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FORMULATION AND ESTIMATION OF A DYNAMIC MODEL OF EXCHANGE RATE DETERMINATION: AN APPLICATION OF GENERAL METHOD OF MOMENTS TECHNIQUES

by

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I. Introduction

The advent of floating exchange rates in 1973 has resulted in a proliferation of papers aimed at formulating and estimating models of spot exchange rate determination. Few of these models have been estimated with techniques that explicitly utilize the identifying restrictions imposed by the assumption of rational expectations. Moreover, those authors who have estimated their models subject to these restrictions (including Driskill and Sheffrin [1981], Salemi [1979] and Hartley [1982])[1/] have explicitly or implicitly made strong assumptions about (a) the exogeneity of the "forcing" variables and (b) the serial correlation properties of reduced form error terms.[2/]

In virtually all of this work the "forcing" variables are assumed to be strictly econometrically exogenous. Recently Meese and Rogoff [1981] and Glaessner [1982A] have found that the necessary conditions for the joint exogeneity of the variables in many of these models are rejected.[3/] Also, in estimation, the error terms in these models typically have been assumed to be either serially uncorrelated or follow a first order autoregressive process (AR(1)). Both Glaessner [1982A] and Hakkio [1981] have noted that very stringent conditions on structural error terms must be made in order to claim that the reduced-form error term in the spot exchange rate equation will take such a simple form. Hakkio [1981] has found evidence of serial correlation (of a greater order than AR(1)) for several different bilateral spot exchange rate equations. Finally Obstfeld [1983] has pointed out that spot exchange rate determination models have been estimated using techniques which presume that equation disturbances are conditionally homoskedastic, an assumption which may have been untenable.4/

In light of these considerations, this paper aspires to improve upon the estimation techniques previously brought to bear upon these types of models. A nonlinear instrumental variables or general method of moments (GMM) estimator developed by Hansen [1979, 1982], Hansen and Sargent [1981] and Hansen and Singleton [1982] is used in order to estimate and test the theoretical restrictions implied by the structure of the model and the assumption of rational expectations. These procedures do not require that (a) the forcing variables be exogenous (b) that equation disturbances be characterized by a specific distribution function or (c) that equation distrubances be conditionally homoskedastic.5/ Also in contrast to previous work restrictive assumptions are not made concerning the structure of the vector autoregression generating the forcing variables. Specifically, the polynomial matrix multiplied by the vector of forcing variables is not assumed to be diagonal, upper triangular, or contain any exclusion restrictions with regard to the other variables within the vector autoregression.6/

The estimation techniques employed in this paper are applied to a "flex-price" model based on the work of Flood [1981] and Mussa [1982A] which has several unique features. First, in contrast to the so called "monetary approach" models developed and estimated by Frenkel [1976, 1979], Hartley [1982] and Bilson [1978] there is more than one good in the model so that purchasing power parity is not assumed to hold at all points in time. Second, in contrast to the monetary models both the money and goods market are modelled explicitly so that real factors are allowed to affect the exchange rate through channels other than the income term in the money demand function. Finally, in contrast, to the "sticky price" models developed by Frankel [1979], Hooper and Morton [1980], Salemi [1979], Dornbusch [1976] Driskill and Sheffrin [1981], Flood [1981, 1982] and Mussa [1982A]7/ the domestic price of domestic goods is not assumed to adjust slowly according to some explicit price adjustment specification.8/ Thus, the flex-price model can be viewed as lying in between the two types of models which have previously been estimated.

Applying the GMM estimation techniques to the "flex-price" model results in a strong rejection of the joint hypothesis of rational expectations and the structure of the model for the Canadian U.S. floating exchange rate period of 1973:3 - 1980:8. In fact, these results indicate that only the autoregressive parameters in both the exchange rate and price equations are statistically significant. These autoregressive parameters are close to unity, a finding consistent with the results of Meese and Singleton [1981] that the univariate process for the log of the Canadian/dollar spot exchange rate may have a unit root. These empirical results should be considered preliminary, however, due to several technical problems discussed in section IV.

The organization of this paper is as follows: Section II describes the equations in the flex-price model and derives closed form solutions for the spot exchange rate and domestic prices, which embody the cross-equation restrictions implied by the assumption of rational expectations. These solutions are obtained by using methods from prediction theory.[9/] Section III presents the orthogonality restrictions implied by the theoretical model which can be used in order to form the GMM estimator. Section IV presents and discusses the empirical results obtained in estimating and testing the theoretical restrictions implied by the flex-price model for Canada and the U.S. over the current floating exchange rate period from March 1973 to August 1980. Finally Section V summarizes the conclusions of this study and suggests topics for future research.

II. The Flex Price Model:

The model presented below is a slightly modified version of the models developed by Flood [1981] and Mussa [1982A]. The country under consideration is assumed to be large only in the markets for its own money and its own output. It is assumed to be small in the world (or foreign) goods and capital markets, so that the foreign price of foreign goods (P_t*) and the foreign interest rate (i_t*) are exogenous variables. Equations (1 - 12) characterize the flex-price model of exchange rate determination. The model presented below is not yet complete in that specific stochastic processes generating the error terms and the hypothesized exogenous variables m_t, Y_t, P_t*, and i_t* (defined below) are not assumed at the outset. The necessary assumptions are made below.

The Model

(1) $$\pi_t = \sigma P_t + (1-\sigma)(S_t + P_t^*)$$

(2) $$q_t = P_t - (S_t + P_t^*)$$

(3) $$m_t^d - \pi_t = \alpha_0 - \alpha_1 i_t + \alpha_2(P_t + Y_t - \pi_t) + u_t, \quad \alpha_0, \alpha_1, \alpha_2 > 0.$$

(4) $$m_t^s = m_t$$

(5) $$m_t^s = m_t^d$$

(6) $$i_t = i_t^* + f_t - s_t$$

(7) $$f_t = {}_tES_{t+1}$$

(8) $$Y_t^s = Y_t$$

(9) $$Y_t^d = b_0 - b_1 q_t - b_2(i_t - ({}_tE\pi_{t+1} - \pi_t)) + b_3 Y_t + \varepsilon_t, \quad b_0, b_1, b_3 > 0$$

(10) $$Y_t^s = Y_t^d$$

(11) $$P_t = Q_t + S_t$$

(12) $$Q_t^* = P_t^* + S_t$$

Definitions of Variables and Notation

π_t is the natural log of the general price level.

σ the domestic consumption share of the domestic good.

P_t is the logarithm of the domestic price of the domestic good.

P_t^* is the logarithm of the foreign price of the foreign good.

S_t is the logarithm of the spot exchange rate, which is expressed in units of domestic currency per unit of foreign currency.

q_t defines the relative price of domestic goods in logs.

m_t is the logarithm of the quantity of money.

i_t is the level or $\ln(1+i_t)$ of the domestic interest rate.

Y_t is the log of real output.

i_t^* is the level of the foreign interest rate.

f_t is the log of the forward exchange rate.

Q_t the log of the foreign price of the domestic good.

Q_t^* the log of the domestic price of the foreign good.

${}_tEX_t$ = ${}_tE[X_t|\Omega_t]$ where Ω_t contains all of the variables and disturbances in the model dated t or earlier and the values of the models parameters.

The flex-price model (eq. (1) - (12)) is identical to that in Flood [1981] except that an additive disturbance term has been introduced into the money market (eq. (3)) and specific stochastic processes for the exogenous variables have not been assumed.[10/] [11/]

Equation (1) defines the log of the general price level (or index (π_t); σ is the domestic consumption share of the domestic good, (P_t^*) is the logarithm of the domestic price of the domestic good and (P^*) is the logarithm of the foreign price of the foreign good. (S_t) is the log of the spot exchange rate which is expressed in units of domestic currency per unit of foreign currency.

Equation (2) defines the terms of trade (q_t) or the relative price of domestic goods in logs.

Equation (3) describes the money demand equation. The demand for real money balances ($m_t - \pi$) depends negatively upon the level of the domestic nominal interest rate (i_t) and positively upon the natural logarithm of real domestic income ($P_t + Y_t - \pi_t$). In contrast to the work of Dooley and Isard [1979] and others on portfolio balance models, wealth does not enter the money demand function explicitly. However, the specification adopted here only neglects the net foreign asset component of wealth due to the presence of real income in the money demand equation. This specification would seem reasonable in countries where net foreign asset holdings do not account for a large proportion of total wealth.[12/]

Equation (4) states that the natural logarithm of the quantity of money (m_t) is an exogenous variable determined by the authorities,[13/] and equation (5) expresses the equilibrium condition for the money market.

