.9	-58.8	-66.5	-61.7	-43.1			
Mean Forecast Error (US$ Billion)											
Model M1	-2.0	-30.5	-26.0	-23.2	-25.5	-35.0	-31.3	-16.7	23.8	4.5	-1.1
Model M2	6.4	-10.2	-1.2	8.8	19.8	23.7	38.7	63.9	21.6	9.3	1.9
Model M3	-3.4	-25.4	-23.7	-17.5	-11.1	-14.3	-6.9	12.1	14.3	4.0	-0.4
Model M4	4.1	-15.1	-3.0	6.6	15.8	15.0	24.5	42.5	15.8	5.0	1.1
Model M5	5.9	-26.3	-6.4	-4.0	-0.4	0.1	9.0	26.2	9.8	4.5	0.3
Model M6	23.4	9.4	25.3	35.6	53.7	63.0	76.8	99.6	48.3	13.4	3.2
Model M7	8.2	-5.3	1.0	13.5	29.8	33.1	41.6	59.4	24.0	6.3	1.8
Model M8	13.8	-6.2	24.1	33.1	49.1	59.6	74.3	94.2	44.3	16.7	3.2
Forecast Standard Error (US$ Billion)											
Model M1	15.4	20.8	24.3	26.2	26.6	27.5	27.9	28.4			
Model M2	14.3	17.9	19.8	20.8	21.1	21.8	21.9	23.0			
Model M3	14.2	18.4	20.4	21.4	21.5	21.9	21.9	22.8			
Model M4	15.2	20.1	23.1	24.5	24.6	25.3	25.5	25.9			
Model M5	17.4	21.7	24.8	25.9	26.1	27.1	27.4	28.7			
Model M6	17.4	21.5	24.3	25.8	26.5	28.0	28.5	30.2			
Model M7	15.4	19.3	21.7	23.2	23.9	25.1	25.5	27.2			
Model M8	17.0	20.8	23.6	24.3	24.2	25.2	25.3	26.3			

[1] The partial trade balance is equal to merchandise exports minus non-oil imports. These measures are based on 1000 stochastic dynamic simulations allowing only residual uncertainty.

	Deflator for Volume	Relative Prices
Model M1	Price Indexes	Relative labor costs
Model M2	Price Indexes	Price indexes relative to foreign GNP deflator
Model M3	Price Indexes	Unit values relative to foreign GNP deflator
Model M4	Price Indexes	Relative consumer price levels
Model M5	Unit Values	Relative labor costs
Model M6	Unit Values	Price indexes relative to foreign GNP deflator
Model M7	Unit Values	Unit values relative to foreign GNP deflator
Model M8	Unit Values	Relative consumer price levels

[2] MAE: Mean Absolute Error

τ: statistic testing against predictive failure; $\tau \sim \chi^2(8)$. If $\tau < 15.5$, then reject the hypothesis of predictive failure.

τ^*: t-statistic testing against non-zero mean forecast error. If $|\tau^*| > 2.4$, then reject the hypothesis of zero mean forecast error.

5. Forecast Encompassing

Forecast encompassing determines the extent to which the forecast errors of one model depend on the predictions from an alternative model. Intuitively, if model h is correctly specified, then its forecast errors are innovations that should be uncorrelated with the forecasts from model ℓ. In that case, model h is said to "forecast-encompass" model ℓ. If model h forecast-encompasses model ℓ, and not vice versa, then using the real exchange rate embodied in model ℓ entails a loss of information.

Implementing the forecast-encompassing test involves regressing the forecast errors of one model (model h) on the forecasts of another model (model ℓ):

$$(6) \qquad \tilde{v}^{(h)}_{T+s} = \kappa_0 + \kappa_\ell \tilde{\mu}^{(\ell)}_{T+s} + \omega^{(h)}_{T+s} \qquad s=1,\ldots,S,$$

