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AN ANALOGUE MODEL OF PHASE-AVERAGING PROCEDURES

by

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ABSTRACT

This paper considers the statistical and econometric effects that fixed n-period phase-averaging has on time series generated by some simple dynamic processes. We focus on the variance and autocorrelation of the data series and of the disturbance term for levels and difference equations involving the phase-average data. Further, we examine the effect of phase-averaging on the exogeneity of variables in those equations and the implications phase-averaging has for conducting statistical inference.

To illustrate our analytical results, we investigate claims by Friedman and Schwartz in their 1982 book Monetary Trends in the United States and the United Kingdom about what the properties of phase-average data and the relationships between those data ought to be. We present certain features of the observed series on velocity, examine how well our analytical model captures them, and contrast them with Friedman and Schwartz's predictions. While our model is an extremely simplified characterisation of the phase-averaging adopted by Friedman and Schwartz, it does offer several insights into the likely consequences of their approach.

Keywords: business cycles. conditional models. conditioning. dynamics. econometrics. exogeneity. marginalising. methodology. money demand. phase averages. quantity theory. statistical inference. temporal aggregation. time series. velocity of circulation.

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1. Introduction

The nature of business cycles has evoked extensive research in economics over the last several decades, including such diverse studies as Beveridge (1921), Slutzky (1937), Schumpeter (1939), Burns and Mitchell (1946), Tinbergen and Polak (1950), Friedman and Schwartz (1963, 1982), Sims (1977), Lucas (1977), and recent investigations on "real business cycles" by (e.g.) Long and Plosser (1983) and King, Plosser, Stock, and Watson (1987). Simplifying (and in some cases, overly so), economic series might be imagined to be composed of a trend and a cycle.[1] Burns and Mitchell (1946), for example, focus on the cycle, abstracting from non-cyclical temporal differences. To do so, they establish the NBER reference business cycles, dating the contraction and expansion phases of the cycles over their sample period, and across cycles average the data at various points in the cycle. Conversely, Friedman and Schwartz in their 1982 book Monetary Trends in the United States and the United Kingdom focus

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[1] Allais (1966, 1972), Neftçi (1984, 1986), Stock (1985), and Falk (1986) *inter alia* present various reasons for the existence of business cycles and give empirical evidence thereon.

on the longer-run features of the data. In an effort to filter out short-run (i.e., cyclical) properties of the data, they transform their (raw) annual series by averaging separately over contraction and expansion phases of the reference business cycles, afterwards presenting detailed graphical and regression studies of the resulting series, both in levels and in differences. They claim that such phase-averaging reduces serial correlation arising from the business cycle (p. 78) and attenuates measurement errors (p. 86), and implicitly claim that phase-averaging does not affect the exogeneity or otherwise of their variables of interest (pp. 35-36, 206): attaining those effects is important to sustain the validity of their statistical analyses.[2] Our paper analytically evaluates statistical and econometric effects that phase-averaging has on time series and considers whether it obtains empirically the results claimed by Friedman and Schwartz.

Use of phase-average data raises three distinct issues: (i) the theoretical effects of phase-averaging (qua time aggregation), (ii) the observed effects of phase-averaging, and (iii) the effects of selecting the intervals over which to average (e.g., by prior analysis of an interrelated data set such as that underlying reference business cycles). In Section 2, we abstract from (iii) and address (i), using as our model fixed n-period phase-averaging of time series generated by a simple dynamic process.[3] We focus on the effects that phase-averaging has on the variance and

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[2]See Maverick (1933) for early advocacy of phase-averaging in econometrics. Unreferenced tables and page numbers refer to Friedman and Schwartz (1982).

[3]Without undertaking a Monte Carlo study, it is difficult to deduce the statistical effects of selecting the phases over which to average when using information from related time series (i.e., (iii)). However, a first approximation is given by examining the effects of fixed-length phase-averaging. The model of phase-averaging with fixed phase lengths below explains salient features of the data for velocity, leaving a small role for the effects of selection of phase lengths.

autocorrelation of the data series and of the disturbance term for levels and difference equations involving phase-average data. Further, we examine the effect of phase-averaging on the exogeneity of variables in those equations and the implications phase-averaging has for conducting statistical inference. To illustrate those analytical results, in Section 3 we examine claims by Friedman and Schwartz (1982) about what the properties of phase-average data and the relationships between those data ought to be. We present certain features of the observed series on velocity (i.e., (ii)), examine how well our analytical model captures them, and contrast them with Friedman and Schwartz's predictions. While our model is an extremely simplified characterisation of the phase-averaging adopted by Friedman and Schwartz, it does offer several insights into the likely consequences of their approach.

2. Analytical effects of phase-averaging

This section describes our model of phase-averaging and considers the effects that phase-averaging has on time series generated by some simple dynamic processes.

Phase-averaging sequentially applies two filters to the annual data: the first averages that data (as with a moving average) and thus implies a re-parameterisation of the data generation process; the second selects the phase-average data from the averaged series (i.e., marginalises the data density with respect to the intermediate observations), thereby entailing a (statistical) reduction in that re-parameterisation. By analogy with seasonal adjustment of quarterly data, the first filter is like the X-11 procedure; the second discards all but the fourth-quarter data points.

To illustrate, consider fixed n-period phase-averaging of the annual series $\{x_t;\ t=..,1,2,...,T\}$ with T a multiple of n. Letting L be the lag operator such that $Lx_t = x_{t-1}$, the first filter is $(1+L+L^2+\ldots+L^{n-1})/n$, so the <u>averaged</u> series is $\{x_t^*;\ t=\ldots,1,2,\ldots,T\}$ where

$$x_t^* = (x_t + x_{t-1} + x_{t-2} + \ldots + x_{t-n+1})/n. \tag{1}$$

The second filter selects every n^{th} observation of x_t^*, so the <u>phase-average</u> series is $\{x_{jn}^*;\ j=\ldots,1,2,\ldots,(T/n)\}$, denoted $\{\bar{x}_j;\ j=\ldots,1,2,\ldots,J\}$. To illustrate, Figure 1 shows the steps involved in going from $\{x_t\}$ to $\{x_t^*\}$ to $\{\bar{x}_j\}$ for fixed 3-period phase-averaging.[4]

There is little loss of information from the use of the moving-average data <u>per se</u> but a distinct loss of information results from the selection process. That loss would be unimportant if (a) all the parameters of interest could be recovered from the phase-average series (and Friedman and Schwartz seem to believe such parameters can be, including the long-run elasticity of interest rates in a money-demand equation) and (b) there was no loss of power in testing the resulting model. However, many of the parameters which we consider to be of interest cannot be obtained from phase-average data, including parameters relevant to tests of Granger (1969) non-causality, short-run variability in the postulated relationships, and the dynamic mechanisms whereby the economy adjusts to "shocks". Point (a) also ties in closely with issues of exogeneity: as will be seen below, variables exogenous at annual observations may no longer be so when phase-averaged. Even for parameters which <u>can</u> be retrieved from the phase-average data, tests about them may be low in

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[4]In fact, Friedman and Schwartz include turning points in both preceding and following phases (but weight them by half the normal weight in each), so the first filter is actually $(.5+L+L^2+\ldots+L^{n-1}+.5L^n)/n$ and the second is unchanged. However, the statistical effects of using phase-averages with (rather than without) overlap appear minor in comparison to those of using phase-average (rather than annual) data (cf. pp. 75, 84-85).

Figure 1. A schema for fixed 3-period phase-averaging.

Original series $\{x_t\}$	... x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	...
Filter 1: Averaging									
Averaged series $\{x_t^*\}$ $x_t^* = (x_t+x_{t-1}+x_{t-2})/3$	... x_0^*	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	...
Filter 2: Sampling									
Phase-average series $\{\bar{x}_j\}$	... $\bar{x}_0$	-	-	$\bar{x}_1$	-	-	$\bar{x}_2$	-	...

power relative to tests based upon the annual data.[5]

To illustrate the statistical effects of phase-averaging, we begin with the time series $\{(y_t:z_t)'; t = 1,\ldots,T\}$ generated by the simple dynamic bivariate process:

$$
\begin{aligned}
y_t &= \alpha_0 + \alpha_1 z_t + u_t && (2)\\
u_t &= \rho u_{t-1} + \varepsilon_t \qquad \begin{bmatrix} \varepsilon_t \\ \nu_t \end{bmatrix} \sim \mathrm{IN}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & 0 \\ 0 & \sigma_\nu^2 \end{bmatrix} \right). && (3)\\
z_t &= \lambda_1 z_{t-1} + \lambda_2 y_{t-1} + \nu_t && (4)
\end{aligned}
$$

Except when otherwise noted, we assume that $(y_t:z_t)'$ is stationary; hence all roots of (2)-(4) are stable, i.e., $|\rho| < 1$ and $|\lambda_1+\alpha_1\lambda_2| < 1$. For convenience, we denote the second root $(\lambda_1+\alpha_1\lambda_2)$ by κ. If $\lambda_2 = 0$, z_t is strongly exogenous for $(\alpha_0:\alpha_1)'$; otherwise it is correlated with u_t.[6] Although (2)-(4) do not include a deterministic cycle, for suitable

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[5]For instance, cf. Banerjee, Dolado, Hendry, and Smith (1986) on co-integration tests with static and dynamic models and note the similarity between those models and the phase-average and annual models herein.

[6]If $(y_t:z_t)'$ is normally distributed, conditional upon its own past, then (2)-(3) is the corresponding conditional density of y_t (conditional upon z_t) and (4) is the corresponding marginal density for z_t, on the assumption that the conditional mean of $(y_t:z_t)'$ is linear in its past and only includes y_{t-1} and z_{t-1}, and that the resulting equation associated with the conditional density of y_t (e.g., $y_t = \delta_0+\delta_1 z_t+\delta_2 y_{t-1}+\delta_3 z_{t-1}+\varepsilon_t$) includes a common factor $(1-\rho L)$. Cf. Hendry and Mizon (1978) and Hendry, Pagan, and Sargan (1984) for further discussion on models with common factors.

Technically speaking, if $\lambda_2 \neq 0$ and so z_t is not strongly exogenous for $(\alpha_0:\alpha_1)'$, then z_t is not weakly exogenous for $(\alpha_0:\alpha_1)'$ either because stationarity requires $|\lambda_1+\alpha_1\lambda_2| < 1$ and so $(\lambda_1:\lambda_2)'$ and $(\alpha_0:\alpha_1)'$ cannot be variation free; cf. Engle, Hendry, and Richard (1983, p. 282). Weak exogeneity could be "restored" either by assuming that $(\lambda_1:\lambda_2)'$ and $(\alpha_0:\alpha_1)'$ lie within a sub-space of the region for stationarity where they are variation free or by relaxing the condition of joint stationarity to that of the conditional stationarity of y_t (requiring only that $|\rho| < 1$). In the latter situation, if z_t is integrable of order 1, then so will be y_t and y_t and z_t will be co-integrated. For further details on co-integration, see Granger and Engle (1987), Phillips and Durlauf (1986), and articles in the August 1986 special issue of the Oxford Bulletin of Economics and Statistics on co-integration, especially Granger (1986) and Hendry (1986).

parameter values, it easily can generate data with cyclical behaviour.[7]

The n-period phase average of an arbitrary series $\{x_t\}$ is:

$$\bar{x}_j = n^{-1} \sum_{i=1}^{n} x_{(j-1)n+i} \qquad j=1,2,\ldots,(\tfrac{T}{n}) , \qquad (5)$$

so

$$\bar{y}_j = \alpha_0 + \alpha_1 \bar{z}_j + \bar{u}_j \qquad (6)$$

from (2). Average two-(phase-)period differences result in:

$$(.5\Delta_2\bar{y}_j) = \alpha_1(.5\Delta_2\bar{z}_j) + (.5\Delta_2\bar{u}_j) \qquad (7)$$

where differencing for phase-average data is defined in terms of phases rather than years so that (e.g.) $\Delta_1\bar{x}_j = \bar{x}_j - \bar{x}_{j-1}$. Our concern is how inference is affected by using (6) (or (7)) rather than (2)-(4).[8] Unless otherwise noted, we limit our discussion to the effects on phase-average data of positive ρ because economic data tend to be highly positively autocorrelated. The effects of negative ρ are

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[7] Slutzky (1937) demonstrates that moving average processes can exhibit cyclical behaviour, and (2)-(4) can be well approximated by a moving average process.