Equation (6) expresses the covered interest rate parity condition, where i_t^* is the level of the foreign interest rate.

Equation (7) states that the log of the forward exchange rate (f_t) equals the expected future spot exchange rate (${}_tES_{t+1}$). Empirical evidence regarding this relationship has been mixed.14/ The weight of the evidence does seem to suggest that this relationship may not hold exactly, and that a time varying risk premium may be present. The present paper follows Flood [1981], Dornbusch [1976], Mussa [1982A], and Rogoff [1979] by asserting that equation (7) can be regarded as "a good approximation" in the present context.

Equation (8) states that the supply of output is exogenous. This simplifying assumption may be unrealistic; in future work it will be relaxed.15/

Equation (9) says that the demand for domestic goods depends negatively upon the log of the relative price of domestic goods (q_t) and the real interest rate (i_t - (${}_tE\pi_{t+1} - \pi_t$) and positively upon the log of real output (or income) Y_t.16/ Finally, equation (10) is simply the equilibrium condition for the goods market.

Equations (11) and (12) state that goods market arbitrage holds for each of the goods (domestic and foreign) in the model.

The flex-price model (eqs. 1 - 12) can be readily solved using difference equation techniques.17/ Using vector notation the solution is:

$$(13) \qquad S_t = d_2\underset{\sim}{\beta}_1' \sum_{k=0}^{\infty} (\eta_1)^k \, E[\underset{\sim}{X}_{t+k}|\Omega_t] + (d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2') \sum_{k=0}^{\infty} (\eta_2)^k$$

$$E[\underset{\sim}{X}_{t+k}|\Omega_t] + V_{1t}$$

$$(14) \qquad P_t = (d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2')\sum_{k=0}^{\infty}(\eta_2)^k \, E[\underset{\sim}{X}_{t+k}|\Omega_t] + d_1\underset{\sim}{\beta}_1' \sum_{k=0}^{\infty}(\eta_1)^k$$

$$E[\underset{\sim}{X}_{t+k}|\Omega_t] + V_{2t}$$

$$(15) \qquad \underset{4\times4}{A(L)}\underset{4\times1}{\underset{\sim}{X}_t} = \underset{\sim}{V}_t^x$$

where

$$\underset{\sim}{\beta}_1' = [0,\ 1,\ -B_2,\ (B_2(1-\sigma)F - B_2(1-\sigma) - B_1)]$$

$$\underset{\sim}{\beta}_2' = [1,\ -\alpha_2,\ \alpha_1,\ ((\sigma-1) + \alpha_2(1-\sigma))],$$

$$\underset{\sim}{X}_t' = [m_t,\ y_t,\ i_t^*,\ P_t^*]$$

$$\eta_1 = \frac{B_2\sigma}{B_1 + B_2\sigma}$$

$$\eta_2 = \frac{\alpha_1}{1+\alpha_1}$$

$$d_1 = \frac{1}{B_1 + B_2\sigma} + d_2$$

$$d_2 = \frac{(\alpha_2(\sigma-1) - \sigma)}{(B_1 + B_2)(1+\alpha_1)}$$

$$d_3 = \frac{1}{1+\alpha_1}$$

$$B_1 = \frac{b_1}{1-b_3}$$

$$B_2 = \frac{b_2}{1-b_3} \qquad \underline{18/}$$

The polynomial matrix A(L) is assumed to be of finite order r so that $\underset{4x4}{A(L)} = I - A_1L - A_2L^2 - \ldots A^rL^r$. The innovations in the vector auto-regression are defined as $\underset{\sim}{V}_t^x = \underset{\sim}{X}_t - E[\underset{\sim}{X}_t \mid \underset{\sim}{X}_{t-1}, \underset{\sim}{X}_{t-2}, \ldots]$ where $E[\underset{\sim}{V}_t^x \underset{\sim}{V}_t^{x'}] = \underset{4x4}{V}$ and $E[\underset{\sim}{V}_t^x \underset{\sim}{V}_{t-j}^{x'}] = \underset{4x4}{0}$ for all $j \neq 0$. These assumptions imply that it is possible for there to be contemporaneous correlation across the equations in the vector autoregression (a nondiagonal V matrix), but serial correlation in the individual components (V_{1t}^x) of $\underset{\sim}{V}_t^x$ are not permitted to be serially correlated. Finally, define F as $FE[Z_t \mid \Omega_t] = E[Z_{t+1} \mid \Omega_t]$ and $LE[Z_t \mid \Omega_t] = E[Z_{t-1} \mid \Omega_{t-1}]$ for some Z_t, where $E[\cdot \mid \,]$ is interpreted as the linear least squares projection operator, whereas $E(\cdot)$ is defined to be the unconditional projection operator.

Finally as in the work of Mussa [1982A] the price of domestic goods, (P_t) and the exchange rate (S_t) depend upon the discounted expected future values of the real and nominal variables included within the vector $\underset{\sim}{X}_t$. In contrast to the work of Frenkel [1976] and Bilson [1978], there are two discount factors η_1, η_2 present in the solutions for P_t and S_t in equations (13) and (14). The discount factor η_1 is a function of only the parameters in the goods market whereas the discount factor η_2 is a function of the parameters in the money market.19/

Closed form solutions for the spot exchange rate and prices which express each of these variables as functions of contemporaneous and past values of the "forcing" variables in $\underset{\sim}{X}_t = (m_t, Y_t, i_t^*, P_t^*)$ are20/

$$(13)' \quad S_t = (d_2\underset{\sim}{\beta}_1' A^{-1}(\eta_1)[\underset{4x4}{I} + \sum_{j=1}^{r-1} (\sum_{k=j+1}^{r} \eta_1^{k-j} A_k) L^j]$$

$$+ (d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2')A^{-1}(\eta_2)[I + \sum_{j=1}^{r-1} (\sum_{k=j+1}^{r} \eta_2^{k-j} A_k) L^j]) \underset{4x1}{\underset{\sim}{X}_t} + V_{1t}$$

(14)' $$P_t = (d_1 \underset{1\times4}{\beta_1'} \underset{4\times4}{A^{-1}(\eta_1)}[I + \sum_{j=1}^{r-1} (\sum_{k=j+1}^{r} \eta_1^{k-j} A_k) L^j]$$

$$+ (d_3 \beta_2' + d_2 \beta_1') A^{-1}(\eta_2)[I + \sum_{j=1}^{r-1} (\sum_{k=j+1}^{r} \eta_2^{k-j} A_k) L^j]) \underset{\sim}{X}_t + V_{2t}$$

(15) $$\underset{4\times4}{A(L)} \underset{\sim}{X}_t = \underset{\sim}{V}_t^x ,$$

Equations (13)', (14)' and (15) represent a compact notation for a six equation system in P_t, S_t, and the "forcing" variables m_t, y_t, i_t^* and P_t^*. The restrictions delivered by the assumption of rational expectations are embodied in equations (13)' and (14)'. To actually express S_t and P_t as functions of all the structural parameters in the model [contained in d_1, d_2, β_1, β_2, d_3, η_1, η_2, $A(L)$] it is necessary to determine the order of the vector autoregression (r) in (15). For example, if r=2 we would have to invert the 4x4 2nd order polynominal matrix $A^{-1}(L)$, and evaluate it at $L = \eta_1$ or $L = \eta_2$. Thus, the order of the vector autoregression will be critical in determining the number of structural parameters to be estimated and the degree of complexity of the cross-equation restrictions in the flex-price model. The important advantage of the representation above is that the structure of the vector autoregression is very general so that each element of $\underset{\sim}{X}_t$ depends on its own past values and upon past values of the other varibles within $\underset{\sim}{X}_t$.

Equations (13)', (14)' and (15) almost provide one with enough information to form an estimator for the flex-price model using GMM procedures; however, it is still necessary to give a specific parameterization to the error terms V_{it}, i=1, 2 in the spot exchange rate and price equations. It is shown in (Glaessner [1982A] Chapter III) that in general the error terms in the exchange rate and price equation will follow some finite order ARMA (p,q) process depending upon the properties of the structural error terms u_t and ε_t. Thus in the section which follows orthogonality restrictions between instruments and error terms are presented which can be used in order to form the GMM or instrumental variables estimator developed by Hansen [1982], Hansen and Sargent [1980, 1981] and Hansen and Singleton [1982].

