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HYSTERESIS IN A SIMPLE MODEL OF CURRENCY SUBSTITUTION

Martín Uribe

ABSTRACT

A simple model of currency substitution is developed in which the private cost of performing transactions in the foreign currency depends upon the aggregate degree of dollarization. This feature generates multiple steady states and hysteresis in an otherwise standard cash-in-advance model of a small open economy. In particular, a temporary increase in the rate of inflation can drive the economy to a dollarized equilibrium in which the velocity of circulation of domestic currency is permanently higher.

Hysteresis in a Simple Model of Currency Substitution

Martín Uribe*

1 Introduction

This paper develops a model that explains why temporary changes in the inflation rate may have permanent effects on money velocity. A striking example of this type of phenomenon is provided by the recent inflationary experience of Peru. Between 1988 and 1990, this economy underwent an unprecedented hyperinflation during which prices grew at an average monthly rate of more than 40 percent. Not surprisingly, real money balances were extremely depressed during this period. What is surprising, however, is that although the stabilization plan of August 1990 brought the inflation rate down to levels comparable to those prevailing during the pre-hyperinflation period,[1] real balances never recovered (see figure 1).

The Peruvian is just one among many examples of this type of seemingly abnormal behavior of real balances. Since the early 70s, many studies have documented the instability of standard econometric money demand specifications. These standard models systematically over and underpredict actual money demand figures. In particular, temporary changes in the nominal interest rate seem to have persistent effects on real money balances, which are not captured by the standard model.

*The author is a Staff Economist in the Division of International Finance Division, Board of Governors of the Federal Reserve System. I would like to thank Guillermo Calvo, Edgardo Favaro, Guillermo Mondino, Stephanie Schmitt-Grohé, Michael Woodford and seminar participants at the University of Maryland, the Board of Governors, and the 1994 meetings of the Latin American Econometric Society (Caracas, Venezuela) for helpful comments. The opinions expressed in this paper are those of the author and do not necessarily reflect the views of the Board of Governors of the Federal Reserve System. Address for correspondence: Mail stop 24, Division of International Finance, Board of Governors of the Federal Reserve System, Washington, DC 20551, (202) 452-3780, e-mail: uribem@frb.gov.

[1] From 1980 to 1987 the average monthly inflation was 5.5 percent. From 91:1 to 93:12 the average monthly inflation was 4.7 percent.

One explanation that has been advanced for these findings is that the standard model does not account for financial innovation. Many empirical studies have found that different proxies for financial innovation help solve the problems of the standard model. Among the variables that have been used as proxies are a time trend (Lieberman (1977)), a past peak interest rate or ratchet variable (Goldfeld (1976), Simpson and Porter (1980), Cagan (1984), Piterman (1988)), the number of electronic fund transfers (Dotsey (1984)) and a stochastic trend (Arrau, et al. (1993)).

In high-inflation countries, an important form of financial innovation, or, more properly, financial adaptation, is the use of a foreign currency as a means of payment (see for example the survey by Calvo and Végh (1992)). This phenomenon is usually called *dollarization* or *currency substitution.*[3] Unfortunately, dollarization is difficult to empirically document due to the lack of data on the amount of foreign currency in circulation. Its importance is mainly judged on casual observation. Empirical studies generally use the ratio of dollar-denominated time deposits to M2 as a proxy for the degree of dollarization (Savastano (1992), Rojas- Suárez (1992), Guidotti and Rodríguez (1992)). An exception is Kamin and Ericsson (1993) who present a new measure of dollar currency circulating outside the US and apply it to the Argentine economy. These studies also find that relatively short periods of high nominal interest rates, may have permanent effects on the demand for domestic currency.

Among the theoretical attempts at modeling this unusual behavior of monetary aggregates is Ireland (1995),[4] who extends the cash-in-advance model of Lucas and Stokey (1983, 1987) to allow for an endogenous determination of the size of the set of goods that can be bought on credit. In his model, irreversible investment in financial capital allows shoppers to buy goods on credit in markets where money was once required. A temporary increase in the nominal interest rate may then induce an expansion in investment in financial capital and as a consequence an increase in money velocity, which will persist even after the interest rate falls back to its long-run level.

[2]This phenomenon is present in data from both developed and developing countries. An early study for the US is Goldfeld (1976). Arrau et al (1993) study ten developing economies.

[3]Calvo and Végh (1992) make a distinction between these two terms. In their terminology, dollarization can also mean the use of a foreign currency as a unit of account or as a store of value.

[4]See also Sturzenegger (1992), who develops a model of endogenous financial adaptation, to study its effects on income distribution, social welfare, and seignorage income, and Krugman (1984), who studies the role of the dollar as an international currency and suggests a model of the dollar as a medium of exchange among countries, displaying multiple equilibria and hysteresis.

Guidotti and Rodríguez (1992) present a continuous-time, liquidity-in-advance model of dollarization in which agents face adjustment costs of changing their current level of dollarization. These adjustment costs are assumed to be increasing and convex in the absolute value of the rate of change of the agent's level of dollarization. That is, they assume that it is costly for the consumer to change his degree of dollarization in either direction, irrespectively of the level of dollarization of the rest of the economy. This creates a tendency for the consumer to stay at his current level of dollarization, unless the inflation rate is either too high or too low. That is, it results in an equilibrium with an inflation band within which the consumer decides not to change his dollarization level and outside of which the consumer either completely dollarizes or completely de-dollarizes.

This paper departs from those described above in that the consumer's decision of what fraction of his transactions to carry out in foreign currency depends, among other things, on the degree of dollarization of the rest of the economy. In particular, if the economy is not dollarized (i.e., if agents are not used to receiving foreign currency in exchange for goods) it is more costly for the consumer to carry out transactions in the foreign currency. Conversely, in an environment in which everybody is used to dealing in dollars, it is easier for the consumer to use dollars as a means of exchange. That is, the aggregate level of dollarization enters as an externality that reduces the marginal cost of performing transactions in dollars at the consumer level.

This paper shows that this feature may produce multiple steady states in an otherwise standard cash-in-advance model. Specifically, at moderate levels of inflation,[5] the model delivers two stable steady states and one unstable steady state. In one of the stable steady states, the economy is not dollarized at all and domestic real balances are relatively high, while in the other, dollar holdings are positive and domestic real balances are relatively small. If agents start with little experience in transacting in the foreign currency, the non-dollarized steady state results. If, on the other hand, agents start with a high level of experience, the dollarized steady state is reached. This implies, in particular, that a *temporary* increase in the rate of inflation that induces agents to become familiar with the use of an alternative currency, may lead the economy to a dollarized equilibrium in which the velocity of circulation of the domestic currency is permanently higher.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3

[5]The terms low, moderate, and high inflation will be defined below.

characterizes its steady states, and sections 4 and 5 analyze the implications for the behavior of money velocity and seignorage income. Section 6 concludes.

