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J-CURVES AND STABILITY OF THE FOREIGN-EXCHANGE MARKET

by

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This paper considers certain aspects of the interaction of the exchange rate and current account during the adjustment of an open economy to an exogenous shock. Its special focus is on the character of the adjustment path when the current account reacts to an exchange-rate change in a delayed and staggered fashion - i.e., with a so-called J-curve response. The study is motivated by the frequent observation in recent popular accounts of exchange-market developments that some currencies -- such as the yen, for example -- have been especially volatile because of the presence of pronounced J-curve effects. According to these accounts, transactors in the foreign-exchange market are "misled" by the near-term, perverse effects of an exchange-rate change on the current account. Their response gives rise, in turn, to further pressures on the exchange rate in the same direction as that of the original shift. As a result, this perverse feedback between the exchange rate and current account will generate a movement of the exchange rate in the "wrong" direction, at least until lagged,stabilizing current-account reactions come into play. Some accounts have gone further to suggest that such a process may generate endogenous cycles that overshoot the ultimate equilibrium and may even give rise to a fundamentally unstable dynamic. In the latter case, some sort of intervention or other braking device is required to maintain the system within acceptable bounds.

One purpose of this paper is to assess the extent to which these properties are found in a more formal model of exchange-rate adjustment both in which J-curve effects are present and which also spells out a more complete (and more supportable) view of expectations formation.[1/] To be specific, I adopt here the now conventional

1/ There have been a limited number of papers that deal with the relationship between lagged trade responses and stability of the foreign-exchange market; leading examples are Williamson (1972) and Britton (1970). In general, these earlier models of the foreign-exchange market are quite partial in nature and regard the determination of the exchange rate and its equilibrium as flow phenomena. A more recent paper that takes a slightly different approach from this one is Driskill and McCafferty (1980).

assumption that transactors make forecasts based on full knowledge of the structure of the economy -- including, in this case, knowledge of present and future J-curve effects.[1/] In this framework, rather than being "misled" by incoming data, agents are assumed to incorporate it, along with all other relevant available data, into forecasts that are consistent (i.e., identical) with actual realized outcomes.

International macro models that presuppose rational expectations (or its close relative, short-run perfect foresight) with respect to exchange rates are now in wide use. Many of these models exhibit a property that I shall refer to here as first-order, conditional stability. This term, when applied to a model, indicates that the system of differential equations by which the model can be represented has exactly one root with a positive real part. The presence of this positive root means that the corresponding economic system is unstable unless at least one variable can make a discrete jump at some point to place the system exactly on its unique stable trajectory. Without such a jump, given enough time the system will spontaneously diverge from equilibrium. In many applications, the present level of the exchange rate provides a convenient jump-variable, although stories to explain how the foreign-exchange market can produce just such an exact shift are tortuous and, in my view, improbable if taken literally.[2/]

1/ The approach in this paper bears a superficial resemblance to a branch of the literature in this area in which market participants fully anticipate the effects of a future shock to the system. (See for example, Dornbusch and Fischer (1980), C. Wilson (1979), or Rogoff (1979).) The "rules of the game" in these papers dictate that a future, fully anticipated event results in a discrete jump in the present exchange rate. Jumps at other (future) moments are excluded because they imply infinite capital gains. Here, lagged responses in the current account play a similar role in that they are fully anticipated (and, because of the specification of the lag as discrete, they impact at a single future moment). They differ, however in that they are generated continuously so that they imply a continuous adjustment of the present exchange rate, rather than a jump. As we shall see below, the discrete specification of the lag is primarily a convenience.
2/ We need not take these explanations literally, of course, and a good deal of work has gone into showing how a different expectations-formation process (such as an adaptive expectations rule) or some endogenous learning process can soften the dynamics and enhance a model's stability.

As we shall see from the findings below, under certain conditions, the introduction of J-curve effects can raise the order of conditional stability of a system from first to second order and beyond. Since in these cases there is an insufficiency of jumping variables to place the system on its stable trajectory, such a system must be inherently unstable. This finding is of some interest in its own right, since it tends to confirm the conjecture drawn from the popular view, but it also has broader implications for this general class of dynamic model as well. Inasmuch as lags and leads are pervasive in any realistic macro system, it suggests either that stability conditions in macro systems may be less easily met than had been thought previously or, at least, that there are some important shortcomings to models with jumping variables which are not yet fully appreciated.

