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Abstract: One of the most widely accepted principles of economics is the existence of gains from trade for every nation under certain conditions including perfect competition. In the last twenty years, trade economists have revolutionized the field by firmly establishing the possibility of modeling imperfectly competitive international markets. Despite this development, most still agree there are good reasons to believe that gains from trade are still present. However, we show that in the absence of international redistributions the presence of a positive profit sector in a general equilibrium model can lead to a situation in which some nations may lose from the reduction of international trade barriers.

Keywords: gains from trade, imperfectly competitive international markets, international price Nash duopoly, continuously differentiated Bertrand competition

JEL: D43, F00, F10, F12

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1 Introduction

Do all countries gain from international trade? Relying on a theory of perfectly competitive markets, the voluntary exchange of goods and services must be an activity that enhances the social welfare of every nation. In the last twenty years, economists have begun to examine trade under imperfect competition. The recent theory of strategic trade (see Brander's Nash quantity model [5] and Eaton and Grossman's Nash price model [9]) has firmly established the possibility of analyzing Nash equilibria that may result from imperfectly competitive international markets. On first blush, it may seem that gains from trade would still be present in simple models of imperfect competition. After all, without trade restrictions, the distortions due to imperfect competition should be minimized in every country, more varieties should be available for consumption in every country, and the incentives for research and development may be enhanced in every country. As a result, the elimination of all trade barriers should still tend to increase the social surplus in every country. However, in an imperfectly competitive model of two countries and two producers, this paper shows that this statement must be made with some caution.

To see why, we must invoke a property of imperfectly competitive international markets, which many have pointed out: producers may make positive profits. In fact, a long series of papers (See Brander & Spencer [5], Cheng [6], Collie [7], Fujimoto & Park [11], and Harris [14]) have detailed how this property provides governments with the profit or rent shifting incentives for export subsidies or taxes. These policies help domestic producers commit to either higher quantities (in the Nash quantity output markets) or higher prices (in the Nash price output market), thus earning more profits for the home country. As this strand of literature has progressed, the issue of whether countries gain from international trade has been set aside. Because producers now make positive profits, welfare gains or losses due to the possibility of trade must adequately take into account the profits earned by producers.[1] Then, imperfect competition can eliminate the gains from free trade, because rent shifting opens the possibility for an exporting producer to extract surplus from sales in a foreign country to the detriment of foreign producers and hence countries. In particular, though lower trade barriers can still make consumers in every country better off and increase world social surplus, enough rent may be extracted through importing to make social surplus in a country fall.

A number of papers have addressed the possibility of losses from trade, each one departing in a unique way from the classical trade model. For instance, in the infamous Graham's paradox ([12], [13]), non-convex production sets can yield a situation where a country may be better off under autarky. Graham's departure assumes that scale economies are external to the producing

[1] That is, for welfare comparison purposes, this paper assumes national welfare is the sum of consumer surplus and the profit of any domestic firms. Because we will employ separable preferences, this assumption is for the most part innocuous.

firm. While this is an important approach, it is also convenient to know whether losses from trade can be had when scale economies are internal to the producer. This is exactly what the current paper does.

More recently, Newberry and Stiglitz [18] address the possibility of losses from trade by showing that when markets are incomplete, trade may be pareto-inferior to autarky in an uncertainty context. Their argument is simple: the opening to trade transfers risk from consumers to producers. So, producers are worse off. In turn, suppliers produce an amount (of the risky good) less than pareto optimal for consumers, making consumers also worse off (despite less risk).

This paper points out another departure from the standard model that may generate losses from trade: positive profits in imperfectly competitive international markets. Brander [4] (see also pages 407-409 of Wong [22]) was the first to consider this departure. However, the possibility of losses from trade in Brander's original paper has received little attention for a number of reasons. First, strictly speaking losses from trade are not present in Brander's paper, because the symmetry of country cancels the effect rent shifting has on national welfare. Nonetheless, as Wong [22] points out if we introduce country differences, then losses from trade will arise. Second, many authors believe it is questionable whether the Brander Cournot model can be convincingly adapted to fit within a general equilibrium setting. After all, the Cournot analysis usually begins by taking the producers' demand functions as primitive. Our analysis will begin by positing a direct utility functional for all consumers in the world. Thus, the preferences of every consumer will be fully specified. Third, the Brander model disallows arbitrage opportunities between markets, so that producers can choose independent quantities for domestic and foreign markets, a fact which has received criticism by some authors. In this paper, it will not be necessary to make this assumption. This paper revisits the point first made by Brander [4] in a different international market structure: continuously differentiated Bertrand competition under mill pricing (i.e. prices reflect delivery of the good at the mill).[2] Then, it will be possible to provide better foundations for the possibility of losses from trade that may follow from international rent shifting. The next section briefly discusses the relevant related literature.

1.1 Review of related literature

The research contained in this paper draws from at least two different strands of literature. The first strand of literature is a subset of what is known as the new Strategic Trade theory, which builds models of imperfectly competitive international markets. In specific, within this large group of models, a growing series of papers analyze Nash equilibria in international markets. The first paper to do so was Brander [4] in 1981, which took the Cournot market structure and applied

[2] This is the name given to the models of imperfect competition first made popular in Hotelling's famous beach example.

it to international markets. Brander broke new ground with this paper. First, he provided an explanation for intra-industry trade in identical commodities. Roughly, intra-industry trade is a situation where two countries trade with each other goods that are in the same industry. According to classical trade theory, such trade can be wasteful and in fact should not occur: after all, countries specialize in producing goods in which they have a comparative advantage. It is for this reason that international trade economists were shocked to find that the overwhelming majority of global trade was intra-industry trade (see Krugman [15]). Before Brander's paper, the usual approach to reconcile these seemingly contradictory facts was to assume that such intra-industry trade arose because of slight differences in the commodities produced to satisfy consumers' taste for variety, differences that were too small to show up in international trade data. By employing the Cournot market structure in international markets, Brander showed that intra-industry trade need not be wasteful (i.e. it can lead to lower prices in all markets) and that there were good reasons to expect two-way trade even in identical products.

Second, he contributed to the issue of why countries trade in the first place. According to classical trade theory, countries trade because they differ in terms of their preferences, technology or endowments. In turn, international trade economists were also shocked to find that a large portion of global trade occurred between similar countries. By the time, Brander wrote this paper, it was well known that increasing returns to scale might imply that it was more efficient to concentrate all production in one country, which in turn implied a need for international trade. While Brander's paper also has increasing returns to scale, because there are Cournot firms in both countries, trade is said to occur mostly because of the strategic interaction between firms.

In 1985, Brander and Spencer [5] expand Brander's previous paper to focus on the phenomenon of rent-shifting (as described in the introduction) by proposing a "third market" model, followed by a long stream of papers in this strand of literature. In specific, they assume three countries. The first two countries each have a Cournot firm, but no consumers. The third country has consumers, but no firm. They analyze a two stage game. In the first stage, governments set tariffs and/or subsidies on producers. In the second stage, firms in the first two countries set quantities competing for consumers in the third country. Brander and Spencer first show that a government in one of the first two countries has a unilateral incentive to provide export subsidies for its home firm (to achieve Stackelberg leader profits for its country) despite the fact that doing so turns the terms of trade against this country. Then, Brander and Spencer confirm that when the governments of the first two countries can both credibly precommit to a policy, exporting governments optimally set subsidies in order to compete for international market share in a subgame perfect Nash equilibrium. Finally, they also allow the third country to precommit to a trade policy. They show the third government typically has an incentive to set a tariff that shifts rent to its country, that would have otherwise gone to one of the first two countries.

In 1986, Eaton and Grossman [9] extend Brander and Spencer's paper by replicating their analysis in a different but related market structure. First, they show that if the market structure is Bertrand rather than Cournot, a government in one of the first two countries has a unilateral incentive to impose export taxes (not subsidies) on its home firm. This tax allows the home firm to commit to a higher price (or act as a Stackelberg leader) and thus obtain higher profits for its country. Then, they confirm that when the governments of the first two countries can both credibly precommit to a policy, exporting governments optimally set taxes (not subsidies) in a subgame perfect Nash equilibrium under the Bertrand market structure. Thus, Eaton and Grossman conclude that the optimal trade policy is sensitive to details of the assumed behavior of firms.

In 1988, Cheng [6] expands on Eaton and Grossman's point. He considers a domestic market that is served by a domestic firm and a foreign firm and derives optimal trade and industrial policies in a general conjectural variations model. Because there is no longer a third market (i.e. because there is domestic consumption), trade and industrial policies are no longer the same and one policy can serve as a substitute for the other. As expected, the optimal policy combination is very sensitive to the details of the assumed behavior of firms. For example, the optimal policy under Cournot competition consists of a domestic production tax and a tariff, but under Bertrand competition, it consists of a production subsidy and free trade.

The second group of papers we draw upon can roughly be called the literature on continuous spatial models of interregional or international trade. By this, I mean models that assume 1) there is a continuous product location space and 2) the production location space may be partitioned into regions or countries. These models consider rigorously what it means for a country and its inhabitants (both consumers and producers) to occupy locations on some continuous space. In fact, a growing number of papers asking important policy questions are employing this environment. For instance, Braid [3], Trandel [21], Kanbur and Keen [16], Ohsawa [19], and Benson and Hartigan [2] are all concerned with issues of tax policy on cross-border sales under different industrial structures. Among others, Braid (1987) considers the equilibrium prices of firms under arbitrary geographic configurations of tax policy. Trandel (1992) considers the welfare effects of use tax evasion between states for a fixed market structure: four equidistant firms around a circle partitioned into two regions. Kanbur and Keen (1993) consider two governments maximizing revenue on a line segment in a non-cooperative sales tax equilibrium under a perfectly competitive industrial structure. Ohsawa (1999) generalizes many of the results in Kanbur and Keen to many countries. Benson and Hartigan (1983) analyze the impact of a tariff employing a Löschian market structure on an interval to derive a version of the Metzler paradox.[3]

Closest to this paper, in 1995, Shachmurove and Spiegel [20] analyze the gains from trade in a model of continuously differentiated Bertrand price competition using unit demands over an unit

[3]Roughly, the Metzler paradox is a situation in which the imposition of a tariff leads to a fall in domestic price.

interval. Their principal contribution is in formulating a reasonable, yet exogenous, location model for countries with one producer. In specific, they assume that each firm is located at the physical center of the country it occupies.

The current paper builds on their model in at least three ways. First, a different location space is proposed: consumers are distributed around the unit circle. Although this will provide the model with stronger microeconomic location foundations (to be discussed below), the location space does not have to be the circle, as the model is isomorphic if we instead employ an unit interval, but take producers to be located at the endpoints.

Second, Shachmurove and Spiegel [20] present a model of imperfectly competitive international markets, where nevertheless identical countries do not trade. Therefore, they allow for countries to be of different sizes, to contain different numbers of consumers and hence have different preferences, so that international trade can occur. We do the same. However, in another section, we also consider a model of international trade where differences in technology (rather than in preferences) cause international trade. We will ask whether less efficient countries should attempt to protect their less efficient industries, when competition is imperfect. But, unlike the differences in preferences (or in size) model, we will find that even in the presence of rent shifting, all countries may prefer complete elimination of barriers to trade. This occurs if the efficiency differences between countries (and hence industries) is sufficiently large.

Finally, we propose a continuous spatial model of international trade, where barriers to international trade are given a continuous treatment. Shachmurove and Spiegel describe two price Nash equilibria and then compare them: one characterized by autarky and one characterized by free trade. Clearly, one would like a model where if the barriers to international trade are large, then international trade cannot occur. Conversely, if there were no barriers to international trade, then we would call this free trade. This paper nests these two extremes in a model which also analyzes intermediate levels of the trade barriers. We handle these cases by allowing the international trade barrier be part of the implicit price paid by the consumer. In specific, consumers purchasing the good from a foreign producer must also pay a fixed cost per unit associated with merely transacting with this firm. For short, we will call this cost, the trade barrier. This cost can be exogenous and geographic (i.e. an ocean can create a gap in the Hotelling line) or it may be legal in the form of trade policy. As such, this cost can be interpreted as the insulation of the two countries' economies. We do not try to explain how the size of the trade barrier is set. Rather, we analyze the pure price Nash equilibria that result for different levels of insulation between the two economies. In fact, as mentioned above, one important contribution is that unlike many quantity Nash models of Strategic Trade which assume markets are segmented, here the degree of market segmentation is endogenized by using the exogenously given international trade barrier.

Section 2 lays down the model's assumptions, first describing them in a general equilibrium setting, then discussing the production location assumptions of the model and ending with some preliminary details that will help in analyzing the equilibrium of the model. Section 3 shows that a Nash equilibrium necessarily exists regardless of the barriers to international trade and characterizes the equilibrium of the model. Section 4 presents a welfare analysis of two variants of the model: one where countries have access to the same production technolgy and instead differ only in their size or preferences and another where countries have identical preferences and instead differ in their production technologies. Section 5 concludes. Some proofs are left to the appendix.