[8] In the context of Friedman and Schwartz (1982), it is natural to interpret "periods" and "phases" as years and half-cycles. However, the analytical properties are quite general and could be applied to other time intervals, e.g., quarterly data aggregated to annual data. In fact, a substantial literature exists on the effects of time aggregation; cf. Theil (1954), Durbin (1960), Mundlak (1961), Telser (1967), Moriguchi (1970), Zellner and Montmarquette (1971), Amemiya and Wu (1972), Sargan (1976), Tiao and Wei (1976), Wei (1978), and Weiss (1984). Those studies show that, inter alia, sizeable inefficiencies result from using least squares even if the regressors are strongly exogenous. Further, if they are only weakly exogenous (or include lagged dependent variables), least squares is generally inconsistent; cf. Wei (1982). Our results below are in line with those general results. However, the motivation for our study is fundamentally different from those of time aggregation. In the latter, the data are generated at unit intervals but observation occurs at multiples of that unit: hence relevant questions include the effects on inference, forecasting, and policy analysis of not having available observations at natural time intervals. In the case of phase-averaging, observations at the unit interval are available, but the investigator chooses to use the aggregated (phase-average) data.

qualitatively different, particularly for n even; cf. (10) and (11) below.[9] However, the formulae derived apply for <u>all</u> $|\rho| < 1$.

<u>Levels of phase averages</u>. As a baseline, we note that the variance and autocovariances of u_t in (2) are:[10]

$$\tau_i \equiv E(u_t u_{t-i}) = \begin{cases} \sigma_\varepsilon^2/(1-\rho^2) & i = 0 \qquad (8) \\ \rho^i \sigma_\varepsilon^2/(1-\rho^2) & i = 1,2,\ldots \qquad (9) \end{cases}$$

Hence the autocorrelation function for u_t is ρ^i $(i>0)$. Equivalent formulae for $\bar{u}_j$ in (6), the levels equation with phase-average data, are:

$$\bar{\tau}_i \equiv E(\bar{u}_j \bar{u}_{j-i}) = \begin{cases} \dfrac{\sigma_\varepsilon^2}{n(1-\rho)^2}\left[1 - \dfrac{2\rho(1-\rho^n)}{n(1-\rho^2)}\right] & i = 0 \qquad (10) \\ \dfrac{\rho^{n(i-1)}\rho(1-\rho^n)^2\sigma_\varepsilon^2}{n^2(1-\rho^2)(1-\rho)^2} & i = 1,2,\ldots \qquad (11) \end{cases}$$

The error variance of (6) is less than that of (2) for all $n > 1$, but both equations manifest residual autocorrelation unless $\rho \approx 0$. The logarithm of the variance of $\bar{u}_j$ (normalised by σ_ε^2) and its first-order autocorrelation r_1 $(= E(\bar{u}_j\bar{u}_{j-1})/E(\bar{u}_j^2))$ are shown in Figures 2 and 3 respectively. The variance falls as n increases, but with an ever smaller proportional effect as ρ increases. As an example, for $\rho = .8$ and $n = 3$, (10) yields $\bar{\tau}_0 = 2.3\sigma_\varepsilon^2$ whereas σ_u^2 from (8) is $2.8\sigma_\varepsilon^2$, so averaging produces only a small decrease in variance.

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[9]Figures 2-7 below are for positive ρ only; Appendix B contains comparable graphs which include negative values of ρ.

[10]Telser (1967, Table 1) obtains parallel results for n=3 (i.e., his m=2), noting that his Table 1 is for a typical "summed process". Thus, his c(k) is our $\bar{\tau}(k/n)\cdot(1-\rho^2)\cdot n^2$ where k a multiple of n and his parameter a is our ρ. Ahsanullah and Wei (1984) obtain equivalent formulae for the AR(1) process; Stram and Wei (1986) generalise those results to ARIMA processes.

Figure 2. The logarithm of the variance of $\bar{u}_j$ (normalised by σ_ε^2) as a function of ρ and n.

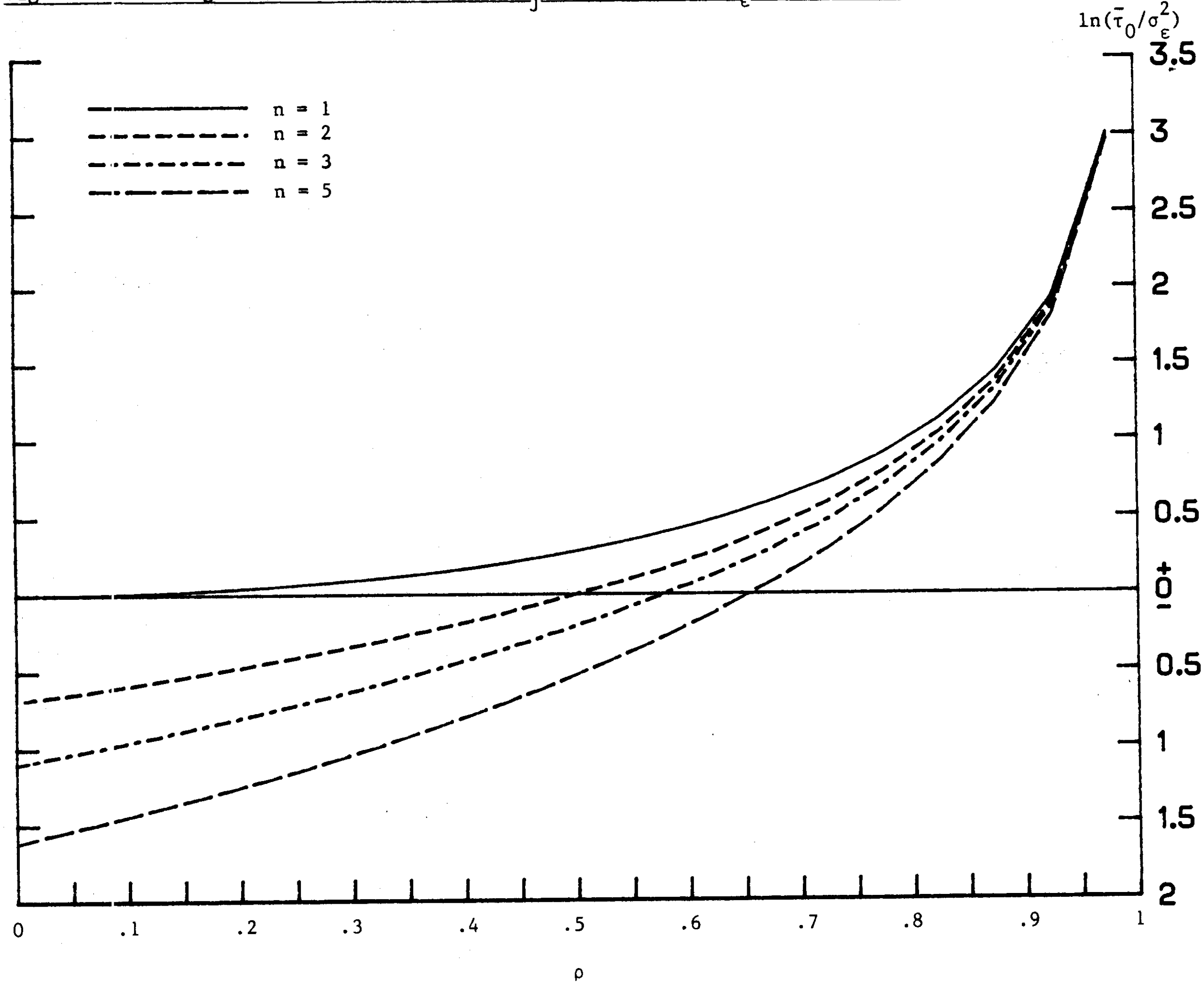


Figure 3. The first-order autocorrelation coefficient of $\bar{u}_j$ (r_1) as a function of ρ and n.

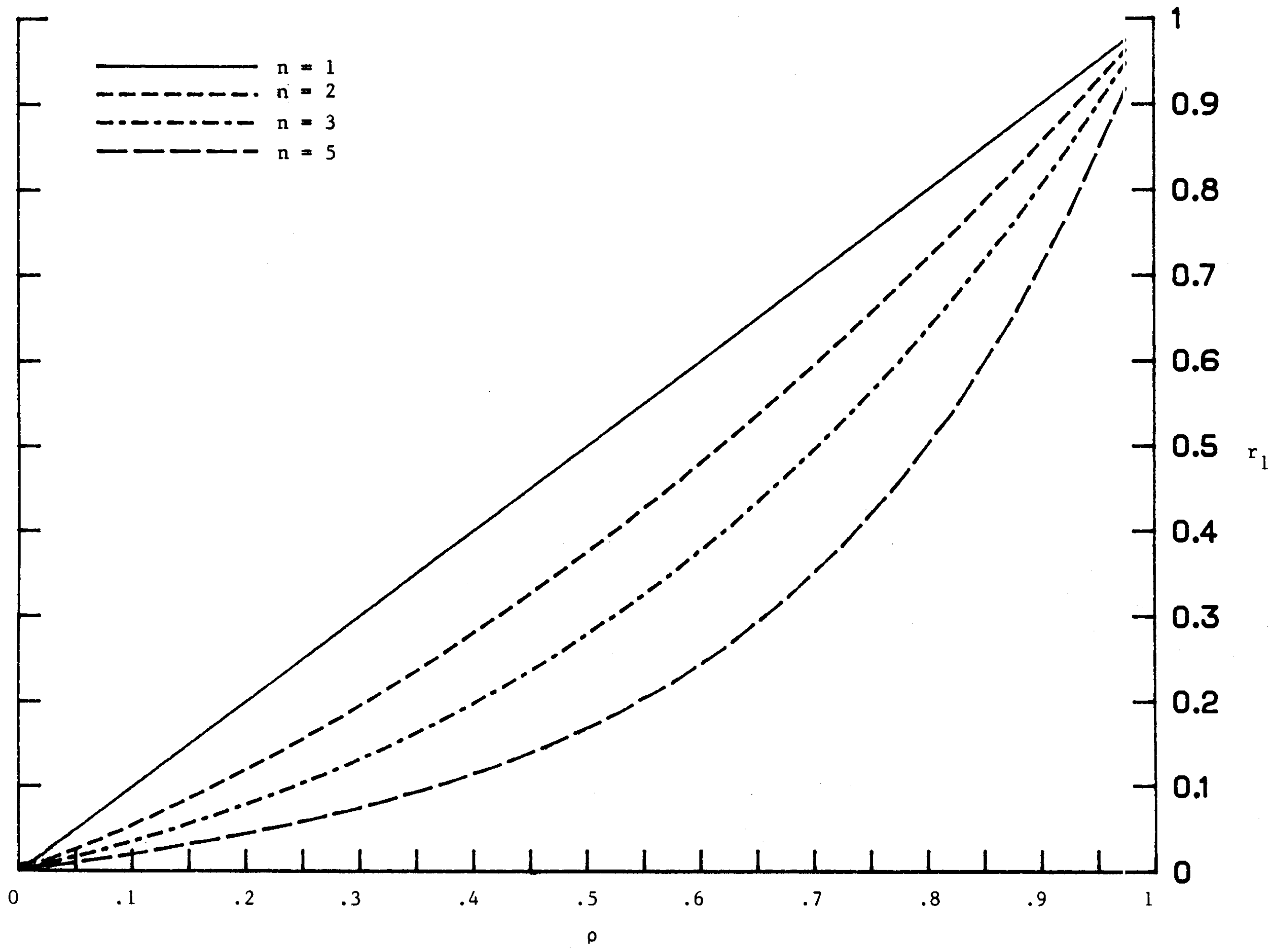
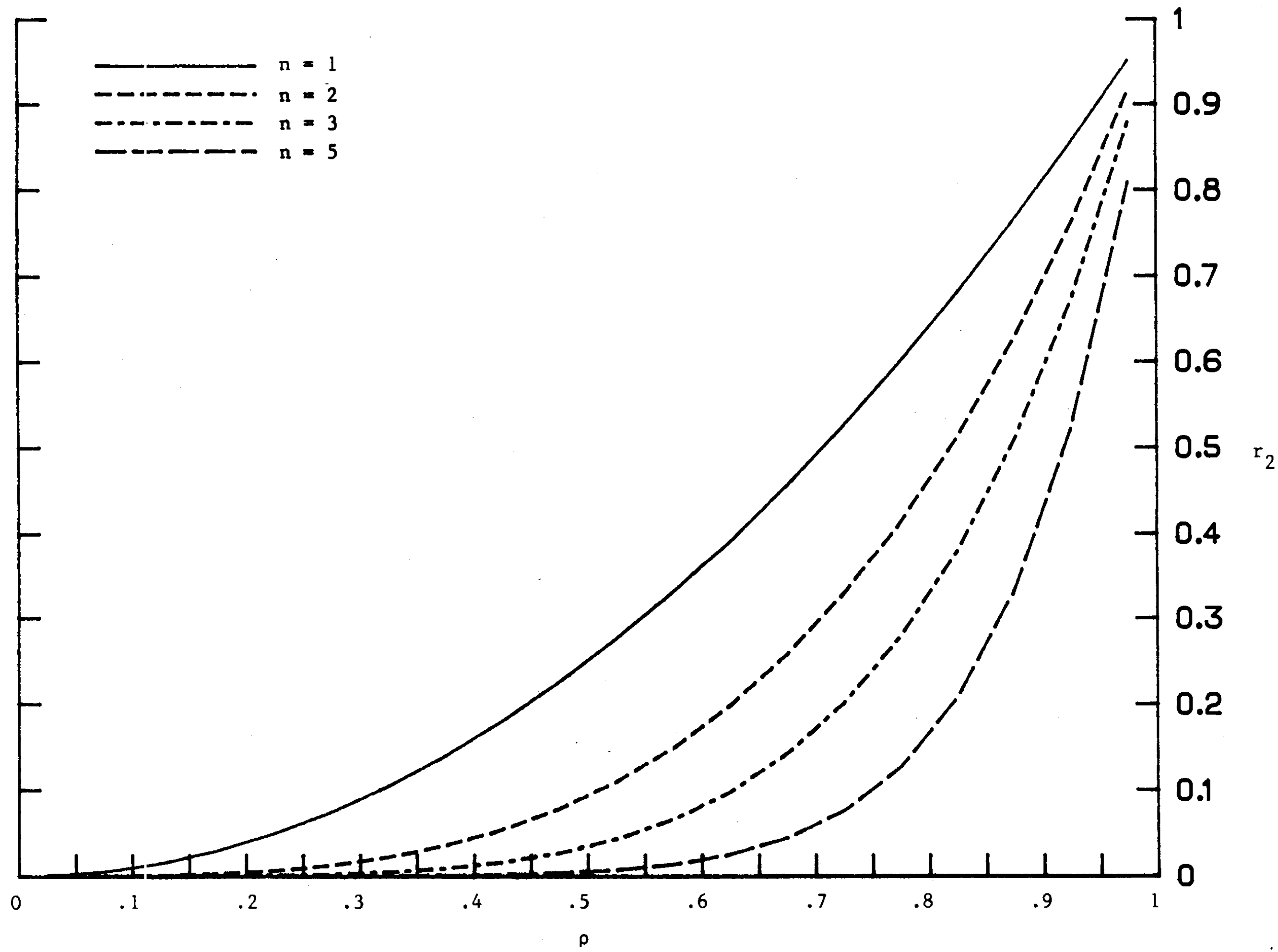