<u>A model of exchange-rate adjustment</u>

At the risk of greatly oversimplifying matters, I shall take the following two semi-reduced-form relationships to be a fair representation of a broad class of international macro models:

(1) $E = F(\varepsilon, V)$, $\partial F/\partial\varepsilon > 0$, $\partial F/\partial V < 0$;

(2) $\dot{V} = G(E, V)$, $\partial G/\partial E > 0$, $\partial G/\partial V < 0$,

where E is the exchange rate (in home currency per unit of foreign exchange), ε is the expected rate of change of the exchange rate, and V is the nominal value of home-country wealth when all assets are expressed in home currency.1/

Equation (1) expresses the relationship between domestic wealth, the exchange rate and its expected rate of change, when asset markets are in equilibrium. (Asset stocks are taken to be fixed, and other financial variables, such

1/ The underlying model could just as well be expressed in terms of real wealth without any important changes in the conclusions.

as interest rates, are assumed to be determined endogenously.) Equation (2) assumes that the present rate of change in wealth, $\dot{V}$, is equal to the current-account position, which is determined, in turn, by the present levels of both the exchange rate and wealth itself.1/

If we now impose the requirement that transactors in the foreign exchange market have perfect foresight (i.e., that $\varepsilon = \dot{E}$), we can rearrange (1) and linearize both (1) and (2) to obtain

(3) $\dot{E} = a_0 + a_1 E + a_2 V, \quad a_1 > 0,\ a_2 > 0;$

(4) $\dot{V} = b_0 + \bar{b}_1 E + b_2 V, \quad \bar{b}_1 > 0,\ b_2 < 0.$

For purposes of later comparisons, it will be expedient to write this pair of first-order differential equations as a single second-order equation in one of the variables. Arbitrarily selecting E as the variable of interest here, we can differentiate (3) and substitute to obtain

(5) $\ddot{E} + c_1 \dot{E} + \bar{c} E + K = 0,$

wherethe constants c_1, $\bar{c}$, and K are given by

$$c_1 = -(a_1 + b_2) > 0,$$ 2/

$$\bar{c} = (b_2 a_1 - \bar{b}_1 a_2) < 0,$$

$$K = (b_2 a_0 - a_2 b_0).$$

1/ Clearly a number of variations on a basic international portfolio model can fit this deliberately general characterization. The literature in this area is extensive, but a recent, representative paper that synthesizes several approaches and provides a useful bibliography is Rodriguez (1980).

To avoid unnecessary complications, interest payments have been ignored in specifying equation (2).

2/ It is usual to assume in these models that $c_1 > 0$ -- that is, that the stabilizing effects of wealth on its own rate of change ($b_2 < 0$) dominate the potentially destabilizing effect of the exchange rate on its expected (and actual) rate of change ($a_1 > 0$). This assumption is made, it appears, largely because, if the opposite were the case and the current-account balance responded perversely to an exchange-rate change, a (first-order conditionally stable) rational expectations solution might not exist. We shall continue to honor this tradition in this analysis but without great conviction.

Stability properties of (5) can be determined by investigating the signs of the real parts of the roots to the homogeneous portion of (5). As is well known, in a simple second-order system of this type, the corresponding roots are

$$r = \frac{-c_1 \pm \sqrt{c_1^2 - 4\bar{c}}}{2}$$

Inasmuch as $\bar{c}$ is negative, it is evident that in this type of model equation (5) must always have two real roots, one negative and one positive; the system exhibits, therefore, first-order conditional stability. A closer look at the components of $\bar{c}$ reveals that the problem in this case (problem, in the sense that the presence of a positive root prevents the system from being globally stable, as would be the case if all roots were negative in their real parts) arises from the positive sign on a_1. This observation should not be surprising, since a positive value for a_1 merely indicates that accurately anticipated expectations of an exchange-rate change tend to put pressure on the rate in the same direction as the expected change itself -- a property that is well known to be destabilizing in much simpler, earlier models of exchange-rate dynamics.[1/] In the version at hand, however, a qualified weak form of stability is attained by introducing the device of an exactly-correct jump in the current exchange rate after any exogenous shock, as described earlier.

