Board of Governors of the Federal Reserve System

International Finance Discussion Papers

Number 399

May 1991

PRECAUTIONARY MONEY BALANCES WITH AGGREGATE UNCERTAINTY

Wilbur John Coleman II

ABSTRACT

This paper studies the dependence of velocity on stochastic monetary growth in a model where households demand money for both its transactions and precautionary services. The setup consists of a cash-in-advance economy in which individual uncertainty leads households to value money for its insurance against adverse endowment shocks. With stochastic monetary growth the distribution of money balances across households does not settle down to a time invariant distribution, so one aim of this paper is to model this distribution as an endogenous state variable.

Precautionary Money Balances with Aggregate Uncertainty

Wilbur John Coleman II[1]

1. Introduction

This paper develops a cash-in-advance model in which the transactions velocity of money depends on a stochastic monetary growth rate. This model, which resembles the pure currency economy developed by Lucas (1980), relies on individual uncertainty to generate a precautionary demand for money, and relies on a dependence of this demand on its opportunity cost to obtain a dependence of velocity on monetary growth. In contrast to Lucas' setup, the model developed here includes uncertainty in national income and the stock of money, as uncertainty in the latter is necessary for studying how velocity responds over time to shocks in the money growth rate.[2] The ability of this model to generate a significant demand for precautionary money balances is in contrast to the representative-household cash-in-advance models of Lucas (1982), Svensson (1985), and Lucas and Stokey (1987), in which little or no cash is voluntarily carried across periods. In these models households usually spend all their cash holdings on goods or assets, where the only sectors that absorb these cash receipts are ones that are physically precluded from trading them away: almost all cash is held overnight

---

[1]The author is a staff economist in the International Finance Division of the Federal Reserve Board. This paper should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or its staff.

[2]Lucas (1980) does not consider monetary growth, but as suggested in Stokey, Lucas, and Prescott (1989) it is straightforward to include a constant money growth rate into his model. The equilibrium is otherwise the same one which Lucas computed, but with a modified rate of time preference (i.e., one deflated by the money growth rate) and an inflation rate equal to the money growth rate. In this sense one can use Lucas' setup to perform a comparative static exercise to determine the dependence of velocity on constant money growth rates, but one would not be able to study the dynamic response of velocity to a stochastic money growth rate.

in cash registers. This result is well documented by both Hodrick, Kocherlakota, and Lucas (1991) and Giovannini and Labadie (1989).

The principal difficulty this paper addresses is modeling the distribution of money balances across households as an endogenous state variable. With aggregate uncertainty this distribution does not settle down to a time invariant distribution, so one needs to model its evolution. To simplify this problem only two types of households are considered, as opposed to the continuum considered by Lucas, and in addition these households are assumed to exhibit log utility. Even with these assumptions it is fairly easy to construct an example in which a one percentage point increase in average monetary growth leads to a roughly one percent increase in the velocity of money, so these assumptions provide a suitable setting with which to begin a study of the role individual uncertainty plays in obtaining a dependence of velocity on monetary policy.

In its reliance on two types of households with log utility this paper is similar to Scheinkman and Weiss (1986), who study a version of this economy without exogenous aggregate uncertainty. Foley and Hellwig (1975) and Bewley (1980) study models in which there is no underlying transactions demand for money, but where individual uncertainty leads to a precautionary demand for money in much the same way as it does in this paper. Recent work by Imrohoroglu and Prescott (1991) attempts to introduce aggregate uncertainty into that framework.

2. The Model

The model consists of a large number of two types of worker-shopper households that produce a single perishable consumption good which can only be traded with fiat money. Households of type i, $i = 1, 2$, are identical and collectively begin a period with one unit of labor and a fraction m_i of the aggregate money supply. These cash balances consist of money acquired in the previous period in addition to lump-sum cash transfers from the government. During the period the workers of type i households inelastically supply their one unit of labor to produce y_i units of the consumption good according to a stochastic technology. These goods are sold to shoppers for cash, where each shopper only has access to the cash he or she has on hand at the beginning of the period. At the end of the period the households consume the purchased goods and consolidate the cash acquired from the sale of goods with the unspent cash.

In addition to the distribution of output (y_1, y_2), the economy consists of an arbitrary number of additional exogenous state variables $(y_3, y_4, \ldots, y_n)$. Denote the entire list of exogenous state variables for a particular period by y.

ASSUMPTION 1: *$y \in Y$, Y is finite. The shocks evolve according to the Markovian transition probabilities $\pi(y'|y)$. Furthermore, $0 < y_i < \infty$, $i = 1, 2$.*

With this assumption there exists an $\alpha > 0$ such that $y_i / \sum_{j=1}^{2} y_j \geq \alpha$, all $y \in Y$. Denote the conditional expectation with respect to $\pi(\cdot|y)$ as E_y.

Denote the aggregate money supply by M and the *equilibrium* shares held by the two types of households by $m = (m_1, m_2)$. Each household assumes the next period's values of these variables evolve according to

$$M' = h(y', y)M, \quad m' = L(m, y),$$

where y' is the next period's values of the exogenous state variables.[3] The monetary-policy function h completely specifies the law of motion for the aggregate money supply, but with

[3]If h depends on y' then the exogenous shocks realized next period are known to the monetary authorities at the beginning of that period, which is when next period's monetary transfer takes place.

individual uncertainty one still needs to specify how the lump-sum transfers of money get distributed across households. To focus this paper on the implications for velocity of varying the opportunity costs to holding precautionary money balances, these transfers are modeled so that they do not alter the distribution of money.[4] This leads to a slight problem, as in equilibrium each household's monetary transfer must be in proportion to its money holdings, and as money holdings differ across households, so must the transfer. To retain the lump-sum nature of the transfer, allow the monetary authorities to identify a household by its type, and thereby constrain them to provide a transfer that is proportional to the average money holdings of households of that type. As a household cannot alter its type, these transfers will appear to be lump sum. The aggregate monetary transfer $M' - M$ is thus distributed such that households of type i receive $(M' - M)m_i'$.

Denote the price of consumption in terms of money by P, which households assume is homogeneous of degree one in M:

$$P(m,y,M) = p(m,y)M.$$

Along with the transition matrix π, the functions h, L, and p are known to all households. The monetary-policy function h is chosen by the government, while the determination of L and p is part of the equilibrium problem. For the households, however, these are simply some arbitrary, fixed functions.

