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David M. Arseneau

Sanjay K. Chugh*

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David M. Arseneau †
Federal Reserve Board

Sanjay K. Chugh‡
University of Maryland
and Federal Reserve Board

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Abstract

A growing body of evidence suggests that an important reason why firms do not change prices nearly as much as standard theory predicts is out of concern for disrupting ongoing customer relationships because price changes may be viewed as "unfair." Existing models that try to capture this concern regarding price-setting are all based on goods markets that are fundamentally Walrasian. In Walrasian goods markets, transactions are spot, making the idea of ongoing customer relationships somewhat difficult to understand. We develop a simple dynamic general equilibrium model of a search-based goods market to make precise the notion of a customer as a repeat buyer at a particular location. In this environment, the transactions price plays a distributive role as well as an allocative role. We exploit this distributive role of prices to explore how concerns for fairness influence price dynamics. Using pricing schemes with bargaining-theoretic foundations, we show that the particular way in which a "fair" outcome is determined matters for price dynamics. The most stark result we find is that complete price stability can arise endogenously. These are issues about which models based on standard Walrasian goods markets are silent.

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†E-mail address: david.m.arseneau@frb.gov.
‡E-mail address: chughs@econ.umd.edu.

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1 Introduction

A growing body of evidence suggests that an important reason why firms do not change prices nearly as much as standard theory predicts is out of concern for disrupting ongoing customer relationships because price changes may be viewed as "unfair." Several recently-developed models try to capture this concern regarding price-setting, but in all of them the underlying model of the goods market is Walrasian. In Walrasian goods markets, transactions are spot, making the idea of ongoing customer relationships somewhat difficult to understand. Instead, models that feature explicit bilateral relationships between customers and firms seem to be called for in order to study the interactions between customer relationships and price rigidities. We develop a simple model that embeds a search-based goods market in an otherwise-standard dynamic general equilibrium model, making the notion of a *customer* as a repeat buyer at a particular location well-defined. In this framework, the transactions price (the terms of trade) plays a distributive role as well as an allocative role. We exploit this distributive role of prices to explore how concerns for fairness influence price dynamics. The most stark result we find is that if pricing is guided by a "fairness norm," complete price stability can arise endogenously. More generally, the consequences of menu costs of price adjustment on the dynamics of prices and allocations depend crucially on the manner in which price and quantity in specific relationships are determined, something about which standard models based on Walrasian goods markets are silent.

The two foundations of our model are a specific notion of customer relationships and menu costs of changing prices in those customer markets. In our model, customer relationships are valuable to both consumers and firms because of search frictions that each must overcome before goods trade can occur. The presence of search frictions leads to a surplus when a customer and a firm meet, and the parties must decide how to share the local monopoly rents. Regarding menu costs, we do not claim we have an explanation any deeper than existing ones for why there may be costs of changing prices; for convenience, such costs can be thought of in the typical fashion of costs associated with recording, reporting, and implementing new prices. By situating menu costs in a clearly-defined concept of a customer relationship, however, we are able to show that the consequences of price rigidities as typically formulated may depend critically on how prices and quantities traded are determined in specific relationships.

We focus on bargaining schemes as the mechanism by which prices and quantities are determined in customer markets. Despite using the formalisms of bargaining theory, we do not need to take literally the idea that customers and firms haggle over prices in every meeting. Rather, it seems to us that the idea that customers wield some "bargaining power" accords with the evidence of Blinder et al (1998) and others that firms often try to avoid upsetting their existing customers.

Given this interpretation, we adopt three bargaining protocols to pin down terms of trade between customers and firms: Nash bargaining, proportional bargaining, and a pricing system that we refer to as fair bargaining. We adopt Nash bargaining as a benchmark because of its familiarity: it has recently become relatively well-understood in macroeconomics due to the ongoing explosion of quantitative labor search models that employ it. Both proportional bargaining and fair bargaining implement an idea of a fairness norm under which parties always split the surplus in a fixed proportion, a feature that Nash bargaining does not always respect. The main difference between proportional bargaining and fair bargaining is the manner in which parties arrive at the fair outcome. Proportional bargaining assumes that the fair outcome is achieved through a mechanical sharing rule. In contrast, our notion of fair bargaining, although not axiomatic like the Nash and proportional outcomes, attempts to retain Nash bargaining's strategic foundation by imposing fairness as a constraint on the standard Nash optimization problem.

Akerlof (2007) makes the case that incorporating norms in macroeconomic models may be an evolutionary step for the field. We view our fairness norm, whether captured through proportional bargaining or fair bargaining, as in this spirit. We also view our idea as complementary to Rotemberg (2005, 2006), who has also stressed the notion that modeling fairness in pricing may be important. The crucial way in which our models of fair-pricing differ from Rotemberg (2005, 2006) is that we embed fairness as a feature of the trading structure of the environment, rather than altering preferences to account for it.

In our model, if there are no menu costs, the Nash-bargaining outcome, the proportional-bargaining outcome, and the fair-bargaining outcome all coincide. In the presence of menu costs, however, the three bargaining protocols imply quite different dynamics of prices and allocations. Under proportional bargaining, menu costs turn out to be completely irrelevant for both quantity and price dynamics, which seems to accord with survey evidence, such as Blinder et al (1998) and Fabiani et al (2006), that menu costs are not a very important friction in practice. Under fair bargaining, prices always remain at their steady-state values and dynamic allocations are completely unaffected by price rigidity. The key to understanding the dynamics under both proportional bargaining and fair bargaining is the fact that under Nash bargaining, price movements cause a time-varying wedge between short-run and long-run shares of the surplus accruing to customers and firms. With a fairness norm, parties eliminate such wedges, but the way in which the wedges are eliminated matters. If the wedges are eliminated according to proportional bargaining's mechanical sharing rule, customers and firms efficiently and equally share the consequences of menu costs and are able to engineer the zero-menu-cost Nash outcome. On the other hand, if the wedges are eliminated with the strategic considerations of fair bargaining in the background, menu costs are borne entirely by firms and eliminating the wedges requires complete price stability.

Due to the presence of local monopoly rents, our model also has something to say about markup behavior irrespective of menu costs. There lately has been a surge of interest in developing models in which markups are endogenously time-varying for reasons other than the presence of price rigidities.[1] Our flexible-price models deliver a time-varying markup; moreover, the flexible-price markup is countercyclical with respect to demand shocks, which seems to accord with empirical evidence and which has been the attention of much modeling effort. However, with fair bargaining and the presence of menu costs, the markup is constant, which is simply a reflection of the fact that both prices and marginal cost are time-invariant under fair bargaining.

In terms of bringing to bear data on our model, we exploit a central idea captured by our model: firms and consumers expend resources looking for trading partners. In our model, firms direct resources towards advertising in order to attract customers, and shoppers spend time looking for and purchasing goods from firms. Empirical evidence shows that the resources expended in such search activities are not negligible. Firm expenditures on advertising constitute over two percent of GDP, and time-use surveys show that individuals spend an average of about one hour per day shopping. Using such evidence, we can calibrate two deep features of our model. A by-product of our structure is that our model reproduces remarkably well the cyclical dynamics of aggregate advertising behavior in U.S. data.

We articulate our ideas in a non-monetary model, meaning the price dynamics on which we focus are those of real (relative) prices. It is apparent that much interest would lie in whether and to what extent our results carry over to monetary environments. We have reason to believe that the crucial aspects of our results — namely, that the consequences of menu costs depend critically on how customers and firms determine prices and quantities and, in particular, the conclusion that some trading arrangements would lead to the endogenous emergence of price stability — would carry over to monetary economies because the core mechanisms at work in our environment do not depend on things being cast in nominal or real terms. However, adding a monetary dimension to our model may not be as straightforward as imposing an ad-hoc cash-in-advance constraint or other typical monetary formulation used in the literature because once customer relationships are modeled explicitly, we may have to be careful about issues such as which consumers carry cash, which firms require payment in cash, etc. A monetary extension seems a logical next step; here, though, we concentrate on understanding some basic principles of the interactions between price rigidity and customer relationships.

Hall (2007) takes a very similar view of product markets as we take here and does use it to think about monetary policy issues. As in our model, the price at which goods change hands in Hall's (2007) model plays a distributive role in additional to the standard allocational role.

[1] Some examples are Jaimovich (2006), Ravn, Schmitt-Grohe, and Uribe (2006), and Gust, Leduc, and Vigfusson (2006).

Different prices inside a customer relationship achieve different distributions of the surplus between the consumer and the firm, but this does not affect the underlying efficiency of a trade. This admits the possibility of, in Hall's (2007) language, equilibrium sticky prices in customer markets. Given what we perceive as a growing sense of frustration with pricing models currently used in macroeconomic models, stemming from the growing body of micro pricing facts that challenge standard time-dependent or state-dependent pricing rules, allowing a distributional role for prices, as both our model and Hall's (2007) model do, may be a useful new direction for macroeconomic models. It seems to us that allowing this additional role for prices accords better with the idea that firms do not re-set prices out of worry for upsetting customers than do standard views of price rigidity. Besides the difference in focus on monetary versus non-monetary issues, we think another aspect of our model that sets it apart from Hall's (2007) is that we embed it from the start in a fully-articulated, quantitative DSGE environment, making comparisons with predictions of existing DSGE models straightforward.

Indeed, there lately has been a general surge of interest in developing simple structures of customer-firm interactions that can be tractably incorporated into state-of-the-art quantitative macroeconomic models. Our work here also falls into this broad category. A few recent examples of work in this broadly-defined area are the deep habits models of Ravn, Schmitt-Grohe, and Uribe (2006) and Nakamura and Steinsson (2007) and the switching-cost model of Kleschelski and Vincent (2007). As we mentioned, in terms of some basic motivation — the idea that fairness norms may interact with or may lead to price rigidity — the studies by Rotemberg (2005, 2006) are the closest in spirit to what we set out to achieve here. We view our work as complementary to all these recent efforts because we model customer relationships and fairness concerns through trading arrangements rather than essentially just through preferences. One could of course go back much further and tie ideas to the rich customer-markets literature, which includes studies by, among many others, Diamond (1971) and Klemperer (1995). Okun (1981) voices some of the ideas — namely, that search frictions in goods markets may have important consequences for aggregate phenomena — that we formalize through a modern search and matching framework. We certainly cannot do justice to the rich history of thought on these topics.

The rest of our paper is organized as follows. In Section 2, we lay out our baseline model with standard Nash bargaining, proportional bargaining, and fair bargaining. In the baseline model, only an extensive margin of consumption exists, and bargaining occurs just over the price paid by the customer. We provide some partial equilibrium analytics in Section 3, which illustrate the core forces at work in our environment. We examine the fully general equilibrium baseline model's numerical properties in Section 4, including a discussion of how and why fair bargaining leads to complete price rigidity in the face of arbitrarily small menu costs of price adjustment.

In Section 5, we enrich our environment to allow for bargaining over both price and quantity in a given customer relationship, thus allowing for extensive and intensive margins of consumption. Finally, in Section 6, we briefly extend our model to include exogenous government purchases to demonstrate that our basic results carry over to an environment with demand shocks. Section 7 concludes and offers some ideas for continuing work. Most of the derivations of key relationships in our model are relegated to the appendices.

2 Baseline Model

Our key point of departure from standard macro models is that, for some goods trades, households and firms each have to expend resources finding individuals on the other side of the market with whom to trade. A fraction of goods market exchange is thus explicitly bilateral, in contrast to all trades happening against the anonymous Walrasian auctioneer. Our model also does feature standard Walrasian goods for which search frictions are absent. We believe this view of goods markets is quite natural — some goods require effort to find and some goods do not. For search goods, households spend time looking for firms from which to purchase goods, while firms direct part of their revenues towards trying to attract customers. We think of these two search activities as shopping and advertising, respectively.

Because we want to avoid taking a specific stand on the details of *why* it is that goods-market trading is costly — there are probably a great many reasons — we adopt the modeling device of an aggregate matching function from the labor search literature. Hall (2007) also takes this route. We describe more fully this matching mechanism below.[2] For our purposes, the important consequence of these search and matching frictions is that once a customer relationship is formed, each party has an incentive to keep the match intact because dissolving the relationship would mean each has to re-enter the costly search process. The existence of a surplus to be shared in a customer relationship means that we must think beyond standard Walrasian marginal pricing conditions because prices play both distributional and allocative roles. In the rest of this section, we describe in detail the households and firms in our model as well as the rest of the economic environment.

2.1 Households

There is a measure one of identical households, with a measure one of individuals that live within each household. In a given period, an individual member of the representative household can be engaged in one of four activities: purchasing goods (shopping) at a firm, working, searching for

[2] Those familiar with the basic labor search model as described by Pissarides (2000, chapter 1), will find close analogies in several of our modeling choices.

goods, or leisure. More specifically, l_t members of the household are working in a given period; s_t members are *searching* for firms from which to buy goods; N_t^h members are *shopping* at firms with which they previously formed relationships; and $1 - l_t - s_t - N_t^h$ members are enjoying leisure. Note our distinction between shopping and searching for goods. Individuals who are searching are looking to form relationships with firms, which takes time. Individuals who are shopping were previously successful in forming customer relationships, but the act of acquiring and bringing home goods itself takes time.[3] We assume that the members of a household share equally the consumption that shoppers acquire.

With this atomistic structure, we assume that lifetime discounted household utility is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(x_t) + \vartheta v \left(\int_0^{N_t^h} c_{it} di \right) + g(1 - l_t - s_t - N_t^h) \right] \tag{1}$$

where x is consumption of a standard Walrasian good and c_i is the quantity of the search good that shopper i brings back to the household. The household costlessly (aside from the direct purchase price) and instantaneously purchases the good x, which is of course what it means for the good to be traded in a Walrasian market. Total consumption $\int_0^{N^h} c_i di$ of the search goods obtained by shoppers is pooled by the household and divided equally amongst all family members. Instantaneous utility over leisure is $g(.)$, and the parameter ϑ governs how the household prefers to divide its total consumption between search and non-search goods.

Note that consumption of the search good potentially has two dimensions in our model: an extensive margin (the number of shoppers who buy goods) and an intensive margin (the number of goods each shopper buys). For the results in this section, we shut down adjustment at the intensive margin by setting $c_i = \bar{c}$. We begin by closing down the intensive margin for two reasons: doing so emphasizes the extensive margin, which is the most novel aspect of our model, and it also allows us some flexibility in calibrating our model. We discuss this issue further below when we present the calibration of the baseline model. In section 5, we endogenize adjustment at the intensive margin of consumption. In the remainder of this section, then, we specialize to the case $c_{it} = \bar{c}$, and our notation reflects this. Finally, we point out that searching and shopping each detract from household leisure in the same, linear, manner.[4]

[3] For example, even if one knows exactly where to go to buy certain goods, one may still have to walk around the aisles, stand in the checkout line, etc.

[4] We recognize that in making N and s perfect substitutes in how they detract from utility — each unit of N or s is equivalent to forgoing one unit of leisure — we are taking a particular stand on the relative disutility of what we term *searching* versus *shopping*. A more general specification that allows for finer distinction between the disutility from shopping versus from searching would be $u(x, l, s, N)$. Because we think we would not have sufficient guidance from data or existing models on how to calibrate, let alone construct, such a function, we adopt the formulation that we do. Despite limitations imposed by data and the lack of previous such modeling efforts, it has been suggested

Whether or not we allow intensive adjustment, note that the aggregator inside $v(.)$ is linear in the total amount of search goods, meaning what we have in mind is a world in which all search goods are perfect substitutes in utility. Our baseline model focuses on this polar case because it means that any "monopoly markups" that arise in our model are due solely to search and matching frictions that create temporary bilateral monopolies, rather than to any ex-ante differentiation of products that create pure monopolies. That is, beginning with this assumption again allows us to isolate effects stemming from the search frictions in goods markets. As we will see in section 5, in order to endogenize the intensive quantity traded, we must add some curvature to the consumption aggregator inside $v(.)$. The economic content of such curvature is that goods obtained from distinct matches are imperfect substitutes. We defer further discussion on this point until we encounter the full model in section 5.

Using $c_i = \bar{c}$ for now, then, the flow budget constraint the household faces is

$$x_t + \int_0^{N_t^h} p_{it}\bar{c}di + b_t = w_t l_t + R_t b_{t-1} + d_t, \tag{2}$$

where b_{t-1} is holdings of a state-contingent one-period real private bond at the end of period $t-1$, which has gross payoff R_t at the beginning of period t, w_t is the real wage, and d_t is firm dividends received lump-sum by the household. The Walrasian good x serves as the numeraire, hence the price p_i of a given search good is measured in units of x. With this structure so far, the household's first-order conditions with respect to Walrasian consumption x_t, labor l_t, and bond holdings b_t are, respectively,

$$u'(x_t) - \lambda_t = 0, \tag{3}$$

$$-g'(1 - l_t - s_t - N_t^h) + \lambda_t w_t = 0, \tag{4}$$

$$-\lambda_t + \beta E_t \left\{\lambda_{t+1} R_{t+1}\right\} = 0, \tag{5}$$

where λ_t is the Lagrange multiplier associated with the time-t flow budget constraint and measures, in equilibrium, the marginal value of wealth to the household. Conditions (3), (4), and (5) are completely standard and imply the usual consumption-leisure optimality condition

$$\frac{g'(1 - l_t - s_t - N_t^h)}{u'(x_t)} = w_t \tag{6}$$

to us that perhaps *shopping* does not entail disutility at all and that it is only *searching* that is associated with disutility. In that case, we may consider writing $u(c, 1 - l - s)$, in which s continues to detract from leisure, which has some precedent from "shopping-time" models, but N does not. We do not have a strong prior regarding this latter view that shopping and searching are sufficiently distinct activities. Thus, rather than present a dizzying array of alternative preference specifications, our approach is to simply lump shopping and searching together in terms of utils and see how much progress we can make on our core ideas. If the primitive trading arrangements our model emphasizes (which are independent of how we wish to write preferences) are deemed sufficiently useful, one may want to consider alternative preference specifications.

and consumption-savings optimality condition

$$u'(x_t) = \beta E_t \left\{ u'(x_{t+1}) R_{t+1} \right\}. \tag{7}$$

The household must also choose how much effort to devote to searching and a desired number of future shoppers; N_t^h is not free to be chosen at the beginning of period t because that depends on how many searchers were previously successful in forming customer relationships. The household faces a perceived law of motion for the number of active customer relationships in which it is engaged,

$$N_{t+1}^h = (1 - \rho^x)(N_t^h + s_t k^h(\theta_t)), \tag{8}$$

where k^h is the probability that a searcher finds a good. This matching probability depends on $\theta \equiv a/s$, which measures the tightness of the goods market — how many advertisements there are per searcher — and is taken as given by the household. With fixed probability ρ^x, which is known to both households and firms, an existing customer relationship dissolves at the beginning of a period. The dissolution of a customer relationship may occur for any of a number of reasons: the customer may move away, the firm may close shop, the customer may simply choose to stop visiting the same store for some reason, and so on. A natural potential future extension would be to endogenize the rate at which customer-firm relationships break up.

Finally, then, the household first-order conditions with respect to s_t and N_{t+1}^h are

$$-g'(1 - l_t - s_t - N_t^h) + (1 - \rho^x)\mu_t^h k^h(\theta_t) = 0 \tag{9}$$

and

$$-\mu_t^h + \beta(1 - \rho^x)E_t\mu_{t+1}^h - \beta E_t \left\{ \lambda_{t+1} p_{Nt+1} \bar{c} \right\} + \beta E_t \left\{ \vartheta v' \left(\int_0^{N_{t+1}^h} \bar{c} di \right) \bar{c} - g'(1 - l_{t+1} - s_{t+1} - N_{t+1}^h) \right\} = 0, \tag{10}$$

where μ_t^h is the Lagrange multiplier on the law of motion for shoppers, p_{Nt+1} is the relative price of the N-th good at time $t+1$, and $\bar{c}$ is the fixed quantity consumed of the N-th good at time $t+1$. As we present below, the price p_i is determined in bargaining. From here on, we conserve on notation by using v_t' to stand for $v'\left(\int_0^{N_t^h} \bar{c} di\right)$, g_t' to stand for $g'(1 - l_t - s_t - N_t^h)$, and u_t' to stand for $u'(x_t)$.

Having taken first-order conditions and given that we restrict attention to symmetric equilibria in which $p_i = p_j$ for all $i \neq j$, the first-order condition on N_{t+1}^h becomes

$$-\mu_t^h + \beta(1 - \rho^x)E_t\mu_{t+1}^h - \beta E_t \left\{ \lambda_{t+1} p_{t+1} \bar{c} \right\} + \beta E_t \left\{ \vartheta v_{t+1}' \bar{c} - g_{t+1}' \right\} = 0, \tag{11}$$

where p_{t+1} now stands for the real (measured in units of x) price of any given good for which there exists a customer-firm relationship. Condensing this expression with the household first-order condition on s, we have

$$\frac{g'_t}{k^h(\theta_t)} = \beta(1-\rho^x)E_t\left\{\vartheta v'_{t+1}\bar{c} - g'_{t+1} - \lambda_{t+1}p_{t+1}\bar{c} + \frac{g'_{t+1}}{k^h(\theta_{t+1})}\right\}. \tag{12}$$

Using (3) and re-grouping terms, we have

$$\frac{g'_t}{k^h(\theta_t)} = \beta(1-\rho^x)E_t\left\{\bar{c}\left[\vartheta v'_{t+1} - p_{t+1}u'(x_{t+1})\right] - g'_{t+1} + \frac{g'_{t+1}}{k^h(\theta_{t+1})}\right\}, \tag{13}$$

which we refer to as the household's shopping condition. The shopping condition simply states that at the optimum, the household should send a number of individuals out to search for goods such that the expected marginal cost of shopping (the left-hand-side of (13)) equals the expected marginal benefit of shopping (the right-hand-side of (13)). The expected marginal benefit of shopping is composed of two parts: the utility gain from obtaining $\bar{c}$ more goods via the search market rather than via the Walrasian market (net of the direct disutility g' of shopping) and the benefit to the household of having one additional pre-existing customer relationship entering period $t+1$. If all trades were frictionless, household optimal choices would imply $\vartheta v'_t = p_t u'(x_t)$. With frictions, in order to engage in costly search, it must be that on the margin, the household expects $\vartheta v'_{t+1} > p_{t+1}u'(x_{t+1})$.[5] This positive flow return ensures that the household finds it worthwhile to send some of its members shopping.

