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THE OUT-OF-SAMPLE FORECASTING PERFORMANCE OF EXCHANGE RATE MODELS WHEN COEFFICIENTS ARE ALLOWED TO CHANGE

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Garry J. Schinasi
P.A.V.B. Swamy

ABSTRACT

This study examines the out-of-sample forecasting performance of models of exchange rate determination without imposing the restriction that coefficients are fixed over time. Both fixed and variable coefficient versions of conventional structural models are considered, with and without a lagged dependent variable. While our results on fixed coefficient models support most of the Meese and Rogoff conclusions, we find that when coefficients are allowed to change, an important subset of conventional models of the dollar-pound, the dollar-deutsche mark, and the dollar-yen exchange rates can outperform forecasts of a random walk model. The structural models considered are the flexible-price (Frenkel-Bilson) and sticky-price (Dornbusch-Frankel) monetary models, and a sticky-price model which includes the current account (Hooper-Morton). We also find that the variable coefficient version of the Dornbusch-Frankel model with a lagged dependent variable generally predicts better than the other models considered including the random walk model.

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I. Introduction

The seminal work of Meese and Rogoff (1983a,b), hereafter referred to as MR, casts serious doubt on the ability of international macroeconomic theory to predict exchange rate movements.[2] These studies concluded that linear fixed-coefficient regressions of exchange rates on variables such as relative money supplies, indices of industrial production, short interest rates, and trade balances, failed to match the out-of-sample forecasting performance of a simple random-walk model. MR's main results were reported as "robust to a variety of [fixed coefficient] estimation techniques, specifications of the underlying money demand functions, alternative serial correlation or lagged adjustment corrections, and measures of forecast accuracy."[3]

One might be tempted to conclude from these studies that economic variables convey little or no useful information about exchange rate

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[2]Some other follow-up studies are the following: MR (1985), Backus (1984), Woo (1985), Finn (1986), Edison (1984), Hakkio (1986), and Samanath (1986).

[3]See MR (1985, p.5).

movements. We find this conclusion unsatisfying. We share the view suggested by MR that "...their disappointing performance is most likely to be attributable to simultaneous equation bias, sampling error, stochastic movements in the ... underlying parameters, or misspecification" (1983a, p.17).[4]

This study evaluates the forecasting performance of models reported by MR (1983a) without imposing the restriction that the regression slopes are fixed over time (misspecification of the dynamics of exchange rate models is also examined). Although there are a number of studies that have relaxed the fixed coefficient assumption, they have done so in rather restrictive ways, requiring extensive and generally unavailable prior information.[5] The present study avoids these problems by applying a general technique for estimating models with stochastic coefficients (see Swamy and Tinsley (1980)) that encompasses as special cases the Kalman filtering technique, the method of Hildreth and Houck (1968), ARCH models (Engle (1982)) and others. Since multi-step-ahead forecasts (to be defined rigorously later in the paper) are not necessarily inferior to one-step-ahead forecasts (Swamy and Schinasi (1986)), we compute the former, using models with fixed and stochastically varying coefficients, to extend the work of MR without duplicating their efforts.

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[4]See Isard (1986) for a complementary view.

[5]Wolff (1985) and Alexander and Thomas (1986) estimate models using the Kalman filtering technique with known covariance matrices and essentially substantiate the MR results. Makin and Sauer (1986) use the Hildreth and Houck (1968) technique for estimating restrictive variable coefficient models to study policy regimes and do not perform the MR experiments.

The major result of our study is that when all coefficients are allowed to vary, the conventional models of exchange rates tested by MR can yield forecasts which are more accurate than their fixed-coefficient counterparts and more accurate than the random walk model. The present study does however support most of the MR conclusions regarding fixed coefficient models, although, contrary to their study, we find major improvements in the forecasts of fixed coefficient models that include lagged adjustment.

The paper is organized as follows. Section 2 presents the exchange rate models tested and describes the forecasting strategy adopted in the MR (1983a) study. It also presents the alternative fixed coefficient estimators used in the present study. Section 3 briefly discusses why variable coefficient models are appropriate for exchange rate modelling. Section 4 presents the stochastic coefficient representation of the exchange rate models tested by MR, and examines the assumptions required to estimate them. It also describes useful approximations to the minimum average mean square error linear predictor of an individual drawing of a dependent variable, given the independent variables, in a stochastic coefficient model. Section 5 describes the forecasting strategy of the present study. Section 6 compares the out-of-sample forecasting performance of the random walk models and the variable and fixed coefficient representations of the exchange rate models. The final section draws conclusions from the study.

