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HYPERINFLATION AND STABILISATION: CAGAN REVISITED

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Abstract

In this paper Cagan's analysis of hyperinflations is adapted to look at situations where the deficits to be financed by money creation are large and variable, but fiscal stabilisation is expected — features found in some of the republics of the Former Soviet Union soon after independence. The impact of various stabilisation policies on real balances and inflation expectations is studied, assuming expectations are rational and deficits follow a geometric Brownian motion until the stabilisation takes place. For a modified form of Cagan's demand function we are able to obtain explicit solutions using Ito's Lemma; these are calibrated to give numerical estimates of the effects of expected fiscal stabilisation.

HYPERINFLATION AND STABILISATION: CAGAN REVISITED

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1 Introduction

For countries in Western Europe, new money creation ("seigniorage") is not typically an important source of public finance. But the former Soviet republics that gained independence in 1991 provide a marked contrast. The main sources of revenue for government in Russia and Ukraine, for example, shrank dramatically as the finances of state owned enterprises deteriorated. Political obstacles to fiscal reform and the subservient role of the new central banks meant that recourse to money creation on a substantial scale was the path of least resistance. The results have been very high and variable rates of inflation.

A key feature of the situation in these republics has been the great unpredictability of deficits pending the fiscal stabilisation required as a condition for external financial support. To examine the link between deficits and inflation in these circumstances, we adopt the monetary approach developed by Cagan (1956) in his classic study of hyperinflation of the 1920's, modified in certain important respects. First it is assumed that deficits evolve as a stochastic process until they are checked by stabilisation, a stylised characterisation of budgetary policy recently used by Bertola and Drazen (1993) in their interesting paper on "Trigger points and budget cuts". Second, we drop Cagan's assumption of adaptive expectations of inflation and replace this with rational expectations as in Sargent and Wallace (1981). Third,

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we simplify Cagan's demand for money equation to obtain explicit analytical results. (Later we provide qualitative results for Cagan's exponential formulation.)

As a result we are able to calculate the rate of expected inflation and the level of real balances associated with any given deficit under a variety of assumptions about stabilisation. The general solutions are found by applying Ito's lemma as in option pricing (and in the recent exchange rate target zone literature, see Krugman and Miller (1991) for example): and the boundary conditions are provided by the types of fiscal stabilisation in prospect.

When budget deficits are stochastic, we find that they can exceed the maximum steady state yield of inflation taxation without necessarily triggering currency collapse. This is because, when stabilisation is expected for sure, things begin to change before the budget is in fact stabilised; i.e., there are anti-inflationary benefits coming ahead of stabilisation. But these benefits can be undermined by uncertainty about policy; and we look in particular at the case where fiscal stabilisation is less than certain and currency collapse threatens.

The paper proceeds as follows. First we spell out the formal model of the money financing of deficits that evolve as a stochastic process and derive the solutions which emerge when stabilisation policy is expected to take the form of a "cap" on the deficit. What happens if stabilisation is uncertain is considered next. Then we look at the effects of budget "cuts". After noting how upper and lower limits on the size of the deficit imply a "band" for expected inflation, we briefly consider the multiple equilibria associated with Cagan's exponential demand for money equation.

2 Deficits and Inflation

We assume that the public sector runs large deficits which can only be financed by printing money, i.e., by "seigniorage"; and that the money creation leads to inflation. (For a case in point see, for example, the figures for Ukraine in 1992/93 reported in Havrylyshyn *et al* (1994).)

Money financing of the deficit implies

$$\frac{dM_t}{P_t} = S_t dt, \tag{1}$$

where M_t is the money supply, P_t is the price level and S_t is the real deficit whose evolution over time will be discussed below.

As for money demand, like Cagan, we assume a stable demand function even under conditions of hyperinflation. To derive analytical results in this section we assume that the demand for money depends inversely on expected inflation and can specifically be written as

$$B_t \equiv \frac{M_t}{P_t} = \frac{N}{1+\alpha\pi_t^e}, \tag{2}$$

where B_t is the real balances, N is the velocity, and π_t^e denotes the expected inflation. Later we will employ the specific functional form he fitted to hyperinflations after the First World War.

It is clear from (2) the demand for real balances falls to zero as anticipated inflation tends to infinity. Where the yield of inflation tax is, as usual, defined as the product of inflation and real balances, πB, we find that increasing anticipated inflation never decreases the yield of the inflation tax, i.e., $[d(\pi B)/d\pi]|_{\pi=\pi^e} \geq 0$. This does not mean, however, that any level of deficit can be sustainably financed by printing money: instead one finds that inflation taxation tends to maximum level when expected inflation increases; specifically $\pi B|_{\pi=\pi^e} \to N/\alpha$ as $\pi = \pi^e \to \infty$, i.e., the inflation tax "Laffer curve" implied by (2) rises with π^e, approaching N/α asymptotically. (In this respect it differs from Cagan's exponential formulation, where beyond some level, higher inflation decreases the tax yield.)

