

International Finance Discussion Papers

Number 182

July 1981

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by

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## The Impact of an Oil Price Increase on Aggregate Supply

by Karen H. Johnson\*

The impact of an exogenous increase in the price of oil on aggregate supply is felt not only through its effect on the demand for oil but also through its effects on the demand for other factors. Except under special assumptions concerning the production technology, the price of one factor is an argument in the demand for all other factors. This paper will explore the impact of an increase in the price of oil, an imported, exogenously-priced input, on the demand for labor and on labor market equilibrium under varying assumptions about the behavior of wages. The response of factor demands to an oil price "shock" will then be used to infer the effect of the shock on the aggregate supply schedule. In order to obtain unambiguous solutions to the analysis, specific assumptions are made concerning production conditions. The paper does not attempt to explain fully the impact of such an oil price shock. Rather, it highlights the possibility of a discrete change in labor demand (and therefore aggregate supply) behavior in response to such a shock when production involves fixed-coefficient relationships. The results reported here suggest that pre-oil price shock experience may provide little or no useful information about the elasticity of labor demand with respect

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\* Economist, Board of Governors of the Federal Reserve System. This paper represents the views of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or other members of its staff.

to the real wage or of output with respect to the price level in the post-oil shock world.

In the model there is only one domestically produced good and three factors of production, capital services, labor, and energy. All energy is imported at a price that is exogenously determined. The model is entirely static. The comparative static properties reported here are offered as indications of what the impact effects would be of an oil price shock in a dynamic setting.

Assume that production is such that capital services and energy are strict complements and can be used only in fixed proportions. Together, as "running" machines, they are jointly substitutes for labor. I personally find this idea more intuitively appealing than the assumption that has been frequently made in the literature that domestic capital and labor are linked to form domestic value added and are then jointly substitutes for energy.<sup>1/</sup> The empirical debate over the complement/substitute relationships amongst these three factors I regard as still unresolved.<sup>2/</sup> Indeed, production functions actually originate at the level of firms and individual goods and presumably differ across goods. Thus aggregate functions for different countries represent aggregations across different groups of industries in most cases and should be expected to differ in the complement/substitute relationships they imply for factors. The implication of an oil price shock on aggregate supply should thus be explored under various

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<sup>1/</sup> An example of an analysis of the oil-price shock based on a model where capital and labor are linked and then jointly substitutes with energy is the paper by Bruno and Sachs. In contrast Mork and Hall, although they do not assume our exact specification, do assume a zero elasticity of substitution between capital and energy.

<sup>2/</sup> Berndt and Wood provide an over-view of this debate and an empirical case for capital-energy complementarity. Griffin and Gregory provide evidence for short-run complementarity but long-run substitutability of capital and energy.

assumptions. What follows in this paper is an examination of one such case.

Let:

$Q$  = units of the domestically produced good

$L$  = flow of labor input

$K$  = flow of capital services that is a constant proportion of the capital stock actively in use

$E$  = flow of energy input

$P_E^*$  = price of energy in foreign currency

$e$  = exchange rate (domestic currency price of foreign currency)

$P_E = eP_E^*$  = price of energy in domestic currency

$P$  = price of a unit of the domestically produced good

$w$  = nominal wage rate

Given the assumptions above, production can be described by:

$$(1) \quad Q = F(L, \min\{K, E\})$$

where the units of  $E$  have been suitably chosen (i.e. one machine - hour's worth of energy). For notational simplicity let

$$(2) \quad Z = \min\{K, E\}$$

Then

$$(3) \quad Q = F(L, Z). \quad F_L > 0 \quad F_Z > 0 \quad F_{LZ} > 0$$

In the short-run the capital stock is given, so capital services can be no greater than some fixed maximum,  $\bar{K}$ . By assumption, the only variable cost of operating capital is the cost of the energy that must be used per unit of capital.

Let us consider the problem facing the representative firm. Such a firm is assumed to be a price taker in all markets. The cost to that firm of one more unit of  $Z$  in (3) is  $P_E$ . Since  $K \leq \bar{K}$ , it follows from (2) that  $Z \leq \bar{Z}$ , where  $\bar{Z} = \bar{K}$ . The firm thus faces the following

constrained maximization problem:

$$\begin{aligned} \text{Max}_{L,Z} \quad & PF(L,Z) - wL - P_E Z \\ \text{s.t.} \quad & Z \leq \bar{Z} \end{aligned}$$

Let the objective function be

$$(4) \quad A = PF(L,Z) - wL - P_E Z + \lambda(\bar{Z} - Z - S)$$

where S is the slack variable. The Kuhn-Tucker conditions for a Maximum are:

$$(5) \quad \begin{aligned} PF_Z - P_E - \lambda &\leq 0 & Z(PF_Z - P_E - \lambda) &= 0 \\ PF_L - w &\leq 0 & L(PF_L - w) &= 0 \\ \bar{Z} - Z - S &= 0 & S \lambda &= 0 \\ Z \geq 0 \quad L \geq 0 \quad S \geq 0 \quad \lambda \geq 0 \end{aligned}$$

Assuming that P, P<sub>E</sub>, and w are such that it pays for the firm to operate and to use positive amounts of both L and Z, we can reduce the conditions in (5) to:

$$(6) \quad \begin{aligned} (i) \quad F_Z(L,Z) &= \frac{P_E}{P} + \frac{\lambda}{P} \\ (ii) \quad F_L(L,Z) &= \frac{w}{P} \\ (iii) \quad S\lambda &= 0 \quad S \geq 0 \quad \lambda \geq 0 \\ (iv) \quad \bar{Z} &= Z + S \end{aligned}$$

In principle (i)-(iv) can always be solved for L, Z, λ, (and S) as functions of P, P<sub>E</sub>, and w. Let us take P and w as given and see how the solution in (6) depends on P<sub>E</sub>. For P<sub>E</sub> = 0 there is no variable cost to operating the capital stock, and so firms would choose to operate such that K =  $\bar{K}$  and so Z =  $\bar{Z}$ . It follows from (iv) that S = 0 and so from (iii) that λ is not necessarily zero. Since Z =  $\bar{Z}$ , (i) and (ii) become

$$(7) \quad \begin{aligned} (i)' \quad F_Z(L, \bar{Z}) &= \frac{P_E}{P} + \frac{\lambda}{P} \\ (ii)' \quad F_L(L, \bar{Z}) &= \frac{w}{P} \end{aligned}$$

(ii)' can be solved directly for  $L$  as a function of  $w/P$ . Thus (ii)' is itself a complete implicit statement of the demand for labor. Once  $L$  has been determined from the familiar marginal product condition (ii)', (i)' can be solved for  $\lambda$ , the shadow price of a unit of capital, given  $\bar{K}$ .