2. The Models and the Method

2.1 Exchange Rate Models

The structural models estimated by MR (1983a,b) and used for forecasting nominal exchange rates are presented below. One general specification covering all these models as special cases is equation (1):

$$(1)\quad s_t = \beta_0 + \beta_1(m_t - m_t^*) + \beta_2(y_t - y_t^*) + \beta_3(r_t - r_t^*) + \beta_4(\pi_t^e - \pi_t^{e*}) + \beta_5(TB_t - TB_t^*) + u_t.$$

where, lower case letters indicate natural logs except for interest rates,

* indicates a foreign variable, and where,
s = the spot exchange rate ($/DM, $/¥, $/£)
m = money supply
y = industrial production
r = short-term interest rate
π^e = expected inflation rate[6]
TB = cumulative trade balance
u = disturbance term which may be serially correlated.

The original MR exercises estimated the following fixed coefficient versions of equation (1): (i) Frenkel-Bilson (purchasing power parity) which assumes $\beta_4=\beta_5=0$; (ii) Dornbusch-Frankel (slow price adjustment) which assumes $\beta_5=0$; and (iii) Hooper-Morton which is equation (1) with unequal coefficients for the trade balances.[7]

These models are all variants of the monetary model of exchange rates

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[6] To try proxies for the unobservable variable $(\pi_t^e - \pi_t^{e*})$ different from those of MR, we replace this variable by a distributed lag in the CPI inflation rate differential.

[7] The most recent vintage of MR (1985) performs root mean square forecast error (RMSE) exercises on the nominal exchange rate models, with three more years of data, and real exchange-rate versions of these same structures.

and differ only in the way they treat price adjustment.[8] In this study we also explicitly report results for these models with a lagged dependent variable as an explanatory variable, a specification that explicitly allows for short run deviations from long run purchasing power parity.

MR(1983a) used monthly data over the period March 1973 through June 1981. The fixed coefficient versions of (1) were initially estimated for each exchange rate using data up through October 1976. Forecasts were generated at four different horizons using the actual realizations of all explanatory variables for a prediction period. Then the data for November 1976 were added to the sample, and the parameters of each model were reestimated. New forecasts were generated at the same horizons, etc. In the MR studies, this sequential estimation yielded fixed-step-ahead forecasts which were generally inferior to those given by the random walk model. The estimation procedures used in this sequential estimation were ordinary least squares, approximate generalized least squares (correcting for serial correlation in the error term), and Fair's instrumental variables. MR also considered six univariate time series models involving a variety of prefiltering

[8]See Bilson (1978,1979), Frenkel (1976), Dornbusch (1976), Frankel (1979, 1981) and Hooper and Morton (1982).

techniques and lag length selection criteria[9], a random walk with drift parameter, and an unconstrained vector autoregression; none could outpredict the random walk model: $s_t = s_{t-1} + a_t$, where $\{a_t\}$ is white noise with mean zero and constant variance.

2.2 Alternative Fixed Coefficient Estimates

This study also applies ordinary least squares and approximate generalized least squares with a correction for serial correlation in the error term to equation (1). To extend MR's work on fixed coefficient models, we also applied alternative estimators of fixed coefficients not applied by them to the structural models. These estimators were: (i) posterior mean for Shiller's smoothness prior on distributed lag coefficients; (ii) Almon's polynomial distributed lag; and (iii) ridge regression estimator with a smoothness restriction on lag coefficients.[10] These three estimators were considered with and without a first-order serial correlation correction for the error term. We experimented with several lag lengths and degrees of polynomials that were required to apply these estimators.

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[9]The six time series models used were the following: (1) an unconstrained autoregression (AR) in which the longest lag considered (M) is set equal to (n/log n), where n is the sample size; (2) AR in which lag lengths are determined by Schwartz's criterion; (3) AR in which lag lengths are determined by Akaike's criterion; (4) long AR estimated by using observations that are arbitrarily weighted by powers of 0.95; (5) the Wiener-Kolmogorov prediction formula; (6) AR estimated by minimizing the sum of the absolute values of errors. The prefiltering techniques involve differencing, deseasonalizing, and removing time trends.

[10]For a theoretical derivation of these alternative estimators, see Thurman, Swamy, and Mehta (1986) and Kashyap, Swamy, Mehta, and Porter (1986).

3. Why Are Variable Coefficient Models Appropriate?

There are several reasons why the coefficients of exchange rate models may change over time. First, even if the explanatory variables capture all information used by traders, there is no reason to believe information is used in the same way over all policy regimes and over all time horizons; parameters can change over time. As argued in another paper (see Swamy and Schinasi (1986a)), sequential estimation of fixed coefficient regressions ("rolling") is not the appropriate technique for capturing variations in coefficients over time.

Secondly, at the high level of aggregation of exchange markets, there is little reason to believe that behavioral parameters are fixed. There is a wide diversity of participants in foreign exchange markets with relatively small and highly variable market shares. Even if each participant reacted to macroeconomic developments according to a stable fixed coefficient reaction function, it is difficult to argue that macroeconomic variables would be related to exchange rates by a simple fixed coefficient relationship, without also assuming that individual reaction functions were identical.