III. The Orthognality Conditions for the Flex-Price Model

In this section two sets of orthogonality restrictions between the errors V_{it} $i=1,2$, $\underset{\sim}{V}_t^x$ and the instruments (i.e. predetermined values of the forcing variables) are presented based on the more detailed derivation in Glaessner [1982A]. It should be observed at the outset that this methodology is adopted because (1) the forcing variables in the flex-price model were not found to meet the necessary conditions for joint exogeneity with respect to S_t and P_t (see appendix 1), (2) the error terms V_{it} $i=1,2$ are probably serially correlated for the reasons mentioned above and (3) the disturbance terms in the spot exchange rate price equations may not be conditionally homoskedastic.

To derive the orthogonality conditions suggested by the flex-price model we need to give a precise parmeterization to the error terms V_{1t} and V_{2t}. This is due to the fact that predetermined values of the "forcing variables" are used as instruments in estimation. For the sake of simplicity we assume that V_{1t} and V_{2t} follow ARMA(1,1) processes.21/

Specifically, write

$$(16) \qquad V_{it} = \frac{(1 - g_i L)}{(1 - \rho_i L)} Q_{it}$$

where $i = 1, 2$, $0 < |\rho_i| < 1$, $0 < |g_i| < 1$, and $E(Q_{it}) = 0$ for all t, $E[Q_{it}Q_{it-j}] = 0$ for all $j \neq 0$, $i = 1, 2$. Also, $E[\underset{\sim}{q}_t \underset{\sim}{q}_t'] = \underset{2\times2}{\Sigma}$ where $\underset{\sim}{q}_t' = [Q_{1t}, Q_{2t}]$ so that the reduced form error terms in the spot exchange rate and price equations are allowed to be contemporaneously correlated.

To derive the orthogonality restrictions present in the flex-price model, let us define the information set $\Omega_{t-1} = \{\underset{\sim}{X}_{t-1}, \underset{\sim}{X}_{t-2}, \ldots\}$ which is composed of predetermined values of the forcing variables.22/ The definition of the innovations in the vector autoregression ($\underset{\sim}{V}_t^x$) stated in section II suggests that $E[\underset{\sim}{V}_t^x | \Omega_{t-1}] = 0$. 23/ This observation implies the following set of orthogonality conditions for the four equation vector autoregression represented by equation (15)

$$(17) \qquad E[\underset{4\times1}{\underset{\sim}{V}_t^x} \underset{\sim}{X}_{t-\nu}'] = \underset{4\times4}{0} \qquad \text{for all } \nu \geq 1.$$

The orthogonality restrictions for the price and exchange rate equations can be obtained by multiplying equations (13) and (14) by $(1-\rho_1 L)$ and $(1 - \rho_2 L)$ respectively and rearranging to obtain

$$(18) \qquad V_{1t}^{*} = (1 - \rho_1 L)(S_t - d_2 \underset{\sim}{\beta}_1' \sum_{k=0}^{\infty} (\eta_1)^k E[\underset{\sim}{X}_{t+k} | \Omega_t]$$

$$- (d_2 \underset{\sim}{\beta}_1' + d_3 \underset{\sim}{\beta}_2') \sum_{k=0}^{\infty} (\eta_2)^k E[\underset{\sim}{X}_{t+k} | \Omega_t])$$

(19)
$$V_{2t}^{*} = (1 - \rho_2 L)(P_t - (d_2 \beta_1' + d_3 \beta_2') \sum_{k=0}^{\infty} (\eta_2)^k \, E[X_{t+k} | \Omega_t]$$

$$- d_1 \beta_1' \sum_{k=0}^{\infty} (\eta_1)^k \, E[X_{t+k} | \Omega_t])$$

where

$$V_{1t}^{*} = (1 - g_1 L) Q_{1t}$$

$$V_{2t}^{*} = (1 - g_2 L) Q_{2t}.$$

Notice that V_{1t}^{*} and V_{2t}^{*} are first order moving average error processes. Hence, it can be shown that $E[V_{1t}^{*} | \Omega_{t-2}] = 0$ and $E[V_{2t}^{*} | \Omega_{t-2}] = 0$.[24/] Thus to estimate the model given the assumption that V_{it} $i = 1, 2$ are generated by an ARMA (1,1) process involves choosing instruments from the set $\Omega_{t-2} = \{X_{t-2}, X_{t-3}, \ldots\}$. In the present context we express the orthogonality conditions for the first order moving average errors V_{it}^{*} $i = 1, 2$ as

(20)
$$\underset{1x1}{E[V_{it}^{*}} \; \underset{4x1}{X_{t-\tau}]} = \underset{4x1}{0} \qquad \text{for } \tau \geq 2, \quad i = 1, 2.$$

The orthogonality conditions have been expressed in terms of the V_{it}^{*} $i = 1, 2$ because the closed form solutions for the spot rate (S_t) and price equations (P_t) (13)' and (14)' can then be used to express these errors (V_{1t}^{*}, V_{2t}^{*}) in terms of observable variables. Expressing the orthogonality restrictions in this fashion allows the investigator to ignore the moving average parameters g_1, g_2. In fact, these parameters will not need to be estimated when the GMM estimator (see Hansen [1982]) is formed below. Thus this estimator has computational advantages vis a vis maximum likelihood techniques.

Equations (13)', (14)' and (15) can be used in conjunction with equations (17) and (20) above to express the orthogonality conditions[25/] in compact notation as

$$(21) \qquad E[\underset{R\times 1}{f_t}(\underset{\sim}{\zeta}_0)] = E[\underset{b\times 1}{\underset{\sim}{d}_t} \otimes \begin{bmatrix} \underset{4\times 1}{\underset{\sim}{X}_{t-2}} \\ \underset{\sim}{X}_{t-3} \\ \vdots \\ \underset{\sim}{X}_{t-P} \end{bmatrix}] = \underset{R\times 1}{\underset{\sim}{0}}$$

where

$$\underset{6\times 1}{\underset{\sim}{d}_t} = \begin{bmatrix} V^*_{1t} \\ V^*_{2t} \\ \underset{\sim}{V}^x_t \end{bmatrix} = \underset{6\times 10}{\lambda(L;\ \underset{\sim}{\zeta}_0)} \underset{10\times 1}{\underset{\sim}{\ell}_t} .$$

The symbol $\otimes$ indicates the Kronecker product, $\underset{\sim}{X}_t$ is defined as above, the vector of observable variables $(S_t,\ P_t,\ \underset{\sim}{X}_t',\ \underset{\sim}{X}_{t-1}')'$ is denoted by $\underset{\sim}{\ell}_t$, $\underset{\sim}{\zeta}_0$ is the (Qxl) vector of true structural parameters $(\alpha_1,\ \alpha_2,\ \sigma,\ b_1,\ b_2,\ b_3,\ \rho_1,\ \rho_2,\ A_1,\ - -,\ A_r)$, and $[\underset{\sim}{X}_{t-3}',\ \text{---}X'_{t-P}]$

is a vector of instruments (predetermined values of the forcing variables).

Finally

$$\lambda(L;\ \underset{\sim}{\xi}_0) = \lambda_0(\underset{\sim}{\xi}_0) + \lambda_1(\underset{\sim}{\xi}_0)L + \dots + \lambda_{r-1}(\underset{\sim}{\xi}_0)L^{r-1}$$

where

$$(22)\qquad \underset{6\times 10}{\lambda_0(\underset{\sim}{\xi}_0)} = \begin{bmatrix} (1-\rho_1), & 0, & \underset{1\times 4}{-(1-\rho_1 L)[d_2\underset{\sim}{\beta}_1'\Psi_0^*(\underset{\sim}{\xi}_0) + (d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2')\Psi^{**}(\underset{\sim}{\xi}_0)]}, & \underset{1\times 4}{\underset{\sim}{0}'} \\ 0, & (1-\rho_2 L), & \underset{1\times 4}{-(1-\rho_2 L)[d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2')\Psi_0^{**}(\underset{\sim}{\xi}_0) + d_1\underset{\sim}{\beta}_1'\Psi_0^*(\underset{\sim}{\xi}_0)]}, & \underset{1\times 4}{\underset{\sim}{0}'} \\ \underset{4\times 2}{0}, & & \underset{4\times 4}{I}, & \underset{4\times 4}{-A_1} \end{bmatrix}$$