Figure 4. The second-order autocorrelation coefficient of $\bar{u}_j$ (r_2) as a function of ρ and n.



Moreover, as is apparent from Figure 3, phase-averaging only marginally reduces the first-order error autocorrelation when ρ is large. By contrast, autocorrelation at higher orders dies off rapidly as n increases (nb. (11), and compare Figure 3 with Figure 4, the latter plotting the <u>second</u>-order autocorrelation of $\bar{u}_j$, denoted r_2).

Least-squares estimation of structural parameters in the presence of autocorrelated disturbances is inefficient. Further, if right-hand side variables are correlated with the disturbance, such estimation is inconsistent:

$$E(z_t u_t) = \rho\lambda_2\sigma_\varepsilon^2/[(1-\rho^2)(1-\rho\kappa)] \equiv \sigma_{zu} \text{ (say)} \tag{12}$$

which is zero only if $\rho\lambda_2 = 0$. $E(z_{t-i}\varepsilon_t) = 0$ for all $i \geq 0$ (and for all ρ and λ_2), so estimation of (2)-(3) by <u>autoregressive</u> least squares is always consistent. Turning to the phase-average data in (6):

$$E(\bar{z}_j\bar{u}_j) = \lambda_2\sigma_\varepsilon^2(1+\rho)/[4(1-\rho)(1-\rho\kappa)] \tag{13}$$

when n = 2 (for illustrative purposes). Thus, even if $\rho = 0$ (and so $\sigma_{zu} = 0$), $E(\bar{z}_j\bar{u}_j) \neq 0$ unless $\lambda_2 = 0$. Conversely, the strict exogeneity of z_t (i.e., $\lambda_2 = 0$) is sufficient for both (12) and (13) to be zero.

<u>Differences of phase averages</u>. Next, we consider the effects of two-period differencing of the phase-averages in (6) to get (7):

$$\mathrm{var}(.5\Delta_2\bar{u}_j) = \frac{\sigma_\varepsilon^2}{2n(1-\rho)^2}\left[1 - \frac{\rho(2-\rho^n)(1-\rho^{2n})}{n(1-\rho^2)}\right] \tag{14}$$

and

$$E[(.5\Delta_2\bar{u}_j)(.5\Delta_2\bar{u}_{j-1})] = (1-\rho^{2n})\rho(1-\rho^n)^2\sigma_\varepsilon^2/[4n^2(1-\rho^2)(1-\rho)^2] \,. \tag{15}$$

After rescaling (14) by 4, the error variances for levels and "rates of change" models will be similar only for quite large values of ρ (e.g., $\rho = .9$), in which case the former will still exhibit substantial first-order autocorrelation although the latter will not; cf. Figure 5 and 6.[11] Further, and in contrast to the results for levels, the differenced series $\{.5\Delta_2\bar{u}_j\}$ exhibits (typically negative) second-order autocorrelation of the form:

$$r_2^* = -.5 + \frac{\rho^{n+1}(1-\rho^{2n})(1-\rho^n)^2}{2n(1-\rho^2)[1 - \frac{\rho(2-\rho^n)(1-\rho^{2n})}{n(1-\rho^2)}]} , \tag{16}$$

as portrayed in Figure 7 for some representative values of n. Only for very large values of ρ is (16) near zero.

For the differenced data, the equation equivalent to (13) is:

$$E(\Delta_2\bar{z}_j \cdot \Delta_2\bar{u}_j) = [2 - (1+\kappa)^2\kappa^2]\lambda_2\sigma_\varepsilon^2/4 \tag{17}$$

when $\rho = 0$ and $n = 2$. So, in general, different biases will result from fitting (6) and (7). Only if z_t is strictly exogenous ($\lambda_2 = 0$) will both biases be zero.

Phase averages of stationary AR(1) data. One particularly interesting special case of (2)-(4) is the stationary AR(1) process, i.e., with $\alpha_1 = 0$ in (2). In mean deviation form (i.e., $\alpha_0 = 0$), that yields:

$$y_t = \rho y_{t-1} + \varepsilon_t \qquad t = 1,\ldots,T \tag{18}$$

with

$$\sigma_y^2 = \sigma_\varepsilon^2/(1-\rho^2) . \tag{19}$$

For n-period averaging,

$$\bar{y}_j = \rho_p\bar{y}_{j-1} + e_j \qquad j = 1,\ldots,J \tag{20}$$

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[11]The variable-length weighting procedures in Friedman and Schwartz (1982) should not alter the substance of this last result; but being data-based to eliminate "cyclical behaviour" (which, e.g., a positively autocorrelated series would appear to manifest), it could seriously alter the nature of the residual autocorrelation. For example, the negative second-order autocorrelation of $\{.5\Delta_2\bar{u}_j\}$ in (16) may not carry over for data-selected phase averages.

Figure 5. The logarithm of the variance of $.5\Delta_2\bar{u}_j$ (normalised by σ_ε^2) as a function of ρ and n.

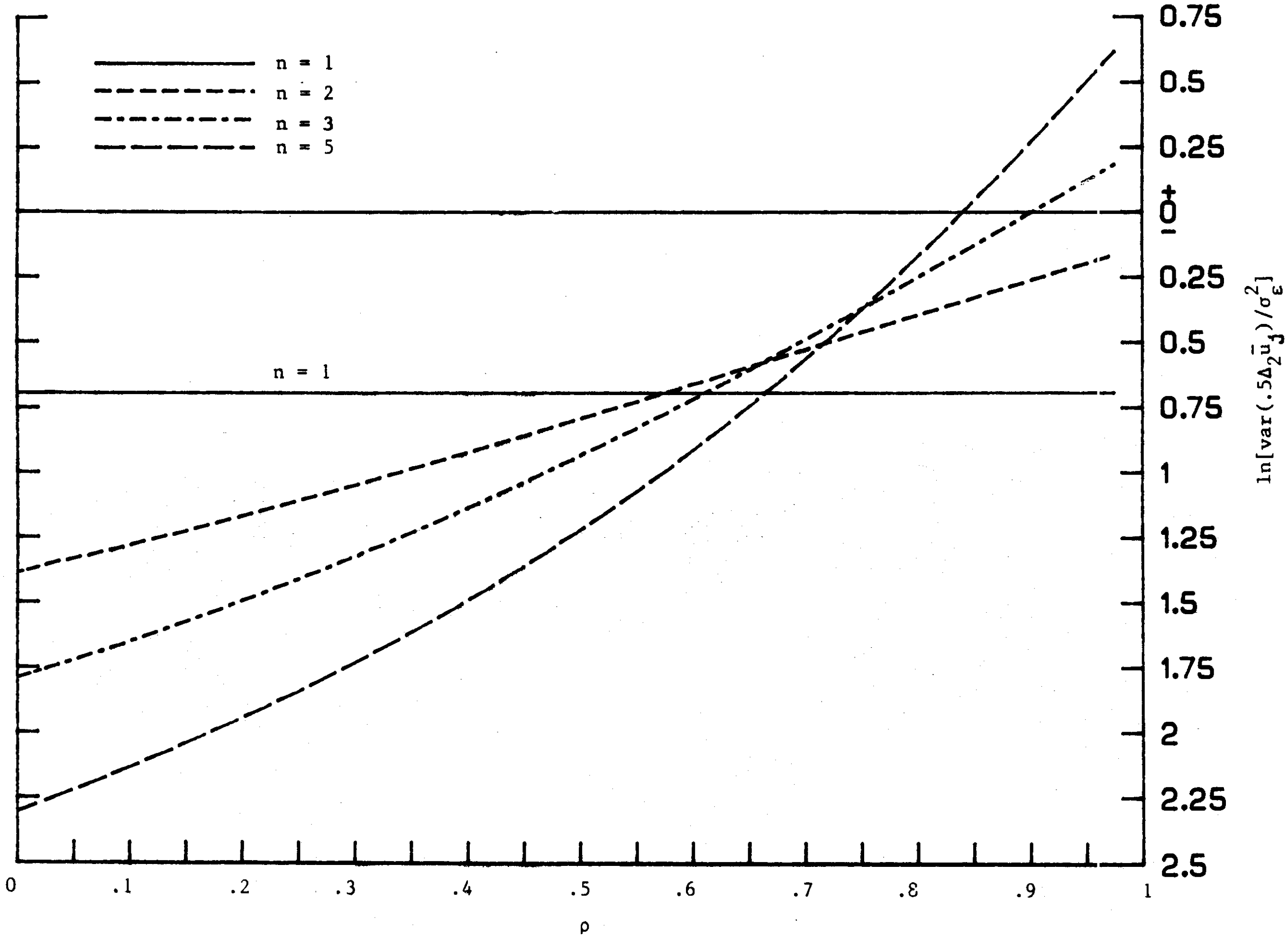


Figure 6. The first-order autocorrelation coefficient of $.5\Delta_2\bar{u}_j$ (r_1^*) as a function of ρ and n.

