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CONSTRAINED SUBOPTIMALITY IN ECONOMIES WITH LIMITED COMMUNICATION

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Abstract

Economies with limited communication contain an externality which typically makes them Pareto inefficient, even taking into account the communication constraints agents face. In a two period model it is shown that an open and dense set of economies with limited communication are constrained Pareto suboptimal. Thus equilibria of economies with voluntary unemployment, search, or other types of limits on communication are unlikely to be Pareto optimal, even in the absence of moral hazard, adverse selection, or search externalities.

Constrained Suboptimality in Economies with Limited Communication

David Bowman[1]

"In a primitive community, where each family, or small group , is more or less self-sufficing and directs the main part of its activity to the production of things to be consumed by itself, the forecast it would have to make on the demand side (i.e. apart from prospective costs of production) would refer exclusively to its own future tastes. In modern conditions, however, with industry conducted on the basis of division of labor and exchange of products, each producer's forecast must refer both to the tastes of other people and also their real income and purchasing power. Naturally this sort of forecast is exposed to much larger error than the primitive forecast; and the difficulty of judgment is enhanced as the relevant market comes to include more and more remote groups of purchasers, about whose circumstances the ordinary producer has great difficulty in informing himself." A.C. Pigou, from *Industrial Fluctuations* (1927) pgs. 82 – 83

1 Introduction

The Arrow-Debreu formulation of general equilibrium can incorporate uncertainty regarding future *exogenous* events. While it is clear that much of the uncertainty in the economy can in fact be ascribed to the unpredictability of choices made by nature regarding the weather, natural disasters, technical possibilities, etc., it also seems clear that much uncertainty must be ascribed to the unpredictability of choices made by individuals within the economy itself. To forecast future demand one must forecast not only moves made by nature but also currently planned moves made by others in the economy. In a small economy this may simply mean that one must forecast the plans of one's neighbors, which may not be subject to much error, but in a large economy one must forecast the plans of a much larger group of actors than one could reasonably hope to communicate with at any point in time. This limited ability to communicate may translate into the possibility of a large forecast error.

While uncertainty about others' plans is outside the scope of the original Arrow-Debreu analysis, models in which communication is limited — and thus in which uncertainty about other's plans could occur — have played a prominent role in macroeconomic theory. Search

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theory, the "islands" models of Lucas [16] and Phelps [21], and monetary models such as Townsend [28] all study environments in which communication between groups at any given point in time is impossible.[2] Limited communication also appears to play a key, though less explicit, role in multiple equilibrium models of "coordination failure" (Cooper and John [4]), since it is difficult to explain why individuals would choose a Pareto dominated equilibrium if they could freely communicate.

This paper demonstrates that economies with limited communication between groups of agents typically contain an externality which makes them Pareto suboptimal, even when the constraints preventing communication are taken into account. Unlike the work of Diamond [5], the externality studied here is pecuniary and quite separate from any search externalities arising from the technology by which agents are matched together over time. Rather, the externality arises from agent's risk aversion. Limited communication can create uncertainty about future trading opportunities by creating uncertainty concerning the current plans and actions of others in the economy, however agents have no incentive to take account of the effects the unpredictability of their plans have on the welfare of those they do not currently communicate with. Because each agent's uncertainty concerns the (endogenous) choices of others in the economy, there is a possibility of Pareto improvement even when existing communication constraints are respected. For example, changes which make each agent's actions more predictable will be Pareto improving if each individual's welfare loss from the constraint that his own choices be more predictable is outweighed by the gain in welfare from the reduction in uncertainty concerning others actions.

It is interesting to note that the limited communication models studied to this point have made one or more of the assumptions necessary for constrained Pareto optimal equilibria. In Lucas [16], no action taken on an island has any effect on future trading opportunities (there are no endogenous state variables). In Diamond [5], agents' decisions *do* affect other trades (or the probability of other trades), but only through a search externality. In the absence of that externality any equilibrium of the model is Pareto optimal, but it is so because agents are risk neutral and (in the absence of the search externality) there are no endogenous state variables. In Townsend [28], there is no uncertainty. In each of these models, had the assumptions been relaxed, constrained Pareto optimality would typically have been lost.

Since the externality generated by limited communication is pecuniary, it has obvious connections to the literature on incomplete financial markets (Radner [24], Hart [9], Stiglitz [26], Geanakopolos and Polemarchakis [8]). In that work agents trade in one location, but the state-contingent contracts that they can trade are limited. In the literature on limited

[2]See Diamond [5], Mortensen [18], Pissarides [22], and Hosios [10] for examples of general equilibrium search models. See Feldman [7], Kiyotaki and Wright [13, 14], Ostroy [19], and Ostroy and Starr [20] for other examples of monetary models.

communication there need be no constraints on the type of contracts agents can trade, but the group of agents they can trade with at any point in time is limited. Unlike economies with incomplete financial markets, each agent faces a single budget constraint in the model studied here; this makes issues of existence simpler since individual demands are continuous under standard regularity conditions. Further, neither the endowments nor preferences of any individual are state dependent, a constraint more reminiscent of sunspot models (Cass and Shell [3]).

The analysis will be carried out in a fairly general two-period model. Section 2 provides an example. Section 3 introduces the general model. Section 4 derives conditions under which an equilibrium will be constrained Pareto suboptimal and proves that all equilibria of an open and dense subset of economies are suboptimal. Section 5 concludes.

2 An Example

There are two time periods, time 1 and time 2; two agents, a and b; and two goods, good 0 and good 1. The two agents will trade competitively at time 2, but at time 1 communication between them is prohibited. Each agent's problem at time 1 is to decide how much of his endowment to consume given that what is not consumed can be carried over to time 2 and traded. At time 1 each agent knows his own preferences and endowment, but may be uncertain about the preferences and/or endowment of the other. Let c_{tj}^i denote i's consumption of good j at time t, and w_j^i denote i's endowment of good j. Let $y_j^i \equiv w_j^i - c_{1j}^i$ denote the amount of good j that agent i brings into time 2.

The utility function of a is:

$$u^a = \ln(c_{11}^a) + \frac{1}{1+\delta}\left(c_{20}^a + \ln(c_{21}^a)\right)$$

and the utility function of b is:

$$u^b = \ln\left((c_{10}^b)^{\beta}(c_{11}^b)^{(1-\beta)}\right) + \frac{1}{1+\delta}\ln\left((c_{20}^b)^{(1-\beta)}(c_{21}^b)^{\beta}\right).$$

Given the amounts $y^i \equiv (y_0^i, y_1^i)$ carried over by a and b, at time 2 the unique competitive equilibrium price is $p = \frac{1+\beta y_0^b}{y_1^a+(1-\beta)y_1^b}$, where p is the relative price of good 1. Taking prices as given, at time 1 agent i chooses y^i to maximize expected utility, $E^i(u^i)$, where $E^i(\cdot) = E(\cdot \mid u^i, y^i)$.

We begin with a set of parameters which imply that there is no uncertainty. Assume that it is common knowledge to both agents that $\delta = 3$, $\beta = 0$, $(w_0^a, w_1^a) = (2,2)$, and $(w_0^b, w_1^b) = (0,1)$. The unique equilibrium is:

$$y^a = \left(2, \frac{6}{25}\right), \; y^b = \left(0, \frac{1}{5}\right).$$

One can easily show that any other choice of y^a or y^b will make at least one of the agents worse off.

Now change the parameters so that there *is* uncertainty. Take $\delta = 3$, but assume that with equal probability either $(w_0^a, w_1^a) = (2,1)$ or $(w_0^a, w_1^a) = (2,3)$. Likewise, assume that with equal probability either $\beta = 0$ and $(w_0^b, w_1^b) = (0,1)$, or $\beta = 1$ and $(w_0^b, w_1^b) = (1,0)$. The unique rational expectations equilibrium is:

$$y^a = \begin{cases} (2, 0.17) \text{ if } w_1^a = 1 \\ (2, 0.59) \text{ if } w_1^a = 3 \end{cases}, \; y^b = \begin{cases} (0, \frac{1}{5}) \text{ if } \beta = 0 \\ (\frac{1}{5}, 0) \text{ if } \beta = 1 \end{cases}.$$

In this case it is straightforward to show that for small ϵ both agents will be strictly better off (regardless of preferences and endowments) if a changes his behavior to:

$$\hat{y}_1^a = y_1^a - \left(\frac{w_1^a}{w_1^a - E^b(w_1^a)} \right) \epsilon$$

Which is to say that if a plans to bring a high amount of good 1 into time 2 then he should bring less, and if he plans to bring a low amount then he should bring more.

These results are robust. While the example is simple, the only thing special about it is that the parameters were chosen so that in the second case a Pareto improvement could be effected by changing a's behavior alone; in general both agent's plans might have to be altered. It is important to note that the proposed intervention can be effected without any more information than agents themselves have — that is, the proposed change in each agent's consumption only depends upon information available to the agent.

The optimality of the first example indicates that limited communication does not affect the optimality of agent's decisions concerning the delivery of goods *per se*. This is true because under perfect foresight agent's forecasts of the future price of each good will equal the actual price, and crucially, each agent will in fact have to pay this price in order to receive delivery of the good.

In the second example agents are uncertain about future prices. Because prices are endogenous, a can effectively insure the income of b without making himself worse off by changing his actions in a way which makes prices less variable. Suppose that at time 1 a were to take this into account and moderate his actions in way that moderates the unpredictability of b's income. Since agents have rational expectations a would correctly forecast the value of this service to b, however at time 2 b has no incentive to pay for this service *because it has been delivered at time* 1. No one will pay for insurance after the fact, however in this case the entire need for insurance arises because agents cannot communicate at time 1 — if they could compensate each other for insurance at time 1 there would be no need for insurance in the first place. With limited communication agents ignore the effect their plans

	loc 1	loc 2
time 1	(a, b)	(a', b')
time 2	(a, b')	(a', b)

Table 1: Trading Patterns.

have on other's uncertainty about future income, and in this sense there will be too much uncertainty.

3 The Economy

We present a generalization of the economic structure considered in the Section 2. Consider an exchange economy with price taking agents. Trading patterns are assumed to take place as shown in Table 1. At time 1 a and b trade at location 1 while a' and b' trade at location 2. At time 2 partners are switched, so that a trades with b' and b trades with a'. No communication is allowed between locations.

At time 1 one of a finite number of events will have occurred at each location, each event specifying a different set of preferences, endowments, and/or information for at least one of the agents at that location.[3] At time 1 each agent is assumed to observe only the event which has occurred at his location; at time 2 the events which have occurred at both locations are made public knowledge. In accordance with the structure of information we differentiate agents by the event which has occurred at their location; (a_i, b_i) and (a_j, b_j) $((a'_i, b'_i)$ and $(a'_j, b'_j))$ $i \neq j$ representing different agents if a different event has occurred at location 1 (location 2). I denotes the set of all agents.[4]

The state of nature describes which agents are matched in the second period. Let S_l represent the set of possible agents at location l at time 1 ($S_1 = \{(a_1, b_1), (a_2, b_2), \ldots, (a_{S_1}, b_{S_1})\}$, $S_2 = \{(a'_1, b'_1), (a'_2, b'_2), \ldots, (a'_{S_2}, b'_{S_2})\}$) and take $S = S_1 \times S_2$ to be the set of possible states. We assume that each $s \in S$ has nonzero probability $\pi(s)$ of occurring and that both S and $\pi(\cdot)$ are public knowledge. We let h_{lt} represent the information available in location l at time t, $h_{l1} \in S_l$ and $h_{l2} \in S$, so that a market is described by the vector (l, t, h_{lt}). Let $I(l, t, h_{lt})$ represent the set of agents in location l at time t given h_{lt}; if $i \in I(l, t, h_{lt})$ then i's decisions are constrained to be measurable functions of h_{lt}. We will write $S^{h_{l1}}$ for the set $\{s \in S \mid proj_l(s) = h_{l1}\}$ and $\pi^{h_{l1}}(s)$ for $prob(s \mid proj_l(s) = h_{l1})$. It will also be convenient

[3]In economies with multiple equilibria one might also define an event as distinguishing the equilibrium choosen at time 1 (the realized event could be imagined to include the possible realization of some set of sunspot variables which only participants at one location have observed at time 1). This will not be explored at any depth in this paper, but the analysis of this section would not differ.