To capture the unpredictable evolution of public sector deficits in economies in transition for example, we further assume that, until stabilisation takes place, deficits follow a geometric Brownian motion

$$\frac{dS_t}{S_t} = \sigma dW_t, \tag{3}$$

where W_t is a standard Brownian motion, σ^2 is the variance of the deficit per unit time.

To capture the effects of future anticipated events, we assume expectations are rational, i.e.,

$$\pi_t^e = \frac{E_t(dP_t)}{P_t dt} \tag{4}$$

where E_t denotes rational expectations. According to equation (1), the flow of new money will depend on the current real deficit and the price level. With rational expectations the price level will depend on expected future deficits (and on stabilisation policy designed to check them).

In what follows, we first derive the general solutions of this stochastic model and then show how expected stabilisation policy ties down particular solutions.

2.1 The general solution for real balances

To find the solution of the real balances when seigniorage is stochastic, we note first that the money supply follows the dynamics below

$$\begin{aligned} dB_t \equiv d\left(\frac{M_t}{P_t}\right) &= \frac{dM_t}{P_t} - \frac{dP_t}{P_t} \cdot \frac{M_t}{P_t} \\ &= S_t dt - \pi_t B_t dt, \end{aligned} \tag{5}$$

where π_t is the actual inflation. Taking expectation on both sides in (5) yields

$$\frac{E_t dB_t}{dt} = S_t - \pi_t^e B_t. \tag{6}$$

Here, we assume that P_t and B_t are adapted to the σ-algebra generated by S_t. Equation (6) means that the expected growth in real balances depends on seigniorage less the expected inflation tax.

Assuming that there is a stable relationship between real balances B_t and seigniorage S_t (i.e., $B_t = B(S_t)$), then using Ito's lemma, (6) becomes

$$\frac{1}{2}\sigma^2 S^2 B''(S) = S - \pi^e B(S).$$

Using the demand for money equation (2) to substitute for π^e, this becomes,

$$\frac{1}{2}\sigma^2 S^2 B''(S) = S + \frac{B(S) - N}{\alpha}. \tag{7}$$

The solution to this linear differential equation is

$$B(S) = N - \alpha S + A_+ S^{\xi_+} + A_- S^{\xi_-}, \tag{8}$$

where $\xi_\pm$ are the positive and negative roots of the quadratic equation

$$\frac{1}{2}\sigma^2 \xi(\xi - 1) = \frac{1}{\alpha}, \tag{9}$$

and $A_\pm$ are constants to be determined by the nature of stabilisation policy and/or the probability of financial collapse.

The nature of these solutions and how they depend on stabilisation policy can be seen from Figure 1, where the deficit S is plotted on the horizontal axis and real balances on the vertical. Consider first the relationship between B and S which would prevail in the absence of stochastic fluctuations in S (i.e., when $\sigma^2 = 0$). This is given by setting $A_{\pm} = 0$, so

$$B(S) \;=\; N \;-\; \alpha S \tag{10}$$

and shown by the locus labelled EL in the figure. This illustrates what we have noted above, namely that increasing levels of seigniorage *can* be financed by the inflation tax, but only by raising inflation expectations and reducing real balances. Real balances B fall linearly with S, reaching zero when $S = S_M = N/\alpha$. Clearly for inflation to be bounded, S must be less than N/α, the upper limit of the inflation tax Laffer curve in this case.

Consider now how the price level, real balances and expected inflation will be determined when σ^2 is positive. Note first that, because there is no trend term in (3), $S_t = 0$ is an "absorbing point" for the deficit (since $dS_t = 0$ when $S_t = 0$); so if the deficit goes to zero, it stays there. Assuming that no specific effort is made to bound the deficit away from zero, then the only solutions we need to consider will emanate from the point labelled E on the vertical axis (where $S = 0$ and $B = N$), i.e., the parameter A_- must be set to zero.

2.2 The implications of "capping" the deficit

To see how the other parameter A_+ depends on the stabilisation policy, consider first the implementation of a cap (or a "reflecting barrier") on the real deficit at S_R. This means that deficit is allowed to evolve without intervention until it is absorbed at the origin *or* until it reaches S_R, at which point it will be checked by marginal stabilisation (i.e., it can rise no further; but it can fall below S_R). The no-profit-arbitrage condition implies that the solution shown by the curve EA in the figure must be horizontal at the point of marginal stabilisation, i.e.,

$$B'(S_R) \;=\; 0. \tag{11}$$

(See Dixit (1991).) The parameter A_+ is determined by applying boundary condition (11) to the general solution (8) (where $A_- = 0$ because $S = 0$ is an absorbing point), i.e., it is given by

$$A_+ = \frac{\alpha}{\xi_+} S_R^{1-\xi_+}. \tag{12}$$

With this value for A_+, we obtain the schedule shown as EA in the figure, which shows how real balances will behave when a cap is expected at S_R. The fact that point A lies above the steady state tradeoff EL is because stabilisation at S_R means $E(dB)/dt > 0$ near S_R: so expected stabilisation dampens inflation expectations and increases the demand for real balances.