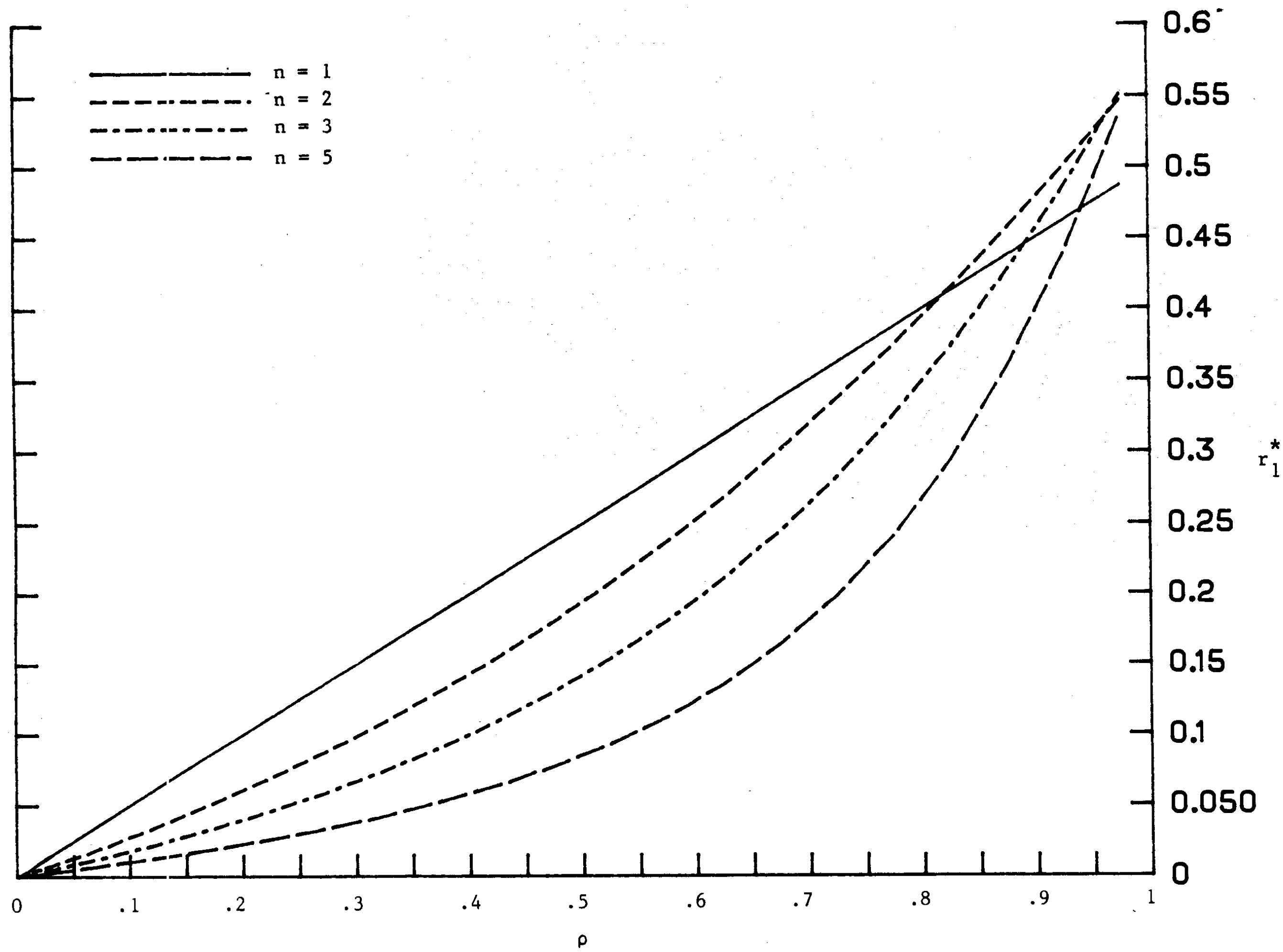
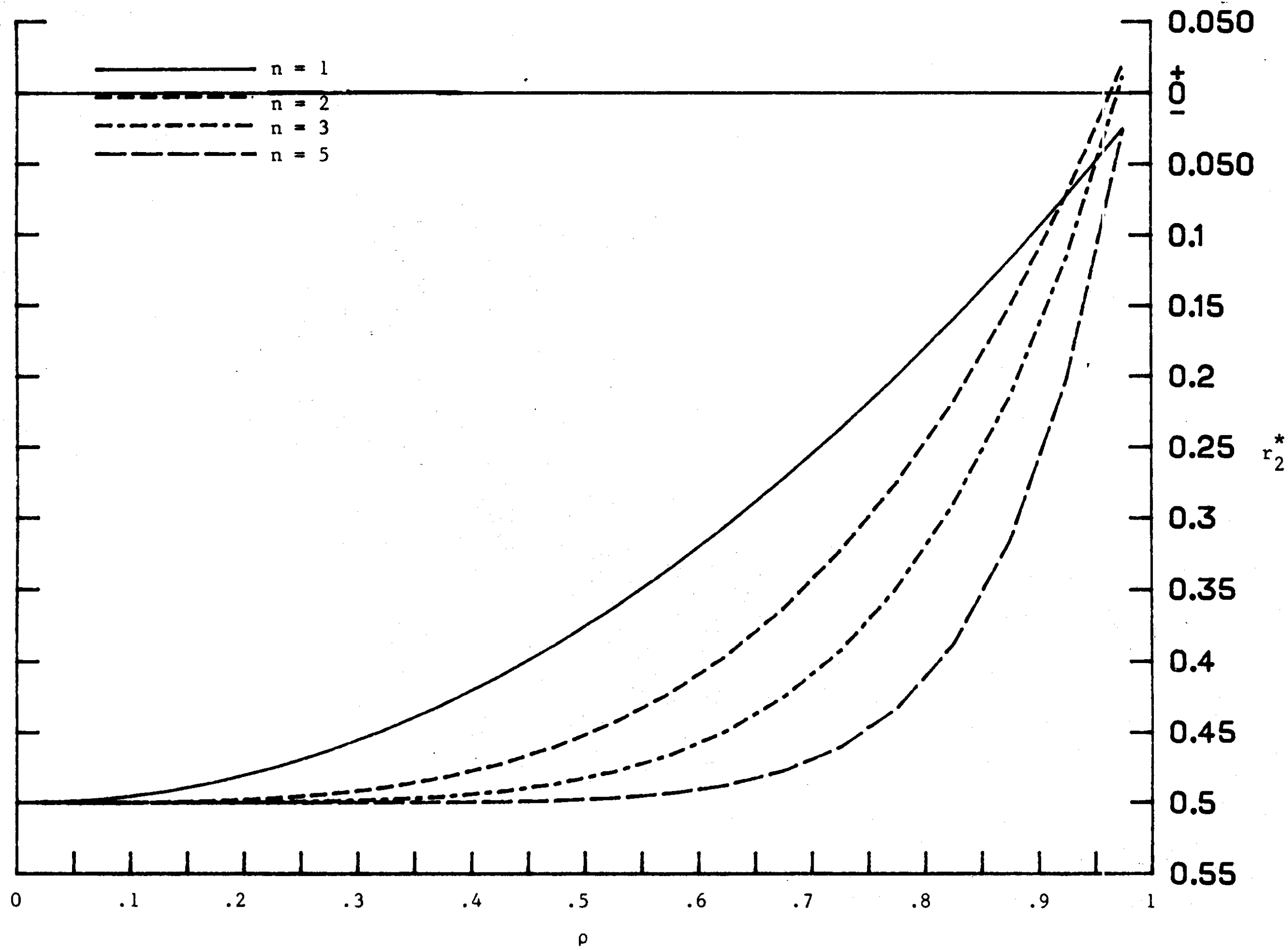


Figure 7. The second-order autocorrelation coefficient of $.5\Delta_2\bar{u}_j$ (r_2^*) as a function of ρ and n.



where ρ_p is defined by $[E(\bar{y}_{j-1}^2)]^{-1}E(\bar{y}_j\bar{y}_{j-1})$, i.e., $E(\bar{y}_{j-1}e_j)=0$. Thus,

$$\sigma^2(\bar{y}) = \sigma_e^2/(1-\rho_p^2) = \sigma_y^2[n(1-\rho^2)-2\rho(1-\rho^n)]/[n^2(1-\rho)^2] \tag{21}$$

(which is equivalent to $\bar{\tau}_0$) and

$$\rho_p = \rho(1-\rho^n)^2/[n(1-\rho^2)-2\rho(1-\rho^n)] \tag{22}$$

(which is $\bar{\tau}_1/\bar{\tau}_0$). Moreover,

$$\Delta_1 y_t = \varepsilon_t + (\rho-1)y_{t-1} \qquad \text{and} \tag{23}$$

$$\Delta_1 \bar{y}_j = e_j + (\rho_p-1)\bar{y}_{j-1} \qquad \text{so that} \tag{24}$$

$$\sigma^2(\Delta_1 y) = 2\sigma_\varepsilon^2/(1+\rho) \qquad \text{and} \tag{25}$$

$$\sigma^2(\Delta_1 \bar{y}) = 2\sigma_e^2/(1+\rho_p) \ . \tag{26}$$

These formulae will be used in Section 3 for analysing the empirical characteristics of velocity and will be contrasted with similar formulae for the random walk model (developed below).

<u>Phase averages of a random walk</u>. The final alternative we consider is that the series is a random walk. If $\rho = 1$ in (18), then $E(y_t^2) = t\sigma_\varepsilon^2$ (for $y_0 = 0$) and the mean of the standard estimator of the variance of y_t is:

$$E\left(\frac{1}{T-1}\sum_{t=1}^{T} y_t^2\right) = \sigma_\varepsilon^2\ T(T+1)/[2(T-1)] \equiv \sigma_T^2(y) \tag{27}$$

(e.g., see Fuller (1976, p. 367)) whereas $\sigma^2(\Delta_1 y) = \sigma_\varepsilon^2$. Working (1960) considers the case of an n-period average of a random walk and shows that, for e_j <u>defined by</u> $\bar{y}_j = \bar{y}_{j-1} + e_j$, its variance is:

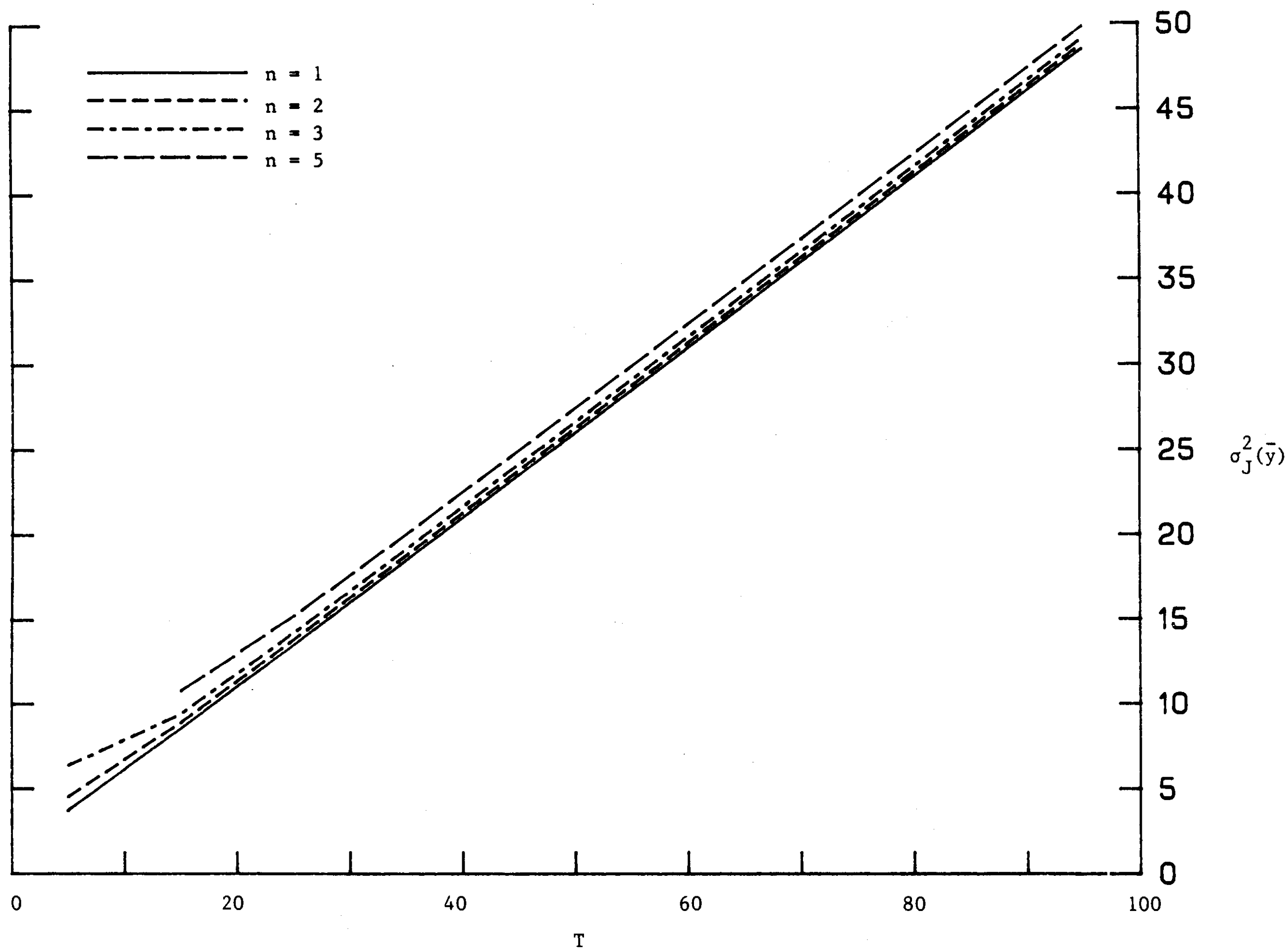
$$\sigma_e^2 = \sigma_\varepsilon^2(2n^2+1)/(3n) \ . \tag{28}$$

However, the mean of the standard estimator of the variance of $\bar{y}_j$ is:

$$E\left(\frac{1}{J-1}\sum_{j=1}^{J} \bar{y}_j^2\right) = \sigma_\varepsilon^2 J(3Jn^2-n^2+3n+1)/[6n(J-1)] \equiv \sigma_J^2(\bar{y}) \ , \tag{29}$$

assuming $y_0 = 0$. Figure 8 plots $\sigma_J^2(\bar{y})$ for different values of <u>T</u>

Figure 8. The mean of the standard estimator of the variance of $\bar{y}_j$ as a function of T and n.



and n with $\sigma_\varepsilon = 1$.[12] It strikingly portrays the linearity of $\sigma_J^2(\bar{y})$ in T and the near invariance of $\sigma_J^2(\bar{y})$ with respect to n, given T. Both follow from (29) which gives $\sigma_J^2(\bar{y}) = \sigma_T^2(y) + O(T^0) = \sigma_\varepsilon^2[(T/2) + O(T^0)]$ where each of the terms $O(T^0)$ depends upon n only, to $O(T^{-1})$.[13]

3. Interpreting Friedman and Schwartz's phase-averaging

To illustrate the value of our analytical results from Section 2 above, we investigate claims by Friedman and Schwartz (1982) about what the properties of phase-average data and the relationships between those data ought to be. We present certain features of the observed series on velocity, examine how well our analytical model captures them, and contrast them with predictions by Friedman and Schwartz. While our model is an extremely simplified characterisation of the phase-averaging adopted by Friedman and Schwartz, it does offer several insights into the likely consequences of their approach.