[4]The same symbol will be used to denote a set and the number of elements in the set.

to define $\pi^i(s) = \pi^{h_{l1}}(s)$ and $S^i = S^{h_{l1}}$, where h_{l1} satisfies $i \in I(l, 1, h_{l1})$. (S^i is the set of states which agent i participates in and $\pi^i(s)$ is the probability i attaches to the occurrence of state s).

At each time and location there is a spot market for $G+1$ goods; free disposal of each good is allowed. In contrast to Section 2 the goods are non-storable — this simplifies notation and does not significantly alter the analysis. Agent i's endowment at time t is $w_t^i \in \mathcal{R}_{++}^{G+1}$. The consumption of agent i at time 1 is $c_1^i \in \mathcal{R}_+^{G+1}$, the consumption of agent i at time 2 in state $s \in S^i$ is $c_2^i(s) \in \mathcal{R}_+^{G+1}$. (Since i is labeled by location and the event which has occurred at his location, we drop any further reference to location and, when dealing with time 1 variables, the state of nature, if a variable is labeled by i.) We let $c^i = (c_1^i, \ldots, c_2^i(s), \ldots)_{s \in S^i} \in \mathcal{R}_+^{(G+1)(S^i+1)}$.

Each agents' preferences will be represented by a time-separable von Neumann-Morgenstern utility function $u^i : \mathcal{R}_+^{(G+1)(S^i+1)} \to \mathcal{R}$,

$$u^i(c^i) = u_1^i(c_1^i) + \sum_{s \in S^i} \pi^i(s) u_2^i(c_2^i(s)),$$

and we make the following assumptions concerning preferences:

A1 u_t^i is C^2 on $\mathcal{R}_+^{(G+1)}$.

A2 u_t^i is differentiably strictly increasing on $\mathcal{R}_{++}^{(G+1)}$.

A3 u_t^i is differentiably strictly concave on $\mathcal{R}_{++}^{(G+1)}$.

A4 For any $(c_1^*, c_2^*) \in \mathcal{R}_+^{2(G+1)} \setminus \{z \in \mathcal{R}_+^{2(G+1)} \mid 0 \leq z < e\}$, where e is a fixed vector satisfying $0 \ll e \ll (w_1^i, w_2^i)$, the set $\{(c_1, c_2) \in \mathcal{R}_+^{2(G+1)} \mid u_1^i(c_1) + u_2^i(c_2) = u_1^i(c_1^*) + u_2^i(c_2^*)\}$ is contained in $\mathcal{R}_{++}^{2(G+1)}$.

The assumptions that preferences are time-separable and that they can be represented in terms of a von Neumann-Morgenstern expected utility function are made for convenience. Assumptions A1-A4 are standard; their primary purpose is to ensure that individual demands exist and are continuously differentiable at an equilibrium. It is important to note that neither the preferences nor endowment of any agent depend upon who the agent is matched with at time 2, and that no agent is uncertain about either his preferences or endowment.

Within each location agents will be allowed to trade a complete set of securities at time 1 as suggested in Arrow [2], each security promising a bundle of one unit of each good if a given state is announced at time 2 and nothing otherwise, with one such security for each possible state.[5] Define y^i as i's purchase of securities, $y^i = (\ldots, y^i(s), \ldots)_{s \in S^i} \in \mathcal{R}^{S^i}$, and let $d = (1, 1, .., 1) \in \mathcal{R}^{G+1}$ so that the bundle of goods i brings to a second period market in state s is $(w_2^i + y^i(s)d)$.

$p(l, t, h_{lt}) = (p_0(l, t, h_{lt}), p_1(l, t, h_{lt}), \ldots, p_G(l, t, h_{lt})) \in \mathcal{R}_{++}^{G+1}$ is the set of spot prices at time t and location l given h_{lt}, and $p = (\ldots, p(l, t, h_{lt}), \ldots)_{\forall l, t, h_{lt}} \in \mathcal{R}_{++}^{(G+1)M}$, where $M = S_1 + S_2 + 2S$ is the total number of possible spot markets. $q(l, h_{l1}) = (\ldots, q(l, s), \ldots)_{s \in S^{h_{l1}}}$ is the set of prices for financial securities at location l and event h_{l1}, and $q = (\ldots, q(l, h_{l1}), \ldots)_{\forall l, h_{l1}} \in R_{++}^{2S}$. Given the structure of the economy, prices at each location, time, and event can be normalized to sum to one. We therefore take

$$\Delta = \left\{ (p, q) \in \mathcal{R}_{++}^{(G+1)M+2S} \mid \sum_{g=0}^{G} p_g(l, 1, h_{l1}) + \sum_{s \in S^{h_{l1}}} q(l, s) = 1, \sum_{g=0}^{G} p_g(l, 2, h_{l2}) = 1 \ \forall l, t, h_{lt} \right\}$$

as the set of prices. At times it will be convenient to refer to the prices faced by some agent i; define $p^i = (p_1^i, \ldots, p_2^i(s), \ldots)_{s \in S^i}$ and q^i as the prices which prevail in the locations, times and events at which $i \in I$ trades; $p_1^i = p(l, 1, h_{l1})$, $p_2^i(h_{l2}) = p(l, 2, h_{l2})$, $q^i = q(l, h_{l1})$, where $i \in I(l, t, h_{lt})$.

Agent i can be viewed as solving the following problem:
Given $(p, q) \in \Delta$, choose $(c^i, y^i) \in \mathcal{R}_{+}^{(G+1)(S^i+1)} \times \mathcal{R}^{S^i}$ to max $u^i(c^i)$ subject to:
$p_1^i \cdot (c_1^i - w_1^i) + \sum_{s \in S^i} q^i(s) p_2^i(s) \cdot (c_2^i(s) - w_2^i) \leq 0$
$y^i(s) = p_2^i(s) \cdot (c_2^i(s) - w_2^i)$

The solution defines i's demand functions for goods and assets,

$$c^i(p, q) = (c_1^i(p, q) \ldots, c_2^i(p, q, s), \ldots)_{s \in S^i}$$
$$y^i(p, q) = (\ldots, y^i(p, q, s), \ldots)_{s \in S^i}.$$

Define the aggregate excess demand function

$$\varphi(p, q) = \Bigg(\left(\sum_{i \in I(l,1,h_{l1})} (c_1^i(p, q) - w_1^i) \right)_{\forall l, h_{l1}},$$
$$\left(\sum_{i \in I(l,2,h_{l2})} (c_2^i(p, q, h_{l2}) - w_2^i - y^i(p, q, h_{l2})d) \right)_{\forall l, h_{l2}}, \left(\sum_{i \in I(l,1,h_{l1})} y^i(p, q) \right)_{\forall l, h_{l1}} \Bigg).$$

[5] Securties markets are included to make the point that an externality exists even when agents trade insurance contracts between themselves (though not across locations). As to how this could occur given the physical environment, consider the following: between time 1 and 2 b takes the time 2 endowment from location 1 and b' takes the time 2 endowment from location 2. Meeting in the middle, b and b' ascertain which state has occured and give each what they owe to a and a' respectively, to be delivered as promised.

An equilibrium is defined as a set of prices $(p,q) \in \Delta$ such that $\varphi(p,q) = 0$.

Proposition 1: An equilibrium exists under A1-A4.[6]

4 Constrained Optimality

Throughout this section agents' endowments are held fixed and the space of economies considered is the space of utility functions for each agent. We will employ local paremeterizations of each agent's utility function based on a technique developed in Kajii [12] (see also Suda et. al. [27]). Since the set of feasible allocations is bounded, we can find $x \in \mathcal{R}^{G+1}$ such that for any feasible allocation $0 \leq c_1^i \leq x$ and $0 \leq c_2^i(s) \leq x$ for all i and $s \in S^i$. Let $X = \{c \in \mathcal{R}^{G+1} \mid 0 \leq c \leq x\}$. Fixing $\pi(\cdot)$, it is natural to identify u^i with $(u_1^i, u_2^i) \in \mathcal{C}^2(X,\mathcal{R}) \times \mathcal{C}^2(X,\mathcal{R})$. We consider the topology of the $\mathcal{C}^2$ uniform convergence on $\mathcal{C}^2(X,\mathcal{R})$ and endow $\mathcal{C}^2(X,\mathcal{R}) \times \mathcal{C}^2(X,\mathcal{R})$ with the product topology. Endow $\mathcal{U}^i = \{(u_1^i, u_2^i) \in \mathcal{C}^2(X,\mathcal{R}) \times \mathcal{C}^2(X,\mathcal{R}) \mid (u_1^i, u_2^i) \text{ satisfies } A1 - A4\}$ with the relative topology from $\mathcal{C}^2(X,\mathcal{R}) \times \mathcal{C}^2(X,\mathcal{R})$. The space of economies considered is $\mathcal{U} = \times_{i \in I} \mathcal{U}^i$ endowed with the product topology. $\mathcal{U}$ is an open subset of a separable Banach space (Mas-Colell [17] (pg. 50) and Dieudonné [6] (pg. 75)). From this point we will consider excess demand as a function of prices and preferences, writing $\varphi(p,q,u)$ (and $\tilde{\varphi}(p,q,u)$), where $(p,q) \in \Delta$ and $u \in \mathcal{U}$.

In defining our notion of constrained optimality it will be convenient to treat agents as solving the problem facing them recursively. Defining agent i's demand for goods at time 2 given spot prices $p_2^i(s)$ and bundle $(w_2^i + y^i(s)d)$:[7]

$$c_2^i(w_2^i + y^i(s)d, p_2^i(s)) \equiv \arg\max\{u_2^i(c_2^i(s)) \mid p_2^i(s) \cdot (c_2^i(s) - w_2^i - y^i(s)d) \leq 0; c_2^i(s) \in \mathcal{R}_+^{G+1}\},$$

agent i's indirect utility function is

$$v^i(w_2^i + y^i(s)d, p_2^i(s)) \equiv u_2^i(c_2^i(w_2^i + y^i(s)d, p_2^i(s))).$$

Using these definitions, agent i's time 1 demand for goods and financial claims can be rewritten as the solution to the following problem:

Given $(p,q) \in \Delta$, choose $(c_1^i, y^i) \in \mathcal{R}_+^{G+1} \times \mathcal{R}^{S^i}$ to
$\max\ u_1^i(c_1^i) + \sum_{s \in S^i} \pi^i(s) v^i(w_2^i(s) + y^i(s)d, p_2^i(s))$
subject to: $p_1^i \cdot (c_1^i - w_1^i) + q^i \cdot y^i \leq 0$.

[6] All proofs are in the Appendix.
[7] It should be clear that $c_2^i(w_2^i + y^i(p,q,s)d, p_2^i(s)) = c_2^i(p,q,s)$

Optimality will be defined in terms of a social planning problem. We will constrain the social planner to choosing only allocations of goods and assets traded at time 1, time 2 allocations will continue to be decided as part of a Walrasian equilibrium. We will also constrain the social planner to choose time 1 allocations in each location without knowledge (or use of the knowledge) of the event which has occurred at the other location. In this way the social planner faces the same information constraints as any private agent.[8] The social planning problem is:

Given $(\ldots,(w_1^i,w_2^i),\ldots)$ and $(\ldots,\gamma^i,\ldots)\in\mathcal{R}_+^I\setminus\{0\}$, choose c_1^i, y^i for each $i\in I$ to
$\max \sum_{i\in I}\gamma^i(u_1^i(c_1^i)+\sum_{s\in S^i}\pi^i(s)v^i(w_2^i+y^i(s)d,p_2^i(s))$
subject to $\forall l,t,h_{lt}$:
$\sum_{i\in I(l,1,h_{l1})}(c_1^i-w_1^i)\leq 0$
$\sum_{i\in I(l,1,h_{l1})} y^i=0$
$\sum_{i\in I(l,2,h_{l2})}(c_2^i(w_2^i+y^i(h_{l2})d,p(l,2,h_{l2}))-w_2^i-y^i(h_{l2})d)=0$
$p(l,2,h_{l2})\in\mathcal{R}_{++}^{G+1},\ \sum_{g=0}^{G}p_g(l,2,h_{l2})=1.$

Any equilibrium allocation which is a solution to the above problem for some $(\ldots,\gamma^i,\ldots)\in\mathcal{R}_+^I\setminus\{0\}$ will be defined to be constrained Pareto optimal;[9] any equilibrium allocation which does not meet this criterion will be defined to be constrained Pareto suboptimal.