Thirdly, most if not all of the empirical literature has assumed that coefficients are fixed over the relevant sample period. Most of this literature decisively rejects economic theory as having any ability to produce accurate predictions. Yet, it would be unreasonable to reject theories that have been tested on only a very limited subset of models, namely linear fixed coefficient models. This study takes one small step in the direction of expanding the set of models for testing theories of exchange rate behavior.

One cannot relax the assumption of fixed coefficients without imposing some other structure on coefficients. It would be preferable to apply existing economic theory to explain variations in coefficients, but there is unfortunately a paucity of such exchange rate theory. As a preliminary investigation, we have chosen a stochastic coefficient model of exchange rate behavior, one in which coefficients have a systematic component, fixed over time, and a stochastic component that can vary in each time period.

While these stochastic coefficient models cannot "explain" variations in coefficients over time -- a phenomenon economists would like to explain for other relationships as well -- estimating such models can nevertheless uncover variations in coefficients.[11] Furthermore, using the estimated variation in coefficients in an efficient statistical forecasting procedure might improve the forecasting ability of the models suggested by economic theory, when compared to fixed coefficient models. Even allowing the coefficients to vary, however, may not result in improved forecasts if the underlying exchange rate equation is not well specified. In such cases we further

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[11]For learning about such variations we cannot rely on the classical F-test (Rao (1973, pp.281-284)) of a one time change in the fixed coefficients within a sample period, and its generalizations, because these tests have poor power against the most general alternative that all coefficients change in every period. This is so because under this most general alternative hypothesis, time-varying coefficients are not consistently estimable (see the uncertainty principle formulated by Swamy and Tinsley (1980, p.117)).

experiment with dynamic versions of the static equations by adding a lagged dependent variable as an explanatory variable.

4. The Stochastic Coefficient Representation of the Exchange Rate Models and its Uses

4.1 The Stochastic Coefficient Models

The stochastic coefficient representation of the models estimated here is presented in equations (2) through (4),

(2) $y_t = x_t' \beta_t$ for all t,

(3) $\beta_t = \bar{\beta} + \varepsilon_t$

(4a) $\varepsilon_t = \Phi \, \varepsilon_{t-1} + v_t$

(4b) $E(v_t) = 0$

(4c) $E(v_t v_s') = \Delta_a$ if t=s and 0 otherwise,

where x_t, β_t, $\bar{\beta}$, ε_t, v_t are all k x 1 vectors, Φ and Δ_a are k x k matrices, x_t represents the vector of the explanatory variables in eq. (1), including a constant term, and y_t is the natural log of the spot exchange rate. Note that equations (2) - (4) represent a special case of a more general variable coefficient specification which allows one to describe variations in coefficients with explanatory variables, allows for "simultaneous equations" complications, and allows for more general specifications of the error processes.[12]

In equation (3), each coefficient in each period, β_{it}, has two components: a time-independent fixed component, $\bar{\beta}_i$, and a time-dependent stochastic component, ε_{it}. ε_t is a vector stationary first-order autoregressive process (which can also be represented as a vector moving average process). Combining (2), (3), and (4a) reveals that the stochastic coefficient representation can be viewed as a fixed coefficient model with errors that are both serially correlated and heteroscedastic, where the form of serial correlation and heteroscedasticity is very general:

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[12]The general model developed by Swamy and Tinsley (1980) is as follows

(i) $y_t = x_t'\beta_t$, for all t

(ii) $\beta_t = \bar{B}z_t + \varepsilon_t$

(iii) $\varepsilon_t = \sum_{i=1}^{p} \Phi_i \varepsilon_{t-i} + \sum_{j=1}^{q} \theta_j a_{t-j} + a_t$,

where $\bar{B}$ is a k x m matrix, Φ_i, i=1, 2,...p, and θ_j, j=1, 2,...q are k x k matrices of fixed but unknown parameters, z_t is a mx1 vector of explanatory variables, and a_t is a white noise variable. Note that if some of the elements of x_t are correlated with β_t because of simultaneous equations complications, then those elements of x_t are also entered into z_t to account for such correlations. Note also that (iii) represents a wide class of time series specifications. Hence, the non-deterministic component of y_t is a nonstationary process (an ARMA(p,q) with time-dependent coefficients); it is a complicated mixture of serially correlated and heteroscedastic error terms. The usual fixed and variable coefficient models can be shown to be special cases of this general model.