Along with a household's knowledge of its type, its state variables consist of the aggregate state variables (m,y) and its beginning-of-period post-transfer money holdings relative to the aggregate money supply, n_i. A household chooses time-stationary state-dependent functions for consumption c_i and next period's pre-transfer money relative to

[4]On the other hand, Grossman and Weiss (1983) and Rotemberg (1984) focus on the ability of monetary shocks to alter the distribution of money balances.

the current money supply, $\hat{n}_i$:[5]

$$c_i = C_i(n_i,m,y), \quad \hat{n}_i = Q_i(n_i,m,y).$$

These choices are subject to the budget and cash-in-advance constraints

$$p(m,y)c_i + \hat{n}_i = p(m,y)y_i + n_i, \tag{2.1}$$

$$p(m,y)c_i \leq n_i. \tag{2.2}$$

At the beginning of the next period the households of type i receive a lump-sum monetary transfer equal to $(h(y',y) - 1)L_i(m,y)M$, so their relative post-transfer money holdings evolve according to

$$n_i' = \frac{Q_i(n_i,m,y) + (h(y',y) - 1)L_i(m,y)}{h(y',y)}.$$

A household's preferences over the above choices are defined by the expected discounted utility over the implied sequence of consumption,

$$\mathrm{E}\left\{\sum_{t=0}^{\infty} \beta^t \, u(c_{it})\right\},$$

where $0 < \beta < 1$, (n_0,m_0,y_0) is known, the expectation is over sequences $\{y_t\}$, and the

[5]The notation reflects the collective choice of households of type i. Disaggregating to reflect the choices of individual households is straightforward.

associated sequence $\{c_t,n_t,m_t\}$ is computed using C, Q, h and L.

Assumption 2: *The single-period utility function* $u:\mathbb{R}_+ \to \mathbb{R}$ *is bounded, continuously differentiable, strictly increasing, strictly concave, and* $u'(0) = \infty$.

Denote $s = (n_i,m,y)$, $S = \mathbb{R}_{++}\times S^1\times Y$ (S^1 is the one-dimensional unit simplex), and for any $s \in S$ define the constraint set $\Omega_i(s) \subset \mathbb{R}^2_+$ as all $(c_i,\hat{n}_i)$ that satisfy

$$p(m,y)c_i + \hat{n}_i \le p(m,y)y_i + n_i,\ \ p(m,y)c_i \le n_i,\ \ (c_i,\hat{n}_i) \ge 0.$$

Note that for any $s \in S$ $\Omega_i(s)$ is nonempty, compact, convex-valued, and continuous and convex in n_i.

Define $V_i(s)$ as the value of the objective function at the optimum, beginning with the indicated state vector, which satisfies

$$(2.3)\quad V_i(n_i,m,y) = \sup_{(c_i,\hat{n}_i)\in\Omega_i(s)} \left\{ u(c_i) + \beta E_y\left[V_i\left[\frac{\hat{n}_i+(h(y',y)-1)L_i(m,y)}{h(y',y)}, L(m,y), y' \right]\right]\right\}.$$

Consider the Banach space of bounded, continuous, real-valued functions $v:S \to \mathbb{R}$ equipped with the sup norm, and let $\mathcal{V}$ denote the subset of functions that are increasing and concave in their first argument. Under Assumptions 1-2, given any continuous monetary-policy function $h:Y^2 \to \mathbb{R}_{++}$, distribution of money function $L:S^1\times Y \to S^1$, and price function $p:S^1\times Y \to \mathbb{R}_{++}$, standard arguments prove the following results. There exists a unique value function $V_i \in \mathcal{V}$ that satisfies (2.3), and this V_i is strictly-increasing and strictly-concave in its first argument. Also, for each $s \in S$ the supremum in (2.3) is attained by unique values $C_i(s)$ and $Q_i(s)$, and these policy functions C_i and Q_i are continuous in their first argument.

Definition: A *stationary equilibrium* consists of (i) value functions $V_i \in \mathcal{V}$, (ii) continuous functions C_i and Q_i mapping $[0,1] \times S^1 \times Y$ into, respectively, $\mathbb{R}_+$ and $[0,1]$, and (iii) continuous functions L and p mapping $S^1 \times Y$ into, respectively, S^1 and $\mathbb{R}_{++}$, such that V_i satisfies (2.3) with the associated policy functions C_i and Q_i, and the following relationships hold for any $m \in S^1$, $y \in Y$, $i = 1, 2$:

(2.4) $$L_i(m,y) = Q_i(m_i,m,y),$$

(2.5) $$p(m,y)C_i(m_i,m,y) + Q_i(m_i,m,y) \le p(m,y)y_i + m_i,$$

(2.6) $$p(m,y)C_i(m_i,m,y) \le m_i,$$

(2.7) $$\sum_{j=1}^{2} C_j(m_j,m,y) = \sum_{j=1}^{2} y_j.$$

Equation (2.4) equates the type i household's demand for money with what was assumed for that type of household, (2.5) requires the household to lie on its budget constraint, (2.6) requires the household to lie within its cash-in-advance constraint, and (2.7) requires the aggregate demand for goods to equal its aggregate supply.

At the equilibrium, since $u'(0) = \infty$ the ranges of the policy functions C_i and Q_i lie in the interior of $\Omega_i(s)$, so by Benveniste and Scheinkman's theorem (1979, Theorem 1) the value function V_i is continuously differentiable in its first argument at (m,y), all $m \in S^1$, $m_i > 0$, $i = 1, 2$, all y. Denote the multipliers associated with (2.1) and (2.2) at the equilibrium by $\lambda_i(m,y)$ and $\varphi_i(m,y)$ respectively. The equilibrium can then be further characterized as consisting of functions c, p, L, λ, and φ such that

$$p(m,y)c_i(m,y) + L_i(m,y) = p(m,y)y_i + m_i, \tag{2.8}$$

$$p(m,y)c_i(m,y) \le m_i \text{ with equality if } \varphi_i(m,y) > 0, \tag{2.9}$$

$$u'(c_i(m,y)) = (\lambda_i(m,y) + \varphi_i(m,y))p(m,y), \tag{2.10}$$

$$\lambda_i(m,y) = \beta E_y[(\lambda_i(L(m,y),y') + \varphi_i(L(m,y),y'))/h(y',y)], \tag{2.11}$$

$$\sum_{j=1}^{2} c_j(m,y) = \sum_{j=1}^{2} y_j. \tag{2.12}$$

By construction, any strictly-positive solution to these equations is an equilibrium.