We wish to determine what happens to factor use and output as  $P_E$  rises, while  $w$  and  $P$  remain constant. For very small values of  $P_E$ ,  $Z$  will remain at  $\bar{Z}$  and since, under those conditions,  $L$  depends only on  $w/P$ , it too will remain unchanged. Let  $\tilde{L}$  be defined by:

$$(8) \quad F_L(\tilde{L}, \bar{Z}) = \frac{w}{P}$$

We can then solve (i)' for  $\lambda$

$$(9) \quad \lambda = PF_Z(\tilde{L}, \bar{Z}) - P_E$$

As  $P_E$  rises above zero,  $\lambda$  will clearly fall. At  $P_E = PF_Z(\tilde{L}, \bar{Z})$ ,  $\lambda$  becomes zero. For any higher  $P_E$ ,  $S$  no longer equals zero (by (iii)), and  $Z$  no longer necessarily equals  $\bar{Z}$  (by (iv)). For some high enough value of  $P_E$ , conditions (6) become:

$$(i)'' \quad F_Z(L, Z) = \frac{P_E}{P}$$

$$(ii)'' \quad F_L(L, Z) = \frac{w}{P}$$

(10)

$$(iii)'' \quad \lambda = 0 \quad S > 0$$

$$(iv)'' \quad \bar{Z} = Z + S$$

(i)'' and (ii)'' can now be jointly solved for  $L$  and  $Z$ . Note that (ii)'' is no longer a complete statement of the demand for labor. Once  $Z$  has been obtained, (iv)'' can be solved for  $S$ .

The above conditions simply state that for some sufficiently low  $P_E$ , the firm will operate at the capacity determined by its capital stock. Labor demand will depend only on the real wage. As  $P_E$  rises, however,

the firm will reach a point where it no longer chooses to use all the capital it owns; the operating costs will have risen enough to push the firm from a corner solution for  $Z$  to an interior solution. Since  $F_{LZ} > 0$  by assumption, when  $P_E$  is high and  $Z < \bar{Z}$ , (ii)" requires that  $L < \tilde{L}$  (for the given  $w/P$ ). Thus, at a sufficiently high  $P_E$ , the firm uses less than its full capacity capital and also demands less labor.

Recall the definition of  $\tilde{L}$  from (8) above.  $\tilde{L}$  is the amount of labor the firm would choose if it were operating all of its capital and the real wage were some  $w/P$ .  $\tilde{L}$  clearly depends on  $w/P$  such that:

$$(11) \quad \tilde{L} = \tilde{L}\left(\frac{w}{P}\right) \quad \frac{\partial \tilde{L}}{\partial w} < 0 \quad \frac{\partial \tilde{L}}{\partial P} > 0$$

We can thus rewrite (9) as

$$(12) \quad \lambda = PF_Z(\tilde{L}\left(\frac{w}{P}\right), \bar{Z}) - P_E$$

$$= \lambda(P_E; \tilde{L}, P) \quad \frac{\partial \lambda}{\partial P_E} < 0$$

$$= \lambda(P_E; w, P)$$

Let us define  $\hat{P}_E$  as that value of  $P_E$  where  $\lambda = 0$ . For a given  $w$  and  $P$ ,  $\hat{P}_E$  is the critical value of the price of energy where the constraint is just binding and the demand for labor goes from being independent of  $P_E$  to being dependent on it.

$$(13) \quad \lambda(\hat{P}_E; w, P) = 0$$

(13) can be solved for  $\hat{P}_E$  as a function of  $(w, P)$ :

$$(14) \quad \hat{P}_E = \hat{P}_E(w, P)$$

We need to know the signs of  $\frac{\partial \hat{P}_E}{\partial w}$  and  $\frac{\partial \hat{P}_E}{\partial P}$ . From (12) it is clear that  $w$  affects  $\lambda$  and thus  $\hat{P}_E$  only through  $\tilde{L}$ . We know from (11) that a rise in  $w$  lowers  $\tilde{L}$ . Since  $F_{ZL} > 0$ , a rise in  $w$  lowers  $F_Z(\tilde{L}, \bar{Z})$ . For  $\lambda$  to remain at zero,  $\hat{P}_E$  must fall, i.e.

$$(15) \quad \frac{\partial P_E}{\partial w} < 0$$

A rise in  $P$  raises  $L$  from (11) and so unambiguously raises  $F_Z(\tilde{L}, \bar{Z})$  and  $PF_Z(\tilde{L}, \bar{Z})$ .

Therefore:

$$(16) \quad \frac{\partial P_E}{\partial P} > 0$$

It is now possible to state all the conditions covering how the demand for labor depends on  $P_E, w, P$ :

By definition:

$$(17) \quad P_E = P_E(w, P) = PF_Z\left(\tilde{L}\left(\frac{w}{P}\right), Z\right) \quad \frac{\partial P_E}{\partial w} < 0 \quad \tilde{L} \text{ defined by (8)}$$

$$\frac{\partial P_E}{\partial P} > 0$$

If  $P_E < P_E(w, P)$ , the demand for labor is given by:

$$(18) \quad F_L(L, \bar{Z}) = \frac{w}{P}$$

If  $P_E \geq P_E(w, P)$  then demand for labor and the demand for "running"

capital are given jointly by:

$$(19) \quad F_L(L, Z) = \frac{w}{P}$$

$$F_Z(L, Z) = \frac{P_E}{P}$$

Thus as the price of oil changes relative to the wage rate and final good's price, the demand for labor function will undergo a discrete change and qualitatively different behavior will govern the decision of how much labor to employ. Conditions (17), (18) and (19) completely describe the decisions of the representative, price-taking firm; they imply:

- I. For values of  $P_E, w, P$  such that  $P_E < P_E(w, P)$ , the quantity of labor demanded by firms is independent of  $P_E$ . For values such that  $P_E \geq P_E(w, P)$  this is not so.