2 General equilibrium assumptions

I) Setup:

There are an uncountable number of potentially producible goods. For convenience, the first good will be an all purpose good which is taken as numeraire and is produced in the first sector. Let $y \in \mathbb{R}_+$ be the number of units of gold purchased by a consumer. The remaining uncountable number of potentially producible goods will be labeled using an index set $l \in \mathbb{L}$ and are produced in the second sector. Of this set, only a subset $\mathbb{S} \subset \mathbb{L}$ will be produced. Assume two home countries labelled h and H. Let C_i be the set of potentially producible goods assigned to country $i \in \{h, H\}$. Assume C_i is a convex set.

II) Preferences:

A consumer maximizes preferences $\succeq_x$ that are quasilinear in the first sector y and unit demand-ideal variety[4] in the second sector $w \in \mathbb{S}$ (in the sense that $x \in \mathbb{L}$ describes the consumer's ideal variety):

$$\widetilde{u}_x(y, w) = y + \phi_x(w)$$

where

$$\phi_x(w) = \max\left\{r - \widetilde{t}\left(|x - w|\right) - k \cdot 1(x, w), 0\right\}$$

and

$$1(x, w) = \left\{ \begin{array}{ll} 1 & \text{if } (x \in C_H \text{ and } w \in C_h) \text{ or } (x \in C_h, w \in C_H) \\ 0 & \text{otherwise} \end{array} \right\}$$

r is the reservation utility of ownership of a second sector good,[5] $-\widetilde{t}\left(|x - w|\right)$ is the ideal variety term that captures transport costs between the location of the consumer and the location of his

[4]This means consumers purchase 0 or 1 unit of the second sector good w and that consumers have ideal varieties for the second sector good characterized by a particular location $x \in \mathbb{L}$.

[5]Note that we have assumed $r_H = r_h = r$, where r_i is the reservation utility from ownership of a second sector good assigned to country i. Following the terminology of industrial organization, this is known as horizontally differentiated goods.

preferred producer $w \in \mathbb{S}$. Finally, $\phi_x(.)$ also includes the international trade barrier k which is paid only by those consumers who purchase a foreign produced good.[6] The barrier k is multiplied by $1(x, w)$, which serves as an indicator function of consumers who purchase a foreign produced good. To make matters as simple as possible, assume that the ideal varieties of consumers are uniformly distributed along $\mathbb{L}$.

Finally, a consumer in country i has access to income given by the sum of non-capital income I and capital income from diffuse ownership of the profits of sales of good l_i. I will assume I is so large that a consumer in country i ignores how his choice between produced goods in $\mathbb{S}$ affects the profits of producers in country i and that r is so large that all consumers are served by the second sector.[7]

III) Technology:

y, the numeraire good, is produced in the first sector which is perfectly competitive. The second sector is imperfectly competitive. In specific, it is assumed that the production of good l_i requires the disbursement of a pre-production fixed setup cost F_i. Producers make choices in two stages, deciding first which good to produce and then which price to set. The analysis of this paper is centered around the second decision. In the next section, we discuss the microeconomic foundations (of the first decision) on which the analysis of this paper is built. Assume F_i is sufficiently large to guarantee that only two goods in $\mathbb{L}$ can be profitably produced, each by one producer at some locations denoted $\mathbb{S} = \{l_h, l_H\}$ and assume $l_i \in C_i$ for $i = H, h$.

Next, producers set prices in a Nash equilibrium competing to attract consumers. Given $\mathbb{S}$, let $q_{l_i}(p_i, p_{-i}; k)$ be the aggregate demand attracted by a producer who supplies good l_i. Producers have access to a one factor Ricardian technology. Let a_{l_i} be the units of input necessary to produce one unit of output of good l_i and r_i be the rental price paid by the producer of good l_i

[6] This generally eliminates the interpretation of k as a tariff. First, there are other potential interpretations for k. Second, we could instead reintroduce k as part of the budget constraint and get the same indirect utility function used in the rest of this paper. However, it would then be necessary to include tariff revenue in national welfare and more importantly find a way to distribute tariff revenue between countries.

[7] Why do we do this? First, it is important to avoid a situation where firms set prices so high that some consumers are not served by the second sector, because in this case producers do not effectively compete for consumers. Following the terminology adopted by the literature on spatial price competition, this is described as a situation where producers in the second sector are spatial monopolists. As a result, if reducing the barrier to international trade still meant that producers acted as spatial monopolists, any adverse welfare results could be attributed to the lack of competition between producers. Second, in order to determine whether rent shifting can affect the usual gains from trade result, it will be necessary for one producer to capture the rent available to another producer as the barriers to trade are reduced. This can only be achieved in a model where all consumers are served by the second sector regardless of the size of the trade barrier. Finally, we argue that welfare results that follow from considering equilibria where some consumers are not served by the second sector are standard and simply not interesting. In the sequel, we will see that this is mainly due to the fact that losses from trade can result from international rent shifting.

to employ one unit of the input. We can always define the cost function for this technology (i.e. the real cost necessary to produce q_{l_i} units of output of good l_i):

$$\widetilde{c}_i(q_{l_i}) = r_i a_{l_i} q_{l_i} + F_i$$

The marginal costs of production are constant at $c_i \equiv r_i a_{l_i}$. Then, profit functions are given by:

$$\pi_i(p_i, p_{-i}; k) = (p_i - c_i) \cdot q_{l_i}(p_i, p_{-i}; k) - F_i$$

2.1 Location and geography

As anticipated in the previous subsection, the label set $\mathbb{L}$ is assumed to be a continuous space. One might be tempted to conclude that different points in $\mathbb{L}$ represent different sectors. Though this is feasible, the typical interpretation of a continuous product differentiation space is that it represents one whole sector within which differentiated products are produced, as discussed in the introduction.

In order to describe the assumptions as general as possible, we have purposely avoided putting structure on the continuous product differentiation space $\mathbb{L}$ in the last subsection. Note that regardless of whether the differentiation takes the variety or the geographic interpretation, whenever a continuous product space is employed, we immediately have to worry about whether the particular products chosen by the producers are supportable as part of some location choice equilibrium. Now, it is time to specify the production differentiation space $\mathbb{L}$ and the location configuration of products $\mathbb{S}$. In doing so, it will be necessary to discuss how the firm's choice of a particular product is affected by different levels of the international trade barrier.

For the moment, suppose there are no trade barriers (i.e. $k = 0$) and that the label set is the unit interval. Assuming the ideal variety transport costs were linear functions of distance (i.e. $\widetilde{t}(|x - w|) = t \cdot |x - w|$), Hotelling in 1929 was the first author to analyze this market structure. As anticipated in the previous section, Hotelling was interested in modelling a market where a producer had to make two choices: 1) a particular product in $\mathbb{L}$ and 2) a price for that particular product. His market structure is diagrammed in figure 1, where l_H is assumed without loss of generality to be to the "left" of l_h. He found it was possible to express profits for firm i, π_i, as a function of both location profiles (l_h, l_H) and price profiles (p_h, p_H). Then, arguing that prices are easier to adjust than product location, he solved for the second stage price Nash equilibrium as a function of locations: $p_i^* = p_i(l_h, l_H)$. Substituting these prices into the producer's objective function, he was ready to analyze the producer's first stage location decision. In specific, he differentiated firm H's profits with respect to l_H and showed the following derivative was positive:

$$\frac{d\pi_H}{dl_H} = \frac{\partial \pi_H}{\partial l_H} + \frac{\partial \pi_H}{\partial p_h}\frac{\partial p_h}{\partial l_H} + \frac{\partial \pi_H}{\partial p_H}\frac{\partial p_H}{\partial l_H} = \frac{\partial \pi_H}{\partial l_H} + \frac{\partial \pi_H}{\partial p_h}\frac{\partial p_h}{\partial l_H}$$

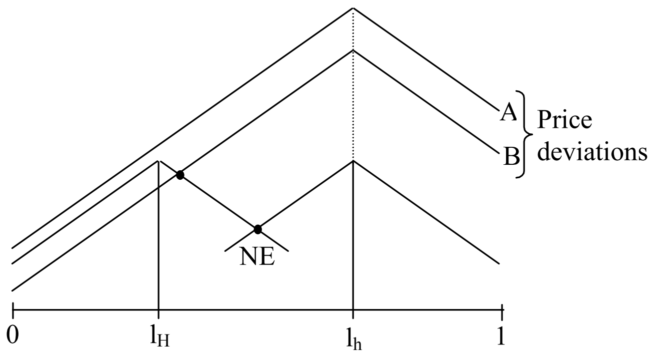

Figure 1: Market structure proposed by Hotelling (1929): the diagonal lines indicate utility levels given a fixed price

where the second equality holds because under the second stage price Nash equilibrium $\frac{\partial \pi_H}{\partial p_H} = 0$ by definition. Hotelling called this the Principle of Minimal Differentiation, as it implied that producers tended to locate close to one another. Fifty years later, D'Aspremont, Gabscewicz and Thisse [8] showed Hotelling's Principle was not quite correct. As producers got closer and closer, the second stage price Nash equilibrium derived by Hotelling ceased to exist because of price undercutting strategies that stole rival's entire market share in a discontinuous fashion (i.e. compare the difference in quantities attracted by firm h under the price deviations labelled A and B in figure 1). D'Aspremont, Gabscewicz and Thisse correctly recognized that this problem arose because the ideal variety transport costs were assumed linear in Hotelling's model. To fix this, they propose using quadratic ideal variety transport costs. Now, the price undercutting strategy was no longer profitable no matter how close the producers were and so second stage price Nash equilibria always existed. Replicating Hotelling's analysis, they show the above derivative was actually negative under quadratic ideal variety transport costs. They called this the Principal of Maximal Differentiation, as it implied that producers tended to locate as far apart from each other as possible. In fact, in their paper, the only location configuration that can be supported as a subgame perfect Nash equilibrium was to have firms locate at the endpoints: $\mathbb{S} = \{0, 1\}$. The model of the current paper is isomorphic to this location configuration, though I assume linear ideal variety transport costs to keep the algebra tractable. Nonetheless, the result that a country may prefer autarky to free trade would continue to hold even if these costs were quadratic. Moreover, the possibility of losses from trade continue even when firms are near, not located exactly at, the endpoints.

This paper analyzes the welfare properties of the pure Nash equilibrium of the two producer pricing game as the barriers to international trade are changed, assuming a constant $\mathbb{S}$. One notorious problem with models of this type is that because the location equilibrium depends on the price equilibrium, which in turn depends on the size of the trade barrier, different loca-

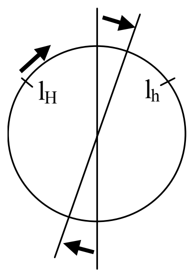

Figure 2: No direct effect when $\mathbb{L}$ is the unit circle

tion/entry configurations can be supported as Nash equilibria when the trade barrier is allowed to vary. Thus, we would like whatever $\mathbb{L}$ and $\mathbb{S}$ is used to remain constant regardless of the international trade barrier. Nonetheless, proposing an $\mathbb{L}$ and $\mathbb{S}$ in which the production location configuration cannot be supported as a location Nash equilibrium for some size of the trade barrier might be questioned. It would also be unfortunate if the Nash status of the location choice configuration $\mathbb{S}$ depended on details of the location process such as whether firms locate simultaneously or sequentially. Thus, we search for an $\mathbb{S}$ that is a location Nash equilibrium in $\mathbb{L}$ regardless of the size of the trade barrier and regardless of whether firms locate simultaneously or sequentially. One location configuration with this property is to let $\mathbb{L}$ be the unit circle and assume that l_i is the midpoint of C_i. That is, the producer locates at the physical center of the country it occupies. This is the reasonable yet exogenous location model proposed by Shachmurove and Spiegel [20]. What makes their exogenous location model attractive is that regardless of the ideal variety transport costs, it can be supported as an autarkic subgame perfect Nash equilibrium. Of course, with linear ideal variety transport costs and with $\mathbb{L} = [0, 1]$, not the circle, their exogenous location model cannot be supported as a free trade subgame perfect Nash equilibrium, because as Hotelling noted there will be a tendency for producers to move closer to each other in an attempt to steal each other's market. This attempt eventually fails as demonstrated by D'Aspremont-Gabscewicz and Thisse, once the producers are sufficiently close. With $\mathbb{L}$ as a circle, the exogenous location model of Shachmurove and Spiegel is still supportable as an autarkic subgame perfect Nash equilibrium, but it is also supportable as a free trade subgame perfect Nash equilibrium. To see this, consider the derivatives computed by Hotelling above. It is clear that in any subgame perfect Nash equilibrium, location has two effects on firm profits: 1) a direct effect (i.e. as l_H is increased, firm H can steal market share from firm h) and 2) a strategic effect (i.e. as l_H is increased, firm h lowers price in an attempt to compete more aggressively for consumers). However, when $\mathbb{L}$ is the unit circle, the direct effect is not present, because whatever market share is gained on one side of the circle is lost on the other side (see figure 2). Thus, with only the strategic effect present, firms tend to differentiate as much as possible under price competition.