At this point, as mentioned in the introduction, assume households exhibit log utility, $u(c) = ln\ c$.[6] Following an argument made by Lucas and Stokey (1987) for a related economy, rewrite the system (2.8)-(2.12) as ($m \in S^1$, $y \in Y$, $i = 1, 2$):

$$\varphi_i(m,y) = \max\{0,\ 1/m_i - \lambda_i(m,y)\}, \tag{2.13}$$

$$c_i(m,y) = \min\{m_i/p(m,y),\ 1/(\lambda_i(m,y)p(m,y))\}, \tag{2.14}$$

$$L_i(m,y) = p(m,y)y_i + m_i - p(m,y)c_i(m,y), \tag{2.15}$$

$$\lambda_i(m,y) = \beta E_y[\max\{1/L_i(m,y),\ \lambda_i(L(m,y),y')\}/h(y',y)]. \tag{2.16}$$

---

[6]A property of the equilibrium will be the existence of $[\underline{c},\bar{c}]$ such that $0 < \underline{c} \le c_i(m,y) \le \bar{c} < \infty$, $i = 1, 2$, all m and y in some ergodic set. It would thus be sufficient at this point to define a utility function that satisfies Assumption 2 and which agrees with log utility on the set $[\underline{c},\bar{c}]$, and then to restrict the analysis to this ergodic set from this point on.

Define $\bar{y} = \sum_{j=1}^{2} y_j$ and $\bar{y}_i = y_i/\bar{y}$. Combine the resource constraint $\sum_{j=1}^{2} c_j(m,y) = \bar{y}$ with (2.14) to arrive at an equation in equilibrium prices:

$$\bar{y} = \sum_{j=1}^{2} \min\{m_j/p(m,y),\ 1/(\lambda_j(m,y)p(m,y))\}. \tag{2.17}$$

Note that (2.14), (2.15) and (2.17) imply $L(m,y) \in S^1$, and thus that money supply equals money demand.

3. The Equilibrium

This section proves the existence of an equilibrium by constructing a sequence whose limit solves (2.13)-(2.17). Each of the functions c, p, L, and φ is first written only in terms of λ, and then λ is written as the solution to a pair of equations only in λ. This pair of equations is used to define a nonlinear monotone operator that maps a compact subset of λ's into itself. Applying Tarski's fixed-point theorem for monotone maps then provides a particular monotone sequence of λ's that converges to an equilibrium.

Use (2.17) to write p as

$$p(m,y) = (1/\bar{y}) \sum_{j=1}^{2} \min\{m_j,\ 1/\lambda_j(m,y)\}, \tag{3.1}$$

and substitute this equation into (2.14) to write c as

$$c_i(m,y) = \frac{\min\{m_i,\ 1/\lambda_i(m,y)\}}{\sum_{j=1}^{2} \min\{m_j,\ 1/\lambda_j(m,y)\}}\ \bar{y}. \tag{3.2}$$

Substitute these equations into (2.15) to derive

$$L_i(m,y) = \bar{y}_i \sum_{j=1}^{2} \min\{m_j,\ 1/\lambda_j(m,y)\} + m_i - \min\{m_i,\ 1/\lambda_i(m,y)\}. \tag{3.3}$$

Along with (2.13), equations (3.1)-(3.3) define c, p, L, and φ solely in terms of λ.

Perhaps the most obvious way to advance is to substitute (3.3) into (2.16) to derive a pair of equations in λ, but it turns out to be more useful to proceed in a less direct way. To proceed from this point requires restricting the set of λ's, which in turn requires the following assumption on monetary policy.

Assumption 3: $0 < h(y',y) < \infty$, all y and y', and $\beta E_y[1/h(y',y)] < 1$, all y.

This assumption essentially ensures that the expected return on money not exceed the rate of time preference, which is an assumption that has become standard in working with cash-in-advance models.

Choose a $\beta^* < 1$ such that $\sup_y \beta E_y[1/h(y',y)] \leq \beta^*$. As long as h satisfies Assumption 3 such a β^* can be chosen. Define $B = 1/(1 - \beta^*) > 1$, and define the function $\lambda^*: S^1 \to \mathbb{R}_+^2$ as

$$\lambda_i^*(m) = B/m_i.$$

Note that λ^* is not a bounded function. Using λ^* define the set $\Gamma(S^1 \times Y) = \Gamma_1(S^1 \times Y) \times \Gamma_2(S^1 \times Y)$, where

$$\Gamma_i(S^1 \times Y) = \left\{ \begin{array}{l} \lambda_i : S^1 \times Y \to \mathbb{R}_+, \quad \lambda_i \text{ is continuous}, \\ 0 \leq \lambda_i(m,y) \leq \lambda_i^*(m), \\ 0 \leq \lambda_i(m,y) - \lambda_i(m',y),\ m_i \leq m_i'. \end{array} \right.$$

Equip $\Gamma(S^1 \times Y)$ with the usual pointwise partial ordering: $\lambda \le \hat{\lambda}$ if $\lambda_i(m,y) \le \hat{\lambda}_i(m,y)$, $i = 1$, 2, all m, y.

Define the function $G: S^1 \times Y \times \Gamma(S^1 \times Y) \to \mathbb{R}^2_+$ as

$$G_i(l,y;\lambda) = \beta E_y[\max\{1/l_i, \lambda_i(l,y')\}/h(y',y)].$$

Note that, in view of (2.16), the equilibrium functions λ and L satisfy $\lambda(m,y) = G(L(m,y),y;\lambda)$. Using (3.3) and G, define $L(m,y;\lambda)$ as the solution to

$$(3.4) \quad L_i(m,y;\lambda) = \bar{y}_i \sum_{j=1}^{2} \min\{m_j,\ 1/G_j(L(m,y;\lambda),y;\lambda)\} + m_i - \min\{m_i,\ 1/G_i(L(m,y;\lambda),y;\lambda)\}.$$

Lemma 1: *Under Assumptions* 1-3, *for any* $\lambda \in \Gamma(S^1 \times Y)$ *there exists a unique continuous* L *that satisfies* (3.4); *furthermore,* $L(m,y;\lambda) \in S^1$, *and for any* m *and* m' *in* S^1,

$$0 \le L_i(m,y;\lambda) - L_i(m',y;\lambda) \le m_i - m_i' \ \text{ if } \ m_i \ge m_i'.$$

Proof: Fix (m,y,λ). The solution $L(m,y;\lambda)$ consists of $(l_1,1-l_1)$ such that

$$Z(l_1,m_1,y;\lambda) = m_1 - \bar{y}_2 \min\{m_1,\ 1/G_1(l_1,1-l_1,y;\lambda)\} + \bar{y}_1 \min\{1 - m_1,\ 1/G_2(l_1,1-l_1,y;\lambda)\} - l_1$$

equals zero. $\lambda_i \in \Gamma_i(S^1 \times Y)$ is decreasing in m_i so G_i is decreasing in l_i, and thus Z is strictly decreasing in l_1. Z is also continuous in l_1, positive for $l_1 = 0$, and negative for $l_1 = 1$ so there exists a unique root $0 \le l_1 \le 1$. L is continuous since Z is continuous in m_1.