where $\omega^{(h)}_{T+s}$ is the error of the regression. Testing the null hypothesis that model h forecast-encompasses model ℓ involves testing the joint significance of κ_0 and κ_ℓ. Chong and Hendry (1986) were the first to develop this test but they set $\kappa_0 = 0$ which, as Marquez and Ericsson (1990) indicate, assumes that the forecast errors of model h are unbiased. Moreover, Chong and Hendry assume one-step ahead predictions from linear models with normal independent disturbances. These assumptions allow the t-ratio from an OLS regression of (6) to have the desired null distribution. But the forecasts in this application come from nonlinear and dynamic models implying that the forecast errors are heteroskedastic and serially correlated. Thus I follow Marquez and Ericsson (1990) and apply GLS to (6) using $[\tilde{\Phi}^{(h)}]^{-1/2}$ as the correction factor. The estimation sample is 1988:2-1990:1 and the forecast-encompassing statistic is distributed as F(2,6).

According to the results (table 6), the forecast errors from one model do not depend on the predictions of the other model. In other words, relative labor costs and relative consumer prices embody the same informational content for predicting U.S. trade over 1988:2-1990:1.[8]

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[8] The results of table (6) are robust to Ericsson's suggestion (Ericsson, 1991) of using $(\tilde{\mu}^{(\ell)}_{T+s} - \tilde{\mu}^{(h)}_{T+s})$ as the explanatory variable in eq. (6).

Table 6
Forecast Encompassing Tests:
Selected Measures of Real Exchange Rates*

	Model to be Encompassed (Mℓ)					
	Exports		Imports		Balance	
	M1	M4	M1	M4	M1	M4
Encompassing Model (Mh)						
M1		3.2 (0.11)		0.4 (0.69)		1.6 (0.27)
M4	0.1 (0.95)		3.7 (0.09)		0.5 (0.64)	

* Entries in the table are the F(2,6)-values associated with the hypothesis that the mean forecast errors of model h cannot be explained by the predictions of model ℓ; entries in parentheses are the associated significance levels.

Differentiating between these two measures of real exchange rates is difficult for two reasons. First, they are highly correlated in the forecast horizon (see figure 1). A solution to this situation involves extending the forecast sample while truncating the estimation sample. This alternative, however, increases the noise-to-signal ratio and ignores that forecasts in policy analyses rarely extend beyond two years. Second, the estimation results for these two models are virtually identical: price elasticities range from -0.79 to -0.69 for exports and from -0.72 to -0.60 for imports; the regressions' standard errors are virtually identical for both exports and imports.

6. Conclusions and Caveats

This paper examines the implications of using alternative measures of real exchange rates for predicting U.S. trade. I focus on how the parameter estimates and the forecast performance of a *given* model vary in response to alternative measures of real exchange rates. Specifically, I ask three questions: Are the elasticity estimates from these models consistent with the predictions from economic theory? Are these models statistically reliable? Are their forecast errors unbiased? The two main conclusions are that relative labor costs and relative consumer prices are equally suited for modeling and predicting U.S. trade and that the choice of real exchange rates accounts for a substantial fraction of the dispersion of trade forecasts found in previous analyses.

These conclusions are conditional on several assumptions: choice of functional form, level of disaggregation, estimation method, length of the forecasting horizon, and the exogeneity of income, prices, and exchange rates. Moreover, maintaining the last assumption amounts to abstracting from the difficulties involved in forecasting these variables. Future work relaxing these assumptions will, in all likelihood, affect the numerical results of this paper but it will also strengthen an important and neglected point--that measuring is just as important as modeling.

Appendix A: Data Sources

This appendix describes the construction of the data and identifies the associated sources which appear as italicized abbreviations and are described at the end of this appendix.

Measures Real Exchange Rates

Exports:

RPX_1 : Relative unit labor costs.

RPX_2 : PX_1 / (Foreign G-10 GNP deflator/G-10 value of the dollar).

RPX_3 : PX_2 / (Foreign G-10 GNP deflator/G-10 value of the dollar).

RPX_4 : Private Consumption deflator/(Foreign G-10 CPI/G-10 value of the dollar).

Imports:

RPM_1 : The inverse of relative unit labor costs (RPX_1).

RPM_2 : (1+average tariff rate/100) PM_1 / GNP deflator.

RPM_3 : (1+average tariff rate/100) PM_2 / GNP deflator.