II. For values of  $P_E, w, P$  such that  $P_E < P_E^\wedge(w, P)$  the demand for labor is homogenous of degree zero in  $w, P$ . For value such that  $P_E \geq P_E^\wedge(w, P)$  this is not so.

III. For value of  $P_E, w, P$  such that  $P_E \geq P_E^\wedge(w, P)$ ,  $\frac{\partial D^L}{\partial P_E} < 0$ .

IV. While  $L$  in (11) is homogenous of degree zero in  $w$  and  $P$ ,  $\lambda$  in (12) is not.  $\lambda$  is homogenous of degree one in  $P_E, w, P$ .

V. From IV it follows that  $P_E^\wedge$  is homogenous of degree one in  $w, P$ .

The above discussion describes the demand for labor decision of the representative firm. I would now like to derive the aggregate supply conditions for the economy as a whole under the following assumptions:

1. All firms are small, identical, price-taking, independently competitive firms. Their aggregated demand for labor behavior is identical to that of the representative firm.<sup>3/</sup>
2. The price of energy is fixed at  $P_E^*$  by the foreign source. The exchange rate is given exogenously as  $e$  so the domestic price of energy is fixed at  $P_E = eP_E^*$ .

Let us first derive the aggregate supply relationship between  $Q$  and  $P$  when  $w$  is fixed at  $\bar{w}$ . The amount of employment will, by assumption, be equal to the amount of labor demanded. With  $P_E$  given to the domestic economy at the level  $P_E$ , we have:

$$(20) \quad P_E^\wedge = P_E^\wedge(\bar{w}, P) \quad \frac{\partial P_E^\wedge}{\partial P} > 0$$

For some high enough values of  $P$ ,  $P_E^\wedge$  will be large enough that  $\bar{P}_E < P_E^\wedge$ , that is the actually prevailing price of energy,  $\bar{P}_E$ , will be sufficiently low, given  $\bar{w}$  and  $P$ , that the entire capital stock will be utilized. Over that range of values of  $P$ , the demand for labor schedule will be given implicitly by:

<sup>3/</sup> The variables  $Q$ ,  $K$ ,  $L$  and  $Z$  now refer to aggregates for all firms taken together. I suppress multiplying everything by the number of firms for notational convenience.

$$(21) \quad F_L(L, \bar{Z}) = \frac{\bar{w}}{P}$$

As  $P$  rises, the amount of labor demanded and thus employment and output will rise as well. Let  $P'$  be defined as:

$$(22) \quad \bar{P}_E = \hat{P}_E(\bar{w}, P')$$

For  $P > P'$ , the aggregate supply will be positively sloped as pictured in Figure 1.

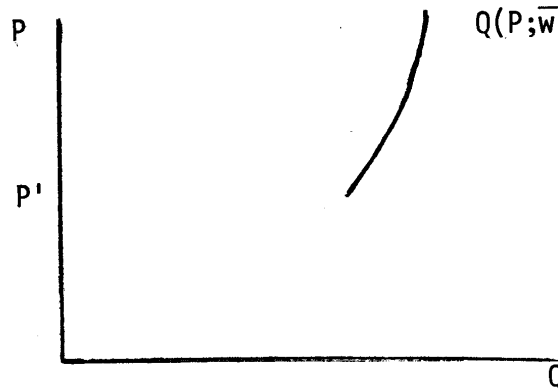


Figure 1

The slope of this curve can be calculated as follows:

$$\begin{aligned}
 Q &= Q(L, Z) = Q(L(P; \bar{w}, \bar{P}_E), \bar{Z}) \quad \text{for } P > P' \\
 Q &= Q(P; \bar{w}, \bar{P}_E, \bar{Z}) \\
 \frac{\partial Q}{\partial P} &= \frac{\partial Q}{\partial L} \cdot \frac{dL}{dP} \\
 (23) \quad F_L(L, Z) &= \frac{\bar{w}}{P} \\
 F_{LL} dL &= -\frac{\bar{w}}{P^2} dP \\
 \frac{dL}{dP} &= \frac{1}{F_{LL}} \frac{-\bar{w}}{P^2} \\
 \frac{\partial Q}{\partial P} &= \frac{\partial Q}{\partial L} \left[ \frac{1}{F_{LL}} \frac{-\bar{w}}{P^2} \right] > 0
 \end{aligned}$$

Over this range of the aggregate supply curve, the capital stock is being fully used. For  $P \leq P'$  it follows that  $\bar{P}_E \geq \hat{P}_E(\bar{w}, P)$ . This means that the given price of energy  $\bar{P}_E$  is sufficiently high relative to  $P$  (and  $\bar{w}$ ) that the capital stock will not be completely in use. From (19) above it follows that the demands for labor and for capital-cum-energy (i.e.  $Z$ )

will be given by:

$$(24) \quad \begin{aligned} F_L(L,Z) &= \frac{\bar{w}}{P} \\ F_Z(L,Z) &= \frac{\bar{P}_E}{P} \end{aligned}$$

The effect of a change in P on the demand for labor than is given by:

$$(25) \quad \begin{aligned} F_{LL} dL + F_{LZ} dZ &= -\frac{\bar{w}}{P^2} dP \\ F_{ZL} dL + F_{ZZ} dZ &= -\frac{\bar{P}_E}{P^2} dP \end{aligned}$$

or

$$(26) \quad \begin{pmatrix} F_{LL} & F_{LZ} \\ F_{ZL} & F_{ZZ} \end{pmatrix} \begin{pmatrix} dL \\ dZ \end{pmatrix} = \begin{pmatrix} -\frac{\bar{w}}{P^2} \\ -\frac{\bar{P}_E}{P^2} \end{pmatrix} dP$$

Thus:

$$(27) \quad dL = \frac{1}{\Delta} \left[ \frac{-F_{ZZ}\bar{w} + F_{LZ}\bar{P}_E}{P^2} \right] dP > 0$$

$$(28) \quad dZ = \frac{1}{\Delta} \left[ \frac{F_{ZL}\bar{w} - F_{LL}\bar{P}_E}{P^2} \right] dP > 0$$

where  $\Delta = (F_{LL}F_{ZZ} - F_{LZ}F_{ZL})$  is assumed to be positive.