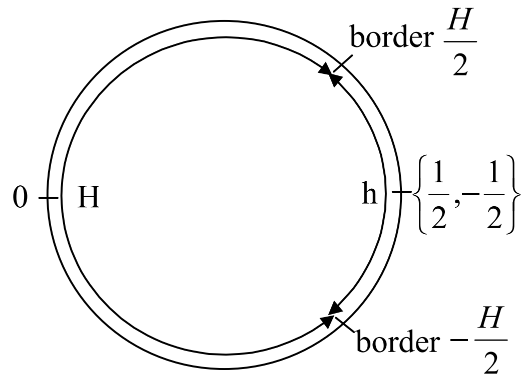

Figure 3: An illustration of the model

Finally, we must deal with some peripheral setup issues. First, locations on the unit circle $\mathbb{L}$ will be labelled $[-\frac{1}{2}, \frac{1}{2}]$. So, $\frac{1}{2}$ and $-\frac{1}{2}$ are the same location. This will yield convenient labelling. Second, let the firm in country H be located at 0 (i.e. $l_H = 0$) and the firm in country h be located at $\frac{1}{2}$ or $-\frac{1}{2}$ (i.e. $l_h = \frac{1}{2}$ or $-\frac{1}{2}$). The model is illustrated in figure 3. The upper semicircle is labelled $[0, \frac{1}{2}]$, while the lower semicircle is labelled $[-\frac{1}{2}, 0]$. Third, in addition to serving as country labels, h and H will refer to numbers in $[0, 1]$ (that sum to one) describing the arclengths of C_h and C_H respectively. Fourth, as indicated in the previous section, identical countries do not trade. So, two variants of this model will be analyzed in this paper. In the first variant, countries will be assumed to have the same technology, but different preferences. This is achieved without loss of generality by assuming: $c_H = c_h = c$ and $H > h$. In the second variant, countries will be assumed to have the same preferences, but different technologies. This is achieved without loss of generality by assuming: $c_H > c_h$ and $H = h = \frac{1}{2}$.

2.2 Preliminary analysis

This subsection presents some basic facts that will allow us to analyze the equilibrium of the model. First, note that searching for pure strategy Nash equilibria when all consumers are served by the second sector simplifies the analysis, because it allows us to focus on finding the location of the marginal consumer in equilibrium in the upper semicircle. In specific, given a pair of prices (p_h, p_H), it is possible to define the location of two marginal consumers, one in the upper semicircle $[0, \frac{1}{2}]$ and the other in the lower semicircle $[-\frac{1}{2}, 0]$, with the property that both are indifferent between purchasing from the domestic and foreign producer. Moreover, once the marginal consumer in the upper semicircle is located at say $\overline{x}$, then the marginal consumer in the lower semicircle is simply found at $-\overline{x} \in [-\frac{1}{2}, 0]$.

However, it will be necessary to pay attention to consumers on either side of the border in the upper semicircle. Consider first a consumer in country H and in the upper semicircle (i.e. a consumer located at $\widehat{x}_H \in [0, \frac{H}{2}]$). In order to purchase from the firm in country H, this consumer must pay transport costs $t\widehat{x}_H$, while in order to purchase from the firm in country h, this consumer must pay transport costs $t(\frac{1}{2} - \widehat{x}_H)$ plus the barrier cost to foreign purchases, k. Thus, conditional upon the marginal consumer being in country H, the location of this consumer in the upper semicircle is defined by:

$$r - p_H - t\widehat{x}_H = r - k - p_h - t(\frac{1}{2} - \widehat{x}_H) \Longrightarrow \widehat{x}_H = \frac{p_h - p_H}{2t} + \frac{k}{2t} + \frac{1}{4}$$

Similarly, a consumer in country h and in the upper semicircle (i.e. a consumer located at $\widehat{x}_h \in (\frac{H}{2}, \frac{1}{2}])$ must pay transport costs $t(\frac{1}{2} - \widehat{x}_h)$ to purchase from the firm in country h, while he must pay transport costs $t\widehat{x}_h$ plus barrier cost k to purchase from the firm in country H. Thus, conditional upon the marginal consumer being in country h, the location of this marginal consumer in the upper semicircle is defined by:

$$r - k - p_H - t\widehat{x}_h = r - p_h - t(\frac{1}{2} - \widehat{x}_h) \Longrightarrow \widehat{x}_h = \frac{p_h - p_H}{2t} - \frac{k}{2t} + \frac{1}{4}$$

Note that because k is non-negative, for any equilibrium the values $\widehat{x}_i$ have the property $\widehat{x}_H \geq \widehat{x}_h$. However, as defined, max $\widehat{x}_H <$ min $\widehat{x}_h$, because $\widehat{x}_H \in [0, \frac{H}{2}]$ and $\widehat{x}_h \in (\frac{H}{2}, \frac{1}{2}]$. Thus, it cannot be the case that both $\widehat{x}_H$ and $\widehat{x}_h$ are in their own country's domain. In turn, this means that the marginal consumer will be at a unique location given by the value of $\widehat{x}_H$ or $\widehat{x}_h$, depending on which one is in its country's domain. Then, define:

$$x_H = \left\{ \begin{array}{ll} 0 & \text{if } \widehat{x}_H \leq 0 \\ \widehat{x}_H & \text{if } 0 < \widehat{x}_H < \frac{H}{2} \\ \frac{H}{2} & \text{if } \widehat{x}_h \leq \frac{H}{2} \leq \widehat{x}_H \end{array} \right\}$$

$$x_h = \left\{ \begin{array}{ll} \widehat{x}_h & \text{if } \frac{H}{2} \leq \widehat{x}_h \leq \frac{1}{2} \\ \frac{1}{2} & \text{if } \frac{1}{2} < \widehat{x}_h \end{array} \right\}$$

In words, there are five cases to consider. First, the location of the marginal consumer may be at 0. This occurs when $\widehat{x}_H \leq 0$. Second, the location of the marginal consumer may be in $(0, \frac{H}{2})$ (i.e. the marginal consumer locates in country H). This occurs when $0 < x_H = \widehat{x}_H < \frac{H}{2}$ (i.e. the true marginal consumer is in H at location x_H given by $x_H = \widehat{x}_H$). Third, the location of the marginal consumer may be at $\frac{H}{2}$ (i.e. the marginal consumer locates on the border). This occurs when $\widehat{x}_h \leq \frac{H}{2} \leq \widehat{x}_H$. The fact that $\widehat{x}_h$ is not consistent with being in country h is the contradiction which leads us to know that the value $\widehat{x}_H \geq \frac{H}{2}$ leads to the corner solution $x_H = \frac{H}{2}$. Fourth, the location of the marginal consumer may be in $(\frac{H}{2}, \frac{1}{2})$ (i.e. the marginal consumer locates in country h). This occurs when $\frac{H}{2} \leq x_h = \widehat{x}_h \leq \frac{1}{2}$. Now $\widehat{x}_H$ is not consistent with being in country H and can be ignored. And finally, the marginal consumer may locate at $\frac{1}{2}$. This occurs when $\frac{1}{2} < \widehat{x}_h$ leading to $x_h = \frac{1}{2}$. Putting these observations together, it is possible to define the location of the marginal consumer over the entire upper semicircle as:

$$\overline{x}(p_h, p_H) = \left\{ \begin{array}{ll} x_H & \text{if } \widehat{x}_h \leq \frac{H}{2} \\ x_h & \text{if } \frac{H}{2} < \widehat{x}_h \end{array} \right\}$$

$$= \left\{ \begin{array}{ll} 0 & \text{if } \frac{t}{2} + k \leq p_H - p_h \\ \frac{p_h - p_H}{2t} + \frac{k}{2t} + \frac{1}{4} & \text{if } \frac{t}{2} + k - tH \leq p_H - p_h \leq \frac{t}{2} + k \\ \frac{H}{2} & \text{if } \frac{t}{2} - k - tH \leq p_H - p_h \leq \frac{t}{2} + k - tH \\ \frac{p_h - p_H}{2t} - \frac{k}{2t} + \frac{1}{4} & \text{if } -\frac{t}{2} - k \leq p_H - p_h \leq \frac{t}{2} - k - tH \\ \frac{1}{2} & \text{if } p_H - p_h \leq -\frac{t}{2} - k \end{array} \right\}$$

Note $\overline{x}(p_h, p_H)$ is a continuous function of (p_h, p_H). With this expression, it is now possible to calculate the producers' demand and profit functions. Formally, $q_{l_H}(p_h, p_H) = 2\overline{x}(p_h, p_H)$ and $q_{l_h}(p_h, p_H) = 1 - 2\overline{x}(p_h, p_H)$ (recall it is assumed that all consumers are served by the second sector). Finally, in order to determine the profit of each firm as a function of price profiles (p_h, p_H), the demand functions are multiplied by the corresponding firm's price-marginal cost margin: $\pi_H(p_h, p_H) = (p_H - c_H) \cdot q_{l_H}(p_h, p_H)$ and $\pi_h(p_h, p_H) = (p_h - c_h) \cdot q_{l_h}(p_h, p_H)$. Because $\overline{x}(p_h, p_H)$ is a continuous function, $\pi_i(p_h, p_H)$ is a continuous function of (p_h, p_H) for $i = H, h.$:

$$\pi_H(p_h, p_H) = \left\{ \begin{array}{ll} (p_H - c_H) \cdot 0 & \text{if } k + \frac{t}{2} \leq p_H - p_h \\ (p_H - c_H)(\frac{p_h - p_H}{t} + \frac{k}{t} + \frac{1}{2}) & \text{if } k + \frac{t}{2} - tH \leq p_H - p_h \leq k + \frac{t}{2} \\ (p_H - c_H)H & \text{if } -k + \frac{t}{2} - tH \leq p_H - p_h \leq k + \frac{t}{2} - tH \\ (p_H - c_H)(\frac{p_h - p_H}{t} - \frac{k}{t} + \frac{1}{2}) & \text{if } -k - \frac{t}{2} \leq p_H - p_h \leq -k + \frac{t}{2} - tH \\ (p_H - c_H) \cdot 1 & \text{if } p_H - p_h \leq -k - \frac{t}{2} \end{array} \right\}$$

$$\pi_h(p_h, p_H) = \left\{ \begin{array}{ll} (p_h - c_h) \cdot 1 & \text{if } k + \frac{t}{2} \leq p_H - p_h \\ (p_h - c_h)(\frac{p_H - p_h}{t} - \frac{k}{t} + \frac{1}{2}) & \text{if } k + \frac{t}{2} - tH \leq p_H - p_h \leq k + \frac{t}{2} \\ (p_h - c_h)(1 - H) & \text{if } -k + \frac{t}{2} - tH \leq p_H - p_h \leq k + \frac{t}{2} - tH \\ (p_h - c_h)(\frac{p_H - p_h}{t} + \frac{k}{t} + \frac{1}{2}) & \text{if } -k - \frac{t}{2} \leq p_H - p_h \leq -k + \frac{t}{2} - tH \\ (p_h - c_h) \cdot 0 & \text{if } p_H - p_h \leq -k - \frac{t}{2} \end{array} \right\}$$

Note that both profit functions are characterized by five branches. Branch j of the firm in country i is characterized by a profit $\pi_i^j(p_h, p_H)$ and some "if" condition $Z_j(p_h, p_H)$. Notice that the intervals defined by $Z_j(p_h, p_H)$ are the same for both profit functions. As described above, this paper analyzes the pure equilibria that result when all consumers are served by the second sector. This has allowed us to focus on the equilibrium location of the marginal consumer as a complete description of the strategy space.

There are of course a number of other regions of this space where some consumers do not purchase the good. For completeness, these 12 regions are displayed in figure 4 and labelled $j = 6, ...17$. The figure also shows the amount demanded from each producer in each region.