2 A simple model of dollarization

Consider a perfect-foresight, small, open economy populated by many identical households with preferences defined over sequences of consumption $\{C_t\}_{t=0}^{\infty}$ and described by the following utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

where $\beta \in (0,1)$ denotes the subjective discount factor and $U(\cdot)$ denotes the period utility function assumed to be strictly increasing, weakly concave and continuously differentiable. The consumption good, C_t, is assumed to be a composite of a continuum of goods, $c_t(\theta)$, indexed by $\theta \in [0,1]$. The aggregator function is assumed to have the following form

$$C_t = \int_0^1 u(c_t(\theta))\, d\theta, \tag{2}$$

where $u : \Re^+ \rightarrow \Re$ is assumed to be continuously differentiable, strictly increasing, strictly concave and to satisfy $\lim_{c\to 0} u'(c) = \infty$.

Purchases of each good $\theta \in [0,1]$ can be made either in domestic or foreign currency. The index θ reflects the degree of difficulty involved in purchasing the good using foreign currency. Specifically, we will assume that in each period $t \geq 0$, each unit of good θ purchased using foreign currency is subject to a transaction cost of $\phi(\theta, k_t)$ units of the foreign currency. The variable k_t, which is taken as given by consumers, reflects the knowledge accumulated by the economy up to period t, in transacting in a foreign currency, and will be called "dollarization capital". In what follows the terms domestic currency and pesos and foreign currency and dollars will be used as synonyms. The following assumption is made about the function $\phi(\cdot,\cdot)$,

Assumption 1 $\phi : [0,1] \times \Re^+ \rightarrow \Re^+$ *is non-negative, twice continuously differentiable and strictly convex with* $\phi_\theta > 0$, $\phi_k < 0$ *and* $\lim_{\theta\to 1} \phi(\theta,k) = \infty \ \forall k \geq 0$.

At the beginning of period t, the household starts with a stock of wealth w_t, and receives a transfer τ_t from the government. Both w_t and τ_t are measured in terms of the foreign currency. At this point a financial market opens in which the household can buy or sell an internationally traded bond denominated in dollars, b_t, that costs one dollar per unit and pays the constant interest rate $r > 0$ in $t+1$. The household can also acquire in this market its desired balances of foreign and domestic currency, d_t and m_t, respectively, both measured in terms of the foreign currency. These two currencies trade at the exchange rate set by the government, who is willing to exchange dollars for pesos and vice versa at that rate. Moreover, at the beginning of period zero the central bank announces the entire path of the nominal exchange rate and commits itself to ensuring free convertibility in each period's financial market. No goods can be traded in this market. The budget constraint of the household is then given by

$$b_t + d_t + m_t = w_t + \tau_t \tag{3}$$

After the financial market is closed, a second market opens in which only goods are traded in exchange for currency and in which the central bank does not intervene. The household sends one of its members, a shopper, to this market to purchase the desired amounts of the perishable consumption goods, $c_t(\theta)$. Every period, on the other hand, the household receives an endowment of y_t units of each good $\theta \in [0,1]$. This endowment has to be sold in the goods market by a member different from the shopper, a firm, in such a way that the shopper can not use the proceeds from the firm's sales to purchase goods in the same period. All goods $\theta \in [0,1]$ are assumed to be internationally traded at the common price of one dollar per unit. Firms can sell their endowments either for pesos or for dollars and can always get one dollar per unit of good by selling it abroad. On the other hand, if they sell goods in pesos, they have to wait until next period's financial market in order to be able to convert those pesos in dollars or dollar-denominated assets. This implies that in order for firms to be indifferent between receiving dollars or pesos in exchange for their goods, the peso price of each unit of good has to equal the exchange rate of the next period's financial market.

Let Θ_t^m denote the set of goods that the household chooses to pay for in pesos in period t, and Θ_t^d the set of goods that the household chooses to pay for in dollars in period t. The household's wealth at the beginning of period $t+1$ is then given by

$$w_{t+1} = (1+r)b_t + \frac{m_t}{1+\pi_{t+1}} + d_t - \int_{\Theta_t^d} [1+\phi(\theta, k_t)]c_t(\theta)d\theta - \int_{\Theta_t^m} c_t(\theta)d\theta + y_t \tag{4}$$

where π_{t+1} is the devaluation rate between the financial markets of period t and $t+1$, which is assumed be finite and high enough as to ensure a positive nominal interest rate, that is $(1+\pi_{t+1})(1+r) > 1 \; \forall t \geq 0$. Combining equations (3) and (4) yields the following expression for the evolution of the household's wealth

$$w_{t+1} = (1+r)(w_t+\tau_t) + [\frac{1}{1+\pi_{t+1}} - (1+r)]m_t - rd_t - \int_{\Theta_t^d} [1+\phi(\theta, k_t)]c_t(\theta)d\theta - \int_{\Theta_t^m} c_t(\theta)d\theta + y_t \tag{5}$$

In addition, the household is subject to the following cash-in-advance constraints

$$\frac{m_t}{1+\pi_{t+1}} \geq \int_{\Theta_t^m} c_t(\theta)d\theta \tag{6}$$

$$d_t \geq \int_{\Theta_t^d} [1+\phi(\theta, k_t)]c_t(\theta)d\theta \tag{7}$$

and to a borrowing constraint of the form $\lim_{t\to\infty} \frac{w_t}{(1+r)^t} \geq 0$ that prevents it from engaging in Ponzi-type schemes. The household's problem consists in choosing sequences for $\{\{c_t(\theta)\}_{\theta\in[0,1]}$, Θ_t^m, Θ_t^d, m_t,d_t, $w_{t+1}\}_{t=0}^{\infty}$ so as to maximize (1) subject to (2), (5)-(7), and to the no-ponzi-game condition, given w_0, r and sequences $\{\pi_{t+1}, \tau_t, k_t, y_t\}_{t=0}^{\infty}$. The first order conditions corresponding to this problem are

$$\lambda_t = \beta(1+r)\lambda_{t+1} \tag{8}$$

$$\gamma_t^m \leq \lambda_t[(1+\pi_{t+1})(1+r) - 1] \text{ with equality if } m_t > 0 \tag{9}$$

$$\gamma_t^d \leq \lambda_t r \text{ with equality if } d_t > 0 \tag{10}$$

$$U'(C_t)u'(c_t(\theta)) = \lambda_t + \gamma_t^m \text{ if } \theta \in \Theta_t^m \tag{11}$$

$$U'(C_t)u'(c_t(\theta)) = (\lambda_t + \gamma_t^d)[1+\phi(\theta, k_t)] \text{ if } \theta \in \Theta_t^d \tag{12}$$

where λ_t is the Lagrange multiplier associated with the budget constraint (5) and denotes the value, in terms of current utility, of one dollar available in the financial market of period t. The Lagrange multipliers γ_t^m and γ_t^d are associated with the cash-in-advance constraints (6) and (7), respectively.