(5a) $y_t = x_t'\bar{\beta} + u_t$

(5b) $u_t = x_t' \varepsilon_t$

(5c) $\varepsilon_t = \Phi \varepsilon_{t-1} + v_t$

4.2 Estimation

Estimation of $\bar{\beta}$ can be viewed as an application of generalized least squares or Aitken estimation, assuming that Φ and Δ_a are known. Swamy and Tinsley (1980) have developed a minimum average risk linear estimator which for given a priori moments of $\bar{\beta}$, Φ and Δ_a, can be shown to be more efficient than the Aitken estimator. Because Φ and Δ_a are not known and must be estimated, Swamy and Tinsley developed an iterative estimation procedure in which Φ and Δ_a are initially arbitrarily chosen but through iteration are consistently and efficiently estimated after initial consistent estimates of $\bar{\beta}$, Φ and Δ_a are obtained.[13]

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[13]Comparisons of the Swamy and Tinsley procedure with the Kalman and Bayesian procedures are given in Narasimham, Swamy and Reed (1986). It should be noted that as emphasized by Narasimham, Swamy and Reed (1986) the statistical notions of consistency and efficiency require the existence of the true values of parameters and hence do not apply to the estimators of $\bar{\beta}$, Φ, and Δ_a (or any other fixed parameter estimators either) if these parameters do not represent "real" physical quantities or if equation (3) does not have a natural interpretation in terms of physical quantities. Since this equation is about the β_t which are never observed, it is necessarily arbitrary. Therefore it is difficult to believe that there are model-free physical quantities associated with each of these arbitrary model parameters. It is difficult to take consistent estimation seriously as an essential procedure if one must always interpret the parameters as "real" physical quantities. Equations (3) - (4c) are used here as a way of obtaining good predictions of the observable exchange rates and are not estimated to interpret the β_t. The β_t are used only to index the set of distributions of y_t.

4.3 Prediction

When $\bar{\beta}$, Φ, and Δ_a are known, it can be shown that the minimum average mean square error linear predictor of y_{T+i} is [14]

$$(6)\quad y_{T+i} = x'_{T+i}\bar{\beta} + x'_{T+i}\Phi^{i}\Omega'_{T}D'_{x}\Sigma_{y}^{-1}(y - X\bar{\beta}),$$

where $\Omega = E(\varepsilon\varepsilon')$,

$\varepsilon = (\varepsilon'_1, \varepsilon'_2, \ldots \varepsilon'_T)'$,

$D_x = \mathrm{diag}(x'_1, x'_2, \ldots, x'_T)$,

$\Sigma_y = D_x\Omega D'_x = E(D_x\varepsilon\varepsilon' D'_x)$, Ω_T consists of the last k columns of Ω,

and

$X = (x_1, x_2, \ldots, x_T)'$.

Since $\bar{\beta}$, Φ, and Δ_a are not known, and in fact estimated, equation (6) with these estimated parameters may no longer represent a minimum average mean square error forecast. It can be shown that $\bar{\beta}$ is more precisely estimated than either Φ or Δ_a, and so it might improve forecasts to drop the second term on the right-hand side of (6) based on estimated $\bar{\beta}$, Φ and Δ_a, since the second term is more heavily influenced by the estimates of Φ and Δ_a than the first term. In the results reported below, the best of the two alternative forecasts, (estimated) $x'_{T+i}\bar{\beta}$ and (estimated) equation (6), were chosen. Furthermore, Φ and Δ_a can take on various forms. Four or five

[14]Note that equation (6) is equal to $E(y_{T+i} \mid y, x_{T+i}, X, \bar{\beta}, \Phi, \Delta_a)$ if $(y_{T+i}, y')'$ is jointly normal.

combinations were tried: both Φ and Δ_a were nondiagonal; both Φ and Δ_a were diagonal; Φ was diagonal and Δ_a was nondiagonal; all elements of Φ except its leading diagonal element were zero and Δ_a was nondiagonal; certain columns and rows of Φ were zero and Δ_a was nondiagonal. In the applications of the present paper, values of $\bar{\beta}$, Φ and Δ_a which implied positive definite covariance matrices for the dependent variables and which produced the most satisfactory forecasts of s_t in a RMSE sense were selected by screening 25 iterations of the Swamy and Tinsley procedure.

5. Forecasting Strategy and Procedures

So that the results of this study complement those of the original MR study, we use (the MR) monthly data. We use the MR data from March 1973 through March 1980 for estimation of variable and fixed coefficient versions of the structural models and then use these estimated models to generate the multi-step-ahead forecasts of the out-of-sample values of the exchange rates for the period April 1980 through June 1981.

5.1 Computable Formulas for the Structural Models

Let T denote the terminal period, March 1980, of the estimation period. The operational versions of (6) are

$$\text{(7a)} \quad \hat{s}_{T+i} = x'_{T+i}\hat{\bar{\beta}}$$

or

$$\text{(7b)} \quad \hat{s}_{T+i} = x'_{T+i}\hat{\bar{\beta}} + x'_{T+i}\hat{\Phi}^i\hat{\Omega}'_T\, D'_x\hat{\Sigma}_y^{-1}(y-X\hat{\bar{\beta}}), \qquad i = 1, 2, \ldots, 15$$

where one of the elements of x'_{T+i} is s_{T+i-1} if a structural model includes a lagged dependent variable, and a "^" above a symbol indicates that it is either a forecast of a variable or an estimate of an unknown constant. In any case x'_{T+i} represents an actual observation vector on all the right-hand side variables in a structural model. Since the stochastic coefficient models (2) cover the fixed coefficient models as special cases, formula (7) with appropriate zero restrictions on Φ and Δ_a applies to the fixed coefficient models. It should be noted that the estimators of $\bar{\beta}$, Φ and Δ_a based on the Swamy-Tinsley procedure involve only the observations for the estimation period and do not involve observations for the forecast period. For this reason, we call the vector $(\hat{s}_{T+1}, \ldots, \hat{s}_{T+15})$ of forecasts generated by (7) a vector of multi-step-ahead forecasts. We would have called this vector a one-step-ahead forecast had we reestimated the fixed parameters every period for predictions using all past data prior to each of the forecast periods T+1, T+2,...,T+15.