For convenience we have drawn in the line EW which is the locus of all points where $B'(S) = 0$. Substituting (12) into (8), we can express this line explicitly as

$$B(S_R) = N - \alpha S_R + \frac{\alpha}{\xi_+} S_R. \tag{13}$$

As the cap on seigniorage is raised, so the parameter A_+ in the solution will fall and the turning point will move along EW to the right of A. So raising the cap towards S_W will shift the solutions from EA down towards ECW. Note that a cap of S_W itself is an extreme or critical value because with a cap of this size, real balances fall to zero and inflation goes to infinity as the cap is approached. (This currency collapse is not expected for certain, however, as S may be absorbed at $S = 0$ before ever reaching S_W.)

The critical value of A_+ when the cap is set at S_W can be expressed as

$$A_+^C = \frac{\alpha}{\xi_+} \left(\frac{N\xi_+}{\alpha(\xi_+ - 1)} \right)^{1-\xi_+}. \tag{14}$$

This is the minimum value of A_+ above which the expected cap at S_R will prevent currency collapse occurring. (Values of A_+ below A_+^C identify solutions lying beneath EC, and one of these is EFL for example, where $A_+ = 0$. Increasing the perceived probability of currency collapse at some time in the future, conditional on $S = S_0 < S_W$, will select one of these solutions.)

As can be seen from the figure, the locus EW always lies to the right of deterministic steady state equilibrium EL. But how large is the maximum sustainable real deficit under a cap relative to that under certainty? We compare them for any given level of real balances. Denote S_D the real deficit in a deterministic steady state equilibrium for any given real balances, then from (10) and (13) we derive for any given B,

$$\frac{S_R}{S_D} = \frac{\xi_+}{\xi_+ - 1} > 1. \tag{15}$$

This is the ratio of points on EW to those on EL (measured horizontally).[1]

To get some ideas of the quantitative impact expected stabilisation might have on real balances and inflation expectations, we calculate the ratio in equation (15) using numerical estimates of the parameters α and σ (which affect the demand for and supply of money respectively). Details are given in the Annex. Briefly, we obtain values for α by matching Cagan's estimate of the semi-elasticity of the demand for money over a range of expected inflation; a range of values for σ is obtained from recent Ukrainian data.

From Table 2 we see that if $\alpha = 4.86$ when inflation is measured on a monthly basis and the monthly standard deviation of real deficit $\sigma = 0.08$, then $S_R/S_D = 1.13$. So under moderate uncertainty a cap can sustain a maximum real deficit 13% higher than that given by deterministic steady state equilibrium. Of course, increasing variability in the real deficit will increase this ratio. For monthly $\sigma = 0.16$, S_R will be 28% higher than S_D.

3 The Risk of No Stabilisation

So far it has been assumed that marginal stabilisation is sure to occur when the deficit reaches the announced trigger point. Now consider what happens if there is *ex ante* uncertainty about the stabilisation policy which is only resolved when S reaches the barrier S_U. Let us assume that, when seigniorage S reaches the trigger value $S_U > S_M$, stabilisation will occur with probability $H < 1$, and no stabilisation with probability $1 - H$. In the former event, the deficit is capped and the barrier taken to be fully credible. But in the latter case, the situation will be desperate: the hoped for stabilisation has not materialised, the current value of the deficit is unsustainable by inflation tax alone ($S_U > S_M$) and the best forecast of the future deficit is its current value. Assume that, in these circumstances, the flight from paper money will cause a "currency collapse", so that money will become worthless. [In their analysis of hyperinflation, Evans and Ramey (1992) argue that "currency collapse at a future date can never be the outcome of a REE (rational expectations equilibrium) path. This follows from a *backward unravelling* argument: if collapse occurs in period t, then the (forecast equation) gives $\pi_t = \infty$, so collapse occurs in $t - 1$, etc." (1992,p7). We have, however, assumed that

[1] Interestingly enough the same formula appears in Dixit and Pindyck (1994) where they derive the ratio of optimal exercise price for a perpetual call option (on a dividend paying stock) to its strike price.

the collapse is *uncertain*, so this unravelling does not occur, as the expected loss in the value of money when the collapse occurs can be balanced by the expected gain on stabilisation.]

To derive the boundary condition that applies at S_U, we define $1/P$ as the "price of money" and assume that this fall to zero when the currency collapses. The "expected value matching condition" is thus

$$\frac{1}{P_B} = \frac{H}{P_A} + (1-H) \times 0, \quad 1 \geq H > 0, \tag{16}$$

where P_B is the price level just before the uncertainty is resolved and P_A is the price level in the event that stabilisation is implemented. Multiplying M on both sides of the above equation gives the alternative formulation

$$B_B = HB_A. \tag{17}$$

Consider specifically the case where marginal stabilisation is anticipated with probability $1 > H > 0$ when S reaches its trigger value. The boundary condition is simply

$$B_B = HB(S_U). \tag{18}$$

where $B(S_U)$ lies on the locus EW.

Thus if the *ex ante* probability that stabilisation will take place at S_U is given by the height J relative to G, this would select ECJ as the relevant solution. So the risk of currency collapse has reduced real balances and exacerbated inflation.