It follows from the form assumed for $U(\cdot)$ and $u(\cdot)$, that $c_t(\theta) > 0\ \forall\theta,\ t$. Also, assumption 1 and conditions (9)-(12) imply that $m_t > 0\ \forall t$. Since $\phi_\theta > 0$, it follows that for any t there exists a cut-off good $\bar{\theta}(k_t, \pi_{t+1}) \in [0,1)$ such that $\Theta_t^d = [0, \bar{\theta}(k_t, \pi_{t+1})]$ and $\Theta_t^m = (\bar{\theta}(k_t, \pi_{t+1}), 1]$. The function $\bar{\theta} : \Re^+ \times \Re^+ \rightarrow [0,1]$ is implicitly given by

$$\bar{\theta}(k,\pi) = \begin{cases} 0 & \text{if } \phi(0,k) \geq \pi \\ \text{the solution of } \phi(\theta,k) = \pi & \text{otherwise} \end{cases} \tag{13}$$

(See also figure 2.) Assumption 1 implies that $\bar{\theta}(\cdot,\cdot)$ is continuous,[6] and that for any pair (k,π) such that $\bar{\theta}(k,\pi) > 0$, $\bar{\theta}$ is strictly increasing in both arguments, continuously differentiable and strictly concave in k. The function $\bar{\theta}(k,\pi)$ represents the degree of dollarization of the economy because it measures the fraction of goods paid for using foreign currency. Figure 3 displays $\bar{\theta}$ as a function of k, for three different levels of inflation.

Finally, we will postulate a law of motion for the stock of dollarization capital, k_t. We will assume that it depreciates at a constant rate $\delta \in (0,1]$ and that it is an increasing function of the current aggregate degree of dollarization

$$k_{t+1} = (1-\delta)k_t + F(\bar{\theta}(k_t, \pi_{t+1})) \equiv G(k_t, \pi_{t+1}) \tag{14}$$

where the function F satisfies,

Assumption 2 $F : [0,1] \rightarrow \Re^+$ *is continuously differentiable, strictly increasing, and strictly concave with* $F(0) = 0$.

Since the time path of inflation is assumed to be exogenously given, (14) is a scalar system in k_t. Assumptions 1 and 2 imply that the function $G(\cdot,\cdot)$ is continuous, strictly increasing in k, non-decreasing in π, bounded above by $(1-\delta)k + F(1)$ and for pairs (k,π) such that $\bar{\theta}(k,\pi) > 0$, $G(k,\pi)$ is strictly concave in k and strictly increasing in π (see figure 4).

[6]The proof, which is straightforward, is in the appendix

3 Multiple steady states and hysteresis

This section is devoted to characterizing the dynamics arising from the difference equation (14). Let $\bar{\pi}^B \equiv \phi(0,0)$. By assumption 1 $\bar{\pi}^B$ is positive. For inflation rates $\pi \leq \bar{\pi}^B$, $k = 0$ is a steady state of (14). To see this assume that the initial capital stock is zero. Then, from (13), we have that $\bar{\theta}(0,\pi) = 0$, which implies that next period's capital stock is given by $G(0,\pi) = F(0) = 0$. On the other hand, if $\pi > \bar{\pi}^B$, $k = 0$ cannot be a steady state because in this case $\bar{\theta}(0,\pi) > 0$ and $G(0,\pi) = F(\bar{\theta}(0,\pi)) > 0$. Moreover, since $G(k,\cdot)$ is strictly concave when $\bar{\theta}(k,\pi) > 0$, it follows that when $\pi > \bar{\pi}^B$, equation (14) has a unique, positive steady state. In other words $\bar{\pi}^B$ is a bifurcation point of the scalar system (14). We define the set of *high* inflation rates, Π^H, as the collection of inflation rates for which the economy converges to a steady state in which some sectors are dollarized regardless of the initial value of k, that is,

$$\Pi^H \equiv \{\pi : \pi > \bar{\pi}^B\} \tag{15}$$

Figure 4 shows the dynamics of k for an inflation rate, π_2, that belongs to Π^H.

We define the set of *low* inflation rates, Π^L, as the collection of those inflation rates for which the economy converges to a completely de-dollarized steady state regardless of the initial value of k, that is,

$$\Pi^L \equiv \{\pi : (1+r)(1+\pi) > 1 \text{ and } (1-\delta)k + F(\bar{\theta}(k,\pi)) < k \ \forall k > 0\} \tag{16}$$

This set is not empty. Consider for instance $\pi = 0$: by definition $\phi(\theta, k) \geq 0$, so by (13) $\bar{\theta}(k,\pi) = 0$ $\forall k$ and hence $(1-\delta)k + F(\bar{\theta}(k,\pi)) = (1-\delta)k < k \ \forall k > 0$. Moreover, for any inflation rate $\pi \in \Pi^L$, $k = 0$ is the only steady state of (14), and is stable. Figure 4 displays the dynamics of k_t for an inflation rate, π_0, that belongs to Π^L.

There might also exist values of $\pi < \bar{\pi}^B$ for which (14) has multiple steady states. This will be the case if the following assumption is imposed

Assumption 3 *The parameter δ and the functions $\phi(\cdot,\cdot)$ and $F(\cdot)$ are such that there exists an inflation rate $\pi < \bar{\pi}^B$ for which*

$$\max_{k>0} \{(1-\delta)k + F(\bar{\theta}(k,\pi)) - k\} \geq 0 \tag{17}$$

A sufficient condition for (17) to be satisfied is that $G_k(0, \bar{\pi}^B) > 1$, that is,

$$1 - \delta - F'(0)\frac{\phi_k(0,0)}{\phi_\theta(0,0)} > 1 \tag{18}$$

For any inflation rate $\pi < \bar{\pi}^B$ for which (17) holds with strict inequality, (14) has exactly three steady states, 0, $k^*(\pi)$ and $k^{**}(\pi)$ with $0 < k^*(\pi) < k^{**}(\pi)$ and only 0 and $k^{**}(\pi)$ are stable (see again figure 4). We define the set of *moderate* inflation rates as

$$\Pi^M \equiv \{\pi : \pi < \bar{\pi}^B \text{ and } \max_{k>0}\{G(k,\pi) - k\} \geq 0\} \tag{19}$$

since $G(k,\pi)$ is continuous and non-decreasing in π, it follows that Π^M is a convex set and that there exists an inflation rate $\underline{\pi}^B$ such that

$$\max_{k>0}\left\{(1-\delta)k + F(\bar{\theta}(k,\underline{\pi}^B)) - k\right\} = 0 \tag{20}$$