Friedman and Schwartz's primary justification for phase-averaging appears to be the claim that it reduces cyclical effects (pp. 13-14, 78), thereby allowing them to focus on their primary concern, monetary trends. By filtering out the cycle, they argue that phase-averaging should reduce serial correlation in the data and lower the data variance. Secondly, they claim phase-averaging reduces the effects of measurement errors (p. 86): that also would imply a reduction in the data variance. Attaining those effects is important to their statistical analysis. In practice, as is

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[12]Note that ρ (=1) is no longer a determinant of the variances whereas T now is. Also, $\sigma_J^2(\bar{y}) = \sigma_T^2(y)$ for n = 1.

[13]Due to the non-stationarity of the series generated, inference in the presence of a unit root requires special attention; cf. Dickey and Fuller (1979, 1981), Evans and Savin (1981, 1984), Sargan and Bhargava (1983), Phillips and Durlauf (1986), and Granger and Engle (1987).

clearly visible in Figure 9 which jointly plots the phase-average and annual data for velocity in the United Kingdom, phase-averaging negligibly reduces either the variance or serial correlation in this time series, a series from which they claim to draw important inferences.[14] The remainder of this section shows that the simple AR(1) model in Section 2 implies properties for the phase-average and annual data closely in line with those observed. The evidence also supports the notion that a substantial loss of information has resulted from phase-averaging.

Time-series evidence. To evaluate some basic time-series properties of velocity v, consider the following first-order autoregressions with annual and phase-average data over the last century for the United Kingdom.[15]

$$\hat{v}_t = \underset{(.016)}{.019} + \underset{(.029)}{.968}\, v_{t-1} \tag{30}$$

$T = 95 \quad \hat{\sigma} = 4.798\% \quad dw = 1.13 \quad dw^* = .08 \quad dw^{**} = 1.15$.

$$\hat{\bar{v}}_j = \underset{(.042)}{.023} + \underset{(.077)}{.972}\, \bar{v}_{j-1} \tag{31}$$

$J = 35 \quad \hat{\sigma} = 7.279\% \quad dw = 1.70 \quad dw^* = .31 \quad dw^{**} = 1.74$.

Values in parentheses (·) are conventionally calculated standard errors. dw denotes Durbin and Watson's (1950, 1951) statistic, dw* is its value

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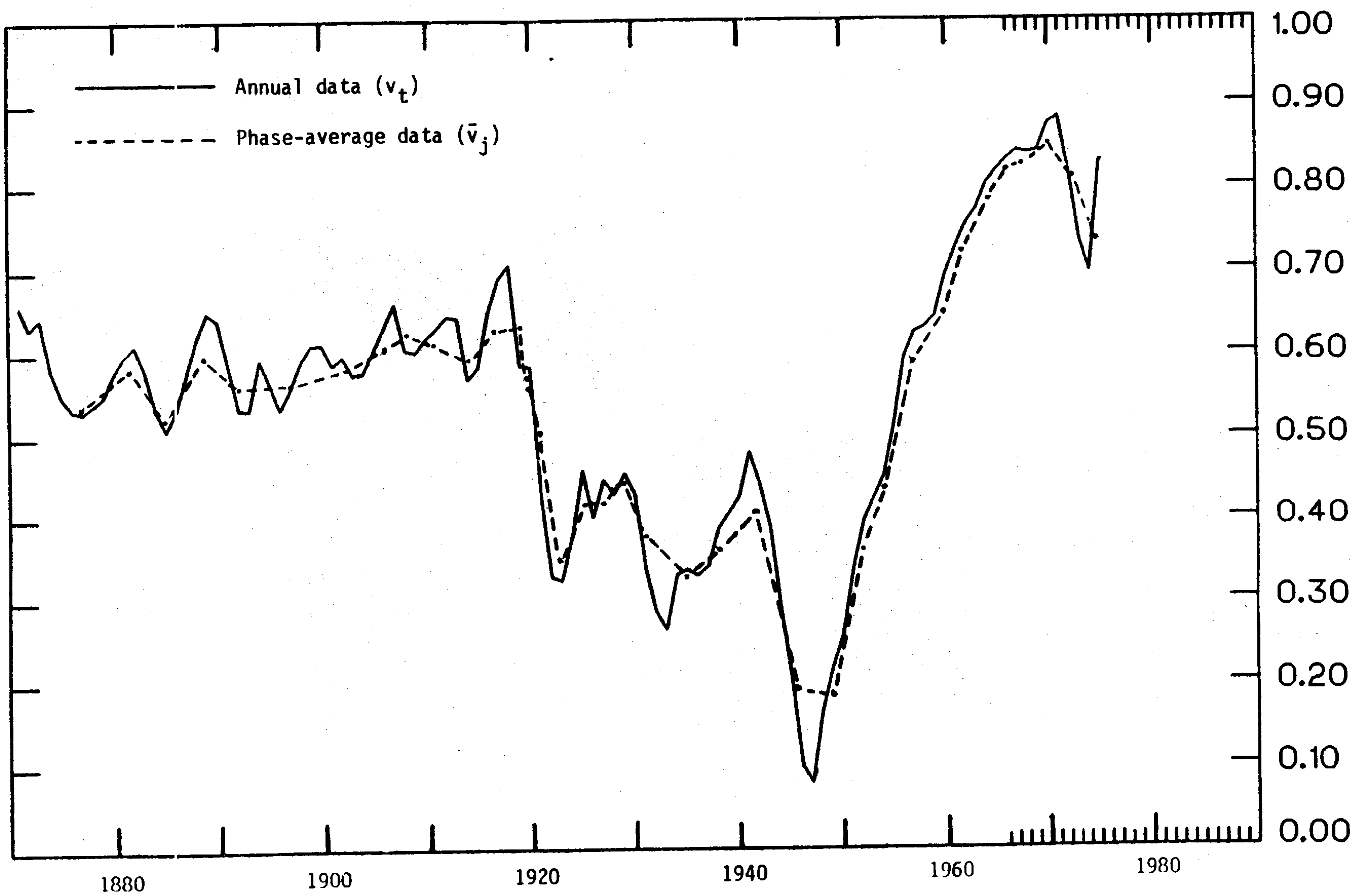
[14] From Table 1 below, the estimated standard error of the annual data is 17.35% whereas that for the phase-average data is 17.31%. The extent of serial correlation in both series is evident from (30) and (31).

[15] The (logarithm of) velocity is $v = \ln(P \cdot I/M)$ where $P \cdot I$ is nominal income and M is the money stock. Appendix A briefly describes these data.

All our estimates using phase-average data are based on weighted least squares, correcting for the different phase lengths. However, parameter estimates are not very different whether ordinary or weighted least squares are used. The summary statistics for phase-average data also are calculated with weights accounting for the differences in phase lengths.

One phase observation is lost in the calculation of lags, so the sample sizes are J = 35 (phase observations 2-36) and T = 95 (annual observations 1879-1973), with the span of the annual data matching that for the phase-average data.

Figure 9. The logarithm of the velocity of money for the United Kingdom: annual and phase-average data.



when the dependent variable is regressed on a constant, and dw** is its value when the first difference of the dependent variable is regressed on a constant.[16] The series in levels (both annual and phase-average) show first-order autocorrelation close to unity with the resulting differenced series being far less positively autocorrelated than the levels, and close to "white noise" for the phase-average data. Those features influence the standard deviations of velocity, listed in the second column of Table 1.

Table 1. Standard deviations of velocity (percentages)[17]

Series	Observed values	Predicted values: AR(1) model	Predicted values: random walk model
v	17.35	17.14	33.43
$\bar{v}$	17.31	16.84	33.60
$\Delta_1 v$	4.80	4.85	4.80
$\Delta_1 \bar{v}$	7.19	6.82	6.67
ε	4.80	4.80	4.80
e	7.28	6.68	6.67

For the annual and phase-average series, the standard deviations for levels are roughly equal; large reductions in standard deviations are obtained by differencing, more so for the annual data. Other series from Friedman and Schwartz (1982) have analogous properties.

Assuming that velocity is a stationary AR(1) process and that $n = 3$ and $\rho = .96$, the analogue model in Section 2 predicts relevant standard deviations for the annual and phase-average data in terms of σ_ε, the standard deviation of the underlying innovation process. Those values are

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[16] The Durbin-Watson statistic dw* and the t-ratio for $(\hat{\rho}-1)$ may be interpreted as statistics for testing for the non-existence of a relationship between money, prices, and income; cf. Sargan and Bhargava (1983), Dickey and Fuller (1979, 1981), and Granger and Engle (1987).

[17] The values in columns three and four of Table 1 have been normalised on the annual AR(1) regression residual standard error because the results in Section 2 are for relative, not absolute, variances.

given in the third column of Table 1 and closely match the corresponding observed values. Further, the implied first-order autocorrelation coefficient of the phase-average data is $\rho_p = .92$, close to that in (31) above. While we do not think velocity is simply a first-order autoregressive process (and give evidence supporting that view in Hendry and Ericsson (1985)), these results nevertheless illustrate the ineffectiveness of phase-averaging in attaining its supposed principal benefits, namely, reduction in the data variance and elimination of serial correlation arising from cyclical components. These results also explain why the models using phase-average data fit much less well than those using annual data.

Alternatively, the series for velocity might be generated by a random walk. Certainly, the graphs of velocity and of several other variables of interest to Friedman and Schwartz (1982) look like those of the random-walk series reported by Working (1934); and (e.g.) $\rho = .96$ is not "far" from unity.[18] Also, that hypothesis for velocity has received considerable attention in the literature (cf. Gould and Nelson (1974), Nelson and Plosser (1982), and Bharghava (1986)). The last column of Table 1 gives various standard deviations for $T = 95$, $J = 35$ (and so $n \equiv T/J \approx 2.71$). Those results are a much poorer match to the observed standard deviations than the predictions from the stationary AR(1) model. Like the significant error-correction coefficients established in Hendry and Ericsson (1985), the evidence here favours a highly autoregressive but "just barely" stationary process for velocity.

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[18]As noted in Hendry and Ericsson (1985), it is difficult to reject the null that $\rho = 1$; but that could be due to the low power of unit-root tests against alternatives like $\rho = .95$ (e.g., see Bhargava (1986)) and does not entail accepting the null.

<u>Econometric evidence</u>. In terms of estimating econometric models, the standard error of Friedman and Schwartz's (1982, p. 282) money-demand equation for the United Kingdom is around 5%. We were able to reproduce closely, but not precisely, their numbers:

$$\begin{aligned}(\widehat{\bar{m}-\bar{p}-\bar{h}})_j = \underset{(.19)}{.012} + \underset{(.049)}{.885}\,(\bar{i}-\bar{h})_j - \underset{(3.3)}{11.21}\,\bar{RN}_j - \underset{(.29)}{.22}\,G(\bar{p}+\bar{i})_j \\ + \underset{(.58)}{1.37}\,\bar{W}_j + \underset{(2.7)}{20.6}\,\bar{S}_j \end{aligned} \qquad (32)$$

$J = 36 \qquad n = 2.75 \qquad \hat{\sigma} = 5.66\%$.