Z is increasing in m_1, hence L_1 is increasing in m_1. From (3.3) $m_1 - L_1(m,y;\lambda)$ is increasing in m_1 if

$$\bar{y}_2\min\{m_1,\ 1/G_1(L(m,y;\lambda),y;\lambda)\} - \bar{y}_1\min\{1-m_1,\ 1/G_2(L(m,y;\lambda),y;\lambda)\}$$

is increasing in m_1, which is true by the monotonicity properties of G and L. Since $L_2 = 1 - L_1$ a similar result holds for L_2. *Q.E.D.*

Use the solution L to (3.4) to define the nonlinear equation $\lambda(m,y) = G(L(m,y;\lambda),y;\lambda)$, which consists of a pair of equations only in λ. To solve this equation, define the operator F on $\Gamma(S^1 \times Y)$ such that $(F\lambda)(m,y) = G(L(m,y;\lambda),y;\lambda)$, which is equivalent to

$$(F\lambda)_i(m,y) = \beta E_y[\max\{1/L_i(m,y;\lambda),\ \lambda_i(L(m,y;\lambda),y')\}/h(y',y)]. \tag{3.5}$$

An equilibrium λ corresponds to a fixed point of F.

Lemma 2: *Under Assumptions* 1-3, *F is monotone on $\Gamma(S^1 \times Y)$.*

Proof: Choose an λ^a and λ^b, both in $\Gamma(S^1 \times Y)$, such that $\lambda^a \geq \lambda^b$. F is monotone if $G(L(m,y;\lambda^a),y;\lambda^a) \geq G(L(m,y;\lambda^b),y;\lambda^b)$, all (m,y) in $S^1 \times Y$. Suppose, for some (m,y) and some i, that $G_i(L(m,y;\lambda^a),y;\lambda^a) < G_i(L(m,y;\lambda^b),y;\lambda^b)$. Without loss of generality, suppose $i = 1$. Note that $G(l,y;\lambda^a) \geq G(l,y;\lambda^b)$, all (l,y), so $G_1(L(m,y;\lambda^a),y;\lambda^a) < G_1(L(m,y;\lambda^b),y;\lambda^b)$ implies $L_1(m,y;\lambda^a) > L_1(m,y,\lambda^b)$, which in turn implies $G_2(L(m,y;\lambda^a),y;\lambda^a) \geq G_2(L(m,y;\lambda^b),y;\lambda^b)$. In view of (3.4), these last three inequalities cannot all be true. *Q.E.D.*

The monotonicity of F can be used to prove the following lemma, which is needed to define a compact subset of $\Gamma(S^1 \times Y)$ that gets mapped into itself by F.

Lemma 3: *Under Assumptions* 1-3, *$F(\Gamma(S^1 \times Y)) \subset \Gamma(S^1 \times Y)$, $F(\lambda^*)$ is bounded, and there exists an $\varepsilon > 0$ such that $L(m,y;\lambda) \geq \varepsilon$, all $(m,y) \in S^1 \times Y$ and all $\lambda \in \Gamma(S^1 \times Y)$.*

Proof: Choose any $\lambda \in \Gamma(S^1 \text{x} Y)$. To prove $F(\lambda) \in \Gamma(S^1 \text{x} Y)$ note first that $F(\lambda)$ is continuous and since $L_i(m,y;\lambda)$ is increasing in m_i it follows from (3.5) that $F_i(\lambda)$ is decreasing in m_i. As F is monotone, the remaining condition $F(\lambda) \leq \lambda^*$ holds if $F(\lambda^*) \leq \lambda^*$. Note that

$$G_i(l,y;\lambda^*) = \beta E_y[\max\{1/l_i, B/l_i\}/h(y',y)] = \beta B E_y[1/h(y',y)]/l_i.$$

It needs to be shown that $\beta B E_y[1/h(y',y)]/L_i(m,y;\lambda^*) \leq B/m_i$, or that $L_i(m,y;\lambda^*) \geq \beta E_y[1/h(y',y)]m_i$. In view of the monotonicity properties of (3.4) that were exploited in the proof of Lemma 1, this inequality holds for L_1 if

$$\beta E_y[1/h(y',y)]m_1 \leq m_1 - \bar{y}_2 \min\{m_1, m_1/B\}, \; all \;\; m, y.$$

A sufficient condition for this inequality to hold is $\beta^* \leq 1 - 1/B$, which is satisfied by the definition of B. A corresponding argument holds for L_2. These results prove $F(\Gamma(S^1 \text{x} Y)) \subset \Gamma(S^1 \text{x} Y)$.

$F_i(\lambda^*)$ is bounded if $L_i(m,y;\lambda^*) > 0$, all m, y, and i. Consider first $i = 1$. Note that $L_1(m,y;\lambda^*) \geq L_1((0,1),y;\lambda^*)$, where $L_1((0,1),y;\lambda^*)$ solves

$$L_1((0,1),y;\lambda^*) = \bar{y}_1 \min\{1, (\beta B E_y[1/h(y',y)])^{-1}(1 - L_1((0,1),y;\lambda^*))\}.$$

Since $\bar{y}_1(\beta B E_y[1/h(y',y)])^{-1} \geq \alpha(\beta^* B)^{-1} > 0$, clearly $L_1((0,1),y;\lambda^*) > 0$. A corresponding argument holds for L_2.

Denote the bound on $F(\lambda^*)$ by γ and let $\varepsilon = \alpha/\gamma > 0$. From (3.4) $L_1((0,1),y;\lambda) \geq \bar{y}_1/\gamma \geq \varepsilon$ and thus $L_1(m,y;\lambda) \geq \varepsilon$. Again, a corresponding argument holds for L_2.

Q.E.D.

Lemma 3 states that even though λ^* is unbounded, $F(\lambda^*) \leq \lambda^*$ is bounded. It is an odd feature of F that in general for no bounded constant function k is it true that $F(k) \leq k$.