It follows then that for  $P \leq P'$ :

$$(29) \quad \frac{dQ}{dP} = \frac{\partial Q}{\partial L} \frac{dL}{dP} + \frac{\partial Q}{\partial Z} \frac{dZ}{dP} > 0$$

where  $\frac{dL}{dP}$  and  $\frac{dZ}{dP}$  are given in (27) and (28) respectively.

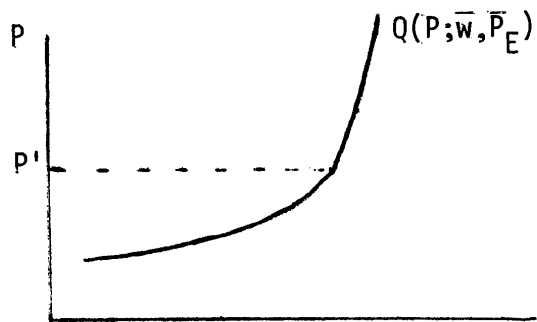


Figure 2

The supply curve is continuous but non-differentiable at  $P'$ . At  $P = P'$  (given  $\bar{w}$  and  $\bar{P}_E$ ), firms would choose to use exactly  $\bar{Z}$  of capital-cum-energy. For  $P > P'$ ,  $\partial Q/\partial P$  reflects the extra output that firms produce as they add laborers to the given  $\bar{Z}$ . For  $P < P'$ , firms are adjusting both labor and capital-cum-energy as  $P$  changes. Moreover, since  $F_{LZ} > 0$  by assumption, as  $P$  falls and firms reduce employment (and capital-cum-energy) because of the direct effect of the change in  $P$ , they also further reduce each factor because of the interaction of less capital-cum-energy reducing the marginal product of labor and vice versa. Thus at  $P'$  the slopes of the two branches of the aggregate supply curve must be as shown in Figure 2.

We need to determine the impact on the schedule  $Q(P; \bar{w}, \bar{P}_E)$  of a shift in each of its parameters  $\bar{w}, \bar{P}_E$ . A higher value for  $P_E$ , say  $\bar{\bar{P}}_E > \bar{P}_E$ , will cause the kink in the curve to be located at a higher  $P$ .

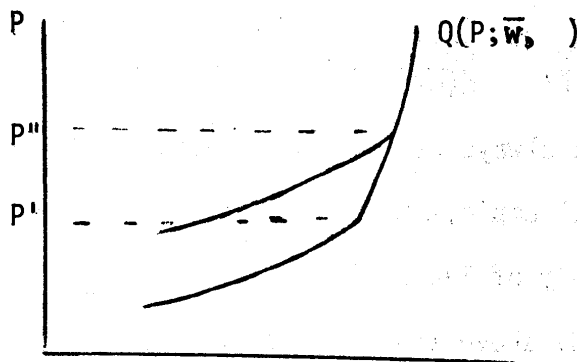


Figure 3

For  $P > P''$ , the value of  $P_E$  is always irrelevant for the levels of  $L$  and  $Z$  used to produce output; and so the schedule is unchanged when  $P_E$  rises from  $\bar{P}_E$  to  $\bar{\bar{P}}_E$ . Once  $P_E = \bar{\bar{P}}_E$ , however, all values of  $P \leq P''$ , where

$$(30) \quad \bar{\bar{P}}_E = \hat{P}_E(\bar{w}, P'')$$

will now cause both the demand for labor and capital-cum-energy to adjust as in (27) and (28).

A shift up in  $\bar{w}$ , from  $\bar{w}$  to  $\bar{\bar{w}}$  will shift the entire curve as well as move up the price at which the curve kinks. The kink will move up

because for any  $P$ , a higher  $w$  will lower  $\hat{P}_E$  (see (15) above). Thus for some range of values of  $P$ ,  $\hat{P}_E$  will be below  $\bar{P}_E$  (instead of above it) when  $w$  rises to  $\bar{w}$ . The curve will shift leftward because for all values of  $P$ , the effect of a higher  $w$  is to reduce the demand for labor and thus output.  $dL/dw < 0$  follows from both (18) and (19) above.

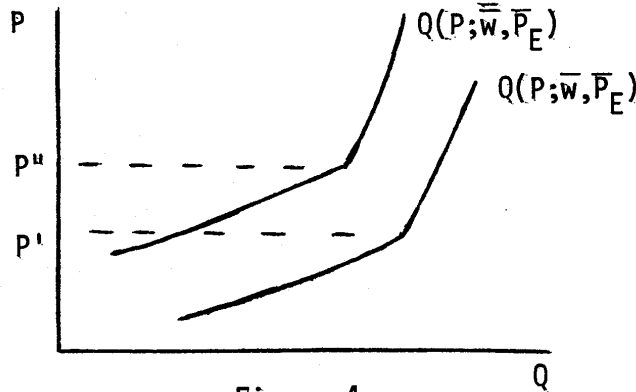


Figure 4

Let us now consider the case where the nominal wage always adjusts instantly to clear the labor market. Let the supply of labor function be given by

$$(31) \quad S^L = L\left(\frac{w}{p}\right) \quad \frac{dS^L}{d(w/p)} > 0$$

The economy is always at full employment in the sense that  $w$  adjusts to keep actual employment at the point where the demand for labor equals the supply of labor.