5.2 Computable Formulas for Random Walk

Define the following formulas for the random walk model,

(8a) $\hat{s}_{T+i} = s_{T+i-1} \qquad i = 1, 2, \ldots, 15$

(8b) $\hat{s}_{T+i} = s_T \qquad i = 1, 2, \ldots, 15$

(8c) $\hat{s}_{T+i} = i\hat{\mu} + s_T \quad i = 1, 2, \ldots, 15$

where $\hat{\mu}$ is the simple arithmetic mean of the changes in the values of s_t in the estimation period.

Equations (8a)-(8c) represent the formulas used to generate predictions from the random walk models. Formulas (8a) and (8b) apply to the random walk model without drift and formula (8c) applies to the model with drift. To distinguish these predictions we call the predictions defined in (8a) the one-step-ahead prediction of the random walk model and those defined in (8b) and (8c) multi-step-ahead predictions of the random walk model without and with drift, respectively. Note that only the random walk model with drift requires estimation of μ for generating predictions.

5.3 Forecasting Strategy

As is clear from the definitions (7) and (8c), by multi-step-ahead we mean that the fixed parameters are estimated only once from the observations for the estimation period and are then used to produce forecasts over the entire forecast period, i.e., sequential estimation is not used. The strategy of sequential estimation and producing one-step-ahead predictions was not employed in this study for several reasons. First, we wanted to extend the interesting comparisons of MR to multi-step-ahead predictions without duplicating their effort. Second, we are not convinced that sequential estimation is the appropriate technique to generate forecasts. We have argued in another paper (see Swamy and Schinasi (1986)) that for fixed coefficient models, sequential estimation does not necessarily improve forecasting performance, and that sequential estimation of fixed coefficient regressions is not the proper technique to use to capture parameter variation.[15] Furthermore, multi-step-ahead forecasts typify the econometric practice of private and public agencies who estimate large-scale econometric models only once in a period, in part because sequentially updating estimates tends to be prohibitively expensive. Multi-step-ahead forecasts more realistically represent the needs of the average user of exchange rate forecasting models. We therefore produced multi-step-ahead random walk forecasts for comparison with multi-step-ahead predictions of the stochastic coefficient models and view this comparison as more useful for evaluating forecast performance.

Following MR, we also use actual realizations of all explanatory variables in the forecast period to generate the out-of-sample forecasts of the exchange rate variables. We then compare multi-step-ahead predictions of the variable coefficient models to multi-step-ahead predictions of the fixed coefficient models and the random walk model with drift. We also compare all these multi-step-ahead forecasts to one-step-ahead forecasts of the random walk model.

We extended the MR computations for the fixed coefficient versions of the Dornbusch-Frankel and the Hooper-Morton models, which involve differences in expectations of inflation, by modelling the unobservable expectations variable as a distributed lag in inflation differentials. We followed the following procedure for constructing the proxies for the inflation expectations differential. First for each

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[15]Sequential estimation is not appropriate for stochastic coefficient models if the variance-covariance matrix of coefficients is fixed over time, as is assumed here.

distrituted lag we chose a few combinations of the values of its lag length permitted by the sample size, degree of differencing appearing in Shiller's smoothness prior on its lag coefficients, and a zero or a nonzero value for its coefficient appearing in a first-order autoregressive process for the error term. We applied the four different estimators of fixed coefficients described in Section 2 for each distributed lag model. Multi-step-ahead forecasts were generated and a root mean square forecast error (RMSE) was computed for each equation and each coefficient estimator. Next we sought whether the smallest of these RMSEs could be reduced further by changing either the lag length, the degree of differencing, or the zero or nonzero value of the autoregressive parameter. By this procedure we found the lag lengths and lag coefficients which yielded the "best" predictions and used these values to combine the current and lagged values of the inflation rate differential for each exchange rate. We used this linear combination in place of $(\pi_t^e - \pi_t^{e*})$ in equation (2), the stochastic coefficient versions of the structural exchange rate models.

The approach described above is not designed, per se, to examine how well a model is likely to perform in the future. It is instead designed to examine how well a model would have performed in a past period. Nor is it suggested here that the model that generates the most accurate forecasts also represents the "truth". The view taken in this paper is rather conservative: if a model cannot accurately predict certain observed events, then it is unlikely to be able to predict future events either. The most a probability forecast from a model can represent is a measure of the confidence with which one expects that model to predict an event in the future, based on currently available evidence, and not based on information yet to be observed.