RPM_4 : (Foreign G-10 CPI/G-10 value of the dollar)/Private Consumption deflator.

PX_1 = Export price index.

PX_2 = Export unit value.

PM_1 = Non-oil import price index.

PM_2 = Non-oil import unit value.

Data Methodology

Relative Labor Costs. The data for the relative labor costs (RPX_1) come from the *IFS*, line reu. Relative labor costs are the ratio between unit labor cost in the United States and a geometric weighted average of the unit labor costs, expressed in U.S. dollars, for the remaining 16 industrial countries. The weights used in the average are based on trade shares in manufacture that, according to the *IFS*, take into account considerations affecting bilateral trade as well as competition in third markets. I use in this application *normalized* relative unit labor costs. The *IFS* uses the term *normalized* to indicate that the indexes of actual hourly compensation per worker are divided by a 5-year moving average index of output per man-hour.

Exports, Imports, and their Prices. Data for merchandise exports in current prices come from the *SCB*, table 4.1; merchandise exports in constant prices come the *SCB*, table 4.2. Data for the export price index (PX_1) are constructed as the ratio between nominal and real exports. (Note that the *SCB* uses price *indexes* generated by the BLS in the construction of trade in constant prices (Walter, 1991).) The unit value for exports (PX_2) comes from the *IFS*, line 74.

Data for merchandise non-oil imports in current prices come from the *SCB*, table 4.1; merchandise non-oil imports in constant prices come the *SCB*, table 4.2. The data for the non-oil import price index (PM_1) are constructed as the ratio between nominal and real trade figures. Data for the unit value for non-oil imports (PM_2) are constructed as the ratio between the nominal and the unit-value based, real value of non-oil imports. I obtain this unit-value based, real value of non-oil imports in two steps. First, I deflate the *SCB* value of merchandise imports by the import unit value to obtain a unit-value based measure of merchandise imports in real terms; the import unit value index come from the *IFS*, line 75. Second, I compute the difference between unit-value based, real imports (constructed in step 1) and oil imports in constant prices, as reported by the *SCB* table 4.2.

Tariffs. Data for the multilateral tariff rate are constructed as the ratio between custom duties and the total value of imports, both in current prices. Data on custom duties come from the *SCB*, table 3.2.

Domestic GNP and Prices. Data for GNP in current prices come from the *SCB*, table 1.1; data for GNP in constant prices (Y) come from the *SCB*, table 1.2. The data for the GNP deflator are constructed as the ratio between GNP in current and constant prices. The data for the private consumption deflator are constructed as the ratio between private consumption in nominal terms and private consumption in constant prices. This measure of consumer prices is not the CPI (fixed weights) but the private consumption deflator (variable weights) from the National Income and Product Accounts. Data for consumption in current prices come from the *SCB*, table 1.1; data for consumption in constant prices come from table 1.2.

Foreign GNP, Foreign Prices, and Exchange Rates. Foreign GNP (Y*) is constructed as a geometric mean of the levels of foreign GNPs, using the weights in Table A.1 below. The sources for foreign GNPs are: *CSR* (Canada); *BOJ* (Japan); *ET* (United Kingdom); Statistical Supplement, *DBB* (Germany); Comptes Nationales, *INSEE* (France); *INSTAT* (Italy); Bulletin de Statistique, Institute National de Statistique (Belgium); Kwartaalrekeningen, Central Bureau of Statistics (Netherlands); Reflets de L'Economie, Office Fédéral de la Statistique (Switzerland); and National Accounts, National Central Bureau of Statistics (Sweden).

The foreign consumer price index (CPI) is constructed as a geometric mean of the country-specific consumer price indices, using the trade weights in Table A.1 below. The sources for the individual CPIs are: *CSR* (Canada); *BOJ* (Japan); Employment Gazette, Department of Employment (United Kingdom); *DBB* (Germany); Bulletin Mensuel de Statistique, *INSEE* (France); *INSTAT* (Italy); Bulletin de la Banque Nationale, National Bank of Belgium (Belgium); Maandstatistiek, van de Pryzen, Central Bureau of Statistics (Netherlands); La Vie Economique, Dept. Fed. Econ. Publ. (Switzerland); and Allman Manadsstatistik, Swedish Official Statistical Office (Sweden).