4 Discrete intervention — deep budget cuts

The effects of capping a deficit are relatively weak as, by assumption, no determined effort is made to *cut* the deficit. In the case where budget cuts are expected, the general solution still takes the form as in (8) above, but the boundary condition required to rule out anticipated jumps in the price level (or equivalently, the real balances) is

$$B(S_H) = B(S_L), \tag{19}$$

where S_H is the point at which the cut is implemented taking the deficit down to S_L.

In Figure 2, we can compare the effect of a cap and a cut. With a cap at S_H the solution lies on EM where π_e rises monotonically as $S \rightarrow S_H$. But with a cut to S_L, the solution will be the curve EDC, which implies a lower level of π^e for any S. Note also that π^e will fall as $S \rightarrow S_H$ as an increase in the deficit brings the cut closer. (Of course, if deep cuts are not fully credible *ex ante*, their effects on expectations will be weakened along the lines already discussed.)

Suppose agents expect the fiscal stabilisation takes the form of a deep budget cut which pulls the seigniorage back to zero when S hits a barrier. If such policy is fully credible, what will be the maximum level of seigniorage? In this case, the boundary condition (19) can be rewritten as

$$B(S_H) \quad = \quad B(0) \quad = \quad N.$$

Applying this to the general solution (8) and using critical value of A_+ in (14), one can derive the maximum sustainable seigniorage as

$$S_H^{Max} \quad = \quad \frac{N}{\alpha}\frac{\xi_+}{\xi_+ - 1}(\xi_+)^{1/(\xi_+ - 1)}. \tag{20}$$

When compared to the maximum level of seigniorage under certainty, we have the following ratio

$$\frac{S_H^{Max}}{S_M} \quad = \quad \frac{\xi_+}{\xi_+ - 1}(\xi_+)^{1/(\xi_+ - 1)}, \tag{21}$$

where $S_M = N/\alpha$ is the maximum sustainable seigniorage under certainty.

To see how large S_H^{Max} is relative to S_M, we calculate this ratio. From Table 3 we see that if, as before, we set $\alpha = 4.86$ and $\sigma = 0.08$, then $S_H^{Max}/S_M = 1.51$. For higher variability in the real deficits ($\sigma = 0.16$), S_H^{Max}/S_M rises to 1.97. So, the confident expectation of a deep cut means that the deficit can rise to 50% (or more) beyond the limit of the Laffer curve without precipitating a collapse of the currency.

5 Bands for deficits and for expected inflation

It seems unlikely that deficits will permanently be absorbed at zero. One way of avoiding this is to put a trend into the deficit so $dS = \sigma S dW + \mu dt$, where $\mu > 0$, see Bertola and Drazen (1993). Another is to bound S away from zero. If, for example, there is a floor on S set by marginal intervention (as well as a ceiling), we obtain the S-shaped solution shown in Figure 3.

These solutions have the form given in (8), but satisfying the boundary conditions

$$B'(S_H) = B'(S_L) = 0, \tag{22}$$

where S_H and S_L are the upper and lower barriers on S respectively. These bands on the deficit will lead to fluctuation bands for real balances and for expected inflation. Because expected future deficits are always closer to the mean than current deficits, the effect of increasing S on π^e is much less than would be implied by the tradeoff EL also shown in the figure (i.e., shifting the deficit *permanently* between S_L and S_H would lead to far greater effects on π^e).

This analysis shows the fiscal limits sufficient to keep expected inflation inside a target band. An analogous study of the limits on monetary policy needed to hit a target band for the price level is to be found in Eichengreen and Garber (1990).

6 Cagan's demand for money and multiple equilibria

It is well known that, with the exponential demand for money function used by Cagan, there is a "backward bending" Laffer curve, so higher inflation ultimately leads to a fall in the inflation tax. In this situation we discuss briefly how the results obtained earlier are modified.

Assuming that there is a stable relationship between real balances B_t and seigniorage S_t (i.e., $B_t = B(S_t)$), we have already seen that

$$\frac{1}{2}\sigma^2 S^2 B''(S) = S - \pi^e B(S).$$

Substituting for π^e using Cagan's demand for money equation, i.e.,

$$B_t = N exp(-\alpha\pi_t^e), \tag{23}$$

we now find that

$$\frac{1}{2}\sigma^2 S^2 B''(S) = S + \frac{B(S)\ln B(S)}{\alpha}. \tag{24}$$

There is no simple analytical solution for this equation, but the form of the solutions can be seen from Figure 4 where the locus of points where real balances are not expected to change is shown as OLE. It is, of course, defined by setting to zero the right hand side (RHS) of equation (6). But equation (24) has the same RHS: so the function $B(S)$ must have a point of inflexion on OLE. To the left, where the RHS is negative, $B''(S) < 0$; and vice versa to the right.

The curvature of the locus OLE indicates that there is a "Laffer curve" effect at work. At E there is no deficit and no inflation, so real balances are high. As one moves along the curve from E to L, inflation is rising and real balances are contracting, but the increasing values of S can still be financed by the product of the two (the "inflation tax"). But between L and O one is evidently on the "wrong side" of the Laffer curve, where higher inflation leads to such a contraction of real balances — the tax base — that the tax yield goes down. So a constant deficit like S_L can be financed in two ways — with high real balances and low inflation: or the reverse, see points A and B in the figure.