The accuracy of multi-step-ahead forecasts can be judged by three statistics: mean forecast error (ME), mean absolute forecast error (MAE) and RMSE. They are defined as follows:

$$\text{(9a)} \qquad ME = \frac{1}{15} \sum_{i=1}^{15} (\hat{y}_{T+i} - y_{T+i}),$$

$$\text{(9b)} \qquad MAE = \frac{1}{15} \sum_{i=1}^{15} |\hat{y}_{T+i} - y_{T+i}|$$

$$\text{(9c)} \qquad RMSE = [\frac{1}{15} \sum_{i=1}^{15} (\hat{y}_{T+i} - y_{T+i})^2]^{1/2}$$

where $\hat{y}_{T+i}$ is an i-step-ahead forecast of y_{T+i} and T is the terminal period of the fitting period.

6. Empirical Results

The major results of this study are summarized in Tables 1 and 2. Note that for each exchange rate, the one-step-ahead forecast of the random walk model is far superior to multi-step-ahead forecasts of the random walk model with or without a drift parameter. The results not reported here show that multi-step-ahead forecasts of the Box-Jenkins (1970) type time series models, ARIMA (1,1,0), ARIMA (0,1,1,) and ARIMA (1,1,1,)[16] are

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[16]The symbol ARIMA(p,d,q) denotes an autoregressive integrated moving average model, where d is the degree of differencing to achieve stationarity, p is the order of the autoregressive part, and q is the order of the moving average part.

inferior to multi-step-ahead forecasts of the random walk model with or without drift.

6.1 How to Compare Predictions of Structural Models and Random Walk Models

Table 1 compares the forecasting performance of the random walk model to fixed and stochastic coefficient structural models without a lagged dependent variable, while Table 2 compares the forecasting performance of the random walk model to the structural models with a lagged dependent variable included. Because each of these two sets of structural models--with and without a lagged dependent variable--use different sets of information in generating multi-step-ahead predictions, one has to be careful in comparing their predictions with those of the random walk.

In Table 1, multi-step-ahead random walk forecasts are probably the proper benchmark to use in judging the relative forecast accuracy of the structural models without a lagged dependent variable. Comparing multi-step-ahead predictions of such structural models with one-step-ahead predictions of the random walk model gives the random walk model the unfair advantage of using possibly as yet "unobserved" spot rates to predict future values of the spot rate (for example, using s_{T+i} to predict s_{T+i+1} for $i \geq 1$). The multi-step-ahead prediction of the random walk model is on more equal footing with the structural model's multi-step-ahead predictions, when the structural models do not include a lagged dependent variable.

In Table 2, multi-step-ahead predictions of the structural models, including a lagged dependent variable, are compared with one-step-ahead predictions of the random walk model. Although equation (7) represents a multi-step-ahead forecast, in the sense that it does not use data beyond period T to estimate fixed parameters used for prediction, its vector x'_{T+i} does contain the observation on the lagged dependent variable for the time periods beyond T up to T+15.

We now turn to the tables and report the major findings of this study.

6.2 <u>The Forecasting Performance of Conventional Models without Lagged Dependent Variables</u>

With one exception (the Hooper-Morton model of the dollar-deutsche-mark rate), when coefficients are allowed to change period by period, multi-step-ahead forecasts of all three structural models of all three currencies consistently outperform multi-step-ahead forecasts of the random walk model (see Table 1). The same cannot be said for the fixed coefficient versions of these conventional models. All three fixed coefficient models of the dollar-pound rate fail to outperform the multi-step-ahead forecast of the random walk model, while some fixed coefficient estimates of the models of the dollar-yen and dollar-deutsche-mark rates can outperform the multi-step-ahead forecast of the random walk model. With one exception (the Hooper-Morton model for the dollar-yen rate), multi-step-ahead forecasts of the time-varying parameter models are more accurate than those of their fixed coefficient counterparts.

Models of the dollar-deutsche-mark rate present the most contrast in forecast accuracy. The Dornbusch-Frankel model, which includes a proxy

Table 1

Root mean square forecast errors[1,2]

	$/£	$/¥	$/DM
Random walk:			
1. multi-step-ahead: with drift	7.34	17.48	10.35
2. multi-step-ahead: without drift	7.24	17.51	10.30
Frenkel-Bilson[3]:			
3. Least squares	20.93	17.36	15.93
4. Serial correlation correction[4]	8.90	17.41	10.29
5. Stochastic coefficient	5.85	11.52	9.31
Dornbusch-Frankel:			
6. Least squares	18.74	17.81	4.81
7. Serial correlation correction[4]	10.53	17.60	14.52
8. Posterior mean	7.80	16.78	4.51
9. Almon	7.93	16.29	4.64
10. Ridge regression	10.68	16.77	4.55
11. Stochastic coefficient	5.75	9.10	3.00
Hooper-Morton:			
12. Least squares	17.70	12.95	20.56
13. Serial correlation correction[4]	10.06	6.71	19.09
14. Posterior mean	9.55	12.62	18.65
15. Almon	9.38	12.01	18.09
16. Ridge regression	10.12	6.48	18.72
17. Stochastic coefficient	7.18	7.08	13.49
Addendum:			
18. One-step-ahead random walk	3.03	3.96	3.69