Recall from (8) above that  $\tilde{L}$  is defined by

$$F_L(\tilde{L}, Z) = \frac{w}{p}$$

$\tilde{L}$  is thus the amount of labor firms would choose to hire when  $Z = \bar{Z}$  as a function of the real wage. Once the level of  $Z$  is given, the demand for and supply of labor relationships can be shown as:

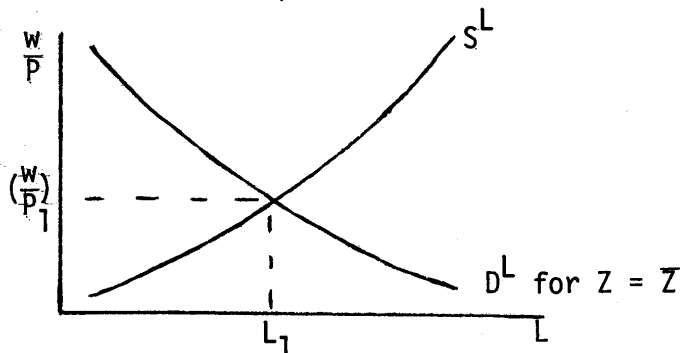


Figure 5

The position of the  $D^L$  schedule depends on the value of  $\bar{Z}$ . Under the assumption that  $w$  adjusts instantly, employment will remain at  $L_1$  and the real wage at  $(\frac{w}{P})_1$ , as long as  $Z$  is at  $\bar{Z}$ . Thus  $\tilde{L}$  now depends only on  $Z$  (and the position of the  $S^L$  schedule) and not effectively on  $w/P$  since it will always adjust. This means that  $w$  in this case does not have an independent effect on  $\hat{P}_E$ . Now  $\hat{P}_E$  is given by

$$(32) \quad \hat{P}_E = \hat{P}_E(P; Z) = P \left[ F_Z(\tilde{L}(Z), Z) \right]$$

The quantity in square brackets is independent of  $P$ , so  $\hat{P}_E$  now has a simple proportional relationship to  $P$ .

For a given  $\bar{P}_E$  set by foreign suppliers, high values of  $P$  will produce the condition that  $\bar{P}_E < \hat{P}_E$ . In this case  $Z$  will be at  $\bar{Z}$  and employment will be determined by:

$$(33) \quad F_L(L, Z) = S^{-1}(L)$$

where  $S^{-1}(L)$  is the inverse of the supply of labor function. The amount of employment so determined is independent of  $P$ , and so the aggregate supply curve for high values of  $P$  will be vertical:

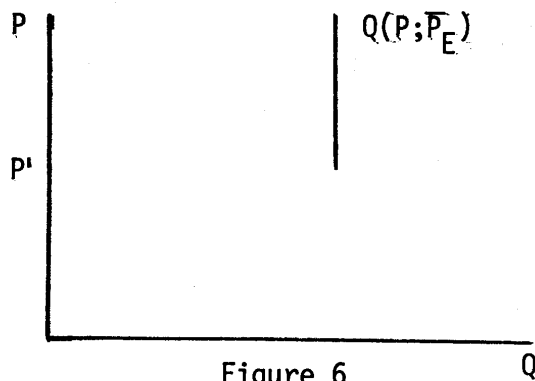


Figure 6

In contrast to the sticky wage case, for  $P > P'$  there will be no nominal price elasticity to the aggregate supply schedule. This is to be expected since all relevant prices are flexible. However, for  $P \leq P'$ , the sticky energy price replaces the sticky wage rate as a reason for positive price elasticity of aggregate supply. For  $P \leq P'$ ,

it is the case that  $\bar{P}_E \geq \hat{P}_E$ . The demand and supply conditions for labor and for capital-cum-energy are then given by:

$$(34) \quad \begin{aligned} F_L(L, Z) &= S^{-1}(L) \\ F_Z(L, Z) &= \frac{\bar{P}_E}{P} \end{aligned}$$

In order to determine  $\frac{dL}{dP}$  and  $\frac{dZ}{dP}$ , we first obtain:

$$(35) \quad \begin{aligned} (F_{LL} - S_L^{-1})dL + F_{LZ}dZ &= 0 \\ F_{ZL}dL + F_{ZZ}dZ &= \frac{-\bar{P}_E}{P^2}dP \end{aligned}$$

It thus follows that:

$$(36) \quad \begin{pmatrix} dL \\ dZ \end{pmatrix} = \frac{1}{\Delta - S_L^{-1}F_{ZZ}} \begin{pmatrix} F_{ZZ} & -F_{LZ} \\ -F_{ZL} & F_{LL} - S_L^{-1} \end{pmatrix} \begin{pmatrix} 0 \\ \frac{-\bar{P}_E}{P^2}dP \end{pmatrix}$$

where  $\Delta = F_{LL}F_{ZZ} - F_{ZL}F_{LZ}$ , as before.

$$(37) \quad \begin{aligned} \frac{dL}{dP} &= \frac{1}{\Delta - S_L^{-1}F_{ZZ}} \frac{F_{LZ}\bar{P}_E}{P^2} > 0 \\ \frac{dZ}{dP} &= \frac{-1}{\Delta - S_L^{-1}F_{ZZ}} \frac{(F_{LL} - S_L^{-1})\bar{P}_E}{P^2} > 0 \end{aligned}$$

Note that  $S_L^{-1} > 0$ . Thus if the determinant in the previous case was positive (as we assumed), it must also be positive here.

Despite the fact that  $w$  adjusts freely, a change in the nominal price of the output good will change employment. This occurs because the change in  $P$  causes firms to choose a different level of  $Z$  and so shifts the demand for labor schedule. Any resulting change in employment is "voluntary." A comparison of (27) with (37) establishes that  $\frac{dL}{dP}$  is smaller in this case, however, when evaluated at mutually consistent values for the parameters.

As  $P$  is reduced, both  $L$  and  $Z$  fall;  $Q$  will fall as well.