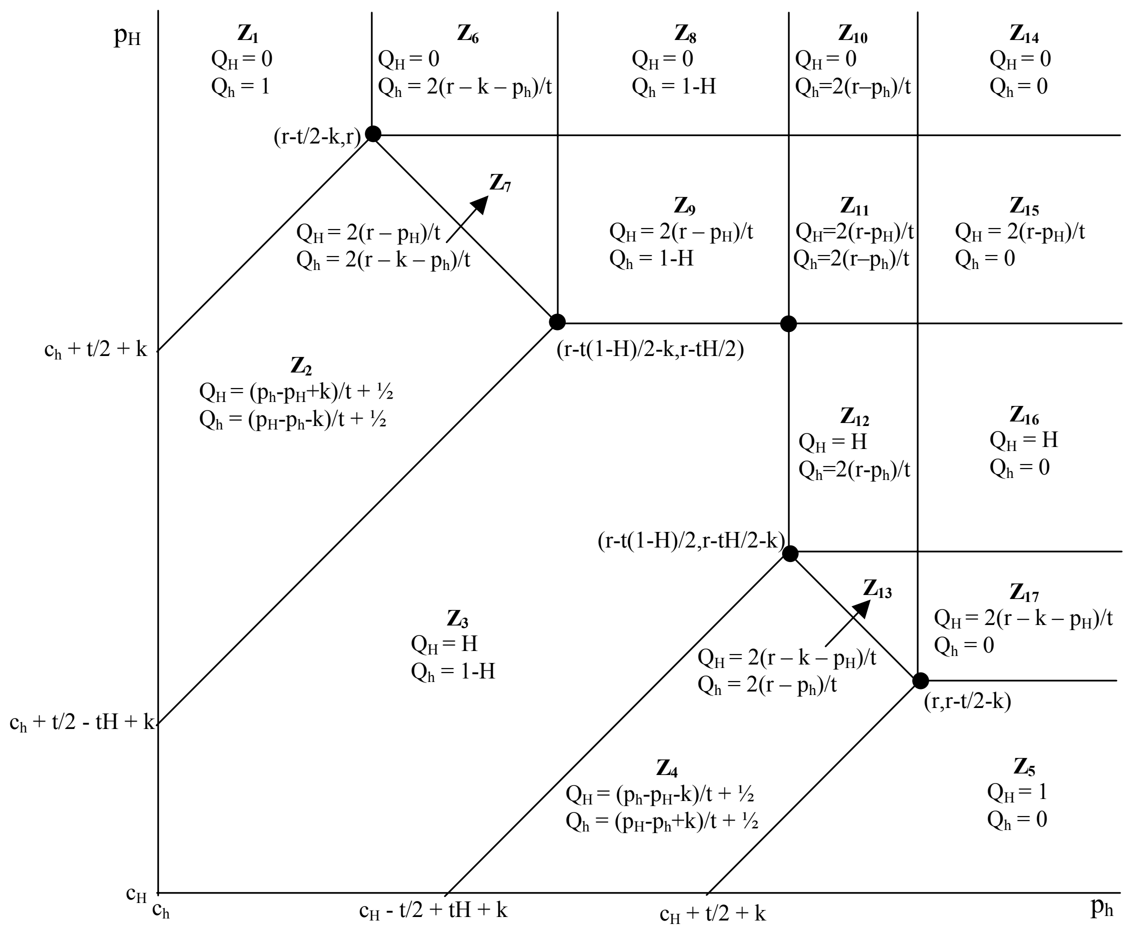

Figure 4: Full strategy space and demand functions

By inspection, it is possible to verify that demand continues to be a continuous function of price profiles in these regions. The next section formally begins our analysis of the model's equilibrium.

3 Equilibrium analysis

3.1 Existence of Nash equilibrium

First, we prove an equilibrium always exists for the two producer pricing game. In this game, the strategies are the prices set by the firms and the payoffs are given by the profit functions above. Thus, the purpose of this section is to find all price profiles (p_h^*, p_H^*) satisfying:

$$\begin{aligned} \pi_H(p_h^*, p_H^*) &\geq \pi_H(p_h^*, p_H) \qquad \forall p_H \\ \pi_h(p_h^*, p_H^*) &\geq \pi_h(p_h, p_H^*) \qquad \forall p_h \end{aligned}$$

The following lemma guarantees a Nash equilibrium exists and is common in papers employing spatial price competition. Its proof is left to the appendix.

Lemma 1 *In any Nash equilibrium of the two producer pricing game* (p_h^*, p_H^*), $p_h^* \geq c_h$, $p_H^* \geq c_H$ *and* $\max\{p_H^*, p_h^*\} \leq r$.

The lemma follows from simple intuition. The first part disallows Nash equilibria, in which firms set negative price-marginal cost margins. This can never be an equilibrium, because producers could always set $p_i^* = c_i$ and earn zero profits, instead of negative profits. The second part disallows situations where a firm sets such a high price that no consumers purchase its good.

In turn, the lemma implies a Nash equilibrium necessarily exists. Because profit is a continuous function of price profiles everywhere and because strategy sets can be restricted to non-empty compact intervals $p_i \in [c_i, r]$, invoking a standard result in game theory (see Fudenberg and Tirole [10]), a Nash equilibrium in prices necessarily exists, though this equilibrium may be mixed.

3.2 Finding Nash equilibria

Next, we begin the search for pure Nash equilibria with the property that all consumers are served by the second sector. In doing so, it will be necessary to establish some parameter conditions that guarantee the equilibria analyzed here have this property. Otherwise, producers do not need to compete for consumers and can simply ignore each other's presence as their markets are never in dispute. In specific, we will require $r \geq c_i + \frac{3t}{2}$ for $i = h, H$.

The simplest way to find Nash equilibria is to first eliminate branches of the profit function, where no pure strategy Nash equilibria can exist. The lemma from the previous subsection helps us in this direction. In specific, the fourth and fifth branch will be eliminated as candidate regions for pure Nash equilibria, in the main proposition of this section. Before I do so, let me foreshadow the reason. In the sections to come, I will analyze the welfare results of two variants of the same model. First, we will assume identical production technologies in each country to focus on differences in physical size (preferences) between countries as causing trade. Next, we will instead assume countries have identical preferences in order to focus on technological differences as causing trade. In both cases, country H imports the good from country h (by construction), though for very different reasons. Because Nash equilibria in the fourth and fifth branch would imply the opposite (i.e. that country h imports the good from country H) these can be eliminated immediately.[8] This is captured in the following proposition, whose proof is left to the appendix.

[8] This is by virtue of the cost and population assumptions. At equal costs, the larger country must be a non-exporter. At equal populations, the less efficient country must be a non-exporter. H is assumed to be either larger than h (section 4.1) with equal costs, or to have a higher marginal cost (section 4.2) with equal populations.

Proposition 2 *In the two producer pricing game, Z_4 and Z_5 cannot occur in a pure strategy Nash equilibrium.*

Then, the only candidates for pure strategy Nash equilibria lie in:

$$\begin{aligned}
Z_1(p_h, p_H) &\equiv \{(p_h, p_H) : \frac{t}{2} + k \leq p_H - p_h\}, \\
Z_2(p_h, p_H) &\equiv \{(p_h, p_H) : \frac{t}{2} + k - tH \leq p_H - p_h \leq \frac{t}{2} + k\} \text{ and} \\
Z_3(p_h, p_H) &\equiv \{(p_h, p_H) : \frac{t}{2} - k - tH \leq p_H - p_h \leq \frac{t}{2} + k - tH\}
\end{aligned}$$

The next step is to identify the only price profiles in each of these regions that can be pure strategy Nash equilibria. We will characterize Nash equilibria for each region. However, this does not imply that we have multiple equilibria; parameter conditions lead to a single equilibrium; changing parameters can move the equilibrium between regions.

First, consider an equilibrium with no international trade (i.e. autarky). This occurs when the marginal consumer locates on the border between the two countries (i.e. in Z_3). Also, note that as long as $Z_3(p_h, p_H)$ occurs in equilibrium, firms face an inelastic segment of their demand functions: small deviations from the price profile (p_h, p_H) yield no changes in demand. As a result, the best response of any producer must be to set the highest price that still yields this demand. For the firm in country H this occurs when $p_H^A = r - \frac{tH}{2}$ and for the firm in country h this occurs when $p_h^A = r - \frac{t}{2}(1 - H)$. In order to see this, suppose the producer in country H increases its price by $\varepsilon > 0$. Consider the utility of the consumer located at the border and in country H. This consumer can purchase from the domestic producer and incur an implicit price of $p_H^A + \varepsilon + \frac{tH}{2} > r$ or he can purchase from the foreign producer and incur an implicit price of $p_h^A + k + \frac{t(1-H)}{2} > r$. In either case, the consumer is better off not purchasing. Hence, any increase in price beyond p_H^A generates a reduction in demand. The same argument holds for the firm in country h. Hence, the only pure strategy Nash equilibrium in Z_3 is located at:

$$(p_h^A, p_H^A) = (r - \frac{t(1 - H)}{2}, r - \frac{tH}{2})$$

Because all consumers purchase from a domestic producer, it is natural to think of this as an Autarkic Nash equilibrium (ANE). Profits under the ANE are easily calculated as:

$$\begin{aligned}
\pi_h^A(k) &= (1 - H)(r - c_h - \frac{t(1 - H)}{2}) \\
\pi_H^A(k) &= H(r - c_H - \frac{tH}{2})
\end{aligned}$$

Note that ANE prices and profits are unaffected by the international trade barrier, k. This makes sense. As long as autarky characterizes the equilibrium, the actual height of the trade barrier is irrelevant. Of course, the existence of the ANE does depend on the trade barrier. In specific, if the trade barrier is small enough, the equilibrium will not be characterized by autarky. Technically, if k is sufficiently small, either firm can profitably deviate to lower prices in an attempt to capture an export market. Finding these deviations is actually quite simple. In figure 4, the ANE is identified. For the firm in country h, profitable deviations may exist to the first and second branch, while for the firm in country H, profitable deviations may exist to the fourth and fifth branch.[9] They are characterized by either the highest price in a region of inelastic demand (i.e. Z_1 and Z_5) or the FOC of a concave quadratic profit function (i.e. Z_2 and Z_4). Computing the deviation profits and subtracting from π_i^A above, we can define:

$$\overline{k}_i \equiv \left\{ \begin{array}{ll} (1-i)(r-c_i) + t\frac{i^2+i-2}{2} & \text{if } \frac{r-c_i}{t} \geq \frac{i^2+2}{2i} \\ r - c_i + \frac{ti}{2} - 2t\sqrt{i(\frac{r-c_i}{t}) - \frac{i^2}{2}} & \text{if } \frac{r-c_i}{t} \leq \frac{i^2+2}{2i} \end{array} \right\} \text{ for } i = h, H$$

such that if $k \geq \overline{k}_i$, then given $p_{-i} = p_{-i}^A$, the best response for the firm in country i is to set its ANE price. Then, letting $\overline{k} \equiv \max\{\overline{k}_h, \overline{k}_H\}$, we have that the ANE exists if and only if $k \geq \overline{k}$.

Next, consider the equilibrium that results when international trade occurs and both producers sell positive amounts of their good (i.e. $Z_2(p_h, p_H)$). In this case, $\overline{x} \in (0, \frac{H}{2})$. This is in a concave quadratic region of both firms' profit functions. Differentiating the profits of the second branch,

$$\begin{aligned} \left(\frac{\partial \pi_H}{\partial p_H}\right)_{\widetilde{Z}_2(p_h,p_H)} &= \frac{p_h - 2p_H + c_H}{t} + \frac{k}{t} + \frac{1}{2} \\ \left(\frac{\partial \pi_h}{\partial p_h}\right)_{\widetilde{Z}_2(p_h,p_H)} &= \frac{p_H - 2p_h + c_h}{t} - \frac{k}{t} + \frac{1}{2} \end{aligned}$$

and solving the system of linear equations, we get:

$$(p_h^{2T}, p_H^{2T}) = (\frac{2c_h + c_H}{3} - \frac{k}{3} + \frac{t}{2}, \frac{c_h + 2c_H}{3} + \frac{k}{3} + \frac{t}{2})$$

In this case, because consumers in country H are choosing to purchase from the foreign producer, it is natural to call this the second branch Trade Nash equilibrium (2TNE). Profits under the 2TNE are easily calculated as:

$$\begin{aligned} \pi_h^{2T} &= \frac{1}{t}(\frac{c_H - c_h}{3} - \frac{k}{3} + \frac{t}{2})^2 \\ \pi_H^{2T} &= \frac{1}{t}(\frac{c_h - c_H}{3} + \frac{k}{3} + \frac{t}{2})^2 \end{aligned}$$

Unlike the ANE, 2TNE prices and profits are affected by both the technological efficiency difference (i.e. $c_H - c_h$) and the international trade barrier (i.e. k). Thus, it is not surprising that

[9] Deviations to prices higher than the ANE are not profitable.

the existence of the 2TNE should depend on both. In specific, in order for the 2TNE to exist, neither the efficiency difference nor the trade barrier can be too large.