By contrast, the equivalent $\hat{\sigma}$ from many dynamic annual models is under 2%. For example, Hendry and Ericsson (1985, equation (22)) obtain the following.[19]

$$\begin{aligned}\Delta_1\widehat{(m-p)}_t = \underset{[.04]}{.37}\Delta_1^2(m-p)_{t-1} - \underset{[.07]}{.06}\Delta_1^2(m-p)_{t-2} - \underset{[.022]}{.20}(m-p-i)_{t-4} \\ + \underset{[.10]}{.66}(\Delta_4 i_t/4) - \underset{[.04]}{.47}\Delta_1^2 p_t - \underset{[.06]}{.14}\Delta_1^2 p_{t-2} - \underset{[.20]}{.78}RS_t \\ - \underset{[1.1]}{3.3}(\Delta_2 R\ell_t/2) + \underset{[1.1]}{1.9}D_1 + \underset{[.6]}{3.6}D_2 + \underset{[.80]}{.64}D_3 - \underset{[.010]}{.086} \end{aligned} \qquad (33)$$

$T = 93 \qquad R^2 = .82 \qquad \hat{\sigma} = 1.711\%$

Thus the additional information leads to a more than <u>tenfold</u> reduction in the residual variance relative to that for their model using variable-length phase averages. This is a larger improvement than predicted by the analysis of fixed-length phase-averaging and may arise in part from the <u>data-selected</u> choices of reference business cycles.

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[19] White (1980) heteroscedasticity-consistent standard errors appear in square brackets [·]; cf. Nicholls and Pagan (1983). Appendix A briefly describes the data in equations (32) and (33) for which $J = 36$ (phase observations 1-36) and $T = 93$ (annual observations 1878-1970) respectively. Hendry and Ericsson (1985) extensively discuss the properties of both regressions.

Exogeneity. Hendry and Ericsson (1985, Appendix G) obtain substantial evidence against Granger non-causality for money, income, prices, and interest rates. Thus, the weak exogeneity of (e.g.) prices, income, and interest rates in (33) may well be lost when a money-demand equation is estimated as in (32) using phase-average data.[20] That happens because some feedback from (e.g.) interest rates to money occurs within a phase, so those dynamics of the annual data introduce simultaneity in the phase-average data.

4. Summary

Business cycles have been the focus of much economic analysis throughout this century: phase-averaging has been proposed as one empirical method of separating cyclical and secular components. This paper analyses the statistical and econometric effects such filtering has on data from some simple dynamic processes and illustrates those effects by comparison of annual and phase-average data from Friedman and Schwartz (1982). For Friedman and Schwartz in particular, phase-averaging empirically does not obtain two results important to their analysis, namely, reductions of serial correlation and of the data variance. In general, phase-averaging involves a loss of information and thereby can result in the inconsistency of previously consistent estimation procedures and in the endogeneity of previously exogenous variables.

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[20]Nb. Engle et al. (1983) on the different concepts of exogeneity; cf. Wei (1982), Hendry and Ericsson (1985, footnote 15), and Longbottom and Holly (1985).

APPENDIX A. The data

Legend

D_1 - A dummy variable for World War I (= 1 for 1914-18 inclusive, zero elsewhere)

D_2 - A dummy variable for 1921-55 (= 1 for 1921-55 inclusive, zero elsewhere)

D_3 - A dummy variable for World War II (= 1 for 1939-45 inclusive, zero elsewhere)

$G(\bar{p}+\bar{i})$ - Growth rate of phase-average nominal income

H - Population (millions)

I - Real net national product (million 1929 £)

M - Money stock (million £)

P - Deflator of I (1929 = 1.00)

$R\ell$ - Long-term interest rate (fraction)

RN - RS·H/M

RS - Short-term interest rate (fraction)

$\bar{S}$ - A dummy variable for phase observations 16-28 (1921-1955; = 1 for observations 16-28 inclusive, zero elsewhere)

$\bar{W}$ - A dummy variable for phase observations 13-15 and 26-28 (1918-1921 and 1946-1955; = -4, -3, -2, 8, 5, and 3 for phase observations 13, 14, 15, 26, 27, and 28 respectively; zero elsewhere)

Coefficients and estimated standard errors of the dummies D_1, D_2, D_3, $\bar{S}$, and $\bar{W}$ are reported times 100 for readability.

These data are as in Friedman and Schwartz (1982, Tables 4.8 and 4.9), but relevant series are rescaled proportionately from 1871 to 1920 to remove the break in 1920 when Southern Ireland ceased to be part of the United Kingdom. A capital letter denotes the level of a variable; lower case denotes the logarithm. Also, P, RS, and $R\ell$ have been divided by 100 so that the values of P in 1929 equal 1.00 (rather than 100) and the interest rates are expressed as fractions (rather than as percentages).

APPENDIX B. Phase-averaging of negatively autocorrelated data

In the text, the effects of phase-averaging are depicted for positive ρ in (3) via variances and autocovariances in Figures 2-7. As noted in Section 2, the effects are qualitatively different for negative ρ. This appendix contains corresponding figures (numbered B.2 through B.7) with ρ both positive and negative. The properties for even n (i.e., n=2 in the figures) are particularly asymmetric.

Figure B.2. The logarithm of the variance of $\bar{u}_j$ (normalised by σ_ε^2) as a function of ρ and n.

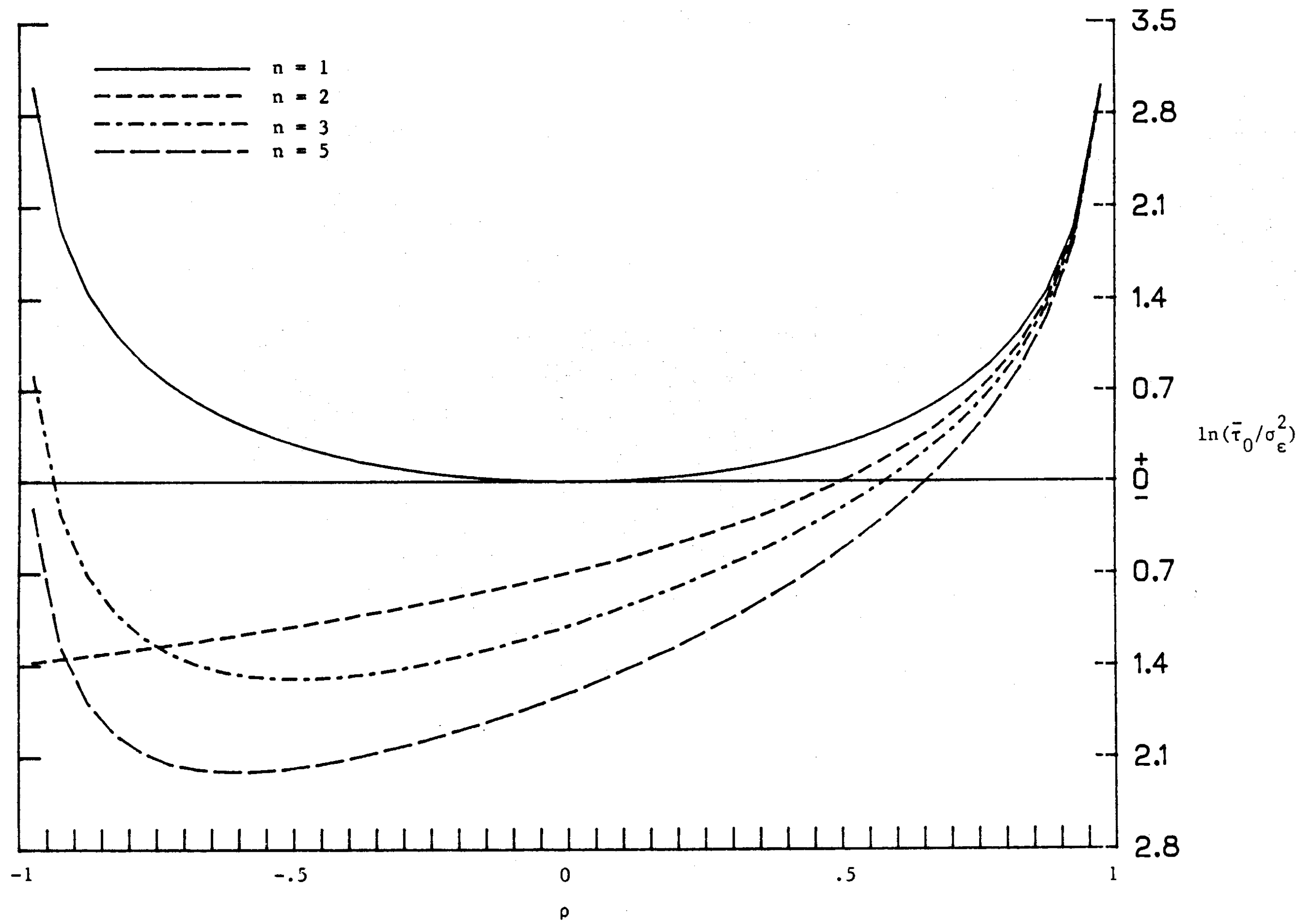


Figure B.3. The first-order autocorrelation coefficient of $\bar{u}_j$ (r_1) as a function of ρ and n.

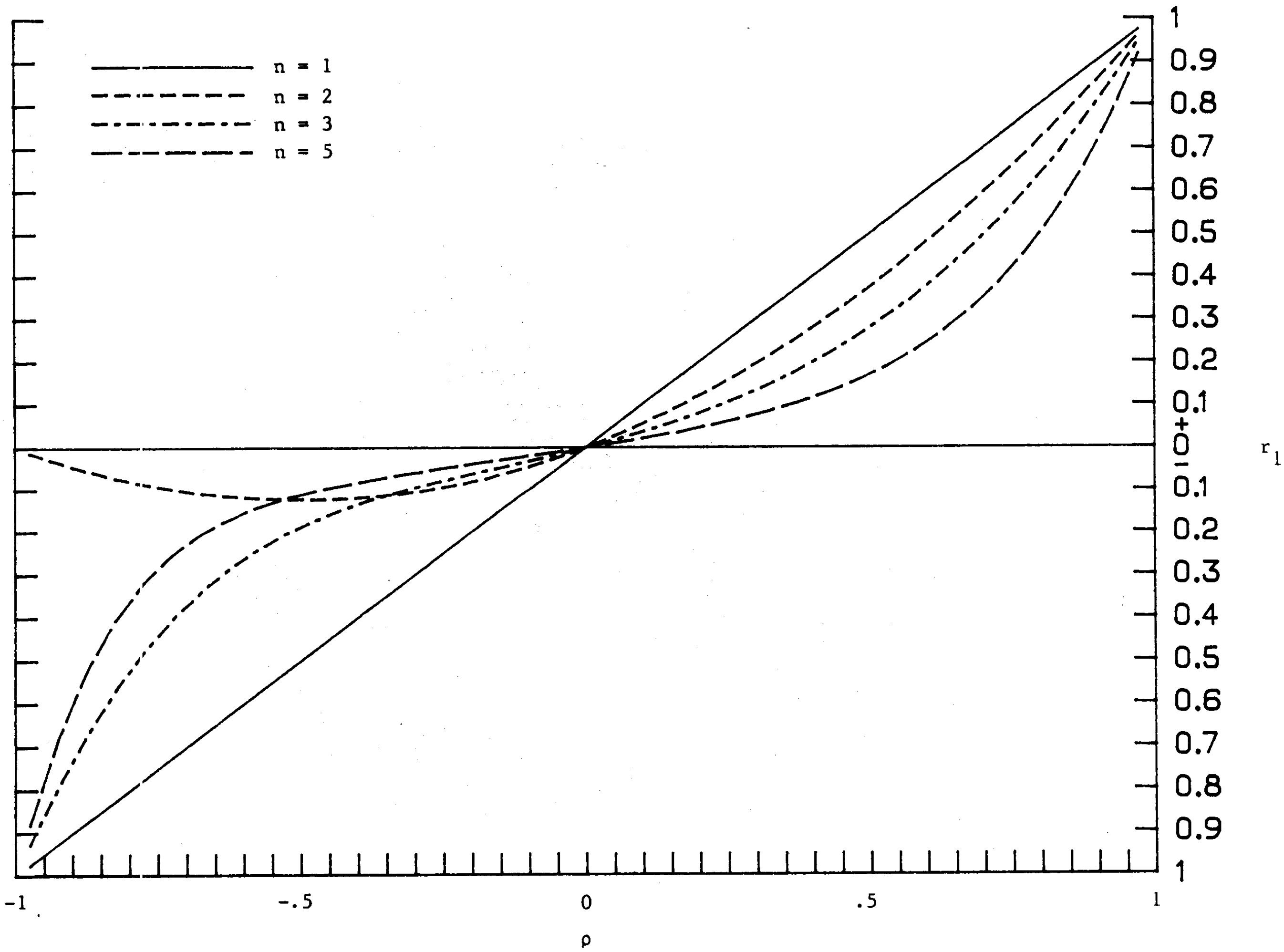


Figure B.4. The second-order autocorrelation coefficient of $\bar{u}_j$ (r_2) as a function of ρ and n.