2.2 Walrasian Firms

To make pricing labor simple, we assume that there is a representative firm that buys labor in and sells the Walrasian good x in competitive spot markets. The firm operates a linear production technology that is subject to aggregate TFP fluctuations, $y_t = z_t l_t^W$, where l^W denotes the labor hired by Walrasian firms. Profit-maximization yields the standard result that

$$w_t = z_t, \tag{14}$$

which all participants in the economy, including the non-Walrasian firms described next, take as given.[6]

[5] In the calibrated version of the model, this condition does indeed hold, but it is likely not a theorem that this condition must hold. For some parameterizations, it is likely that the condition fails, in which case a participation constraint would be needed to ensure an equilibrium in which search goods are desirable. We echo and expand on this point in the context of firm incentives in this environment when we discuss the model with intensive adjustment in section 5.

[6] A structure isomorphic to our division into Walrasian firms and non-Walrasian firms described next is to suppose that there is a single representative firm that hires labor to produce output, some of which it sells directly to consumers via Walrasian markets and some of which it sells via search-based channels. One could labels these two channels of sales to consumers as "wholesale" and "retail" channels, which would make the environment look more similar to that of Hall (2007).

2.3 Non-Walrasian Firms

We also assume that there is a representative firm that sells a large number of goods in bilateral trades. For each good that it sells, the representative search firm must first attract customers. To attract customers, the firm must advertise, and how any given level of advertisements it posts maps into how many customers it finds is governed by a matching technology to be described below. Owing to frictions associated with finding customers, the firm views existing customers as assets. Its total stock of customers evolves according to the perceived law of motion

$$N_{t+1}^f = (1-\rho^x)(N_t^f + a_t k^f(\theta_t)), \tag{15}$$

where a_t is the number of advertisements the firm posts in period t, and k^f is the probability that one of the firm's advertisements attracts a customer, which depends on goods market tightness θ; θ is taken as given by the firm.

As with competitive firms, the production technology is linear in labor and subject to an exogenous aggregate productivity shock. Total output of the non-Walrasian firm is thus $y_t = z_t l_t^A$, where l^A denotes the labor hired by the non-Walrasian firm. Because we assume a constant-returns production technology with no fixed costs of production (there is a fixed cost of advertising, but no fixed cost of producing), its real marginal cost of production is constant and coincides with average cost.[7] Denoting marginal production cost by mc_t in period t, we can express the firm's total production costs as the sum of production costs across all of its customer relationships, $\int_0^{N_t^f} mc_t c_{it} di$.

The firm also faces a menu cost of adjusting the per-unit price of each good it sells to a given customer. Specifically, we use a Rotemberg-type quadratic cost of price adjustment, which is a fairly conventional way of modeling menu costs. As we mentioned earlier, our goal is not to provide a compelling micro-foundation for why price adjustment may entail costs; rather, adopting a typical reduced-form specification is just a tractable way to get at our ultimate objective.

With this structure in place, total profits of the representative search firm in a given period t are

$$\int_0^{N_t^f} p_{it} c_{it} di - \int_0^{N_t^f} mc_t c_{it} di - \int_0^{N_t^f} \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 - \gamma a_t, \tag{16}$$

where p_{it} is the relative price (which is, recall again, measured in units of x) of good i, γ is the flow cost of an advertisement, thus γa_t is the total flow advertising cost the firm incurs. We again specialize right away to the case $c_i = \bar{c}$ and, as we have already mentioned, defer considering intensive adjustment until section 5. The parameter κ measures how large menu costs are; setting $\kappa = 0$ of course means there are no menu costs. The firm's customer base N_t^f is pre-determined

[7] To preview the equilibrium, $mc_t = 1\ \forall t$ in our model because we have $w_t = z_t$.

entering period t. Discounted lifetime profits of the firm are thus

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[\int_0^{N_t^f} p_{it} \bar{c} di - \int_0^{N_t^f} mc_t \bar{c} di - \int_0^{N_t^f} \frac{\kappa}{2} \left(\frac{p_{it}}{p_{it-1}} - 1 \right)^2 - \gamma a_t \right], \tag{17}$$

where $\Xi_{t|0}$ is the period-0 value to the household of period-t goods, which we assume the firm uses to discount profit flows because the households are the ultimate owners of firms.[8]

The problem of the firm is thus to maximize (17) subject to the evolution of its customer base (15) by choosing $\{a_t, N_{t+1}^f\}$. In the firm's pursuit of customers, it takes p_i and $\bar{c}$ as given because those will be determined in the trading protocols to be described below. Note an important point of departure from standard macro models of goods markets: the firm is *not* a unilateral price-setter here.[9] The first-order conditions with respect to $\{a_t, N_{t+1}^f\}$ thus are

$$-\gamma + (1 - \rho^x) \mu_t^f k^f(\theta_t) = 0 \tag{18}$$

and

$$-\Xi_{t|0} \mu_t^f + (1 - \rho^x) E_t \left\{ \Xi_{t+1|0} \mu_{t+1}^f \right\} + E_t \left\{ \Xi_{t+1|0} \left[p_{N,t+1} \bar{c} - mc_{t+1} \bar{c} - \frac{\kappa}{2} \left(\frac{p_{N,t+1}}{p_{N,t}} - 1 \right)^2 \right] \right\} = 0, \tag{19}$$

where μ_t^f is the Lagrange multiplier on the firm's customer constraint. Condensing these first-order conditions, we arrive at the firm's optimal advertising condition,

$$\frac{\gamma}{k^f(\theta_t)} = (1 - \rho^x) E_t \left\{ \Xi_{t+1|t} \left[p_{t+1} \bar{c} - mc_{t+1} \bar{c} - \frac{\kappa}{2} \left(\frac{p_{N,t+1}}{p_{N,t}} - 1 \right)^2 + \frac{\gamma}{k^f(\theta_{t+1})} \right] \right\}, \tag{20}$$

where $\Xi_{t+1|t} \equiv \Xi_{t+1|0} / \Xi_{t|0}$ is the household discount factor (again, technically, the real interest rate) between period t and $t+1$. In equilibrium, $\Xi_{t+1|t} = \frac{\beta \lambda_{t+1}}{\lambda_t}$, which in turn, by the household optimality condition (3), is $\Xi_{t+1|t} = \frac{\beta u'(x_{t+1})}{u'(x_t)}$. In writing (20), we have also imposed symmetric equilibrium, in which $p_{it} = p_{jt} = p_t$ for any $i \neq j$.

The advertising condition states that at the optimum, the expected marginal cost of posting an ad (the left-hand-side of (20)) equals the expected marginal benefit of forming a relationship with a new customer (the right-hand-side of (20)). The expected marginal benefit takes into account the revenue from selling to one extra customer, the production costs incurred for producing to sell those

[8] Technically, of course, it is the real interest rate with which firms discount profits, and in equilibrium the real interest rate between time zero and time t is measured by $\Xi_{t|0}$. Because there will be no confusion using this equilibrium result "too early," we skip this intermediate level of notation and structure.

[9] The fact that price is "taken as given" as the firm optimally chooses its level of advertising might lead some to interpret this model as one in which firms have a primitive concern for maximizing their market share, an objective that some models of customer relations do posit. While this is somewhat a semantic point, we note that it is in fact *profits* that the firms in our environment maximize. It is just that firms are not free to *unilaterally* set prices to achieve maximum profits, rather they must form relationships before determining prices jointly with customers.

extra units, future menu costs in that customer relationship, and the cost savings of finding another customer in the future due to the pre-existing (in time $t+1$) customer relationship. Condition (20) is a free-entry condition in advertising. The fact that an entry decision must be made before a firm can enjoy any profit flows means that profit flows from sales of goods are not pure rents as they are in commonly-employed formulations of goods markets.

2.4 Price Determination

With bilateral relationships between customers and firms, there is an array of ways to think about how prices are determined. We focus on three bargaining schemes, two that are axiomatic and one that, although it is not axiomatic, seems to us to capture important intuitive elements underlying each of the two axiomatic schemes. In considering bargaining, we do not need to take literally the idea that customers and firms haggle over prices in every meeting, even though that is the formalism we use. Rather, it seems to us that the idea that customers wield some "bargaining power" accords with the evidence that firms often try to avoid "upsetting their existing customers."

To make progress with this idea, then, we consider three bargaining protocols. The first protocol is Nash bargaining, the second is proportional bargaining, and the third is a modified Nash problem in which the two parties always divide the match surplus in a fixed proportion. We refer to this third trading protocol as *fair bargaining*, although we recognize that bargaining theorists would not accord this trading protocol "bargaining" status. Conceding this point, we nonetheless use the term fair bargaining to make the discussions symmetric.

Proportional bargaining and fair bargaining implement the idea of "fairness" in trading outcomes in similar, but distinct, ways. Our specific notion of fairness is one in which the surplus accruing to customers is always a fixed ratio of the surplus accruing to firms. Clearly, as discussed by Binmore (2007) and many others, there are a great many ways to operationalize the concept of fairness. Given our environment of explicit relationships between consumers and firms, we think ours is at least one natural definition. As we show below, and is well-known in bargaining applications (see Aruoba, Rocheteau, and Waller (2007) for a particularly recent application), the Nash solution does not generally satisfy this definition of fairness.

It is well-understood that Nash bargaining has explicit strategic foundations. Specifically, Rubinstein (1982) shows that the Nash bargaining solution is the limiting solution of a strategic alternating-offers bargaining environment. In contrast, proportional bargaining, while it does, as we show below, capture our definition of fairness, does not have as clear a strategic foundation. Instead, it accords better with the concept of a focal-point equilibrium, an outcome that, perhaps for some evolutionary reason, simply *is* the accepted social norm. Experimental evidence on bilateral games people play, summarized by, among others, Binmore (2007, Chapter 6), suggests that both strategic and focal-point elements are typically at work. This motivates us to construct our fair-bargaining scheme in an attempt to retain the strategic element inherent in Nash bargaining as well as the fairness/focal-point element inherent in proportional bargaining.

2.4.1 Nash Bargaining

In Nash bargaining over the i-th product, the firm and the customer jointly choose p_{it} to maximize the Nash product

$$(\mathbf{M_t} - \mathbf{S_t})^{\eta} \mathbf{A_t}^{1-\eta}, \tag{21}$$

where $\mathbf{M_t}$ is the value to a household of having a member engaged in a relationship with a firm, $\mathbf{S_t}$ is the value to a household of having a member searching for goods, $\mathbf{A_t}$ is the value to a firm of being engaged in a relationship with a customer, and η is a standard time-invariant Nash bargaining weight. The value to a firm of an advertisement that failed to attract any customers is normalized to zero. As we show in Appendix B (where we present the definitions of $\mathbf{M}$, $\mathbf{S}$, and $\mathbf{A}$), the price p_{it} that emerges from Nash bargaining in any particular customer-firm relationship satisfies the sharing rule

$$(1 - \omega_t)(\mathbf{M_t} - \mathbf{S_t}) = \omega_t \mathbf{A_t}, \tag{22}$$

in which ω_t is the effective bargaining power of the customer and $1 - \omega_t$ is the effective bargaining power of the firm. Specifically,

$$\omega_t \equiv \frac{\eta}{\eta + (1-\eta)\Delta_t^F / \Delta_t^H}, \tag{23}$$

where Δ_t^F and Δ_t^H measure marginal changes in the value of a customer relationship for the firm and the household, respectively.

We provide more details in Appendix B, but three points are worth mentioning here. First, time-variation in ω_t means that the household's surplus $\mathbf{M_t} - \mathbf{S_t}$ from an active customer relationship is not a fixed ratio of the firm's surplus $\mathbf{A_t}$. The Nash solution thus does not generally satisfy our definition of fairness. Second, with zero costs of price adjustment in period t (which may arise for two reasons: either $\kappa = 0$ or $\kappa > 0$ but $p_{it} = p_{it-1}$), $\omega_t = \eta$ (because in that case $\Delta_t^F / \Delta_t^H = 1$). It is thus variation in prices *coupled with the presence of menu costs* that drives a time-varying wedge between effective bargaining power and the Nash bargaining weights. Indeed, if the Nash product the customer and firm maximized were $(\mathbf{M_t} - \mathbf{S_t})^{\omega_t} \mathbf{A_t}^{1-\omega_t}$ rather than (21), the outcome would be the sharing rule (22). Third, in the long run (i.e., the deterministic steady state), $\omega = \eta$ because $p_{it} = p_{it-1} = \bar{p}_i$ (recall these are real prices). Thus, the wedge between η and ω_t is a business-cycle phenomenon.

With time-varying effective bargaining power, the price solves

$$\frac{\omega_t}{1-\omega_t} \left[p_{it}\bar{c} - mc_t\bar{c} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)} \right] = \frac{\tilde{u}(\bar{c})}{\lambda_t} - p_{it}\bar{c} + \tag{24}$$

$$+ \quad (1-\theta_t k^f(\theta_t))E_t \left[\Xi_{t+1|t}\left(\frac{\omega_{t+1}}{1-\omega_{t+1}}\right)(1-\rho^x)\left[p_{it+1}\bar{c} - mc_{t+1}\bar{c} - \frac{\kappa}{2}\left(\frac{p_{it+1}}{p_{it}}-1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right]\right].$$

where $\tilde{u}(\bar{c})$ is the utility function defined over the quantity consumed of a good obtained from a given customer relationship (rather than the utility $v(.)$ defined over the household's aggregate consumption of search goods $\int_0^N \bar{c}di.$).[10] The pricing condition (24) shows that price-setting is forward-looking for two distinct reasons. One reason is a standard sticky-price reason: with costs of price adjustment, a setting for p_{it} has ramifications for future setting of p_{it+1}. But note that even with $\kappa = 0$, p_{it} is affected by expectations regarding p_{it+1}. This forward-looking aspect of pricing has to do with the long-lived customer relationship: with probability $1-\rho^x$, the customer and firm will bargain over the same good again in the future. In typical models, price-setting is static in the absence of menu costs; in our framework, it is dynamic even in the absence of menu costs.[11]

Finally, we note that with $\kappa = 0$, the Nash sharing rule (22) reduces to the more standard $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta\mathbf{A_t}$, and the pricing equation (24) simplifies dramatically to

$$p_{it} = (1-\eta)\left(\frac{\tilde{u}(\bar{c})}{\lambda_t}\right) + \eta(mc_t - \gamma\theta_t), \tag{25}$$

which one can obtain by working through the derivations in Appendix B.

2.4.2 Proportional Bargaining

Condition (22) shows that in the presence of menu costs, the surplus is split between consumers and firms according to time-varying shares. Suppose instead that some fairness norm in pricing were in place in which the customer and firm ensure that they always split the total surplus in a time-invariant ratio. An axiomatic solution (see Kalai (1977) for the original exposition) that guarantees constant splits is the proportional bargaining solution. Under proportional bargaining, the price p_{it} solves

$$(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta\mathbf{A_t}, \tag{26}$$

[10] With intensive adjustment not allowed here in our basic model, we could just as well use the notation $\bar{u} \equiv \tilde{u}(\bar{c})$ rather than the structure we present. Looking forward to our full model in section 5, though, the additional notation here will prove useful in comparing features across models.

[11] In Ravn, Schmitt-Grohe, and Uribe's (2006) and Nakamura and Steinnson's (2007) models, pricing is also forward-looking despite the absence of menu costs. In their setups, "deep habits," which are long-lived preference relationships consumers have with particular goods, are the source of forward-looking pricing. At the core of their models, though, is still the typical Walrasian goods market, making, in our view, relationships perhaps a more tenuous idea than in our framework. No matter the relative merits of our approach versus others, a broader idea that emerges is that one can model long-lived customer relationships through preferences, as Ravn, Schmitt-Grohe and Uribe (2006) and Nakamura and Steinsson (2007) both do, or through trading arrangements, as we do here.

without any reference to an underlying maximization problem.[12] Substituting the definitions of $\mathbf{M_t}$, $\mathbf{S_t}$, and $\mathbf{A_t}$ presented in Appendix B, the proportional-bargaining price solves

$$\frac{\eta}{1-\eta}\left[p_{it}\bar{c}-mc_t\bar{c}-\frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}}-1\right)^2+\frac{\gamma}{k^f(\theta_t)}\right]=\frac{\tilde{u}(\bar{c})}{\lambda_t}-p_{it}\bar{c}+ \tag{27}$$
$$+\quad(1-\theta_t k^f(\theta_t))\left(\frac{\eta}{1-\eta}\right)E_t\left[\Xi_{t+1|t}(1-\rho^x)\left[p_{it+1}\bar{c}-mc_{t+1}\bar{c}-\frac{\kappa}{2}\left(\frac{p_{it+1}}{p_{it}}-1\right)^2+\frac{\gamma}{k^f(\theta_t)}\right]\right],$$

which differs from (24) only in that $\eta/(1-\eta)$ replaces $\omega_t/(1-\omega_t)$.

2.4.3 Fair Bargaining

As we discussed above, Nash bargaining has clear strategic foundations, while those underlying proportional bargaining are less clear. To try to capture both the strategic element underlying Nash bargaining and the focal-point element underlying proportional bargaining — both elements that the evidence of Binmore (2007) suggests are important for understanding bilateral interactions — we construct a pricing scheme that draws on both.

The way in which we implement this idea is to continue using the Nash product as the objective the parties seek to maximize; however, constant shares are now enforced. Specifically, p_{it} is chosen to maximize

$$(\mathbf{M_t}-\mathbf{S_t})^{\eta}\mathbf{A_t}^{1-\eta} \tag{28}$$

subject to a constant-split rule

$$(1-\varphi)(\mathbf{M_t}-\mathbf{S_t})=\varphi\mathbf{A_t}. \tag{29}$$

The most straightforward constant-split rule to understand is the case $\varphi=\eta$, which amounts to enforcing the standard Nash sharing condition despite the presence of menu costs. We focus just on this case.[13]

We provide the details behind this problem in Appendix C, but the outcome of fair bargaining over price is

$$\kappa(\pi_{it}-1)\pi_{it}-(1-\rho^x)\kappa E_t\left[\Xi_{t+1|t}(\pi_{it+1}-1)\pi_{it+1}\right]=0, \tag{30}$$

in which π_{it} is defined as the gross rate of price change between period $t-1$ and t, $\pi_{it}\equiv p_{it}/p_{it-1}$. If this were a monetary model, we would of course call π inflation and thus may be tempted to

[12] Technically, one could write an underlying maximization problem that gives rise to this sharing rule, as well, but for most practical applications, one does not need to do so. For more on the relationship between Nash bargaining and proportional bargaining, see Kalai (1977).

[13] Although Nash bargaining is ostensibly at the core of this pricing protocol, we point out that while Nash bargaining, due to its axiomatic nature, is a cooperative equilibrium concept, fair bargaining is a non-cooperative equilibrium concept. In particular, because by construction Nash bargaining places the bargaining parties on the Pareto frontier, the fact that the fair-bargaining solution differs necessarily means that it does not place the parties on the Pareto frontier.

refer to condition (30) as a modern Phillips curve because it links period-t price growth (inflation) to expected future price growth (inflation).

What prevents condition (30) from being a Phillips curve (aside from the fact that our model is not cast in nominal terms) is that it contains nothing at all about allocations. To show how tantalizingly close (30) is to a standard New Keynesian Phillips curve, compare it with the standard pricing equation that emerges from New Keynesian models; for example, the analogous condition in Chugh (2006, equation 7) is

$$f(mc_t) + \kappa(\pi_t - 1)\pi_t - \kappa E_t\left[\Xi_{t+1|t}(\pi_{t+1} - 1)\pi_{t+1}\right] = 0, \tag{31}$$

where f is some function that depends on the marginal cost of production.[14] The fact that marginal cost, which reflects something about allocations, appears in a standard New Keynesian Phillips curve of course provides the linkage between real activity and price movements in that class of models.

Our fair-pricing condition (30), missing marginal costs, is thus of course not a Phillips curve because it does not link price changes to real activity. Indeed, it is quite the opposite of the spirit of a Phillips Curve: condition (30) describes only the dynamics of prices and demonstrates that the dynamics of allocations are divorced from the dynamics of prices. Thus, price rigidity coupled with a fairness norm in bargaining effectively decouples prices from quantities in our model.

Finally, we point out that our notion of fairness in pricing is an assertion about the bargaining protocol. We are not modeling deeper-rooted reasons that may underlie *why* fair bargaining (or proportional bargaining, for that matter) may be adopted in the first place. One potential candidate explanation is the recent model of Rotemberg (2006), in which customer "anger" over prices that are perceived to be exploitative may lead to a fairness norm. More broadly, Akerlof (2007), in his overture to macroeconomics to adopt more "norm-based" behavior into standard models, discusses why incorporating norms and, at least as a first step, ignoring endogeneity of norm adoption may be an important evolutionary step for the field. Such issues are quite interesting to consider, and if our model proves to be useful in thinking about some aspects of goods-market relationships, one may want to embed such mechanisms in our model in future work. For now, our focus is on the consequences of menu costs of price adjustment *given* a fairness norm in pricing.

2.5 Goods Market Matching

The number of new customer-firm relationships that are formed in any period t is described by an aggregate matching function $m(s_t, a_t)$. We assume the matching technology is Cobb-Douglas,

[14] In New Keynesian models, the Phillips curve is essentially just the first-order condition of firms facing rigidities in their price-setting choices and of course has nothing to do with bargaining.