[1] Approximately in percentage terms.
[2] Forecasts for all structural models were multi-step-ahead forecasts for 15 periods beyond the last sample point which was March 1980.
[3] The estimators labelled Posterior Mean, Almon, and Ridge Regression are designed to estimate distributed lag models subject to smoothness restrictions and therefore do not apply to the Frankel-Bilson model.
[4] First-order serial correlation correction.

for the differential in inflation expectations, has the lowest RMSEs. The RMSE of the stochastic coefficient version of the Dornbusch-Frankel model is low enough to outrank the one-step-ahead forecast of the random walk model, despite the extreme informational advantage (of using the actual previous value to predict the next observation of the spot rate) of the one-step-ahead random walk model forecast presented in Table 2. In contrast, the Hooper-Morton model, which further adds the differential between cumulated trade balances of the United States and Germany as an additional explanatory variable, fails to outperform the multi-step-ahead forecast of the random walk model or any other model. These results indicate the importance of inflation expectations differentials (and therefore real interest rate differentials) and the unimportance of trade balances between Germany and the United States. Similar conclusions, though less extreme, can be drawn from the results for the dollar-pound rate, although the stochastic coefficient estimator does not outperform the one-step-ahead forecast of the random walk model.

Models of the dollar-yen rate are ranked quite differently. Only in the case of the dollar-yen rate does the inclusion of the differential between cumulated trade balances substantially lower RMSEs and only for the dollar-yen rate is the Hooper-Morton model superior to multi-step-ahead forecasts of the other models, probably owing to the relatively significant trade flows between Japan and the United States. For the dollar-yen rate, two of the fixed coefficient estimators for the Hooper-Morton model outperform its stochastic coefficient version.

Note that when coefficients are fixed, the Dornbusch-Frankel model outperforms the Frankel-Bilson model (when the posterior mean estimator is used in our sample). Forecasts generated by the posterior mean can be improved by using the Almon or the ridge regression estimator in some cases.

To summarize this section, multi-step-ahead forecasts of the stochastic coefficient versions of the structural models examined in this study are generally more accurate than multi-step-ahead forecasts of fixed coefficient models and multi-step-ahead forecasts of the random walk model. The Dornbusch-Frankel model is the most accurate multi-step-ahead forecasting model of the dollar-pound and the dollar-deutsche-mark rates, while the Hooper-Morton model is the best multi-step-ahead predictor of the dollar-yen rate.

6.3 Models with a Lagged Dependent Variable

Turning to Table 2, and comparing its values with those reported in Table 1, a striking observation one can make is that adding a lagged dependent variable makes a substantial difference in the forecasting ability of all three structural models. Both fixed and stochastic coefficient models for all three currencies produce multi-step-ahead forecasts that are more accurate than the multi-step-ahead forecast of the random walk model. Also note that stochastic coefficient representations of the structural models uniformly outperform the fixed coefficient versions.

Further inspection reveals that with one exception (the Hooper-Morton model for the dollar-pound rate), multi-step-ahead forecasts

Table 2

Models with lagged dependent variable:
Root mean square forecast errors[1,2]

	$/£	$/¥	$/DM
Random-walk:			
1. One-step-ahead Random Walk	3.03	3.96	3.69
Dornbusch-Frankel:			
2. Least squares	3.66	5.29	2.69
3. Serial correlation correction[3]	3.63	5.10	2.87
4. Posterior mean	3.61	4.98	2.79
5. Almon	3.54	4.03	2.56
6. Ridge regression	3.63	4.96	2.76
7. Stochastic coefficient	2.17	3.50	2.17
Hooper-Morton:			
8. Least squares	4.49	5.79	6.58
9. Serial correlation correction[3]	4.49	5.66	7.91
10. Posterior mean	4.45	5.53	6.60
11. Almon	4.30	5.03	6.00
12. Ridge regression	4.46	5.51	6.59
13. Stochastic coefficient	3.85	3.27	3.17

[1] Approximately in percentage terms.
[2] All structural models were estimated with a lagged dependent variable as an explanatory variable. Forecasts for all structural models were multi-step-ahead forecasts for 15 periods beyond the last sample point which was March 1980.
[3] First-order serial correlation correction.

of the stochastic coefficient versions of all the structural models are more accurate than the one-step-ahead forecast of the random walk model. Except for the fact that the structural models are used to produce multi-step-ahead forecasts and the random walk model is used to produce one-step-ahead forecasts, comparison of their respective RMSEs is a "pure" measure of the contribution of economic variables (theory) in improving forecast accuracy.