$$(23)\qquad \underset{6\times 10}{\lambda_j(\underset{\sim}{\xi}_0)} = \begin{bmatrix} 0, & 0, & \underset{1\times 4}{-(1-\rho_1 L)[d_2\underset{\sim}{\beta}_1'\Psi_j^*(\underset{\sim}{\xi}_0) + (d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2')\Psi_j^{**}(\underset{\sim}{\xi}_0)]}, & \underset{1\times 4}{\underset{\sim}{0}'} \\ 0, & 0, & \underset{1\times 4}{-(1-\rho_2 L)[(d_2\underset{\sim}{\beta}_1' + d_3\underset{\sim}{\beta}_2')\Psi_j^{**}(\underset{\sim}{\xi}_0) + d_1\underset{\sim}{\beta}_1'\Psi_j^*(\underset{\sim}{\xi}_0)]}, & \underset{1\times 4}{\underset{\sim}{0}'} \\ \underset{4\times 2}{0}, & & \underset{4\times 4}{0}, & \underset{4\times 4}{-A_{j+1}} \end{bmatrix}$$

for $j = 1, \text{---}, r-1$, and the coefficients Ψ_0^*, Ψ_j^* or Ψ_0^{**} Ψ_j^{**} can be expressed as

$$(24)\qquad \Psi_0^* = A^{-1}(\eta_1)$$

$$\Psi_j^* = A^{-1}(\eta_1)(\eta_1{}^j A_{j+1} + \eta_1{}^{j+1}A_{j+2} + \dots + \eta_1{}^{r-j}A_r)$$

and

(25) $$\Psi_0^{**} = A^{-1}(\eta_2)$$

$$\Psi_j^{**} = A^{-1}(\eta_2)(\eta_2^{\,j}A_{j+1} + \eta_2^{\,j+1}A_{j+2} + \ldots + \eta_2^{\,r-j}A_r)$$

for $j = 1, \ldots, r-1$.

These expressions for the coefficients Ψ_j^* and Ψ_j^{**} for $j = 0, ---, r-1$ can be derived from the closed form solutions given in equations (13)' and (14)' above.[26] Thus, equations (24) and (25) provide one with an explicit representation for the rational expectations restrictions across the parameters in equations (13) - (15), which can be imposed when constructing the GMM estimator.[27] Equations (24) and (25) also focus attention on the importance of the choice of r (the order of the vector autoregression) in determining the number of parameters to be estimated (Q) and the complexity of the cross equations restrictions. Appendix 2 below explains and implements procedures for choosing finite lag lengths based on work by Parzen [1975]. These procedures suggest a lag order of unity (or r = 1) for the vector autoregression generating the "forcing" variables in $\underset{\sim}{X}_t$, for Canada and the U.S.[28] Thus, if r = 1, only contemporaneous values of $\underset{\sim}{X}_t$ would appear in the closed form solutions. In addition only twenty-four structural parameters would have to be estimated (i.e., sixteen structural parameters in the vector autoregression and eight additional structural parameters $\{\alpha_1, \alpha_2, q, b_1, b_2, b_3, \rho_1, \rho_2\}$.

Equation (21), also suggests that the total number of orthogonality conditions present in the model will depend upon (1) the number of instruments, denoted by h (2) the number of error terms denoted by N and (3) how many lagged values (i.e. lag order) of each "instrument" is chosen, denoted by P. Equation (21) defines $f_t(\zeta_0)$ as an $N((P-1) \cdot h) \times 1 = R \times 1$ vector of random variables where $E[f_t(\zeta_0)] = 0$. Thus in the present context given N=6 and h=4, the number of orthogonality restrictions is determined by P. Hansen [1982] and Hansen and Singleton [1982] have shown that models estimated using GMM procedures will be overidentified if $R > Q$ and if certain regularity conditions hold.[29] In the present context $R > Q$ if $P \geq 3$. Note that if P=3, R=48 which is a large number of orthogonality restrictions relative to the number of observations 89 in the model. Hansen and Singleton [1982] have pointed out that as the number of orthogonality restrictions gets large relative to the number of observations there is a loss of precision in both the estimated coefficients and asymptotic standard errors.

To avoid the problems associated with using a representation for the orthogonality restrictions like that in (21) an alternative, less efficient estimation procedure, which conserves on (1) the number of orthogonality restrictions and (2) upon the number of simultaneously estimated parameters was pursued. The structural parameters in the vector autoregression were estimated using OLS and were held fixed when computing the GMM estimator for the eight additional structural parameters $\{\alpha_1, \alpha_2, \sigma, b_1, b_2, b_3, \rho_1, \rho_2\}$. In this case the orthogonality restrictions appeared as

(26)

$$\underbrace{E[f_t^1(\zeta_0^1)]}_{\substack{(2((P-1)\cdot 4)\times 1 \\ R}} = E[\lambda^1(L;\, \zeta_0)\, \ell_t^1 \otimes \begin{pmatrix} X_{t-2} \\ X_{t-3} \\ \vdots \\ X_{t-P} \end{pmatrix}] = 0$$

where

$$\underset{2\times6}{\lambda_0^1(\underline{\zeta}_0)} = \begin{bmatrix} (1-\rho_1),\ 0,\ -(1-\rho_1 L)[d_2\underline{\beta}_1'\Psi_0^*(\underline{\zeta}_0) \\ +\ d_2\underline{\beta}_1' + d_3\underline{\beta}_2')\Psi^{**}(\underline{\zeta}_0)] \\ \\ 0,\ (1-\rho_2 L),\ -(1-\rho_2 L)[d_2\underline{\beta}_1' + d_3\underline{\beta}_2')\Psi^{**}(\underline{\zeta}_0) \\ +\ d_1\underline{\beta}_1'\Psi^*(\underline{\zeta}_0)] \end{bmatrix}$$

$$\underset{2\times6}{\lambda_j^1(\underline{\zeta}_0)} = \begin{bmatrix} 0,\ 0,\ -(1-\rho_1 L)[d_2\underline{\beta}_1'\Psi_j^*(\zeta_0) + (d_2\underline{\beta}_1' + d_3\underline{\beta}_2')\Psi_j^{**}(\underline{\zeta}_0)] \\ \\ 0,\ 0,\ -(1-\rho_2 L)[(d_2\underline{\beta}_1' + d_3\underline{\beta}_2')\Psi_j^{**}(\underline{\zeta}_0) + d_1\underline{\beta}_1\Psi_j^*(\underline{\zeta}_0) \end{bmatrix}$$

for j = 1, ... , r-1 and $\underline{\ell}_t^1 = [S_t, P_t, \underline{X}_t]$. Observe that N=2 so that given P=3 and h=4 there are only sixteen orthogonality restrictions.

In sum, this procedure will not result in estimates of parameters which will be as efficient as those obtained if one could estimate all twenty-four structural parameters jointly as in (21).[30/]

IV. Estimation of the Flex-Price Model for Canada and the U.S. 1973:3 - 1980:8

4.5(a) Description of Estimation Procedure

Tables I and II below present the results of estimating the flex-price model represented by the orthogonality restrictions in equation (26) given a choice of lag length for the "forcing" variables (see Appendix 2) of r = 1, and a maximum lag length for the instruments of P = 3. Thus the set of eight instruments contained only predetermined values of the forcing variables $\underset{\sim}{X}_{t-2}$, $\underset{\sim}{X}_{t-3}$, where the vector $\underset{\sim}{X}_t$ is defined as before.

Heuristically the application of Hansen's [1982] GMM estimation procedures within the present multi-equation context, involves the minimization of a criterion function which is a weighted average of the small sample estimates of the population orthogonality conditions in equation (26). The weighting scheme is chosen so as to obtain the minimum asymptotic variance covariance matrix for estimators that can be viewed as exploiting the same fixed set of orthogonality restrictions. A formal derivation of the GMM estimator and an explanation of the actual procedures used to obtain the parameter estimates and standard errors in Tables I and II below can be found in Glaessner [1982A].

Tables I and II reveal that the autoregressive parameters in the spot exchange rate (ρ_1) and price (ρ_2) equations are statistically significant. Both of the coefficients ρ_1 and ρ_2 have a value close to unity. The other six structural parameters in the model α_1 (the semi-interest-elasticity of the demand for money), α_2 (the income elasticity of the demand for money), b_1 (the relative price elasticity of the demand for domestic goods), b_2 (the elasticity of the demand for domestic goods with respect to the real interest rate), b_3 (the real income elasticity of the demand for domestic goods) and σ (the domestic consumption share of the domestic good) are all found to be insignificant in Table I. However, these first stage estimates of the additional structural parameters do seem to have the correct signs and be of the proper magnitudes. The coefficients on the first own lags of the left hand side variables (m_t, y_t, i_t, P_t^*) in the vector autoregression (see Table I) are usually close to unity and statistically significant.