$m(s_t, a_t)$. With Cobb-Douglas matching, the probabilities that shoppers and firms, respectively, find partners is

$$k^h(\theta) = \frac{m(s,a)}{s} = m\left(1, \frac{a}{s}\right) = m(1, \theta) \tag{32}$$

$$k^f(\theta) = \frac{m(s,a)}{a} = m\left(\frac{s}{a}, 1\right) = m(\theta^{-1}, 1), \tag{33}$$

with $\theta \equiv a/s$ a measure of how thick (the ratio of firms searching for customers to individuals searching for goods) the goods market is.

As in the labor search literature, the matching function is meant to be a reduced-form way of capturing the idea that it takes time for parties on opposite sides of the market to meet. Rogerson, Shimer, and Wright (2005, p. 968) note that the ability to be agnostic about the actual mechanics of the process by which parties make contact with each other may be a virtue. Our modeling motivation is very much in line with this idea.

With the matching function describing the flow of new customer relationships, the aggregate number of active customer relationships evolves according to

$$N_{t+1} = (1 - \rho^x)(N_t + m(s_t, a_t)). \tag{34}$$

2.6 Resource Constraint

Total output of the economy is absorbed by Walrasian consumption, non-Walrasian consumption, price adjustment costs, and advertising costs. The resource constraint is thus

$$x_t + \int_0^{N_t} c_{it} di + \int_0^{N_t} \frac{\kappa}{2}(\pi_t - 1)^2 + \gamma a_t = z_t l_t. \tag{35}$$

2.7 Equilibrium

We restrict attention to a symmetric equilibrium in which the price p is identical across all active customer relationships. Because (part of) goods trade is carried out in non-Walrasian markets, the core notion of equilibrium in our model is that of a search equilibrium, rather than a competitive equilibrium. This motivates the definition of a symmetric search equilibrium for our model.

A symmetric search equilibrium is endogenous processes for the household's choice $\{x_t, l_t, s_t, N^h_{t+1}, b_t\}_{t=0}^{\infty}$, the Walrasian firm's choice $\{l^W_t\}_{t=0}^{\infty}$, the non-Walrasian firm's choice $\{a_t, N^f_{t+1}, l^A_t\}_{t=0}^{\infty}$, the real wage $\{w_t\}_{t=0}^{\infty}$, bond returns $\{R_t\}_{t=0}^{\infty}$, dividends $\{d_t\}_{t=0}^{\infty}$, prices in the non-Walrasian goods market $\{p_t\}_{t=0}^{\infty}$, and matching probabilities $\{k^h_t, k^f_t\}_{t=0}^{\infty}$ such that

- given $\{p_t, w_t, R_t, d_t, k^h_t, k^f_t\}_{t=0}^{\infty}$, the household maximizes (1) subject to (2) and (8);
- given $\{w_t\}_{t=0}^{\infty}$, the Walrasian firm chooses labor to maximize profit;
- given $\{p_t, w_t, k^h_t, k^f_t\}_{t=0}^{\infty}$, the non-Walrasian firm maximizes (17) subject to (15);

- $\{p_t\}_{t=0}^{\infty}$ satisfies either (24) (Nash bargaining) or (30) (fair bargaining);
- the resource constraint (35) holds for $t = 0, 1, ...$;
- the labor market clears, $\{l_t = l_t^W + l_t^A\}_{t=0}^{\infty}$;
- the customer market clears, $\{N_t^h = N_t^f\}_{t=0}^{\infty}$;
- the aggregate law of motion for active customer relationships is given by (34) for $t = 0, 1, ...$;
- the bond market clears, $\{b_t = 0\}_{t=0}^{\infty}$;
- dividends $\{d_t\}_{t=0}^{\infty}$ are determined residually from the non-Walrasian firm's profit function;
- $\{k_t^h, k_t^f\}_{t=0}^{\infty}$ are given by (32) and (33).

Collecting conditions that summarize the equilibrium, equilibrium is endogenous processes $\{x_t, N_t, p_t, s_t, a_t, l_t, w_t, R_t\}_{t=0}^{\infty}$ that satisfy (6), (7), (13), (14), (20), either (24) or (30), (34), and (35), for given exogenous process $\{z_t\}_{t=0}^{\infty}$.

3 Partial Equilibrium Analytics

Our main interest lies in some general equilibrium business cycle consequences of search frictions in goods trade and price rigidity. However, we can gain a lot of intuition for the economic forces at work in our model by examining both analytically and numerically its steady-state equilibrium. The triple (p, θ, s) are the most important endogenous variables describing our frictional goods market.[15] Features such as the intensive quantity c, presence of a production technology, and endogenous labor force participation are all present to make our model quantitatively realistic and readily comparable to other quantitative macroeconomic models. We again emphasize that the fundamental notion of goods-market equilibrium here is that of a *search* equilibrium, not a Walrasian (or Walrasian-based) equilibrium.[16]

To make some analytical progress, then, for the moment suppose the general equilibrium features of our model were shut down. Specifically, suppose $c = 1$ and labor is not needed ($l = 0$).[17] In

[15] In this regard, there is a very tight analogy with the basic static labor search model, as described by Pissarides (2000, chapter 1), as we mentioned earlier. Those familiar with the textbook labor search model will find the analytical exposition here extremely familiar; because there are a couple of details that do not carry over completely (but, admittedly, nearly completely) identically, we think it worthwhile to explain the basics.

[16] As is apparent, what we discuss in this section is equilibrium in the search-goods market; Walrasian equilibrium is of course the relevant equilibrium concept for the good x.

[17] Really all we require for this partial equilibrium steady-state analysis is just that labor is fixed at $l = \bar{l}$. Going all the way to essentially an endowment model — in which when a firm finds a customer to whom to sell, product magically appears — in which $l = 0$ just eases the ensuing exposition a bit.

this partial equilibrium version of our model, then, imposing steady-state on the firm advertising condition gives us

$$\frac{\gamma}{k^f(\theta)} = \frac{\beta(1-\rho^x)}{1-\beta(1-\rho^x)}(p - mc), \tag{36}$$

in which we have used the fact that in the deterministic steady state, prices do not change (remember, these are real prices), hence there are no menu costs of price adjustment. With Cobb-Douglas matching, the probability a firm that has advertised matches with a customer is $k^f(\theta) = \theta^{-\xi}$. Imposing this and rearranging, we have

$$\frac{\beta(1-\rho^x)}{1-\beta(1-\rho^x)}p = \frac{\beta(1-\rho^x)}{1-\beta(1-\rho^x)}mc + \gamma\theta^{\xi}, \tag{37}$$

which shows that the markup of price over marginal cost of production is governed by the advertising cost γ and customer market tightness θ. If $\gamma = 0$, we have $p = mc$, which makes perfect sense because in that case it is costless for firms to find customers and we have assumed that goods are ex-ante perfect substitutes; in other words, the goods market is (nearly) Walrasian with $\gamma = 0$. With $\gamma = 0$, firms make normal profits by charging simply their marginal production cost as in a textbook model. Clearly, then, it is the search friction embodied in $\gamma\theta^{\xi}$ that drives a wedge between price and marginal production cost. Indeed, because $\gamma\theta^{\xi} \geq 0$, $p \geq mc$. Figure 1 shows that the advertising condition is upward-sloping in (p, θ) space.[18] To preview some of the intuition behind the dynamics of markups we present soon: because θ in general will vary over time as part of business cycle fluctuations, the markup of price over marginal production cost should be expected to be time-varying, and this markup is endogenous. Time-variation in search costs drives time-variation in markups in our framework.[19]

To determine the steady-state (p, θ), we must also examine (the steady-state version of) the pricing condition (24), which is the relevant pricing condition for both Nash and fair bargaining (because, recall again, both bargaining schemes deliver the same outcome when there is no cost of price adjustment, which is true in the steady-state of all our models). Again assuming $c = 1$ and after also using the steady-state version of the advertising condition, we can express the steady-state pricing condition as

$$p = (1-\eta)\left(\frac{\tilde{u}(1)}{\lambda}\right) + \eta(mc - \gamma\theta), \tag{38}$$

where we have used the fact that in steady state, $\omega = \eta$. The first term on the right-hand-side of this expression, because it itself depends on θ in general equilibrium (because the marginal utility of wealth λ is a general equilibrium object), makes analyzing this expression prohibitively more

[18] In constructing Figures 1 and 2, we use the parameter values described in Section 4.1.

[19] In a different notion of "consumer search" (one in which search means that consumers explicitly face a distribution of prices from which to choose), Alessandria (2005) makes the very similar point that search costs are reflected in prices.

complicated than analyzing the steady-state advertising condition. To focus ideas, though, suppose it were simply a constant, A. In that case, we have the very simple pricing equation

$$p = (1-\eta)A + \eta(mc - \gamma\theta), \tag{39}$$

which states that the price lies inside an interval bounded above by the firm's production cost net of savings on search costs and bounded below by the household's utility of consumption A. The convex weights are simply the Nash bargaining weights.[20] The locus (39) is downward-sloping in (p, θ) space, as illustrated in Figure 1; the steady-state equilibrium (p, θ) is determined where it and the pricing condition cross in Figure 1.

Conditions (37) and (39) characterize (p, θ) in terms of deep parameters. It is fairly straightforward to show (we provide the details in Appendix D) that the steady-state price is strictly decreasing in customer bargaining power η. This result makes a lot of intuitive sense: the more bargaining power customers wield, the less rents the firm can extract out of the match surplus, meaning the smaller is the markup of unit price over unit marginal production cost.

So far, we have characterized the equilibrium pair (p, θ). To complete the description of the steady-state equilibrium core of our model, it remains to characterize s (or, equivalently, a, because $\theta \equiv a/s$) for a given (p, θ). To do this, we begin by noting that in a steady-state equilibrium, the flow of individuals from search into customer relationships equals the flow of individuals from customer relationships (gone sour) back into search. Equating these flows, $k^h(\theta)s = \rho^x N = \rho^x(1-s)$.[21] Rearranging,

$$s = \frac{\rho^x}{\rho^x + k^h(\theta)}. \tag{40}$$

Cobb-Douglas matching implies $k^h(\theta) = \theta k^f(\theta) = \theta^{1-\xi}$. In (a, s) space, then, the flow condition (40), which characterizes activity on the household side of the goods market, defines the downward-sloping locus shown in Figure 2.

Finally, to describe equilibrium in (a, s) space, we need a description of activity on the firm side of the goods market; such adescription comes from combining (37) and (39). Specifically, solving (37) for p, substituting in (39), and using the implicit function theorem to compute the slope of a with respect to s (we again relegate the details to Appendix D), we can show that the resulting form of the advertising condition is the upward-sloping ray in (a, s) space shown in Figure 2; the steady-state equilibrium (a, s) is determined where it and the locus (40) cross

[20] Once again, to those familiar with the modern theory of labor markets, this is all very familiar. Indeed, as we pointed out at the outset, our steady-state analysis is parallel to the analysis of the basic labor search model in Pissarides (2000, Chapter 1).

[21] Here it is critical that it is only searching and shopping that are the two possible activities for individuals, meaning $s + N = 1$. If individuals could also work/take leisure and work/leisure were a possible state to transit to after a failed customer experience, it would be much more difficult to conduct the ensuing analysis.

in Figure 2. In combination, then, Figures 1 and 2 fully characterize the steady-state (partial) equilibrium triple (p, θ, a) that describes goods-market trade.

4 Quantitative Results in Baseline Model

With these analytics in mind, we turn to characterizing the full deterministic steady state of our model numerically (i.e., fully endogenizing $\tilde{u}/\lambda$ by re-introducing elastic labor supply). We begin by describing the calibration we use.

4.1 Calibration

For instantaneous utility, we choose the common functional forms

$$u(x) = \log x, \tag{41}$$

$$v(y) = \log y, \tag{42}$$

and

$$g(z) = \frac{\zeta}{1-\nu} z^{1-\nu}. \tag{43}$$

The time unit of our model is meant to be one quarter, so we set $\beta = 0.99$, in line with an average annual real interest rate of about four percent. We fix the curvature parameter for the subutility function over leisure to $\nu = 0.4$. We set the preference parameter $\vartheta = 1$ as a baseline. With this baseline setting and given the rest of the calibration described below, the fraction of equilibrium total consumption that is comprised of consumption obtained through search is 43 percent. That is, $\vartheta = 1$ delivers $\frac{Nc}{Nc+x} = 0.43$, which does not seem unreasonable. Varying ϑ varies this share; in the limit, of course, $\vartheta = 0$ collapses our model to one in which all goods are exchanged via Walrasian trade.

As we mentioned earlier, we choose a Cobb-Douglas specification for the matching function,

$$m(s, a) = \psi s^{\xi_s} a^{1-\xi_s}, \tag{44}$$

and set the elasticity to $\xi_s = 0.5$. We choose Cobb-Douglas because of its convenient properties — in particular, the fact that matching probabilities depend only on goods-market tightness $\theta \equiv a/s$, as we already mentioned — but recognize that it would be desirable to test how empirically useful the Cobb-Douglas description is for goods-market frictions.[22] For the Nash bargaining weight η, we choose a middle-of-the-road calibration $\eta = 0.50$ for most of the results we report, but do vary it in some of our experiments. One virtue of setting $\eta = \xi_s$, well-known to search theorists, is that, as

[22] Petrongolo and Pissarides (2001) survey the empirical usefulness of Cobb-Douglas matching for labor markets.

Hosios (1990) first showed in the context of labor search models, the underlying search equilibrium is socially efficient. We of course do not know if an efficient search equilibrium in the goods market is the best description of the data, but it seems useful as a starting point.

We calibrate a number of other parameters in the version of our model in which there are zero costs of price adjustment ($\kappa = 0$). In this flexible-price version of our model, we set $\zeta = 4.3$ so that the household spends 30 percent of its time working (equivalently, the household sends 30 percent of its family members to work) and then hold this value fixed as we move to other versions of our model. We also calibrate γ, $\bar{c}$, and ψ, all of which we discuss in more detail immediately below, in the zero-menu-cost version of our model and hold the resulting values constant throughout all versions of our model.

Two novel properties of our model about which we can obtain some empirical evidence are the amount of time consumers spend shopping and firms' expenditures on advertising. According to the American Time Use Survey (ATUS), conducted annually by the U.S. Bureau of Labor Statistics, the average American consumer spends just under one hour per day shopping, which is roughly one-fourth as much time spent working.[23] The questionnaire that is the basis for the survey does not distinguish between, in the terminology of our model, "searching" for goods and "shopping" for goods. Thus, we calibrate our model so that, in the deterministic steady state, $(N + s)/l = 0.25$ and allow the model to endogenously determine N and s separately. Hitting this target requires setting the intensive quantity traded in a customer relationship to $\bar{c} = 1.4$.

Regarding advertising expenditures, we use data from the BLS and Universal McCann, an advertising agency that tracks and projects developments in the industry.[24] According to these data, total nominal advertising expenditures in the U.S. in 2005 were about \$276 billion and are estimated to have been about \$290 billion in 2006, putting the share of advertising in total nominal GDP around 2.25 percent. This figure strikes us as non-neglible: firm spend quite a lot attracting and retaining customers. Going back to 1950, this share has generally fluctuated within the range 2 percent to 2.5 percent. Because the notion of advertising in our model is likely a bit more general than activities typically expensed as advertising, we use the upper limit of 2.5 percent as our guidepost.[25] We thus calibrate γ so that γa in our model is 2.5 percent of GDP in steady state. In Appendix F, we provide for interested researchers annual aggregate advertising data.

Unfortunately, neither the shopping data nor the advertising data gives us any guidance (at least not that we have been able to discern) about how to calibrate our model's matching and separation probabilities. Lacking solid evidence, we calibrate the matching function parameter ψ

[23] Data on the American Time Use Survey, which began in 2003, can be accessed at http://www.bls.gov/tus/.

[24] A summary of the data through 2005 can be found in the BLS's 2007 Statistical Abstract of the United States, Table 1261.

[25] In our model, any activity that potentially helps a firm attract customers is "advertising."

in the flexible-price version of our model so that $k^h = 0.4$. The resulting value is $\psi = 0.45$, which we hold fixed as we move to other versions of our model. We simply set the parameter that governs the breakup of a customer relationship at $\rho^x = 0.10$, which states that a firm loses ten percent of its existing customers in any given period. Equivalently, this parameter setting means that a newly-formed customer-firm relationship is expected to last for $1/\rho^x = 10$ periods (quarters), which we think does not seem implausible.

We also face a problem in terms of calibrating the price-adjustment parameter κ. Ideally, we would like to set it so that in the deterministic steady state, resources devoted to price adjustment absorb some empirically-relevant portion of output. However, because all prices are real in our model, price-adjustment costs are always zero in any deterministic steady state, making κ irrelevant for the steady state. Hence, we report results for several values of κ, corresponding to no, small, moderate, and large menu costs. We think this strategy is sufficient because the main results we wish to convey are conceptual rather than quantitative.

Finally, the exogenous TFP process follows an AR(1) in logs,

$$\log z_{t+1} = \rho_z \log z_t + \epsilon^z_{t+1}, \tag{45}$$

with $\epsilon^z \sim iidN(0, \sigma_z)$. We choose $\rho_z = 0.95$ and $\sigma_z = 0.007$ in keeping with the RBC literature — see, for example, King and Rebelo (1999, p. 955). As our results show, this setting for the volatility of the shock to TFP delivers volatility of total output in our model of about 1.6 percent, in line with empirical evidence. Thus, the amplification of TFP shocks to GDP fluctuations in our model is no different than in a standard RBC model.

4.2 Steady State Numerical Results

Steady-state prices and quantities are reported in Table 1 for our baseline calibration. At our benchmark value $\eta = 0.50$, the markup in the search goods sector is just above eight percent, in line with empirical evidence and with the settings for product-market markups employed by many quantitative macroeconomic models.

The basic motivation of our work is that customer bargaining power may be important in pricing (and other) decisions. As such, one would want to know the predictions of our framework regarding key endogenous variables as customer bargaining power changes. Figure 4 illustrates how a number of steady-state variables vary with customer bargaining power η regardless of whether or not there are menu costs of price adjustment. The results in Figure 4 are invariant to both the menu cost parameter κ and the specific bargaining protocol because in the steady state, $\pi = 1$ (i.e., prices are unchanging) because the interesting price in our model is a real price — in the steady state, real prices are of course constant.

As shown in Figure 4, an increase in consumer bargaining power depresses the bargained price (the top left panel), confirming our analytical results. Because marginal production cost is constant at unity by construction, the markup declines as η rises, as well. A lower price leaves the household with a larger gain $\mathbf{M} - \mathbf{S}$ from forming a match (the top right panel), which also induces it to put more effort into search. The firm, on the other hand, loses from a rise in η, as the fall in $\mathbf{A}$ shows. The firm reduces its advertising because lower prices eat directly into profits, making customer relationships less valuable to it. With lower a and higher s, market thickness θ ($\equiv a/s$) unambiguously falls.

Regarding the flow of new relationships formed, the reduction in advertising expenditures dominates the increase in search activity, and the number of customer-firm relationships (N) falls. For our calibration, even though s rises, the total amount of time that households spend engaged in shopping-related activities ($N+s$) declines. Households optimally reallocate this additional time between labor and leisure according to the consumption-leisure optimality condition (expression (6)), and it turns out that total labor declines, which, in turn, leads to lower total output.

The latter result highlights an interesting point of contrast with the standard Dixit-Stiglitz model of monopolistic competition. In the standard model, reducing the degree of firms' pricing power (in the form of a higher elasticity of demand for its output) pushes price closer to marginal cost, causing output to rise. Figure 4 shows that in our model, reducing firms' pricing power (here, in the form of lower bargaining power for the firm) causes households to devote less time to production activities and enjoy more leisure, causing output to decline.

The reason for the difference in the response of output as the markup of price over marginal cost falls is of course a direct result of the non-Walrasian features of our model. Advertising activities open up a supply-side channel missing in a standard model. When the firm has less to gain from trading in the non-Walrasian market, it simply chooses to reduce its activity there by cutting back on advertising. The resulting number of transactions in the non-Walrasian market declines. Shopping becomes less of a burden for households, who enjoy more leisure and devote less time to production activities.[26]

4.3 Dynamics

To study dynamics, we approximate our model by linearizing in levels the equilibrium conditions of the model around the non-stochastic steady-state. Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004b). We conduct

[26] One way to interpret this is that firms can indirectly manage labor demand through the non-Walrasian market. By changing advertising behavior, the firm can influence the number of long-term relationships in the economy and, hence the amount of time households spend in shopping activities. Shopping time, in turn, is closely linked to the household labor supply decision.

5000 simulations, each 100 periods long. For each simulation, we compute first and second moments and report the medians of these moments across the 5000 simulations. To make the comparisons meaningful as we vary model parameters, the same realizations for productivity shocks are used across versions of our model. Because by construction marginal production cost is always $mc_t = 1$ in our model, we will speak interchangeably about the dynamics of prices and markups. Depending on the issue at hand, one or the other language will typically be more convenient to use, which we think is clear in the ensuing discussion.

4.3.1 Nash Bargaining with No Menu Costs

Table 2 presents simulation-based first and second moments for key variables under Nash bargaining and zero menu costs. Examining basic business cycle statistics reveals that the search goods market is a source of consumption smoothing, in the sense that the volatility of total consumption is lower than the volatility of GDP.[27] In our model, total consumption is $Nc + x$. The volatility of total search consumption Nc, at 1.13 percentage points, is noticeably lower than the volatility of Walrasian consumption x. In our baseline calibration, search consumption makes up 43 percent of total consumption in the steady-state $Nc + x$. The volatility of total consumption, at 1.55 percent (not shown in the table), is thus a bit lower than volatility of total output. The consumption smoothing, although obviously not as quantitatively strong as in an RBC model with capital, arises because active customer-firm relationships are a state variable in our environment, making part of total consumption pre-determined. At roughly 0.94% (= 1.55/1.66), the relative volatility of aggregate consumption in our model is clearly not as low as that which obtains in an RBC model with capital (see, for example, Table 3 in King and Rebelo (1999, p. 957)). Thus, we are clearly not claiming that search consumption induces as quantitatively strong a smoothing effect as does capital. The fact that consumption smoothing *can* arise in a model with completely perishable goods and no capital, though, is novel.