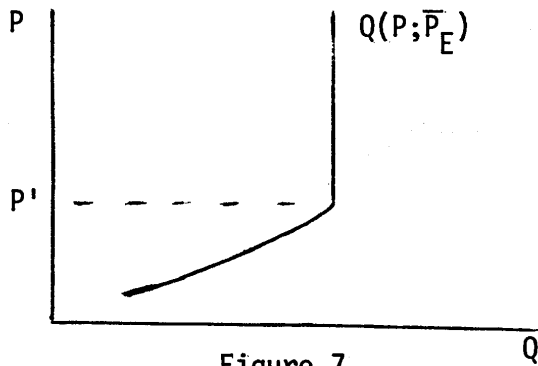


Figure 7

If  $P_E$  rises to  $\bar{P}_E$ , the kink in the curve will again shift up. From (32), let  $P''$  be defined by

$$(38) \quad \bar{P}_E = \hat{P}_E(P''; Z)$$

For  $P > P''$ , output is always independent of  $P$ . For  $P \leq P''$ ,  $L$  and  $Z$  will again both adjust as  $P$  declines, given the new higher value for  $P_E$ .

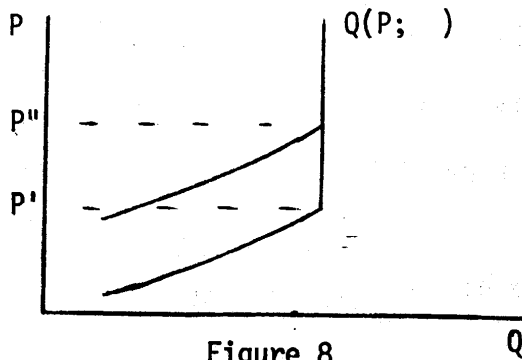


Figure 8

There remains one additional case we shall consider, that when the labor market is characterized by sticky real wages. This case is particularly important. Real wage (downward) rigidity, as a result of explicit or implicit indexing of wages, at least partially characterized the European response to the 1973 oil price shock. Indeed in the European OECD countries real wages<sup>4/</sup> on average rose about seven percent from 1973 to 1975. In contrast in the United States it is nominal wages which are regarded as somewhat inflexible. U.S. real wages fell by about three percent over the same interval.

This case is similar to the one just considered. When the capital stock is fully in use the demand for labor is a function of  $w/P$  ( as in Figure 5 above):

<sup>4/</sup> Real wages here are measured as hourly earnings in manufacturing deflated by consumer prices.



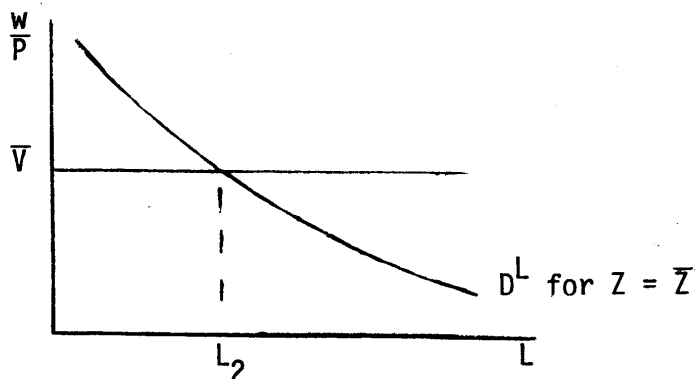


Figure 9.

Let the rigid level of the real wage be  $\bar{V}$ . It is as if the effective supply of labor schedule were perfectly elastic at  $\bar{V}$ , at least up to some limit for total labor hired. There is implicitly a notional supply of labor schedule, as in Figure 5, which would imply an equilibrium real wage lower than (or equal to)  $\bar{V}$  and employment higher than (or equal to)  $L_2$ . Given the assumption of the real wage fixed at  $\bar{V}$ , employment will remain at  $L_2$  as long as the capital stock is fully in use.  $\tilde{L} = L_2$  depends only on  $\bar{V}$  (and  $\bar{Z}$ ). Therefore for this case  $\hat{P}_E$  is:

$$(39) \quad \hat{P}_E = \hat{P}_E(P; \bar{Z}, \bar{V}) = P\{F_Z(L_2(\bar{V}), \bar{Z})\}$$

Again  $\hat{P}_E$  depends directly on  $P$ , and there is no independent role for  $w$  in determining  $\hat{P}_E$ . For a range of sufficiently high  $P$ ,  $\hat{P}_E > \bar{P}_E$ . As shown in Figure 10 output will be determined by  $\bar{V}$  and  $\bar{Z}$ . Higher values of  $P$  produce

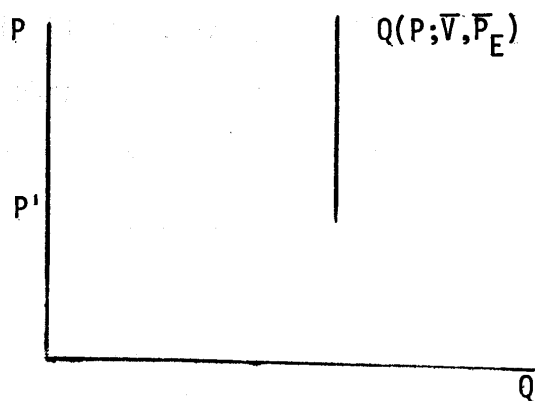


Figure 10

$P'$  defined by:

$$\bar{P}_E = \hat{P}_E(P'; \bar{Z}, \bar{V})$$

immediate adjustments in  $w$  to keep  $\frac{w}{P} = \bar{V}$ , so no additional output is forthcoming. For  $P \leq P'$  it follows that  $\hat{P}_E \leq \bar{P}_E$ . Demand for the two

factors will then be given by:

$$(40) \quad \begin{aligned} F_L(L,Z) &= \bar{V} \\ F_Z(L,Z) &= \frac{\bar{P}_E}{P} \end{aligned}$$

To determine the effect on Q of a change in P for this range of values, we conclude:

$$(41) \quad \begin{pmatrix} F_{LL} & F_{LZ} \\ F_{ZL} & F_{ZZ} \end{pmatrix} \begin{pmatrix} dL \\ dZ \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{\bar{P}_E}{P^2} dP \end{pmatrix}$$

From this it follows that:

$$(42) \quad \begin{aligned} \frac{dL}{dP} &= \frac{1}{\Delta} \frac{F_{LZ} \bar{P}_E}{P^2} > 0 \\ \frac{dZ}{dP} &= \frac{1}{\Delta} \frac{-F_{LL} \bar{P}_E}{P^2} > 0 \end{aligned}$$

where  $\Delta = F_{LL}F_{ZZ} - F_{ZL}F_{LZ} > 0$ , by assumption above.