First, consider what happens when the efficiency difference is large. To see why a large efficiency difference can eliminate the 2TNE, we calculate the location of the marginal consumer in the second branch during the 2TNE.

$$\overline{x}^T = \frac{p_h^T - p_H^T}{2t} + \frac{k}{2t} + \frac{1}{4} = -\left(\frac{c_H - c_h}{6t}\right) + \frac{k}{6t} + \frac{1}{4}$$

From this equation, it is clear that if the technological difference (i.e. $c_H - c_h$) is sufficiently large, then $\overline{x}^T \leq 0$. What happens to the equilibrium when the efficiency difference is so large that $\overline{x}^T = 0$? Technically, the equilibrium point reaches the first branch and the 2TNE ceases to exist. However, this does not mean that once $\overline{x}^T = 0$, we have no Nash equilibrium. Instead, we have a different trade equilibrium in the first branch. Then, in order to know what happens to the equilibrium for large technological differences, we need to first search for Nash equilibria in the first branch. This is actually quite easy to do. Note that the first branch is an inelastic segment of demand. So, the firm in country h sets the highest price that still attracts all consumers: $p_h^* = p_H^* - k - \frac{t}{2}$. Now, we argue that $p_H^* > c_H$ cannot be part of an equilibrium. If so, the firm in country H has a profitable deviation $\widehat{p}_H = p_H^* - \varepsilon$, where ε is sufficiently small. Note that $\widehat{p}_H - p_h^* = k + \frac{t}{2} - \varepsilon$ implies that under the deviation the firm in country H attracts a positive measure of consumers (because the deviation moves the price profile to the second branch). Finally, if ε is chosen sufficiently small, then $\widehat{p}_H > c_H$ and the firm earns a positive price-marginal cost margin. By the lemma of the previous section, the only pure Nash equilibrium in the first branch is given by

$$(p_h^{1T}, p_H^{1T}) = (c_H - k - \frac{t}{2}, c_H)$$

yielding profits of:

$$\begin{aligned} \pi_h^{1T} &= c_H - c_h - k - \frac{t}{2} \\ \pi_H^{1T} &= 0 \end{aligned}$$

Again, because consumers in country H are choosing to purchase from the foreign producer, it is natural to call this the first branch Trade Nash equilibrium (1TNE). It is important to note that the 2TNE and the 1TNE coincide exactly when $\overline{x}^T = 0$.[10] That is, once $\overline{x}^T \leq 0$ (or $k \leq c_H - c_h - \frac{3t}{2}$), a Nash equilibrium continues to exist, though it is now characterized by 1TNE prices, rather than 2TNE prices. In turn, equilibrium profit and surplus also coincide at $\overline{x}^T = 0$. Thus, we conclude that as the efficiency difference rises, the equilibrium moves

[10] To verify this, note that $\overline{x}^T = 0$ occurs when $k = c_H - c_h - \frac{3t}{2}$. Then, plug this expression for k, into the 2TNE and 1TNE price profiles.

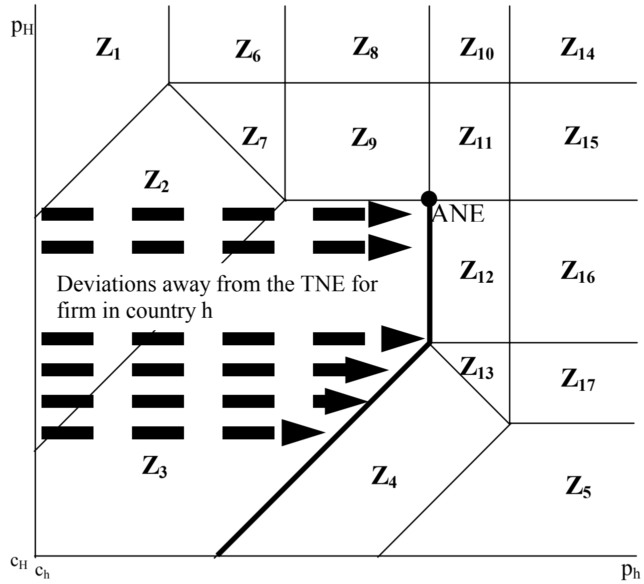

Figure 5: Deviations away from the TNE for the firm in country h

continuously between the 2TNE and the 1TNE. In fact, it is easiest if we take the 2TNE and 1TNE to be parts of the same equilibrium, with the understanding that the 1TNE occurs only when the countries differ greatly in their technological efficiency. For simplicity, I will call this simply as the TNE. In specific, we can write:

$$(p_h^T, p_H^T) = \left\{ \begin{array}{ll} (c_H - k - \frac{t}{2}, c_H) & \text{if } k \leq c_H - c_h - \frac{3t}{2} \\ (\frac{2c_h + c_H}{3} - \frac{k}{3} + \frac{t}{2}, \frac{c_h + 2c_H}{3} + \frac{k}{3} + \frac{t}{2}) & \text{if } k \geq c_H - c_h - \frac{3t}{2} \end{array} \right\}$$

yielding profits of:

$$(\pi_h^T, \pi_H^T) = \left\{ \begin{array}{ll} (c_H - c_h - k - \frac{t}{2}, 0) & \text{if } k \leq c_H - c_h - \frac{3t}{2} \\ (\frac{1}{t}(\frac{c_H - c_h}{3} - \frac{k}{3} + \frac{t}{2})^2, \frac{1}{t}(\frac{c_h - c_H}{3} + \frac{k}{3} + \frac{t}{2})^2) & \text{if } k \geq c_H - c_h - \frac{3t}{2} \end{array} \right\}$$

Second, we are now in a position to examine how large international trade barriers can eliminate the existence of the TNE. Careful examination shows that if the trade barrier is large enough, the TNE may not be characterized by international trade. Technically, if k is sufficiently large, the firm in country h (the more efficient or smaller country) can profitably deviate to higher prices that attract only domestic consumers and no foreign consumers. Finding these deviations is actually quite simple. In figure 5, the TNE lies in either Z_1 or Z_2. For the firm in country h, profitable deviations may exist to the third branch. They are characterized by the highest price in this region of inelastic demand. A quick glance at figure 5 shows there are four possibilities. First, the firm in country h can deviate to the line of unit slope separating the third and fourth region from its 1TNE price. Secondly, the same can happen, but from its 2TNE price. Thirdly,

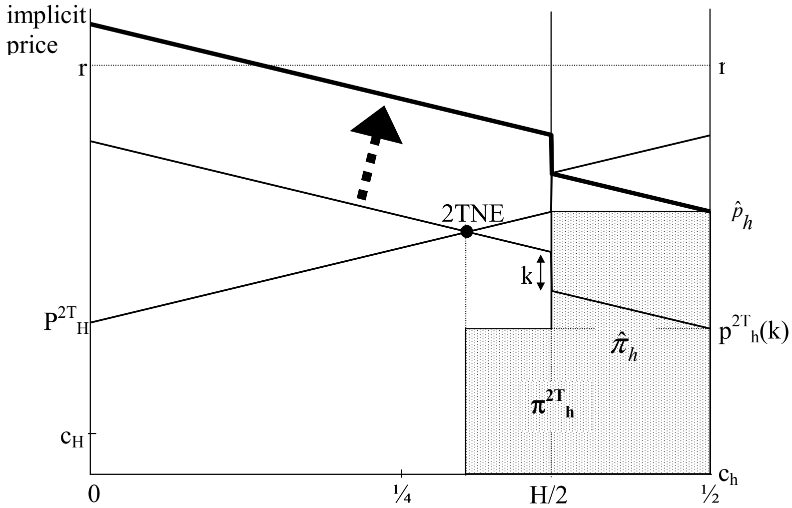

Figure 6: Profitable deviation for the producer in country h that eliminate the 2TNE

the firm in country h can deviate to its ANE price from its 1TNE price. And, finally, it can do the same from its 2TNE price. Figure 6 illustrates the second case. The vertical axis in this diagram measures the implicit price paid by consumers. The diagram illustrates only the upper semicircle. Clearly, the diagram of the lower semicircle is symmetric to this one. In order to characterize all the deviations that can eliminate the TNE, we also need to consider both cases for the 1TNE. Computing the deviation profits, subtracting from π_h^T above and letting $\Delta \equiv \frac{c_H - c_h}{t}$, we can define:

$$\underline{k} \equiv \left\{ \begin{array}{ll} t(2-H)^{-1}[H\Delta - \frac{1}{2} - (H - \frac{1}{2})(1-H)] & \text{if } k \leq t(\Delta - \frac{3}{2}) \\ & \text{and } B \\ t\left[\Delta - (1-H)(\frac{r-c_h}{t}) + \frac{(1-H)^2}{2} - \frac{1}{2}\right] & \text{if } k \leq t(\Delta - \frac{3}{2}) \\ & \text{and } \sim B \\ t\left[\Delta + \frac{3}{2} - \frac{3}{2}\sqrt{4(1-H)(\frac{r-c_h}{t}) - 2(1-H)^2}\right] & \text{if } k \geq t(\Delta - \frac{3}{2}) \\ & \text{and } A \\ t\left[\Delta + 6(1-H) + \frac{3}{2} - 3\sqrt{(1-H)[2\Delta + 3(2-H)]}\right] & \text{if } k \geq t(\Delta - \frac{3}{2}) \\ & \text{and } \sim A \end{array} \right\}$$

where $\sim A$ means the converse of A and where A and B are the following conditions:[11]

$$A \; : \; \frac{c_H - c_h}{t} - \frac{r - c_H}{t} + \frac{H}{2} + \frac{5}{2} \geq 2\sqrt{4(1-H)(\frac{r-c_h}{t}) - 2(1-H)^2}$$

$$B \; : \; H\frac{r-c_h}{t} - 2\frac{r-c_H}{t} + \frac{H^2}{2} - \frac{H}{2} \leq 0$$

[11] Note the first two branches of $\overline{k}$ represent deviations away from 1TNE and the second two branches represent deviations away from 2TNE.

It follows that if $k \leq \underline{k}$ then the deviations showcased above are not profitable and the TNE exists.[12]

Finally, it can be shown that $\overline{k} > \underline{k}$ in the two variants of the model considered here. As a result, when $\overline{k} > k > \underline{k}$, the model necessarily exhibits some "turbulence" in the sense of there being a mixed strategy Nash equilibrium. This equilibrium is rather complicated. Though a better understanding of the equilibrium in this region might be in order, we believe that in terms of the interesting welfare questions that can be answered, little new would be learned. By concentrating the analysis on the welfare properties of the pure strategy Nash equilibria derived here, we can answer the interesting welfare questions raised in the Introduction. The next section does just this.

4 Welfare Analysis

This section analyzes the welfare implications of changing the international trade barrier in the two variants of this model. In the first, we simplify the analysis by assuming identical production technologies in each country. This allows us to focus on a model where countries differ in terms of their size. Using the classical terminology, we will focus on different preferences as a cause of trade. Of course, the interpretation given here is much broader. Notice that countries that are larger have more consumers. They have a larger consumer pool.[13] In the second, we simplify the analysis by assuming countries are of the same size (have the same preferences). This allows us to focus on a model in which international trade occurs because countries differ technologically. For both variants, we analyze the welfare properties of the model's pure strategy equilibrium for all $k \geq 0$ for which a pure strategy equilibrium exists.

4.1 Differences in Preferences

Suppose countries have identical technologies: $c_h = c_H \equiv c$. By arguments made in the preceding section, we know there cannot be a Nash equilibrium in the first branch. As a result, the TNE reduces simply to just the 2TNE. We will however assume countries strictly differ in terms of their preferences in order to generate international trade. This is accomplished by assuming $H > h$. As a result, in this subsection, we will refer to country H and h as the "large" and

[12] Recall that if $c_H = c_h$ and $H = \frac{1}{2}$, countries are identical and we should not expect trade as an equilibrium. Indeed, when these parameter restrictions are imposed, we get that $\underline{k} \leq 0$, implying the TNE never exists. In chapters 4 and 5, $\underline{k}$ is drawn as strictly positive, because this is the non-trivial case, in which international trade occurs.

[13] It is also important to note that this is not the only way we could introduce preference asymmetry. Indeed, one variant might be to analyze a model where countries differ only on their uniform level of density. The welfare results are essentially the same.

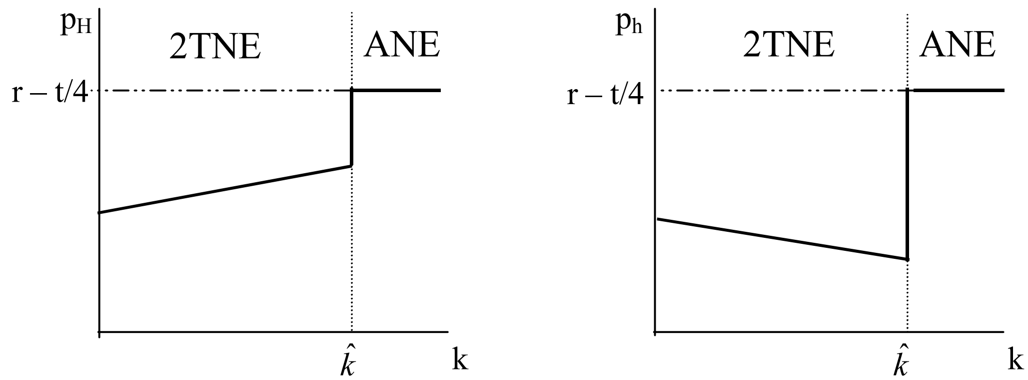

Figure 7: Equilibrium price path (differences in preferences)

"small" country respectively.