1/ For an early discussion of some of these issues see Baumol (1957). Although most of the modern papers that deal with conditional stability in this context seem to trace their intellectual origins to Dornbusch (1976) and Kouri (1976), the pedigree of this concept is really much longer. Earlier, partial models of the foreign-exchange market were also conditionally stable in the obvious sense that an exactly-correct jump to the equilibrium rate would keep the market stable following any exogenous shock. In a certain sense, the more modern approach has simply imbedded this basic property in a higher-order system which includes a feedback into the foreign exchange market from other sectors. One cannot help but notice, however, that most earlier writers avoided the temptation to simply assert that such a stability-ensuring jump in the exchange rate would take place automatically.

Introducing J-curve Effects

Equations (2), (4), and (5) presume that a depreciated exchange rate (a numerically higher value of E) is associated with a more positive level of the contemporary current account. That is, holding wealth constant, the counterpart in this model of the extended Marshall-Lerner condition is satisfied, and no J-curve effects are present. To introduce a workable approximation to a J-curve, let us modify equations (2) and (4) to

(2.a.) $\dot{V}(t) = G^*(E(t), E(t-\theta), V(t)), \quad \partial G^*/\partial E(t) \leq 0, \partial G^*/\partial E(t-\theta) > 0, \partial G^*/\partial V(t) < 0;$

(4.a.) $\dot{V}(t) = b_0 + b_1 E(t) + b_2 V(t) + b_3 E(t-\theta), \quad b_1 \leq 0, b_2 < 0, b_3 > 0,$

where the argument $(t-\theta)$ indicates values of a variable taken θ time units ago. Although there are obvious limitations to such a simple two-period version of the J-curve,[1/] nonetheless it does allow us to adjust θ, b_1 and b_3 to generate a rather wide range of "J" shapes. In this notation, a conventional J-curve is produced when the present exchange rate is negatively related to the present current-account position ($b_1 < 0$), while the exchange rate of one period previous is positively related to the present current-account position.[2/] In order to make fair comparisons, however, we shall

1/ Ideally, we should like to express the present current account as functionally dependent on all the exchange-rate levels over some relevant time interval, rather than at only two specific points. Even the simplest specification of this sort, however, leads to equations somewhat similar to so-called "renewal" equations and unmanageable technical complexities. For a discussion of equations of this type, see Bellman and Cooke (1963), Chapter 7.

2/ As is pointed out by Magee (1973), there is no theoretical reason that the response curve must have a J- shape. The response of the measured current account can be decomposed into "currency contract" effects (due to valuation effects on contracts already in place), "pass-through" effects (related to effects on the pricing of traded goods), and eventual volume adjustments. For any given exchange-rate change, these elements can overlap in a complex fashion giving a veritable "alphabet soup" (to quote Magee) of potential patterns. There is some evidence, however, that the simple J-shape is commonly found. (See, for example, Spitaller, (1980).)

customarily keep the long-run exchange-rate elasticity constant during any orthographic contortions -- i.e., we require the sum of b_1 and b_3 in (4.a.) to be constant, positive, and equal to $\bar{b}_1$ in (2).

Following the same general pattern as before, we can now combine (3) and (4.a.) to obtain a second-order equation in E, i.e.,

$$\ddot{E}(t)+c_1\dot{E}(t)+c_0E(t)+d_0E(t-\theta)+K=0, \tag{5.a.}$$

where

$$c_1= -(a_1+b_2)>0,$$
$$c_0=(b_2a_1-b_1a_2),$$
$$d_0= -(b_3a_2)<0,$$
$$K=(b_2a_0-a_2b_0).$$

Notice that according to our conventions the sum of c_0 and d_0 above is negative and equal to the value of $\bar{c}$ in equation (5). Furthermore, although d_0 is certainly negative, there is no clear presumption as to the sign of c_0.

Stability Properties of the Model with a J-Curve

Equation (5.a.) differs from its predecessor, equation (5), primarily by the inclusion of the lagged term and is, thereby, a mixed differential-difference equation. As such, finding and expressing its solution is considerably more complicated than in the previous case, but the general procedures for determining stability and other basic properties follow lines parallel to those of the simpler, non-mixed case.[1/] The stability properties of (5.a.) can be determined by examining the sign of the real parts of the roots of the characteristic equation associated with the homogenous part of (5.a.). Since solutions to (5.a.) take the exponential form,

$$E(t)=e^{rt} \quad ,$$

the characteristic equation for (5.a.) can be written as

$$(6) \quad H(r) = r^2 + c_1 r + c_o + d_o e^{-r\theta} = 0.$$

Furthermore, if we let $z=r\theta$, we can transform (6) to the more convenient form,