$\underline{\pi}^B$ is another bifurcation point of the scalar system (14). For $\underline{\pi}^B < \pi < \bar{\pi}^B$ (i.e., $\pi \in \Pi^M$), (14) has one de-dollarized steady state, and two partially dollarized steady state. One of the partially dollarized steady states is stable and the other one unstable. At $\pi = \underline{\pi}^B$ the two dollarized steady states merge and for $\pi < \underline{\pi}^B$ (i.e., $\pi \in \Pi^L$), both dollarized steady states disappear. This is shown in figure 5, where the set of steady states of (14) is displayed as a function of π. In systems like (14), the bifurcation points $\underline{\pi}^B$ and $\bar{\pi}^B$ are also referred to as points of catastrophe, because small changes in π away from these values change the qualitative nature of the system's orbit structure.[7]

The scalar system (14) displays hysteresis. A temporary increase in the rate of inflation can permanently increase the degree of dollarization and the velocity of circulation of the domestic currency. To see this, suppose that the economy starts at a steady state with no dollarization capital, $k = 0$, and a moderate level of inflation $\pi_1 \in \Pi^M$ as in point a in figure 5. Suppose now that the government decides to raise the inflation rate temporarily to a high level, $\pi_2 \in \Pi^H$. The economy will immediately start moving from point b in figure 5 to the dollarized steady state d. If the government decided to stabilize by setting the inflation rate back to π_1 at a moment in which

[7]See for example Azariadis (1993), technical appendix.

k is below point d but beyond point c, the economy will not converge to the initial steady state a, but rather to the partially dollarized steady state f.

The government could induce complete de-dollarization by setting the inflation rate below the catastrophe point $\underline{\pi}^B$. This "aggressive" anti-inflationary policy, however, does not have to be permanent. It has to last until the dollarization capital is below the level corresponding to point c in figure 5. At that moment, the inflation rate can go back to π_1 and the economy will continue to de-dollarize until only domestic currency is used in transactions.

4 Implications for money velocity

We started this paper by mentioning a number of studies documenting the instability of standard econometric money demand models. The practical solutions proposed for this problem consist in adding to the standard model some proxy for financial innovation or financial adaptation. The framework presented above captures this idea by implying that the velocity of circulation of domestic currency depends not only on expected inflation, but also on the stock of dollarization capital accumulated by the economy.

Specifically, the GNP velocity of circulation of the domestic currency implied by the model is given by the following expression,

$$v_t = \frac{d_t + \frac{m_t}{(1+\pi_{t+1})}}{\frac{m_t}{(1+\pi_{t+1})}} \tag{21}$$

where v_t denotes money velocity (recall that $\frac{m_t}{(1+\pi_{t+1})}$ and not just m_t, is the real value of money in t terms of goods purchased in that period's goods market). One can get a simple expression for v_t by assuming that $u(c) = \ln c$ in (2). Equilibrium conditions $(8)-(11)$ then imply that equation (21) can be written as

$$v_t = \frac{1 + \pi_{t+1}\bar{\theta}(k_t, \pi_{t+1})}{1 - \bar{\theta}(k_t, \pi_{t+1})} \tag{22}$$

Substituting repetitively (14) into this expression, one can write v_t as

$$v_t = f_j(k_{t-j}, \pi_{t+1}, \pi_t, \ldots, \pi_{t+1-j})$$

but the fact that the model displays hysteresis, means that the partial derivatives of f_j with respect to k_{t-j} and π_{t+1-j} do not necessarily vanish as j gets large. Therefore the standard econometric model of money velocity cannot be fixed by simply increasing the lag length of inflation. A proxy for k_t has to be used.

We close this section with a simple simulation exercise that intends to give a graphic idea of the implications of the model for the behavior of money velocity in the time space. It consists in choosing functional forms for $\phi(\cdot,\cdot)$ and $F(\cdot)$, assigning values to all parameters of the model, assigning an initial value k_0, and feeding equation (14) with the actual time series for inflation in Peru shown in figure 1.[8] Finally, the sequence for k_t obtained in this way together with the actual series for inflation, are used to compute the time series for v_t implied by the model, given by equation (22). The functional forms used are,

$$\phi(\theta, k) \equiv \frac{1}{(\gamma + k)(1-\theta)^{\rho}}$$

$$F(x) \equiv A\sqrt{x}$$

and the parameter values assigned were, $\delta = 0.01$, $\rho = 5$, $\gamma = 3$, $A = 50$. The initial value k_0 was assumed to be zero. The upper panel of figure 6 shows the actual and theoretical money velocity series. The theoretical time series was multiplied by a constant so as to make its mean equal to that of the actual series. For convenience, the lower panel of the figure reproduces the plot of inflation from figure 1. The correlation coefficient between the two series is 0.75. Note that the model "correctly" predicts that after the stabilization plan of August 1990, velocity stays way above the levels it reached in the period 1983-87, even though the inflation rates in the two periods are similar. The difference is accounted for by the accumulation of dollarization capital induced by the high inflation rates of the second half of the 80s.[9]

[8] The series was truncated at $\pi_t = 40\%$ and the range used was 1983:12-1993:12.

[9] Note that when the economy is completely de-dollarized, the model is identical to that of Lucas (1980), in which money velocity is completely inelastic to changes in inflation, that is why the simulated velocity is completely flat between 1984 and mid 1988. This could be fixed in an easy way by introducing leisure as a credit good like in the model of Lucas and Stokey (1987).

5 Implications for seignorage income

Consider now the implications of the model for the relationship between the rate of inflation and the amount of real resources appropriated by the government through the inflation tax on domestic currency Let represent the ratio of seignorage income to GNP by η_t. Then

$$\eta_t = \frac{1}{v_t} - \frac{1}{v_{t-1}} \frac{1}{(1+\pi_{t-1})} \frac{GNP_{t-1}}{GNP_t}$$

In steady state, and assuming no long-run growth, this expression reduces to

$$\eta = \frac{1}{v} \frac{\pi}{1+\pi}$$

This relation is depicted in figure 7. At low levels of inflation, the economy is completely de-dollarized and real domestic balances are insensitive to the inflation rate, so the slope of η is just $1/(1+\pi)^2$. At moderate levels of inflation, there are two stable steady-state values for η. The one corresponding to the dollarized equilibrium, lies below that associated with $k = 0$. Moreover, the slope of η is lower when $k > 0$, because velocity is increasing in inflation. For high levels of inflation there is only one steady state value of η. The model then captures the observation made at both theoretical and empirical levels, that one of the effects of currency substitution on inflationary finance is to make the inflation rate required to finance a given level of fiscal deficit higher.[10]

6 Concluding remarks

This paper develops a model in which temporary changes in the inflation rate may produce permanent effects on money demand. The main departure from other models that try to capture this effect is that in this paper the process of currency substitution is assumed to produces external effects by making it easier for consumers to use the foreign currency to perform transactions when the rest of the economy is also using it. So agents take into account the aggregate level of currency substitution in deciding their demands for domestic and foreign currency.