Does the raising of barriers to international trade necessarily lead to higher prices in both countries? Figure 7 illustrates how changing k affects the equilibrium price in each country. Price in the large country is monotonically increasing in k. This is straightforward to see during the 2TNE, where $p_H^T = c + \frac{t}{2} + \frac{k}{3}$. However, because we are interested in all $k \geq 0$, we must also know how the equilibrium strategy changes in moving between the 2TNE and the ANE. Some simple calculations verify that under our parameter assumptions the price in the large country is highest under the ANE.

On the other hand, price in the small country follows a non-monotonic path before the ANE. Initially, increases in k make the firm in the small country reduce its price, in order to entice foreign consumers to continue purchasing the good from it. During the 2TNE, this follows from $p_h^T = c + \frac{t}{2} - \frac{k}{3}$. However, once k is large enough, this firm requires offering so low a price to attract foreign consumers that it is better off to raise price and have no export sales. At this point, focusing on the domestic market allows the firm in the small country to raise its price substantially above the going rate of even the large country.

Can lower international trade barriers shift rent from one country to another? Recall that increases in k move the equilibrium location of the marginal consumer towards the border. That is, as k increases, the firm in the large country attracts a larger demand and the firm in the small country attracts a smaller demand. These observations imply that as long as countries are in a 2TNE, increases in k increase the profits of the firm in the large country and decrease the profits of the firm in the small country. This is exactly what we mean by rent shifting. As the barriers to international trade decrease, the producer in the small country extracts more surplus from the large country. This of course does not tell us what will happen to profits once the ANE is encountered. Figure 8 illustrates how producer surplus is affected by changes in the international trade barrier.

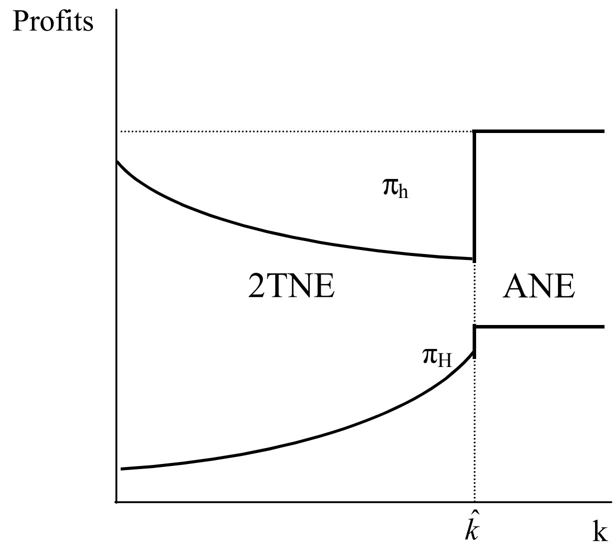

Figure 8: Equilibrium profits (differences in preferences)

Can rent shifting eliminate the gains from lower trade barriers? In order to assess this, we calculate the consumer surplus that results in each country under both the 2TNE and ANE:

$$CS_h(k) = \left\{ \begin{array}{ll} (1-H)(r-c+\frac{k}{3}+\frac{tH}{4}-\frac{3t}{4}) & \text{in 2TNE} \\ \frac{t(1-H)^2}{4} & \text{in ANE} \end{array} \right\}$$

$$CS_H(k) = \left\{ \begin{array}{ll} H(r-c)-tH+\frac{tH^2}{4}+\frac{t}{8}+\frac{k^2}{18t}+\frac{k}{6}-\frac{2kH}{3} & \text{in 2TNE} \\ \frac{tH^2}{4} & \text{in ANE} \end{array} \right\}$$

Consumer surplus in each country is smallest when trade barriers are so high that exporting is not profitable. This is not too surprising as consumers in all countries face higher prices under the ANE. Adding profits and consumer surplus, we obtain welfare in each country,

$$W_h(k) = \left\{ \begin{array}{ll} \frac{k^2}{9t}-\frac{kH}{3}+(1-H)(r-c)-\frac{tH^2}{4}-\frac{t}{2}+tH & \text{in 2TNE} \\ (1-H)(r-c)-\frac{t(1-H)^2}{4} & \text{in ANE} \end{array} \right\}$$

$$W_H(k) = \left\{ \begin{array}{ll} \frac{k^2}{6t}-\frac{2kH}{3}+\frac{k}{2}+H(r-c)-tH+\frac{tH^2}{4}+\frac{3t}{8} & \text{in 2TNE} \\ H(r-c)-\frac{tH^2}{4} & \text{in ANE} \end{array} \right\}$$

Because the small country exports the good, rent shifting can only enhance the gains from trade for this country. Indeed, as in figure 9, the welfare in this country decreases as a function of the size of the trade barrier. However, can rent shifting affect the usual results on gains from trade in the large country? Yes, the large country finds that erecting large barriers to trade is surplus maximizing, as can be seen in figure 10. Notice in the left panel that if the countries

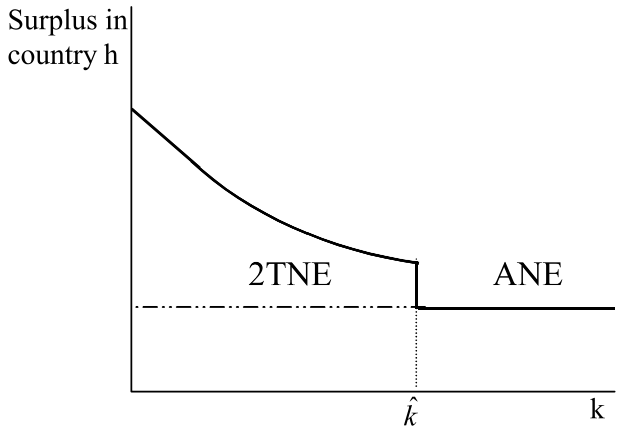

Figure 9: Country h equilibrium surplus (differences in preferences)

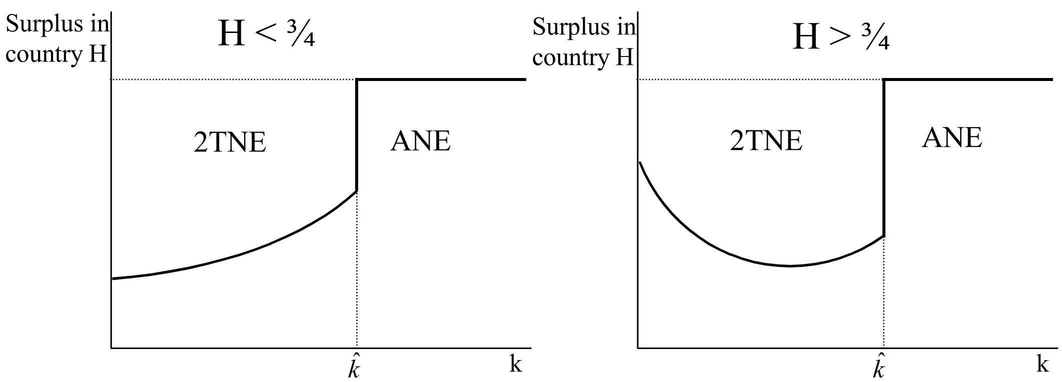

Figure 10: Country H equilibrium surplus (differences in preferences)

are similar in size (i.e. $H < \frac{3}{4}$), then welfare is a monotonically increasing function of the trade barrier. However, this is not true when countries are sufficiently different in size as illustrated in the right panel. The welfare observations made in this subsection are summarized in the remark below.

Summary 3 *Assume international trade is caused by differences in country size or preference. Price in the large country is monotonically increasing in the size of the trade barrier. On the other hand, price in the small country is decreasing in the size of the trade barrier, as long as the barrier is sufficiently low as to permit profitable exporting. However, once the trade barrier is so high that international trade is not profitable, price in the small country discontinuously increases as the firm in this country profitably ignores the export market. As would be expected, welfare in the small country is maximized when no barriers to international trade exist, because* *in this case the small country enjoys a more competitive market structure and an export market. On the other hand, despite the fact that without trade barriers consumers in the large country enjoy a more competitive market structure and lower transport costs, the international trade that occurs between the countries shifts enough producer surplus from one country to another to reduce the large country's welfare. As a result, welfare in the large country is maximized when trade barriers are large enough to prevent international trade from occurring in equilibrium.*

4.2 Differences in Technology

Suppose countries are of the same size (i.e. contain same number of consumers or $H = h$). So that international trade occurs in this subsection, we assume countries have access to strictly different technologies (i.e. $c_H > c_h$). Then, in this subsection, we will refer to country H and h as the less efficient and more efficient country respectively.

Define $\Delta \equiv \frac{c_H - c_h}{t}$. This is a measure of the efficiency difference between the technologies available to each country. Recall from section 3 that if the technologies available in each country are sufficiently different, then the 1TNE (and not the 2TNE) exists. This possibility will require we consider the existence of all three pure strategy Nash equilibria. In specific, depending on the size of the efficiency difference, the equilibrium for all $k \geq 0$ can take three different possible paths:

Case 1: $0 < \Delta \leq \frac{3}{2}$. 1TNE plays no role.[14] That is, for all $k \geq 0$, the only pure strategy Nash equilibria are the 2TNE and the ANE.

Case 2: $\frac{3}{2} < \Delta \leq \frac{7}{4}$: when $k = 0$, the equilibrium is characterized by the 1TNE. However, as k rises, prices move continuously between the 1TNE and the 2TNE at $k = c_H - c_h - \frac{3t}{2}$. That is, deviations away from the 1TNE to the third autarkic branch are never profitable. Instead, once the 2TNE is reached, as k continues to rise, deviations away from the 2TNE to the third autarkic branch eventually become profitable. At this point, the TNE ceases to exist. As k continues to rise, eventually the ANE will exist.

Case 3: $\frac{7}{4} \leq \Delta$: As before, when $k = 0$, the equilibrium is characterized by the 1TNE. However, as k rises, deviations away from the 1TNE to the third branch eventually become profitable, before k reaches $c_H - c_h - \frac{3t}{2}$. At this point, the 1TNE ceases to exist. The equilibrium path never passes through the 2TNE. Instead, as k continues to rise, eventually the ANE will exist.

We could analyze prices, profits and surplus in each country and in each case. However, many of the welfare effects showcased in the model of the previous subsection are the same here. For expositional purposes, the diagrams will illustrate all three equilibria (case 2 above) although the 1TNE does not occur to the left of the 2TNE in case 1 and the 2TNE does not

[14] Otherwise, $k \geq 0 > c_H - c_h - \frac{3t}{2}$ would imply the 1TNE does not occur

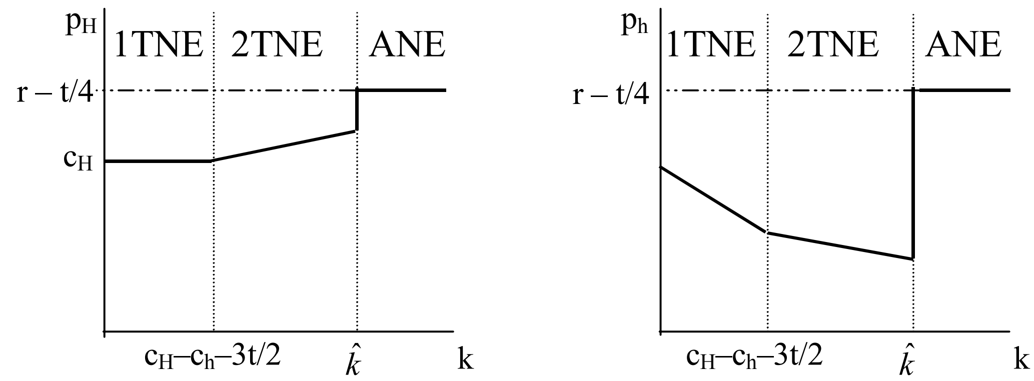

Figure 11: Equilibrium price path (differences in technology)

occur between 1TNE and ANE in case 3. Welfare in the less efficient country will on the other hand be discussed in depth.