$$(7) \quad H(z) = z^2 + m_1 z + m_o + n_o e^{-z} = 0 \ ,$$

where

$$m_1 = c_1\theta > 0 \ ,$$
$$m_o = c_o \ \theta^2 \ ,$$
$$n_o = d_o\theta^2 < 0 \ ,$$

and

$$m_o + n_o = \bar{m} \equiv \bar{c}\theta^2 < 0.$$

1/ An extensive treatment of mixed-type equations is found in Bellman and Cooke (1963). Some simple, practical guidelines on determining their properties is provided in the appendix to Gray and Turnovsky (1979). The analysis in this paper focuses mainly on the strongest conjecture from the popular accounts -- namely, the presence of a fundamental (endogenous) instability. To deal with the weaker conjectures -- i.e., the possibilities of initial perverse exchange-rate movements and overshooting -- one needs to consider initial conditions and the corresponding particular solution to the system. For even the simplest examples, it is not easy to obtain a concise closed-form solution to a mixed-type equation. In general, however, initial conditions on both e and V must be specified over an <u>interval</u> of length θ (or conditions that are equivalent); the solution will usually take the form of a series of smooth functions, joined at every integral multiple of θ at a point where the first derivative is discontinuous. Properties of the solution in its early stages can be inferred for elementary examples by building up the solution by a simple step-by-step "continuation" technique. (See Bellman and Cooke (1963), Chapter 3.) In the long-run, for a stable system, the trajectory is essentially independent of the intial conditions. For the system in this paper, it is fairly easy to confirm that in many circumstances, depending in part on initial conditions, exactly the properties described in popular accounts are seen -- namely, perverse movements and overshooting -- but a detailed treatment is beyond the scope of this paper.

Equation (7) is a transcendental equation in z which typically has an infinite number of roots. Some of these may be real; the others are complex and in conjugate pairs. The properties of the roots of (7) can be analyzed somewhat more easily by considering its quadratic and exponential parts separately. If we define

$$Q(z) \equiv z^2 + m_1 z + m_0 \quad ,$$

and

$$P(z) \equiv -n_0 e^{-z} \quad ,$$

then a solution of (7) satisfies

$$Q(z) = P(z) \quad .$$

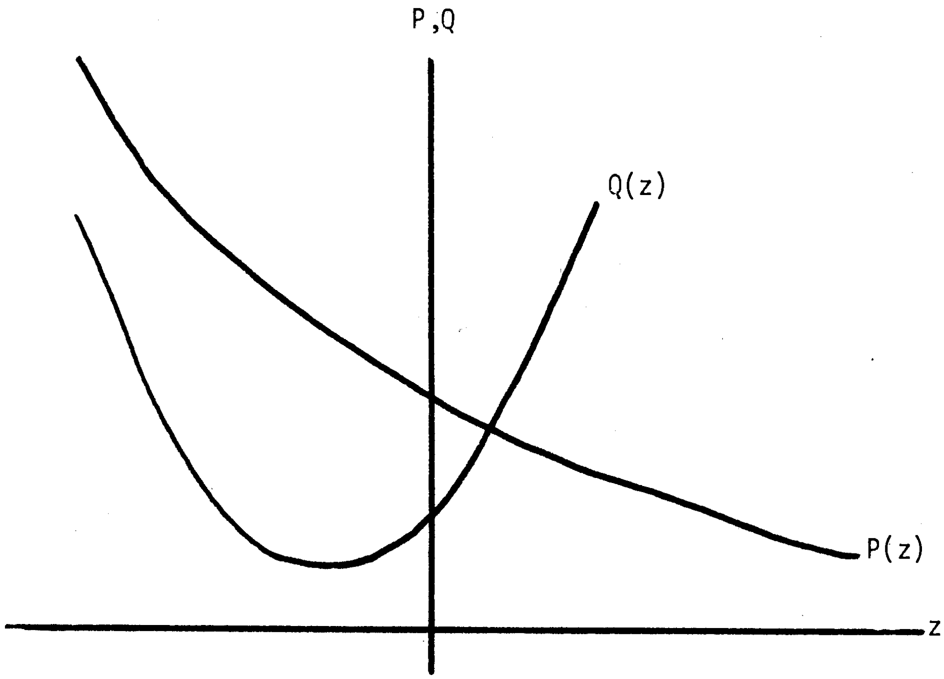
The two functions, Q(z) and P(z), are graphed in Figure 1. Since P(z) is negatively sloped throughout, it is evident that it can intersect Q(z) at no more than one point to the right of the minimum of Q(z); to the left of the minimum, there may be two intersections, a tangency, or none at all, depending on parameter values. The intercepts of P(z) and Q(z) are $-n_0$ and m_0, respectively; since we require, $(m_0+n_0)<0$, evidently there must be exactly one positive real root of (7), and the system is no less than first-degree conditionally stable.