To see how stabilisation policy can identify particular solutions consider the implementation of a cap at $\bar{S}$, for example: so the deficit can rise as far as $\bar{S}$ but no further. The boundary condition $B'(S) = 0$ identifies a solution passing through M for instance, see the curve EAM in the figure. When we generate the locus EW of all points where $B'(S) = 0$, we find that it bends back rather like OLE. Consequently, one may be able to find another solution, such as that shown by the curve labelled EN in the figure.

This same phenomenon occurs when there is two sided stabilisation, see Figure 5. The bounds on S are shown as S_L and S_U: and as the figure shows they imply bounds on π^e labelled U and L. (If S_L and S_U are identical then π^e would be at a point on the steady state locus labelled EL.) The existence of such multiple equilibria suggests that fiscal policy may need to be complemented by efforts to coordinate expectations on the lower inflation equilibria, see Barnett (1995), Davies and Vines (1995) and Sachs (1995).

7 Conclusion

We have studied the effects of expected stabilisation in a Cagan-style model of hyperinflation, modified to allow for stochastic deficits, rational expectations and an inflation-tax Laffer curve which is non-decreasing. What do we find?

Essentially that the prospect of stabilisation allows the deficit to go beyond the maximum yield of the Laffer curve without precipitating an immediate collapse of the currency: but that this extra "degree of freedom" is limited (as the deficit can only be expected to stay beyond the Laffer curve for temporary period).

When stabilisation takes the form of a budget "cap" we found a simple formula for determining the extra degree of freedom: and calculated this to be over 13%, using the semi-elasticity of money demand Cagan reported for seven hyperinflations taken together. This is not a large factor because a budget cap is a fairly weak form of stabilisation: even if fully credible, it is not a promise to do very much. So we also looked at deep budget cut; and found that a fully credible prospect of completely eliminating the deficit meant that the deficit could rise to over 50% beyond the maximum associated with the Laffer curve. (Promise of deep cuts at trigger points beyond this are, however, rather like promises to lock the stable door after the horse has bolted; they are useless because the currency will have collapsed long before the stabilisation is due!)

Diminishing the credibility of promised stabilisation weakens its *ex ante* effects, as was seen when we allowed for the possibility that a failure to act at the preannounced trigger point might lead to immediate currency collapse. This prospect changed the boundary condition used to determine the solution, with the effect of reducing real balances and raising expected inflation; and we found that, as a result, deficits could be associated with inflation rates higher than those implied by the Laffer curve.

The analytical results were obtained for a demand equation where the Laffer curve is always increasing. But we were able to show that the same qualitative features apply to Cagan's exponential demand function (where the Laffer curve reaches a peak, typically well before hyperinflation is reached). In addition, as one might expect, there are in this case multiple equilibria.

Findings of this sort, which purport to show the potential effects of different strategies for stabilisation, lead naturally to questions in "political economy". When, for example, might a government choose to "cap" the deficit? or to cut it? What promises could it make that were credible? Could institutional factors play a role?

Although these are not the issues addressed in this paper, they do perhaps call for some comment in conclusion. Given the nature of the political costs facing the government, one could, for example, derive the optimal discretionary policy (where and when to "cap" or "cut"); and see how institutional or other factors might improve this outcome (by mitigating the "time consistency" problems relevant in this situation). We are pursuing this approach elsewhere, adding aspects of "political economy" to the Cagan-style model of this paper.[2]

We end by noting two aspects of the problem which have been highlighted recently: namely the scope for the *delegation* of monetary policy and for the *coordination* of market expectations. The first addresses the critical institutional issue of who controls the printing press and has been examined in a study of transition economies by Loungani and Sheets (1995). The authors note that, in these economies, the degree of central bank independence is negatively correlated with inflation and that "high inflation adversely affects real activity in subsequent years" and conclude that "the establishment of an independent central bank is an institutional reform that should be implemented early in the reform process". (Interestingly enough, Russia and Ukraine feature among those with the highest inflation and most subservient central banks in the sample of a dozen transition economies they examine.)

It is, however, the issue of multiple equilibria which Jeffrey Sachs emphasises in the 1995 Frank Graham lecture. The implication of his talk is that institutional factors can, by focusing market expectations on the low inflation equilibrium, help to prevent the economy getting on the wrong side of the Laffer curve.

Annex

In the text, we have already seen that various expected fiscal stabilisation policies can sustain larger real deficits than those implied by the Laffer curve. In this note, we give some numerical examples to illustrate their quantitative significance. To obtain the ratios as S_R/S_D and S_H^{Max}/S_M in the text, we first briefly outline how we obtain the estimates for parameters α and σ.