Table I

First Stage Nonlinear Instrumental Variable (or GMM) Estimates of the Constrained Flex Price Model (73:3 - 80:8)[A] in Eq. (26)

$\hat{\zeta}_{-1,T}$	α_1	-.1533	(-.2359)
	α_2	1.0944	(.3903)
	b_1	-.5020	(-.0885)
	b_2	-.4314	(-.1112)
	b_3	.5857	(.1402)
	ρ_1	.9304	(6.173)**
	ρ_2	.9273	(4.222)**
	σ	.8199	(.3697)
M_t equation	a_{11}	.8086	(13.1593)**
	a_{12}	-.13909	(-2.2909)*
	a_{13}	.0022	(.3462)
	a_{14}	-.1034	(-1.9251)*
Y_t equation	a_{21}	.0993	(1.2328)
	a_{22}	.7129	(8.9568)**
	a_{23}	.0153	(1.8598)
	a_{24}	-.1969	(-2.794)**
i_t^* equation	a_{31}	1.0849	(1.9506)*
	a_{32}	.9540	(1.7361)*
	a_{33}	.9051	(15.9926)**
	a_{34}	.6248	(1.2844)
P_t^* equation	a_{41}	-.0704	(-1.4862)
	a_{42}	.0360	(.7699)
	a_{43}	.0121	(2.1213)*
	a_{44}	.9239	(22.317)**

A) Statistics in parenthesis are ratios of coefficient estimates to their asymptotic standard errors.

** Indicates significance at the .01 and .05 level.

* Indicates significance at the .05 level.

Table II

Final GMM Estimates of the Constrained Flex-price Model

Using a Consistent Estimate of the Optimal Weighting Matrix for the

Orthogonality Restrictions in Eq. (26) [A]

Parameter		Estimate		$\chi^2(8)$
$\hat{\zeta}_{2,T}$	α_1	.1164	(.0966)	
	α_2	.9361	(1.239)*	
	b_1	-.6270	(-.0864)	21.473**
	b_2	-.3919	(-.1003)	
	b_3	.7055	(.1949)	
	ρ_1	.9223	(14.575)**	
	ρ_2	1.0884	(16.232)**	
	σ	.9153	(2.5816)**	

A) Statistics in parentheses are ratios of coefficient estimates to their asymptotic standard errors.

** Indicates significance at both the .05 and .01 significance levels.

* Indicates significance at the .1 significance level.

The parameter estimates presentred in Table II were obtained by using the initial estimates $\hat{\underline{\zeta}}_{1,T}$ to form a second criterion function which was weighted by an optimal weighting matrix based on the orthogonality restrictions in equation (26). Heuristically, these parameter estimates can be thought of as more efficient than those in Table I because the weighting matrix reflects the investigators assumptions concerning the structure of the error vector.31/

The parameter estimates in Table II are not appreciably different in sign or magnitude than those in Table I. However, α_1 (the semi interest elasticity of money demand) now has the wrong sign, α_2 (the income elasticity of money demand) is now significant at the .2 level and σ (the domestic consumption share of the domestic good) is significant at the .05 and .01 significance levels. It is possible to conduct a joint test of (a) the structure of the flex-price model (including assumptions made about the serial correlation properties of reduced form error terms) and (b) the nonlinear rational expectations restrictions. This test statistic is distributed χ^2 (8) since there are sixteen orthogonality restrictions (R = 16) and eight structural parameters (Q = 8) given the representation of the orthogonality restrictions in equation (26) with a maximal lag for the instruments of P=3.32/ The $\chi^2(8)$ statistic of 21.473 rejects the theoretical restrictions implied by the flex-price exchange rate determination model.

The findings above may in part be explained by the recent work of Meese and Singleton [1981] which suggests that the log of the Canadian dollar rate may not be a covariance stationary process (i.e. the univariate process on S_t may have a unit root). Several other considerations of a more technical nature suggest that the results obtained above should be considered preliminary.

First, the chi-square statistic (see Glaessner [1982A]) will tend to be biased toward rejection of the theoretical restrictions implicit in the flex-price model to the extent that the true minimum value of the second stage criterion function has not been obtained. This consideration is important in the present context since the convergence criterion for minimizing the criterion functions used to obtain the parameter estimates in Table I or Table II, had to be relaxed substantially, to obtain convergence. Specifically, the convergence criterion was 1.0E-2 or 1.0E-3 for changes in the coefficients. Often these convergence criterions for changes in the coefficients were not met after two hundred iterations even though the change in the function at each iteration was only occurring out to five places (e.g. 1.0E-6). Thus convergence for each criterion function was achieved by relaxing the convergence criterion from 1.0E-2 to 1.0E-1, after seeing that very little change was occurring in the actual value of the function being minimized.[33/]

Second, the fact that in the final (GMM) estimates reported in Table II the value of the autoregressive parameter in the price equation (ρ_2) is greater than unity (1.0884) is quite troubling in light of the theoretical restriction noted in equation (16) that $0 < | \rho_i | < 1$ for i = 1,2. Such a finding could be due to either (1) problems in minimizing the second stage criterion function or (2) possible problems in characterizing the restrictions. Thus, it may be necessary to <u>not</u> allow ρ_1 i=1, 2 to take on a value greater than unity in the process of minimizing the criterion function. This would simply be a way of imposing the inequality constraint on the ρ_i's in equation (16).

Third, even though the sixteen parameters in the vector autoregression were treated as fixed in obtaining estimates of the eight additional structural parameters it was extremely difficult to calculate the partial derivative matrix $D_T = \frac{\partial f_t(\underline{\zeta})}{\partial(\underline{\zeta})} \Big| \hat{\underline{\zeta}}_{1,T}$ analytically. This was due to the appearance of the terms $A^{-1}(\eta_1)$ and $A^{-1}(\eta_2)$ within the orthogonality restrictions in (26). The asymptotic standard errors calculated above in Tables I and II were sensitive to how the derivatives were approximated, since if the perturbations to $f_t(\hat{\underline{\zeta}}_T)$, were too small (e.g. out to the eighth or ninth decimal place) the D_T matrix tended to not be of full rank, so that the D_T' D_T matrix used in the calculation of standard errors, was singular. This numerical problem was solved by perturbing $f_t(\hat{\underline{\zeta}}_T)$ by somewhat larger amounts in approximating derivatives so as to ensure that the partial derivative matrix D_T was of full rank.[34/]

Fourth, attempts to estimate all of the parameters in the model jointly using the orthogonality restrictions in equation (21) were not successful because the large number of orthogonality restrictions (R = 48 in this case) resulted in severely ill conditioned estimates of the weighting matrix which made it impossible to calculate asymptotic standard errors, or calculate the second stage criterion function.[35/]

V. Conclusions and Possible Extensions

The results presented in this study suggest that the theoretical restrictions implied by the Flex-price model of exchange rate determination are rejected soundly. This is the case even though the restrictions implied by the assumption of rational expectations are imposed in estimating the model, and the assumption of purchasing power parity is not a foundation of the flex-price model estimated. In addition the rejection of the flex-price model cannot simply be attributed to an incorrect exogeneity specification since the use of GMM procedures in estimation allows one not to have to make strong assumptions about the exogeneity of "forcing" variables. This is in contrast to all previous work aimed at estimating and testing the rational expectations restrictions embodied in exchange rate determination models.

In light of the above conclusions one might ask two questions. First, what might explain the poor performance of the flex-price model for Canada and the U.S. over the current floating exchange rate period, and what alternative specifications for exchange rate determination models would be interesting to estimate? Second, to what extent should future research be devoted to solving some of the technical problems mentioned at the end of section IV.?

It is difficult to answer the first question, because the test of the theoretical restrictions conducted is a joint test of rational expectations and the structure of the model. Thus it is difficult to determine whether it is the assumption of (1) rational expectations

(2) a flexible domestic price of domestic goods (3) the money demand or goods demand specification (4) the possible instability of the vector autoregression generating the forcing variables or (5) assumptions made about the serial correlation properties of error terms which result in the rejection of the model. One possible method of attacking this problem is to look at these foundations explicitly (see Meese and Rogoff [1981, 1982]. Two other implications of these findings are (1) to make more variables in the model endogenous (e.g. allow for an intervention function), and (2) assume that domestic prices adjust slowly (as in Glaessner [1982A] Chapter III). This latter assumption in conjunction with the assumption of rational expectations leads to price and exchange rate equations with different nonlinear restrictions than those in the flex-price model which are testable using the procedures developed in this paper.