Whether or not equation (7) also has complex roots with positive real parts remains to be answered. To gain some insight into when this is the case, let us write a representative complex root in the form

$$\rho = \beta + \alpha i \quad ,$$

where both α and β are real. Substituting this expression into (7), we find that any complex root of (7) must meet the following pair of conditions:

Figure 1



Quadratic and exponential part of:

$$H(z)=z^2+m_1 z+m_o+n_o e^{-z};$$

$$P(z)= -n_o e^{-z} \quad , \; n_o <0,$$

$$Q(z)= z^2+m_1 z+m_o, \; m_o >0,$$

$$m_o+n_o=\overline{m} <0.$$

(8) $2\alpha\beta + m_1\alpha - n_0 e^{-\beta}\sin\alpha = 0$,

(9) $\beta^2 + m_1\beta + m_0 = -n_0 e^{-\beta}\cos\alpha + \alpha^2$

So as to be somewhat more systematic, consider first the special case in which the present current account is positively related to the lagged exchange rate but unrelated to the present exchange rate -- a situation that might be described as a (reverse) "L-curve". In such a case, we specify $m_0=0$ and $n_0=\bar{m}<0$. Equation (8) defines implicitly a (non-unique) relationship between α and β, which we designate by

(10) $\alpha = \phi(\beta)$.

Using (10), we can now express the right-hand side of (9), for any particular α that satisfies (10), as

(11) $\tilde{P}(\beta) = -\bar{m}e^{-\beta}\cos(\phi(\beta)) + \phi^2(\beta)$,

where the notation, $\tilde{P}(\beta)$, is meant to indicate that α is now endogenous.

The existence of a complex root to equation (7) with a positive real part requires that

(12) $Q(\beta)=\tilde{P}(\beta)$, $\beta > 0$,

where $\phi(\beta)$ stands for one of the values of α that satisfies (10) (the same α in both its appearances in (11)).

To establish sufficient conditions for (12) to hold, we shall consider only the relevant values of α in the first full cycle -- i.e., where $\pi<\alpha<2\pi$. 1/

1/ There may well be other solutions for $\alpha > 2\pi$; we confine the analysis to the range where $\alpha \leq \pi$ because $(\sin\alpha/\alpha)$ has the greatest amplitude in this range.

First, for there to be <u>any</u> values of α in this (or any subsequent cycles) that satisfy (8), it is easy to determine (from (8)) that the following necessary condition on m_1 and $\bar{m}$ must be met:

$$(13)\quad \left|\frac{m_1}{\bar{m}}\right| < \frac{\sin\alpha^*}{\alpha^*} \quad ,$$

where α^* is the value of α in the interval, $[\pi, 2\pi]$, for which

$$\alpha = \tan \alpha \ .$$

Condition (13) simply says that a positive complex root requires that the algebraic sum of own-variable effects in (3) and (4) not be of too large a scale.1/

Consider now the two intercepts of $\tilde{P}(\beta)$, (again referring only to the interval, $[\pi, 2\pi]$). We designate these as $\tilde{P}_1(0)$ and $\tilde{P}_2(0)$, corresponding to $\tilde{P}(\beta)$ at $\beta=0$, with α taken at the smaller and larger values that satisfy (10) in the indicated range, respectively. These values of α, I shall designate as $\alpha_1(0)$ and $\alpha_2(0)$. Since $\pi<\alpha_1(0)<\alpha_2(0)<2\pi$, it can be confirmed that

$$\tilde{P}_1(0)<\tilde{P}_2(0).$$

The inverse of the relationship $\tilde{P}(\beta)$ is continuous in the domain $[\tilde{P}_1(0), \tilde{P}_2(0)]$, positive in β for at least some $\tilde{P}$ in this domain, and has at most two zeros. Accordingly, a simply expressed sufficient condition (in addition

1/ Though it is rather restrictive, condition (13) is by no means impossible to satisfy in a practical example. Recall from earlier discussions that a_1 and b_2, two components of m_1 that enter additively, are of opposite sign. Accordingly, they could very nearly offset one another and, thereby, keep m_1 very small.

to (13) above) for (12) to hold is

(14) $\tilde{P}_1(0) < 0$ and $\tilde{P}_2(0) > 0$.