As noted in section 3, no agent faces uncertainty about either his preferences or endowment, therefore states of nature are distinguishable to an agent only if the prices facing the agent differ across states. This is a feature of models of sunspot equilibria as well (Cass and Shell [3]), however here the states of nature are intrinsic in the sense that they describe which agents are matched. Nonetheless, for the model to be of interest we must, as with sunspot models (see for instance, Suda et.al. [27]), find some way to guarantee that equilibria in which prices differ across states do in fact exist, or else agents in the economy face no uncertainty. This is provided in Proposition 2.

[8]One might also consider allowing a social planner to intervene at time 2, however such interventions may not be time consistent (see Kydland and Prescott [15]).

[9]This definition corresponds to the notion of *weak* optimality (an allocation is weakly optimal if there is no other attainable allocation which all agents strictly prefer). The proof of Proposition 4 establishes that an allocation is weakly optimal in this economy iff it is *strongly* optimal (an allocation is strongly optimal if there is no other attainable allocation which all agents weakly prefer and at least one agent strictly prefers).

Proposition 2: There is an open and dense subset $\mathcal{U}^*$ of $\mathcal{U}$ such that for any $u \in \mathcal{U}^*$, if (p,q) is an equilibrium price set then for every $i \in I$ and $s, s' \in S^i$, $\frac{q^i(s)}{\pi^i(s)} p_2^i(s) \neq \frac{q^i(s')}{\pi^i(s')} p_2^i(s')$ if $s \neq s'$.

In characterizing the solution to the social planners problem we will assume that second period prices are differentiable functions of agents' asset holdings, which is justified by Proposition 3. (The basic structure of the proof is similar to one in Geanakopolos and Polemarchakis [8]).

Proposition 3: There is an open and dense subset $\mathcal{U}^{**}$ of $\mathcal{U}^*$ such that at any equilibrium time 2 spot prices in each location and state are locally differentiable with respect to the bundles of goods brought by the agents participating in that market.

Since $\mathcal{U}^*$ is open and dense in $\mathcal{U}$, $\mathcal{U}^{**}$ is open and dense in $\mathcal{U}$. To ease notation, we write $v^i(s) \equiv \pi^i(s) v^i(w_2^i + y^i(s)d, p_2^i(s))$. An equilibrium allocation can be an optimum of the social planning problem only if it solves the first order (necessary) conditions:

$\forall\ l, h_{l1}, s \in S^{h_{l1}}$ and $j, k \in I(l, 1, h_{l1})$ ($j \in I(1,2,s), k \in I(2,2,s)$):

$$\gamma^j \partial u_1^j(c_1^j) - \gamma^k \partial u_1^k(c_1^k) = 0 \tag{1}$$

$$\gamma^j \partial_{y^j(s)} v^j(s) - \gamma^k \partial_{y^k(s)} v^k(s) = \partial_{p(2,2,s)}[\sum_{i \in I(2,2,s)} \gamma^i v^i(s)] \partial_{y^k(s)} p(2,2,s) - \partial_{p(1,2,s)}[\sum_{i \in I(1,2,s)} \gamma^i v^i(s)] \partial_{y^j(s)} p(1,2,s). \tag{2}$$

In addition, any equilibrium allocation will solve the individual's first order conditions:

$$\partial u_1^i(c_1^i) = \alpha^i p_1^i, \qquad \alpha^i > 0 \tag{3}$$

$$\partial_{y^i(s)} v^i(s) = \alpha^i q^i(s) \quad \forall s \in S^i. \tag{4}$$

Comparison of equations (3) and (4) to (1) and (2) reveals that an equilibrium allocation can be an optimum of the social planning solution only if the right hand side of equation (2) is zero and $\gamma^j \alpha^j = \gamma^k \alpha^k$ for all l, h_{l1} and $j, k \in I(l, 1, h_{l1})$. Assuming that the latter condition holds and using the fact that

$$\partial_{p_2^i(s)} v^i(s) = -\lambda^i(s)(c_2^i(s) - w_2^i - y^i(s)d)',$$

where

$$\lambda^i(s) \equiv \partial_{y^i(s)} v^i(s)$$

is i's second period marginal utility of income in state $s \in S^i$, this implies that an equilibrium allocation is constrained Pareto optimal only if $\forall\ l, h_{l1}, s \in S^{h_{l1}}$:

$$r(s)[(c_2^j(s) - w_2^j - y^j(s)d)'\partial_{y^j(s)}p(1,2,s) - (c_2^k(s) - w_2^k - y^k(s)d)'\partial_{y^k(s)}p(2,2,s)] = 0 \quad (5)$$

where $j, k \in I(l, 1, h_{l1})$ ($j \in I(1,2,s), k \in I(2,2,s)$) and

$$r(s) = (\gamma^j \alpha^j q^j(s) - \gamma^h \alpha^h q^h(s)); \quad j, h \in I(1,2,s), j \neq h.$$

The final proposition is:

Proposition 4: If $S_1 = 1$, $S_2 = 1$, or $G = 0$ then all equilibria are constrained Pareto optimal. If $S_1 \geq 2$, $S_2 \geq 2$, and $G > 0$ then there is an open and dense subset of $\mathcal{U}$ such that all equilibria are constrained Pareto suboptimal.

The following examples should help provide intuition.

Example 1: $S_1 = S_2 = 1$. In this situation agents face no uncertainty about who they will be matched with at time 2, so that there is spatial separation but no limited communication. It is simple to see that choosing $\gamma^a = \frac{1}{\alpha^a q_1(s)}$ and $\gamma^{b'} = \frac{1}{\alpha^{b'} q_2(s)}$ will make $r(s) = 0$, so that equation (5) is satisfied. The private equilibrium is therefore constrained Pareto optimal.

Example 2: $S_1 = 2, S_2 = 1$. In this example agents in location 2 are uncertain about what type of agent they will be matched with at time 2, but the agents in location 1 know that they will be matched with (a_1', b_1'), so uncertainty is one-sided. Suppose that in state 1 (a_1, b_1') and (a_1', b_1) are matched and in state 2 (a_2, b_1') and (a_1', b_2) are matched. One can determine that choosing $\gamma^{a_1'} = 1, \gamma^{a_1} = \frac{\alpha^{a_1'} q_2(1)}{\alpha^{a_1} q_1(1)}$, and $\gamma^{a_2} = \frac{\alpha^{a_1'} q_2(2)}{\alpha^{a_2} q_1(2)}$ will set $r(1) = r(2) = 0$, so that equation (5) is satisfied. The example is constrained Pareto optimal because while agents in location 1 can take actions which lessen the uncertainty of agents in location 2, agents in location 2 cannot return the favor since agents in location 1 face no uncertainty. Because the actions which (a, b) can take to insure (a', b') are costly, there is no Pareto improving intervention which satisfies the imposed constraints.

Example 3: $S_1 = S_2 = 2$. In this example agents in both locations face uncertainty about what type of agent they will be matched with at time 2. Suppose that in state 1 (a_1, b_1') and (a_1', b_1) are matched, in state 2 (a_1, b_2') and (a_2', b_1) are matched, in state 3 (a_2, b_1') and (a_1', b_2)

are matched, and in state 4 (a_2, b_2') and (a_2', b_2) are matched. Examination of equation (5) reveals that if

$$\frac{q_1(1)}{q_1(2)} = \frac{q_2(1)}{q_2(3)}\frac{q_1(3)}{q_1(4)}\frac{q_2(4)}{q_2(2)},$$

then the equilibrium is constrained Pareto optimal, if not then there is at least one state s such that $r(s) \neq 0$. The condition has a simple logic — if marginal rates of substitution are equal then the equilibrium is optimal — but in this example no two people matched at time 2 desire income in the same two states: a_1 will trade income in state 1 for income in state 2, b_2' will trade income in state 2 for income in state 4, a_2 will trade income in state 4 for income in state 3, and b_1' will trade income in state 3 for income in state 1. If the above condition holds then there is no trade of state-dependent incomes that a_1, b_1', a_2, b_2' would care to make if they were given the option (which is impossible by themselves since only two of them ever actually meet). If it does not hold then the allocation of incomes across states is inefficient, and the equilibrium will be Pareto constrained suboptimal if reallocations of assets can affect prices, and via prices affect agents uncertainty about future income.

From the logic of this analysis, one can see that constrained Pareto optimality is satisfied only if one or more of a fairly restrictive set of conditions hold: (1) Agent's marginal utilities of income across states are such that there is no room for any further trade. This would occur if all agents were on one island, or under certainty, or if at least one agent on each island were risk neutral, but otherwise there is no economic force which would cause it to occur. (2) No action that agents take in the first period affects second period prices. This would occur if there were only spot markets in the first period, i.e. no trades of contingent claims was allowed.[10] It would also occur if there were only a single good (since the relative price must be one). (3) There is no trade in the second period (which is also satisfied if there is only a single good). These conditions seem unlikely to hold in most models of interest.

One might conclude that this externality occurs because agents ignore the impact their actions have on prices, and, through prices, incomes. This in fact is not the case. What prevents optimality is not so much that agents ignore their effect on prices, but that agents will quite reasonably (under any circumstances) ignore the effect their actions have on other's uncertainty unless they are properly compensated. The problem is that individuals cannot create a system of compensation; if an agent changes his time 1 actions to insure those he will meet at a future time period, those who benefit have no incentive to compensate him

[10]There are of course other channels through which first period decisions could affect second period markets, some type of storage technology or preferences which were not time separable are obvious candidates. These were not included in the analysis in order to ease an already cumbersome amount of notation. Extension of the analysis to these cases is straightforward and similar results on constrained Pareto suboptimality could be attained.

once they meet since the action has already been taken, and no one will pay for insurance after the fact. Agents would be willing to pay for insurance before the fact (at time 1), but the entire desire for insurance arises from the fact that those who are uncertain about some agent's actions are those who do not communicate with him at time 1. If they *did* communicate with him there would be no uncertainty to insure against.

These problems *would* be solved if there were a time 0 at which all agents trade in one location before any privately observed event has occurred. In that environment agents could set up a proper means of compensation by agreeing at time 0 to pay at time 2 for actions (taken at time 1) which reduce income uncertainty. A variant of such a model has been analyzed by Radner [23] , and it is straightforward to show that equilibria must be Pareto optimal. As a practical matter, however, such a model would appear to be at odds with the empirical observations which have led macroeconomists to study models of limited communication in the first place. For instance, job seekers do not generally have contracts from firms promising to hire them if specific states of nature are revealed to have occurred.

5 Conclusion

Models which seek to explain unemployment or valued fiat money typically must have some sort of limited communication. Although previous models which have incorporated limited communication have made sufficient assumptions to guarantee constrained Pareto optimal equilibria, the results of this paper imply that this feature rests upon quite special assumptions and need not be the norm.

There is obviously a question as to the amount of information a social planner would need in order to institute a policy which could correct for the externality. Nonetheless, the results are at least suggestive of some type of countercyclical fiscal policy, albeit at a highly disaggregated level. The possible role of externalities of this type in increasing economic fluctuation in a way that is not Pareto optimal, and in generating Pareto ranked equilibria, may also be of interest. However, whether or not one believes that the government could plausibly correct the externality, the analysis of this paper indicates that it is likely to affect welfare analysis in many situations of interest in economics.