We close the paper by noting three policy implications derived from our theoretical framework.

[10] See for instance Nichols (1974), Bufman and Leiderman (1991), Imrohoroglu (1994), and McNellis and Asiles (1992).

First, in general there will not be a unique level of money velocity associated with each level of inflation: the inflationary history of the economy can have permanent effects on money velocity. Second, in dollarized economies, stabilization plans aimed at increasing the degree of monetization and of seignorage income have to be accompanied by an initial period of very low inflation, so as to induce agents to de-dollarize and lose their skills in using the foreign currency in their transactions. Finally, the model presented above, unlike the inaction-band model of Guidotti and Rodríguez, predicts that the government does not need to generate deflation in order to induce complete de-dollarization. The reason is that in our model the relevant opportunity cost of using a foreign currency as a means of payment, includes the transactions costs incurred in the process of exchanging it for goods. To the extent that these transactions costs are higher for the foreign currency than for the domestic currency, the domestic currency can dominate the foreign currency even at positive levels of inflation.

Appendix

Proof that $\bar{\theta}(\cdot,\cdot)$ is continuous. We have to check for continuity in three cases: (i) Consider a point $(\bar{k},\bar{\pi})$ such that $\phi(0,\bar{k}) > \bar{\pi}$. It follows that $\bar{\theta}(\bar{k},\bar{\pi}) = 0$. Take any sequence (k_n,π_n) converging to $(\bar{k},\bar{\pi})$. Since ϕ is continuous, $\exists N$ such that $\phi(0,k_n) > \pi_n$ $\forall n > N$, so $\bar{\theta}(k_n,\pi_n) = 0$ $\forall n > N$ and $\bar{\theta}(k_n,\pi_n) \to 0 = \bar{\theta}(\bar{k},\bar{\pi})$. (ii) Consider next a point $(\bar{k},\bar{\pi})$ such that $\phi(0,\bar{k}) < \bar{\pi}$. Since ϕ is continuous, there exists an open ball B containing $(\bar{k},\bar{\pi})$ such that $\phi(0,k') < \pi'$ for all (k',π') $\in$B. Since $\phi(\theta,k) - \pi = 0$ satisfies all the conditions of the implicit function theorem, we have that $\bar{\theta}$ is actually continuously differentiable at any $(k',\pi') \in B$. (iii) Finally, consider a point $(\bar{k},\bar{\pi})$ such that

$$\phi(0,\bar{k}) = \bar{\pi} \tag{A1}$$

Then $\bar{\theta}(\bar{k},\bar{\pi}) = 0$. Let $(k_n,\pi_n) \to (\bar{k},\bar{\pi})$. Consider the subsequence (k_n',π_n') of (k_n,π_n) such that $\phi(0,k_n') < \pi_n'$. To this subsequence corresponds a subsequence $\theta_n' \equiv \bar{\theta}(k_n', pi_n')$ which is implicitly given by $\phi(\theta_n',k_n') = \pi_n'$. We want to know whether θ_n' converges to zero. Suppose not. Then $\exists \epsilon > 0$ such that $\forall n$ there exists an $N > n$ for which $\theta_N' \geq \epsilon$. Consider now the subsequence (k_n'',π_n'') of (k_n',π_n') such that

$$\theta_n'' \equiv \bar{\theta}(k_n'',\pi_n'') > \epsilon \tag{A2}$$

where $\bar{\theta}(k_n'',\pi_n'')$ is implicitly given by,

$$\phi(\theta_n'',k_n'') = \pi_n'' \tag{A3}$$

subtract A1 from A3 to get

$$\pi_n'' - \bar{\pi} = \phi(\theta_n'',k_n'') - \phi(0,\bar{k}) > \phi(\epsilon,k_n'') - \phi(0,\bar{k}) \tag{A4}$$

where the last inequality follows from A2 and the fact that $\phi_\theta > 0$. Taking limits in A.4 we get

$$0 = \lim_{n\to\infty} [\phi(\theta_n'',k_n'') - \phi(0,\bar{k})] \geq \phi(\epsilon,\bar{k}) - \phi(0,\bar{k}) > 0$$

which is a contradiction. Then $\bar{\theta}(k_n'',\pi_n'') \to 0 = \bar{\theta}(\bar{k},\bar{\pi})$. All other elements of (k_n,π_n) are associated with $\bar{\theta}(k_n,\pi_n) = 0$ ■.

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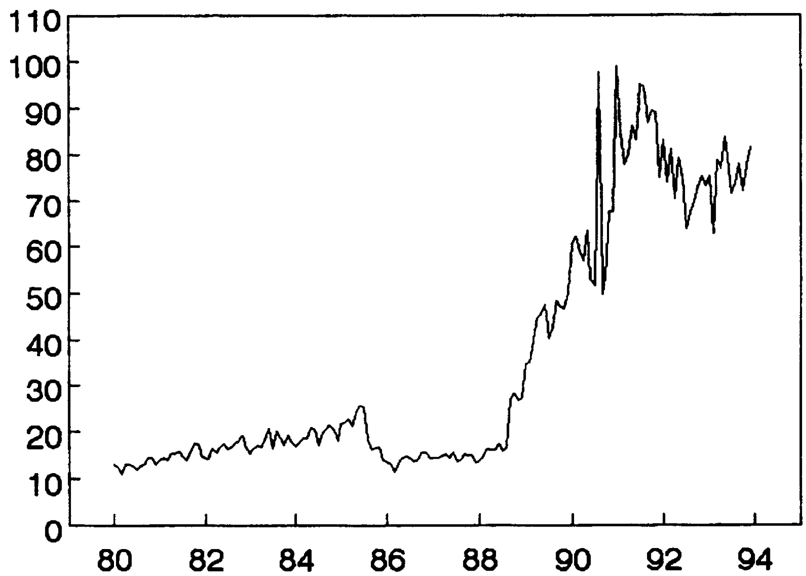
Figure 1
PERU: M1 VELOCITY AND INFLATION

M1 Velocity
monthly data 1980:1 – 1993:12



CPI Inflation (truncated at 40%)
monthly data 1980:1 – 1993:12

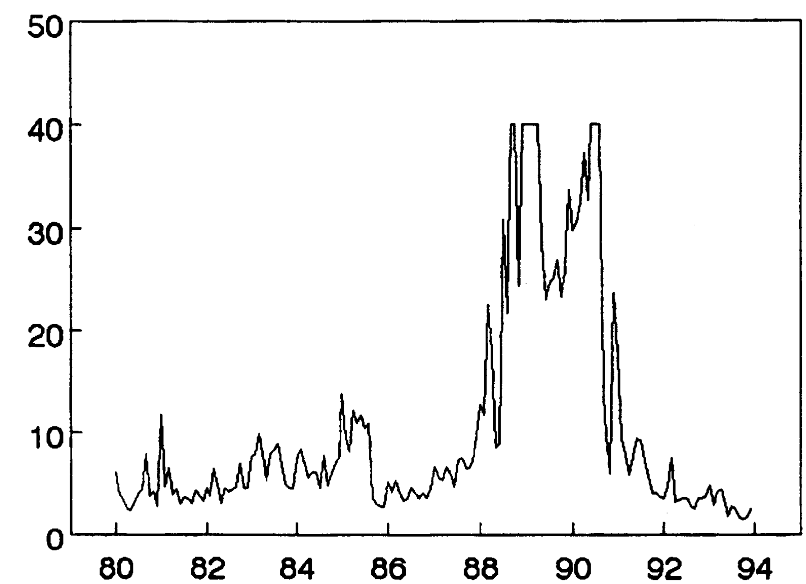


Figure 2
The degree of dollarization for two levels of dollarization capital, k_0 and $k_1 > k_0$