Because one cannot derive the exact finite sample distributions of the RMSE statistics (for even much simpler models) one cannot make probability statements about how significant these differences are. But we do observe that including economic variables (i.e., using economic theory) along with a lagged dependent variable generally improves upon the one-step-ahead forecast of the random walk model when coefficients are allowed to vary, and generally does not improve upon the one-step-ahead forecast of the random walk model when coefficients are constrained to be fixed. Note however that the fixed coefficient estimators of the Dornbusch-Frankel model (with a lagged dependent variable) for the dollar-deutsche-mark rate gave more accurate forecasts of the (log) spot rate than the random walk model, regardless of whether multi-step- or one-step-ahead forecasts are considered.

The relative ranking of the various structural models of the dollar-pound and the dollar-deutsche-mark rates match those of Table 1, while the ranking of models of the dollar-yen rate is reversed for the fixed coefficient case. Hence when one includes a lagged dependent variable, the Dornbusch-Frankel model is generally superior for all three currencies tested in this study. The major improvements in the models with stochastically varying coefficients and a lagged value of the dependent variable indicate that departures from purchasing power parity and instability in the money demand functions, which underpin the structural models of the exchange rate, may explain a significant part of the instability in the exchange rates that apparently cannot be adequately captured by the fixed coefficient models.

6.4 Further Results

The mean absolute forecast error statistics, which are generally smaller than RMSE, are not listed here since they yielded the same rankings as the RMSEs. The mean forecast errors, which are also not listed here, do not suggest systematic over- or underprediction for the models yielding the "best" predictions.

It is worth emphasizing that for each stochastic coefficient specification one of 25 different estimates of $\bar{\beta}$, Φ and Δ_a, the one with the lowest RMSE, is selected by screening 25 iterations of the Swamy and Tinsley procedure. This is necessary because this iterative procedure does not converge unless sufficient a priori restrictions are imposed on Φ. For example, when all the elements of Φ except the leading diagonal element are restricted to be zero, the iteration converged in several cases. But imposing constraints of this nature did not improve forecast accuracy. The forecasts generated by the initial estimates were better than those generated by the final iteration. One of the 25 different sets of estimates of nondiagonal Φ and nondiagonal Δ_a yielded the lowest RMSE in general. In a few cases the diagonality restriction on Φ improved forecast accuracy.

The reason for following an iterative procedure which converges is to find maximum likelihood or least squares estimates of nonlinear models. If such estimates do not exist, however, then no iterative procedure converges to those estimates. In cases where Φ and Δ_a are both nondiagonal, the conditions for the existence of maximum likelihood estimates are not satisfied. In these cases our procedure of choosing the best predicting estimates of $\bar{\beta}$, Φ and Δ_a from among 25 different estimates is analogous to that of comparing inferences given by a class of prior distributions in a Bayesian analysis.

These computations show that arbitrary choices of the values of $\bar{\beta}$, Φ and Δ_a do not guarantee improvements in forecast accuracy. Perhaps this explains why Alexander and Thomas (1986) could not reverse the MR conclusions using the Kalman filtering technique with the prior values $\bar{\beta} = (\bar{\beta}_0,0,..,0)'$ and $\Phi = I$. Wolff (1985) also evaluates the Kalman filter using the prior values $\bar{\beta} = (\bar{\beta}_0,0,..,0)'$ and $\Phi = I$ and finds that the forecasts generated by this filter are poor relative to the forecasts of the random walk model for the dollar-yen and dollar-pound rate. Our computations show that the use of these a priori values reduces the accuracy of forecasts.

7. Conclusion

The main result of this paper is that once one is willing to relax the conventional assumption of fixed regression slopes, it is possible to estimate structural models of exchange rate determination which perform substantially better than the random walk model in predicting out-of-sample values of exchange rates. As was shown decisively by Meese and Rogoff, and others, fixed coefficient models of exchange rates -- with and without a lagged dependent variable, and with a variety of proxies for the differential of inflation expectations -- could not outperform the random walk model. Our findings are traceable, in varying degrees, to three sources, namely, relaxation of the assumption that coefficients are fixed, inclusion of a lagged dependent variable as an explanatory variable, and our detailed construction of a proxy on inflation expectations differentials. But the results reported here indicate that relaxing the fixed coefficient assumption is the crucial source of the superior forecasting ability of the exchange rate models tested.

We would have preferred reporting results that explain the variability of coefficients with sound and rigorous economic principles. But until economic theory postulates empirically implementable hypotheses addressing why exchange markets are so volatile and why model coefficients vary over time, one can at least examine the type of stochastic coefficient models presented here before rejecting existing exchange rate models out of hand. This study demonstrates that stochastic coefficient models of exchange rate determination can be useful in improving the accuracy of forecasts of exchange rates.

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