Although the analysis was carried out within a two-period, two location structure, with an exogenous pattern of communication, generalizing the results to a larger number of time periods or locations should be straightforward once a pattern of communication is specified. When the choice of communication or matching is endogenous there is a question as to whether agents will make correct choices (for instance, Diamond [5]), however even when agents make optimal matching decisions the externality studied in this paper should remain.

A Appendix

Proof of Proposition 1:

Under A1-A4, $c^i(p,q)$ is a $\mathcal{C}^1$ function on the set $\{(p,q) \in \bar{\Delta} \mid (p^i, q^i) \gg 0\}$, and therefore $y^i(p,q) = (\ldots, p_2^i(s)\cdot(c_2^i(s) - w_2^i), \ldots)$ is a $\mathcal{C}^1$ function on this set. $c^i(p,q)$ is also bounded from below, and since $p_2^i(s)$ is bounded from above, $y^i(p,q)$ is bounded from below. If $(p_n, q_n) \in \Delta$ tends to (p,q) in the boundary of $\bar{\Delta}$ then $\|c^i(p_n,q_n)\ \|$ tends to ∞ for some i. All of this implies that $\varphi(p,q)$ is continuous on Δ, is bounded from below, and as $(p_n, q_n) \in \Delta$ tends to (p,q) in the boundary of $\bar{\Delta}$, $\|\ \varphi(p_n,q_n)\ \|$ tends to ∞. Monotonicity, agent's budget constraints, and the definition of $y^i(p,q)$ also guarantee that $(p,q)\cdot\varphi = 0$ for all $(p,q) \in \Delta$. Define

$$\Lambda(z) = \begin{cases} \left\{h \in \{1,\ldots,M(G+1)+2S\} \mid \varphi_h(z) = \max\{\varphi_g(z)\}_{g=1,\ldots,M(G+1)+2S}\right\} & \text{if } z \in \Delta \\ \{h \in \{1,\ldots,M(G+1)+2S\} \mid z_h = 0\} & \text{if } z \in \partial\bar{\Delta}, \end{cases}$$

and

$$\mu(z) = \{x \in \bar{\Delta} \mid x_h = 0 \text{ if } h \notin \Lambda(z)\},\ z \in \bar{\Delta},$$

where φ_g represents the g'th element of φ. One can adapt the argument of Hildenbrand and Kirman ([9], pg. 111) to show that there is a fixed point of μ. This implies the existence of an equilibrium price vector $(p,q) \in \Delta\Box$

For any vector $z \in \mathcal{R}^{G+1}$ we define the vector $\tilde{z} \in \mathcal{R}^G$ by dropping the first element of z. Using the version of Walras Law appropriate for this economy and the definition of y^i, we can drop the excess demand equation for good 0 in each spot market from our definition of an equilibrium. Accordingly we define the function:

$$\begin{aligned}\tilde{\varphi}(p,q) \ = \ & \Bigg(\Bigg(\sum_{i\in I(l,1,h_{l1})} (\tilde{c}_1^i(p,q) - \tilde{w}_1^i)\Bigg)_{\forall l,h_{l1}}, \\ & \Bigg(\sum_{i\in I(l,2,h_{l2})} (\tilde{c}_2^i(p,q,h_{l2}) - \tilde{w}_2^i - y^i(p,q,h_{l2})\tilde{d})\Bigg)_{\forall l,h_{l2}}, \Bigg(\sum_{i\in I(l,1,h_{l1})} y^i(p,q)\Bigg)_{\forall l,h_{l1}}\Bigg).\end{aligned}$$

$\varphi(p,q) = 0$ iff $\tilde{\varphi}(p,q) = 0$.

It will be convenient to define

$$\varphi_2^i(s) = (c_2^i(p,q,s) - w_2^i - y^i(p,q,s)d)$$

as i's excess demand at time 2 in state s. Also define

$$\varphi(l,2,s) = \sum_{i\in I(l,2,s)} \varphi_2^i(s).$$

It is straightforward to prove that at any equilibrium under assumptions A1-A4,

$$c_2^i(p,q,s) = c_2^i(p,q,s') \text{ iff } \frac{q^i(s)}{\pi^i(s)}p_2^i(s) = \frac{q^i(s')}{\pi^i(s')}p_2^i(s') \; \forall i \text{ and } s,s' \in S^i.$$

With this in hand we employ Lemmas 1 and 2 to form a recursive proof of Proposition 2.

Lemma 1: There is an open and dense subset $\mathcal{U}_1$ of $\mathcal{U}$ such that for all $(p,q,u) \in \Delta \times \mathcal{U}_1$, $\tilde{\varphi}(p,q,u) = 0$ implies $\frac{q^i(s)}{\pi^i(s)}p_2^i(s) \neq \frac{q^i(s')}{\pi^i(s')}p_2^i(s')$ for some $i \in I$ and $s,s' \in S^i$.

Proof of Lemma 1: The condition $\frac{q^i(s)}{\pi^i(s)}p_2^i(s) = \frac{q^i(s')}{\pi^i(s')}p_2^i(s')$ $\forall i$ and $s,s' \in S^i$ can be rewritten as

$$\begin{aligned} \tilde{p}(l,2,s) &= \tilde{p}_{l2} && \forall l, s \in S, \\ \pi^{h_{l1}}(s)q(l,s) &= \pi^{h_{l1}}(s')q(l,s') && \forall l, h_{l1} \text{ and } s,s' \in S^{h_{l1}}. \end{aligned}$$

Let R_1 be a $(2S(G+1) - S_1 - S_2 - G) \times (M(G+1) + 2S)$ matrix comprised of these linear restrictions on $(p,q) \in \Delta$ such that $R_1(p,q) = 0$ iff they hold. By construction R_1 has full row rank.

Fix some state $\hat{s} = (\hat{s}_1, \hat{s}_2) \in S$ and define $\hat{S} \subset S$ by $\hat{S} = S^{\hat{s}_1} \times S^{\hat{s}_2}$, $\omega^{s_l} \in \hat{S}$ by $\omega^{s_l} = (s_1, \hat{s}_2)$ if $l = 1$ and $\omega^{s_l} = (\hat{s}_1, s_2)$ if $l = 2$. We first demonstrate that for all $(p,q,u) \in \Delta \times \mathcal{U}$, $(\tilde{\varphi}(p,q,u), R_1(p,q)) = 0$ iff $(\hat{\varphi}(p,q,u), R_1(p,q)) = 0$, where

$$\begin{aligned} \hat{\varphi}(p,q,u) = \Bigg(&\left(\sum_{i \in I(l,1,h_{l1})} (\tilde{c}_1^i(p,q) - \tilde{w}_1^i) \right)_{\forall l, h_{l1}}, \\ &\left(\sum_{i \in I(l,2,\hat{s})} (\tilde{c}_2^i(p,q,\hat{s}) - \tilde{w}_2^i - y^i(p,q,\hat{s})\tilde{d}) \right)_{\forall l, \hat{s} \in \hat{S}}, \left(\sum_{i \in I(l,1,h_{l1})} y^i(p,q,\omega^{h_{l1}}) \right)_{\forall l, h_{l1}} \Bigg). \end{aligned}$$

Since the equations comprising $\hat{\varphi}(p,q,u) = 0$ are a subset of those comprising $\varphi(p,q,u) = 0$, the necessity of $(\hat{\varphi}(p,q,u), R_1(p,q)) = 0$ for $(\tilde{\varphi}(p,q,u), R_1(p,q)) = 0$ is obvious. Sufficiency is shown by noting

(1) For any $s \in S^{h_{l1}}$, if $R_1(p,q) = 0$ then

$$\sum_{i \in I(l,1,h_{l1})} y^i(p,q,h_{l1}) = \sum_{i \in I(l,1,h_{l1})} y^i(p,q,\omega^{h_{l1}}).$$

(2) Let $\hat{j}$ be matched with $\hat{k}$ at location l in state $\hat{s}$. For any $s = (s_1, s_2) \in S$, let j be matched with k at location l in state s, $\hat{j}$ matched with k in state $\omega^{s_1} = (s_1, \hat{s}_2)$ and j

matched with $\hat{k}$ in state $\omega^{s_2} = (\hat{s}_1, s_2)$. If $R_1(p,q) = 0$ then

$$\begin{aligned}
\varphi(l,2,s) &= \varphi_2^j(s) + \varphi_2^k(s) \\
&= \varphi_2^j(\omega^{s_1}) + \varphi_2^k(\omega^{s_2}) \\
&= (\varphi(l,2,\omega^{s_1}) - \varphi_2^{\hat{k}}(\omega^{s_1})) + (\varphi(l,2,\omega^{s_2}) - \varphi_2^{\hat{j}}(\omega^{s_2})) \\
&= (\varphi(l,2,\omega^{s_1}) - \varphi_2^{\hat{k}}(\hat{s})) + (\varphi(l,2,\omega^{s_2}) - \varphi_2^{\hat{j}}(\hat{s})) \\
&= (\varphi(l,2,\omega^{s_1}) + \varphi(l,2,\omega^{s_2}) - (\varphi(l,2,\hat{s})).
\end{aligned}$$

Let $\psi(p,q,u) = (\hat{\varphi}(p,q,u), R_1(p,q))$, we prove that $\partial\psi(p,q,u)$ has full row rank when evaluated at any $(p,q,u) \in \Delta \times \mathcal{U}$ satisfying $\psi(p,q,u) = 0$. (The perturbation employed will closely follow a similar argument in Kajii [12].)

For each $i \in I$, let B_1^i and X_1^i be open sets of $\mathcal{R}^{(G+1)}$ such that $c_1^i(p,q,u) \in B_1^i \subset \bar{B}_1^i \subset X_1^i \subset X$ and ρ_1^i be a $\mathcal{C}^\infty$ function from $\mathcal{R}^{(G+1)}$ to $\mathcal{R}$ such that $\rho_1^i(c) = 0$ on B_1^i and $\rho_1^i(c) = 1$ on the complement of X_1^i. In the same way, let B_2^i and X_2^i be open sets of $\mathcal{R}^{(G+1)}$ such that $c_2^i(p,q,u,s) \in B_2^i \subset \bar{B}_2^i \subset X_2^i \subset X$ $(s \in S^i)$ and ρ_2^i be a $\mathcal{C}^\infty$ function from $\mathcal{R}^{(G+1)}$ to $\mathcal{R}$ such that $\rho_2^i(c) = 0$ on B_2^i and $\rho_2^i(c) = 1$ on the complement of X_2^i. For $\epsilon^i = (\epsilon_1^i, \epsilon_2^i) \in \mathcal{R}^{2(G+1)}$ define a function $\varsigma^i(\epsilon^i)$ from $\mathcal{R}_+^{(G+1)(S^i+1)}$ to $\mathcal{R}$ by

$$\begin{aligned}
\varsigma^i(\epsilon^i)(c^i) = [\rho_1^i(c_1^i)u_1^i(c_1^i) + (1-\rho_1^i(c_1^i))u_1^i(c_1^i - \epsilon_1^i)] + \\
\sum_{s\in S^i} \pi^i(s)\,[\rho_2^i(c_2^i(s))u^i(c_2^i(s)) + (1-\rho_2^i(c_2^i(s)))u^i(c_2^i(s) - \epsilon_2^i)]\,.
\end{aligned}$$

We can find an open neighborhood N^i of $0 \in \mathcal{R}^{2(G+1)}$ such that $\varsigma^i(\epsilon^i) \in \mathcal{U}^i$ for all $\epsilon^i \in N^i$. $\varsigma = (\ldots, \varsigma^i, \ldots)$ is a smooth parameterization of a $2(G+1)I$ submanifold of $\mathcal{U}$ about $(\ldots, c^i(p,q,u), \ldots)$, and one can regard $\psi(p,q,\cdot)$ as a function from $\Delta \times N$, $N = \times_{i\in I} N^i$, writing $\psi(p,q,\epsilon)$. If $\partial_{(p,q,\epsilon)}\psi$ has full row rank, then so does $\partial_{(p,q,u)}\psi$.