The foreign GNP deflator is constructed as a geometric mean of the country-specific GNP deflators, using the trade weights in Table A.1 below. The sources for the individual deflators are: *CEO* (Canada); *BOJ* (Japan); Monthly Digest, Central Statistical Office (United Kingdom); *DBB* (Germany); Bulletin Mensuel de Statistique, *INSEE* (France); *IFS* (Italy); *IFS* (Belgium); *IFS* (Netherlands); La Vie Economique, Dept. Fed. Econ. Publ. (Switzerland); and *IFS* (Sweden).

The G-10 value of the dollar is a geometric mean of bilateral exchange rates (foreign currency/US$) using multilateral trade weights and come from *FRB*. This measure of the value of the dollar excludes exchange rates of developing countries to ensure comparability of results with the measure of the real exchange rate constructed by the IMF.

Table A.1. Average Trade Weights for the G-10 Countries

Country	Foreign GNP	Prices and Exchange Rate
Canada	9.1%	7.8%
Japan	13.6%	15.4%
United Kingdom	11.9%	11.8%
Germany	20.8%	20.4%
France	13.1%	13.0%
Italy	9.0%	9.5%
Belgium	6.4%	7.1%
Netherlands	8.3%	8.1%
Switzerland	3.6%	3.5%
Sweden	4.2%	3.4%

The trade weights for foreign GNP are averages over 1978-83 of the given country's share in the group's total imports. The trade weights for exchange rates and consumer prices are averages over 1972-76 of the given country's share in the group's total imports. The source of these nominal imports is the *IFS*, Line 77abd.

Data Sources

BOJ	*Economic Statistics Monthly*, Bank of Japan, Tokyo, monthly.
CEO	*Canadian Economic Observer*, Statistics Canada, Ottawa, monthly.
CSR	*Canadian Statistical Review*, Statistics Canada, Ottawa, quarterly.
DBB	*Monthly Report*, Deutsche Bundesbank, Frankfurt am Main, monthly.
ET	*Economic Trends*, U.K. Central Statistical Office, London, monthly.
FRB	*Federal Reserve Bulletin*, Board of Governors of the Federal Reserve System, Washington, D.C., monthly.
IFS	*International Financial Statistics*, International Monetary Fund, Washington, D.C., monthly.
INSEE	*Comptes Nationales* and *Bulletin Mensuel de Statistique*, Institute National de la Statistique et des Etudes Economique, Paris, quarterly and monthly respectively.
INSTAT	*Indicatori Mensili*, Instituto Centrale di Statistica, Rome, monthly.
SCB	*Survey of Current Business*, U.S. Department of Commerce, Bureau of Economic Analysis, Washington, D.C., quarterly.

Appendix B: Monte Carlo Simulations

Following Marquez and Ericsson (1990), I express eqs. (1)-(5) as

(B.1) $$y_{T+s} = g(y_{T+s-1}, z_{T+s}, \hat{\theta}, u_{T+s}), \qquad s=1, ..., S,$$

where y_{T+s} is the 5×1 vector of endogenous variables, T is the last observation in the estimation sample (1988:1), S is number of observations in the forecast horizon (12), $g(\cdot)$ is the 5×1 vector of equations in the model, z_{T+s} is the 8×1 vector of exogenous variables, $\hat{\theta}$ is the 12×1 vector of parameter estimates (excluding the variance-covariance matrix of the disturbances), and $u'_{T+s} = (\varepsilon_{T+s} \ \ \xi_{T+s})$. Note that $g(\cdot)$ is nonlinear because eqs. (1) and (2) are logarithmic in form.