Define $\bar{\lambda} = F(\lambda^*)$ and choose an $\varepsilon > 0$ such that $L(m,y;\lambda) \geq \varepsilon$, all $\lambda \in \Gamma(S^1 \times Y)$. According to Lemma 3 such an ε can be chosen. Define the subset $\varGamma(S^1 \times Y) = \varGamma_1(S^1 \times Y) \times \varGamma_2(S^1 \times Y)$ of $\Gamma(S^1 \times Y)$, where

$$\varGamma_i(S^1 \times Y) = \begin{cases} \lambda_i : S^1 \times Y \to \mathbb{R}_+, \quad \lambda_i \text{ is continuous}, \\ 0 \leq \lambda_i(m,y) \leq \bar{\lambda}_i(m,y), \\ 0 \leq \lambda_i(m,y) - \lambda_i(m',y) \leq \varepsilon^{-2}|m_i - m_i'|, \; m_i \leq m_i'. \end{cases}$$

Equip $\varGamma_i(S^1 \times Y)$ with the sup norm and $\varGamma(S^1 \times Y)$ with the product topology. $\varGamma(S^1 \times Y)$ is an equicontinuous family of functions that are defined on a compact set. We can now prove the main theorem.

THEOREM 4: *Under Assumptions* 1-3, *there exists a fixed point* $\lambda = F(\lambda)$, $\lambda \in \varGamma(S^1 \times Y)$. *Moreover, both* $F^n(0)$ *and* $F^n(\bar{\lambda})$ *uniformly converge to a fixed point, say* λ_{min} *and* λ_{max} *respectively, and if* λ *is any other fixed point in* $\varGamma(S^1 \times Y)$, *then* $\lambda_{min} \leq \lambda \leq \lambda_{max}$.

PROOF: Lemma 2 proves that F is monotone. F is continuous since $\varGamma(S^1 \times Y)$ is an equicontinuous family of functions defined on a compact set, and for any sequence $\{\lambda_i\}$ that converges to some λ, where $\lambda_i, \lambda \in \varGamma(S^1 \times Y)$, the sequence $\{F(\lambda_i)\}$ converges pointwise to $F(\lambda)$. To prove $F(\varGamma(S^1 \times Y)) \subset \varGamma(S^1 \times Y)$, choose any $\lambda \in \varGamma(S^1 \times Y)$, any $i = 1, 2$, any m, m' in S^1 such that $m_i \leq m_i'$, and any $y \in Y$. By Lemma 3 and the monotonicity of F, $0 \leq F(\lambda) \leq \bar{\lambda}$ and $0 \leq (F\lambda)_i(m,y) - (F\lambda)_i(m',y)$. To prove

$$(F_i\lambda)(m,y) - (F_i\lambda)(m',y) \leq \varepsilon^{-2}|m_i - m_i'|,$$

note that

$$(F_i\lambda)(m,y) - (F_i\lambda)(m',y)$$

$$\leq \beta^* E_y[\max\{1/L_i(m,y;\lambda),\lambda_i(L(m,y;\lambda),y')\} - \max\{1/L_i(m',y;\lambda),\lambda_i(L(m',y;\lambda),y')\}].$$

In the expectations operator there are four combinations that may result depending on which argument is selected by each of the two max operators. Each combination satisfies the desired inequality:

$$1/L_i(m,y;\lambda) - 1/L_i(m',y;\lambda) \leq \varepsilon^{-2}(L_i(m,y;\lambda) - L_i(m',y;\lambda)) \leq \varepsilon^{-2}|m_i - m_i'|,$$

$$\lambda_i(L(m,y;\lambda),y') - \lambda_i(L(m',y;\lambda),y') \leq \varepsilon^{-2}(L_i(m,y;\lambda) - L_i(m',y;\lambda)),$$

$$1/L_i(m,y;\lambda) - \lambda_i(L(m',y;\lambda),y') \leq 1/L_i(m,y;\lambda) - 1/L_i(m',y;\lambda),$$

$$\lambda_i(L(m,y;\lambda),y') - 1/L_i(m',y;\lambda) \leq 1/L_i(m,y;\lambda) - 1/L_i(m',y;\lambda).$$

The latter two inequalities used the property that the terms on the left are always positive (recall $m_i \leq m_i'$).

F is thus monotone, continuous, and maps a partially-ordered compact set into itself. The assumptions of Tarski's fixed point theorem are met, which completes the proof. ***Q.E.D.***

While Theorem 4 does not assert there only exists one fixed point of F, it does assert that if $F^n(0)$ and $F^n(\lambda)$ converge to the same fixed point then there exists no other fixed point in $\Gamma(S^1 \times Y)$. This condition can be verified in practice.

4. Velocity and Monetary Policy

Towards the end of this section explicit solutions constructed along the lines suggested by Theorem 4 are used to study simulated time paths of money, income, and velocity. These simulations reveal that this model exhibits the following two features: (i) the velocity of money is appreciably less than its institutional maximum of one, and (ii), the dependence of velocity on monetary policy is quantitatively important. Both results are quantitative in nature, so it is difficult to prove theoretically that they necessarily follow from this type of model, and for this reason it is difficult to get a precise sense as to why this model exhibits quantitatively-important effects while a similar representative-household setting does not. What can be proven, however, is that under some circumstances velocity is less than one, and that velocity is higher in economies with higher monetary growth. This is in contrast to velocity always equaling one in the representative-household version of this economy, which is essentially Svensson's (1985) economy restricted to log utility households. The role of individual uncertainty in obtaining these results is made explicit in their proof, and in this way one can begin to understand the link between individual uncertainty and velocity. This link is further developed in a less formal argument in terms of money's insurance value.