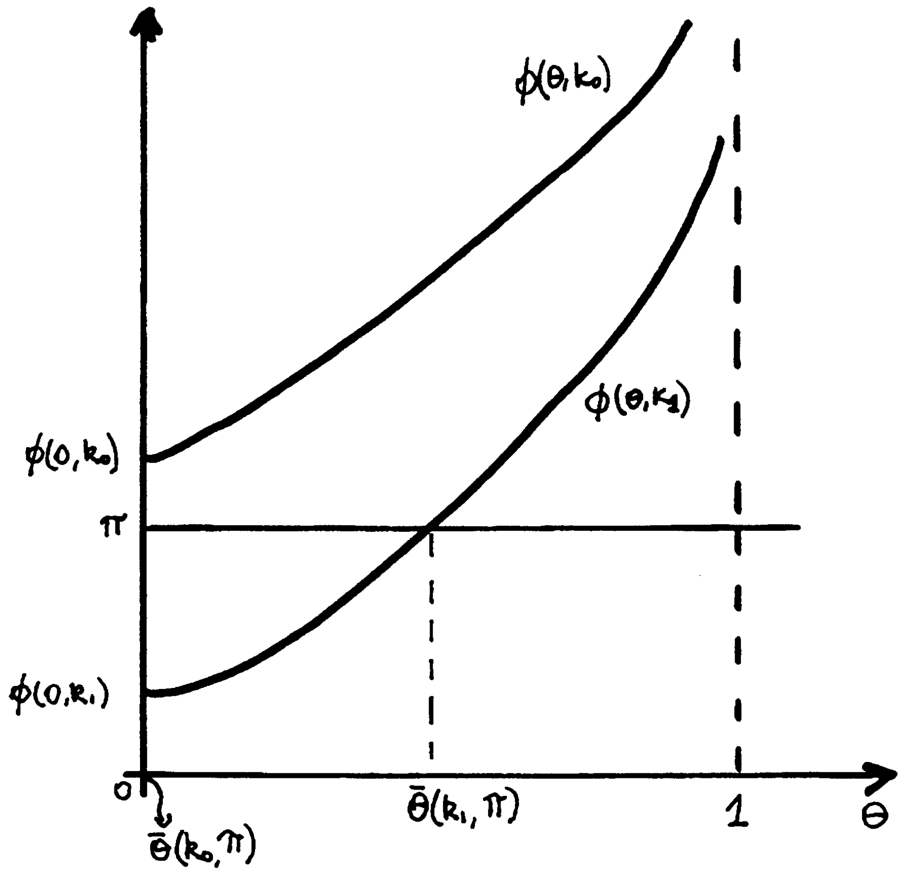


Figure 3

The function $\bar{\theta}(k,\pi)$ for two levels of inflation, π_1 and $\pi_2>\pi_1$