First, how do prices behave when differences in technological efficiency cause trade? Figure 11 illustrates the behavior of prices as a function of the height of the international trade barrier. Price in the less efficient country is (weakly) monotonically increasing in k. This is straightforward to see during the 1TNE (in which case $p_H^{1T} = c_H$), during the 2TNE (in which case $p_H^{2T} = \frac{c_h+2c_H}{3} + \frac{k}{3} + \frac{t}{2}$) and during the ANE (in which case $p_H^A = r - \frac{t}{4}$). As mentioned previously, the equilibrium prices move continuously between the 1TNE and the 2TNE, when they both occur. However, when moving between the TNE and the ANE, matters are more complicated. As in the differences in preference variant, it can be shown that under our parameter assumptions the price in the less efficient country is highest under the ANE.

On the other hand, price in the more efficient country follows a non-monotonic path. Initially, increases in k make the firm in the more efficient country reduce its price, in order to entice foreign consumers to continue purchasing the good from it. During the 1TNE this follows from $p_h^{1T} = c_H - k - \frac{t}{2}$ and during the 2TNE this follows from $p_h^{2T} = \frac{2c_h+c_H}{3} - \frac{k}{3} + \frac{t}{2}$. However, once k is large enough, this firm requires offering so low a price to attract foreign consumers that it is better off to raise price and have no export sales. At this point, the ANE occurs and price in the more efficient country is constant as a function of k at $p_h^A = r - \frac{t}{4}$.

Second, how strong is the case for rent shifting when differences in technology cause trade? Can lower international trade barriers shift rent from one country to another? Recall that increases in k (weakly) move the equilibrium location of the marginal consumer towards the border. That is, as k increases, the firm in the less efficient country attracts a larger demand and the firm in the more efficient country attracts a smaller demand. These observations imply that as long as countries are in a TNE, increases in k increase the profits of the firm in the less efficient country and decrease the profits of the firm in the more efficient country. This is exactly

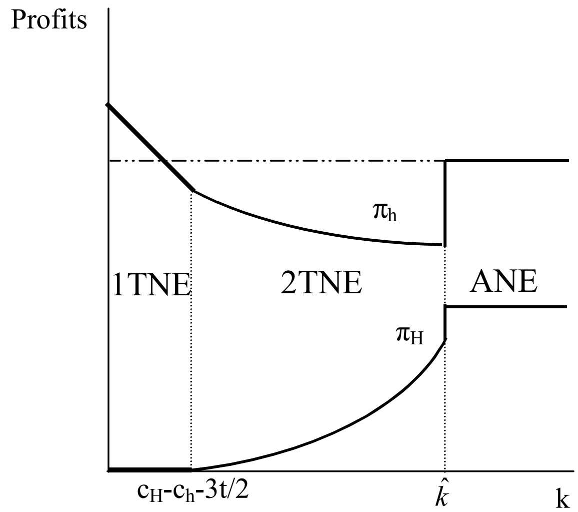

Figure 12: Equilibrium profits (differences in technology)

what happened in the difference in preferences variant. As the barriers to international trade decrease, the producer in the more efficient country extracts more surplus from the less efficient country. In fact, the case for rent-shifting is strengthened in this variant, as very small trade barriers can lead to the 1TNE, in which case the producer in the less efficient country earns no profits. This of course does not tell us what will happen to profits once the ANE is encountered. Figure 12 illustrates how producer surplus is affected by changes in the international trade barrier.

Does rent shifting eliminate the gains from eliminating all trade barriers when technological differences cause trade? In order to assess this, we calculate the consumer surplus that results in each country under each equilibrium.

$$CS_h = \left\{ \begin{array}{ll} \frac{1}{2}((r - c_H) + k + \frac{3t}{8}) & \text{in 1TNE} \\ \frac{1}{4}(2r - \frac{4c_h+2c_H}{3} + \frac{2k}{3} - \frac{5t}{4}) & \text{in 2TNE} \\ \frac{t}{16} & \text{in ANE} \end{array} \right\}$$

$$CS_H = \left\{ \begin{array}{ll} \frac{1}{2}(r - c_H) + \frac{t}{16} & \text{in 1TNE} \\ \frac{r}{2} - \frac{c_h+c_H}{4} - \frac{c_H-c_h}{12} + \frac{(c_H-c_h)^2}{18t} - \frac{k(c_H-c_h)}{9t} + \frac{k^2}{18t} - \frac{k}{6} - \frac{5t}{16} & \text{in 2TNE} \\ \frac{t}{16} & \text{in ANE} \end{array} \right\}$$

Again, consumer surplus in each country is smallest when trade barriers are so high that exporting is not profitable. Adding profits and consumer surplus, we obtain welfare in each country,

$$W_h = \left\{ \begin{array}{ll} \frac{1}{2}(r - c_H) + c_H - c_h - \frac{k}{2} - \frac{5t}{16} & \text{in 1TNE} \\ \frac{(c_H - c_h)^2}{9t} - \frac{2}{9}\frac{c_H - c_h}{t}k + \frac{k^2}{9t} + \frac{r}{2} + \frac{c_H - 4c_h}{6} - \frac{k}{6} - \frac{t}{16} & \text{in 2TNE} \\ \frac{1}{2}(r - c_h) - \frac{t}{16} & \text{in ANE} \end{array} \right\}$$

$$W_H = \left\{ \begin{array}{ll} \frac{1}{2}(r - c_H) + \frac{t}{16} & \text{in 1TNE} \\ \frac{r}{2} - \frac{c_h + c_H}{4} - \frac{5}{12}(c_H - c_h) + \frac{(c_H - c_h)^2}{6t} - \frac{k(c_H - c_h)}{3t} + \frac{k^2}{6t} + \frac{k}{6} - \frac{t}{16} & \text{in 2TNE} \\ \frac{1}{2}(r - c_H) - \frac{t}{16} & \text{in ANE} \end{array} \right\}$$

Welfare in the more efficient country behaves almost identically to the previous subsection: this country finds that no trade barriers is surplus maximizing. See figure 13.

What about welfare in the less efficient country? Should the less efficient country protect its industry? The answers to these questions depend on the size of the efficiency difference. When the efficiency difference is small, the less efficient country finds that erecting large barriers to international trade is surplus maximizing. This occurs when $0 < \Delta < 1$. This is illustrated in both panels of figure 14 and in the left panel of figure 15 and mimics the diagrams of the previous subsection.

However, once $\Delta > 1$, the less efficient country finds that eliminating all barriers to international trade is surplus maximizing. The intuition is quite straightforward. As in any standard model of international trade, consumers gain from lower barriers to trade. However, the larger the efficiency difference, the larger this gain. When the efficiency difference is very large, consumers in the less efficient country gain more than the producer in this country loses from lower trade barriers. In effect, raising trade barriers when the efficiency difference is large is equivalent to denying consumers in the less efficient country access to the much more efficiently produced good.

We have not yet mentioned 1TNE. Recall that 1TNE does not play a role if $\Delta \leq \frac{3}{2}$. The right panel of figure 16 shows how welfare in the less efficient country is affected by the possibility of having equilibrium 1TNE exist when trade barriers are very low. Finally, recall that once $\frac{7}{4} \leq \Delta$, the 2TNE never exists. Welfare in the less efficient country in this case is illustrated in figure 17. The welfare observations made in this subsection are summarized in the remark below.

Summary 4 *Assume international trade is caused by differences in the technologies countries can access. Price in the less efficient country is (weakly) increasing in the size of the trade barrier. On the other hand, price in the more efficient country is decreasing in the size of the trade barrier, as long as the barrier is sufficiently low as to permit profitable exporting. However, once the trade barrier is so high that international trade is not profitable, price in the more efficient country discontinuously increases as the firm in this country profitably ignores the export market. As would be expected, welfare in the more efficient country is maximized when*

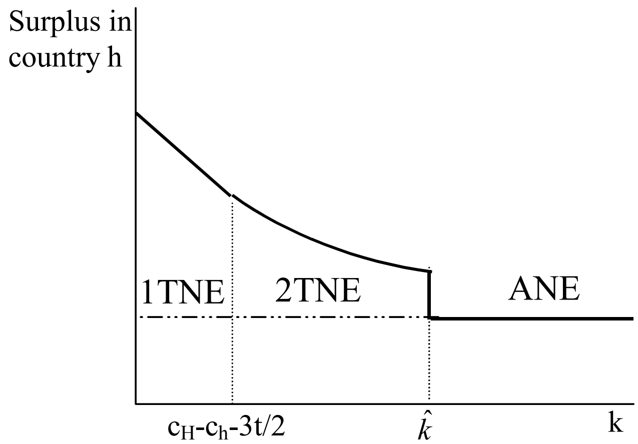

Figure 13: Country h equilibrium surplus (differences in Technology)

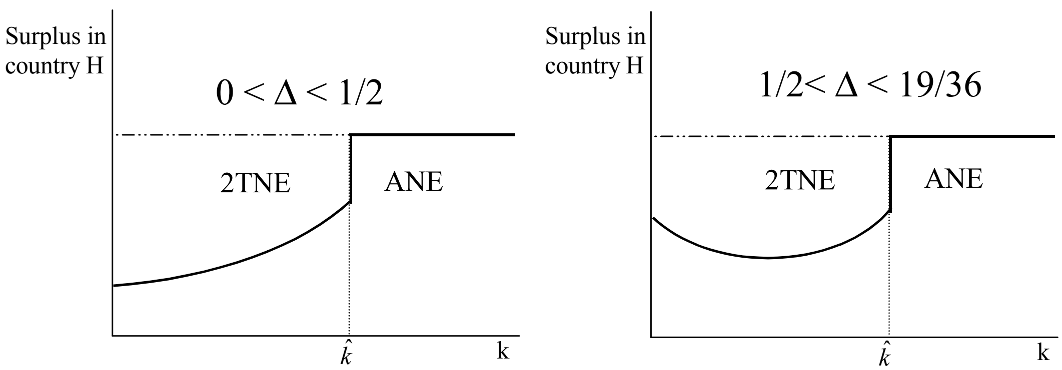

Figure 14: Country H equilibrium surplus (differences in technology)

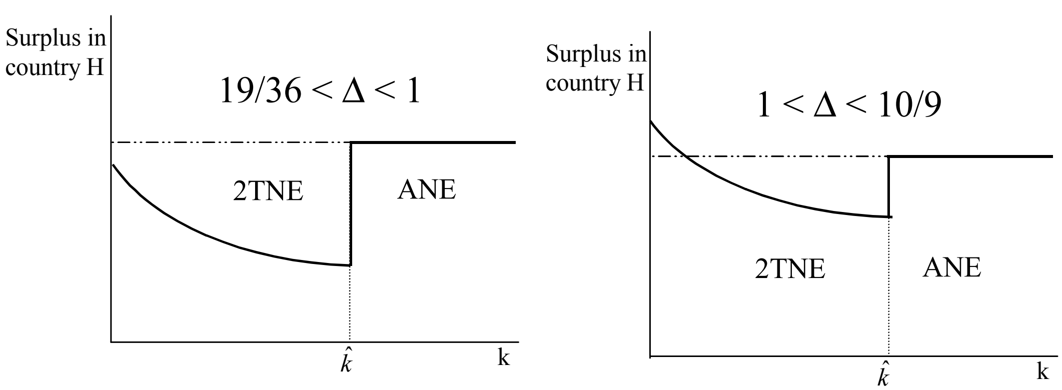

Figure 15: Country H equilibrium surplus (differences in technology)

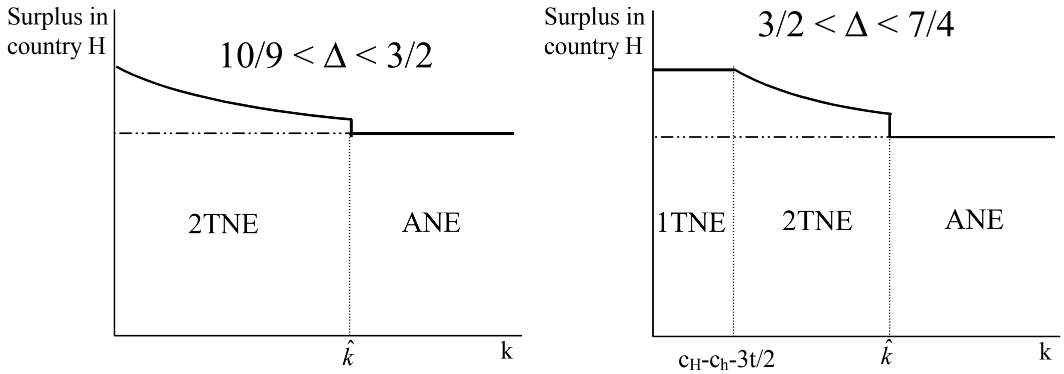

Figure 16: Country H equilibrium surplus (differences in technology)

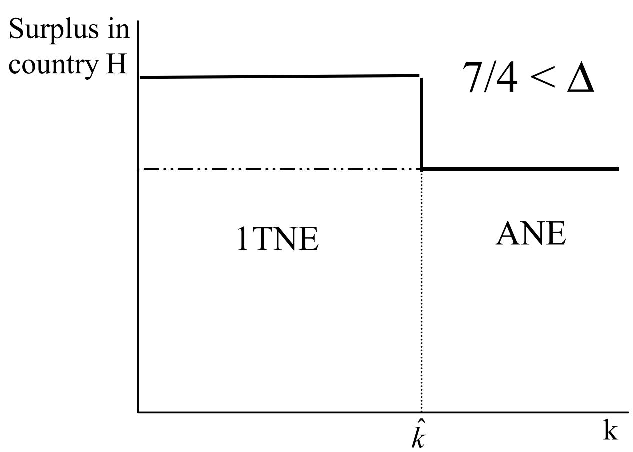

Figure 17: Country H equilibrium surplus (differences in technology)

no barriers to international trade exist. On the other hand, welfare in the less efficient country is maximized by either the elimination of all trade barriers or the erection of sufficiently large trade barriers that prevent international trade from occurring in equilibrium. The former occurs when the countries have access to sufficiently different technologies and the latter occurs when the countries employ sufficiently similar technologies.