Finally, to show when (14) is met, let us write (8), when $\beta = 0$, as

$$\sin \alpha(0)/\alpha(0) = m_1/\bar{m} ,$$

and substitute into (11) to get

(15) $\tilde{P}(0) = \alpha(0) m_1 [-\tan^{-1}\alpha(0) + (\alpha(0)/m_1)]$.

It is easy to confirm that as the value of $(m_1/\bar{m})$ approaches zero, the values of $\alpha_0(0)$ and $\alpha_1(0)$ approach π and 2π, respectively, and that $-\tan^{-1}\alpha(0)$ approaches, respectively, $-\infty$ and $+\infty$. Consequently, for a given value of m_1, it is always possible to find a value of $\bar{m}$ large enough in absolute size (and not necessarily infinite) so as to satisfy both (13) and (14), and thereby give rise to at least second-order conditional stability in (7).

A particularly interesting example of this occurs if we consider simply extending the lag in (7) (i.e., making θ larger) without changing any other parameters. For the "L-curve" example above, this means that both m_1 and $\bar{m}$ must increase, but the latter more rapidly; hence, the ratio $(m_1/\bar{m})$ must decline in absolute value as θ is extended, and $\alpha_1(0)$ and $\alpha_2(0)$ approach π and 2π as before. In this case, the expression equivalent to (15) above is

(16) $\tilde{P}(0) = \alpha^2(0) \left\{ -(c_1^2/\bar{c}) \left\{ \dfrac{\cos\alpha(0)}{1-\cos^2\alpha(0)} \right\} + 1 \right\}$.

Again, it is easy to show that, as θ gets large, $[\cos\alpha(0)/(1-\cos^2\alpha(0)]$ approaches $-\infty$ and $+\infty$, for $\alpha_1(0) \simeq \pi$ and $\alpha_2(0) \simeq 2\pi$, respectively. Hence, with unchanged elasticities, a sufficiently long lag, by itself, is enough to raise a system such as that in equation (7) to conditional stability of at least second-order. Unless some variable in addition to the present exchange rate is allowed to make an appropriate exact jump, such a system cannot automatically return to a stable equilibrium following an exogenous shock.

The remarks above deal with the effects of a change in only one dimension of the J-curve, its duration. We can also look, however, at the effect on stability of altering the degree of "stagger" in the J by strengthening the near-term, perverse effects, while at the same time adjusting lagged effects to keep constant the long-run cumulative effect of an exchange-rate change. When $m_0 \neq 0$, the expression equivalent to (15) is

$$(17) \quad \tilde{P}(0) = -\bar{m} + \alpha^2(0) + \alpha m_1[\sin^{-1}\alpha(0) - \tan^{-1}\alpha(0)] ,$$

where $\alpha(0)$ is given by

$$\sin\alpha(0) / \alpha(0) = m_1/m_0 ,$$

Again, as m_0 gets sufficiently large (with m_1 and $\bar{m}$ constant), $\alpha(0)$ approaches π and 2π, and $[\sin^{-1}\alpha(0) - \tan^{-1}\alpha(0)]$ approaches $-\infty$ and 0, respectively. Since $(-\bar{m} + \alpha^2(0))$ is positive and finite, it is evident that absolutely larger values of m_0 will tend to raise the level of an otherwise first-order stable system to second order and beyond. Hence, the greater the "stagger" in the J-curve, the greater is the likelihood of an unstable system.