An individual's demands for goods and assets must solve the first order (necessary and sufficient) conditions

$$\begin{aligned}
\partial u_1^i(c_1^i) &= \alpha^i p_1^i, \qquad \alpha^i > 0 \\
\pi^i(s)\partial u_2^i(c_2^i(s)) &= \alpha^i q^i(s) p_2^i(s) \quad \forall s \in S^i \\
p_1^i \cdot (c_1^i - w_1^i) &\le \sum_{s\in S^i} q^i(s) p_2^i(s) \cdot (c_2^i(s) - w_2^i) \\
y^i(s) &= p_2^i(s) \cdot (c_2^i(s) - w_2^i).
\end{aligned}$$

$\partial_{c^i}\varsigma^i(\epsilon^i) = \partial_{c_1^i} u_1^i(c_1^i - \epsilon_1^i) + \sum_{s\in S^i} \pi^i(s)\partial_{c_2^i(s)} u^i(c_2^i(s) - \epsilon_2^i)$ for c^i sufficiently close to $c^i(p,q)$. Let $\epsilon^i \in N^i$ satisfy $p_1^i \cdot \epsilon_1^i + \sum_{s\in S^i} q^i(s) p_2^i(s) \cdot \epsilon_2^i = 0$, then $(\hat{c}^i, \hat{y}^i)$ defined by $\hat{c}_1^i = c_1^i(p,q,u) + \epsilon_1^i$, $\hat{c}_2^i(s) = c_2^i(p,q,u,s) + \epsilon_2^i$ and $\hat{y}^i(s) = y^i(p,q,u,s) + p_2^i(s) \cdot \epsilon_2^i$, $s \in S^i$, satisfies i's maximization problem when i's utility function is $\varsigma^i(\epsilon^i)$. This implies that by perturbing ϵ^i one can arbitrarily perturb $(\tilde{c}_1^i(p,q,\epsilon^i) - \tilde{w}_1^i, y^i(p,q,\epsilon^i,\omega^{h_{l1}}), \tilde{c}_2^i(p,q,\epsilon^i,\omega^{h_{l1}}) - \tilde{w}_2^i - y^i(p,q,\epsilon^i,\omega^{h_{l1}})\tilde{d})$, $i \in I(l,1,h_{l1})$. By perturbing $(\tilde{c}_1^i(p,q,\epsilon^i) - \tilde{w}_1^i, y^i(p,q,\epsilon^i,\omega^{h_{l1}}), \tilde{c}_2^i(p,q,\epsilon^i,\omega^{h_{l1}}) - \tilde{w}_2^i - y^i(p,q,\epsilon^i,\omega^{h_{l1}})\tilde{d})$ for each $i \in \{a_1, a_2, \ldots, a_{S_1}, a_1', a_2', \ldots, a_{S_2}'\}$ in this manner we can arbitrarily perturb $\hat{\varphi}(p,q,u)$. (Note, however, that we cannot arbitrarily perturb $\tilde{\varphi}(p,q,u)$ in this manner.) $\partial\psi$ has the form

$$\partial\psi = \begin{bmatrix} \partial_{(p,q)}\hat{\varphi} & \partial_\epsilon\hat{\varphi} \\ R_1 & 0 \end{bmatrix},$$

by our argument $\partial_\epsilon\hat{\varphi}$ has full row rank, and by construction R_1 has full row rank, therefore 0 is a regular value of ψ. By the transversality theorem (Abraham and Robbin [1]), there exists a dense subset $\mathcal{U}_1$ of $\mathcal{U}$ such that for all $u \in \mathcal{U}_1$, 0 is a regular value of $\psi_u(p,q) = \psi(p,q,u)$. But since the domain of ψ_u has lower dimension than its range, this implies that there is no solution to $\psi_u(p,q) = 0$.

Let χ be the projection from $\psi^{-1}(\{0\})$ into $\mathcal{U}$. Let H be a closed subset of $\psi^{-1}(\{0\})$. For any sequence $\{u_n\}$ satisfying $u_n \in \chi(H)$ and $u_n \to u$, there is (by definition of χ) a sequence $\{(p_n, q_n, u_n)\}$ satisfying $(p_n, q_n, u_n) \in H$. Because $\bar{\Delta}$ is compact there is a convergent subsequence $\{(p_{n'}, q_{n'})\}$ of $\{(p_n, q_n)\}$, $(p_{n'}, q_{n'}) \to (p,q)$. Since ψ is continuous, $\psi(p,q,u) = 0$. Since if $(p_{n'}, q_{n'}, u_{n'}) \to (p,q,u)$ and (p,q) lies on the boundary of $\bar{\Delta}$ then $\|\varphi(p,q,u)\| \to \infty$, we must have $(p,q) \in \Delta$. Hence $(p,q,u) \in \psi^{-1}(\{0\})$, and since H is closed, $(p,q,u) \in H$. Therefore $u \in \chi(\psi^{-1}(\{0\}))$, which establishes that for any closed set $H \subset \psi^{-1}(\{0\})$, $\chi(H)$ is closed. Since ψ is continuous, $\psi^{-1}(\{0\})$ is closed. We can define the set $\mathcal{U}_1$ by

$$\mathcal{U}_1 = \mathcal{U} \setminus \chi(\psi^{-1}(\{0\})).$$

Which establishes that $\mathcal{U}_1$ is open.□

We can generalize the restriction $\frac{q^i(s)}{\pi^i(s)}p_2^i(s) = \frac{q^i(s')}{\pi^i(s')}p_2^i(s')$ $\forall$ $i \in I$ and $s, s' \in S^i$ by constructing partitions $(\Xi_1^i, \ldots, \Xi_{M^i}^i)$ of S^i for each $i \in I$ and considering restrictions of the form

$$\frac{q^i(s)}{\pi^i(s)}p_2^i(s) = \frac{q^i(s')}{\pi^i(s')}p_2^i(s') \ \forall i \in I \text{ and } s, s' \in \Xi_m^i \text{ for some } m \in \{1, .., M^i\}.$$

Construct a sequence $\{\Xi_k\}$ $(k = 1..K)$ by forming all possible collections (collecting over i) of partitions of S^i, $i \in I$. Since the number of states of nature is finite, this sequence is finite. Each element of this sequence, $\Xi_k = (\ldots, (\Xi_{k1}^i, \ldots, \Xi_{kM_k^i}^i), \ldots)_{i \in I}$, will be taken to represent a set of r_k linear restrictions on $(p,q) \in \Delta$ such that

$$\frac{q^i(s)}{\pi^i(s)}p_2^i(s) = \frac{q^i(s')}{\pi^i(s')}p_2^i(s') \ \forall i \in I \text{ and } s, s' \in \Xi_{km}^i \text{ for some } m \in \{1, .., M_k^i\}.$$

As in Lemma 1, we can construct a full row rank $r_k \times (M(G+1)+2S)$ matrix R_k such that $R_k(p,q)=0$ iff $(p,q)\in\Delta$ satisfies the set of restrictions represented by Ξ_k. Order the sequence $\{\Xi_k\}$ so that if Ξ_k and $\Xi_{k'}$ are elements and $R_{k'}(p,q)=0$ implies $R_k(p,q)=0$, then $k'<k$.

Lemma 2: There is an open and dense subset $\mathcal{U}_k$ of $\mathcal{U}_{k-1}$ such that for all $(p,q,u)\in\Delta\times\mathcal{U}_k$, $\varphi(p,q,u)=0$ implies $R_j(p,q)\neq 0$, $j=1..k$.

Proof of Lemma 2:

Let $z^i(p,q) = (\tilde{c}_1^i(p,q)-\tilde{w}_1^i,\ldots,c_2^i(p,q,s)-w_2^i-y^i(p,q,s)d,\ldots)\in\mathcal{R}^{(G+1)S^i+G}$ and $z(p,q,u)=(\ldots,z^i(\cdot),\ldots)$, we can construct a matrix F such that $\tilde{\varphi}(p,q,u)=0$ iff $Fz(\cdot)=0$. Since any equilibrium allocation will satisfy $c_2^i(p,q,s)=c_2^i(p,q,s')$ iff $\frac{q^i(s)}{\pi^i(s)}p_2^i(s)=\frac{q^i(s')}{\pi^i(s')}p_2^i(s')$ for all i and $s,s'\in S^i$, the set of restrictions $R_k(p,q)=0$ places a corresponding set of restrictions $R_k^z z(\cdot)=0$ on $z(\cdot)$. For each $i\in I$ and $m\in\{1,..,M_k^i\}$, choose an element $\omega_m^i\in\Xi_{km}^i$ and define the subvector $\hat{z}^i(p,q)$ of $z^i(p,q)$ by

$$\hat{z}^i(p,q)=(\tilde{c}_1^i(p,q)-\tilde{w}_1^i,\ldots,c_2^i(p,q,\omega_m^i)-w_2^i-y^i(p,q,\omega_m^i)d,\ldots)\in\mathcal{R}^{(G+1)M_k^i+G}.$$

Let $\hat{z}(p,q,u)=(\ldots,\hat{z}^i(\cdot),\ldots)$. We can construct a selector matrix E such that $R_k^z z(\cdot)=0$ iff $z(\cdot)=E\hat{z}(\cdot)$. If, as in Lemma 1, some equations in $\varphi(p,q,u)=0$ are redundant given $R_k(p,q)=0$, then FE will not be full row rank and we can construct a matrix $\hat{F}$ by dropping the redundant rows of FE, so that $\hat{F}$ is full row rank and $\hat{F}\hat{z}(\cdot)=0$ iff $FE\hat{z}(\cdot)=0$. By construction $(\tilde{\varphi}(p,q,u),R_k(p,q))=0$ iff $(\hat{F}\hat{z}(p,q,u),R_k(p,q))=0$.

Let $\psi(p,q,u)=(\hat{F}\hat{z}(p,q,u),R_k(p,q))$, we prove that $\partial\psi(p,q,u)$ has full row rank when evaluated at any $(p,q,u)\in\Delta\times\mathcal{U}_{k-1}$ satisfying $\psi(p,q,u)=0$. (As in Lemma 1, the perturbation employed will closely follow a similar argument in Kajii [12].)

For each $i\in I$, let B_1^i and X_1^i be open sets of $\mathcal{R}^{(G+1)}$ such that $c_1^i(p,q,u)\subset B_1^i\subset\bar{B}_1^i\subset X_1^i\subset X$ and ρ_1^i be a $\mathcal{C}^\infty$ function from $\mathcal{R}^{(G+1)}$ to $\mathcal{R}$ such that $\rho_1^i(c)=0$ on B_1^i and $\rho_1^i(c)=1$ on the complement of X_1^i. By assumption if $(p,q,u)\in\Delta\times\mathcal{U}_{k-1}$, then $\varphi(p,q,u)=0$ implies $R_j(p,q)\neq 0$, $j=1..k-1$. The ordering of the sequence $\{\Xi_k\}$ implies that if $R_j(p,q)\neq 0$, $j=1..k-1$ then for all $i\in I$ and $s\in\Xi_{km}^i$, $s'\in\Xi_{km'}^i$ ($m,m'\in\{1,..,M_k^i\}$), $\frac{q^i(s)}{\pi^i(s)}p_2^i(s)=\frac{q^i(s')}{\pi^i(s')}p_2^i(s')$ only if $m=m'$. In particular, if $(p,q,u)\in\Delta\times\mathcal{U}_{k-1}$ is such that $\varphi(p,q,u)=0$, then $c_2^i(p,q,u,\omega_m^i)\neq c_2^i(p,q,u,\omega_{m'}^i)$ if $m\neq m'$. We can therefore also construct open sets, B_{2m}^i and X_{2m}^i, of $\mathcal{R}^{(G+1)}$ for each $i\in I$ and $m\in\{1,..,M_k^i\}$ such that $c_2^i(p,q,u,\omega_m^i)\subset B_{2m}^i\subset\bar{B}_{2m}^i\subset X_{2m}^i\subset X$ and $X_{2m}^i\cap X_{2m'}^i=\emptyset$ if $m\neq m'$. Let ρ_{2m}^i be a $\mathcal{C}^\infty$ function from $\mathcal{R}^{(G+1)}$ to $\mathcal{R}$ such that $\rho_{2m}^i(c)=0$ on B_{2m}^i and $\rho_{2m}^i(c)=1$ on the complement of X_{2m}^i. For $\epsilon^i=(\epsilon_1^i,\ldots,\epsilon_{2m}^i,\ldots)\in\mathcal{R}^{(M_k^i+1)(G+1)}$ define a function $\varsigma^i(\epsilon^i)$ from