Nevertheless, in terms of developing intuition for the basic dynamics of the model, we find it useful to think of the formation of customer-firm relationships as an investment decision for participants on both sides of the market. Households invest time searching for retailers, and firms invest advertising dollars in order to attract shoppers. A comparison of the volatility of advertising expenditures in our model to the volatility of investment in a standard RBC model with capital supports this analogy. We have more to say about the business cycle properties of advertising expenditures below.

The final point worth emphasizing is that even with flexible prices, our model delivers a time-

[27] Trivially, in a model with no search markets and in which labor is the only production input, the volatility of GDP would be identical to the volatility of consumption simply because all output is absorbed by consumption.

varying markup, a result that is of some interest on its own. Our baseline model predicts that markups are procyclical, as the fifth column in Table 2 documents. However, most empirical evidence — for example, Rotemberg and Woodford (1999) or, more recently, Jaimovich (2006) — suggests that markups are countercyclical. We do not view this as a shortcoming of our model for two reasons. First, our basic goal was not to model markup behavior per se. Second, our baseline model features only TFP shocks. Our reading of the empirical evidence is that it is not even clear whether countercyclicality of markups observed in the data is due to demand shocks or supply shocks. In Section 6, we briefly extend our model to include demand shocks and demonstrate that our model is in fact capable of delivering countercyclical markups.

4.3.2 Nash Bargaining with Menu Costs

We now turn on menu costs. Table 3 presents simulation-based first and second moments for key variables under Nash bargaining and for several values of κ. Comparing results across models, one feature that stands out is that the volatility and correlation properties of the typical quantity variables GDP, (Walrasian) consumption x, and time spent working l are virtually invariant to the size of the menu cost κ. The same is true of the correlation and volatility properties of time spent shopping N and time spent searching s. Quantity variables as a whole are thus largely unaffected by the magnitude of menu costs. An exception is goods-market tightness θ, which becomes a bit less volatile as κ rises. Because, as we just noted, the volatility of s changes little with κ, changes in the volatility of θ are driven by fluctuations in firms' incentives to advertise. We document and discuss this latter feature of our model further at the close of this section.

With most quantity variables largely unaffected by the degree of price rigidity, then, we focus most of the rest of our discussion on the dynamics of prices (equivalently, markups) and effective bargaining power. As Table 2 shows, with zero costs of price adjustment, effective bargaining power is constant over the business cycle at $\omega_t = \eta$. We pointed out in Section 2.4.1 that this must be the case with no costs of price adjustment, and Appendix B proves this. Nevertheless, prices, and hence markups, do vary. Recall that for the case $\kappa = 0$, the Nash pricing condition can be expressed as in (25), which makes clear that in general prices vary over time despite constant effective bargaining power because the marginal utility of wealth λ_t and aggregate goods-market tightness θ_t are both time-varying.

As Table 3 shows, prices (markups) become more volatile and more persistent as κ rises, mirroring what happens to customers' effective bargaining power ω_t. In fact, ω_t and p_t are negatively correlated over the business cycle: their mean correlation across simulations is -0.11 with $\kappa = 5$, -0.64 with $\kappa = 20$, -0.79 with $\kappa = 50$, and -0.81 with $\kappa = 100$. Figure 5 provides more evidence of this phenomenon. We think this result is quite easy to understand: at times when consumers' effective bargaining power is low, firms are more likely to be to able to push through higher prices. The relationship holds in the opposite direction as well, of course: at times when consumers' effective bargaining power is high, firms must lower their profit margins. Our model predicts that such phenomena hold at a business-cycle frequency, perhaps opening a new way to empirically thinking about how and why markups change over time. Regardless of the ability to empirically test this idea with currently available data, eliminating the phenomenon that prices are higher the lower is consumers' bargaining power (and vice-versa) is precisely the goal of the fair-bargaining norm that we examine below.

Finally, we noted above that firms' incentives to advertise fluctuate over time. Changes in these incentives are reflected in shifts of the firm advertising condition, the steady-state version of which, recall, is the downward-sloping locus in Figure 1. In Figure 6, we scatter the dynamic realizations of (θ, p) for a representative simulation with $\kappa = 0$. Comparing the results with Figure 1, clearly both the advertising condition and the Nash pricing condition shift over the business cycle, but shifts in the advertising condition dominate because the resulting scatter is upward-sloping. This result carries over to the environment with positive menu costs, as the comparable results presented in Figure 7 show. For brevity, we omit plotting the dynamic realizations of (a, s) in the space of Figure 2, which all show that it is dynamic shifts in the flow condition (40) that dominate. The reason that a fluctuates so much more than s is that firm profits are linear in a, while household utility displays diminishing returns in leisure (which depends on s). All else equal, the household has an incentive to limit fluctuations in its search activity in a way that firms do not.

Whether in the face of zero menu costs (Table 2) or positive menu costs (Table 3), the standard deviation of advertising, at over 4 percent, lines up well with the cyclical volatility of advertising behavior in the U.S. economy. In Appendix F, we present data and summary statistics for U.S. aggregate advertising. The cyclical volatility of advertising over the past 50 years is 4.2 percent, and its contemporaneous correlation with GDP is 0.73; our model predictions are quite close to these. We did not set out to match advertising dynamics per se, but the fact that our basic model matches quite well the volatility and correlation of advertising behavior means that it is not subject to the same critique Shimer (2005) leveled at labor search models.[28]

[28]Specifically, in work that has sparked much subsequent modeling effort, Shimer (2005) demonstrated that the standard Mortensen-Pissarides structure coupled with Nash bargaining fails to match the volatilities of unemployment and vacancies, the two inputs to the aggregate matching function, observed in the data. Our model employs a Mortensen-Pissarides structure coupled with Nash bargaining, yet at least one of the inputs to the aggregate matching function — advertising — is as volatile as observed in the data. Regarding the volatility of the other input to the matching function — household search behavior — the American Time Use Survey only began in 2003, so we cannot compute meaningful time-series summary statistics for it. We can only note that over the four years of its survey, household time spent shopping has been remarkably stable, which also qualitatively matches our model's prediction regarding the volatility of N and s.

In summary, regardless of the presence of menu costs, the dynamics of prices and quantities are largely divorced from each other in the search goods market. The price p plays much more of a distributive role (determining how the surplus in a customer relationship is split) than the purely allocative role it plays in standard Walrasian views of goods markets. This point is made quite clearly in Figure 8, which shows impulse responses to a one-time, persistent TFP shock for $\kappa = 0$ and $\kappa = 20$. Output dynamics are essentially identical (the two responses in the top panel of Figure 8 lie virtually on top of each other) despite large differences in price dynamics.

Because the price plays a distributive role, our model seems ideally suited to study notions of fairness in pricing. With some understanding of the dynamic results under Nash bargaining in hand, we thus turn next to dynamics under our two fairness schemes, proportional bargaining and fair bargaining. We again point out that these fairness schemes are only relevant in the presence of menu costs ($\kappa > 0$) because, as we discussed in Section 2.4, all three bargaining schemes collapse to standard Nash bargaining, both in steady state and dynamically, if $\kappa = 0$.

4.3.3 Proportional Bargaining

The proportional-bargaining price always solves (27), which is identical to the Nash sharing condition (22) when $\omega_t = \eta$, which occurs, recall, if $\kappa = 0$. Thus, no matter the magnitude of κ, all dynamics under proportional bargaining, both those of prices and quantities, are identical to the dynamics under Nash bargaining and $\kappa = 0$ reported in Table 2. The way in which fairness is captured by proportional bargaining thus renders menu costs completely irrelevant.

To understand this, recall from the analysis of the results under Nash bargaining and positive menu costs that the primary channel through which menu costs affected price dynamics was through time-variation in effective bargaining power ω_t. In proportional bargaining, in contrast, there is no notion of "bargaining power" — indeed, this is what we mean when we say that proportional bargaining lacks clear strategic foundations. In proportional bargaining, parties' adherence to the ad-hoc rule that splits the surplus according to fixed shares no matter what shuts down the transmission of menu costs to prices intermediated through effective bargaining power. As a consequence, price dynamics are governed entirely by the dynamics of the underlying surplus, and any price adjustment is simply an efficient response to a shock and neutralizes the distributive role of prices.

Finally, to square proportional bargaining with Nash bargaining in a different way and to motivate fair bargaining from a quantitative perspective, we conduct the following thought experiment. For a given value of κ and using Nash bargaining, we can construct the sequence $\{\omega_t^{PROP}\}$ of effective bargaining power that would make the Nash outcome coincide with the proportional-bargaining outcome. In other words, $\{\omega_t^{PROP}\}$ is a series we can construct residually to make the Nash dynamics for a given κ identical to the Nash dynamics for $\kappa = 0$. Using $\kappa = 20$, the last row of Table 2 reports the dynamics of this counterfactual effective bargaining power series. This residual measure of effective bargaining power is much more volatile than any of the actual effective bargaining power series reported in Table 3. In proportional bargaining, ω^{PROP} is indeed nothing but a residual; there is no notion of bargaining power at all in proportional bargaining. This fact, coupled with the observation that if there *were* a notion of bargaining power behind fairness it would apparently fluctuate a great deal, is part of our motivation for constructing our fair-bargaining protocol, the results of which we now turn to.

4.3.4 Fair Bargaining

Table 4 compiles results using fair bargaining for the same set of experiments as in Table 3. The central result here is that the presence of menu costs makes the price of the search good completely rigid. In the discussion surrounding expression (23), we noted that $\omega_t = \eta$ if and only if the cost of price adjustment in period t is zero. The menu cost can be zero either because $\kappa = 0$ or because $p_t = p_{t-1}$. Under fair bargaining, the customer and firm are constrained to share the surplus according to $\omega_t = \eta\ \forall t$. With $\kappa > 0$, the only way this sharing norm can be achieved is if prices never vary. One way of understanding this result is that the "fairness constraint" on the Nash bargaining problem effectively eliminates the efficient adjustment of the price that obtains under proportional bargaining. Echoing the result under proportional bargaining, however, the magnitude of menu costs (here, as long as they are positive) is irrelevant for price, as well as quantity, dynamics.

However, it is *not* a theorem that would hold in any goods-search model of the type we propose that fair bargaining necessarily requires complete price stability. With the features present in our model, it is only time-variation of prices that create a wedge between η and ω_t. One can easily extend our model to include other features that would affect this wedge. For example, in Arseneau and Chugh (2007), time-varying labor taxes also induce such a wedge (albeit in a model of wage bargaining, but we think the idea would translate readily to our environment). Suppose exogenous tax movements and (endogenous) price movements were both present in our model. In such an environment, there may be situations in which variations in the two offset each other in such a way as to leave no wedge at all between η and ω_t. Indeed, the fair-bargaining norm may *require* customers and firms to engineer price movements in precisely the right way to offset exogenous tax shifts. We have not conducted such experiments with our model and so cannot assert this definitively, but based on our work here and the results and intuition in Arseneau and Chugh (2007), this hypothesis seems sound.

Quantity dynamics, which are also invariant to κ, are very nearly, but not completely, identical to the flexible-price Nash baseline. The most obvious difference is in the volatility of goods-market tightness θ. Its standard deviation of about 1.2 percent in Table 4 is roughly half that of all the results under Nash bargaining displayed in Tables 2 and 3. Because the volatility of s is virtually the same across models, the smaller fluctuations in θ under fair bargaining must be due to smaller fluctuations in a. To verify this conjecture, we plot in Figure 9 the dynamic realizations of the pairs (θ, p) and (a, s) for a representative simulation.[29] The reason that advertising does not fluctuate as much under fair bargaining is simple: because p does not fluctuate, a firm's incentives to advertise do not vary nearly as much as they do under Nash bargaining. With the return to advertising thus much less variable over time, actual advertising is less variable as well.

Related to a point we made earlier, we do not need to take literally the idea that customers and firms calculate deviations of ω from η and use that to inform what prices they consider acceptable. Our model is obviously a metaphor for more subjective forces underlying the kinds of evidence that Blinder et al (1998) and others tabulate that suggest that avoiding customer anger is often a major concern of firms when determining prices.

4.3.5 Customer Bargaining Power and Price Volatility

In the dynamic results so far, we have focused on the case $\eta = \xi_s$, which corresponds to the Hosios (1990) parameterization. In our model with Nash bargaining, even though effective bargaining power ω_t generally is different from η along the business cycle, in the long-run (i.e., in steady-state), $\omega = \eta$. Thus, using the Hosios setting implies that customers' long-run bargaining power is such that the underlying search equilibrium is efficient, which is a useful benchmark.

Of course, long-run bargaining power may not satisfy the Hosios condition. A basic goal of our study is to shed some light on the consequences of bargaining and bargaining power for pricing outcomes. We need not limit ourselves to short-run changes in bargaining power; we can also easily investigate how changes in long-run bargaining power affect pricing. We thus repeat our basic experiments for alternative values of long-run customer bargaining power η. Table 5 reports results for Nash bargaining (the issue at hand here is moot with fair bargaining because fair bargaining renders prices constant), fixing $\kappa = 0$, using a lower setting for customer bargaining power ($\eta = 0.20$) and a higher setting for customer bargaining power ($\eta = 0.50$) (the middle panel repeats results from the top panel of Table 2).

As the results make apparent, the means, volatilities, and correlations of almost all variables change little as we vary long-run bargaining power. The exception, though, is prices (equivalently, the markup): the standard deviation of prices in the search goods sector falls about four-fold as long-run customer bargaining power rises from $\eta = 0.20$ to $\eta = 0.80$. Thus, not only does higher

[29] The setting for κ does not matter here because equilibrium dynamics are invariant to κ.

average customer bargaining power lower the average markup (from 15 percent to less than 5 percent in Table 5), it lowers its volatility as well. To demonstrate this point a bit further, we plot in Figure 10 the mean volatility of prices across simulations for a wide range of values of η and for several values of κ.[30] The rate at which price volatility declines as long-run customer bargaining power rises is larger the higher is κ. Figure 10 thus illustrates two implications of our model that may be testable using time-series data on, say, sectoral-level prices: the relationship between volatility of prices and the fraction of repeat customers (which may serve as a proxy for average customer bargaining power), and how this relationship changes with the importance of pure menu costs in that sector. Such empirical investigation is left for future work. The main point that emerges from our experiments here, though, is that average bargaining power has implications not only for average prices (as illustrated in Figure 4) but also for price volatility.

5 Intensive Quantity Adjustment

In order to highlight the consequences of the formation and dissolution of customer relationships, we have so far limited fluctuations in consumption of search goods to fluctuations at the extensive margin (fluctuations in N). In goods markets, it is natural to think that fluctuations also occur at the intensive margin, the quantity traded per transaction; indeed, this is the only margin at which fluctuations occur in standard models. We now relax the assumption in our baseline model that $c_{it} = \bar{c}$ in every trade i and instead endogenize c.

In order to open up the intensive margin of adjustment, we modify one feature of our baseline model. In our baseline model, the aggregator over search goods over which subutility $v(.)$ is defined is linear in the number of goods obtained, $y = \int_0^N c_i di$. In a symmetric equilibrium, $y = Nc$, which makes clear that from the point of view of just preferences, the household does not care whether a given amount of total search consumption comes from many matches, each with a small per-match quantity, or a small number of matches, each with a large per-match quantity. From a cost perspective, however, the latter is cheaper than the former because finding and engaging in many customer relationships takes time. Thus, were we to stick with the formulation of y we have used thus far as we endogenize c, the number of matches would be driven very low and quantity traded per match would be very large. This poses both a conceptual and a quantitative problem. Conceptually, our model is meant to be one in which consumers "must" engage in search and shopping to obtain goods; a model in which N and s are extremely small and c extremely large goes against this spirit. The agents in our model would cleverly circumvent the frictions we have placed in their way. Quantitatively, it could easily be the case that we would need to employ an

[30]The experiments we conduct in this section of course would be trivial under fair bargaining because prices never change under fair bargaining.

incentive-compatibility constraint in the baseline model to ensure that firms would actually want to participate in such an equilibrium because if a given customer is able to bargain a large c, price could easily be driven below marginal production cost, in which case it of course does not make sense for firms to want to engage in advertising to find customers in the first place.[31]

To sidestep such conceptual and quantitative issues, we modify the aggregator inside $v(.)$ to be a CES composite of the goods obtained from distinct matches,

$$y_t = \left[\int_0^{N_t} c_{it}^{\rho} \right]^{1/\rho}, \tag{46}$$

with $\rho < 1$, which indicates that households have a preference for obtaining consumption from different matches. With sufficiently diminishing returns to consumption at the intensive margin, N will not be driven too low. Aside from the reasons we have already mentioned, introducing such diminishing returns to intensive consumption may be natural in its own right. For example, were sweaters to never carry labels, two sweaters from Banana Republic may be completely indistinguishable from two sweaters from JCrew. With labels, however, it is plausible that a household may desire one sweater from Banana Republic *and* one sweater from JCrew. This type of preference idea often goes under the name "preference for variety," and the label, so to speak, applies well enough to our idea here, as well.[32] Finally, note that in a symmetric equilibrium, $y = cN^{1/\rho}$.

We briefly describe the main modifications to the baseline model of Section 2 and leave most details to Appendix B and C. In principle, the modifications to the model are simple: in addition to the introduction of curvature in the consumption aggregator, $\bar{c}$ from the baseline model is replaced by c_t in all equilibrium conditions, and we must describe the protocol by which quantity traded

[31] A bit more precisely, we tried, holding fixed the calibration in our baseline model, endogenizing c (using the mechanisms described below) with the linear aggregator and indeed found that firms need to be *subsidized* (i.e., $\gamma < 0$) to be induced to advertise and produce for customers (and the resulting equilibrium featured $p < mc$). It could be the case that for some other constellation of parameters of our baseline model, this would not occur, but we did not find one. Assuming that such a parameter constellation does not exist or that such a parameter constellation is unrealistic on other grounds, one would need to impose a constraint that would guarantee that firms enjoy non-negative profits from *paying for* advertising. Because this entry decision occurs every period, such a constraint would in principle be an occasionally-binding constraint, which would require quite different numerical tools than we use to solve the model and would likely be difficult to implement in any case due to the size of our model.

[32] We could also call this formulation "differentiation" of products, as is typical in models that use this type of consumption aggregator. We hesitate to use this terminology, though, because what we have in mind is *not* an ex-ante notion of differentiation, but rather an ex-post notion of differentiation. To continue with the example, what we have in mind is that at a primitive level the household does not care whether it finds sweaters at Banana Republic or JCrew or does not even care whether it obtains either at all. More subtly, *given* that one sweater has been purchased from Banana Republic, in our formulation the household would prefer if the second sweater were purchased from JCrew. In the end, however, this is a somewhat semantic point, and if one wants to call this formulation of our model one in which there is differentiation of goods, we do not strongly object.

in a customer relationship is determined. We assume that the customer and firm bargain (either Nash-bargain or fair-bargain) simultaneously over price and quantity.

5.1 Households

Discounted household utility is now

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(x_t) + \vartheta v \left(\left[\int_0^{N_t^h} c_{it}^{\rho} di \right]^{\frac{1}{\rho}} \right) + g(1 - l_t - s_t - N_t^h) \right], \tag{47}$$

and in the flow budget constraint (2), we simply replace $\bar{c}$ by c_{it}. Because the household takes as given c_{it} in its unilateral utility maximization problem, all household first-order conditions are identical to those in the baseline model, with appropriate replacement of $\bar{c}$ by c_{it}. For example, in a symmetric equilibrium, the shopping condition is now

$$\frac{g'_t}{k^h(\theta_t)} = \beta(1-\rho^x) E_t \left\{ c_{t+1} \left[\vartheta v'_{t+1} - p_{t+1} u'(x_{t+1}) \right] - g'_{t+1} + \frac{g'_{t+1}}{k^h(\theta_{t+1})} \right\}. \tag{48}$$

5.2 Firms

Walrasian firms are no different than in our baseline model. Search firms are also no different, except that in the profit-maximization problem $\bar{c}$ is replaced by c_{it}, which has the consequence that c_{t+1} appears in the period-t advertising condition,

$$\frac{\gamma}{k^f(\theta_t)} = (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left[p_{t+1} c_{t+1} - mc_{t+1} c_{t+1} - \frac{\kappa}{2} \left(\frac{p_{t+1}}{p_t} - 1 \right)^2 + \frac{\gamma}{k^f(\theta_{t+1})} \right] \right\}. \tag{49}$$

Just as households do, search firms take c_{it} as given in their unilateral optimzation problem.

5.3 Price and Quantity Determination

We extend both Nash bargaining and fair bargaining to now include simulataneous bargaining over (p_{it}, c_{it}).

5.3.1 Nash Bargaining

In simultaneous Nash bargaining over (p_{it}, c_{it}), the customer and firm continue to maximize the Nash product (21). The bargained price continues to satisfy condition (24), with appropriate replacement of $\bar{c}$ by c_{it}. The bargained quantity c_{it} solves a condition that takes a similar form,

$$\begin{aligned} \frac{\phi_t}{1-\phi_t} \left[p_{it} c_{it} - mc_t c_{it} - \frac{\kappa}{2} \left(\frac{p_{it}}{p_{it-1}} - 1 \right)^2 + \frac{\gamma}{k^f(\theta_t)} \right] &= \frac{\tilde{u}(c_{it})}{\lambda_t} - p_{it} c_{it} + \\ + \quad (1-\theta_t k^f(\theta_t)) E_t \left[\Xi_{t+1|t} \left(\frac{\phi_{t+1}}{1-\phi_{t+1}} \right) (1-\rho^x) \left[p_{it+1} c_{it+1} - mc_{t+1} c_{it+1} - \frac{\kappa}{2} \left(\frac{p_{it+1}}{p_{it}} - 1 \right)^2 + \frac{\gamma}{k^f(\theta_t)} \right] \right], \end{aligned} \tag{50}$$

except with bargaining weights $\o_t$ and $1 - \o_t$, and $\o_t \neq \omega_t$. The main conceptual difference between $\o_t$ and ω_t is that the former depends on $\tilde{u}'(c_{it})$ whereas the latter does not. If $\kappa = 0$, a more convenient characterization of the solution for the quantity traded is available. With $\kappa = 0$, c_{it} satisfies

$$\frac{g_t'}{\tilde{u}'(c_{it})} = z_t. \tag{51}$$

Further details are provided in Appendix B and Appendix E.