5 Conclusion

Consider an industry characterized by market power in two nearby countries. The countries can be insulated from each other for many reasons. Oceans, mountain ranges and deserts can provide natural examples of geographic insulation. But, they need not be the only examples. Language can on occasion serve to insulate two economies as it raises the costs needed to transact with foreign counterparts. But, most generally, consider all the additional costs associated with purchasing from a foreign firm over a domestic firm that help to insulate one economy from another. These are exogenous barriers to international trade. Finally, there can also be legal barriers to trade which can serve to insulate two economies, such as trade policy. What we examine are the effects of altering the degree of insulation between two countries.[15]

We show that depending on the height of trade barriers and the difference in technological efficiency, one of three pure strategy Nash equilibria can result. When barriers to international trade are small, a Nash equilibrium with trade between countries results. On the other hand, when barriers to international trade are prohibitively high, a Nash equilibrium without international trade results. Moreover, when countries employ very different technologies, a Nash equilibrium in which the less efficient producer sells nothing occurs.

Following standard practice in classical international trade theory, we analyze the welfare properties of two variants of the same model: one in which differences in country size or (preferences) cause trade and one in which differences in technology cause trade. For both variants, the key results are that consumers gain from lower trade barriers as producers across the countries compete more strongly. Producers are however disadvantaged by this competition, with the firm in the large (or less efficient) country losing share and profits to its international rival. This is known as rent shifting and is a standard feature of strategic trade models. In the first variant, it is shown that international trade causes rent to be shifted from the large to the small country and that regardless of country size, the large country is better off erecting large barriers to international trade. In the second variant, it is shown that international trade causes rent to be shifted from the less efficient country to the more efficient country. When the efficiency difference is

[15] Regarding trade policy, we only analyze cases which would "burn money" in the sense that neither population reaps tax benefits. Examples include stringent border tests on imports, where the same tests are not applied to domestic products.

small, the less efficient country is again better off erecting larger barriers to international trade. However, when the efficiency difference is large, protecting its less efficient industry makes this country worse off, because its citizens are denied access to a more efficiently produced good.

Does the model of this paper reflect absolute reality in all sectors? Certainly not. There are many sectors which do not meet the assumptions of this model. There are a few, however, that might. It might be a sector that is subject to a severe case of increasing returns, so that the case for positive profits can be made relatively easy. It must also be a sector in which international trade could occur. One sector that might apply is commercial airplane manufacturing, where the hegemony of Airbus and Boeing seem to fit nicely with the assumption of only two producers in this paper. Electric generation is another potential sector to which the model of this paper may apply, though international trade in electricity is still very small. Possibly the best industry to which the model of this paper might apply is the steel industry. It is well known that the U.S. steel industry has for a long time enjoyed protection against foreign imports. A long time ago, it was argued that foreign steel imports were of lower quality. Today, this is no longer the case. Instead, the on-going protection of the U.S. steel industry seems to rely on the argument of unfair competition.[16] For instance, it is commonly argued that Mexican steel companies do not care about the environment, pay their workers very little and do not take action to care for the health of their workers. This allows Mexican steel companies to have low marginal costs, with which U.S. steel companies claim they cannot compete.

In this paper, we build a model which allows an industry to argue in favor of protection: countries may lose from trade because of rent shifting. And, despite the recent optimism with the elimination of trade barriers globally, many countries still have significant trade protection schemes. So, we need a model such as the one in this paper in order to explain the current state of affairs. However, what the model of this paper shows is that when the marginal costs of production are very different, less efficient countries have nothing to fear from rent shifting: consumers will end up winning even if domestic industries are wiped out. So, in arguing that Mexican steel is produced at a much lower marginal cost, the U.S. steel industry may actually be making the case for free trade: a revindication of the original arguments of comparative advantage in favor of free trade.

6 Appendix

Proof Lemma 1. [Lemma 1.1] First, note that any price profile (p_h^*, p_H^*) such that $p_i^* < c_i$ cannot be a NE. If $p_i^* < c_i$ in some NE, then the firm with the lower price attracts some positive measure of consumers at a negative price-marginal cost margin. As a result, it earns negative profits. However, this firm has a profitable deviation. Suppose instead it sets a price equal to

[16]Security concerns are also cited as an argument in favor of protection of the U.S. steel industry.

c_i. Then, it earns zero profits. As a result, in any Nash equilibrium of the two firm pricing game $p_i^* \geq c_i$. Next, note that any price profile (p_h^*, p_H^*) such that $\max\{p_h^*, p_H^*\} > r$ cannot be a NE either. Let this be the firm in country i. Then, this firm has a profitable deviation. By setting a price of $\min\{r - \varepsilon, p_{-i}^* + (c_i - c_{-i})\}$ (where $\varepsilon > 0$ is small), firm i can sell a positive quantity, at a positive price-marginal cost margin and hence earn positive profits. In conclusion, we have that in any price NE (p_h^*, p_H^*) it must be that $\min\{p_h^*, p_H^*\} \geq c$ and $\max\{p_h^*, p_H^*\} \leq r$. ■

Proof Prop. 2. [Prop. 1.1] Define the interior of the fourth branch as:

$$\widetilde{Z}_4(p_h, p_H) \equiv \{(p_h, p_H) \in \mathbb{R}_+^2 : -\frac{t}{2} - k < p_H - p_h < \frac{t}{2} - k - tH\}$$

What happens when $\widetilde{Z}_4(p_h^*, p_H^*)$ occurs in equilibrium? Then, $\overline{x} \in (\frac{H}{2}, \frac{1}{2})$: consumers in the small country purchase the good from the firm in the large country. Notice that on this branch, π_i is concave quadratic in p_i. Finally, in order for there to be an equilibrium in the interior of the fourth branch, it must be characterized by the FOCs of each firm's maximization problem. Taking the appropriate derivatives,

$$\begin{aligned}\left(\frac{\partial \pi_H}{\partial p_H}\right)_{\widetilde{Z}_4(p_h, p_H)} &= \frac{p_h - 2p_H + c_H}{t} - \frac{k}{t} + \frac{1}{2} \\ \left(\frac{\partial \pi_h}{\partial p_h}\right)_{\widetilde{Z}_4(p_h, p_H)} &= \frac{p_H - 2p_h + c_h}{t} + \frac{k}{t} + \frac{1}{2}\end{aligned}$$

Setting them to zero and solving the system of linear equations, one gets $(p_h^*, p_H^*) = (\frac{2c_h + c_H}{3} + \frac{k}{3} + \frac{t}{2}, \frac{c_h + 2c_H}{3} - \frac{k}{3} + \frac{t}{2})$. Now, we ask whether the potential equilibrium just derived is included in the set $\widetilde{Z}_4$. Notice that because $k \geq 0$ and $H \geq \frac{1}{2}$, $p_H^* - p_h^* = \frac{c_H - c_h}{3} - \frac{2k}{3} \geq -\frac{2k}{3} \geq \frac{t}{2} - k - tH$ and hence $\widetilde{Z}_4(p_h^*, p_H^*)$ is false: a contradiction. Hence, $\widetilde{Z}_4$ cannot occur in equilibrium.

To eliminate Z_4 as a location for pure Nash equilibria, it suffices to show that $p_H^* - p_h^* = \frac{t}{2} - k - tH$ cannot occur in equilibrium. If it did, $\overline{x} = \frac{H}{2}$ and no international trade occurs and the firm in country H has a profitable deviation. For the moment, notice that under this circumstance, this firm earns $H(p_h^* + \frac{t}{2} - k - tH - c)$. Suppose instead it sets a price of $p_H = p_h^* + \frac{t}{2} + k - tH$. Then, demand is unchanged: all consumers purchase domestically, but price is higher. In this case, it earns $H(p_h^* + \frac{t}{2} + k - tH - c)$. Thus, it only remains to show that branch 5 cannot occur in equilibrium.

Finally, suppose (p_h^*, p_H^*) is a Nash equilibrium such that Z_5 occurs (i.e. satisfies $p_H^* - p_h^* \leq -\frac{t}{2} - k$). According to the above lemma, in order for (p_h^*, p_H^*) to be a Nash equilibrium, it must be that $p_i^* \in [c_i, r]$ for $i \in \{h, H\}$. Given this situation, it follows that the firm in country h attracts no consumers and hence earns zero profit. Then, the firm in country h has a profitable deviation. By setting a price $p_h = \min\{p_H^* + \frac{t}{2} + k, r\} - \varepsilon$, where ε is arbitrarily small and positive, it attracts some consumers and retains a positive price-marginal cost margin. This allows the firm in country h to earn strictly positive profits. ■

References

[1] Beath, John, 1991, Economic Theory of Product Differentiation (Cambridge University Press, Cambridge, UK).

[2] Benson, Bruce L. and James C. Hartigan, 1983, Tariffs which Lower Price in the Restricting Country: An analysis of spatial markets, Journal of International Economics 15, 117-133.

[3] Braid, Ralph M., 1987, The Spatial Incidence of Local Retail Taxation, Quarterly Journal of Economics 102, 881-891.

[4] Brander, James A., 1981, Intra-industry Trade in Identical Commodities, Journal of International Economics 11, 1-14.

[5] Brander, James A. and Barbara J. Spencer, 1985, Export Subsidies and International Market Share Rivalry, Journal of International Economics 18, 83-100.

[6] Cheng, Leonard K., 1988, Assisting Domestic Industries under International Oligopoly: The Relevance of the Nature of Competition to Optimal Policies, American Economic Review 78, 746-758.

[7] Collie, David R., 1997, Bilateralism is Good: Trade Blocs and Strategic Export Subsidies, Oxford Economic Papers 49, 504-520.

[8] d'Aspremont, C., Gabszewicz, J. and J. F. Thisse, 1979, On Hotelling's Stability in Competition, Econometrica 47, 1045-1050.

[9] Eaton, J. and G.M. Grossman, 1986, Optimal Trade and Industrial Policy under Oligopoly, Quarterly Journal of Economics 101, 383-406.

[10] Fudenberg, Drew and Jean Tirole, 1998, Game Theory (MIT Press, Cambridge, Mass.).

[11] Fujimoto, Hiroaki & Eun-Soo Park, 1997, Optimal Export Subsidy when Demand is Uncertain, Economic Letters 55, 383-390.

[12] Graham, Frank D., 1923, Some Aspects of Protection Further Considered, Quarterly Journal of Economics 37, 199-227

[13] Graham, Frank D., 1923, The Theory of International Values Re-examined, Quarterly Journal of Economics 38, 54-86.

[14] Harris, Richard G., 1989, The New Protectionism Revisited, Canadian Journal of Economics 12, 751-778.

[15] Krugman, Paul R., 1981, Intraindustry Specialization and the Gains from Trade, Journal of Political Economy 89, 959-973.

[16] Kanbur, Ravi and Michael Keen, 1993, Jeux Sans Frontieres: Tax Competition and Tax Coordination When Countries Differ in Size, American Economic Review 83, 877-892.

[17] Marazzi, Mario, 2001, Gains from Trade under Spatial Price Competition, Cornell University.

[18] Newberry, David M. G. & Joseph E. Stiglitz, 1984, Pareto Inferior Trade, Review of Economic Studies LI, 1-12.

[19] Ohsawa, Yoshiaki, 1999, Cross-border shopping and commodity tax competition among governments, Regional Science and Urban Economics 29, 33-51.

[20] Shachmurove, Y. and Uriel Spiegel, 1995, On Nations' Size and Transportation Costs, Review of International Economics 3, 235-243.

[21] Trandel, Gregory A., 1992, Evading the Use Tax on Cross-border sales, Journal of Public Economics 49, 313-331.

[22] Wong, Kar-yiu, 1995, International Trade in Goods and Factor Mobility (MIT Press, Cambridge, Mass.).