$\mathcal{R}_+^{(G+1)(S^i+1)}$ to $\mathcal{R}$ by

$$\varsigma^i(\epsilon^i)(c^i) = [\rho_1^i(c_1^i)u_1^i(c_1^i) + (1-\rho_1^i(c_1^i))u_1^i(c_1^i - \epsilon_1^i)] +$$
$$\sum_{s\in S^i}\pi^i(s)\left[(\rho_{21}^i(c_2^i(s))\cdots\rho_{2M_k^i}^i(c_2^i(s)))u^i(c_2^i(s)) + \sum_{m=1}^{M_k^i}(1-\rho_{2m}^i(c_2^i(s)))u^i(c_2^i(s)-\epsilon_{2m}^i)\right].$$

For each i we can find an open neighborhood N^i of $0 \in \mathcal{R}^{(M_k^i+1)(G+1)}$ such that $(\ldots,\varsigma^i(\epsilon^i),\ldots) \in \mathcal{U}_{k-1}$ for all $(\ldots,\epsilon^i,\ldots) \in N = \times_{i\in I}N^i$. $\varsigma = (\ldots,\varsigma^i,\ldots)$ is a smooth parameterization of a $\sum_{i\in I}(M_k^i+1)(G+1)$ submanifold of $\mathcal{U}$ about $(\ldots,c^i(p,q,u),\ldots)$, and one can regard $\psi(p,q,\cdot)$ as a function from $\Delta\times N$, writing $\psi(p,q,\epsilon)$. If $\partial_{(p,q,\epsilon)}\psi$ has full row rank, then so does $\partial_{(p,q,u)}\psi$.

Now $\partial_{c^i}\varsigma^i(\epsilon^i) = \partial_{c_1^i}u_1^i(c_1^i-\epsilon_1^i) + \sum_{m=1}^{M_k^i}\sum_{s\in\Xi_{km}^i}\pi^i(s)\partial_{c_2^i(s)}u^i(c_2^i(s)-\epsilon_{2m}^i)$ for c^i sufficiently close to $c^i(p,q)$. Let $\epsilon^i \in N^i$ satisfy $p_1^i\cdot\epsilon_1^i + \sum_{m=1}^{M_k^i}\sum_{s\in\Xi_{km}^i}q^i(s)p_2^i(s)\cdot\epsilon_{2m}^i = 0$, then $(\hat{c}^i,\hat{y}^i)$ defined by $\hat{c}_1^i = c_1^i(p,q,u)+\epsilon_1^i$, $\hat{c}_2^i(s) = c_2^i(p,q,u,s)+\epsilon_{2m}^i$ and $\hat{y}^i(p,q,u,s) = y^i(s)+p_2^i(s)\cdot\epsilon_{2m}^i$, $s\in\Xi_{km}^i$, satisfies i's maximization problem when i's utility function is $\varsigma^i(\epsilon^i)$. This implies that by perturbing ϵ^i one can arbitrarily perturb $\hat{z}(p,q,\epsilon^i)$. By perturbing $\hat{z}(p,q,\epsilon^i)$ for each $i\in I$ in this manner we can arbitrarily perturb $\hat{z}(p,q,\epsilon)$. (Note, however, that we cannot arbitrarily perturb $z(p,q,u)$ in this manner.) $\partial\psi$ has the form

$$\partial\psi = \begin{bmatrix} \partial_{(p,q)}\hat{F}\hat{z} & \hat{F}\partial_\epsilon\hat{z} \\ R_k & 0 \end{bmatrix},$$

by our argument $\partial_\epsilon\hat{z}$ has full row rank, and by construction $\hat{F}$ and R_k have full row rank, therefore 0 is a regular value of ψ. By the transversality theorem (Abraham and Robbin [1]), there exists a dense subset $\mathcal{U}_k$ of $\mathcal{U}_{k-1}$ such that for all $u\in\mathcal{U}_k$, 0 is a regular value of $\psi_u(p,q) = \psi(p,q,u)$. But since the domain of ψ_u has lower dimension than its range, this implies that there is no solution to $\psi_u(p,q) = 0$. Since $\mathcal{U}_k \subset \mathcal{U}_{k-1}$, if $\varphi(p,q,u) = 0$ then $R_j(p,q) \neq 0$, $j = 1..k-1$ for any $(p,q,u)\in\Delta\times\mathcal{U}_k$, and we have just proved that if $\varphi(p,q,u) = 0$ then $R_k(p,q)\neq 0$ for any $(p,q,u)\in\Delta\times\mathcal{U}_k$. A argument similar to the one made in the proof of Lemma 1 shows that $\mathcal{U}_k$ is open in $\mathcal{U}_{k-1}$□

Proof of Proposition 2: Lemmas 1 and 2 are sufficient for a recursive proof of Proposition 2. Note that since $\mathcal{U}_K$ is open and dense in $\mathcal{U}_{K-1}$, which is open and dense in $\mathcal{U}_{K-2}$, which is...open and dense in $\mathcal{U}$, $\mathcal{U}^* = \mathcal{U}_K$ is open and dense in $\mathcal{U}$□

Proof of Proposition 3: Given $i,h\in I(l,2,s)$ and $y_i(s)$, $y_h(s)$, $p(l,2,s)$ must be such that

$$\tilde{\varphi}(l,2,s) \equiv (\tilde{c}_2^i(w_2^i + y^i(s)d, p(l,2,s)) - \tilde{w}_2^i - y^i(s)\tilde{d}) +$$
$$(\tilde{c}_2^h(w_2^h + y^h(s)d, p(l,2,s)) - \tilde{w}_2^h - y^h(s)\tilde{d}) = 0$$

By the implicit function theorem $p(l,2,s)$ can be written (locally) as a differentiable function of $y_i(s)$ and $y_h(s)$ if $\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s)$ is invertible. Defining the function

$$\rho = \left(\prod_{l\in\{1,2\}}\prod_{s\in S}\det(\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))\right)^{\frac{1}{2S}},$$

second period prices will be locally differentiable functions of the assets holdings of market participants for each state and location if and only if $\rho \neq 0$. Define ψ: $\Delta \times \mathcal{U} \to \mathcal{R}^{MG+2S+1}$ by $\psi = (\tilde{\varphi}, \rho)$.

For each $i \in I$, let B_1^i and X_1^i be open sets of $\mathcal{R}^{(G+1)}$ such that $c_1^i(p,q) \subset B_1^i \subset \bar{B}_1^i \subset X_1^i \subset X$ and ρ_1^i be a $\mathcal{C}^\infty$ function from $\mathcal{R}^{(G+1)}$ to $\mathcal{R}$ such that $\rho_1^i(c) = 0$ on B_1^i and $\rho_1^i(c) = 1$ on the complement of X_1^i. By Proposition 2, if $(p,q,u) \in \Delta \times \mathcal{U}^*$ then $\varphi(p,q,u) = 0$ implies that for all i and $s,s' \in S^i$, $\frac{q^i(s)}{\pi^i(s)}p_2^i(s) = \frac{q^i(s')}{\pi^i(s')}p_2^i(s')$ only if $s = s'$, which in turn implies $c_2^i(p,q,s) = c_2^i(p,q,s')$ only if $s = s'$. We can therefore also construct open sets, B_{2s}^i and X_{2s}^i, of $\mathcal{R}^{(G+1)}$ for each $i \in I$ and $s \in S^i$ such that $c_2^i(p,q,s) \subset B_{2s}^i \subset \bar{B}_{2s}^i \subset X_{2s}^i \subset X$ and $X_{2s}^i \cap X_{2s'}^i = \emptyset$ if $s \neq s'$. Let ρ_{2s}^i be a $\mathcal{C}^\infty$ function from $\mathcal{R}^{(G+1)}$ to $\mathcal{R}$ such that $\rho_{2s}^i(c) = 0$ on B_{2s}^i and $\rho_{2s}^i(c) = 1$ on the complement of X_{2s}^i.

Let $\mathcal{L}$ be the set of symmetric $(G+1)$ dimension matrices ($\mathcal{L}$ is a manifold). For $\epsilon^i = (\epsilon_1^i, \ldots, \epsilon_{2s}^i, \ldots) \in \mathcal{R}^{(G+1)(S^i+1)}$ and $A^i = (\ldots, A_s^i, \ldots) \in \mathcal{L}^{S^i}$, define a function $\varsigma^i(\epsilon^i, A^i)$ from $\mathcal{R}_+^{(G+1)(S^i+1)}$ to $\mathcal{R}$ by

$$\begin{gathered}\varsigma^i(\epsilon^i, A^i)(c^i) = [\rho_1^i(c_1^i)u_1^i(c_1^i) + (1-\rho_1^i(c_1^i))u_1^i(c_1^i - \epsilon_1^i)] + \\ \sum_{s\in S^i}\pi^i(s)[(\rho_{21}^i(c_2^i(s))\cdots\rho_{2S^i}^i(c_2^i(s)))u^i(c_2^i(s)) + \sum_{s\in S^i}(1-\rho_{2s}^i(c_2^i(s)))u^i(c_2^i(s)-\epsilon_{2s}^i) + \\ \sum_{s\in S^i}(1-\rho_{2s}^i(c_2^i(s)))((c_2^i(p,q,s) - c_i^2(s))'A_s^i c_2^i(s))].\end{gathered}$$

For each i we can find an open neighborhood N^i of $0 \in \mathcal{R}^{(G+1)(S^i+1)} \times \mathcal{L}^{S^i}$ such that $(\ldots, \varsigma^i(\epsilon^i, A^i), \ldots) \in \mathcal{U}^*$ for all $(\ldots, (\epsilon^i, A^i), \ldots) \in N = \times_{i\in I}N^i$. $\varsigma = (\ldots, \varsigma^i, \ldots)$ is a smooth parameterization of a $M(G+1) + S(G+1)(G+2)$ dimensional submanifold of $\mathcal{U}$ about $(\ldots, c^i(p,q,u), \ldots)$, and one can regard $\psi(p,q,\cdot)$ as a function from $\Delta \times N$, writing $\psi(p,q,\epsilon,A)$. If $\partial_{(p,q,\epsilon,A)}\psi$ has full row rank, then so does $\partial_{(p,q,u)}\psi$.

$\partial_{c^i}\varsigma^i(\epsilon^i, 0) = \partial_{c_1^i}u_1^i(c_1^i - \epsilon_1^i) + \sum_{s\in S^i}\pi^i(s)\partial_{c_2^i(s)}u^i(c_2^i(s) - \epsilon_{2s}^i)$ for c^i sufficiently close to $c^i(p,q)$. Let $(\epsilon^i, 0) \in N^i$ satisfy $p_1^i \cdot \epsilon_1^i + \sum_{s\in S^i} q^i(s)p_2^i(s) \cdot \epsilon_{2s}^i = 0$, then $(\hat{c}^i, \hat{y}^i)$ defined by $\hat{c}_1^i = c_1^i(p,q,u) + \epsilon_1^i$, $\hat{c}_2^i(s) = c_2^i(p,q,u,s) + \epsilon_{2s}^i$ and $\hat{y}^i(s) = y^i(p,q,u,s) + p_2^i(s) \cdot \epsilon_{2s}^i$, $s \in S^i$, satisfies i's maximization problem when i's utility function is $\varsigma^i(\epsilon^i, 0)$. This implies that by perturbing ϵ^i for $i \in \{a_1, a_2, \ldots, a_{S_1}, a_1', a_2', \ldots, a_{S_2}'\}$, one can arbitrarily perturb $\tilde{\varphi}(p,q,\epsilon,A)$.