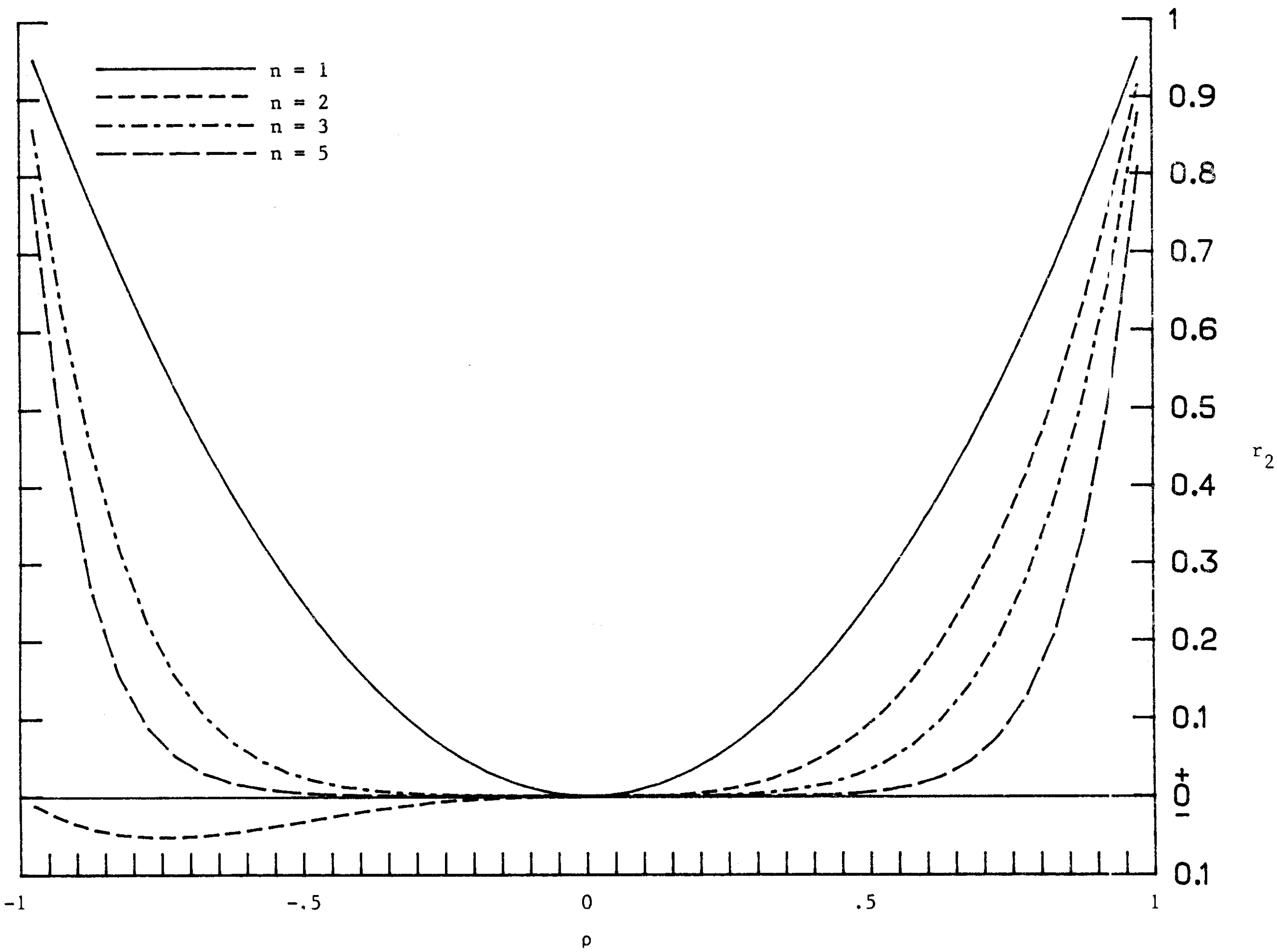


Figure B.5. The logarithm of the variance of $.5\Delta_2\bar{u}_j$ (normalised by σ_ε^2) as a function of ρ and n.

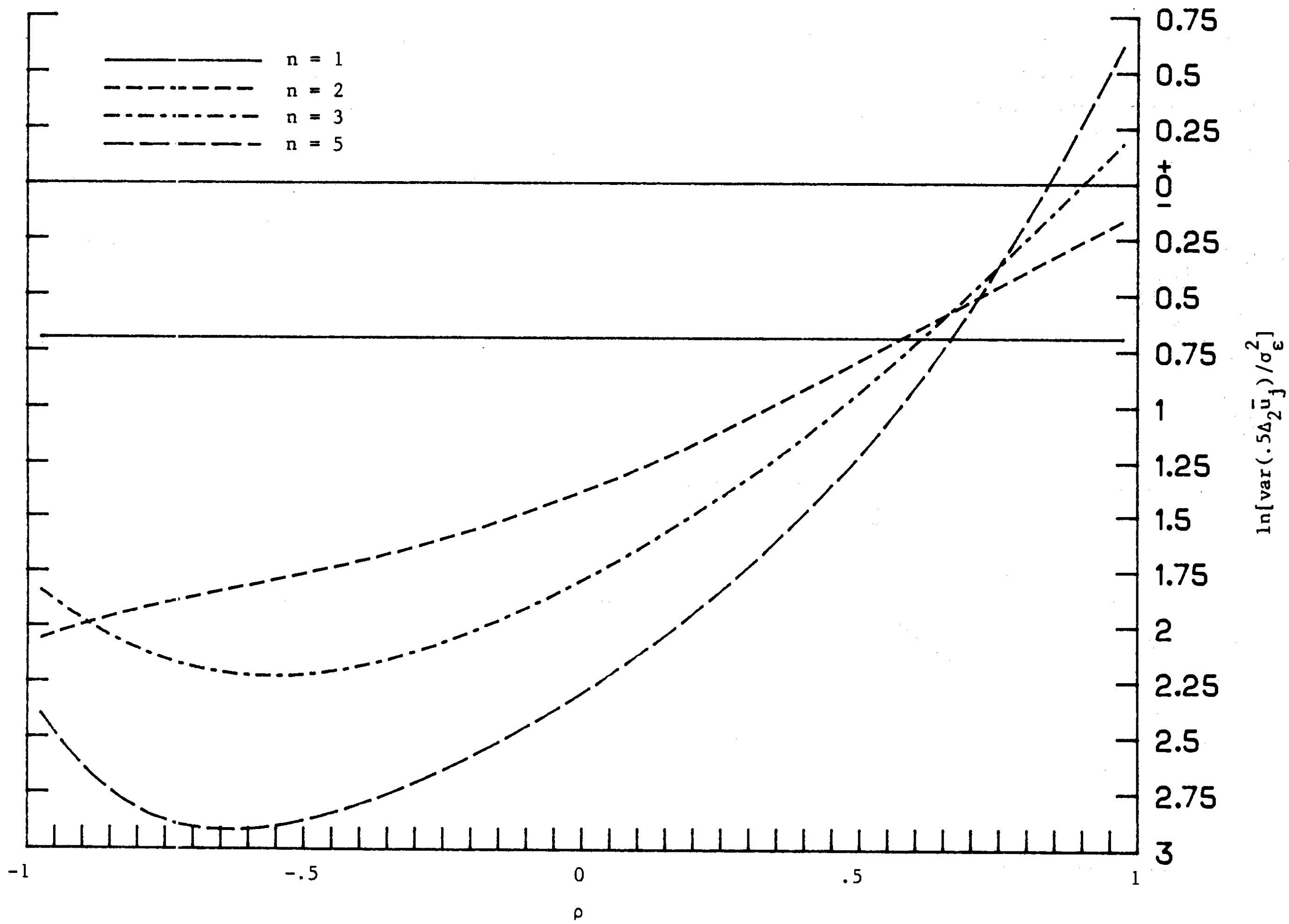


Figure B.6. The first-order autocorrelation coefficient of $.5\Delta_2\bar{u}_j$ (r_1^*) as a function of ρ and n.

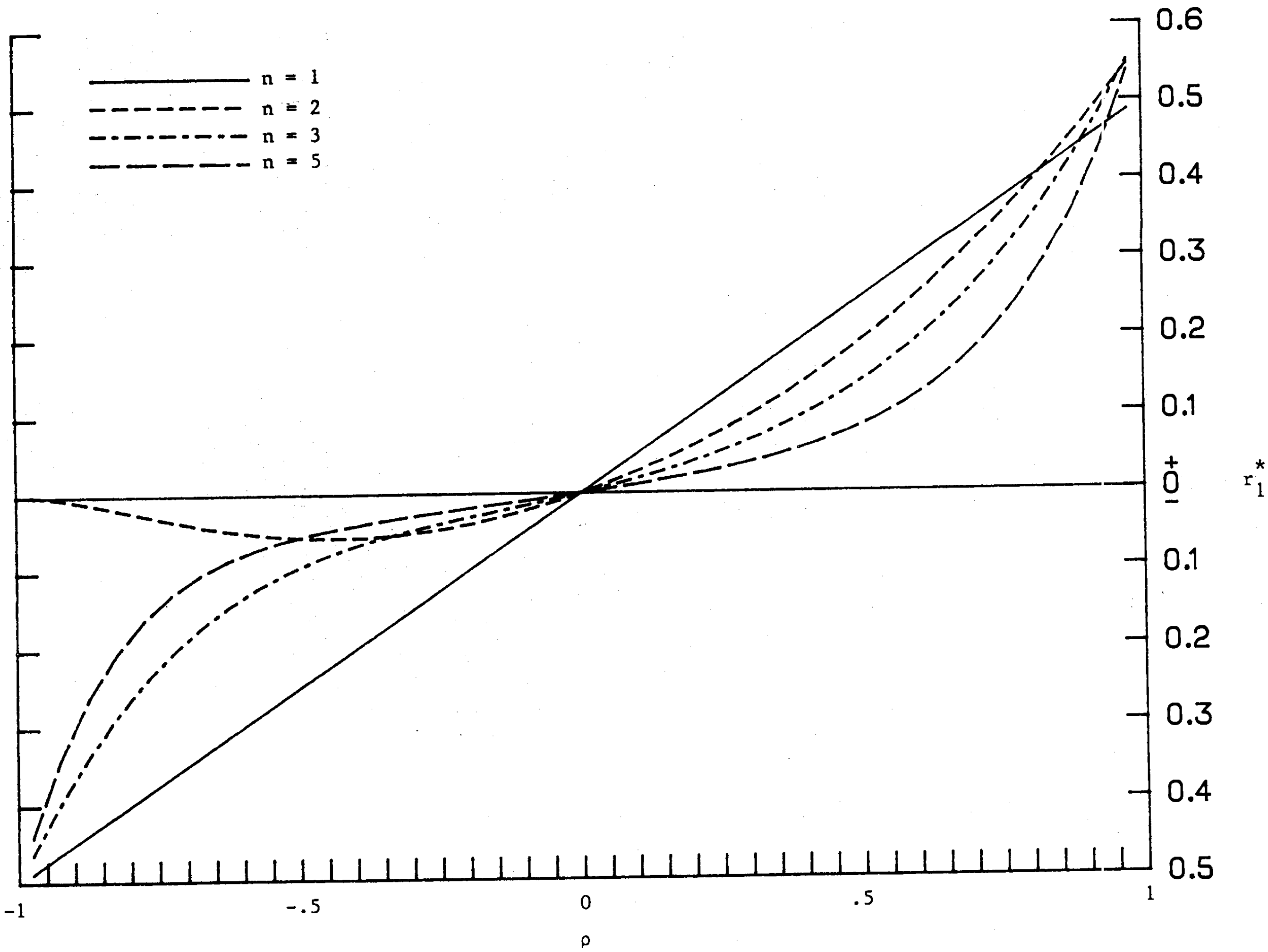
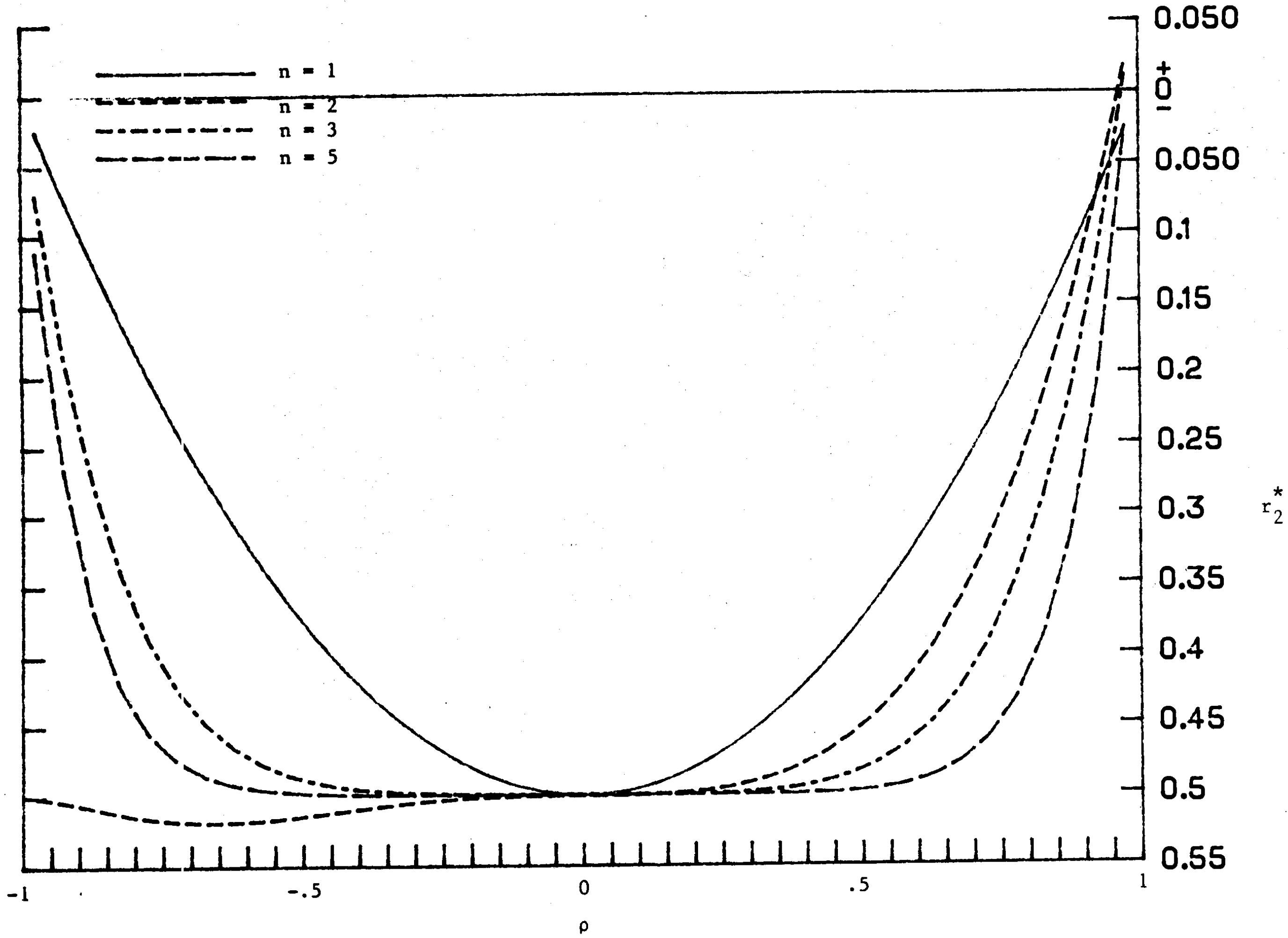