5.3.2 Proportional Bargaining

As we know from the presentation and discussion of proportional bargaining in the baseline model, the outcome replicates that of Nash bargaining with $\kappa = 0$. The irrelevance of menu costs under proportional bargaining thus readily extends to simultaneous bargaining over price and quantity.

5.3.3 Fair Bargaining

Simultaneous fair bargaining over price and quantity means that the customer and firm choose (p_{it}, c_{it}) to maximize $(\mathbf{M_t} - \mathbf{S_t})^{\eta} \mathbf{A_t}^{1-\eta}$ subject to the constant-split rule $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}$. Price continues to be characterized by (30), while the quantity satisfies

$$\frac{g_t'}{\tilde{u}'(c_{it})} = z_t, \tag{52}$$

exactly as in the Nash case with $\kappa = 0$. Thus, fair bargaining, in eliminating the wedge in price-bargaining, also eliminates the wedge in quantity-bargaining. Further details appear in Appendix C and Appendix E.

5.4 Equilibrium

The variables and conditions defining a symmetric search equilibrium are the same as in Section 2.7 (with appropriate replacement of $\bar{c}$ by c_t), with the addition of c_t as an endogenous stochastic process and, depending on whether we are using Nash bargaining or fair bargaining, either condition (50) or (52) to pin down quantity traded.

5.5 Quantitative Results

5.5.1 Parameterization

The functional forms we use are the same as in the baseline model, and, as we already discussed, we use the aggregator $\left[\int_0^{N_t} c_{it}^{\rho}\right]^{\frac{1}{\rho}}$ inside the subutility function $v(.)$. We keep all parameter settings fixed at their values in the baseline model to try to achieve some comparability. However, the models are not readily comparable because we also must choose $\rho < 1$; by thus changing a structural parameter of the economy relative to the baseline economy, cross-model comparisons inherently become difficult to interpret. In our setting for ρ, we choose to make the steady-state markup the same as in the baseline model when $\eta = 0.50$. Setting $\rho = 0.19$ delivers a gross markup of 1.0835 here, identical to that in the baseline calibration of the model of Section 2.

5.5.2 Steady State

Steady-state prices and quantities are reported in Table 6 for the model with intensive adjustment. Comparing with Table 1, the main difference is that the fraction of time spent in shopping-related activities is much higher. Both the number of active customer relationships and time spent searching are much higher. Larger N reflects the curvature we introduced in the search goods aggregator, which, as we discussed at the start of this section, gives households a preference for obtaining consumption from many matches. Absent some other counteracting feature of the environment, this version of our model thus cannot account for the time-use evidence that shows $(N+s)/l = 0.25$. Because relationships occasionally dissolve, households must also search more in order to maintain the larger N, explaining the higher level of s in the model here compared to the baseline model.

Figure 11 illustrates how key steady-state variables change with customer bargaining power η. As the middle left panel shows, the extensive quantity N falls, as it did in Figure 4; the difference here is that the intensive quantity c simultaneously rises the larger is η. This substitution reflects the fact that with higher η, households know they can leverage a higher quantity out of each customer relationship. Given that match formation is costly, it makes sense for the household to essentially substitute out of extensive consumption and into intensive consumption. The levels and responses of all other variables are very similar to those in Figure 4, so we do not discuss them further.

5.5.3 Dynamics

Tables 7 and 8 catalog results for the same set of dynamic experiments we conducted in Section 4.3; endogenizing the intensive quantity does not change the main quantitative results presented there. A few new notable results do arise, though. First, endogenizing the intensive quantity imparts more persistence to household search behavior. Second, as shown in Tables 2, 3, and 4, with fixed $\bar{c}$, search activity is weakly procyclical no matter the bargaining protocol. In contrast, with endogenous intensive quantity, the cyclicality of s depends crucially on the bargaining protocol in place. In turn, the cyclicality of search mirrors the cyclicality of the intensive quantity c.

With Nash bargaining, c is always countercyclical with respect to total GDP. This result is the dynamic manifestation of the substitution between N and c just discussed above: because of the "preference for variety" in this version of the model, all else equal, the household prefers that expansions in search consumption come at the extensive margin rather than at the intensive margin. It is also striking how much less volatile c is than N is: between five- and six-fold less volatile depending on the value of κ. A standard model's notion of consumption-smoothing is thus pushed down to the level of intensive consumption in this version of our model. We think this makes sense because it is intensive consumption that is essentially governed by a MRS-type of optimality condition; this can be seen most clearly in condition (51), which is also the core of condition (50).

As in the baseline model without intensive adjustment, the proportional bargaining outcome, no matter the value of κ, is identical to the Nash outcome with $\kappa = 0$, so this notion of fairness leaves c countercyclical. In contrast, under fair bargaining, c is procyclical with respect to total GDP. This seems to be related to the result that the volatility of c relative to the volatility of N is not nearly as small under fair bargaining as it is under Nash bargaining. As in the baseline model, fair bargaining mutes the volatility of p, which in turn governs the incentives of firms to cyclically alter their advertising behavior. With advertising not nearly as volatile as under Nash bargaining, the equilibrium stock of active customer relationships does not fluctuate nearly as much, which in turn means that the scope for substitution between N and c is dramatically reduced. Thus, despite the primitive "preference for variety," households must accept procyclical movements of c to vary their consumption of search goods.

6 Demand Shocks

We noted in Section 4.3 that, although our basic model driven by only TFP shocks does not deliver countercyclical markups, extending our model to allow for demand shocks can overturn this prediction. To the extent that it is not clear whether the countercyclicality of markups observed in the data is due predominately to demand shocks or supply shocks, we think it is useful to at least know that our model is capable of matching this stylized fact for some shocks.

We introduce demand shocks by allowing exogenous government purchases. For the sake of simplicity, we only conduct the experiments in this section in the baseline model with fixed $\bar{c}$. Total output is now absorbed by consumption (of both Walrasian and search goods), advertising costs, price adjustment costs, and government spending, so the resource constraint is

$$x_t + N_t\bar{c} + g_t + \gamma a_t + \frac{\kappa}{2}(\pi_t - 1)^2 N_t = z_t l_t. \tag{53}$$

No other features of the baseline model of Section 2 change, hence all equilibrium conditions, with the obvious exception of the resource constraint, are unchanged. Government spending is assumed to follow an AR(1) in logs,

$$\log g_{t+1} = (1 - \rho_g)\log \bar{g} + \rho_g \log g_t + \epsilon^g_{t+1}, \tag{54}$$

with $\epsilon^g \sim iidN(0, \sigma_g)$, and the shock to government spending is uncorrelated with the shock to TFP. We set $\bar{g} = 0.05$ so that government spending makes up about 19 percent of total output, set $\rho_g = 0.97$, in line with Schmitt-Grohe and Uribe (2004a), and set σ_g so that the standard deviation of government purchases is 6 percent of the mean level $\bar{g}$ — the resulting value is $\sigma_g = 0.03$.

We examine the dynamics of this model in the face of just shocks to government spending. The upper panel of Table 9 shows that with $\kappa = 0$ and Nash bargaining as the pricing mechanism, the contemporaneous correlation of the markup with GDP is -0.32. As κ increases, however, the correlation turns positive. So our claim is not that our model with demand shocks always predicts a countercyclical markup, just that it can. Finally, we do not need to consider fair bargaining here because markups are time-invariant under that protocol. We leave to future work a full investigation of the cyclicality properties of the markup in our model.

7 Conclusion

We constructed a model in which long-lived customer relationships allow one to think about an array of pricing schemes in goods-market transactions. Our focus here was on bargaining arrangements between firms and customers in which fairness was of paramount concern. Depending on exactly how our concept of fairness is operationalized, menu costs may be either completely irrelevant for dynamics (proportional bargaining) or may lead endogenously to complete price stability (fair bargaining). Under proportional bargaining, consumers and firms effectively share the menu costs efficiently, making them irrelevant. Under fair bargaining, price changes are the only source of time-variation in effective bargaining power between a firm and a customer. Avoiding this time-variation in effective bargaining power requires ensuring price stability. As we noted, in a richer model (one that includes, for example, labor or consumption taxes), stabilizing surplus shares need not require complete price stability.

Understanding how pricing decisions are affected by bargaining power is an understudied topic, at least in the context of modern quantitative macroeconomic models. Survey evidence seems to repeatedly support the view that firms often avoid changing prices out of concern for upsetting their existing customers. It is difficult to articulate very precisely such views in standard models of goods markets because in the basic Walrasian framework, of course, there simply are no clear notions of customers and bargaining. We think we have made some progress in at least pointing out a potentially useful direction, one that is immediately tractable in modern quantitative macroeconomic models. An even deeper bargaining-theoretic explanation for price rigidity is provided by Menzio (2007), but the tension there is that his framework seems likely not readily tractable in standard DSGE models.

More broadly than the fair-bargaining result around which we centered much of our analysis, we think the general framework we developed of search-based frictions in goods markets holds the promise of being usefully incorporated into standard quantitative macroeconomic models. Such a line of research immediately allows one to think beyond the standard marginal pricing conditions present in most models and therefore is likely to be useful in bringing to bear new ideas about a host of issues in macroeconomics. As we have admitted, more work needs to be done in thinking about the empirical counterparts of and calibration of some elements of our model. But at least these are testable parts of our model.

It is well-understood from standard macro models that wedges between consumption-leisure marginal rates of substitution and marginal rates of transformation must stem from frictions in goods markets or from frictions in labor markets or both. Chari, Kehoe, and McGrattan (2006), among others, have emphasized that understanding this (to use their terminology) "labor wedge" is quite important for macro modeling efforts. Our model posits search frictions in product markets alongside a perfectly-competitive labor market, so the wedge between the MRS and the consumption-leisure MRT stems from goods market frictions. Labor search models, especially their recent DSGE incarnations, have made clear that such wedges arise from primitive labor matching frictions as well. A model featuring labor search and goods search frictions may have even more to offer in terms of explaining labor wedges. In addition, a nominal version of such a model, in which it is nominal prices and nominal wages over which parties bargain, would have some potentially very interesting things to say regarding dynamics of real wages. Hall's (2007) model does incorporate both goods search and labor search frictions and points out some interesting tradeoffs regarding stabilizing goods markets versus stabilizing labor markets that monetary policy may face.

We see a great many other possible applications of our framework. One application is to asset pricing: the fact that marginal utility of consumption is tied to consumer search frictions opens up a new mechanism for thinking about asset prices. Another possible use for our framework is to provide a micro-foundation behind habit-based consumption models: part of consumption is a state variable in our model, as it is in habit-based models, because it directly reflects the number of pre-existing customer relationships. Our framework, sufficiently enriched on the pricing/bargaining side of the model, may also provide a different way to think about time-varying markups in goods markets, an issue we only scratched the surface of here.

Finally, another important motivation behind the construction of our model is the eventual design of optimal macroeconomic policy in such an environment. Arseneau and Chugh (2006, 2007) study optimal policy in the presence of deep-rooted frictions in labor markets, with all other macro markets quite standard. Aruoba and Chugh (2006) study optimal policy in the presence of deep-rooted frictions in money markets, with all other macro markets quite standard. Both of these studies uncover policy channels and implications about which standard models used to study dynamic optimal policy are silent, and several of the policy prescriptions obtained in these studies are indeed opposite those reached using standard frameworks. However, each of these studies considers goods trade in more or less standard Walrasian fashion. We conjecture that thinking about deep-rooted frictions in goods markets is likely to also yield new insights about how macroeconomic policy ought to be conducted, a topic rising on our research horizon.

A Tables and Figures

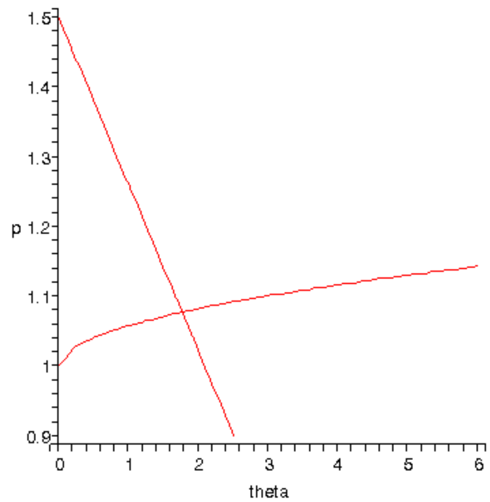

Figure 1: Steady-state firm advertising condition and Nash pricing condition.

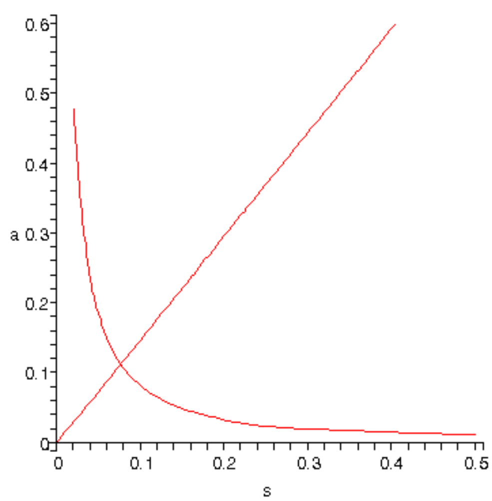

Figure 2: Steady-state flow condition and firm advertising condition

μ	a	s	θ	N	Nc	x	gdp
1.0835	0.0151	0.0189	0.8000	0.0680	0.0952	0.1919	0.2943

Table 1: Steady-state prices and quantities under benchmark calibration.

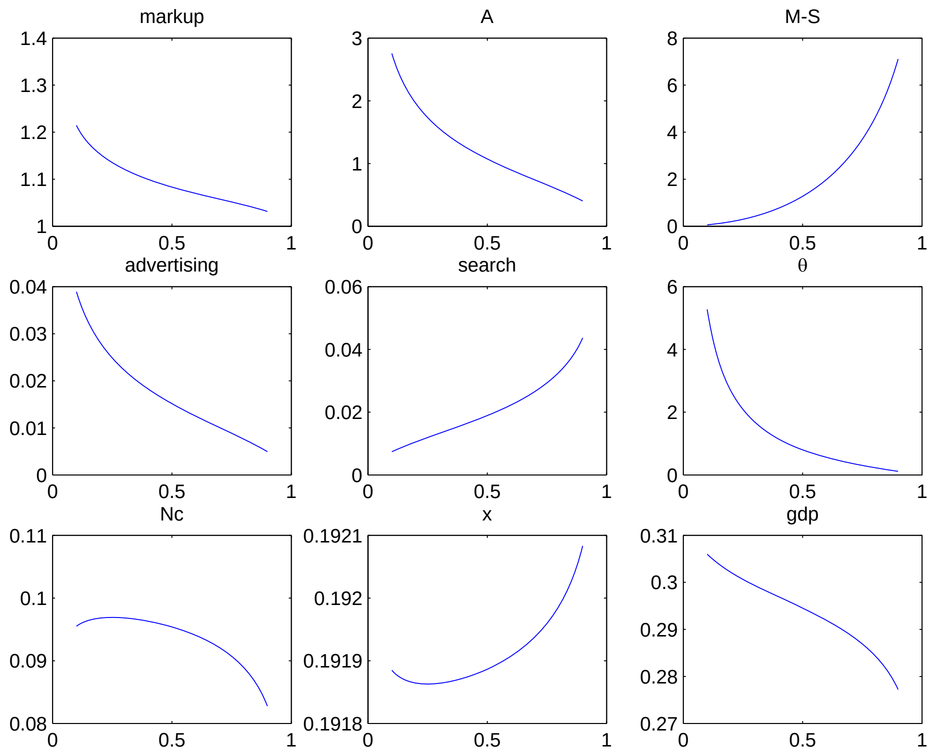

Figure 3: Steady-state allocation as function of customer bargaining power η.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x, Y)	Corr(x, Z)
			No menu cost, $\kappa = 0$			
μ	1.0835	0.0036	0.0033	0.1751	0.5557	0.5850
gdp	0.2945	0.0049	0.0166	0.9342	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9246	0.9996	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0008	0.0113	0.9274	0.9345	0.9212
Nc	0.0954	0.0011	0.0113	0.9274	0.9345	0.9212
l	0.2945	0.0007	0.0025	0.7866	-0.9626	-0.9718
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0.0036	0.0033	0.1751	0.5557	0.5850
s	0.0189	0.0007	0.0360	0.0187	0.0885	0.1237
θ	0.7996	0.0233	0.0292	0.9209	0.9992	1.0000
ω	0.5000	0.0000	0.0000	—	—	—
a	0.0152	0.0007	0.0494	0.2868	0.6624	0.6878
			Addendum			
ω^{PROP}	0.5000	0.0138	0.0277	-0.3752	-0.2030	-0.1776

Table 2: Simulation-based moments in baseline model with Nash bargaining, zero menu costs, and TFP shocks as driving force.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x,Y)	Corr(x,Z)
			Menu cost, $\kappa = 5$			
μ	1.0835	0.0032	0.0030	0.8008	0.9387	0.9488
gdp	0.2945	0.0049	0.0166	0.9344	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9245	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0008	0.0113	0.9285	0.9346	0.9211
Nc	0.0954	0.0011	0.0113	0.9285	0.9346	0.9211
l	0.2945	0.0007	0.0025	0.7864	-0.9623	-0.9716
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0.0032	0.0030	0.8008	0.9387	0.9488
s	0.0189	0.0007	0.0356	0.0256	0.0940	0.1295
θ	0.7996	0.0230	0.0287	0.9227	0.9994	1.0000
ω	0.5000	0.0015	0.0031	0.1087	-0.3572	-0.3246
a	0.0152	0.0007	0.0488	0.2934	0.6642	0.6898
			Menu cost, $\kappa = 20$			
μ	1.0834	0.0050	0.0047	0.9775	0.9533	0.9467
gdp	0.2945	0.0049	0.0165	0.9344	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9244	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0008	0.0113	0.9293	0.9346	0.9211
Nc	0.0954	0.0011	0.0113	0.9293	0.9346	0.9211
l	0.2945	0.0007	0.0025	0.7882	-0.9627	-0.9720
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0834	0.0050	0.0047	0.9775	0.9533	0.9467
s	0.0189	0.0007	0.0353	0.0296	0.1076	0.1432
θ	0.7996	0.0221	0.0277	0.9242	0.9995	1.0000
ω	0.5000	0.0031	0.0061	0.5501	-0.4542	-0.4233
a	0.0152	0.0007	0.0481	0.2940	0.6618	0.6876
			Menu cost, $\kappa = 100$			
μ	1.0833	0.0093	0.0086	0.9967	0.5995	0.5909
gdp	0.2945	0.0048	0.0164	0.9344	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9244	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0008	0.0111	0.9298	0.9345	0.9210
Nc	0.0954	0.0011	0.0111	0.9298	0.9345	0.9210
l	0.2945	0.0008	0.0026	0.7967	-0.9658	-0.9747
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0833	0.0093	0.0086	0.9967	0.5995	0.5909
s	0.0189	0.0007	0.0349	0.0293	0.1570	0.1922
θ	0.7997	0.0190	0.0237	0.9256	0.9996	0.9999
ω	0.5001	0.0058	0.0117	0.8008	-0.1332	-0.1113
a	0.0152	0.0007	0.0459	0.2750	0.6429	0.6691

Table 3: Simulation-based moments in baseline model with Nash bargaining, positive menu costs, and TFP shocks as driving force.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x,Y)	Corr(x,Z)
			Menu cost, $\kappa = 5$			
μ	1.0835	0	0	—	—	—
gdp	0.2945	0.0048	0.0161	0.9341	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9242	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0007	0.0106	0.9299	0.9342	0.9210
Nc	0.0954	0.0010	0.0106	0.9299	0.9342	0.9210
l	0.2945	0.0009	0.0029	0.8184	-0.9737	-0.9812
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0	0	—	—	—
s	0.0189	0.0007	0.0346	0.0474	0.3010	0.3341
θ	0.7998	0.0098	0.0123	0.9294	0.9999	0.9998
ω	0.5000	0	0	—	—	—
			Menu cost, $\kappa = 20$			
μ	1.0835	0	0	—	—	—
gdp	0.2945	0.0048	0.0161	0.9341	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9242	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0007	0.0106	0.9299	0.9342	0.9210
Nc	0.0954	0.0010	0.0106	0.9299	0.9342	0.9210
l	0.2945	0.0009	0.0029	0.8184	-0.9737	-0.9812
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0	0	—	—	—
s	0.0189	0.0007	0.0346	0.0474	0.3010	0.3341
θ	0.7998	0.0098	0.0123	0.9294	0.9999	0.9998
ω	0.5000	0	0	—	—	—
			Menu cost, $\kappa = 50$			
μ	1.0835	0	0	—	—	—
gdp	0.2945	0.0048	0.0161	0.9341	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9242	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0007	0.0106	0.9299	0.9342	0.9210
Nc	0.0954	0.0010	0.0106	0.9299	0.9342	0.9210
l	0.2945	0.0009	0.0029	0.8184	-0.9737	-0.9812
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0	0	—	—	—
s	0.0189	0.0007	0.0346	0.0474	0.3010	0.3341
θ	0.7998	0.0098	0.0123	0.9294	0.9999	0.9998
ω	0.5000	0	0	—	—	—
			Menu cost, $\kappa = 100$			
μ	1.0835	0	0	—	—	—
gdp	0.2945	0.0048	0.0161	0.9341	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9242	0.9995	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0007	0.0106	0.9299	0.9342	0.9210
Nc	0.0954	0.0010	0.0106	0.9299	0.9342	0.9210
l	0.2945	0.0009	0.0029	0.8184	-0.9737	-0.9812
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0	0	—	—	—
s	0.0189	0.0007	0.0346	0.0474	0.3010	0.3341
θ	0.7998	0.0098	0.0123	0.9294	0.9999	0.9998
ω	0.5000	0	0	—	—	—