Recall that

$$\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s) = \sum_{i\in I(l,2,s)} K^i(s) + \tilde{m}^i(s)\tilde{\varphi}^i(s)',$$

where $K^i(s) = \lambda^i(s)\left[\partial^2_{c_2^i(s)}\varsigma^i\right]^{-1} - \frac{\lambda^i(s)}{\partial_{y^i(s)}\lambda^i(s)}\tilde{m}^i(s)'\tilde{m}^i(s)$ is a symmetric, negative definite Slutsky matrix, $m^i(s) = \partial_{y^i(s)}\lambda^i(s)\left[\partial^2_{c_2^i(s)}\varsigma^i\right]^{-1} p_2^i(s)$ is the vector of income effects for i. By perturbing $\partial^2_{c_2^i(s)}\varsigma^i$ one can choose $K^i(s)$ and $m^i(s)$.

$$\partial_{c^i}\varsigma^i(0, A^i) = \partial u_1^i(c_1^i) + \sum_{s \in S^i} \pi^i(s)\partial u^i(c_2^i(s))$$

and

$$\partial^2_{c^i}\varsigma^i(0, A^i) = \partial^2 u_1^i(c_1^i) + \sum_{s \in S^i} \pi^i(s)(\partial^2 u^i(c_2^i(s) - A_s^i)$$

when evaluated at $c^i = c^i(p,q)$. By perturbing A_s^i (in a symmetric manner) one can perturb $\partial^2_{c_2^i(s)}\varsigma^i(0, A^i)$ (and therefore choose $K^i(s)$) without affecting either $\tilde{\varphi}$ or $\partial_{\tilde{p}(l,2,s')}\tilde{\varphi}(l,2,s')$ for any $s' \neq s$. By perturbing A_s^i in this manner one can perturb $\det(\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))$ (see Geanakopolos and Polemarchakis [8]), and in particular one can always choose a perturbation such that $\det(\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))$ is perturbed by $\delta \neq 0$. This implies that by perturbing A^i for $i \in \{a_1, a_2, \ldots, a_{S_1}, a_1', a_2', \ldots, a_{S_2}'\}$, one can perturb each $\det(\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))$ by $\delta \neq 0$ without perturbing $\tilde{\varphi}$, and therefore one can arbitrarily perturb ρ without perturbing $\tilde{\varphi}$. After a suitable change in coordinates $\partial_{(\epsilon,A)}\psi$ has the form

$$\partial_{(\epsilon,A)}\psi = \begin{bmatrix} \partial_\epsilon\tilde{\varphi} & 0 \\ * & 1 \end{bmatrix}.$$

By our argument $\partial_\epsilon\tilde{\varphi}$ has full row rank, therefore 0 is a regular value of ψ. By the transversality theorem (Abraham and Robbin [1]), there exists a dense subset $\mathcal{U}^{**}$ of $\mathcal{U}^*$ such that for all $(p,q,u) \in \Delta \times \mathcal{U}^{**}$, 0 is a regular value of $\psi_u(p,q) = \psi(p,q,u)$. Since the range of ψ_u is one dimension higher than its domain, this implies that for any $(p,q,u) \in \Delta \times \mathcal{U}^{**}$, if $\tilde{\varphi}(p,q,u)$= 0 then $\rho \neq 0$. An argument similar to the one made in the proof of Lemma 1 shows that $\mathcal{U}^{**}$ is open in $\mathcal{U}^*$. Since $\mathcal{U}^{**}$ is open and dense in $\mathcal{U}^*$, which is open and dense in $\mathcal{U}$, $\mathcal{U}^{**}$ is open and dense in $\mathcal{U}$□

In order to prove Proposition 4 we employ the following lemma:

Lemma 3: If $G > 0$, then there is an open and dense subset $\mathcal{U}_1^{***}$ of $\mathcal{U}^{**}$ such that for all $(p,q,u) \in \Delta \times \mathcal{U}_1^{***}$, $\varphi(p,q,u) = 0$ implies

$$[(c_2^j(s) - w_2^j - y^j(s)d)'\partial_{y^j(s)}p(1,2,s) - (c_2^k(s) - w_2^k - y^k(s)d)'\partial_{y^k(s)}p(2,2,s)] \neq 0$$

for all $s = (s_1, s_2) \in S$ and $j, k \in I(1,1,s_1)$, $j \in I(1,2,s)$, $k \in I(2,2,s)$.

Proof of Lemma 3:

Let $\varphi^i_{2g}(s) = (c^i_{2g}(s) - w^i_{2g} - y^i(s))$ represent i's excess demand for good g at time 2 in state s. Since $p(l,2,s)$ is normalized to satisfy $\sum_{g=0}^{G} p_g(l,2,s) = 1$, we will write $\partial p_0(l,2,s) = -\sum_{g=1}^{G} \partial p_g(l,2,s)$. Under this convention

$$(c^i_2(s) - w^i_2 - y^i(s)d)'\partial p^i_2(s) = (\tilde{\varphi}^i_2(s) - \varphi^i_{20}(s)\tilde{d})'\partial \tilde{p}^i_2(s).$$

Now for $i, h \in I(l,2,s)$,

$$\partial_{y^i(s)}\tilde{p}(l,2,s) = (\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))^{-1}(\tilde{m}^i(s) - \tilde{d})$$

where

$$\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s) = (K^i(s) + K^h(s)) + (\tilde{m}^i(s) - \tilde{m}^h(s))\tilde{\varphi}^i_2(s)'.$$

The proof will proceed in two steps.

Step 1:

In this step we will prove that there is a open and dense set $\mathcal{U}^{***}_0 \subset \mathcal{U}^{**}$ such that at any equilibrium of this set there is no i and $s \in S^i$ for which $(\tilde{\varphi}^i_2(s) - \varphi^i_{20}(s)\tilde{d}) = 0$. Consider the smooth parameterization $\varsigma = (\ldots, \varsigma^i, \ldots)$ employed in the proof of Proposition 3. Under that parameterization for each i we can find an open neighborhood N^i of $0 \in \mathcal{R}^{(G+1)(S^i+1)}$ such that $(\ldots, \varsigma^i(\epsilon^i), \ldots) \in \mathcal{U}^{**}$ for all $(\ldots, \epsilon^i, \ldots) \in N = \times_{i\in I} N^i$, where

$$\varsigma^i(\epsilon^i)(c^i) = [\rho^i_1(c^i_1)u^i_1(c^i_1) + (1 - \rho^i_1(c^i_1))u^i_1(c^i_1 - \epsilon^i_1)] +$$
$$\sum_{s\in S^i} \pi^i(s)[(\rho^i_{21}(c^i_2(s)) \cdots \rho^i_{2S^i}(c^i_2(s)))u^i(c^i_2(s)) + \sum_{s\in S^i}(1 - \rho^i_{2s}(c^i_2(s)))u^i(c^i_2(s) - \epsilon^i_{2s})].$$

Let $I_l = \{i \in I \mid i \in I(l,1,h_{l1}) \text{ for some } h_{l1}\}$ represent the set of agents who trade at location l at time 1. Define the function $\rho = \left(\prod_{i\in I_1}\prod_{s\in S^i}(\varphi^i_{2G}(s) - \varphi^i_{20}(s))\right)^{\frac{1}{S}}$ and let $\psi : \Delta \times N \to \mathcal{R}^{(S_1+S_1+2S)G+2S+1}$ be given by $\psi = (\tilde{\varphi}, \rho)$. If $\epsilon^i \in N^i$ satisfies $p^i_1 \cdot \epsilon^i_1 + \sum_{s\in S^i} q^i(s)p^i_2(s) \cdot \epsilon^i_{2s} = 0$, then $(\hat{c}^i, \hat{y}^i)$ defined by $\hat{c}^i_1 = c^i_1(p,q,u) + \epsilon^i_1$, $\hat{c}^i_2(s) = c^i_2(p,q,u,s) + \epsilon^i_{2s}$ and $\hat{y}^i(s) = y^i(p,q,u,s) + p^i_2(s) \cdot \epsilon^i_{2s}$, $s \in S^i$, satisfies i's maximization problem when i's utility function is $\varsigma^i(\epsilon^i)$. By choosing $\epsilon^i_{2sG} = \frac{p^i_{20}(s)\delta}{p^i_{2G}(s)+p^i_{20}(s)}$ and $\epsilon^i_{2s0} = -\frac{p^i_{2G}(s)\delta}{p^i_{2G}(s)+p^i_{20}(s)}$ for each $i \in I_1$ and $s \in S^i$, one can perturb $(\varphi^i_{2G}(s) - \varphi^i_{20}(s))$ by δ. If in the same way and at the same time for each $h \in I_2$ and $s \in S^h$, one perturbs $(\varphi^h_{2G}(s) - \varphi^h_{20}(s))$ by $-\delta$, then $\tilde{\varphi}$ will remain undisturbed but ρ has been perturbed by δ. Therefore after a suitable change in coordinates $\partial_\epsilon\psi$ has the form

$$\partial_\epsilon\psi = \begin{bmatrix} \partial_\epsilon\tilde{\varphi} & * \\ 0 & 1 \end{bmatrix}.$$

By our argument in the proof to proposition 3, $\partial_\epsilon \tilde{\varphi}$ has full row rank, therefore 0 is a regular value of ψ. By the transversality theorem (Abraham and Robbin [1]), there exists a dense subset $\mathcal{U}_0^{***}$ of $\mathcal{U}^{**}$ such that for all $(p,q,u) \in \Delta \times \mathcal{U}_0^{***}$, 0 is a regular value of $\psi_u(p,q) = \psi(p,q,u)$. Since the range of ψ_u is one dimension higher than its domain, this implies that for any $(p,q,u) \in \Delta \times \mathcal{U}_0^{***}$, if $\tilde{\varphi}(p,q,u)=0$ then there is no $\rho \neq 0$. This in turn implies that there is no i and $s \in S^i$ in which $(\tilde{\varphi}_2^i(s) - \varphi_{20}^i(s)\tilde{d}) = 0$. An argument similar to the one made in the proof of Lemma 1 shows that $\mathcal{U}_0^{***}$ is open in $\mathcal{U}^{**}$.