In answer to the second question posed above it would seem useful to pursue several approaches to remedying the technical problems suggested at the end of section IV. First, experimentation with different minimization and derivative approximation routines to see how sensitive the results are would seem proper before describing the results above as conclusive. Second, constraining the autoregressive parameters in the price and exchange rate equations to be less than unity in estimating the model would be an important improvement in the preliminary procedures followed. Third, it would also seem useful to try to estimate all twenty-four parameters jointly by reducing the number of orthognality conditions to twenty-five, (e.g. this is equivalent to reducing the number of instruments used). This would result in more efficient parameter estimates and more importantly might provide a way of obtaining a consistently estimated weighting matrix which is not illconditioned. However, given a sample of only eighty-nine observations, obtaining the joint estimates of twenty-four structural parameters will tend to reduce the precision of both parameter estimates and asymptotic standard errors.[36/] Fourth, further experimentation regarding the optimal choice of the value by which one perburbs parameters in approximating derivatives should be explored. Finally, Hansen and Singleton [1982] have suggested that the GMM estimator formed above is often sensitive to the choice of maximal lag length chosen for the instruments. Thus it would be interesting to conduct the estimation of the orthognality restrictions in (26) under different assumptions about the maximum lag length (P) chosen for the instruments.

Exogeneity Specification Tests of the Flex-Price Exchange Rate Determination Model for Canada and the U.S. March 1973 - August 1980

This appendix presents the results of performing joint exogeneity tests for the flex-price model of exchange rate determination using the procedures suggested by Geweke [1978] or Geweke and Dent [1978]. Specifically, the joint exogeneity of the variables in the vector autoregression $\underset{\sim}{X}_t' = [m_t, y_t, i_t^*, P_t^*]$ is tested with respect to the hypothesized endogenous variables in the flex-price model, P_t the price of domestic (i.e. Canadian) goods and S_t the Canadian U.S. dollar exchange rate. The variables m_t and Y_t are defined as Canadian money and real income while i_t^* and P_t^* are the U.S. eurodollar interest rate and the U.S. price level respectively. The data series used are described in more detail in appendix 3. The results of conducting these tests for the detrended data alone versus data which has been deseasonalized are presented in Table A1 below. The tests were conducted by estimating a restricted and unrestricted version of the following four equation system, written in vector notation as

$$(A1.1) = \underset{4x1}{\underset{-}{X}_t} = \sum_{s=1}^{w} \underset{4x4}{F_s^*} \underset{-}{X}_{t-s} + \sum_{s=1}^{\ell} \underset{2x2}{G_s^*} \underset{-}{Y}_{t-s} + \underset{4x1}{\underset{-}{\varepsilon}_t^*}$$

where $\underset{-}{X}_t$ is defined as above, $\underset{-}{Y}_t' = [S_t \; P_t]$, F_s^* and G_s^* are matrices of coefficients and $COV(\underset{-}{\varepsilon}_t^* \; \underset{-}{X}_{t-s}) = 0$ for all t and s = 1, ..., w,

$COV(\underset{-}{\varepsilon}_t^* \; Y_{t-s}) = 0$ for all t and s = 1, ..., ℓ. Also

$\underline{\varepsilon}^*_t$ will be serially uncorrelated if w is chosen to be large enough. In choosing w and ℓ properly the investigator must choose a parameterization that offers a compromise between the criteria of unbiasedness which suggests a generous parameterization vs. power which will diminish as the parameter space expands. The results presented below employ a value of w = 6 and ℓ = 2 as the choice of lag length for the hypothesized exogenous and endogenous variables respectively. The results in Table A1 test whether $G^*_s = 0$ through the use of the likelihood ratio statistic

(A1.2) $L = T \mid \ln \text{Det}(\Sigma_1) - \ln \text{Det}(\Sigma_2) \mid \sim \chi^2(q)$ where Σ_1 is the variance covariance matrix of residuals for the restricted model where $G^*_s = 0$, Σ_2 is the unrestricted variance covariance matrix of residuals obtained by estimating (A1.1) with no restrictions imposed, T is the number of observations, and q is the number of restrictions imposed. Table A1 also presents the results of applying Sim's [1980] correction to these likelihood ratio statistics since the chi square statistics computed can be biased toward rejecting the exogeneity specification when the number of degrees of freedom left in the data is not a different order of magnitude than the number of restrictions being tested.

Table A1

Test of the Joint Exogeneity Specification of the Flex-Price Model

for Canada and the U.S.

1973 - 1980

Flex Price Model	Likelihood Ratio[A]	Sims Correction[B]
	$\chi^2(16)$	$\chi^2(16)$
Data Series detrended only	43.145**	28.596*
Data series detrended and deseasonalized	53.336**	28.528*

* indicates significance at the .05 significance level

** indicates significance at the .05 and .01 significance levels.

A The likelihood ratio is defined as $L = T|\ell n \text{ Det } \Sigma_1 - \ell n \text{ Det } \Sigma_2|$ where Σ_1 is the restricted model variance covariance matrix of residuals (i.e. the case where all hypothesized endogenous variables are assumed to have zero coefficients) and Σ_2 is the unrestricted variance covariance matrix of residuals.

B The sims correction takes the form $(T-K)/T \cdot L$ given the definition of L in A. Also T is the number of observations (86 in the present case) and K = the number of regression coefficients per equation. Thus k = 29 in the case where only the detrended series were used and k = 40 when the data series were deseasonalized and detrended.

The results presented in Table A1 test whether the sixteen coefficients on the hypothesized endogenous variables in the four equation system are significantly different from zero. For both the raw data (only detrended using a linear trend) and the deseasonalized and detrended series the joint exogeneity specification of the model was rejected at both the .05 and .01 levels using the likelihood ratio test of the restrictions described above. Applying "sims" correction still results in test statistics which are significantly different from zero at the .05 level for the flex-price model.

Appendix 2

Tests for the Order of the Vector Autoregression $\underset{4x4}{A(L)} \underset{-t}{X} = \underset{-t}{V}^{x}$.

This appendix presents the results of conducting tests for the finite lag length of the vector autoregression generating the forcing variables in the vector $\underset{-t}{X}' = [m_t, Y_t, i_t^*, P_t^*]$ where the elements in $\underset{-t}{X}$ are defined in Appendix 3. Table A2 below presents the results of applying two different multivariate criterions for choosing lag length for the vector autoregression of "forcing" variables described in the text.

Table[a] A2

lag length.	CAT(r)[c]	Likelihood Ratio[b]
1	-31,947.2	$75.25^* \sim \chi^2(48)$
2	-30,885.9	$50.328^* \sim \chi^2(32)$
3	-29,886.0	$25.953 \sim \chi^2(16)$
4	-29,155.4	

* Indicates rejection at the .05 significance level.

a) The author wishes to thank Richard Meese for providing software which was modified in order to obtain the results in this table.

b) The likelihood ratio statistics represent tests of four vs. one lag distributed $\chi^2(48)$, 4 vs.2, and 4 vs. 3 lags respectively.

c) The CAT criterion attributable to Parzen [1975] chooses lag length n to minimize the quantity.

$$\text{trace } [4/T \sum_{j=1}^{h} \hat{V}_j^{-1} - \hat{V}_h^{-1}], \; n = 1, ---, M,$$

where T is sample size, M is the maximal lag considered (eg. M=4) and $\hat{V}_j$ is an estimate of the contemporaneous covariance matrix of distrubances for the model with j lags.

The CAT criterion attributable to Parzen [1975] and the more standard likelihood ratio tests for choosing the finite unknown order of a vector autoregressive process, presented in Table A2 have been described by Meese [1980].

To limit computational costs the maximum value of lag length (r) was chosen to be 4. Thus in Table A2 the multivariate Cat criterion suggests a lag length of r=1. This is because the minimum vlaue of CAT(r*) occurs at r* = 1 and thereafter increases monitonically for r = 2, 3, 4. In contrast the likelihood ratio tests tend to suggest a lag length of three since the restrictions imposed by a one or two lag model are rejected at the .05 significance level. Thus, the results in Table A2 suggest that it is not unreasonable to assume that the order of the vector autoregression generating the forcing variables is unity.

Appendix 3

The data for this study were obtained from the following sources:

(1) M_t = Canadian money supply, M1 (currency + demand deposits) IFS DATA TAPES, not seasonally adjusted.

(2) Y_t = Canadian Industrial production index from the OECD (MEI), not seasonally adjusted.

(3) i_t^* = U.S. eurodollar interest rate, obtained from DRI. These are London Interbank offered rates (LIBOR) for Eurodollars. Not seasonally adjusted.