Table 4: Simulation-based moments in baseline model with fair bargaining and TFP shocks as driving force.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x,Y)	Corr(x,Z)
			Low customer bargaining power ($\eta = 0.20$)			
μ	1.1526	0.0059	0.0052	0.2616	0.6868	0.6778
gdp	0.3021	0.0051	0.0168	0.9167	1.0000	0.9999
x	0.1918	0.0036	0.0190	0.9236	0.9998	1.0000
c	1.4000	0	0	—	—	—
N	0.0691	0.0007	0.0107	0.9256	0.9163	0.9213
Nc	0.0968	0.0010	0.0107	0.9256	0.9163	0.9213
l	0.3021	0.0007	0.0022	0.9498	-0.9942	-0.9955
w	0.9997	0.0190	0.0190	0.9216	0.9999	1.0000
p	1.1526	0.0059	0.0052	0.2616	0.6868	0.6778
s	0.0105	0.0004	0.0342	0.0311	0.0357	0.0235
θ	2.6707	0.0916	0.0343	0.9215	0.9999	1.0000
ω	0.2000	0	0	—	—	—
a	0.0281	0.0014	0.0494	0.3186	0.7270	0.7188
			Equal bargaining power ($\eta = 0.50$)			
μ	1.0835	0.0036	0.0033	0.1751	0.5557	0.5850
gdp	0.2945	0.0049	0.0166	0.9342	1.0000	0.9993
x	0.1918	0.0036	0.0190	0.9246	0.9996	1.0000
c	1.4000	0	0	—	—	—
N	0.0681	0.0008	0.0113	0.9274	0.9345	0.9212
Nc	0.0954	0.0011	0.0113	0.9274	0.9345	0.9212
l	0.2945	0.0007	0.0025	0.7866	-0.9626	-0.9718
w	0.9997	0.0190	0.0190	0.9216	0.9993	1.0000
p	1.0835	0.0036	0.0033	0.1751	0.5557	0.5850
s	0.0189	0.0007	0.0360	0.0187	0.0885	0.1237
θ	0.7996	0.0233	0.0292	0.9209	0.9992	1.0000
ω	0.5000	0	0	—	—	—
a	0.0152	0.0007	0.0494	0.2868	0.6624	0.6878
			High customer bargaining power ($\eta = 0.80$)			
μ	1.0452	0.0015	0.0015	0.2316	0.5364	0.5951
gdp	0.2846	0.0047	0.0165	0.9448	1.0000	0.9974
x	0.1919	0.0036	0.0189	0.9282	0.9985	0.9998
c	1.4000	0	0	—	—	—
N	0.0635	0.0007	0.0116	0.9368	0.9459	0.9202
Nc	0.0889	0.0010	0.0116	0.9368	0.9459	0.9202
l	0.2846	0.0008	0.0027	0.6392	-0.8747	-0.9072
w	0.9997	0.0190	0.0190	0.9216	0.9974	1.0000
p	1.0452	0.0015	0.0015	0.2316	0.5364	0.5951
s	0.0326	0.0011	0.0343	0.0824	0.1325	0.2022
θ	0.2338	0.0059	0.0251	0.9196	0.9970	1.0000
ω	0.8000	0	0	—	—	—
a	0.0076	0.0004	0.0468	0.3301	0.6403	0.6914

Table 5: Simulation-based moments, for various values of η, in basic model with Nash bargaining, $\kappa = 0$, and TFP shocks as driving force.

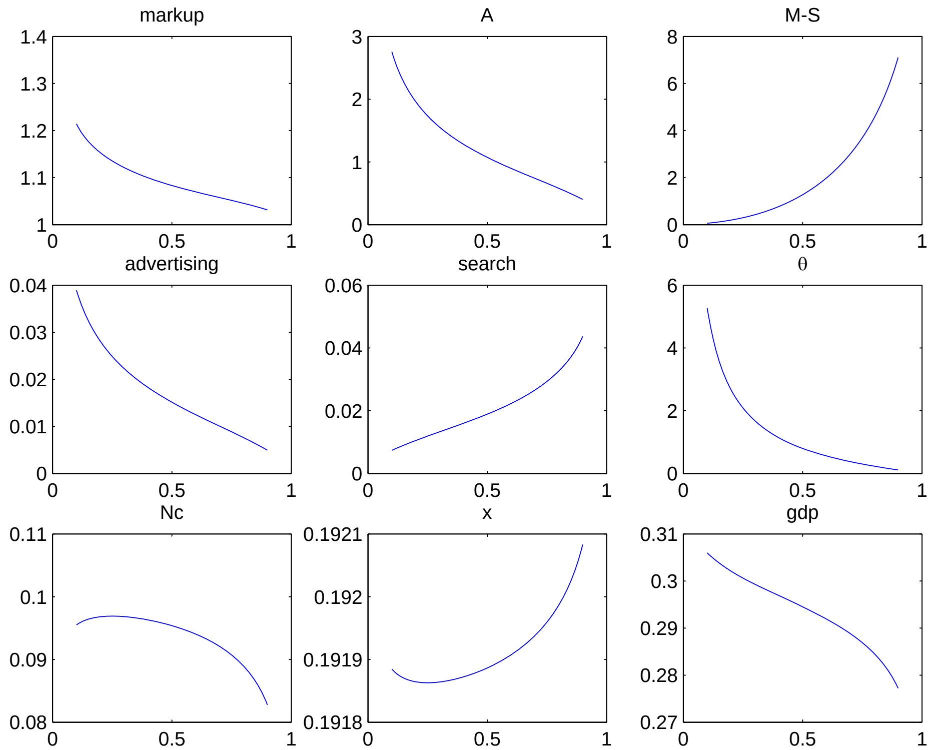

Figure 4: Steady-state allocation as function of customer bargaining power η.

μ	a	s	θ	N	c	Nc	x	gdp
1.0835	0.0267	0.1851	0.1443	0.2830	0.5951	0.1684	0.1276	0.3088

Table 6: Steady-state prices and quantities with intensive adjustment.

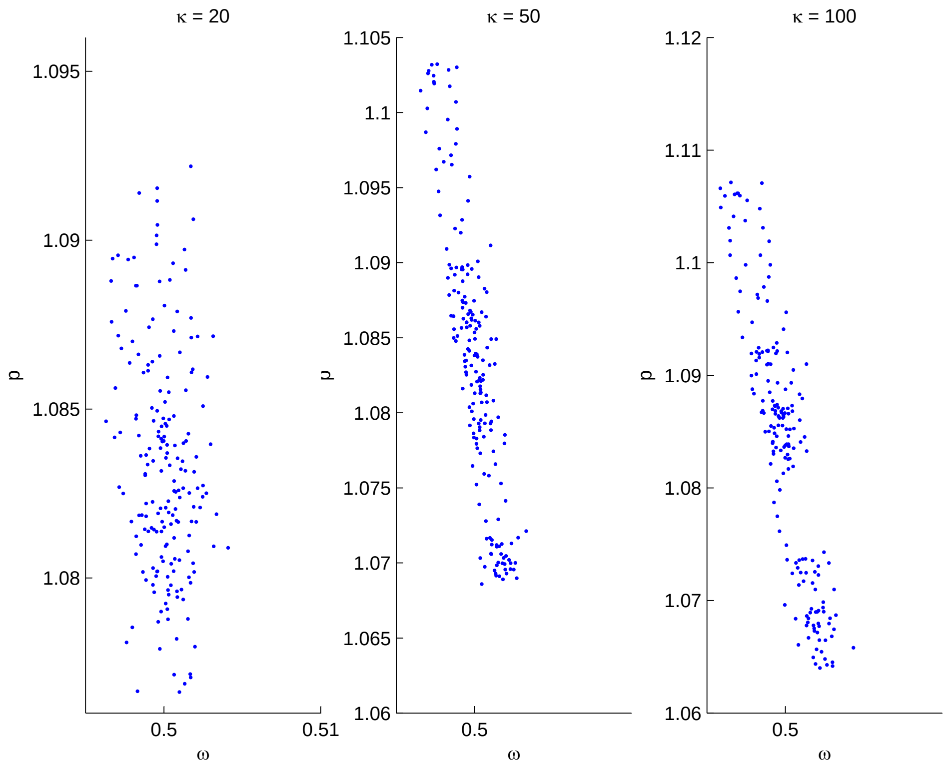

Figure 5: Dynamic relationship between customer effective bargaining power ω and Nash price for various degrees of price rigidity.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x, Y)	Corr(x, Z)
			Flexible prices, $\kappa = 0$			
μ	1.0834	0.0030	0.0027	0.8317	0.9196	0.9701
gdp	0.3088	0.0044	0.0143	0.9613	1.0000	0.9874
x	0.1276	0.0022	0.0173	0.9484	0.9982	0.9952
c	0.5951	0.0012	0.0020	0.8868	-0.7597	-0.6487
N	0.2829	0.0035	0.0123	0.9783	0.9466	0.8840
Nc	0.1684	0.0018	0.0105	0.9827	0.9672	0.9151
l	0.3088	0.0016	0.0053	0.7549	-0.8355	-0.9115
w	0.9997	0.0190	0.0190	0.9216	0.9874	1.0000
p	1.0834	0.0030	0.0027	0.8317	0.9196	0.9701
s	0.1851	0.0032	0.0173	0.6639	-0.3275	-0.1772
θ	0.1442	0.0067	0.0466	0.9224	0.9878	1.0000
ω	0.5000	0.0000	—	—	—	—
a	0.0267	0.0013	0.0473	0.7884	0.8672	0.9329
			Menu cost, $\kappa = 5$			
μ	1.0833	0.0084	0.0078	0.9802	0.9892	0.9553
gdp	0.3088	0.0044	0.0141	0.9601	1.0000	0.9882
x	0.1276	0.0022	0.0174	0.9459	0.9979	0.9960
c	0.5951	0.0012	0.0020	0.8827	-0.7379	-0.6274
N	0.2829	0.0034	0.0120	0.9791	0.9432	0.8816
Nc	0.1684	0.0017	0.0101	0.9834	0.9668	0.9165
l	0.3088	0.0017	0.0054	0.7710	-0.8540	-0.9233
w	0.9997	0.0190	0.0190	0.9216	0.9882	1.0000
p	1.0833	0.0084	0.0078	0.9802	0.9892	0.9553
s	0.1851	0.0031	0.0165	0.6694	-0.3077	-0.1616
θ	0.1442	0.0064	0.0447	0.9258	0.9903	0.9999
ω	0.5000	0.0034	0.0068	0.9397	-0.8392	-0.7470
a	0.0267	0.0012	0.0454	0.7967	0.8759	0.9375
			Menu cost, $\kappa = 20$			
μ	1.0832	0.0173	0.0160	0.9947	0.8147	0.7554
gdp	0.3088	0.0043	0.0138	0.9580	1.0000	0.9898
x	0.1276	0.0022	0.0176	0.9428	0.9978	0.9970
c	0.5951	0.0012	0.0020	0.8750	-0.6628	-0.5526
N	0.2829	0.0032	0.0112	0.9799	0.9375	0.8788
Nc	0.1684	0.0016	0.0095	0.9836	0.9676	0.9220
l	0.3088	0.0018	0.0057	0.7932	-0.8851	-0.9423
w	0.9997	0.0190	0.0190	0.9216	0.9898	1.0000
p	1.0832	0.0173	0.0160	0.9947	0.8147	0.7554
s	0.1851	0.0028	0.0151	0.6623	-0.2531	-0.1158
θ	0.1442	0.0058	0.0404	0.9294	0.9933	0.9996
ω	0.5001	0.0089	0.0178	0.9685	-0.5538	-0.4626
a	0.0267	0.0011	0.0416	0.8032	0.8857	0.9410
			Menu cost, $\kappa = 50$			
μ	1.0830	0.0225	0.0208	0.9973	0.5805	0.5337
gdp	0.3088	0.0041	0.0134	0.9559	1.0000	0.9912
x	0.1276	0.0023	0.0178	0.9401	0.9978	0.9977
c	0.5951	0.0011	0.0018	0.8415	-0.5144	-0.4056
N	0.2829	0.0029	0.0103	0.9803	0.9324	0.8771
Nc	0.1684	0.0015	0.0089	0.9837	0.9689	0.9277
l	0.3088	0.0018	0.0060	0.8133	-0.9109	-0.9574
w	0.9997	0.0190	0.0190	0.9216	0.9912	1.0000
p	1.0830	0.0225	0.0208	0.9973	0.5805	0.5337
s	0.1851	0.0025	0.0136	0.6410	-0.1672	-0.0388
θ	0.1442	0.0050	0.0347	0.9318	0.9953	0.9993
ω	0.5002	0.0126	0.0252	0.9681	-0.1732	-0.1076
a	0.0267	0.0010	0.0368	0.8038	0.8900	0.9407

Table 7: Simulation-based moments in model with intensive adjustment, Nash bargaining, and TFP shocks as driving force.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x,Y)	Corr(x,Z)
			Flexible prices, $\kappa = 5$			
μ	1.0835	0	0	—	—	—
gdp	0.3088	0.0037	0.0120	0.9475	1.0000	0.9952
x	0.1276	0.0024	0.0186	0.9325	0.9982	0.9993
c	0.5951	0.0007	0.0012	0.6530	0.6820	0.7494
N	0.2829	0.0018	0.0064	0.9805	0.9193	0.8768
Nc	0.1684	0.0012	0.0070	0.9821	0.9743	0.9477
l	0.3088	0.0022	0.0071	0.8677	-0.9664	-0.9869
w	0.9997	0.0190	0.0190	0.9216	0.9952	1.0000
p	1.0835	0	0	—	—	—
s	0.1851	0.0018	0.0095	0.5763	0.3775	0.4645
θ	0.1443	0.0017	0.0121	0.9428	0.9998	0.9969
ω	0.5000	0	0	—	—	—
a	0.0267	0.0005	0.0181	0.7691	0.8605	0.9062
			Menu cost, $\kappa = 20$			
μ	1.0835	0	0	—	—	—
gdp	0.3088	0.0037	0.0120	0.9475	1.0000	0.9952
x	0.1276	0.0024	0.0186	0.9325	0.9982	0.9993
c	0.5951	0.0007	0.0012	0.6530	0.6820	0.7494
N	0.2829	0.0018	0.0064	0.9805	0.9193	0.8768
Nc	0.1684	0.0012	0.0070	0.9821	0.9743	0.9477
l	0.3088	0.0022	0.0071	0.8677	-0.9664	-0.9869
w	0.9997	0.0190	0.0190	0.9216	0.9952	1.0000
p	1.0835	0	0	—	—	—
s	0.1851	0.0018	0.0095	0.5763	0.3775	0.4645
θ	0.1443	0.0017	0.0121	0.9428	0.9998	0.9969
ω	0.5000	0	0	—	—	—
a	0.0267	0.0005	0.0181	0.7691	0.8605	0.9062
			Menu cost, $\kappa = 50$			
μ	1.0835	0	0	—	—	—
gdp	0.3088	0.0037	0.0120	0.9475	1.0000	0.9952
x	0.1276	0.0024	0.0186	0.9325	0.9982	0.9993
c	0.5951	0.0007	0.0012	0.6530	0.6820	0.7494
N	0.2829	0.0018	0.0064	0.9805	0.9193	0.8768
Nc	0.1684	0.0012	0.0070	0.9821	0.9743	0.9477
l	0.3088	0.0022	0.0071	0.8677	-0.9664	-0.9869
w	0.9997	0.0190	0.0190	0.9216	0.9952	1.0000
p	1.0835	0	0	—	—	—
s	0.1851	0.0018	0.0095	0.5763	0.3775	0.4645
θ	0.1443	0.0017	0.0121	0.9428	0.9998	0.9969
ω	0.5000	0	0	—	—	—
a	0.0267	0.0005	0.0181	0.7691	0.8605	0.9062
			Menu cost, $\kappa = 100$			
μ	1.0835	0	0	—	—	—
gdp	0.3088	0.0037	0.0120	0.9475	1.0000	0.9952
x	0.1276	0.0024	0.0186	0.9325	0.9982	0.9993
c	0.5951	0.0007	0.0012	0.6530	0.6820	0.7494
N	0.2829	0.0018	0.0064	0.9805	0.9193	0.8768
Nc	0.1684	0.0012	0.0070	0.9821	0.9743	0.9477
l	0.3088	0.0022	0.0071	0.8677	-0.9664	-0.9869
w	0.9997	0.0190	0.0190	0.9216	0.9952	1.0000
p	1.0835	0	0	—	—	—
s	0.1851	0.0018	0.0095	0.5763	0.3775	0.4645
θ	0.1443	0.0017	0.0121	0.9428	0.9998	0.9969
ω	0.5000	0	0	—	—	—
a	0.0267	0.0005	0.0181	0.7691	0.8605	0.9062

Table 8: Simulation-based moments in model with intensive adjustment, fair bargaining, and TFP shocks as driving force.

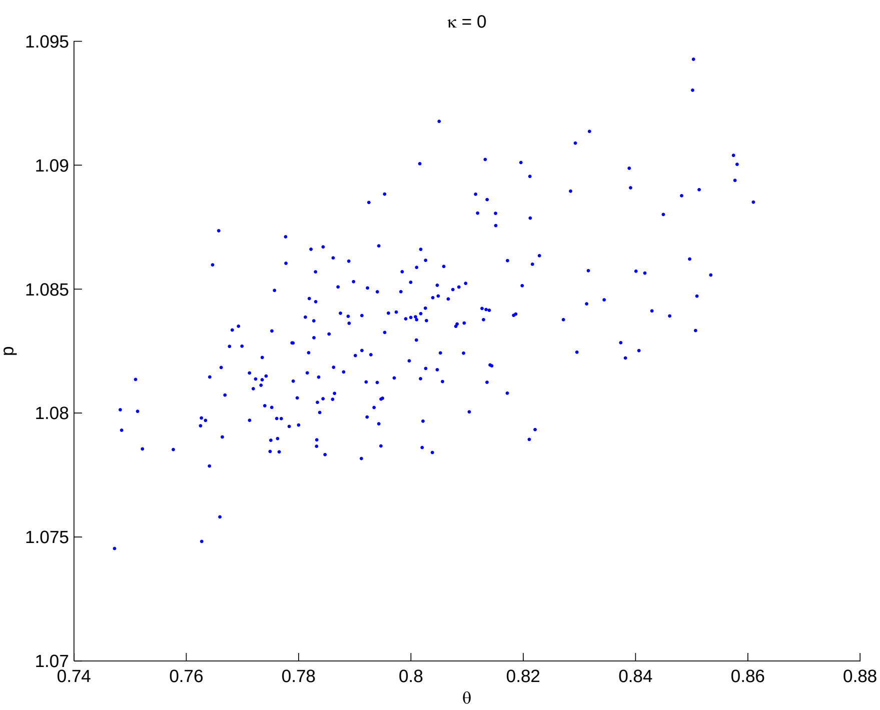

Figure 6: Dynamic relationship between goods market tightness θ and Nash price p in baseline model with $\kappa = 0$.

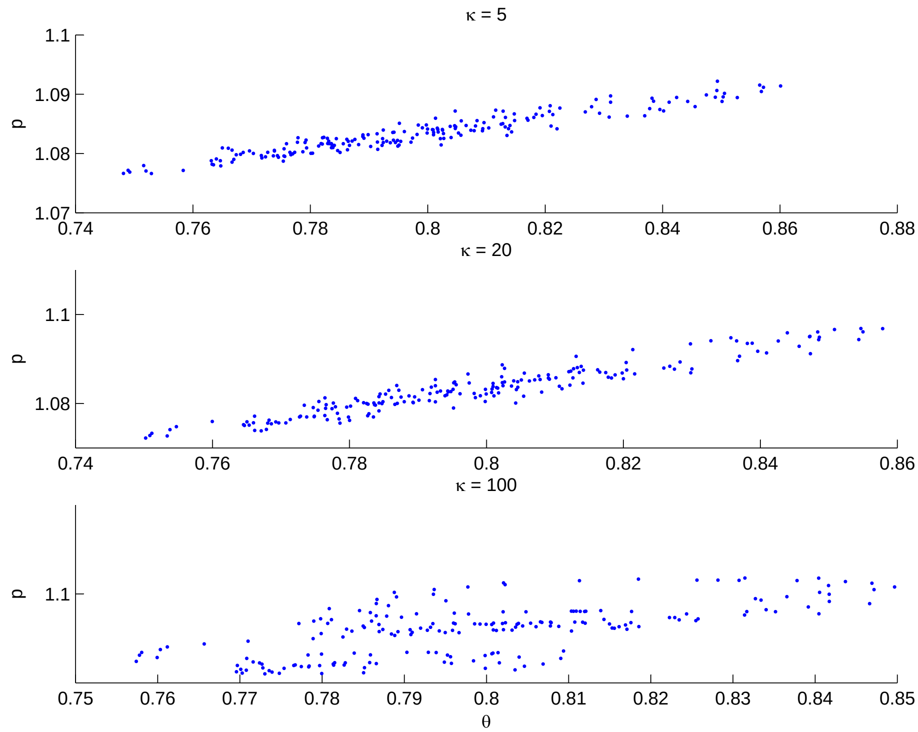

Figure 7: Dynamic relationship between goods market tightness θ and Nash price p in baseline model with $\kappa > 0$.