Velocity, Individual Uncertainty, and Monetary Growth

The velocity of money $v(m,y)$ corresponding to a fixed point λ of F can in general be written as

$$v(m,y) = \sum_{j=1}^{2} \min\{m_j, 1/\lambda_j(m,y)\}. \tag{4.1}$$

To prove that under some circumstances velocity is strictly less than one, fix any $\beta > 0$, suppose $h(y',y)$ is constant at $\bar{h} > \beta$, and, for a constant aggregate output $\bar{y} = 1$, suppose the three divisions of this output across households, $(\alpha,\ 1 - \alpha)$, $(1/2,\ 1/2)$, and $(1 - \alpha,\ \alpha)$, occur with probabilities π_1, π_2, and π_3 respectively. The parameters α and π reflect the level of individual uncertainty. To standardize the comparison with the representative-household economy, consider velocity when money and income happen to be evenly distributed. What will be proven is if $\alpha < (1/2)(\beta/\bar{h})$ and

$$\pi_1 > (1 - \beta/\bar{h}) \div \left[\left[\frac{(1/2)(1 + \alpha)}{((1/2)(\beta/\bar{h}) - \alpha) + (1/2)(1 + \alpha)}\right]^{-1} - \beta/\bar{h}\right],$$

then for any fixed point λ of F it follows that $v(1/2,\bar{y}/2) < 1$ (note that as $\bar{h}$ approaches β these restrictions approach $\alpha < 1/2$ and $\pi_1 > 0$). Due to the monotonicity of F, if λ is a fixed point then $\lambda \geq F(F(0)) = F^2(0)$, and so

$$v(1/2,\bar{y}/2) \leq \sum_{j=1}^{2} \min\{1/2,\ 1/(F^2 0)_j(1/2,\bar{y}/2)\}.$$

The proof is complete if $(F^2 0)_1(1/2,\bar{y}/2) > 2$. It is straightforward to show

$$(F0)_1(1/2,y) = (\beta/\bar{h})/L_1(1/2,y;0),$$

where $L_1(1/2,y;0)$ solves

$$L_1(1/2,y;0) = 1/2 - \bar{y}_2\min\{1/2,\ L_1(1/2,y;0)/(\beta/\bar{h})\} + \bar{y}_1\min\{1/2,(1-L_1(1/2,y;0))/(\beta/\bar{h})\}.$$

The solution is $L_1(1/2,\alpha,1-\alpha;0) = (1/2)((1+\alpha)/(1 + (1-\alpha)(\beta/\bar{h})^{-1}))$, $L_1(1/2,\bar{y}/2;0) = 1/2$, and $L_1(1/2,1-\alpha,\alpha;0) = 1 - L_1(1/2,\alpha,1-\alpha;0)$. Since $L_1(1/2,\bar{y}/2;0) = 1/2$, it follows that

$$(F^2 0)_1(1/2,\bar{y}/2) = (\beta/h)E_{\bar{y}/2}[\max\{2, (F0)_1(1/2,y')\}]$$

$$= 2\left[\left[\frac{(1/2)(1 + \alpha)}{((1/2)(\beta/h) - \alpha) + (1/2)(1 + \alpha)}\right]^{-1} \pi_1 + (\beta/h)(1 - \pi_1)\right].$$

The assumptions on α and π_1 ensure $(F^2 0)_1(1/2,\bar{y}/2) > 2$.

Velocity is thus less than one for an apparently wide range of individual uncertainty. For example, if $\beta = .9975$, $h = 1$, and $\alpha = .45$, then velocity is less than one for any $\pi_1 >$ .0358.

To prove that higher monetary growth leads to higher velocity, consider two economies, a and b, such that $h_a(y',y) < h_b(y',y)$, all y, y' (functions for economy a are denoted by a subscript a, and likewise for economy b). In economy a monetary growth is uniformly lower than that for economy b. The following lemma is essential for deriving the result.

LEMMA 5: *For any* $\lambda \in \Gamma_a(S^1 \times Y) \cap \Gamma_b(S^1 \times Y)$, $F_a(\lambda) \geq F_b(\lambda)$.

PROOF: Note that $G_a(l,y;\lambda) \geq G_b(l,y;\lambda)$, and a proof similar to that of Lemma 2 establishes this result. *Q.E.D.*

It follows from Lemma 5 that $F_a(0) \geq F_b(0)$, $F_a(F_a(0)) \geq F_b(F_a(0)) \geq F_b(F_b(0))$, and by induction that $F_a^n(0) \geq F_b^n(0)$. In the limit, then, the minimum fixed point of F for economy a is higher than that of economy b. On the other hand, starting from the maximum fixed point of F for economy b, say $\hat{\lambda}_b$, it also follows from Lemma 5 that $F_a(\hat{\lambda}_b) \geq F_b(\hat{\lambda}_b) = \hat{\lambda}_b$, and thus that the maximum fixed point of F for economy a is higher than that for economy b. In this sense, as seen from (4.1), a monetary policy of uniformly higher monetary growth leads to higher velocity.

The Precautionary Demand for Money

One's intuition suggests that households only hold excess cash balances if its insurance value offsets its low return. To see this formally, note that since cash is perfectly liquid, and as derived from eqs. (2.9)-(2.11), excess cash is held by type i households when

$$u'(c_i(m,y)) = \beta E_y[u'(c_i(L(m,y),y'))(p(m,y)/p(L(m,y),y')h(y',y))],$$

and no excess cash is held when

$$u'(c_i(m,y)) \geq \beta E_y[u'(c_i(L(m,y),y'))(p(m,y)/p(L(m,y),y')h(y',y))].$$

Money's rate of return is captured by the rate of deflation $(p(m,y)/p(L(m,y),y')h(y',y))$, and money's insurance value is captured by the correlation between (the marginal utility of) consumption and money's ex post real return. A necessary condition for excess cash balances to be held is either a relatively high return on money, or, given the return on money, a relatively low correlation between consumption and money's return.

The ability of individual uncertainty to lower the correlation between household consumption and the return on money gives it the ability to lower velocity. Without individual uncertainty consumption next period equals $\bar{y}'$ and money's ex post real return equals $\bar{y}'/\bar{y}h(\bar{y}',\bar{y})$. Given a monetary policy that satisfies a condition like Assumption 3, the positive correlation between consumption and money's rate of return that stems from the effect of output shocks on prices is sufficient to drive the level of precautionary money balances to zero.

Some Simulations

At the level of theory what has been proven is that velocity can be less than one and that velocity increases under a monetary policy of uniformly higher monetary growth. Associating some quantitative magnitude to these two statements requires studying equilibria that are explicitly computed for particular values of the model's parameters. Throughout this exercise $\beta = .9975$, which, if one thinks of the model's time period as monthly, corresponds to a 3 percent annual rate of time preference. For the stochastic process describing the endowments, assume

$$y_{it} = \bar{y}_t e^{z_{it}}/(1 + e^{z_{it}}),$$

where $z_{1t} + z_{2t} = 0$ and thus $\bar{y}_t$ equals the aggregate endowment. For studying velocity only $y_{it}/\bar{y}_t$ matters, so the stochastic process for $\bar{y}_t$ drops out (in part, this result is due to the log utility assumption). Assume z_{it} is iid over time and is drawn from a discrete distribution that approximates the Normal(0, σ_z^2) distribution.[7]

To isolate the effect of individual uncertainty on the velocity of money, Table 1 reports average velocity $\bar{v}$ for a constant money supply and various values of σ_z.[8] The statistics in this table are based on a simulated time series of 10,000 starting from a uniform distribution of money and income. The columns labeled cv_y and cv_v are the coefficients of variation of household income ($y_i/\bar{y}$) and velocity. Velocity falls from .9392 for $\sigma_z = .1$ to .7218 for $\sigma_z = .3$, so that on average a one percentage-point increase in σ_y leads to a roughly two

[7]For a finite number n, the possible values $(z_1, z_2, \ldots, z_n)$ are the points from a Hermite-Gauss quadrature rule and the probabilities $(\pi_1, \pi_2, \ldots, \pi_n)$ are the weights from this rule. For the following simulations $n = 11$. Tauchen (1987) generalized this approach to Markov processes.