(4) P_t^* = Wholesale U.S. price index, obtained from Federal Reserve Board Data Base. Not seasonally adjusted.

(5) S_t = The spot exchange rate expressed in units of the domestic currency (i.e. Canadian dollars) per unit of foreign currency (i.e. the U.S. Dollar). This series was obtained from DRI. These are interbank rates (bid prices). Not seasonally adjusted.

(6) P_t = the industrial selling price index for Canada from the Canadian Statistical Review or the STAT Canada data base at the Federal Reserve Board. Not seasonally adjusted.

Footnotes

*/Economist, Division of International Finance, Board of Governors of the Federal Reserve System. The views expressed in this paper are solely those of the author and should not be interpreted as those of the Board or other members of its staff. The paper is from the author's doctoral dissertation ("Theoretical and Emprical Essays on the Determination of Spot and Forward Exchange Rates," University of Virginia - Charlottesville, 1982). The author is particularly indebted to his thesis advisors Robert Flood and Richard Meese for encouragement and support. Kenneth Rogoff also made many helpful comments on an earlier draft of this paper presented at the 1981 Winter Econometric Society Meetings. Discussions with Robert Cumby, Dale Henderson, Robert Hodrick, Peter Hooper, Maurice Obstfeld, Kenneth Singleton and Steve Symansky led to improvements in the paper.

1/Although, these authors adopt a methodology similar to the one used in the present study in order to derive the cross-equation restrictions implied by the rational expectations assumption, their work is aimed at estimating different models of exchange rate determination than the flex-price model developed in the present study. (See Glaessner [1982A] Chapter III). Moreover the methodology used in estimating their models, the joint assumptions of exogeneity of the "forcing" variables and the nonallowance of complicated interactions between the "forcing" variables in the vector autoregression are not necessary in the present study.

2/Authors such as Frenkel [1976, 1979], Bilson [1978] and Frankel [1979, 1981] have also made simplifying assumptions with respect to (a) and (b) in their empirical work. Frankel [1979] does admit the possibility of a quasi-reduced form error term in the spot exchange rate equation following a first order autoregressive process. However Hakkio [1981] finds evidence which suggests that the stochastic process generating the error term is somewhat more complicated than an AR(1).

3/This was the case for the models of Bilson [1978], Hartley [1982], Frankel [1979] and others. (See Glaessner [1982A] Chapter II).

4/ See Obstfeld [1983] and Cumby and Obstfeld [1983] for a discussion of how and why conditional heteroskedasticity of error terms can present problems.

5/ As will be noted below (Section III) and as is discussed in more detail in Glaessner [1982A] Chapter IV, the GMM estimator as applied in the present context does require that the investigator make some assumption regarding the order of the serial correlation in quasi-reduced form error terms. Such an assumption is necessary when predetermined values of the forcing variables are used as instruments. This assumption, is weaker than that typically assumed, i.e. that the error term is characterized by a specific distribution function. It should also be observed however, that to the extent that the distribution function assumed for the error terms in using maximum likelihood procedures is the true one, the estimates obtained will be more efficient than those obtained using GMM procedures.

6/ These assumptions about the lack of interaction between "forcing" variables are frequently made so as to obtain closed form solutions more easily. For example Driskill and Sheffrin follow Frankel [1979] by assuming that the log of the growth rate of the money supply follows a random walk. Papell [1981] assumes either an independent first order autoregressive process on each exogenous variable or an upper traingular vector autoregressive process on the exogenous variables. Finally Hartley [1982] assumes that foreign income and money do not effect domestic income and money in obtaining closed form solutions so that he need only invert 2x2 polynomial matrices in obtaining closed form-solutions. However, given that in Hartley's model agents see both foreign and domestic variables it is not clear why they would use only a subset of this information in order to form optimal predictors of domestic vs. foreign variables. This dichotomy does not seem reasonable.

7/ The price adjustment specifications adopted by these authors are not all alike. Dornbusch [1976] assumes that the rate of price change depends only upon excess demand in the goods market. Flood [contrast assumes that pricing is completely anticipatory so that the rate of price change depends only upon the expected rate of change in the price which would clear the domestic goods market. The price adjustment specifications adopted by Salemi [1979], Hooper and Morton [1980], Frenkel [1979] and Driskill and Sheffrin are similar to that in Dornbusch [1976] with some minor modifications (see Glaessner [1982A] Chapter III. The price adjustment specifications of Mussa [1982A, 1982B] and Flood [1982] tend to subsume both of the above approaches to price adjustment. See Glaessner [1982A] for a more detailed explanation of these different price adjustment rules and for a critical review of some of these different specifications. The exchange rate overshooting properties of these models can also be obtained by assuming that wages are sticky. (See Obstfeld [1981] or Rehm [1981].

8/ It should be observed that the "flex-price" model derived below can also be viewed as a special case of a more general "sticky-price" model (see Glaessner [1982A]); thereby permitting empirical tests of different price adjustment specifications posited by various authors. This model is solved in Chapter III of Glaessner [1982A] where closed form solutions for the exchange rate and prices are derived which can be used to estimate the model.

9/ For a treatment of prediction theory, see Sargent [1979] or Hansen and Sargent [1980].

10/The error term in the model could also have been introduced on the supply side of the market once specific stochastic processes have been specified for the "forcing" variables m_t, Y_t, i_t^* and P_t^*. (See Glaessner [1982A] Chapter III).

11/ Specifically using difference equation techniques to solve the model allows one to make assumptions about the stochastic processes generating the exogenous variables later in the analysis. More importantly, following such a procedure allows for assuming that the exogenous variables follow a general vector autoregressive process.

12/ Also allowing for wealth to enter the model explicitly would result in the need to model savings behavior which will increase the order of the dynamics in the system. This point has also been made by Mussa [1982A].

13/ This assumption is, of course, a major simplification. The work of Flood [1980] suggests that intervention can lead to more volatility in exchange rates. A more detailed treatment of the effects of intervention are provided by Henderson [1979, 1980] who handles the two country case and Canzoneri [1980] who discusses the multiple country case. Allowing for intervention in the present model might be an interesting extension (see Henderson [1980], p. 41).

14/ Meese and Singleton [1980B] Hansen and Hodrick [1980, 1981], Stockman [1978], Tryon [1979] and Geweke [1979] find that this relationship does not tend to hold while Hakkio [1980] and Frenkel [1979] find that it does.

15/ As mentioned in Chapter III of Glaessner [1981A] letting output be endogenous without changing the order of the difference equation system may be useful in efforts to derive the rational expectations restrictions explicitly. In addition, Meese and Singleton [1980A] have pointed out that a richer specification for an exchange rate model (with less exogenous variables) may explain a good deal of the volatility of the exchange rate without appealing to exogenous driving processes that have infinite variances.

16/ Note that there is an asymmetry here between the definition of real income used in the money demand equation vs. goods demand. In future work real income will be defined similarly for each equation, however, the correct specification does not affect the results. In addition, the specification of the demand for domestic goods equation does not allow for the impact of government on the demand for domestic goods. Incorporating government into the model in the demand function for domestic goods would be a further extension of previous work. For example, see Flood [1981], Mussa [1982A], and Dornbusch [1976].

17/ Specifically, the model can be written as a pair of simultaneous first order difference equations in P_t and S_t with the roots

$$\lambda_1 = \frac{\alpha_1}{1+\alpha_1} > 1, \quad \lambda_2 = \frac{B_1 + B_2\sigma}{B_2\sigma} > 1$$

as shown in Glaessner [1982A] Chapter III. The model is solved by following the conventional practice of choosing terminal conditions which make the general rational expectations solution and the particular solution coincide (i.e. bubbles in the sense of Flood and Garber [1980B] are excluded).

18/ Note that constant terms do not appear in these solutions. This is because each series and various lagged values of each series are detrended and deseasonalized so that the resulting series are assumed to be mean zero, linearly indeterministic, covariance stationary processes.

19/ When output is allowed to be an endogenous variable, this property of the model no longer holds. Thus, authors who have emphasized this point (see Mussa [1982A]) have been drawing conclusions which are too strong given the specific structure of the model.

20/ See Glaessner [1981] Chapter III, (Section 3.4(a) and Appendix 4) for a detailed derivation of the equations presented below. Note also that to be able to apply the prediction theory discussed in Hansen and Sargent [1980, 1981] or Sargent [1979] requires that the variables in the vector $\underset{\sim}{X}_t$ be jointly covariance stationary. To insure that this was the case it was assumed that the roots of det A(Z) = 0, for Z complex, were outside the unit circle. This insured that the polynomial matrix A(L) was invertible. Weaker conditions which suffice for applying these prediction formulas are discussed in Hansen and Sargent [1980].