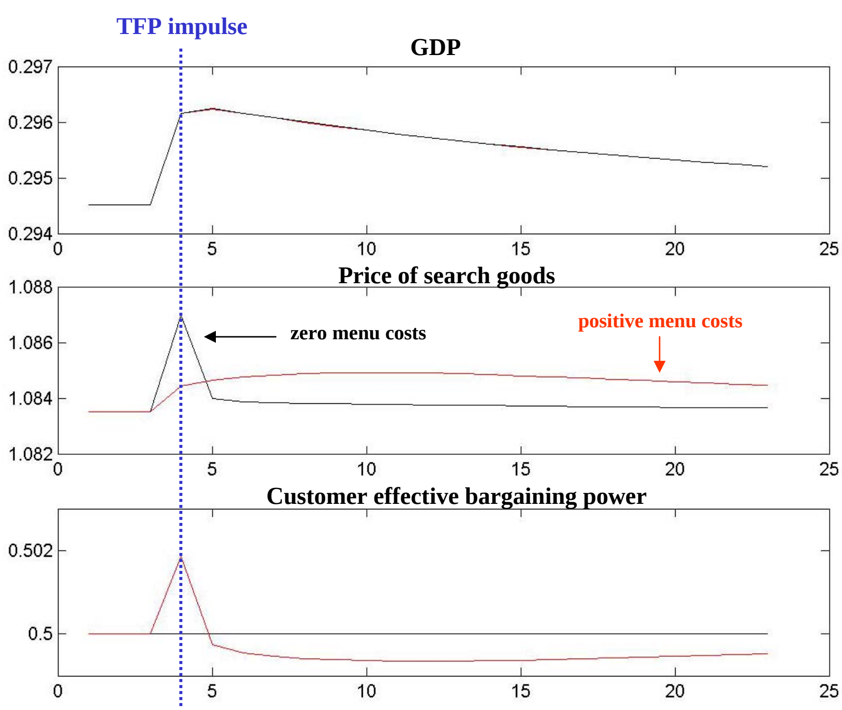

Figure 8: Impulse response in baseline model.

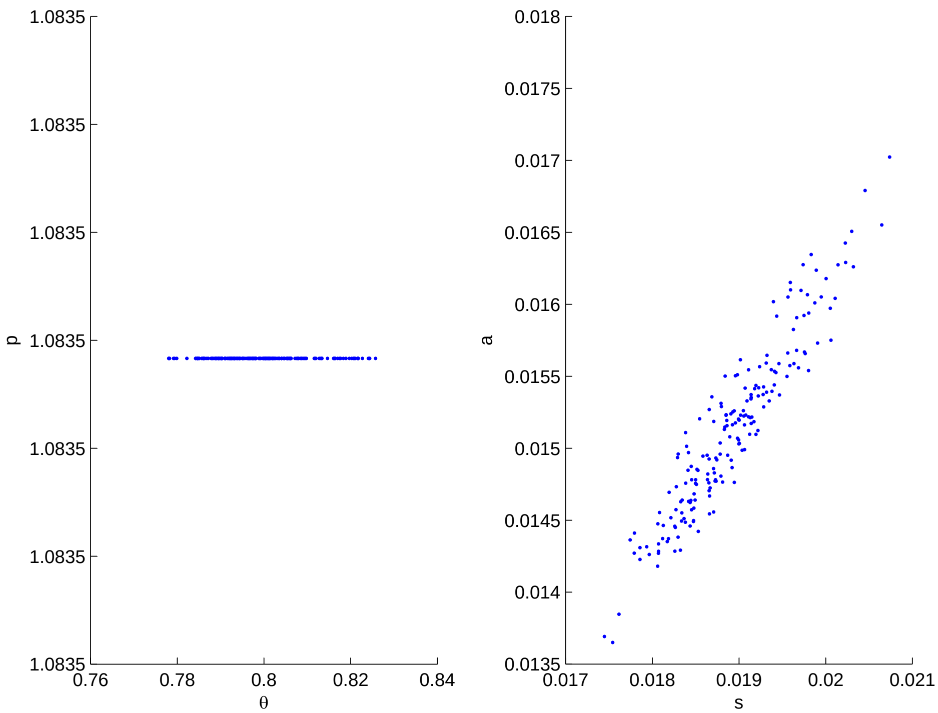

Figure 9: Dynamic relationship between goods market tightness θ and Nash price p (left panel) and advertisements a and consumer search activity s in baseline model with fair bargaining.

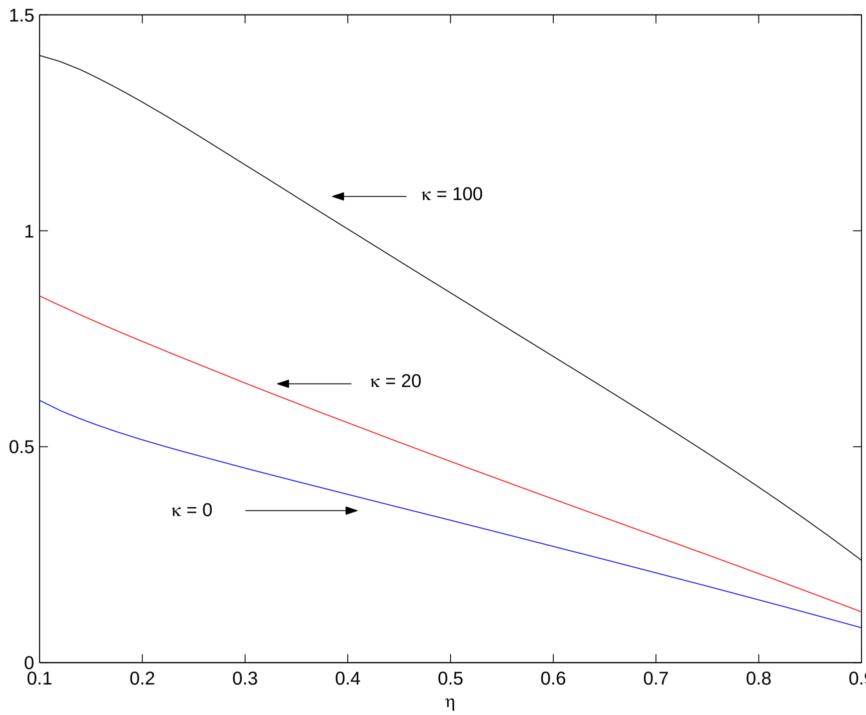

Figure 10: Volatility of price (expressed in standard deviation percentage points) as function of customer bargaining power η in baseline model with Nash bargaining for various values of κ.

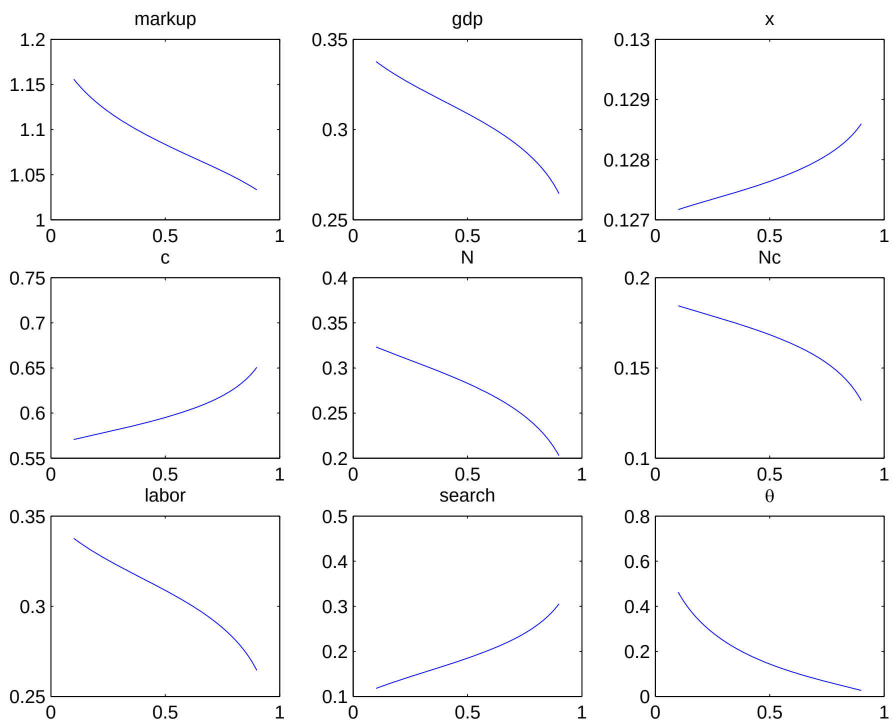

Figure 11: Steady-state allocation as function of customer bargaining power η in model with intensive adjustment.

Variable	Mean	Std. Dev.	SD %	Auto corr.	Corr(x,Y)	Corr(x,Z)	Corr(x,g)
				Zero menu cost			
μ	1.0835	0.0006	0.0006	-0.0650	-0.3222	—	-0.3065
gdp	0.3368	0.0020	0.0061	0.7678	1.0000	—	0.9999
x	0.1868	0.0002	0.0013	0.7927	-0.9992	—	-0.9997
c	1.4000	0	0	—	—	—	—
N	0.0663	0.0001	0.0014	0.7949	-0.7672	—	-0.7777
Nc	0.0929	0.0001	0.0014	0.7949	-0.7672	—	-0.7777
l	0.3368	0.0020	0.0061	0.7678	1.0000	—	0.9999
w	1.0000	0	0	—	—	—	—
p	1.0835	0.0006	0.0006	-0.0650	-0.3222	—	-0.3065
s	0.0184	0.0002	0.0084	-0.0386	-0.5131	—	-0.4988
θ	0.8000	0.0002	0.0002	0.8108	0.9974	—	0.9984
ω	0.5000	0	0	—	—	—	—
a	0.0147	0.0001	0.0084	-0.0354	-0.4917	—	-0.4773
				Menu cost, $\kappa = 5$			
μ	1.0835	0.0004	0.0003	0.6454	0.1137	—	0.1277
gdp	0.3368	0.0020	0.0061	0.7676	1.0000	—	0.9999
x	0.1868	0.0002	0.0013	0.7923	-0.9992	—	-0.9997
c	1.4000	0	0	—	—	—	—
N	0.0663	0.0001	0.0014	0.7978	-0.7669	—	-0.7776
Nc	0.0929	0.0001	0.0014	0.7978	-0.7669	—	-0.7776
l	0.3368	0.0020	0.0061	0.7676	1.0000	—	0.9999
w	1.0000	0	0	—	—	—	—
p	1.0835	0.0004	0.0003	0.6454	0.1137	—	0.1277
s	0.0184	0.0002	0.0084	-0.0334	-0.5106	—	-0.4961
θ	0.8000	0.0001	0.0001	0.6178	0.9807	—	0.9773
ω	0.5000	0.0003	0.0005	-0.1895	-0.4863	—	-0.4741
a	0.0147	0.0001	0.0084	-0.0259	-0.5016	—	-0.4871
				Menu cost, $\kappa = 20$			
μ	1.0835	0.0006	0.0006	0.9711	0.5143	—	0.5230
gdp	0.3368	0.0020	0.0061	0.7674	1.0000	—	0.9998
x	0.1868	0.0002	0.0013	0.7922	-0.9992	—	-0.9997
c	1.4000	0	0	—	—	—	—
N	0.0663	0.0001	0.0014	0.8000	-0.7664	—	-0.7774
Nc	0.0929	0.0001	0.0014	0.8000	-0.7664	—	-0.7774
l	0.3368	0.0020	0.0061	0.7674	1.0000	—	0.9998
w	1.0000	0	0	—	—	—	—
p	1.0835	0.0006	0.0006	0.9711	0.5143	—	0.5230
s	0.0184	0.0002	0.0084	-0.0329	-0.4990	—	-0.4840
θ	0.8000	0.0002	0.0002	0.8626	-0.9820	—	-0.9851
ω	0.5000	0.0004	0.0008	0.2115	-0.5358	—	-0.5267
a	0.0147	0.0001	0.0085	-0.0150	-0.5191	—	-0.5044
				Menu cost, $\kappa = 100$			
μ	1.0835	0.0008	0.0007	0.9923	0.3406	—	0.3461
gdp	0.3368	0.0020	0.0060	0.7668	1.0000	—	0.9998
x	0.1868	0.0002	0.0013	0.7926	-0.9991	—	-0.9997
c	1.4000	0	0	—	—	—	—
N	0.0663	0.0001	0.0015	0.8012	-0.7658	—	-0.7774
Nc	0.0929	0.0001	0.0015	0.8012	-0.7658	—	-0.7774
l	0.3368	0.0020	0.0060	0.7668	1.0000	—	0.9998
w	1.0000	0	0	—	—	—	—
p	1.0835	0.0008	0.0007	0.9923	0.3406	—	0.3461
s	0.0184	0.0002	0.0086	-0.0390	-0.4679	—	-0.4518
θ	0.8000	0.0008	0.0010	0.8114	-0.9972	—	-0.9984
ω	0.5000	0.0005	0.0011	0.5698	0.0485	—	0.0582
a	0.0147	0.0001	0.0090	0.0013	-0.5519	—	-0.5368

Table 9: Simulation-based moments in basic model with government spending, Nash bargaining, and government purchase shocks as driving force.

B Nash Bargaining

Here we derive the Nash-bargaining solution between an individual customer and the firm. We present the most general case in which bargaining occurs over both price and quantity, and at the end of the section we show how to simplify things if the quantity is fixed and bargaining occurs only over price. For notational simplicity, we omit the conditional expectations operator E_t where it is understood.

The marginal value to the household of a family member who is already engaged in a relationship (a shopper) with a firm is

$$\mathbf{M_t} = \frac{\tilde{u}(c_{it})}{\lambda_t} - \frac{g'(1 - l_t - s_t - N_t)}{\lambda_t} - p_{it}c_{it} + E_t\left[\Xi_{t+1|t}\left((1-\rho^x)\mathbf{M_{t+1}} + \rho^x\mathbf{S_{t+1}}\right)\right]. \tag{55}$$

Here, p_{it} denotes the price of the consumption good traded between a firm and a customer, and c_{it} denotes its quantity. The function $\tilde{u}$ is the *marginal* utility to the household of obtaining consumption from the i-th match. The precise expression for $\tilde{u}$ depends on whether the household's aggregator over search consumption goods is linear (as in the baseline model) or displays curvature (as in the model with intensive adjustment). For simplicity, here we assume that the aggregator is linear, and the expression for $\tilde{u}$ for the case with curvature is derived in Appendix E. Thus, let y be defined as simply the sum of the c_i's, $y \equiv \int_0^N c_i di$, in which case $\tilde{u}(c_i)$ is defined as $u_1(.)\frac{\partial y}{\partial c_i}$, which reduces to $\tilde{u}(c_i) = u_1(.)$. Because the units of $\tilde{u}$ are utils, we convert it into units of the final composite by dividing by the period-t marginal utility of wealth for the household, λ_t, which has units of utils per final good.

The marginal value to the household of an individual who is searching for goods is

$$\mathbf{S_t} = -\frac{g'(1 - l_t - s_t - N_t)}{\lambda_t} + E_t\left[\Xi_{t+1|t}\left(\theta_t k^f(\theta_t)(1-\rho^x)\mathbf{M_{t+1}} + (1 - \theta_t k^f(\theta_t)(1-\rho^x))\mathbf{S_{t+1}}\right)\right]. \tag{56}$$

The only instantaneous components of $\mathbf{S_t}$ is the marginal reduction in utility due to the marginal reduction in total household leisure time because of searching.

The value to a firm of an existing customer is

$$\mathbf{A_t} = p_{it}c_{it} - mc_t c_{it} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 + E_t\left[\Xi_{t+1|t}(1-\rho^x)\mathbf{A_{t+1}}\right], \tag{57}$$

where the instantaneous components takes into account the revenue from sales to the customer, the total cost of production, and the cost of price adjustment.

Bargaining occurs every period over the price p_{it} and quantity c_{it}. The firm and customer maximize the Nash product

$$(\mathbf{M_t} - \mathbf{S_t})^\eta \mathbf{A_t}^{1-\eta}, \tag{58}$$

where $\eta \in (0,1)$ is the fixed weight given to the customer's individual surplus. The first-order condition of the Nash product with respect to p_{it} is

$$\eta(\mathbf{M_t} - \mathbf{S_t})^{\eta-1}\left(\frac{\partial \mathbf{M_t}}{\partial p_{it}} - \frac{\partial \mathbf{S_t}}{\partial p_{it}}\right)\mathbf{A_t}^{1-\eta} + (1-\eta)(\mathbf{M_t} - \mathbf{S_t})^{\eta}\mathbf{A_t}^{-\eta}\frac{\partial \mathbf{A_t}}{\partial p_{it}} = 0, \tag{59}$$

which can be condensed as usual to

$$(1-\eta)\left(\mathbf{M_t} - \mathbf{S_t}\right)\frac{\partial \mathbf{A_t}}{\partial p_{it}} = -\eta \mathbf{A_t}\left(\frac{\partial \mathbf{M_t}}{\partial p_{it}} - \frac{\partial \mathbf{S_t}}{\partial p_{it}}\right). \tag{60}$$

We have $\frac{\partial \mathbf{M_t}}{\partial p_{it}} = -c_{it}$, $\frac{\partial \mathbf{S_t}}{\partial p_{it}} = 0$, and

$$\frac{\partial \mathbf{A_t}}{\partial p_{it}} = c_{it} - \kappa\left(\frac{p_{it}}{p_{it-1}} - 1\right)\frac{1}{p_{it-1}} + (1-\rho^x)E_t\left[\Xi_{t+1|t}\kappa\left(\frac{p_{it+1}}{p_{it}} - 1\right)\frac{p_{it+1}}{p_{it}}\frac{1}{p_{it}}\right], \tag{61}$$

which reveals a second forward-looking element to pricing, independent of the forward-looking element that arises due to the long-lived customer relationship.

Defining $\pi_{it} \equiv p_{it}/p_{it-1}$ as the gross growth rate of the price of the i-th good,

$$\frac{\partial \mathbf{A_t}}{\partial p_{it}} = c_{it} - \kappa(\pi_{it} - 1)\frac{1}{p_{it-1}} + (1-\rho^x)\kappa E_t\left[\Xi_{t+1|t}(\pi_{it+1} - 1)\frac{\pi_{it+1}}{p_{it}}\right]. \tag{62}$$

Next, define $\Delta_t^H \equiv -\partial \mathbf{M_t}/\partial p_{it}$, $\Delta_t^F \equiv \partial \mathbf{A_t}/\partial p_{it}$, and thus

$$\omega_t \equiv \frac{\eta}{\eta + (1-\eta)\Delta_t^F/\Delta_t^H} \tag{63}$$

$$1 - \omega_t = \frac{(1-\eta)\Delta_t^F/\Delta_t^H}{\eta + (1-\eta)\Delta_t^F/\Delta_t^H} \tag{64}$$

as time-varying bargaining weights. With these definitions, the sharing rule that determines p_{it} is

$$(1-\omega_t)(\mathbf{M_t} - \mathbf{S_t}) = \omega_t \mathbf{A_t}. \tag{65}$$

The surplus is split proportionally according to time-varying weights, which is a generalization of the usual Nash-sharing rule. Note that if $\kappa = 0$ (i.e., there are no menu costs), $\omega_t = \eta$, in which case the typical Nash sharing rule $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}$ emerges.

Using the Bellman equation for the value to a firm of an existing customer along with the advertising condition, we have $\mathbf{A_t} = p_{it}c_{it} - mc_t c_{it} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2$. Using this along with the definitions of $\mathbf{M_t}$ and $\mathbf{S_t}$ and going through several algebraic rearrangements, we can, after some tedious algebra, show that the price p_{it} is characterized by

$$\begin{aligned}\frac{\omega_t}{1-\omega_t}\left[p_{it}c_{it} - mc_t c_{it} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right] &= \frac{\tilde{u}(c_{it})}{\lambda_t} - p_{it}c_{it} + \\ + \quad (1-\theta_t k^f(\theta_t))E_t\left[\Xi_{t+1|t}\left(\frac{\omega_{t+1}}{1-\omega_{t+1}}\right)(1-\rho^x)\left[p_{it+1}c_{it+1} - mc_{t+1}c_{it+1} - \frac{\kappa}{2}\left(\frac{p_{it+1}}{p_{it}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right]\right]&.\end{aligned} \tag{66}$$

Note there are two reasons why price-setting is forward-looking. One reason is a standard sticky-price reason: with costs of price adjustment, a setting for p_{it} has ramifications for future setting of p_{it+1}. But note that even with $\kappa = 0$, p_{it} is affected by expectations regarding p_{it+1}. This has to do with the long-lived customer relationship: with probability $1 - \rho^x$, the customer and firm will bargain over the same good again in the future. If $\kappa = 0$, we have that $\partial \mathbf{A_t}/\partial p_{it} = c_{it}$, and the bargaining weight collapses to $\omega_t = \eta$.