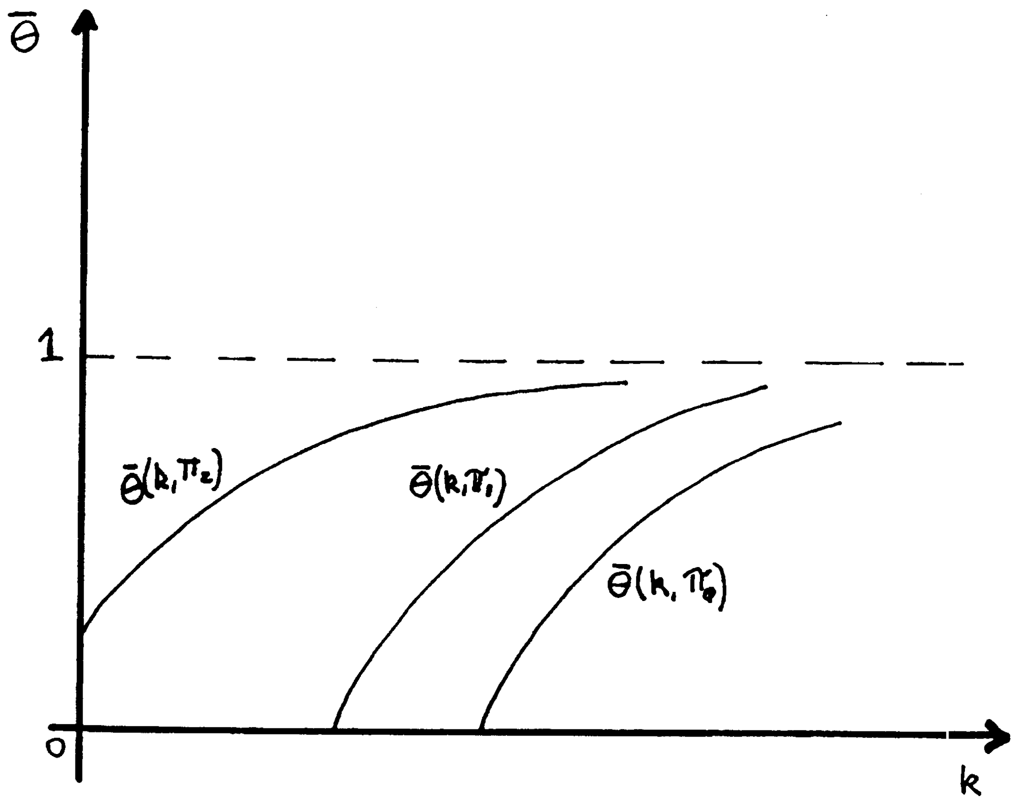
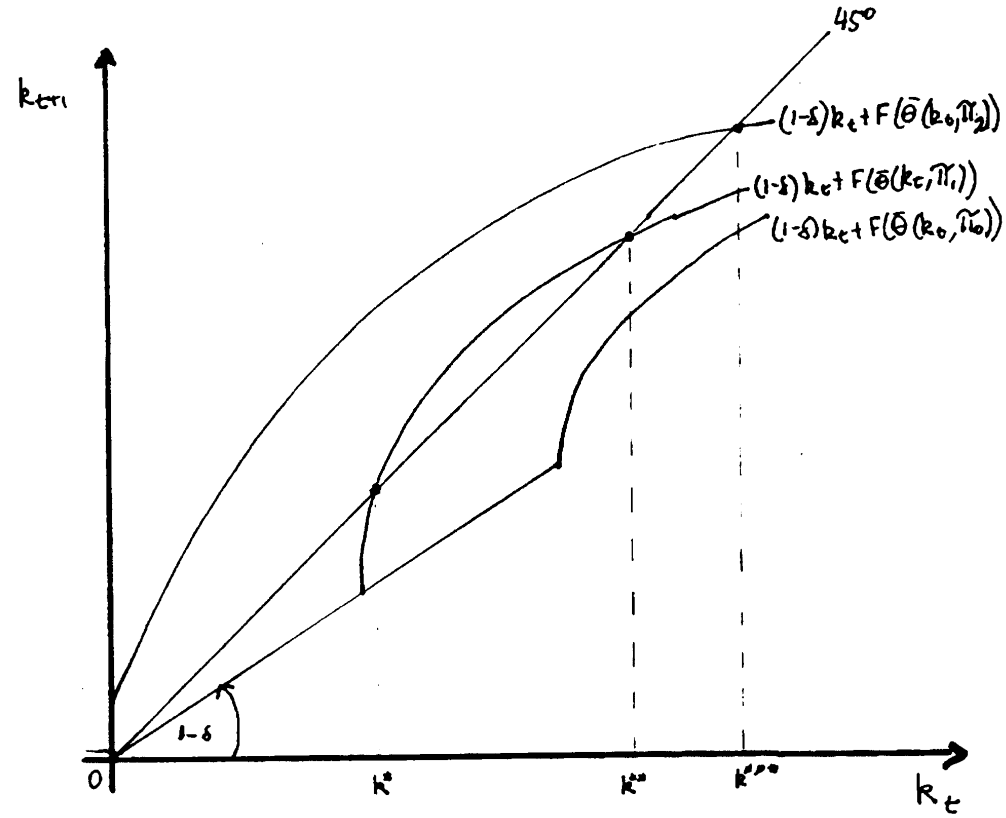
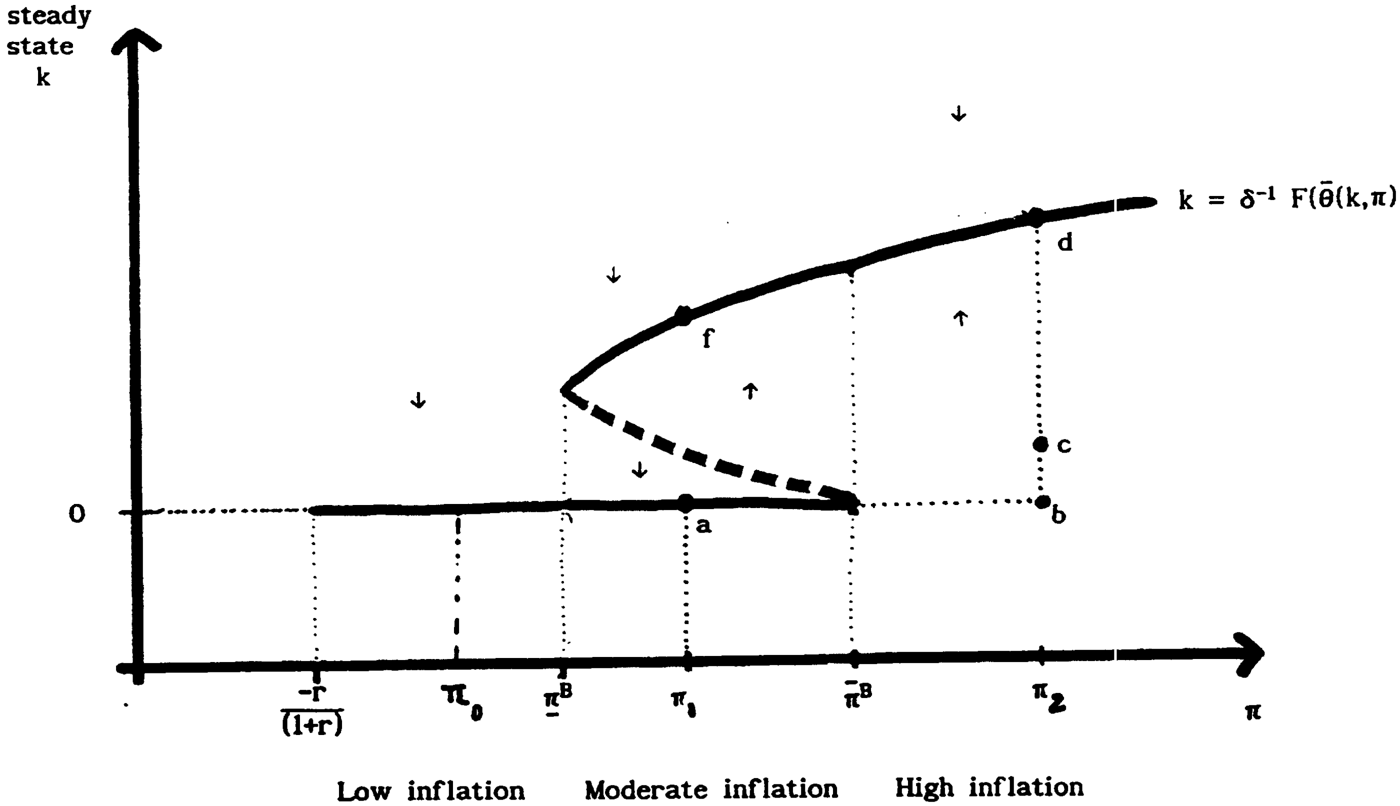