[8]Both limits $F^n(0)$ and $F^n(\lambda^*)$ were computed and found to be the same. Some of the computational details of this approach are described in Coleman (1990).

percent decrease in velocity.

Table 1

Average Velocity and Household-Specific Uncertainty

Constant Money Supply, $h = 1$

σ_z	cv_y	$\bar{v}$	cv_v
.1	.0502	.9392	.0102
.2	.0998	.8317	.0170
.3	.1480	.7218	.0219

With velocity well away from its upper limit ($\bar{v}$ = .8317 for cv_y = .0998), this model opens up the possibility that velocity responds to a wide variety of forces. To study how average velocity differs under various constant monetary growth rates, Table 2 reports average velocity for three values of h (again, these statistics are based on a simulated time series of 10,000 starting with a uniform distribution of money and income) The column labeled h_a is simply the annual money growth factor (h^{12}). As the annual monetary growth rate increases from 0 to 5 percent velocity increases from .8317 to .9048, and as monetary growth increases to 10 percent velocity increases to .9317. A one-percentage point increase in the average annual money growth rate thus leads to a roughly 1 1/2 percent (1.49%) increase in velocity for relatively low levels of monetary growth, and a roughly 1/2 percent (.53%) increase in velocity for relatively high levels of monetary growth.[9] The one percent mentioned in the introduction is the average over these two numbers.

[9]In a somewhat different terminology, the semi-elasticity of velocity with respect to the average money growth rate is .0149 and .0053 respectively.

Table 2

Average Velocity and Monetary Policy

$\sigma_z = .2$

h	h_a	$\bar{v}$	cv_v
1.000	1.000	.8317	.0170
1.004	1.049	.9048	.0207
1.008	1.100	.9317	.0220

Tables 1 and 2 report properties of the distribution of velocity at any point in time, but studying the dynamics between velocity and money requires knowledge of the joint distribution of velocity at various points in time and, for a stochastic monetary growth rate, the joint distribution of monetary growth and current and future values of velocity. Table 3 reports velocity's autocorrelation function for the three constant money growth rates in Table 2. Although monetary growth is constant and income is iid, velocity is significantly autocorrelated (for $h = 1$ velocity's autocorrelation at lag 1 is .72). This is due to the distribution of money being a state variable, and evidently the evolution of this state variable is fairly persistent.[10] Following a negative income shock which leads households to consume out of money balances, households reaccumulate money gradually. Somewhat paradoxically, although for different reasons, for this type of example velocity is not autocorrelated (it is constant) when there is either only one type of household or a continuum of different households. In both cases the distribution of money balances does not vary over time.

[10]The autocorrelation function of m_1 is similar to that of v.

Table 3

Velocity's Autocorrelation Function

$\sigma_z = .2$

Lag	$h = 1.000$	$h = 1.004$	$h = 1.008$
1	.72	.61	.52
2	.40	.24	.17
3	.24	.10	.05
4	.14	.05	.02
5	.08	.03	.01
6	.05	.01	.00

To study the joint distribution of monetary growth and velocity, consider a discrete state Markov process for the monetary growth rate with transition matrix

$$\begin{array}{r} \\ h_1 = 1.000 \\ h_2 = 1.004 \\ h_3 = 1.008 \end{array} \begin{array}{c} \begin{array}{ccc} h_1 & h_2 & h_3 \end{array} \\ \begin{bmatrix} \gamma & (3/4)(1-\gamma) & (1/4)(1-\gamma) \\ (1/2)(1-\gamma) & \gamma & (1/2)(1-\gamma) \\ (1/4)(1-\gamma) & (3/4)(1-\gamma) & \gamma \end{bmatrix} \end{array}.$$

Let h_t be the monetary growth rate between the current period and the next period, which is known at time t. For $\gamma = .9$ (which implies there is a roughly 30 percent chance that a choice of h will remain in effect for one year) Table 4 reports the autocorrelation function for both money and velocity along with the crosscorrelation function for velocity with lagged money growth rates. The contemporaneous correlation between monetary growth and velocity is .47 (recall that h_t is the money growth rate between t and t+1), and the correlation between lagged money growth rates and velocity dies out very slowly (at about the same rate as the autocorrelation function for monetary growth). Although the full range of possibilities is not explored, evidently the dependence of velocity on monetary growth displayed in Figure 2 can be exploited in a stochastic monetary growth setting to obtain a wide variety of dynamic relationships between velocity and monetary growth.

Table 4

Auto- and Cross-Correlation Functions for Money and Velocity

$\sigma_z = .2, \gamma = .9$

Lag	ACF(h)	ACF(v)	CCF(lag h with v)
0	1.00	1.00	.54
1	.88	.69	.47
2	.77	.41	.41
3	.67	.28	.36
4	.59	.21	.31
5	.52	.17	.27
6	.46	.14	.24

5. Concluding Remarks

Although this paper sought to explain why household's hold precautionary money balances, in some sense this demand is a fiction as we no more observe it than we do a risk premium. We think there exists such a demand, but our only evidence is its ability to explain the observed relationship between real money balances and a variety of factors. What we observe is summarized by a joint distribution that captures the dynamics of such a relationship, and with this in mind this paper studied the dynamics between velocity and a stochastic monetary growth rate that a precautionary demand for money induces. That this demand yields a rich dynamic relationship between velocity and monetary policy seems promising.

In this model money provided the only means to smooth consumption, so it is natural to criticize this setup as failing to acknowledge the many other ways in which households smooth consumption. Debt contracting, or for that matter any other financial contract, could fundamentally alter the equilibrium. Without an explicit model it is difficult to address the many issues that come to mind, but a sensible outcome is nevertheless one that retains some individual uncertainty in which money plays a unique role for insuring against short-term risks. In addition to its assumed transactions role money would then retain its precautionary role, which this paper has shown can be a quantitatively important component of the demand for money within a cash-in-advance context.