Step 2:

In this step we will prove that there is a generic set $\mathcal{U}_1^{***} \subset \mathcal{U}_0^{***}$ such that at any equilibrium of this set there is no $l, h_{l1}, s \in S^{h_{l1}}$ and $j,k \in I(l,1,h_{l1})$ $(j \in I(1,2,s), k \in I(2,2,s))$ for which

$$\eta(s) = (c_2^j(s) - w_2^j - y^j(s)d)'\partial_{y^j(s)}p(1,2,s) - (c_2^k(s) - w_2^k - y^k(s)d)'\partial_{y^k(s)}p(2,2,s) = 0.$$

Consider the smooth parameterization $\varsigma = (\ldots, \varsigma^i, \ldots)$ employed in the proof of Proposition 3. Under that parameterization for each i we can find an open neighborhood N^i of $0 \in \mathcal{R}^{(G+1)(S^i+1)} \times \mathcal{L}^{S^i}$ such that $(\ldots, \varsigma^i(\epsilon^i, A^i), \ldots) \in \mathcal{U}_0^{***}$ for all $(\ldots, (\epsilon^i, A^i), \ldots) \in N = \times_{i \in I} N^i$, where

$$\begin{gathered}\varsigma^i(\epsilon^i, A^i)(c^i) = [\rho_1^i(c_1^i)u_1^i(c_1^i) + (1-\rho_1^i(c_1^i))u_1^i(c_1^i - \epsilon_1^i)] + \\ \sum_{s \in S^i} \pi^i(s)[(\rho_{21}^i(c_2^i(s)) \cdots \rho_{2S^i}^i(c_2^i(s)))u^i(c_2^i(s)) + \sum_{s \in S^i}(1-\rho_{2s}^i(c_2^i(s)))u^i(c_2^i(s) - \epsilon_{2s}^i) + \\ \sum_{s \in S^i}(1-\rho_{2s}^i(c_2^i(s)))((c_2^i(p,q,s) - c_i^2(s))'A_s^i c_2^i(s))].\end{gathered}$$

Define the function $\rho = \left(\prod_{s \in S} \eta(s)\right)^{\frac{1}{S}}$ and let $\psi : \Delta \times N \to \mathcal{R}^{(S_1+S_1+2S)G+2S+1}$ be given by $\psi = (\tilde{\varphi}, \rho)$. Since $(\ldots, \varsigma^i(\epsilon^i, A^i), \ldots) \in \mathcal{U}_0^{***} \subset \mathcal{U}^{**}$, we know that $(\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))^{-1}$ exists and $(\tilde{\varphi}_2^i(s) - \varphi_{20}^i(s)\tilde{d}) \neq 0$, therefore $(\tilde{\varphi}_2^i(s) - \varphi_{20}^i(s)\tilde{d})'(\partial_{\tilde{p}(l,2,s)}\tilde{\varphi}(l,2,s))^{-1} \neq 0$. By perturbing A_s^i and A_s^h one can perturb $m^i(s)$ and $m^h(s)$ by the same vector, and therefore one can arbitrarily perturb $(\tilde{m}^i(s) - \tilde{d})$ without disturbing either $\tilde{\varphi}$, $(\tilde{\varphi}_2^i - \varphi_{20}^i\tilde{d})$, or $\partial_{p(l,2,s)}\varphi(l,2,s)$. In this manner one can perturb each $\eta(s)$ by δ, so that $\tilde{\varphi}$ is left undisturbed but ρ is perturbed by δ. Therefore after a suitable change in coordinates $\partial_{(\epsilon,A)}\psi$ has the form

$$\partial_{(\epsilon,A)}\psi = \begin{bmatrix} \partial_\epsilon \tilde{\varphi} & * \\ 0 & 1 \end{bmatrix}.$$

By our argument in the proof to proposition 3 $\partial_\epsilon \tilde{\varphi}$ has full row rank, therefore 0 is a regular value of ψ. By the transversality theorem (Abraham and Robbin [1]), there exists a dense subset $\mathcal{U}_1^{***}$ of $\mathcal{U}_0^{***}$ such that for all $(p,q,u) \in \Delta \times \mathcal{U}_1^{***}$, 0 is a regular value of $\psi_u(p,q) =$

$\psi(p,q,u)$. Since the range of ψ_u is one dimension higher than its domain, this implies that for any $(p,q,u) \in \Delta \times \mathcal{U}_1^{***}$, if $\tilde{\varphi}(p,q,u)=0$ then there is no $\rho \neq 0$. This in turn implies that there is no $s \in S$ in which $\eta(s)=0$. An argument similar to the one made in Lemma 1 shows that $\mathcal{U}_1^{***}$ is open in $\mathcal{U}_0^{***}$. Since $\mathcal{U}_1^{***}$ is open and dense in $\mathcal{U}_0^{***}$ which is open and dense in $\mathcal{U}$, $\mathcal{U}_1^{***}$ is open and dense in $\mathcal{U}\square$

Proof of Proposition 4:

The necessity of the condition $G > 0$ should be obvious, since if $G = 0$ then we must have

$$(c_2^i(s) - w_2^i - y^i(s)d)' = \partial_{y^i(s)}p_2^i(s) = 0$$

for all i and $s \in S^i$. This immediately proves that if $G = 0$ then any equilibrium must be constrained Pareto optimal.

Let $s_1, s_2, \ldots, s_S$ be some ordering of the set S, and let $i_1, i_2, \ldots, i_{S_1+S_2}$ be some ordering of the set $\{i \in I \mid i \in I(1,2,s) \text{ for some } s \in S\}$. Define the $(S_1+S_2)\times S$ matrix $Q = [Q_{jk}]$ by

$$Q_{jk} = \begin{cases} \kappa(i_j)q^{i_j}(s_k) & \text{if } s_k \in S^{i_j} \\ 0 & \text{if } s_k \notin S^{i_j}, \end{cases}$$

where $\kappa(i) = 1$ if i trades at location 1 at time 1 and $\kappa(i) = -1$ if i trades at location 2. If we let $\hat{\gamma}$ represent the vector $(\alpha^{i_1}\gamma^{i_1}, \alpha^{i_2}\gamma^{i_2}, \ldots, \alpha^{i_{S_1+S_2}}\gamma^{i_{S_1+S_2}})$, then the condition $r(s) = 0$ $\forall s \in S$ can be rewritten as $\hat{\gamma}'Q = 0$. Given its structure, if Q does not have full row rank then there must be a $\lambda \in \mathcal{R}_+^{S_1+S_2} \setminus \{0\}$ such that $\lambda' Q = 0$, which implies the existence of a $(\ldots, \gamma^i, \ldots) \in \mathcal{R}_+^I \setminus \{0\}$ such that $\hat{\gamma}'Q = 0$. If $S_1 + S_2 < S$ then Q obviously cannot have full row rank. Since $S = S_1S_2$, $S_1 + S_2 < S$ only if $S_1 = 1$ and/or $S_2 = 1$, which establishes that if $S_1 = 1$ or $S_2 = 1$ then all equilibria are constrained Pareto optimal.

If Q does have full row rank, then obviously $\hat{\gamma}'Q \neq 0$. Q will have full row rank only if there is no sequence of states s_1, s_2, s_n and ordering of I such that in state s_1 (i_1, i_2) are matched at time 2, in state s_2 (i_2, i_3) are matched at time 2, $\ldots$, in state s_n (i_n, i_{n+1}) are matched at time 2, $i_{n+1} = i_1$, and

$$\frac{q^{i_1}(s_1)}{q^{i_1}(s_n)} = \frac{q^{i_2}(s_1)}{q^{i_2}(s_2)}\frac{q^{i_3}(s_2)}{q^{i_3}(s_3)}\cdots\frac{q^{i_n}(s_{n-1})}{q^{i_n}(s_n)}.$$

In this step we prove that if $S_1 \geq 2$ and $S_2 \geq 2$ then there is an open and dense subset $\mathcal{U}^{***}$ of U_1^{***} such that any $(p,q,u) \in \Delta \times \mathcal{U}^{***}$ satisfying $\varphi(p,q,u) = 0$ must have $r(s) \neq 0$ for some $s \in S$ (i.e, Q has full row rank).

Recall that

$$r(s) = (\gamma^j\alpha^j q^j(s) - \gamma^h\alpha^h q^h(s)); \quad j,h \in I(1,2,s), j \neq h.$$

Note that the condition $r(s) = 0 \ \forall s \in S$ implies that if $\gamma^i = 0$ for some $i \in I$, then $\gamma^i = 0$ for all $i \in I$ (since $\alpha^i q^i(s) > 0 \ \forall i$ — this is essentially a statement that an equilibrium allocation satisfies the "no isolated communities condition" of Smale [25]). Therefore either $r(s) \neq 0$ for some s, or we can restrict ourselves to considering $(\ldots, \gamma^i, \ldots) \in \mathcal{R}^I_{++}$ in the social planner's problem (or both). This implies that we may assume that $\hat{\gamma} \in \mathcal{R}^{S_1+S_2}_{++}$, and we can normalize γ so that $\hat{\gamma} \in \Lambda^{S_1+S_2-1}_{++} = \{\lambda \in \mathcal{R}^{S_1+S_2}_{++} \mid \sum \lambda_i = 1\}$.

Consider a submanifold of the smooth parameterization $\varsigma = (\ldots, \varsigma^i, \ldots)$ employed in the proof of Proposition 3. Under that parameterization for each i we can find an open neighborhood N^i of $0 \in \mathcal{R}^{(G+1)(S^i+1)}$ such that $(\ldots, \varsigma^i(\epsilon^i), \ldots) \in \mathcal{U}^{***}_1$ for all $(\ldots, \epsilon^i, \ldots) \in N = \times_{i \in I} N^i$, where

$$\varsigma^i(\epsilon^i)(c^i) = [\rho^i_1(c^i_1)u^i_1(c^i_1) + (1 - \rho^i_1(c^i_1))u^i_1(c^i_1 - \epsilon^i_1)] +$$
$$\sum_{s \in S^i} \pi^i(s)[(\rho^i_{21}(c^i_2(s)) \cdots \rho^i_{2S^i}(c^i_2(s)))u^i(c^i_2(s)) + \sum_{s \in S^i}(1 - \rho^i_{2s}(c^i_2(s)))u^i(c^i_2(s) - \epsilon^i_{2s})].$$

Assume that $S_1 \geq 2$ and $S_2 \geq 2$ and define ψ: $\Delta \times N \times \Lambda^{S_1+S_2-1}_{++} \to \mathcal{R}^{MG+2S} \times \mathcal{R}^{S_1+S_2}$ by $\psi = (\tilde{\varphi}(p, q, \epsilon, A), \lambda' Q)$. Since $\lambda \gg 0$, by perturbing $q^{i_j}(s)$ for each i_j who trades at location 1 at time 1 and each $s \in S^{i_j}$, one can arbitrarily perturb $\lambda' Q$. $\partial_{(\epsilon,q)}\psi$ has the form

$$\partial_{(\epsilon,q)}\psi = \begin{bmatrix} \partial_\epsilon \tilde{\varphi} & 0 \\ * & \partial_q \lambda' Q \end{bmatrix}.$$

By our argument in the proof to proposition 3, $\partial_\epsilon \tilde{\varphi}$ has full row rank and we have just shown that $\partial_q \lambda' Q$ has full row rank, therefore 0 is a regular value of ψ. By the transversality theorem (Abraham and Robbin [1]), there exists a dense subset $\mathcal{U}^{***}$ of $\mathcal{U}^{***}_1$ such that for all $(p, q, u, \lambda) \in \Delta \times \mathcal{U}^{***} \times \Lambda^{S_1+S_2-1}_{++}$, 0 is a regular value of $\psi_u(p, q, \lambda) = \psi(p, q, u, \lambda)$. Since the range of ψ_u is one dimension higher than its domain, this implies that for any $(p, q, u) \in \Delta \times \mathcal{U}^{***}$, if $\tilde{\varphi}(p, q, u) = 0$ then there is no $\lambda \in \Lambda^{S_1+S_2-1}_{++}$ such that $\lambda' Q = 0$. This in turn implies that there can be no vector $(\alpha^{i_1}\gamma^{i_1}, \alpha^{i_2}\gamma^{i_2}, \ldots, \alpha^{i_{S_1+S_2}}\gamma^{i_{S_1+S_2}}) \in \mathcal{R}^{S_1+S_2}_{++}$ satisfying $r(s) = 0 \ \forall s$. An argument similar to the one made in the proof of Lemma 1 shows that $\mathcal{U}^{***}$ is open in $\mathcal{U}^{***}_1$. Since $\mathcal{U}^{***}_1$ is open and dense in $\mathcal{U}$, $\mathcal{U}^{***}$ is open and dense in $\mathcal{U}$.

We know that at any equilibrium of this set there is some state $s = (s_1, s_2)$ such that $r(s) \neq 0$. From Proposition 2 we also know that at any equilibrium of this set $p(1, 2, s)$ and $p(2, 2, s)$ are (locally) differentiable functions of asset holdings. From equation (5) one can see that if $r(s) \neq 0$ then an equilibrium can only be constrained Pareto optimal if

$$[(c^j_2(s) - w^j_2 - y^j(s)d)'\partial_{y^j(s)}p(1, 2, s) - (c^k_2(s) - w^k_2 - y^k(s)d)'\partial_{y^k(s)}p(2, 2, s)] = 0$$

for all l, and $j, k \in I(l, 1, s_l)$, ($j \in I(1, 2, s)$ and $k \in I(2, 2, s)$). Since Lemma 3 implies that at any equilibrium of $(p, q, u) \in \Delta \times \mathcal{U}^{***}_1$ this condition does not hold for $l = 1$, no equilibrium can be constrained Pareto optimal□

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