Finally, it is worth taking a brief look at one other manipulation of the J-curve -- specifically, a movement forward. There is persuasive evidence that some traders may accelerate or delay current-account transactions in anticipation of a future exchange-rate change.[1/] If such a tendency is widespread, the consequence

1/ Anticipation effects in trade among the industrialized countries have been investigated empirically in a recent paper by J. Wilson and Takacs (1980). Additional discussion of this issue is found in Magee (1978).

could be a negative association between the current account and the expected exchange rate of, say, one period forward. These anticipation effects also would have their counterpart in a more than offsetting positive relationship between the contemporary levels of the current account and exchange rate as anticipation effects are made up for in the subsequent period. In our notation, if we now ignore lagged terms, the presence of anticipation effects could be shown in a modified version of (5.a.) as

$$(5.b.)\quad \ddot{E}(t)+ c_1\dot{E}(t) + c_0E(t) + f_0E(t+\theta) + K = 0$$

where

$$c_1>0\ ,$$
$$c_0<0\ ,$$
$$f_0>0\ ,$$

and

$$c_0+f_0=\bar{c}<0$$ 1/

Following the same procedures as before, the characteristic equation for (5.b.) can be written as

$$(7.b.)\quad H(z)=z^2 + m_1z + m_0 + s_0e^{-z} = 0\ .$$

where

$$m_1=\theta c_1>0\ ,$$
$$m_0=\theta^2c_0\lessgtr 0\ ,$$
$$s_0=\theta^2f_0>0\ ,$$

and

$$s_0+m_0=\theta^2\bar{c}<0\ .$$

1/ This specification assumes, in effect, short-run perfect foresight on the part of traders in goods markets -- an assumption which seem reasonable in view of its application to foreign-exchange market transactions elsewhere in the model.

Inasmuch as the analysis of this equation is very similar to that of the preceeding section, we shall not repeat it here. Suffice it to say that (7.b.) always has exactly one positive real root; in addition, if f_0 is sufficiently large, there are also complex roots with positive real parts, and the economic system cannot be stabilized by a single discrete jump in the exchange rate.

Concluding Remarks

Based on the findings above, I have to conclude that the popular view -- namely, that the presence of a J-curve response can lead to endogenous instability -- has been largely confirmed. In fact, it appears that the mere presence of a lag in the current-account reaction to exchange-rate changes, even without the extra destablizing kick provided by near-term perverse effects, can be destabilizing. Moreover, this can be the case even if the trade response satisfies the conventional Marshall-Lerner stability conditions. Furthermore, the findings also suggest that forward-looking trade effects related to anticipated changes in exchange rates can produce a similar instability in the exchange-rate adjustment mechanism.

Nor do these findings appear to be special cases without empirical relevance. Although we cannot very well attach numbers to the key parameters above without specifying more details of a particular model, it is apparent that the unstable result is more likely in every instance when m_1 is a small number. The coefficients that measure own-variable effects on $\dot{e}$ and $\dot{V}$ (i.e., a_1 and b_2) are of opposite sign and enter into m_1 additively, and there is no apparent reason to exclude the possibility that they might nearly offset one another to produce a sufficiently small (but still positive) value for m_1.

Although these findings suggest that an unstable system is a legitimate theoretical and empirical possibility, the implicit threat of an explosion or collapse of the foreign-exchange market (or macro economy, for that matter) is made a little less urgent if we recall some of the limitations of this exercise. For one thing, official intervention has been ruled out by assumption; for another, the model has been made strictly linear. In view of the latter simplification, the model is strictly valid only in the neighborhood of its long-run equilibrium. If it were to veer too far away from this point, its basic relationships would be altered by non-linearities (due to risk aversion in portfolios, for example) -- possibly so as to maintain the system within bounds. When instability of the type described above is present, however, this point might not be reached until an unacceptably large swing in the exchange rate has occurred.

The findings above also raise some unsettling questions about the use and interpretation of jumping variables as a device to ensure stability. We have focussed here on lags and leads in trade, but similar instability problems could well arise, with even more force, in models with lags from other sources of which there are many candidates. Since there is usually only a small number of jump-variables available to accommodate shocks in more complex frameworks, this suggests that a high degree of instability may be endemic -- at least from a theoretical point of view.

Clearly the need for discrete jumps in variables is bound closely to the convenient analytical device of distinguishing markets according to whether they adjust slowly or very rapidly (i.e., instantaneously). In such a framework, instantaneous jumps may be understood as a compressed, stylized version of a more protracted process, the details of which are usually not shown. It may be, therefore, that sufficient slowing of the rate of adjustment of such a market or of the expectations formation process (by introducing a learning process, an adaptive mechanism, or some other "friction") could rid these models of some of their less appealing features,[1/] but whether or not this is the case is an open question.

1/ For an example in which a conditionally stable system can be made globally stable (at a higher order) by introducing an adaptive rule consistent with rational expectations, see Mussa (1976).

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