The conditional expectation of the ith component of y_{T+s} is

(B.2) $$\mu_{i,T+s} \equiv A_i \cdot E(y_{T+s}) = A_i \cdot \int g(y_{T+s-1}, z_{T+s}, \hat{\theta}, u_{T+s}) \cdot f(u) \cdot du, \quad s=1, ..., S,$$

where $E[\cdot]$ is the expectation operator and A_i is a selection matrix that extracts the ith variable from the vector function $g(\cdot)$. The forecast errors associated with (B.2) are

(B.3) $$v_i = (v_{i,T+1}, ..., v_{i,T+S})' = ([A_i \cdot y_{T+1} - \mu_{i,T+1}], ..., [A_i \cdot y_{T+S} - \mu_{i,T+S}])',$$

and their variance-covariance matrix is $\Phi_i = E[v_i v_i']$. Solving analytically the integrals associated with $\mu_{i,T+s}$ and Φ_i is difficult because $g(\cdot)$ is nonlinear. To bypass this difficulty, I rely on Monte Carlo simulations as a statistical analogue to the analytical formula implicit in (B.2). Specifically, assuming that u is distributed as

$$u_t \sim NI(0, \Omega) \qquad t = 1,...,T+S,$$

I draw random numbers u_k $[\equiv (u_{k,T+1},...,u_{k,T+S})']$ from the distribution $NI(0, \hat{\Omega})$ where the subscript k indicates the replication number of K possible replications and the empirical estimates replace the unknown population values in the distribution functions. Given the paths for the exogenous variables, z_{T+s}, the coefficient estimates, $\hat{\theta}$, and a drawing of the random shocks affecting exports and imports, $u_{k,T+s}$, eqs. (1)-(5) are solved sequentially over 1988:2-1990:1 (S=8) to generate the kth replication as

(B.4) $$y_{k,T+s} = g(y_{k,T+s-1}, z_{T+s}, \hat{\theta}, u_{k,T+s}), \qquad s = 1,...,S,$$

where the values of the lagged endogenous variables are generated by the model and $y_{k,T} \equiv y_T \ \forall k$. This process is repeated 1000 times (K=1000) to generate a sample of 1000 forecast paths for nominal exports, nominal non-oil imports, and the trade balance. Given this sample, I compute four statistics for each of these variables:

(B.5) $$\tilde{\mu}_{i,T+s} = \Sigma_k A_i \cdot y_{k,T+s}/K, \qquad s=1,...,S,$$

(B.6) $$\tilde{v}_{i,T+s} = A_i \cdot y_{T+s} - \tilde{\mu}_{i,T+s}, \qquad s=1,...,S,$$

(B.7) $$\tilde{\sigma}^2_{i,T+s} = \Sigma_k (A_i \cdot y_{k,T+s} - \tilde{\mu}_{i,T+s})^2/K, \qquad s=1,...,S,$$

(B.8) $$\tilde{\Phi}_i = \Sigma_k \phi_{ik}\phi_{ik}'/K,$$

where $\phi_{ik} = ([A_i \cdot y_{k,T+1} - \tilde{\mu}_{iT+1}], ..., [A_i \cdot y_{k,T+S} - \tilde{\mu}_{i,T+S}])'$, $\tilde{\mu}_{i,T+s}$ is the estimated mean forecast; $\tilde{v}_{i,T+s}$ is the estimated forecast error; $\tilde{\sigma}^2_{i,T+s}$ is the estimated variance around $\tilde{\mu}_{i,T+s}$; and $\tilde{\Phi}_i$ is the corresponding measure of uncertainty around $(\tilde{\mu}_{iT+1}, ..., \tilde{\mu}_{i,T+S})'$. Note the important difference between $\tilde{v}$ and ϕ_k. The elements of $\tilde{v}$ are the discrepancies between *outcomes* and the mean forecasts, whereas the elements of ϕ_k are the discrepancies between the kth set of *simulated forecasts* and the mean forecasts. The $\tilde{v}$ indicate the actual forecast performance of the model, whereas the ϕ_k (via $\tilde{\Phi}$) are the basis for estimating how well the model should forecast if it were well-specified.

Given (B.5)-(B.8), the *predictive failure* test statistic is (Hendry, 1979)

(B.9) $$\tau_i = \tilde{v}_i' [\tilde{\Phi}_i]^{-1} \tilde{v}_i \sim \chi^2 (S).$$

The biasedness test-statistic is

(B.10) $$\tau_i^* = (\surd S)(\tilde{v}_i' [\tilde{\Phi}_i]^{-1/2} \iota / S) \sim t(S-1),$$

where ι is an Sx1 vector of ones.