From (2) in the text, the semi-elasticity of demand for money is

$$\left|\frac{dB}{Bd\pi^e}\right| = \frac{\alpha}{1+\alpha\pi^e}. \tag{A1}$$

Let α_C be the semi-elasticity in Cagan's exponential form of demand for money equation as in (23), and let π^e_{Max} be the expected inflation rate where

[2] We have earlier carried out a rather similar exercise for currency intervention, Miller and Zhang (1994).

the Laffer curve reaches a maximum in this case, i.e.,

$$\pi^e_{Max} = \frac{1}{\alpha_C}. \tag{A2}$$

Suppose we let $0 \leq \pi^e \leq \pi^e_{Max}/2$ and match the semi-elasticity given in (A1) to that of Cagan's (α_C), then we obtain the range of parameter α given by $[\alpha_C, 2\alpha_C]$. Of course, increasing the expected inflation increases the value of α. As the estimated mean value of α_C from Cagan (1956) is 4.86 for monthly data, the range we choose for α is therefore [4.86, 9.72].

To obtain an estimate for σ, we use Ukrainian data and do the following calculations. Let S_t be the ratio of real deficit to GDP and assume that S_t satisfies the stochastic process given in (3) in the text, applying Ito's lemma to $\ln(S_t)$ we derive

$$d\ln(S_t) = -\frac{1}{2}\sigma^2 dt + \sigma dW_t. \tag{A3}$$

Equation (A3) has the solution given by

$$\ln(S_t) - \ln(S_0) = -\frac{1}{2}\sigma^2 t + \sigma W_t. \tag{A4}$$

Discretising (A4) using annual observations and taking expectation on both sides yield

$$\frac{1}{N}\sum_{i=1}^{N} E_{t_i}[\ln(S_{t_{i+1}}) - \ln(S_{t_i})] = -\frac{1}{2}\sigma^2, \tag{A5}$$

where N is the sample size and σ^2 is the annual variance of S_t. So the estimate for the variance becomes

$$\hat{\sigma}^2 = -\frac{2}{N}\sum_{i=1}^{N}[\ln(S_{t_{i+1}}) - \ln(S_{t_i})]. \tag{A6}$$

Given annual data for Ukraine in Table 1, we obtain the estimate for the annual variance of real deficits $\hat{\sigma}^2 = 0.31$, or equivalently its monthly standard deviation

$$\hat{\sigma}_m = \sqrt{\frac{\hat{\sigma}^2}{12}} = 0.16.$$

As this is a very rough estimate of σ_m, we use σ_m in the range of [0.08, 0.24].

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Tables

Table 1: Real deficits in Ukraine 1991–94.

	91	92	93	94
Real deficit as % of GDP	13.6	29.3	9.7	8.6

Source: IMF Annual Reports 94 (p95) and 95 (p116).

Table 2: Response of S_R/S_D to α and σ.

	σ_m=0.08	0.16	0.24
α = 4.86	1.13	1.28	1.45
9.72	1.19	1.42	1.68

Table 3: Response of S_H^{Max}/S_M to α and σ.

	σ_m=0.08	0.16	0.24
α = 4.86	1.51	1.97	2.46
9.72	1.69	2.37	3.13

Figures

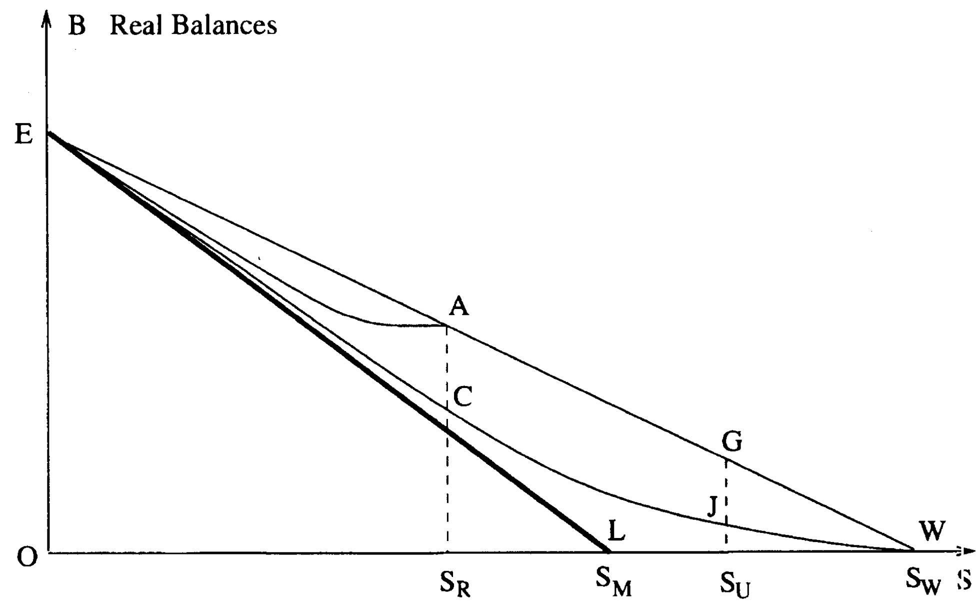

Figure 1: Real balances and budget "caps".