REFERENCES

Benveniste, L. M. and J. A. Scheinkman (1979): "On the Differentiability of the Value Function in Dynamic Models of Economics," *Econometrica*, 47, 727-732.

Bewley, Truman (1980): "The Optimum Quantity of Money," In *Models of Monetary Economics*, eds. J. H. Kareken and N. Wallace, pp. 169-210. Minneapolis: Federal Reserve Bank of Minneapolis.

Clower, Robert W. (1967): "A Reconsideration of the Microfoundations of Monetary Theory," *Western Economic Journal*, 6, 1-8.

Coleman, Wilbur John II (1990): "Solving the Stochastic Growth Model by Policy-Function Iteration," *Journal of Business and Economic Statistics*, 8, 27-29.

Foley, Duncan K. and Martin F. Hellwig (1975): "Asset Management with Trading Uncertainty," *Review of Economic Studies*, 42, 327-346.

Giovannini, Alberto and Pamela Labadie (1989): "Asset Prices and Interest Rates in Cash-In-Advance Models," NBER Working Paper No. 3109.

Grossman, Sanford and Laurence Weiss (1983): "A Transactions-Based Model of the Monetary Transmission Mechanism," *The American Economic Review*, 73, 871-880.

Hodrick, Robert, Narayana Kocherlakota, and Deborah Lucas (1991): "The Variability of Velocity in Cash-In-Advance Models," *Journal of Political Economy*, 99, 358-384.

Imrohoroglu, Ayse and Edward C. Prescott (1991): "Seignorage as a Tax: A Quantitative Evaluation," Federal Reserve Bank of Minneapolis, Research Department Staff Report 132.

Lucas, Robert E., Jr. (1980): "Equilibrium in a Pure Currency Economy," *Economic Inquiry*, 28, 203-220.

__________ (1982): "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, 10, 335-360.

Lucas, Robert E., Jr. and Nancy L. Stokey (1987): "Money and Interest in a Cash-in-Advance Economy," *Econometrica*, 55, 491-514.

Rotemberg, Julio J. (1984): "A Monetary Equilibrium Model with Transactions Costs," *Journal of Political Economy*, 92, 40-58.

Scheinkman, Jose A. and Laurence Weiss (1986): "Borrowing Constraints and Aggregate Economic Activity," *Econometrica*, 54, 23-45.

Stokey, Nancy L., Robert E. Lucas, Jr., and Edward C. Prescott (1989): *Recursive Methods in Economic Dynamics*. Cambridge, Massachusetts: Harvard University Press.

Svensson, Lars (1985): Money and Asset Prices in a Cash-in-Advance Economy, *Journal of Political Economy*, 93, 919-944.

Tauchen, George (1987): "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models," unpublished manuscript, Duke University, Dept. of Economics.

International Finance Discussion Papers

IFDP NUMBER	TITLES	AUTHOR(s)
	1991	
399	Precautionary Money Balances with Aggregate Uncertainty	Wilbur John Coleman II
398	Using External Sustainability to Forecast the Dollar	Ellen E. Meade Charles P. Thomas
397	Terms of Trade, The Trade Balance, and Stability: The Role of Savings Behavior	Michael Gavin
396	The Econometrics of Elasticities or the Elasticity of Econometrics: An Empirical Analysis of the Behavior of U.S. Imports	Jaime Marquez
395	Expected and Predicted Realignments: The FF/DM Exchange Rate during the EMS	Andrew K. Rose Lars E. O. Svensson
394	Market Segmentation and 1992: Toward a Theory of Trade in Financial Services	John D. Montgomery
	1990	
393	*Post* Econometric Policy Evaluation A Critique	Beth Ingram Eric M. Leeper
392	Mercantilism as Strategic Trade Policy: The Anglo-Dutch Rivalry for the East India Trade	Douglas A. Irwin
391	Free Trade at Risk? An Historical Perspective	Douglas A. Irwin
390	Why Has Trade Grown Faster Than Income?	Andrew K. Rose
389	Pricing to Market in International Trade: Evidence from Panel Data on Automobiles and Total Merchandise	Joseph E. Gagnon Michael M. Knetter
388	Is the EMS the Perfect Fix? An Empirical Exploration of Exchange Rate Target Zones	Robert P. Flood Andrew K. Rose Donald J. Mathieson
387	Estimating Pass-through: Structure and Stability	William R. Melick
386	International Capital Mobility: Evidence from Long-Term Currency Swaps	Helen Popper
385	Is National Treatment Still Viable? U.S. Policy in Theory and Practice	Sydney J. Key

Please address requests for copies to International Finance Discussion Papers, Division of International Finance, Stop 24, Board of Governors of the Federal Reserve System, Washington, D.C. 20551.

International Finance Discussion Papers

IFDP NUMBER	TITLES 1990	AUTHOR(s)
384	Three-Factor General Equilibrium Models: A Dual, Geometric Approach	Douglas A. Irwin
383	Modeling the Demand for Narrow Money in the United Kingdom and the United States	David F. Hendry Neil R. Ericsson
382	The Term Structure of Interest Rates in the Onshore Markets of the United States, Germany, and Japan	Helen Popper
381	Financial Structure and Economic Development	Ross Levine
380	Foreign Currency Operations: An Annotated Bibliography	Hali J. Edison Dale W. Henderson
379	The Global Economic Implications of German Unification	Lewis S. Alexander Joseph E. Gagnon
378	Computers and the Trade Deficit: The Case of the Falling Prices	Ellen E. Meade
377	Evaluating the Predictive Performance of Trade-Account Models	Jaime Marquez Neil R. Ericsson
376	Towards the Next Generation of Newly Industrializing Economies: The Roles for Macroeconomic Policy and the Manufacturing Sector	Catherine L. Mann
375	The Dynamics of Interest Rate and Tax Rules in a Stochastic Model	Eric M. Leeper
374	Stock Markets, Growth, and Policy	Ross Levine
373	Prospects for Sustained Improvement in U. S. External Balance: Structural Changes *versus* Policy Change	Catherine L. Mann
372	International Financial Markets and the U.S. External Balance	Deborah Danker Peter Hooper
371	Why Hasn't Trade Grown Faster Than Income? Inter-Industry Trade Over the Past Century	Joseph E. Gagnon Andrew K. Rose
370	Contractionary Devaluation with Black Markets for Foreign Exchange	Steven B. Kamin