Bargaining over the quantity c_{it} traded yields

$$\eta(\mathbf{M_t} - \mathbf{S_t})^{\eta-1}\left(\frac{\partial \mathbf{M_t}}{\partial c_{it}} - \frac{\partial \mathbf{S_t}}{\partial c_{it}}\right)\mathbf{A_t}^{1-\eta} + (1-\eta)(\mathbf{M_t} - \mathbf{S_t})^{\eta}\mathbf{A_t}^{-\eta}\frac{\partial \mathbf{A_t}}{\partial c_{it}} = 0. \tag{67}$$

We have $\frac{\partial \mathbf{M_t}}{\partial c_{it}} = \frac{\tilde{u}'(c_{it})}{\lambda_t} - p_{it}$, $\frac{\partial \mathbf{S_t}}{\partial c_{it}} = 0$ (because of our maintained assumption that u is separable in its two arguments), and $\frac{\partial \mathbf{A_t}}{\partial c_{it}} = p_{it} - mc_t$. Thus, the sharing rule that determines c_{it} is

$$(1-\eta)(\mathbf{M_t} - \mathbf{S_t})\frac{\partial \mathbf{A_t}}{\partial c_{it}} = -\eta\mathbf{A_t}\frac{\partial \mathbf{M_t}}{\partial c_{it}}. \tag{68}$$

Similar to how we proceeded above in Nash bargaining over the price, we can define the following: $\delta_t^H \equiv -\partial \mathbf{M_t}/\partial c_{it}$, $\delta_t^F \equiv \partial \mathbf{A_t}/\partial c_{it}$, and thus

$$\o_t \equiv \frac{\eta}{\eta + (1-\eta)\delta_t^F/\delta_t^H} \tag{69}$$

$$1 - \o_t \equiv \frac{(1-\eta)\delta_t^F/\delta_t^H}{\eta + (1-\eta)\delta_t^F/\delta_t^H} \tag{70}$$

as time-varying bargaining weights. Note that $\o_t \neq \omega_t$. Proceeding completely analogously as above (i.e., using the Bellam equations for the value to a firm of an existing customer along with the advertising condition and going through several tedious steps of algebra), we can show that the quantity c_{it} is characterized by

$$\begin{aligned}\frac{\o_t}{1-\o_t}\left[p_{it}c_{it} - mc_t c_{it} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right] &= \frac{\tilde{u}(c_{it})}{\lambda_t} - p_{it}c_{it} + \\ + \quad (1-\theta_t k^f(\theta_t))E_t\left[\Xi_{t+1|t}\left(\frac{\o_{t+1}}{1-\o_{t+1}}\right)(1-\rho^x)\left[p_{it+1}c_{it+1} - mc_{t+1}c_{it+1} - \frac{\kappa}{2}\left(\frac{p_{it+1}}{p_{it}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right]\right]&,\end{aligned} \tag{71}$$

which is expression (50) in the text. Clearly, the only difference between (66) and (71) is in the relevant bargaining weights. In particular, note that $\o_t$ depends on $\tilde{u}'(c_{it})$ through $\partial \mathbf{M_t}/\partial c_{it}$, whereas ω_t does not. With curvature in the aggregator over search consumption goods, deriving the expression for $\tilde{u}'(c_{it})$ requires some tedious algebra, the details of which are provided in Appendix E.

If $\kappa = 0$, a more convenient representation of the solution for the quantity traded is available. Using the derivatives of $\mathbf{M_t}$ and $\mathbf{A_t}$, (68) becomes

$$(1-\eta)(p_t - mc_t)(\mathbf{M_t} - \mathbf{S_t}) = \eta\left(p_t - \frac{\tilde{u}'(c_t)}{\lambda_t}\right)\mathbf{A_t}. \tag{72}$$

Next, recognize that, with $\kappa = 0$, the Nash solution for the price satisfies $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}$. Imposing this on the previous expression, we have that the quantity c_t traded solves

$$mc_t = \frac{\tilde{u}'(c_t)}{\lambda_t}. \tag{73}$$

Recall that households' optimal choice of labor implied $\lambda_t = g'_t / w_t$, meaning

$$\frac{mc_t}{w_t} = \frac{\tilde{u}'(c_t)}{g'_t}. \tag{74}$$

Finally, because marginal cost is identically equal to the ratio of the real wage to the marginal product of labor w_t/z_t, in equilibrium, the quantity traded in any given customer-firm relationship satisfies

$$\frac{g'_t}{\tilde{u}'(c_t)} = z_t, \tag{75}$$

which is a goods-market efficiency condition stating that the household's MRS between leisure and consumption of a particular good equals the marginal product of labor. Thus, with simultaneous bargaining over price and quantity traded of any given good, quantity (the intensive margin of consumption) is privately efficient, which is a standard outcome of Nash bargaining. One implication of this result is that if consumption were subject to proportional taxation, the intensive margin of trade would be unaffected by the consumption tax, given a number of customer-firm relationships (which in general would be distorted by a consumption tax).

In our baseline model without the intensive margin, the customer and firm asset values $\mathbf{M_t}$ and $\mathbf{A_t}$ simplify to

$$\mathbf{M_t} = \frac{\tilde{u}(\bar{c})}{\lambda_t} - \frac{g'(1-l_t-s_t-N_t)}{\lambda_t} - p_{it}\bar{c} + E_t\left[\Xi_{t+1|t}\left((1-\rho^x)\mathbf{M_{t+1}} + \rho^x \mathbf{S_{t+1}}\right)\right] \tag{76}$$

and

$$\mathbf{A_t} = p_{it}\bar{c} - mc_t\bar{c} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 + E_t\left[\Xi_{t+1|t}(1-\rho^x)\mathbf{A_{t+1}}\right]. \tag{77}$$

That is, quantity exchanged in a customer match is fixed at $\bar{c}$. Maximization of the Nash product is now with respect to only p_{it}. Proceeding similarly as above,

$$\begin{aligned} \frac{\omega_t}{1-\omega_t}\left[p_{it}\bar{c} - mc_t\bar{c} - \frac{\kappa}{2}\left(\frac{p_{it}}{p_{it-1}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right] &= \frac{\tilde{u}(\bar{c})}{\lambda_t} - p_{it}\bar{c} + \qquad (78) \\ + \quad (1-\theta_t k^f(\theta_t))E_t\left[\Xi_{t+1|t}\left(\frac{\omega_{t+1}}{1-\omega_{t+1}}\right)(1-\rho^x)\right. & \left.\left[p_{it+1}\bar{c} - mc_{t+1}\bar{c} - \frac{\kappa}{2}\left(\frac{p_{it+1}}{p_{it}} - 1\right)^2 + \frac{\gamma}{k^f(\theta_t)}\right]\right], \end{aligned}$$

which is expression (24) in the text.

C Fair Bargaining

Rather than the surplus being split according the time-varying rule $(1-\omega_t)(\mathbf{M_t} - \mathbf{S_t}) = \omega_t \mathbf{A_t}$, here we impose fair bargaining, in which it is required that $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}$. That is, suppose that the customer and firm arrange the effects of inflation so that the shares of the surplus each gets are not time-varying.

In fair bargaining, the customer and firm jointly choose (p_{it}, c_{it}) to maximize the Nash product

$$(\mathbf{M_t} - \mathbf{S_t})^{\eta} \, \mathbf{A_t}^{1-\eta} \tag{79}$$

subject to the constraint

$$(1-\eta)\,(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}. \tag{80}$$

Letting ι_t be the Lagrange multiplier on the constraint, the first-order conditions of the bargaining problem are

$$\eta \mathbf{A_t} \left[\frac{\partial \mathbf{M_t}}{\partial p_{it}} - \frac{\partial \mathbf{S_t}}{\partial p_{it}} + \frac{\partial \mathbf{A_t}}{\partial p_{it}} \right] + \eta \iota_t \frac{\partial \mathbf{A_t}}{\partial p_{it}} \frac{\mathbf{A_t}^{\eta}}{(\mathbf{M_t} - \mathbf{S_t})^{1-\eta}} = (1-\eta)\iota_t \left[\frac{\partial \mathbf{M_t}}{\partial p_{it}} - \frac{\partial \mathbf{S_t}}{\partial p_{it}} \right] \frac{\mathbf{A_t}^{\eta}}{(\mathbf{M_t} - \mathbf{S_t})^{1-\eta}} \tag{81}$$

and

$$\eta \mathbf{A_t} \left[\frac{\partial \mathbf{M_t}}{\partial c_{it}} - \frac{\partial \mathbf{S_t}}{\partial c_{it}} + \frac{\partial \mathbf{A_t}}{\partial c_{it}} \right] + \eta \iota_t \frac{\partial \mathbf{A_t}}{\partial c_{it}} \frac{\mathbf{A_t}^{\eta}}{(\mathbf{M_t} - \mathbf{S_t})^{1-\eta}} = (1-\eta)\iota_t \left[\frac{\partial \mathbf{M_t}}{\partial c_{it}} - \frac{\partial \mathbf{S_t}}{\partial c_{it}} \right] \frac{\mathbf{A_t}^{\eta}}{(\mathbf{M_t} - \mathbf{S_t})^{1-\eta}}. \tag{82}$$

In obtaining these, we used the constraint to make the substitution $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}$. Next, because $(1-\eta)(\mathbf{M_t} - \mathbf{S_t}) = \eta \mathbf{A_t}$, clearly $(1-\eta)\left[\frac{\partial \mathbf{M_t}}{\partial p_{it}} - \frac{\partial \mathbf{S_t}}{\partial p_{it}}\right] = \eta \frac{\partial \mathbf{A_t}}{\partial p_{it}}$, and similarly for derivatives with respect to c_{it}. The two first-order conditions thus reduce to

$$\eta \mathbf{A_t} \left[\frac{\partial \mathbf{M_t}}{\partial p_{it}} - \frac{\partial \mathbf{S_t}}{\partial p_{it}} + \frac{\partial \mathbf{A_t}}{\partial p_{it}} \right] = 0 \tag{83}$$

and

$$\eta \mathbf{A_t} \left[\frac{\partial \mathbf{M_t}}{\partial c_{it}} - \frac{\partial \mathbf{S_t}}{\partial c_{it}} + \frac{\partial \mathbf{A_t}}{\partial c_{it}} \right] = 0. \tag{84}$$

Because $\eta \mathbf{A_t} \neq 0$, the terms in brackets characterize the fair bargaining solutions for p_i and c_i.

With the definitions of $\mathbf{M_t}$, $\mathbf{S_t}$, and $\mathbf{A_t}$, $\frac{\partial \mathbf{M_t}}{\partial c_{it}} = \frac{\tilde{u}'(c_{it})}{\lambda_t} - p_{it}$, $\frac{\partial \mathbf{S_t}}{\partial c_{it}} = 0$ (because of our maintained assumption that u is separable in its two arguments), and $\frac{\partial \mathbf{A_t}}{\partial c_{it}} = p_{it} - mc_t$. Inserting these in the latter FOC,

$$\frac{\tilde{u}'(c_{it})}{\lambda_t} = mc_t. \tag{85}$$

Once again, using the equilibrium condition $\lambda_t = g_t'/w_t$ and the fact that $mc_t = w_t/z_t = 1$,

$$\frac{g_t'}{\tilde{u}'(c_t)} = z_t \tag{86}$$

characterizes the solution for the quantity traded, just as in the Nash case with $\kappa = 0$. Thus, in fair bargaining, eliminating the wedge in price-bargaining has the associated effect of eliminating the wedge in quantity-bargaining as well. As was the case in Nash bargaining, the derivation of $\tilde{u}'(c_{it})$ is presented in Appendix E.

Proceeding similarly, the price is characterized by

$$\kappa\left(\pi_{it}-1\right)\pi_{it}-(1-\rho^{x})E_t\left[\Xi_{t+1|t}\kappa\left(\pi_{it+1}-1\right)\pi_{it+1}\right]=0, \tag{87}$$

which reveals something very interesting. If this were a monetary model and thus π were the rate of change of the nominal price level, this condition would be identical to a standard New Keynesian Phillips curve except for the fact that nothing at all about allocations appears.

D Steady-State Analytics

Using conditions (37) and (39), we first prove our claim in Section 4.2 that the steady-state price (and hence the steady-state markup) decreases as customer bargaining power η rises. Using the fact that Cobb-Douglas matching implies the probability a firm finds a customer is $k^f(\theta)=\theta^{\xi}$, from (37) we can solve for θ:

$$\theta=\left[\frac{\beta(1-\rho^{x})}{1-\beta(1-\rho^{x})}\frac{p-mc}{\gamma}\right]^{\frac{1}{\xi}}. \tag{88}$$

Inserting this in (39):

$$p=(1-\eta)A+\eta\left[mc-\gamma\left(\frac{\beta(1-\rho^{x})}{1-\beta(1-\rho^{x})}\frac{p-mc}{\gamma}\right)\right]. \tag{89}$$

Defining an implicit function $F(p,\eta)=0$ using this last expression, we can compute the partials

$$F_p=1+\frac{\eta\gamma}{\xi}\left(\frac{\beta(1-\rho^{x})}{\gamma[1-\beta(1-\rho^{x})]}\right)\left(\frac{\beta(1-\rho^{x})}{1-\beta(1-\rho^{x})}\frac{p-mc}{\gamma}\right)^{\frac{1}{\xi}-1} \tag{90}$$

and

$$F_\eta=A-\left[mc-\gamma\left(\frac{\beta(1-\rho^{x})}{1-\beta(1-\rho^{x})}\frac{p-mc}{\gamma}\right)^{\frac{1}{\xi}}\right]. \tag{91}$$

With search frictions, $\gamma\theta^{\xi}>0$; the advertising condition then guarantees that $p>mc$. Hence, given that all other parameters are of appropriate sign and magnitude (i.e., $\beta\in(0,1)$, $\rho^{x}\in(0,1)$, $\eta\in(0,1)$, $\xi\in(0,1)$, and $\gamma>0$), $F_p>0$. Next, note that we can write F_η compactly as $F_\eta=A-(mc-\gamma\theta)$. If $A<(mc-\gamma\theta)$, forming customer relationships would not even be socially beneficial because the social marginal benefit (the utility A of consuming) would not cover the social marginal cost of forming relationships and producing. Thus, we simply assume that $A>(mc-\gamma\theta)$, implying $F_\eta>0$, Finally, then, we have by the implicit function theorem that the bargained price decreases the higher is customer bargaining power, $\frac{dp}{d\eta}=-\frac{F_\eta}{F_p}<0$.

Next, to characterize the firm advertising condition in (a,s) space, first solve for p from (37):

$$p=mc+\frac{1-\beta(1-\rho^{x})}{\beta(1-\rho^{x})}\gamma\theta^{\xi}. \tag{92}$$

Inserting this in (39) gives us a version of the advertising condition that embeds the pricing condition:

$$mc + \frac{1-\beta(1-\rho^x)}{\beta(1-\rho^x)}\gamma\theta^\xi = (1-\eta)A + \eta(mc - \gamma\theta). \tag{93}$$

Replacing θ by a/s, we can define an implicit function $G(a,s) = 0$ using this last expression and compute the partials

$$G_a = \frac{1-\beta(1-\rho^x)}{\beta(1-\rho^x)}\gamma\xi a^{\xi-1}s^{-\xi} + \eta\gamma s^{-1} \tag{94}$$

and

$$G_s = -\frac{1-\beta(1-\rho^x)}{\beta(1-\rho^x)}\gamma a^\xi s^{-\xi-1} - \eta\gamma a s^{-2}. \tag{95}$$

By the implicit function theorem, the slope of the advertising condition in (a,s) space is thus $\frac{da}{ds} = -\frac{G_s}{G_a}$. With a couple of steps of algebra, it is not difficult to show that $\frac{da}{ds} = \theta$, meaning that in (a,s) space, the advertising condition is a ray through the origin with angle θ to the s axis, as illustrated in Figure 2.

E Intensive Quantity Adjustment

For use in the Nash-bargaining and fair-bargaining solutions for quantity, we require an expression for $\tilde{u}'(c_{it})$. Recall that with curvature, the household subutility function over search consumption goods is

$$v\left(\left[\int_0^{N_t} c_{jt}^\rho dj\right]^{1/\rho}\right). \tag{96}$$

With $\tilde{u}(c_{it})$ defined as the *marginal* utility to the household of obtaining consumption from the i-th match, we have

$$\widetilde{u}(c_{it}) = v'\left(\left[\int_0^{N_t} c_{jt}^\rho di\right]^{1/\rho}\right)\frac{1}{\rho}\left[\int_0^{N_t} c_{jt}^\rho dj\right]^{\frac{1}{\rho}-1}\rho c_{it}^{\rho-1}. \tag{97}$$

Note the distinction between the indices i and j: j is a dummy index of integration, while i denotes a good obtained from the i-th customer relationship.

For use in the bargaining solutions for the intensive quantity, what requires some work is obtaining $\tilde{u}'(c_{it})$ because, note, c_{it} appears three times in expression (97): as one of the consumption terms in the argument to $v(.)$, as one of the consumption terms in the integral $\int_0^N c_i^\rho di$, and by itself in the last term on the right-hand-side. To make the problem manageable, define

$$f(c_{it}) \equiv v'\left(\left[\int_0^{N_t} c_{jt}^\rho dj\right]^{1/\rho}\right), \tag{98}$$

$$g(c_{it}) \equiv \frac{1}{\rho}\left[\int_0^{N_t} c_{jt}^\rho dj\right]^{\frac{1}{\rho}-1}, \tag{99}$$

$$h(c_{it}) \equiv \rho c_{it}^{\rho-1}. \tag{100}$$

Letting $z \equiv \tilde{u}(c_{it})$, what we are interested in deriving is $\frac{\partial z}{\partial c_{it}} = f'gh + fg'h + fgh'$. Proceeding,

$$f' = v''\left(\left[\int_0^{N_t} c_{jt}^{\rho} dj\right]^{1/\rho}\right) \frac{1}{\rho}\left[\int_0^{N_t} c_{jt}^{\rho} dj\right]^{\frac{1}{\rho}-1} \rho c_{it}^{\rho-1}, \tag{101}$$

$$g' = \frac{1}{\rho}\left(\frac{1}{\rho} - 1\right)\left[\int_0^{N_t} c_{jt}^{\rho} dj\right]^{\frac{1}{\rho}-2} \rho c_{it}^{\rho-1}, \tag{102}$$

$$h' = \rho(\rho-1)c_{it}^{\rho-2}. \tag{103}$$

We limit attention to symmetric equilibria, so having computed derivatives, we can impose symmetry, $c_i = c$, on f, g, h, f', g', and h', yielding

$$f = v'\left(c_t N_t^{1/\rho}\right), \tag{104}$$

$$g = \frac{1}{\rho} c_t^{1-\rho} N_t^{\frac{1-\rho}{\rho}}, \tag{105}$$

$$h = \rho c_t^{\rho-1}, \tag{106}$$

$$f' = v''\left(c_t N_t^{1/\rho}\right) N_t^{\frac{1-\rho}{\rho}}, \tag{107}$$

$$g' = \frac{1-\rho}{\rho} c_t^{-\rho} N_t^{\frac{1-2\rho}{\rho}}, \tag{108}$$

$$h' = \rho(\rho-1)c_t^{\rho-2}. \tag{109}$$

Constructing the symmetric version of $f'gh + fg'h + fgh'$ and rearranging,

$$\tilde{u}'(c_t) = N_t^{\frac{1-\rho}{\rho}}\left[v''\left(c_t N_t^{1/\rho}\right) N_t^{\frac{1-\rho}{\rho}} + v'\left(c_t N_t^{1/\rho}\right)(1-\rho)c_t^{-1}\left(\frac{1}{N_t} - 1\right)\right], \tag{110}$$

which goes into the Nash- and fair-bargaining solutions for the intensive quantity traded. Note that if we we were to remove curvature by setting $\rho = 1$, this collapses to $\tilde{u}'(c_t) = v'\left(c_t N_t^{1/\rho}\right)$.

F Advertising Data

The aggregate advertising data are from two separate sources. The data for 1951 to 1999 are obtained from an updated version of Robert J. Coen's (McCann-Erikson, Inc.) original data published in *Historical Statistics of the United States, Colonial Times to 1970.* The data for 2000 to 2005 are obtained from the Newspaper Association of America (NAA). The aggregate data include spending for advertising in newspapers, magazines, radio, broadcast television, cable television, direct mail, billboards and displays, Internet, and other forms. The GDP figures are from the US Bureau of Economic Analysis (BEA).

Using the nominal data in Table 10, we construct a real advertising series by deflating by the all-items Consumer Price Index (results were quite similar deflating by the GDP deflator). Logging and HP-filtering the resulting series, we find that over the entire sample the cyclical volatility of real advertising is 4.2 percent, the contemporaneous correlation with GDP is 0.73, and its first-order serial correlation is 0.6.

Year	Aggregate nominal advertising expenditure ($B)	Nominal GDP ($B)	Share of GDP (%)
1950	5.7	293.8	1.9
1951	6.4	339.3	1.9
1952	7.1	358.3	2.0
1953	7.7	379.4	2.0
1954	8.2	380.4	2.1
1955	9.2	414.8	2.2
1956	9.9	437.5	2.3
1957	10.3	461.1	2.2
1958	10.3	467.2	2.2
1959	11.3	506.6	2.2
1960	12.0	526.4	2.3
1961	11.9	544.7	2.2
1962	12.4	585.6	2.1
1963	13.1	617.7	2.1
1964	14.2	663.6	2.1
1965	15.3	719.1	2.1
1966	16.6	787.8	2.1
1967	16.9	832.6	2.0
1968	18.1	910.0	2.0
1969	19.4	984.6	2.0
1970	19.6	1,038.5	1.9
1971	20.7	1,127.1	1.8
1972	23.2	1,238.3	1.9
1973	25.0	1,382.7	1.8
1974	26.6	1,500.0	1.8
1975	27.0	1,638.3	1.7
1976	33.3	1,825.3	1.8
1977	37.4	2,030.9	1.8
1978	43.3	2,294.7	1.9
1979	48.8	2,563.3	1.9
1980	53.6	2,789.5	1.9
1981	60.5	3,128.4	1.9
1982	66.7	3,255.0	2.0
1983	76.0	3,536.7	2.1
1984	88.0	3,933.2	2.2
1985	94.9	4,220.3	2.2
1986	102.4	4,462.8	2.3
1987	110.3	4,739.5	2.3
1988	118.8	5,103.8	2.3
1989	124.8	5,484.4	2.3
1990	129.6	5,803.1	2.2
1991	127.6	5,995.9	2.1
1992	132.7	6,337.7	2.1
1993	139.5	6,657.4	2.1
1994	151.7	7,072.2	2.1
1995	162.9	7,397.7	2.2
1996	175.2	7,816.9	2.2
1997	187.5	8,304.3	2.3
1998	201.6	8,747.0	2.3
1999	215.3	9,268.4	2.3
2000	243.3	9,817.0	2.5
2001	231.3	10,128.0	2.3
2002	236.9	10,469.6	2.3
2003	245.6	10,960.8	2.2
2004	263.8	11,712.5	2.3
2005	271.1	12,455.8	2.2

Table 10: Advertising Data

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