References

Bryant, R., G. Holtham, P. Hooper (eds.), 1988, *External Deficits and the Dollar* (Brookings Institution: Washington, DC).

Campbell, J., and P. Perron, 1991, "Pitfalls and opportunities: what macroeconomists should know about unit roots," NBER Technical Paper No. 100 (National Bureau of Economic Research: Cambridge).

Chong, Y., and D. Hendry, 1986, "Econometric evaluation of linear macroeconomic models," *Review of Economic Studies*, 53, 671-690.

Engle, R., 1982, "Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom inflation," *Econometrica*, 50, 987-1008.

Ericsson, N. R., 1991, "Parameter constancy, mean square forecast errors, and measuring forecast performance: an exposition, extensions, and illustration," International Finance Discussion Paper No. 412 (Board of Governors of the Federal Reserve System: Washington, DC).

Fair, R., 1984, *Specification, Estimation, and Analysis of Macroeconometric Models* (Harvard University Press: Cambridge).

Goldstein, M., and M. Khan, 1985, "Income and price effects in foreign trade," in R. Jones and P. Kenen (eds.) *Handbook of International Economics*, vol. 2 (North-Holland: Amsterdam).

Hakkio, C., and M. Rush, 1991, "Cointegration: how short is the long run?," *Journal of International Money and Finance*, 10, 571-581.

Helkie, W., and P. Hooper, 1988, "An empirical analysis of the external deficit, 1980-1986," in R. Bryant, G. Holtham, P. Hooper (eds.) *External Deficits and the Dollar* (Brookings Institution: Washington, DC).

Hendry, D., 1979, "Predictive failure and econometric modeling in macroeconomics: the transactions demand for money," in P. Ormerod (ed.) *Economic Modeling* (Heinemann Education Books: London).

Hendry, D., 1989, *PC-GIVE: An Interactive Econometric Modelling System* (University of Oxford: Oxford).

Hendry, D., A. Pagan, and D. Sargan, 1984, "Dynamic specification," in Z. Griliches and M. Intriligator (eds.) *Handbook of Econometrics*, vol. 2 (North-Holland: Amsterdam).

Houthakker, H., and S. Magee, 1969, "Income and price elasticities in world trade," *Review of Economics and Statistics*, 51, 111-125.

Jarque, C., and A. Bera, 1980, "Efficient tests for normality, homoscedasticity, and serial independence of regression residuals," *Economic Letters*, 6, 255-259.

Lawrence, R., 1990, "U.S. current account adjustment: an appraisal," *Brookings Papers on Economic Activity*, No. 2, 343-392.

Lipschitz, L., and D. McDonald, 1991, "Real exchange rates and competitiveness: a clarification of concepts, and some measurements for Europe," IMF working paper WP/91/25 (International Monetary Fund: Washington, DC).

Marquez, J., 1992, "The autonomy of trade elasticities: choice and consequences," International Finance Discussion Paper No. 422 (Board of Governors of the Federal Reserve System: Washington, DC).

Marquez, J., and N. R. Ericsson, 1990, "Evaluating the predictive performance of trade-account models," International Finance Discussion Paper No. 377 (Board of Governors of the Federal Reserve System: Washington, DC).

Marston, R., 1987, "Real exchange rates and productivity growth in the United States and Japan," in S. Arndt and J. D. Richardson (eds.) *Real-Financial Linkages among Open Economies* (MIT Press: Cambridge).

Moffet, M., 1989, "The J-curve revisited: an empirical examination for the United States," *Journal of International Money and Finance*, 8, 425-444.

Pauls. D., and W. Helkie, 1987, "A reassessment of measures of the dollar's effective exchange value," International Finance Discussion Paper No. 306 (Board of Governors of the Federal Reserve System: Washington, DC).

Walter, B., 1991, "Quality issues affecting the compilation of the U.S. merchandise trade statistics," in P. Hooper and J. D. Richardson (eds.) *International Economic Transactions: Issues in Measurement and Empirical Research* (University of Chicago Press: Chicago).

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