Thus it follows that once again the general shape of the aggregate supply schedule is:

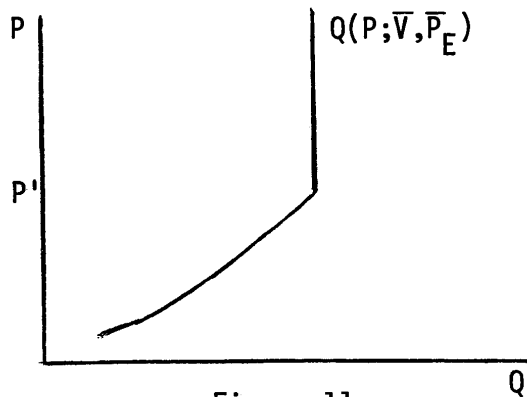


Figure 11

The effect of an increase in  $\bar{P}_E$  to  $\bar{P}'_E$  would be the same as that shown in Figure 8 for the previous case.

Rigid Nominal Wage	Rigid Real Wage	Flexible Wage
$\frac{dL}{dP} = \frac{1}{\Delta} \left( \frac{-F_{ZZ}\bar{w} + F_{LZ}\bar{P}_E}{p^2} \right)$	$\frac{1}{\Delta} \frac{F_{LZ}\bar{P}_E}{p^2}$	$\frac{1}{(\Delta - S^{-1}F_{ZZ})} \frac{F_{LZ}\bar{P}_E}{p^2}$

$$\Delta = F_{LL}F_{ZZ} - F_{LZ}F_{ZL}$$

Comparison of (42) with (27) and (37) (repeated above) shows that  $\frac{dL}{dP}$  is smallest in the flexible wage case and largest in the rigid nominal wage case, if the derivatives are evaluated at a common P and a mutually consistent set of parameters. In all three cases, the use of capital-cum-energy falls as P decreases (recall that  $\bar{P}_E$  is fixed so the "real" cost of capital-cum-energy rises in all three cases as P falls), but to differing extents due to interaction with the labor market. The decrease in the use of capital-cum-energy tends to reduce labor use as well. In the fixed nominal wage case this impact is augmented by the fact that the real wage must actually increase as P falls. In the rigid real wage case this does not happen while in the flexible wage case (given the assumed shape of the supply of labor function), the equilibrium real wage will be lower for a lower P and so will mitigate the effect of the reduced use of capital on the demand for labor.

Figures 3, 8, and 11 display the impact of an oil price shock on aggregate supply under different assumptions. In the real world wages (both real and nominal) are somewhere between absolutely rigid and perfectly flexible. All three aggregate supply curves have a kink (which coincides with a structural shift in the demand for labor behavior) because of the impact of the oil price change on firms' use of capital-cum-energy.

The full impact on output and employment of the oil price increase even in this simple model of course also requires a specification of aggregate demand behavior. The model presented below is one of many different possible aggregate demand stories one could tell. While the precise conclusions I draw about the over-all equilibrium in this model following an oil price shock depend on the particular details of both the supply and demand sides of the model, the aggregate supply analysis presented above would have generally the same sorts of implications for various demand-side models.

Let consumption demand for the domestic final good be given by:

$$(43) \quad C = C(Q,P) \quad \begin{array}{l} 1 > C_Q > 0 \\ C_P < 0 \end{array}$$

Consumption is assumed to fall as  $P$  rises because the higher  $P$  lowers the real value of outstanding "money" (assets fixed in nominal value and regarded by the private sector as net wealth). The lower real money stock reduces consumption directly (real balance effect) and/or through interest rate channels (so-called Keynes effect). Although assuming the derivate  $C_P$  is non-zero clearly implies some sort of assets exist, only the real sector of the economy is modeled in the paper.

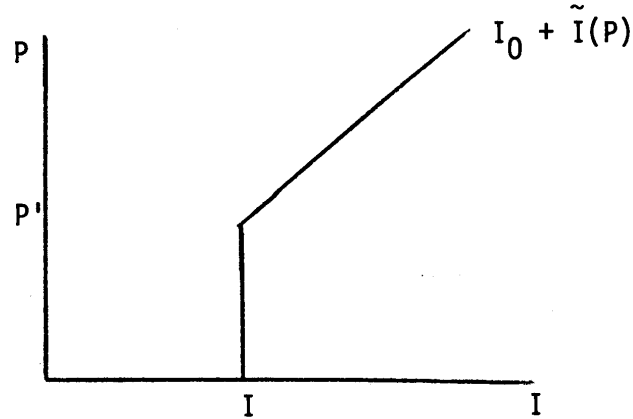
Assume investment is given by:

$$(44) \quad I = I(\lambda) + I_0 \quad \text{for } \lambda \geq 0$$

$$(45) \quad I(0) = 0 \quad I_\lambda > 0$$

where  $\lambda$  is the shadow price of capital as before. Investment thus responds when capital is "scarce" in the short run. Let us assume also that  $I(\lambda)$  goes to zero continuously as  $\lambda$  goes to zero from above. Recall from above that for a given  $\bar{P}_E$ ,  $\lambda$  depends positively on  $P$  and

that  $\lambda \geq 0$  is equivalent to  $P \geq P'$ . Thus the investment schedule is:



$$P' \text{ defined by } \bar{P}_E = P'_E(P', \quad)$$

$$\tilde{I}(P') = I(0) = 0$$

Figure 12

Aggregate demand can then be represented by the expressions:

$$(46) \quad C(Q,P) + \tilde{I}(P) + A_0 - Q = 0 \quad \text{for } P \geq P'$$

$$C(Q,P) + A_0 - Q = 0 \quad \text{for } P < P'$$

where  $A_0$  is the sum of all autonomous components of demand.