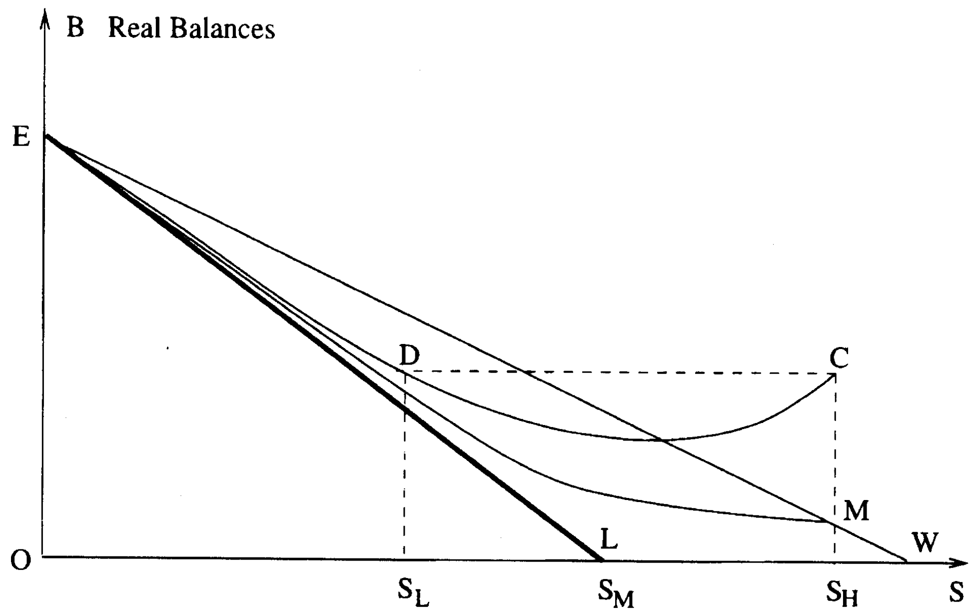

Figure 2: Real balances and fiscal stabilisation.

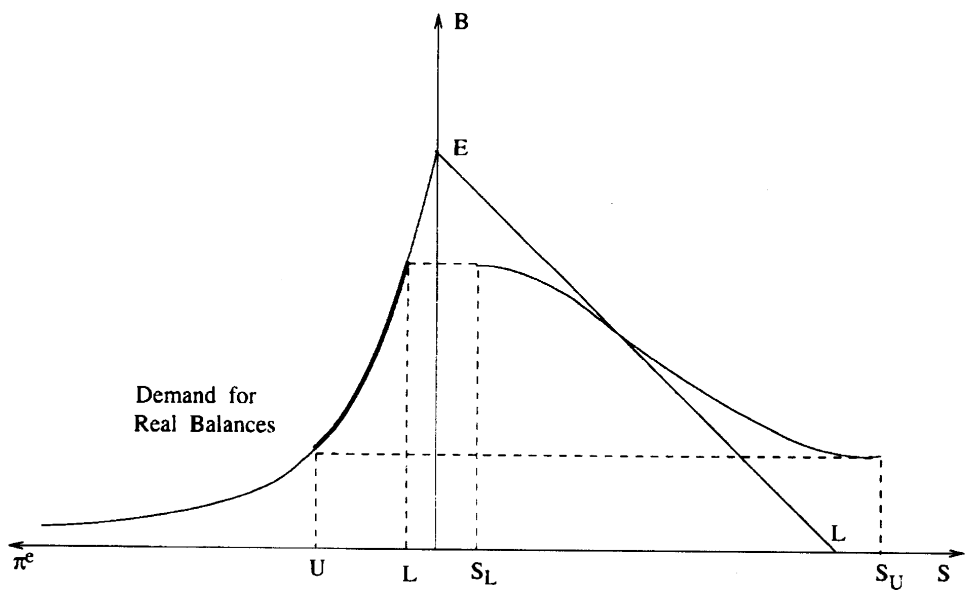

Figure 3: A band on expected inflation.