21/ The conditions under which V_{1t} and V_{2t} will follow ARMA(1,1) processes are explored in Section 3.4(a) of Chapter III in Glaessner [1982A]. Specifically, this will be the case if one of the structural errors is white noise (e.g., u_t) and the other error (e.g., ε_t) follows an AR(1) process. In this case, the error terms (V_{1t}, V_{2t}) appearing in the price and spot exchange rate equations will each be sums of uncorrelated white noise and AR(1) processes which Granger and Morris [1976] show to be ARMA(1,1) processes. If both u_t and ε_t are assumed to follow independent AR(1) processes, the V_{it} $i=1,2$ will be an ARMA(2,1). The assumption of an ARMA(2,1) process is not adopted because the higher order of the autoregressive component of the error term will complicate the nature of the restrictions greatly and will increase the number of structural parameters to be estimated. An additional assumption which is spelled out in more detail in Glaessner (1982A) Chapter IV, concerns the definition of the information set Ω_t. In the context of estimating the model it was assumed that $\Omega_t = \{ X_t, X_{t-1}, --- \}$. where both economic agents and the econometrician use Ω_t to form optimal predictors of $\hat{X}_{t+k}$. Assuming that agents observe a different information set than the econometrician, (see Shiller [1972]) introduces an additional forecast error into the set-up described below. However, it can be shown that allowing for this forecast error will not alter the orthogonality restrictions derived below.

22/ The set of instruments could also have included lagged values of the variables P_t and S_t in addition to the lagged values of the "forcing" variables. Thus, an implication of this observation would be to use a different set of instruments in estimation. Future work may address this issue.

23/ $E[V_t^*|\Omega_{t-1}] = E[X_t - E[X_t|\Omega_{t-1}]|\Omega_{t-1}] = \underset{4x1}{0}$.

24/ Note that $E[V_{it}^*|\Omega_{t-2}] = E[Q_{it} - g_i Q_{it-1}|\Omega_{t-2}] = 0$ for $i = 1, 2$ if it is assumed that the elements in $\Omega_{t-2} = \{X_{t-2}, X_{t-3}, \ldots\}$ are uncorrelated with Q_{it} and Q_{it-1} for $i = 1, 2$. This will be the case since the fundamental noises Q_{it} $i = 1, 2$ are not assumed to be serially correlated. Note, however, that in general the MA(1) error is not projected on Ω_{t-1} because we allow for contemporaneous correlation between the elements of Ω_{t-1} and Q_{it-1}, $i = 1, 2$.

25/ In Glaessner [1982A] a more detailed explanation for the notation adopted here is given. Also the observant reader will have noticed that one orthogonality condition is missing in equation (21) to the extent that we want to use the fact that

$$\underset{4x1}{E[V_t^x X_{t-\nu}']} = 0 \text{ for } \nu \geq 1 \text{ instead of } \nu \geq 2. \text{ This}$$

consideration could be allowed for in estimating the model. There is, however, no sense in which using $\nu \geq 2$ will lead to inconsistent parameter estimates.

26/ For example, to see how the coefficients (Ψ_j^*) in $\Psi^*(L) = \sum_{j=0}^{r-1} \Psi_j^* L^j$ were derived, note that from equation (13)' and (14)' we can derive expressions of the form

$$\Psi^*(L) = \sum_{j=0}^{r-1} \Psi_j^* L^j = A^{-1}(\eta_1)[I + \sum_{j=1}^{r-1} (\sum_{k=j+1}^{r} \eta_1^{k-j} A_k) L^j]$$

Note that when $j = 0$ we have $A^{-1}(\eta_1)$ so $\Psi_0^* = A^{-1}(\eta_1)$, however, for $j = 1, \ldots, r-1$ the double sum in the far right expression can be rewritten in terms of the Ψ_j^*'s as

$$\Psi_j^* = A^{-1}(\eta_1)(\eta_1{}^{j}A_{j+1} + \eta_1{}^{j+1}A_{j+2} + \ldots + \eta_1{}^{r-j}A_r)$$

for $j = 1, \ldots, r-1$. This can be seen by writing out the double sum $\sum_{j=1}^{r-1} (\sum_{k=j+1}^{r} \eta_1{}^{k-j}A_k)$ explicitly.

27/ In practice it is possible to impose these restrictions by inverting the polynomial matrix A(L) in the frequency domain by using procedures suggested by Sargent [1979]. The moving average representation thus obtained is then truncated at values for the moving average parameters which are extremely small and the inverted polynomial matrix is evaluated at the discount factor of interest (η_1) or (η_2). Having done this, the restrictions embodied in the Ψ_j's $j = 0, \ldots, r-1$ can be built up forming the criterion function to be minimized.

28/ These lag length specification tests are done for Canada and the U.S. over the period March 1973 to August 1980 for monthly data. See Appendix 3 for a description of the data and Appendix 2 for a more detailed explanation of the tests for finite lag length.

29/ More formally, for the theoretical model to be identified Hansen and Singleton [1982] argue that the QxR partial derivative matrix $D_0 = E[\frac{\partial f_t}{\partial \zeta_0}(\zeta_0)]$ must be of full rank and the nonlinear f function RxQ characterizing the orthogonality restrictions must be differentiable. Moreover an implicit exclusion restriction in the current specification of the flex-price model concerns the fact that the variables in the vector autoregression are not allowed to be dependent on contemporaneous values of the spot exchange rate or domestic prices. In addition, the overidentifying restrictions implied by the theoretical model can be tested by constructing fairly straightforward statistics which only involve estimation of the model subject to the nonlinear restrictions in $\Psi^*(\cdot)$ and $\Psi^{**}(\cdot)$ unlike likelihood ratio tests. See Glaessner [1982A] Chapter IV, or Hansen and Singleton [1982].

30/ This is primarily due to the fact that joint estimation of all equations using the orthogonality restrictions in equation (21) would take into account the nonlinear restrictions across the vector autoregression and the price and exchange rate equations. An additional implication of this less efficient estimation procedure is that the vector of innovations $\tilde{V}^{x}_{it}$ is assumed to be contemporaneously uncorrelated with the error terms $\tilde{V}^{*}_{it}$ i=1, 2 within $\tilde{d}_t$ so that

$$E[\underset{6x1}{\tilde{d}_t} \underset{1x6}{\tilde{d}_t'}] = \begin{bmatrix} \underset{2x2}{\Sigma} & \underset{2x4}{0} \\ \underset{4x2}{0} & \underset{4x4}{V} \end{bmatrix}$$

where Σ and V are as defined in the text.

Also it is necessary that the innovations in $\tilde{V}^{x}_{t}$ not be correlated with the fundamental white noise components Q_{it} i=1,2 at all leads and lags, a condition which holds given the assumptions about the Q_{it} i=1,and $\tilde{V}^{x}_{t}$ disturbance terms made in the text.

31/ A more formal discussion of this point can be found in Glaessner [1982A] Chapter IV. Also see Hansen [1982], Hansen and Sargent [1981] or Hansen and Singleton [1982] for a discussion of how to form GMM estimators within a variety of different contexts.

32/ A formal discussion of this test statistic is given in Glaessner [1982A], Hansen and Sargent [1981] and Hansen and Hodrick [1981].

33/ Use of a different algorithm (other than DFP) to minimize the criterion functions in question would seem useful in this context. Also using different procedures to approximate derivatives would be useful since the search procedure used to find a minimum depends critically on the calculation of these derivatives. Moreover meeting the convergence criteria on each coefficient can be difficult when one is minimizing a function over a fairly large number of parameters (eight or twenty-four).

34/ In practice perturbations of about ((.0001)* coefficient value) or larger were sufficient to ensure that the D_T matrix would be of full rank. Also the asymptotic standard errors tended to get smaller as the perturbation became larger (i.e. as the approximation to the "true" derivatives became worse.) More experimentation with regard to developing some way of optimally choosing the amount by which to shock $f_t(\hat{\zeta}_T)$ in approximating D_T would seem useful.

35/ This weighting or distance matrix and the criterion functions referred to are discussed in detail in Glaessner [1982A] Hansen [1982] and Hansen and Singleton [1982].

36/ This is due to the fact that as the number of orthogonality conditions grows relative to sample size, the estimates of the parameters, and asymptotic standard errors tend to become very imprecise (see Hansen and Singleton [1982] and Section IV above).

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