The slope of the aggregate demand schedule can then be calculated as:

$$C_Q dQ + C_P dP + \tilde{I}_P dP - dQ = 0$$

$$\frac{dQ}{dP} = \frac{C_P + \tilde{I}_P}{1 - C_Q} \quad P \geq P'$$

$$C_Q dQ + C_P dP - dQ = 0$$

$$\frac{dQ}{dP} = \frac{C_P}{1 - C_Q} \quad P < P'$$

Let us assume that  $|C_P| > \tilde{I}_P$ . Therefore the curve is negatively sloped throughout, but steeper for  $P \geq P'$  as shown in Figure 13.

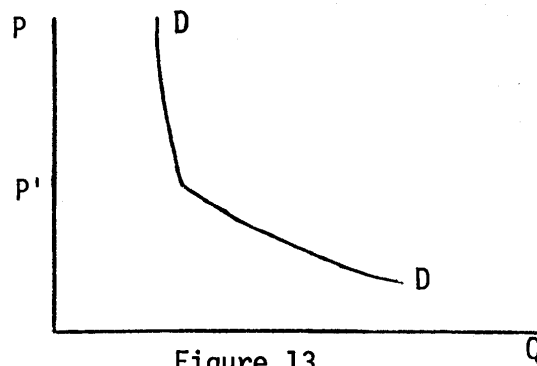


Figure 13

For  $P \geq P'$  consumption demand is rising but investment demand is falling as  $P$  decreases. By assumption, the net effect is for demand to rise. Once  $P < P'$ , however, capital is in excess supply; and the induced component of investment has fallen to zero. The effect of a lower  $P$  on consumption demand is thus not offset (partially) by the response of investment, and the response of output demanded is thus greater for this range of values of  $P$ .

A rise in  $\bar{P}_E$  to  $\bar{P}'_E$  will raise  $P'$  as before (to  $P''$ ) and reduce investment spending along the portion of the curve where  $I(P) \neq 0$ .

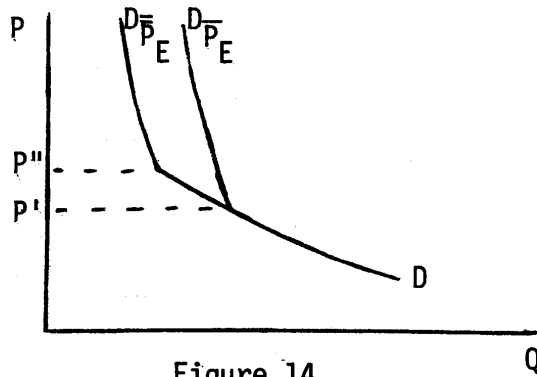


Figure 14

The value  $P'$  in this model is critical for both supply and demand. A change in the price of oil can produce a structural shift in supply (because of its effects on factor demand) and a structural shift in demand (because of its effect on investment spending). Assume that the initial equilibrium is that shown as point A in Figure 15:

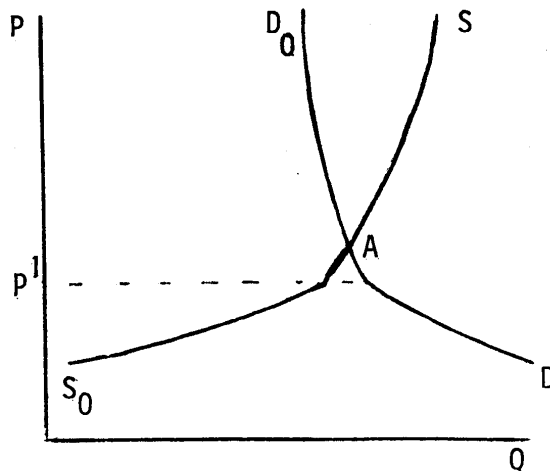


Figure 15

Let the price of oil rise and shift the curves from  $S_0S$  and  $D_0D$  to  $S_1S$  and  $D_1D$ :

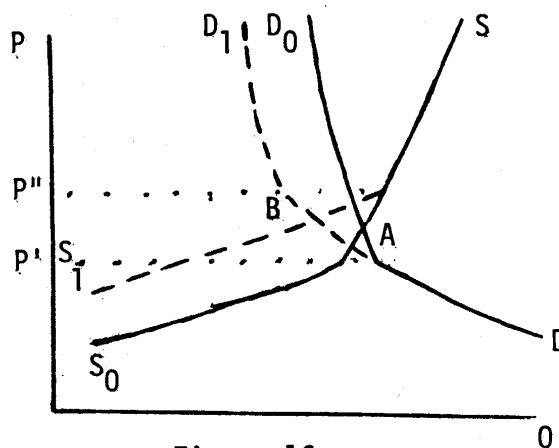


Figure 16

Equilibrium will move from A to B, implying a higher price level and reduced output (especially for investment). In addition it implies a substantial shift in the elasticity of supply (and of employment) with respect to the price level. If policy-makers were to contract demand further in the neighborhood of B in order not to accommodate the induced price rise they would discover much larger output effects and much smaller price effects than experience in the neighborhood of A would have led them to expect.<sup>5/</sup>

This paper has explored some of the implications of an oil price shock in the context of a fixed coefficient production model. While no new theoretical results for that model are obtained, the application of that model to the oil shock problem did yield some interesting insights. The simultaneous character of factor demand is emphasized by the analysis of the model in this paper. Changes in firms' use of capital in response to a rise in the price of energy induce shifts in the structure of demand in the labor market. As a result, the structure of the aggregate supply schedule changes as well.

<sup>5/</sup> Despite the simplicity of this model, I believe there were elements of the above in the experiences of the major European economies in 1975, when output fell dramatically, and, perhaps, in the U.K. in 1980 under the anti-inflation policies of the Thatcher government.

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