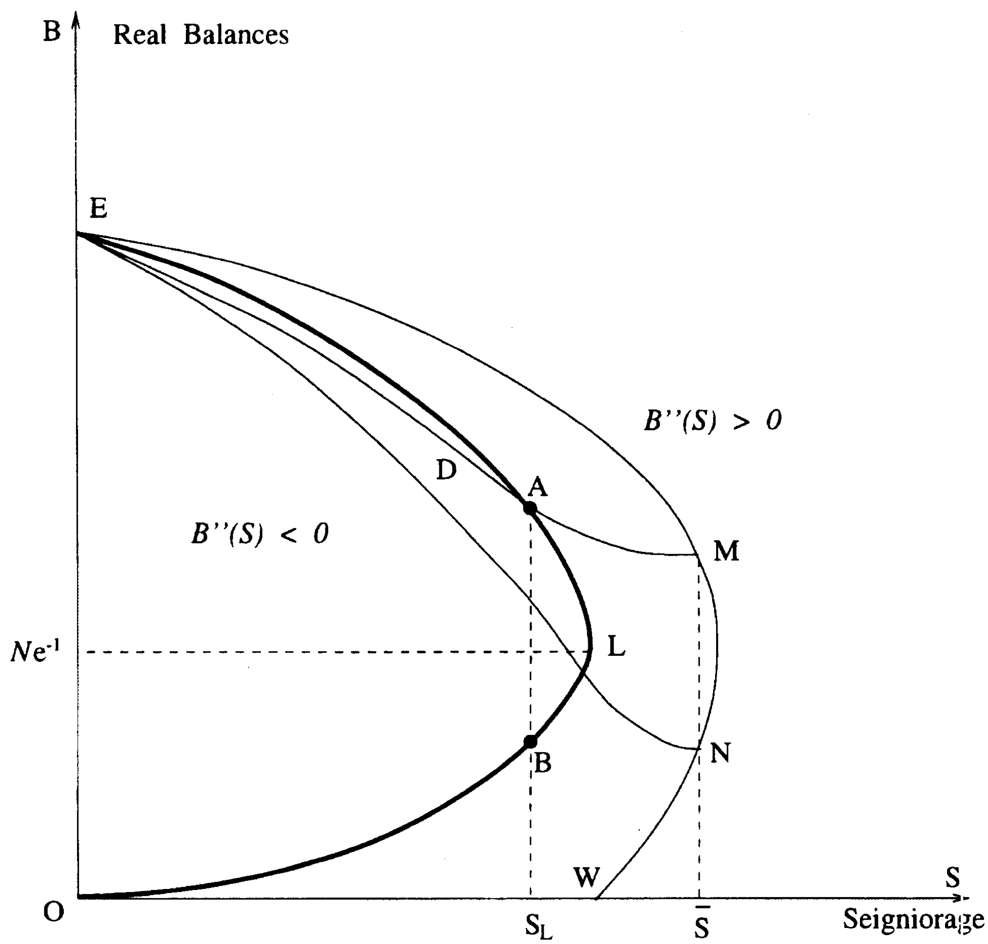

Figure 4: Real balances and fiscal stabilisation: Cagan's demand for money formulation.

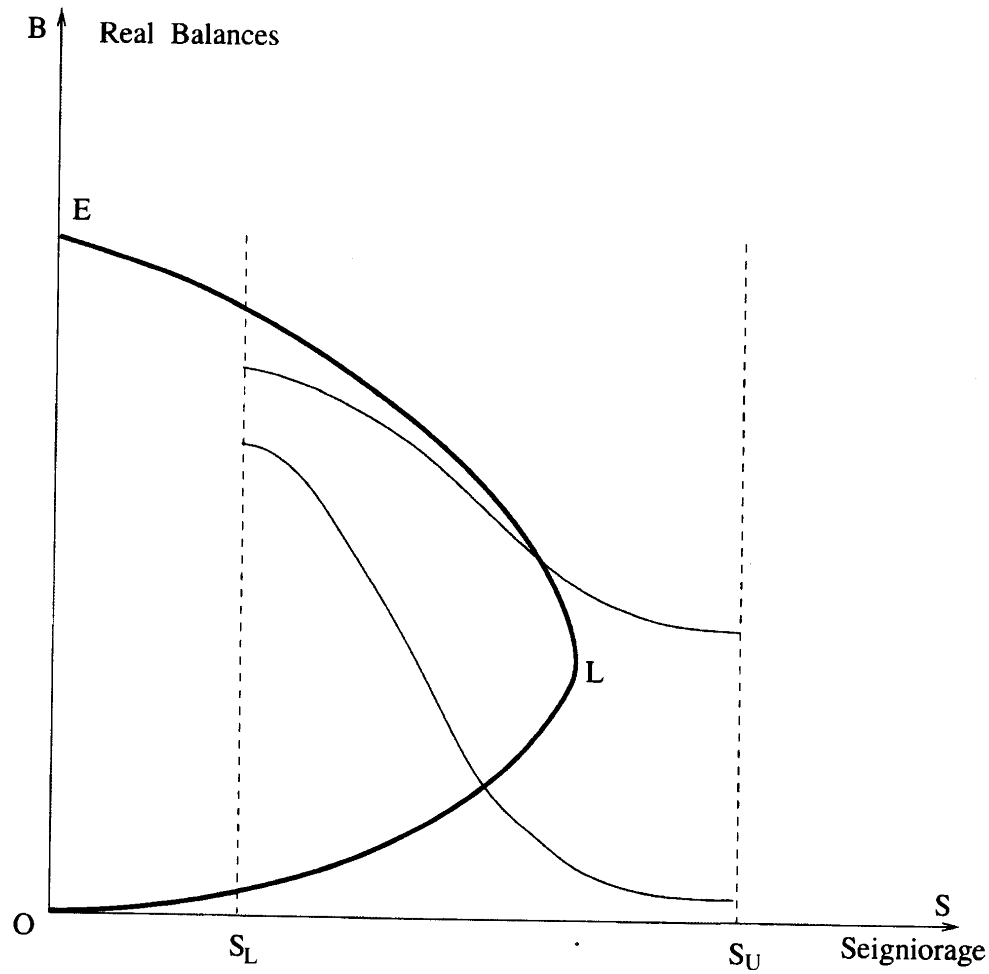

Figure 5: Two-sided stabilisation and multiple equilibria.

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