Figure B.7. The second-order autocorrelation coefficient of $.5\Delta_2\bar{u}_j$ (r_2^*) as a function of ρ and n.



References

Ahsanullah, M. and W.W.S. Wei (1984) "The Effects of Time Aggregation on the AR(1) Process", Computational Statistics Quarterly, 1, 4, 343-352.

Allais, M. (1966) "A Restatement of the Quantity Theory of Money", American Economic Review, 56, 5, 1123-1157.

Allais, M. (1972) "Forgetfulness and Interest", Journal of Money, Credit, and Banking, 4, 1, 40-73.

Amemiya, T. and R.Y. Wu (1972) "The Effect of Aggregation on Prediction in the Autoregressive Model", Journal of the American Statistical Association, 67, 339, 628-632.

Banerjee, A., J.J. Dolado, D.F. Hendry, and G.W. Smith (1986) "Exploring Equilibrium Relationships in Econometrics Through Static Models: Some Monte Carlo Evidence", Oxford Bulletin of Economics and Statistics, 48, 3, 253-277.

Beveridge, W.H. (1921) "Weather and Harvest Cycles", Economic Journal, 31, 124, 429-452.

Bhargava, A. (1986) "On the Theory of Testing for Unit Roots in Observed Time Series", Review of Economic Studies, 53, 3, 369-384.

Burns, A.F. and W.C. Mitchell (1946) Measuring Business Cycles, New York, National Bureau of Economic Research.

Dickey, D.A. and W.A. Fuller (1979) "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", Journal of the American Statistical Association, 74, 366, 427-431.

Dickey, D.A. and W.A. Fuller (1981) "Likelihood Ratio Statistics for Autoregressive Time Series with a Unit Root", Econometrica, 49, 4, 1057-1072.

Durbin, J. (1960) "Efficient Fitting of Linear Models for Continuous Stationary Time Series from Discrete Data", Bulletin de L'Institute International de Statistique, 38, 4, 273-281.

Durbin, J. and G.S. Watson (1950) "Testing for Serial Correlation in Least Squares Regression. I", Biometrika, 37, 3 and 4, 409-428.

Durbin, J. and G.S. Watson (1951) "Testing for Serial Correlation in Least Squares Regression. II", Biometrika, 38, 1 and 2, 159-178.

Engle, R.F., D.F. Hendry, and J.-F. Richard (1983) "Exogeneity", Econometrica, 51, 2, 277-304.

Evans, G.B.A. and N.E. Savin (1981) "Testing for Unit Roots: 1", Econometrica, 49, 3, 753-779.

Evans, G.B.A. and N.E. Savin (1984) "Testing for Unit Roots: 2", Econometrica, 52, 5, 1241-1269.

Falk, B. (1986) "Further Evidence on the Asymmetric Behavior of Economic Time Series over the Business Cycle", Journal of Political Economy, 94, 5, 1096-1109.

Friedman, M. and A.J. Schwartz (1963) A Monetary History of the United States, 1867-1960, Princeton, Princeton University Press.

Friedman, M. and A.J. Schwartz (1982) Monetary Trends in the United States and the United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867-1975, Chicago, University of Chicago Press.

Fuller, W.A. (1976) Introduction to Statistical Time Series, New York, John Wiley and Sons.

Gould, J.P. and C.R. Nelson (1974) "The Stochastic Structure of the Velocity of Money", American Economic Review, 64, 3, 405-418.

Granger, C.W.J. (1969) "Investigating Causal Relations by Econometric Models and Cross-spectral Methods", Econometrica, 37, 3, 424-438.

Granger, C.W.J. (1986) "Developments in the Study of Cointegrated Economic Variables", Oxford Bulletin of Economics and Statistics, 48, 3, 213-228.

Granger, C.W.J. and R.F. Engle (1987) "Dynamic Model Specification with Equilibrium Constraints: Co-integration and Error-correction", Econometrica, forthcoming.

Hendry, D.F. (1986) "Econometric Modelling with Cointegrated Variables: An Overview", Oxford Bulletin of Economics and Statistics, 48, 3, 201-212.

Hendry, D.F. and N.R. Ericsson (1983) "Assertion without Empirical Basis: An Econometric Appraisal of 'Monetary Trends in ... the United Kingdom' by Milton Friedman and Anna Schwartz" in Monetary Trends in the United Kingdom, Bank of England Panel of Academic Consultants, Panel Paper No. 22, 45-101 (with additional references).

Hendry, D.F. and N.R. Ericsson (1985) "Assertion without Empirical Basis: An Econometric Appraisal of Monetary Trends in ... the United Kingdom by Milton Friedman and Anna J. Schwartz", International Finance Discussion Paper No. 270, Federal Reserve Board of Governors, Washington, D.C.

Hendry, D.F. and G.E. Mizon (1978) "Serial Correlation as a Convenient Simplification, Not a Nuisance: A Comment on a Study of the Demand for Money by the Bank of England", Economic Journal, 88, 351, 549-563.

Hendry, D.F., A.R. Pagan, and J.D. Sargan (1984) "Dynamic Specification", Chapter 18 in Z. Griliches and M.D. Intriligator (eds.) Handbook of Econometrics, Amsterdam, North-Holland, Volume II, 1023-1100.

King, R.G., C.I. Plosser, J.H. Stock, and M.W. Watson (1987) "Stochastic Trends and Economic Fluctuations", manuscript, University of Rochester.

Long, Jr., J.B. and C.I. Plosser (1983) "Real Business Cycles", Journal of Political Economy, 91, 1, 39-69.

Longbottom, A. and S. Holly (1985) "Econometric Methodology and Monetarism: Professor Friedman and Professor Hendry on the Demand for Money", Discussion Paper No. 131, London Business School.

Lucas, Jr., R.E. (1977) "Understanding Business Cycles" in K. Brunner and A.H. Meltzer (eds.) Stabilization of the Domestic and International Economy, Carnegie-Rochester Conference Series on Public Policy, Amsterdam, North-Holland, Volume 5, 7-29.

Maverick, L.A. (1933) "Time Series: Their Analysis by Successive Smoothings", Econometrica, 1, 3, 238-246.

Moriguchi, C. (1970) "Aggregation over Time in Macroeconomic Relations", International Economic Review, 11, 3, 427-440.

Mundlak, Y. (1961) "Aggregation over Time in Distributed Lag Models", International Economic Review, 2, 2, 154-163.

Neftçi, S.N. (1984) "Are Economic Time Series Asymmetric over the Business Cycle?", Journal of Political Economy, 92, 2, 307-328.

Neftçi, S.N. (1986) "Is There a Cyclical Time Unit?" in K. Brunner and A.H. Meltzer (eds.) The National Bureau Method, International Capital Mobility and Other Essays, Carnegie-Rochester Conference Series on Public Policy, Amsterdam, North-Holland, Volume 24, 11-48.

Nelson, C.R. and C.I. Plosser (1982) "Trends and Random Walks in Macroeconomic Time Series", Journal of Monetary Economics, 10, 2, 139-162.

Nicholls, D.F. and A.R. Pagan (1983) "Heteroscedasticity in Models with Lagged Dependent Variables", Econometrica, 51, 4, 1233-1242.

Phillips, P.C.B., and S.N. Durlauf (1986) "Multiple Time Series Regression with Integrated Processes", Review of Economic Studies, 53, 4, 473-495.

Sargan, J.D. (1976) "Some Discrete Approximations to Continuous Time Stochastic Models", Chapter 3 in A.R. Bergstrom (ed.) Statistical Inference in Continuous Time Economic Models, Amsterdam, North-Holland, 27-79.

Sargan, J.D. and A. Bhargava (1983) "Testing Residuals from Least Squares Regression for Being Generated by the Gaussian Random Walk", Econometrica, 51, 1, 153-174.

Schumpeter, J.A. (1939) Business Cycles: A Theoretical, Historical, and Statistical Analysis of the Capitalist Process, New York, McGraw-Hill.

Sims, C.A. (ed.) (1977) New Methods in Business Cycle Research: Proceedings from a Conference, Minneapolis, Federal Reserve Bank of Minneapolis.

Slutzky, E. (1937) "The Summation of Random Causes as the Source of Cyclic Processes", Econometrica, 5, 2, 105-146.

Stock, J.H. (1985) "Measuring Business Cycle Time", mimeo, Harvard University.

Stram, D.O., and W.W.S. Wei (1986) "Temporal Aggregation in the ARIMA Process", Journal of Time Series Analysis, 7, 4, 279-292.

Telser, L.G. (1967) "Discrete Samples and Moving Sums in Stationary Stochastic Processes", Journal of the American Statistical Association, 62, 318, 484-499.

Theil, H. (1954) Linear Aggregation of Economic Relations, Amsterdam, North-Holland.

Tinbergen, J. and J.J. Polak (1950) The Dynamics of Business Cycles, Chicago, University of Chicago Press.

Tiao, G.C., and W.W.S. Wei (1976) "Effect of Temporal Aggregation on the Dynamic Relationship of Two Time Series Variables", Biometrika, 63, 3, 513-523.

Wei, W.W.S. (1978) "The Effect of Temporal Aggregation on Parameter Estimation in Distributed Lag Model", Journal of Econometrics, 8, 2, 237-246.

Wei, W.W.S. (1982) "The Effects of Systematic Sampling and Temporal Aggregation on Causality --- A Cautionary Note", Journal of the American Statistical Association, 77, 378, 316-319.

Weiss, A.A. (1984) "Systematic Sampling and Temporal Aggregation in Time Series Models", Journal of Econometrics, 26, 3, 271-281.

White, H. (1980) "A Heteroskedasticity-consistent Covariance Matrix Estimator and a Direct Test for Heteroskedasticity", Econometrica, 48, 4, 817-838.

Working, H. (1934) "A Random-difference Series for Use in the Analysis of Time Series", Journal of the American Statistical Association, 29, 185, 11-24.

Working, H. (1960) "Note on the Correlation of First Differences of Averages in a Random Chain", Econometrica, 28, 4, 916-918.

Zellner, A., and C. Montmarquette (1971) "A Study of Some Aspects of Temporal Aggregation Problems in Econometric Analyses", Review of Economics and Statistics, 53, 4, 335-342.

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