Figure 4



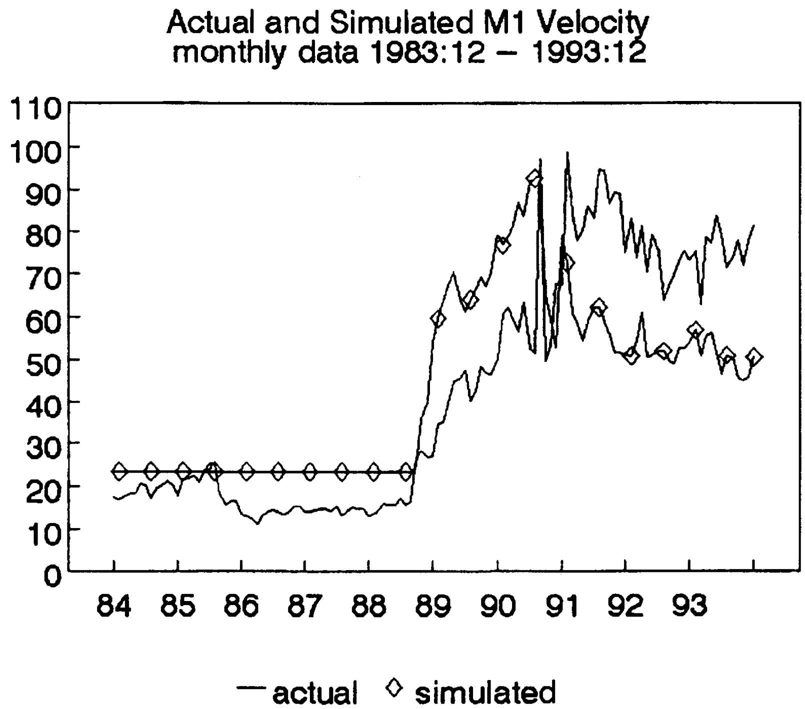
This figure displays the dynamics of the dollarization capital, k_t, for three inflation rates: for the low rate $\pi_0 \in \Pi^L$, the only steady state is $k=0$. At the high inflation rate $\pi_2 \in \Pi^H$, there is also only one steady state $k^{***}>0$. At the moderate inflation rate $\pi_1 \in \Pi^M$, the system displays three steady states $k=0$, $k=k^*$ and $k=k^{**}$, with $0<k^*<k^{**}<k^{***}$.

Figure 5
Bifurcation Diagram



The diagram shows the steady states as a function of the inflation rate. The stable steady states are drawn as solid lines and the unstable ones are shown as broken lines. The arrows indicate the direction of motion of the system around the steady state.

Figure 6
ACTUAL AND SIMULATED M1 VELOCITY AND ACTUAL CPI INFLATION



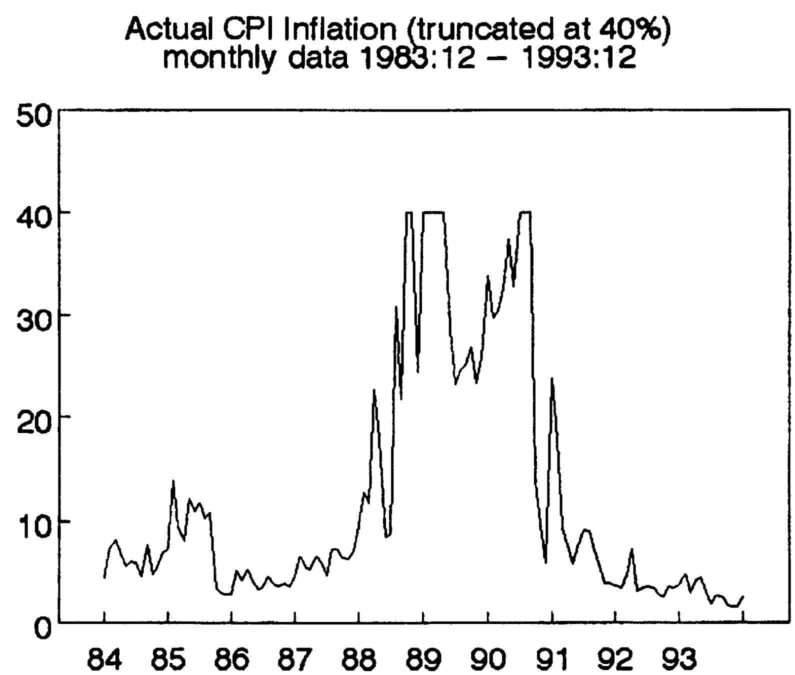
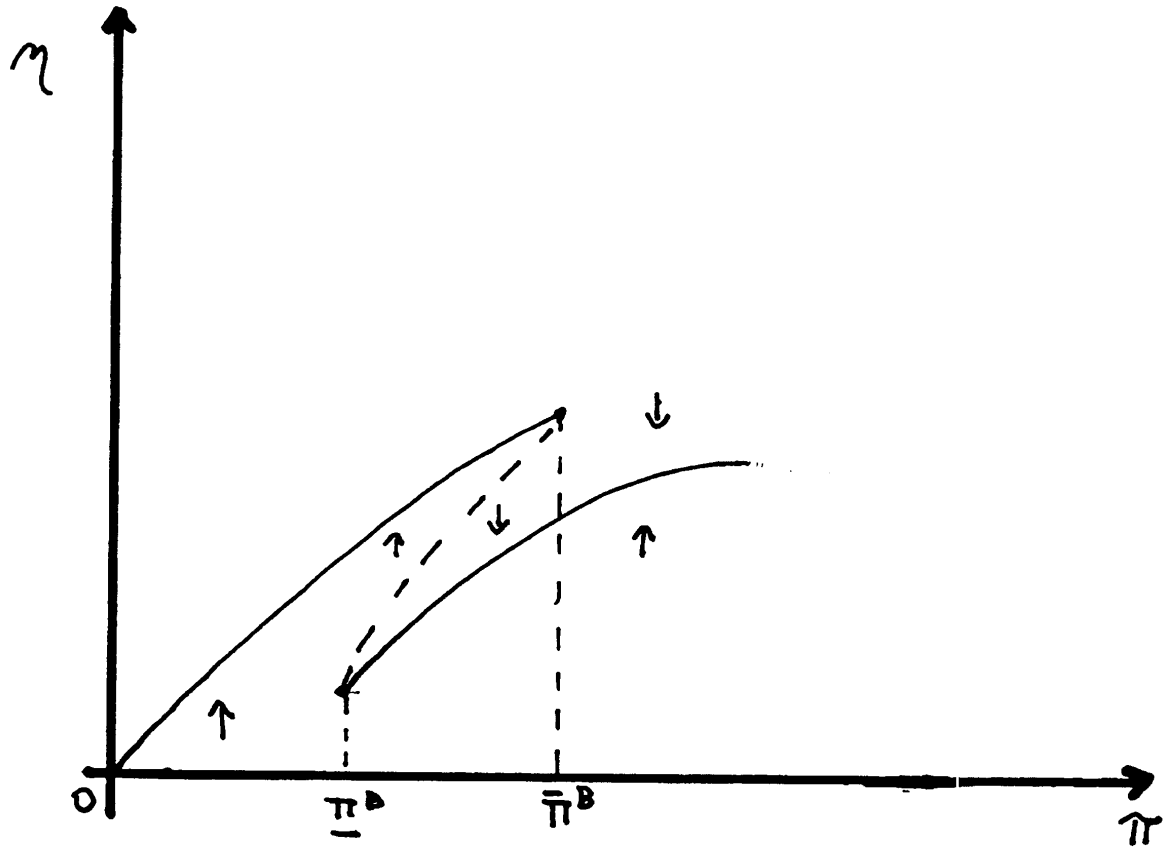


Figure 7
Seigniorage income, η, as a function of
the inflation rate π



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